

Geotechnical Earthquake Engineering
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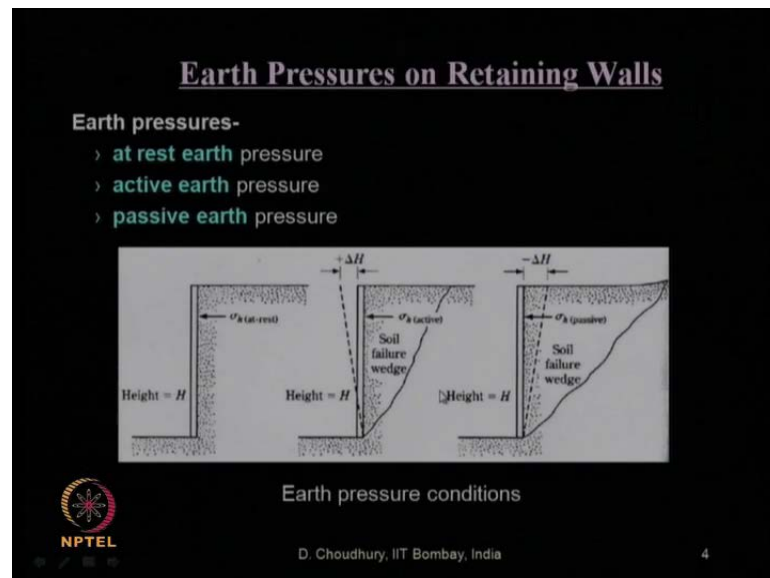
Module - 9

Lecture - 35

**Seismic Analysis and Design of Various
Geotechnical Structures (Contd...)**

Welcome to the NPTEL lecture on video course, geotechnical earthquake engineering. We were going through module number nine of this course, that is, seismic analysis and design of various geotechnical structures. So, let us have a quick recap, what we have learnt in our previous lecture. We started with this subtopic within this module, that is, seismic design of retaining wall.

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We have mentioned that, what are the various states of earth pressures acting on the retaining wall based on the movement of the wall.

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Failure of Retaining Walls

- ⦿ Retaining walls may **fail** during an **earthquake**, if they are **not** designed to resist **additional** destabilizing earthquake forces.



Failure of gravity retaining wall

 Source: <http://www.geofffox.com>(Source: <http://www.parmeleegeology.com/>)


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Same remains in the case of earthquake condition also. There are various examples worldwide – the failures of retaining wall due to the additional destabilizing earthquake forces, which comes on the retaining wall during the earthquake process. So, unless we estimate this extra or additional destabilizing earthquake forces on the retaining wall and consider those for the design of retaining wall, then failure cannot be avoided.

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Seismic Analysis and Design of Retaining Walls

- ⦿ **Seismic** analysis/design of retaining walls mainly consists of
 - > Determining magnitude of **additional** destabilizing forces that act during an earthquake
 - > Determining **seismic active and passive** earth pressures due to all destabilizing forces (static + seismic)
 - > Design section based on above parameters using
 1. **Force based approach**
 2. **Displacement based approach – for performance based design**

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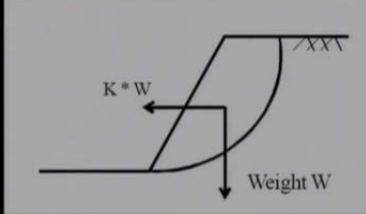
For the seismic analysis and design of retaining wall, we need to compute the combined earth pressure, that is, static plus seismic whether it is an active state or passive state of

earth pressure. And, to do that, we basically follow two different types of approaches: one approach is called force-based approach, where we compute the earth pressure using the force equilibrium, etcetera; another is displacement-based approach, where in addition to the computation of the seismic earth pressure, we also compute how much amount of displacement of the wall is going to take place; which is necessary for performance-based design.

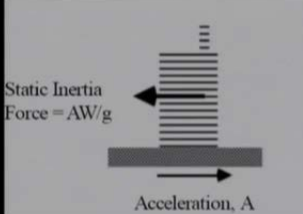
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Pseudo-static Method

- Theoretical background of **seismic coefficient** lies in the application of **D'Alembert's** principle of mechanics.



Example of elementary seismic slope stability analysis (Towhata, 2008)



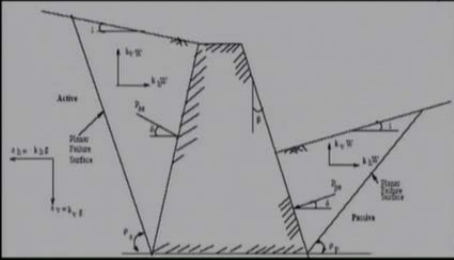
D'Alembert's principle of mechanics

NPTEL Towhata, I., (2008), Geotechnical Earthquake Engineering, Springer, Tokyo, Japan. 7

Then, we had introduced what is the pseudo-static method, which was originally proposed by Terzaghi in the year 1950. And, this is nothing but a coefficient, which is multiplied with respect to the failure soil mass, which will give the seismic inertia force. So, this is the additional destabilizing force. As we can see from the D'Alembert's principle of mechanics, if there is acceleration, A , the corresponding static inertia force or equivalent static inertia force will be $A W$ by g , where this small g is acceleration due to gravity. So, for this, this book Towhata can be referred to for the details.


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Conventional Seismic Design of Retaining Walls



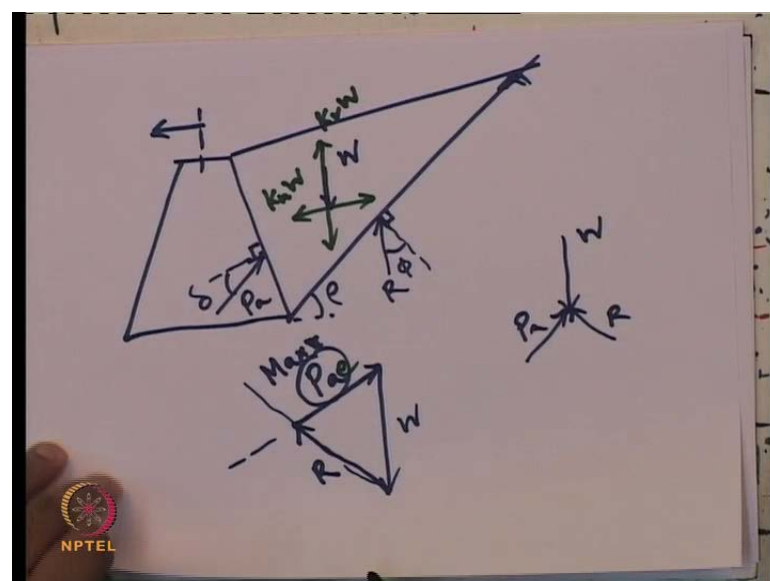
Pseudo-Static Approach Proposed by Mononobe-Okabe (1929)

Questions: Value of k_h and k_v to be used for design?
Soil amplification?
Variation of seismic acceleration with depth and time?
Effect of dynamic soil properties? etc.

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Then, we have seen what is the conventional seismic design of retaining wall; which is nothing the pseudo-static approach as proposed by Mononobe and Matsu in 1929, and by Okabe in 1926, which is combinedly and commonly known as Mononobe-Okabe method of 1929, which is the pioneering work for computation of seismic earth pressure on retaining wall. So, this is how the Mononobe-Okabe method was proposed; which is nothing but an extension of Coulomb's static earth pressure theory. So, in Coulomb's static earth pressure theory, as we all know, we have already seen this one.

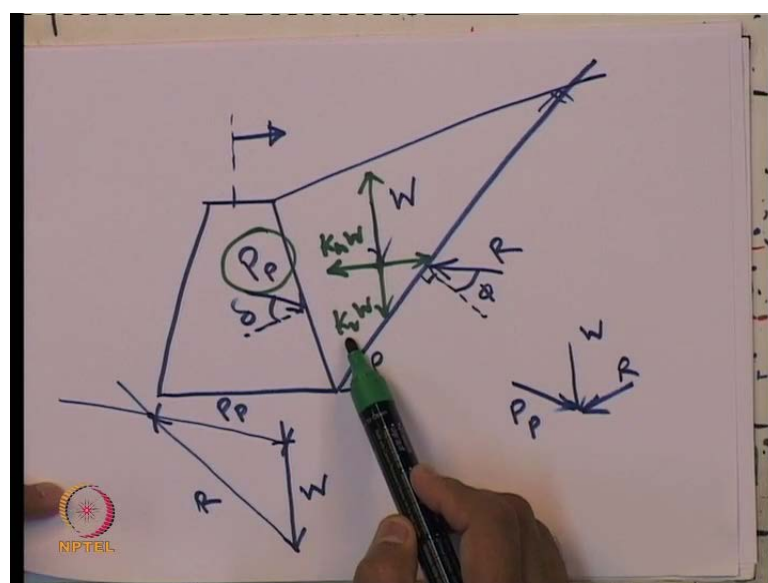
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Suppose for the active state of earth pressure, the wall tends to move in this direction for an assumed failure plane with respect to a failure angle like ρ over here, we know what is the value of this failure mass or weight w . This vector totally is known, because its magnitude and direction is known. Then, we have the active earth pressure P_a acting at an angle δ ; where, δ is the wall friction angle; and, this soil reaction R , which acts at an angle ϕ , which is the soil friction angle. Now, these three forces should be concurrent forces to maintain equilibrium. And, if we draw the force polygon by knowing the line of action of this P_a and R , we will get the force polygon or closed triangle like this. So, this is w , which is completely known.

Now, by drawing this line and line here, wherever they intersect, we will get the value of P_a over here, value of soil reaction R over here. So, for design, what we do? For conventional static analysis of Coulomb earth pressure theory, we maximize this value of P_a to get the design value of active earth pressure by changing this angle – failure angle, ρ , so that this maximum value of P_a is obtained. Now, in the Mononobe-Okabe method, they added these values of $K_h w$ in both the directions for analysis and this $K_v w$ in both the directions and the critical direction; whichever gives the maximum value of P_a , that is needs to be estimated under the earthquake condition using this pseudo static approach. Also, for the passive state... So, this is for the active state as you can see over here.

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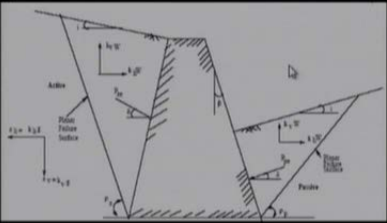
Similarly, for the passive state also, we have seen that, this is the Coulomb's passive earth pressure closed triangle or force polygon, where w is again known; R and P_p – their directions are known, but magnitude was not known. But, using this force polygon, that is, all the forces are concurrent; we can get their magnitude also. This is P_p – passive earth pressures; this is the soil resistance. And, in addition to that, this $K_h w$ and $K_v w$ has been introduced by Mononobe-Okabe for pseudo-static analysis.


But, limitations of this pseudo-static approach – we had already learnt in our previous lecture, that is, first of all, this value of K_h and K_v – what should be used for design is not clear. Also, soil amplification we cannot consider in this pseudo-static approach; variation of the seismic acceleration with respect to depth cannot be considered in this pseudo-static approach; and, effect of the dynamic soil properties are not possible to consider; that means, suppose if you are designing a retaining wall in a soft soil, maybe the loose sand and another retaining wall in the dense sand; for both of them, as far as the chosen soil properties are considered, you can only take care of the static soil property, that is, ϕ value and wall friction value etcetera. But, you cannot take any dynamic soil property, which is also an important factor as far as earthquake engineering is concerned, as we have already learnt.

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Available Literature

- Mononobe-Okabe (1926, 1929)
- Madhav and Kameswara Rao (1969)
- Richards and Elms (1979)
- Saran and Prakash (1979)
- Prakash (1981)
- Nadim and Whitman (1983)
- Steedman and Zeng (1990)
- Ebeling and Morrison (1992)
- Das (1993)
- Kramer (1996)
- Kumar (2002)
- Choudhury and Subba Rao (2005)
- Choudhury and Nimbalkar (2006)
- and many others.....




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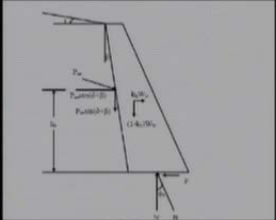
What are the various available literatures in the literature for the seismic earth pressure estimation using pseudo-static approach and the recent pseudo-dynamic approach? There

are many researchers, who have proposed this work starting from Mononobe and Okabe as I said, 1926 and 29; then Madhav and Kameswara Rao, 1969; Richard and Elms, 1979; Saran and Prakash, 1979; Prakash, 1981; Nadim and Whitman in 1983; Steedman and Zeng in 1990; Ebeling and Morrison, 1992; Das, 1993; Kramer, 1996; Kumar, 2002; Choudhury and Subba Rao, 2005; Choudhury and Nimbalkar, 2006; and many many other researchers. So, we will talk about some of the recent research work on this seismic earth pressure, not this pioneering or the older or former research work, because that mostly closely relates to the Mononobe-Okabe method, the alteration of that method. So, we will more focus on the recent development of these seismic earth pressure theories for both active as well as passive condition.

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Pseudo-static Method

- **Richards and Elms (1979)** proposed a method for seismic design of **gravity retaining walls** which is based on **permanent** wall displacements. (**Displacement based approach**)




Seismic forces on gravity retaining wall
Richards and Elms (1979)

Displacement should be calculated by following formula, and should be checked against allowable displacement.

$$d_{perm} = 0.087 \frac{v_{max}^2 a_{max}^3}{a_y^4}$$

where, v_{max} is the peak ground velocity, a_{max} is the peak ground acceleration, a_y is the yield acceleration for the wall-backfill system.



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Richards, R. Jr., and Elms, D. G. (1979). Seismic behavior of gravity retaining walls. *Journal of Geotechnical Engineering, ASCE*, 105: 4, 449-469.

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Now, when we talk about the pseudo-static method of approach, the Mononobe-Okabe method was force-based method, because in that case, as we have already seen, we consider only the force equilibrium and we estimate the value of P_a or P_p . But, as I have mentioned, for performance-based design, we need to look into the displacement-based approach, where we talk about the displacement of the wall also. So, the pioneering work in the displacement-based approach was proposed by Richard and Elms in 1979. This is the detail of the paper one can easily get. You can see the details of the paper, Richard and Elms, 1979 – seismic behavior of gravity retaining walls. It was published in journal of Geotechnical Engineering ASCE, volume 105, issue 4; page number is given over here.

What it proposed, in addition to the computation of this active earth pressure, P_a , under earthquake condition using the same pseudo-static approach; but, additionally, it mentioned that, it can be checked with respect to the permissible displacement of the wall. And, how to estimate the permissible displacement? You can see, this is the seismic active earth pressure. Can you see in the picture over here? Its point of action also is considered like this. Now, within the wall, wall is also subjected to... If wall is having weight of w , then wall is subjected to seismic inertia force of $K_h w$ and $K_v w$. So, these are the wall inertia forces including the wall-self weight. And, for the stability, this wall can either move in the sliding pattern; that is, it can translate or it can rotate or it can have a combination of these two. So, these are nothing but the displacements or movement – sliding or rotational.

Richard and elms proposed the sliding mode of movement; that is, for the amount of movement, how to estimate it? This is the force of resistance; this is the normal reaction; this is the reaction or resultant of this f and n acting at an angle ϕ_b ; where, ϕ_b is nothing but the angle of friction between this base of the wall and the foundation soil. So, displacement should be calculated using the formula and should be checked against the allowable displacement. So, the permissible displacement can be computed with respect to $0.087 v_{max}^2 a_{max}^3$ by a_y to the power 4; where v_{max} is the peak ground velocity; a_{max} is the peak ground acceleration. And, what is a_y ? a_y is known as yield acceleration for the wall-backfill system.

What is yield acceleration? Let me describe it. What is the definition of yield acceleration? Yield acceleration is that acceleration when the factor of safety with respect to sliding of this wall will be equals to 1. So, a_y refers to that seismic acceleration; beyond which, if the acceleration increases, then wall starts moving, because then factor of safety with respect to sliding will be less than 1. So, a_y gives us that limiting value or that threshold value of acceleration at which factor of safety is equals to 1. That is known as yield acceleration. So, how to estimate that? Obviously, as we know, this will be nothing but factor of safety against sliding; will be the stabilizing force with respect to sliding divided by the disturbing force. Disturbing force we know; it is the horizontal component of this seismic active earth pressure and this seismic inertia of the wall, which are causing the disturbance of the wall or tends the wall to move in

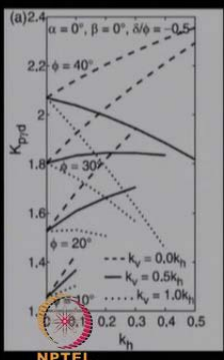
this direction; whereas, what are the stabilizing forces? This frictional force, which can be computed as tan of this angle times the normal reaction.

What will be the normal reaction? That is nothing but if you do the vertical equilibrium, this normal reaction is nothing but equals to weight of the wall and the vertical component of the pressure, which helps to find it out. So, it has to be done algebraically to find out the normal reaction. And, normal reaction times tan of this base friction angle will give you the stabilizing force. So, that stabilizing force divided by the disturbing force will give you factor of safety against sliding. That has to be equated with respect to 1 to get the a y value. Once you get the a y value, you can always get the permissible displacement. Now, this is the permissible displacement under a particular given value of a max and corresponding value of v max. Now, if the allowable displacement for a wall – if it is specified at a project, then automatically you should keep the permissible displacement within the range of the allowable displacement. And accordingly, you can design the wall section, so that the a y value, etcetera comes accordingly, so that this permissible value of displacement is within the allowable range. So, these are the basic steps of the displacement-based approach of the pseudo-static method of design.

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Pseudo-static Method

- **Choudhury and Subba Rao (2002)** gave design charts for the estimation of **seismic passive** earth pressure coefficient for **negative wall friction** case. (*Canadian Geotech. Journal*, 2002)



- The application of this case is for **anchor uplift capacity**.
- **Design charts** are given for calculation of various parameters like k_{psd} , which can be used to compute **seismic passive resistance**, which is given by,

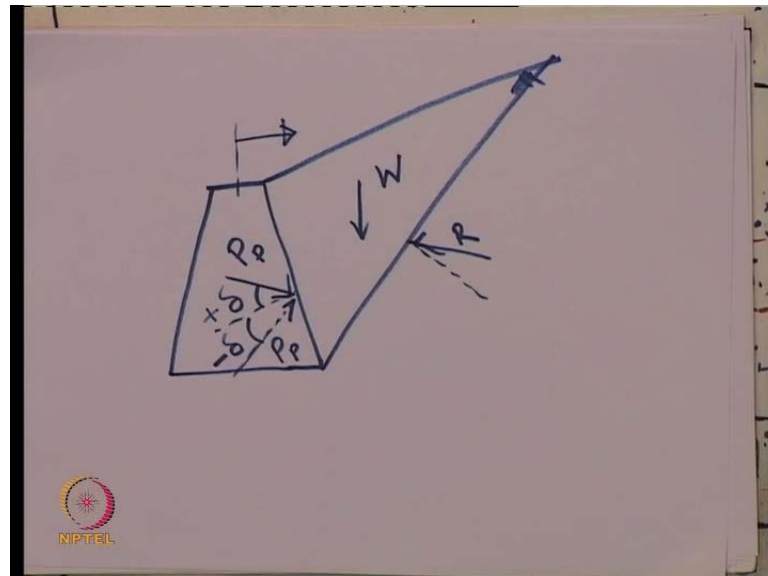
$$P_{pd} = \left(2cHK_{psd} + qHK_{psd} + \frac{1}{2} \gamma H^2 K_{psd} \right) \frac{1}{\cos \delta}$$
 where P_{pd} is seismic passive resistance, K_{psd} , K_{psqd} , and K_{pscd} are the seismic passive earth pressure coefficients.

Choudhury, D. and Subba Rao, K. S. (2002). Seismic passive earth resistance for negative wall friction, *Canadian Geotechnical Journal*, 39:5, 971–981. 11

Now, for pseudo-static method, further, the method was extended by other researchers. As I said, the recent research work I will describe. Choudhury and Subba Rao in 2002 gave the design chart for the estimation of seismic passive earth pressure coefficient for

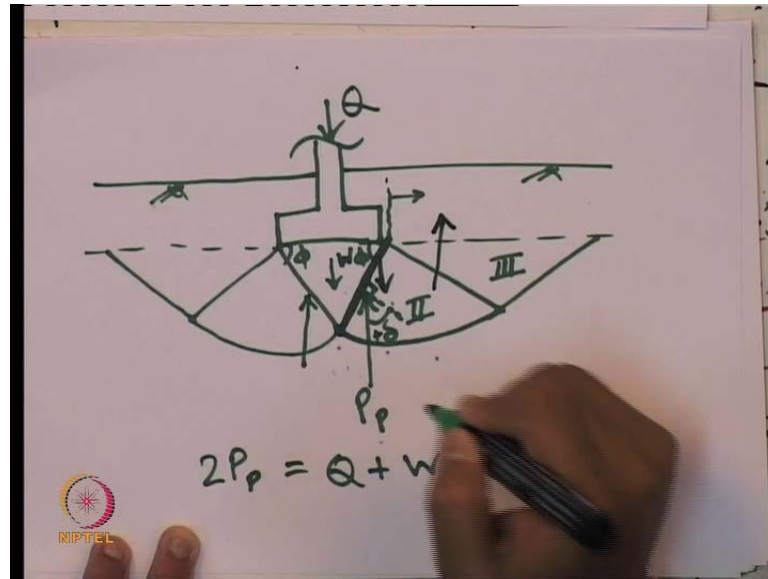
negative wall friction case. What is negative wall friction case? Let me describe it little bit.

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When we are talking about passive condition of earth pressure; as we have already mentioned, for passive state of earth pressure on a rigid retaining wall; if we have ground like this and this is the failure plane, we have weight of this; for passive condition, this is the R. Now, if we have seen already, the P_p value will be acting over here at an angle delta. This passive earth pressure when wall tends to move in this direction is nothing but called positive delta. But, there can be few cases, where this passive force may act in this direction also, which is called as negative delta. This passive earth pressure is nothing, but an application of this. Where we can apply this passive earth pressure? Not only for the design of retaining wall; this passive earth pressure we can apply for bearing capacity of foundation as well as anchor uplift capacity. How? Let me describe you.

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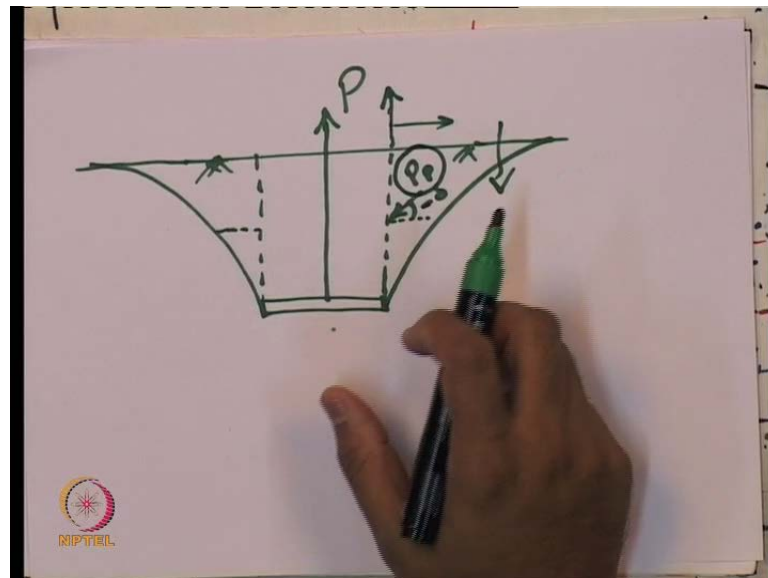
When we are talking about some footing – shallow footing problem like this; which is let us say embedded at a depth – shallow depth like this. As we all know, let us consider the Terzaghi's bearing capacity failure theory, which is all known to all of us from the basic geotechnical engineering course. So, this is the Terzaghi's theory, which says we have one zone over here, which is known as this angle is π for static case; this is a zone 1, which is named as active zone; this is zone 2, which is log-spiral zone; and, this zone 3 is Rankine passive zone. This is log-spiral zone and it is a symmetrical under the static condition.

Now, if we consider this line; let us look at here; this line has an imaginary wall. What we can say? There it will be passive earth pressure acting at an angle δ with respect to normal to this. Why it is a passive earth pressure? First of all, when we are talking about this imaginary wall, let us see how the displacement is occurring. When the load is acting on the footing from the column, etcetera, this soil tries to punch inside and go inside. So, this surrounding soil moves this direction. This wall tends to move in this direction towards the backfill soil. That is why it is a passive state of earth pressure. Why it is a passive state of earth pressure? Because this wall tends to move in this direction.

Now, why it is in this direction of passive earth pressure? Because it is acting at an angle δ ; and, considering this composite failure surface, that is, log-spiral followed by a Rankine passive zone, we get the passive earth pressure acting in this direction, which

will balance this W and this outside load. So, basically, $2 P p$ equals to Q plus W . This is the application of positive wall friction angle of passive earth pressure, where... What is the movement of the wall with respect to surrounding soil? This wall moves down compared to surrounding soil; surrounding soil moves up. That is why it is named as positive wall friction angle. What is negative wall friction angle? So, for positive wall friction angle, what happens? Wall moves down; surrounding soil moves up. That is the relative movement between wall and soil for positive wall friction. For negative wall friction, it is reverse. Reverse in the sense, wall moves up and this soil goes down. So, where is the application of that? Its application comes in the form of anchor uplift capacity.

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Where we can get that? Let us say this is an anchor plate, which is being pulled by this force. This is the ground surface; we all know from our static theory. So, this will be failure surface let us say. So, if we consider this as imaginary retaining wall, what is happening? First of all, if we take this normal on this side, there will be force in this direction; which we will call as negative $\delta P p$. Why it is passive earth pressure? Because this imaginary wall again it tends to move in this direction. When we are pulling out the anchor; when we are taking out the anchor like this, obviously, this wall tends to move in this direction towards the backfill. So, when we talk about this backfill and this imaginary wall, it is nothing but passive state.

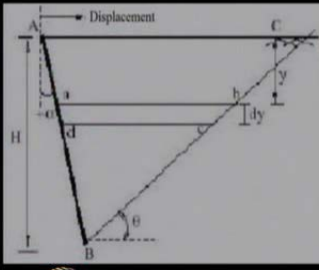
And, why we called it as negative delta? Because in this case, this wall moves up and this surrounding soil moves down, because this entire central area goes up; this goes down. So, that is referred as negative wall friction angle case. So, passive earth pressure is very much utilized in geotechnical engineering for anchor up lift capacity whether it is a negative wall friction angle case or for shallow foundation bearing capacity theory if it is a positive wall friction. So, that is what in this paper, if we look here, for this negative wall friction means where this anchor uplift capacity is the application as it is mentioned. We had obtained the design chart for this passive earth pressure coefficient under the dynamic condition with respect to the unit weight component. There are three components if you consider the generalized $c \phi$ soil, where cohesion is also present and friction angle is also present including the surcharge on the ground surface. So, then, the total seismic passive resistance can be computed using this expression; and each of these K values are nothing but seismic passive earth pressure coefficient with respect to cohesion with respect to surcharge and with respect to unit weight.

This chart gives, how this unit weight-related seismic passive earth pressure varies with respect to pseudo-static seismic acceleration K_h and K_v . You can see, K_v is 0; K_v is half of K_h ; and, K_v is 1 K_h , because these are typical ranges of K_v . And, as per AASHTO code it says about two-third of K_h should be considered for design as K_v value. So, for different values of ϕ , that is, soil friction angle, this design chart gives for a particular wall inclination, ground inclination and for a particular ratio of wall friction angle to soil friction angle. And, that value should be negative. As I have already mentioned, this is the case for negative wall friction angle. So, the details about this concept can be obtained using pseudo-static method in this journal paper by... This is my work – Choudhury with my supervisor Professor Subba Rao in 2002 – seismic passive earth resistance for negative wall friction, which is published in Canadian Geotechnical journal, volume 39, issue 5; these are the page numbers.

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Pseudo-static Method

- Choudhury et al. (2004) presented a comprehensive review for different methods to calculate seismic earth pressures and their point of applications. (Current Science, 2004)



Expression for the point of application of seismic passive resistance by,

$$h_a = H - \frac{2\pi^2 H^2 \cos \omega \zeta + 2\pi \lambda H \sin \omega \zeta - \lambda^2 (\cos \omega \zeta - \cos \omega t)}{2\pi \lambda H \cos \omega \zeta + \pi \lambda (\sin \omega \zeta - \sin \omega t)}$$

$$\lambda = \frac{2\pi V_s}{\omega} \text{ and } \zeta = \frac{2\pi V_p}{\omega}$$

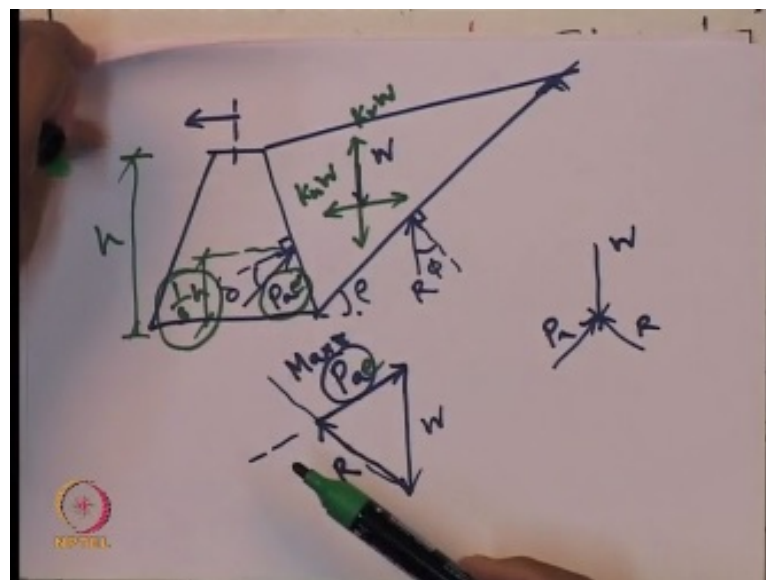
ω excitation frequency, V_s and V_p are shear and primary wave velocities respectively, H is height of retaining wall.

Choudhury, D., Sitharam, T. G. and Subba Rao, K. S. (2004) Seismic design of earth retaining structures and for NPTEL Current Science, India, 87:10, 1417-1425.

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Now, further, that pseudo-static method again applied to compute the point of application of the earth pressure, because earlier, if you remember, when we talked about this earth pressure...

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Let us look at where this total value of P acts or in dynamic condition P acts, where they will act is not known. Mononobe-Okabe had considered that, let us say, this will act at as one-third of height of the wall from the base. As they considered, it is a hydrostatic distribution of the pressure. So, they considered one-third from the base. But, there is no

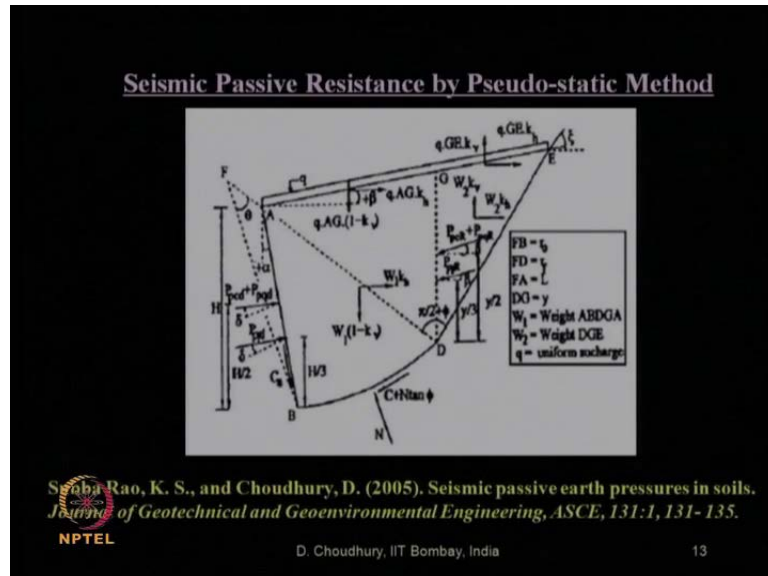
logical explanation, why it should be at one-third. It was based on an assumption; like in Coulombic theory also, we assumed that, it is hydrostatic pressure; but, in static condition, we know, yes, it is a hydrostatic condition. So, it has to be one-third from the base. But, in this case, it is dynamic case also, why it should be at one-third; that should be questioned. So, it needs to be obtained where this point of application of this design value of this $P_a e$ is acting, because for design of this retaining wall, we should know not only the value of this force. This value of force will give us to design the section.

But, to put the reinforcement in the wall, suppose if you are going to use the reinforced concrete wall as a retaining wall, you need to know how much reinforcement at different levels you are going to put. And, for that, you should know, what is the distribution or exact point of application of this $P_a e$, so that accordingly, you can place the reinforcement. So, point of application also should be known. And, that also takes care of the sliding analysis, rotational analysis, everything, because every case, the point of application is an important parameter. So, to find out that, what has been done? You can see in this paper – Choudhury et al., 2005, we worked this comprehensive review of different methods; how to estimate the seismic passive earth pressure using pseudo-static approach and their point of application. This is published in the journal, Current Science in 2004.

You can see, these are the details – Choudhury, Sitharam and Subba Rao, 2004 with my teachers – Professor Sitharam and Professor Subbarao. This paper was published in the journal, Current Science, volume 87, issue 10; these are the page numbers. So, what we did, we considered an inclined rigid wall section. So, this is the face of the wall; this is the ground surface – horizontal ground surface for passive state of earth pressure we determined. That is why displacement of the wall is showing in this direction. The entire failure zone, which has to be optimized of course, with respect to this failure angle as I have already mentioned; but, in addition to that, the entire zone we have not taken as a single mass, but we have divided it into number of small horizontal slices – infinitesimal small slices like this ABCD of thickness dy at a depth of y from the ground surface. So, for each of these slices, we considered what are the forces acting: static force as well as the pseudo-static forces. Then, considering equilibrium of each slices and doing the analysis and integrating over the entire height, we got the expression for point of application for the seismic passive resistance in this form; where, ω is the excitation

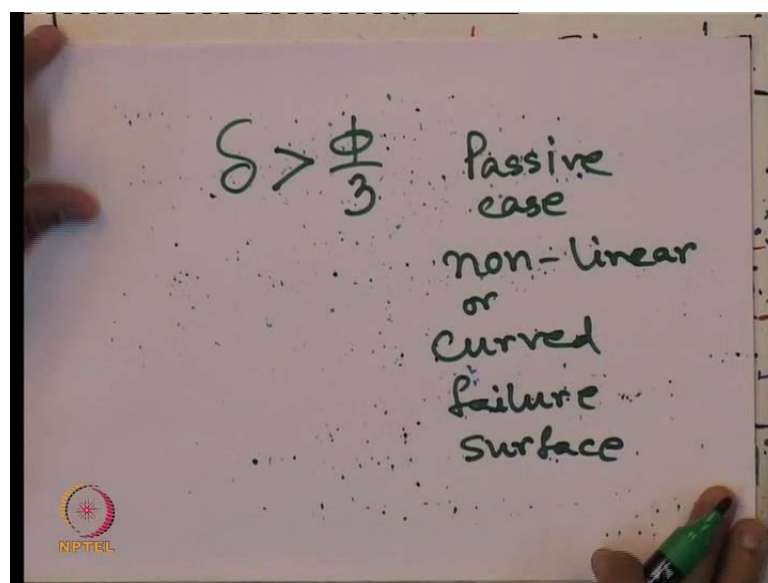
frequency; V_s , V_p are the shear and primary wave velocities, etcetera, can be used for estimation of this point of application.

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Then, seismic passive resistance by pseudo-static method for a generalized c-phi soil was also estimated by us considering composite failure surface. Why composite we have taken? Because in the passive state of earth pressure, we know planar failure surface seriously overestimates the value; for passive case, we should get the minimum value.

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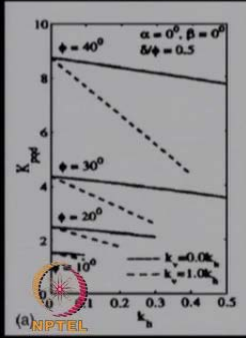
But, planar failure surface – when the wall friction angle, that is, delta value exceeds phi by 3, that is, one-third of the soil friction angle; in that case, we should not consider... For passive case, it is always non-linear or curved failure surface. It is no longer a planar failure surface. That Terzaghi himself has mentioned way back in 1943. The same is true for the seismic condition also. So, that is why, instead of taking planar rupture surface, we considered the curved rupture surface like this, which is a composite rupture surface; like this portion BD is a part of log spiral and DE is a part of planar failure surface. So, that combination has been taken for this retaining wall faces AB of height H.

For the passive earth pressure condition considering the surcharge on the ground – uniform surcharge small q, this is the c-phi soil. So, there are three components of passive earth pressure: one is with respect to gamma, that is, unit weight component, which acts at one-third from the base; then, cohesion component, P p c d, which acts at mid-height of the wall; and, P p q d is the surcharge component of passive earth pressure, which also acts at the mid-height of the wall. Then, summing them up, we will get the total seismic passive resistance. The details about this study is available in this journal paper, seismic passive earth pressure in soil, journal of Geotechnical and Geoenvironmental Engineering of ASCE, published in 2005, volume 131, issue 1; these are the page numbers.

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Pseudo-static Method

- **Subba Rao and Choudhury (2005)** gave design charts for seismic passive earth resistance coefficients by using a composite failure surface for positive wall friction angle. (*Jl. of Geotechnical and Geoenvironmental Engg., ASCE, 2005*)
- Design charts were proposed for the direct computation of seismic passive earth resistance coefficients for various input parameters.
- The application of this case is for bearing capacity of foundations.



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So, what we had proposed in this case, finally, we gave the design chart, that is, how to use this coefficient of passive earth pressure for different input values of K_h and K_v . So, how people can use it very easily? For a particular seismic zone in India or anywhere in the world, they should get what should be the input parameter of design value of K_h maybe based on design basis earthquake or something like that to estimate the K_h value and K_v value. Based on that input K_v and K_h value at the site, what is the value of ϕ – friction angle of the soil, you can go to this chart; get the value of this seismic passive earth pressure coefficient; and, directly using the formula as I had already mentioned – this formula, you can get the passive earth pressure. So, that way, the design of the wall you can do once you know the passive earth pressure, their total value and also their point of application for further design.

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Pseudo-static Method

- Shukla et al. (2009) have described the derivation of an analytical expression for the total active force on the retaining wall for c- ϕ soil backfill considering both the horizontal and vertical seismic coefficients.
- The seismic active earth pressure is given by,

$$P_{ae} = \frac{1}{2} \gamma H^2 (1 - k_v) K_{ae\gamma} - c H K_{aec}$$

where,

$$K_{ae\gamma} = \frac{\cos(\phi - \theta) - \frac{\sin(\phi - \theta)}{\tan \alpha_c}}{\cos \theta (\cos \phi + \tan \alpha_c \sin \phi)} \quad \text{and} \quad K_{aec} = \frac{\cos \phi (1 + \tan^2 \alpha_c)}{\tan \alpha_c (\cos \phi + \tan \alpha_c \sin \phi)}$$

$\tan \theta = \frac{k_h}{1 - k_v}$ and α_c is critical failure plane angle, ϕ shearing resistance of soil

Shukla, S. K., Gupta, S. K. and Sivakugan, N. (2009). Active earth pressure on retaining wall for c- ϕ soil backfill under seismic loading condition. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 135:5, 690-696.*

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Now, in pseudo-static method, other researchers also worked. As I have already mentioned, only the very recent research works I am showing you here; like Shukla et al. in 2009 – they described the derivation of an analytical expression for the estimation of total active force, that is, static plus seismic active force on a retaining wall with a backfill of c- ϕ , that is, both cohesion and frictional soil considering both the horizontal and vertical seismic acceleration. So, what is the expression for the computation of total active earth pressure under seismic condition is this one. This active earth pressure coefficient $K_{ae\gamma}$ is with respect to the unit weight component of the soil; whereas, K_{aec} is the coefficient of active earth pressure with respect to cohesion

component. And, their expressions for K_a and K_c is given by these expressions, where in this expression, $\tan \theta$ is nothing but k_h by $1 - k_v$; like for Mononobe-Okabe also, this is the same expression. And, α_c is the critical failure angle with ϕ is the frictional angle of the soil. So, this is the details about the paper, that is, Shukla, Gupta and Sivakugan. It was published in journal of Geotechnical and Geoenvironmental Engineering of ASCE; this volume, these page numbers in 2009.

Now, let us see, what are the advantages and disadvantages of this pseudo-static method when we are doing the analysis for retaining wall.

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Pseudo-static Method

- Major limitations
 - > Representation of the **complex, transient, dynamic** effects of earthquake shaking by **single constant unidirectional** pseudo-static acceleration is **very crude**.
 - > Relation between K and the maximum ground acceleration is **not clear** i.e. 1.9 g acceleration does not mean $K = 1.9$
- Advantages
 - > **Simple and straight-forward**
 - > **No** advanced or complicated analysis is necessary.

It uses **limit state equilibrium** analysis which is routinely conducted by Geotechnical Engineers.

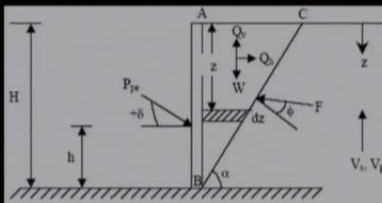
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What are the major limitations? First, let us look at that. Like representation of the complex, transient, dynamic effects of earthquake shaking by a single constant unidirectional pseudo-static acceleration is very crude. So, that is why, it is one of the major limitation of the pseudo-static method. It neither consider the complex nature of the dynamic load; it does not consider the time duration or transient nature of the dynamic load and other dynamic effects like frequency of excitation, shear wave velocity, primary wave velocity – all these seismic waves, which are traveling through the soil during this earthquake shaking. None of these dynamic properties are considered in this pseudo-static method; only a single constant value is used. So, that is why, it is a very crude estimation of the dynamic nature of a problem in quasi-static manner.

Also, the relation between that K and the maximum ground acceleration is also not clear. Suppose 1.9 g acceleration; it does not mean that K value should be 1.9. But, what are the advantages, why people still use this age-old pseudo-static method? Because it is very simple to use as we have seen; it is nothing but an extension of static analysis only; it is quasi-static or pseudo-static as the name suggest. And, it is very straightforward. So, there is hardly any complexity involved in the analysis. That is why many people, many researchers, many practitioners – they prefer this method because of its simplicity. No advanced or complicated analysis is necessary because of the nature of the problem. So, it uses the simple limiting state of equilibrium of the analysis, which is routinely conducted by geotechnical engineers mostly the practitioners, etcetera when they want to design any retaining wall in the seismically active region to consider the seismic earth pressure on the retaining structures.

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Development of Modern Pseudo-Dynamic Approach



Pseudo-Dynamic Approach by
Steedman and Zeng (1991),
Choudhury and Nimbalkar (2005)

$$a_g(z, t) = \{1 + (H - z) \cdot (I_s - 1) / H\} a_g \sin [\omega(t - (H - z) / V_s)]$$


$$a_s(z, t) = \{1 + (H - z) \cdot (I_s - 1) / H\} a_s \sin [\omega(t - (H - z) / V_p)]$$

Soil amplification is considered.

Frequency of earthquake excitation is considered.

Advantages

- Time duration of earthquake is considered.
- Phase differences between different waves can be considered.
- Amplitude of equivalent PGA can be considered.
- Considers seismic body wave velocities traveling during earthquake.



Choudhury and Nimbalkar, (2005), in *Geotechnique*, ICE Vol. 55(10), pp. 949-953.

Now, let us move forward and now let us talk about development of modern pseudo-dynamic approach. This pseudo-dynamic approach was originally proposed by Steedman and Zeng in 1991, which has been further modified and the generalized solution for pseudo-dynamic approach was given by Choudhury and Nimbalkar in 2005 for the first time. So, what is pseudo-dynamic method? Let us look at this problem. So, this is again we are taking about the basic rigid retaining wall analysis; this is the line diagram. AB is nothing but the vertical face of the retaining wall. This is the ground surface. This condition is showing the passive state of earth pressure; that means, wall – it tends to

move towards the backfill side. That is why this is the seismic passive earth pressure; this is the soil reaction; this is the weight of the failure mass ABC at a particular failure assumed angle of α .

And, these are the seismic inertia forces: Q_h and Q_v ; Q_h is horizontal seismic inertia force and Q_v is vertical seismic inertia force. In the pseudo-static method, how we estimated this Q_h and Q_v ? Nothing but Q_h is coefficient of seismic acceleration, that is, K_h times this W ; and, this Q_v , that is, vertical seismic inertia force was estimated as K_v times W ; where, K_v is vertical seismic acceleration coefficient. But, now, we are proposing this pseudo-dynamic approach, which takes care of all these dynamic effects. So, what are the advantages of this modern pseudo-dynamic approach over the conventional pseudo-static approach? Let us look at it first. Like soil amplification, which may happen in some of the cases of the soil as we have already studied that in the previous module on site response analysis, that can be considered in the design. That is the one biggest advantage of this pseudo dynamic approach, which the pseudo static approach cannot do.

Then, frequency of earthquake excitation is also considered. So, this dynamic nature is considered at what frequency the earthquake motion is coming. So, that can be taken care of. Time duration of the earthquake, that is, the transient nature of the earthquake can be considered in this method. Phase differences between different waves can be considered. Amplitude of equivalent peak ground acceleration can be considered. And, it considers the seismic body wave velocities; body waves means s wave and p wave both traveling during the earthquake. So, let us look at this basic picture once again. During the earthquake, we have this vertically propagating upwards this shear wave velocity V_s and primary wave velocity V_p . Now, why this is vertical? That already we have described; that if it comes from a large depth below the ground surface, after several layers, it will be almost close to vertical. So, it is a good assumption to consider the vertically propagating seismic wave like that.

Now the seismic accelerations, which are considered for the analyses are expressed in the form of sinusoidal motion. So, that is why the name pseudo is still there. One can always say why this dynamic approach is still having this component of the name that pseudo; because of the reason earthquake motion will be random in nature; it will not follow any sinusoidal or any particular mathematical function; it will be fully random in

nature. But, to have a generalized design approach, we cannot do that design with random motion. In that case, it will be a case-specific design; that is, for each taken random motion, you will get different values. Instead of that the practitioners or designers or the codal provisions will always look for a closed form design solution or generalized design solution. So, to do that what we have proposed, we have considered the equivalent sinusoidal motion; like once you have the random motion of earthquake, take the area under the curve of that acceleration versus time history and find out the equivalent area in the sinusoidal form of the same acceleration versus time history, which can be represented in this format. So, horizontal acceleration a_h now, no longer a constant, but it varies with respect to depth from the ground surface, that is, z measured from the ground surface and time t , that is, up to which this earthquake is occurring; which is expressed in this form – that is, $1 \text{ pulse } h \text{ minus } z$; that is, at a depth of z , a small infinitesimal slice of thickness dz is considered.

How it varies? $1 \text{ plus } H \text{ minus } z \text{ times } f_a \text{ minus } 1 \text{ by } H$; that means, f_a considers the soil amplification factor. Soil amplification factor is nothing but from the bed rock to ground surface, how much the PGA value got amplified or the maximum value of earthquake acceleration got amplified. So, that gives us the amplification factor. That non-dimensional value of amplification factor is assumed here to follow a linear distribution. That obviously, people can argue and can take different variation. But, when there is no other solution, one can fairly estimate the variation of f_a as a linear variation with this expression. This is nothing but a linear equation with multiplication of a_h ; a_h is nothing but amplitude of the horizontal earthquake acceleration. And, $\sin(\omega t \text{ minus } H \text{ minus } z \text{ by } V_s)$; where, this t is the duration of earthquake; $h \text{ minus } z \text{ by } V_s$ – it gives nothing but the phase; as you can see, this is nothing but the phase. So, that automatically gives us the expression, how this sinusoidal horizontal seismic acceleration varies with respect to z , because there is a function of z over here; and, varies with respect to time, because it is a function of time. And, frequency is also involved. So, frequency is also taken care. Soil amplification is taken care. And, shear wave velocity is also taken care of, because shear wave velocity mostly contribute on the horizontal seismic acceleration as we already know.

Similarly, for the vertical seismic acceleration, what is the equation? The a_v is also a function of z and t , which is expressed as $1 \text{ plus } H \text{ minus } z \text{ times } f_a \text{ minus } 1 \text{ by } H \text{ times}$

a_v . This a_v is nothing but amplitude of vertical seismic acceleration time sine of ω times t minus H minus z by V_p , because as we know, the primary wave velocity – that will contribute to the vertical seismic acceleration. So, that is why, this expression is considered for both horizontal as well as vertical seismic acceleration, which are now function of depth and function of time. Now, once you know the acceleration, what you can do? For this infinitesimally small slice, what is chosen over here, you can find out the mass of it. And, once you know from the geometry its mass of it, you can multiply that mass with respect to corresponding acceleration to get the inertia forces. So, that is what it has been done.

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$a_h(z, t) = a_h \sin [\omega \{t - (H - z)/V_s\}]$ and $a_v(z, t) = a_v \sin [\omega \{t - (H - z)/V_p\}]$
 where ω = angular frequency; t = time elapsed; V_s = shear wave velocity;
 V_p = primary wave velocity

$$Q_h(t) = \int_0^H m(z) a_h(z, t) dz = \frac{\lambda \gamma a_h}{4 \pi^2 g \tan \alpha} [2\pi H \cos \omega \zeta + \lambda (\sin \omega \zeta - \sin \omega t)]$$

 where, $\lambda = TV_s$ is the wavelength of the vertically propagating shear wave and $\zeta = t - H/V_s$.

$$Q_v(t) = \int_0^H m(z) a_v(z, t) dz = \frac{\eta \gamma a_v}{4 \pi^2 g \tan \alpha} [2\pi H \cos \omega \psi + \lambda (\sin \omega \psi - \sin \omega t)]$$

 where, $\eta = TV_p$ is the wavelength of the vertically propagating primary wave and $\psi = t - H/V_p$.
 The total (static plus dynamic) passive resistance is given by,

$$P_{ps} = \frac{W \sin(\alpha + \phi) - Q_h \cos(\alpha + \phi) - Q_v \sin(\alpha + \phi)}{\cos(\alpha + \delta + \phi)}$$

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 Choudhury, D. and Nimbalkar, S. (2005), in *Geotechnique*, ICE London, U.K., Vol. 55, No. 10, 949-953.

Let us look at here. If you do not consider any amplification, then the equation boils down to this; that is, amplification is 1; basically, there is no amplification. So, in that case, the expression for the seismic horizontal inertia force, $Q_h t$, will be nothing but mass of that infinitesimally small slice, that is, m of z times the acceleration $a_h z$, will give you the seismic horizontal inertia force. If you integrate that over the entire height of the wall, that will give you the total horizontal seismic inertia force; which on simplification, can be expressed like this; where, this value of λ is nothing but T times V_s ; which is nothing but wave length of the vertically propagating shear wave. And, this parameter η is nothing but t minus H by V_s .

Then, the total seismic vertical inertia force can also be estimated in the similar fashion; that is, integrate over the entire height of the wall 0 to H m z times a v times a z t; that will give us on simplification, this expression; where, this eta is nothing but T times V p, is the wave length of the vertically propagating primary wave. And, this psi is nothing but t minus H by V p. After doing this, what is done here? The details can be obtained in this paper – Choudhury and Nimbalkar; Dr Sanjay Nimbalkar was my first Ph.D. student. This is a part of his Ph.D. thesis work under my supervision at IIT Bombay. In 2005, this paper has been published in the journal, Geotechnique, published by Institute of Civil Engineers, London; this is the volume number, page number. So, what has been done, after finding out the Q h and Q v expression, W is already known as I said; then, the limiting equilibrium of all these forces involved was considered. And, once you do the limiting equilibrium of all the forces involved in this two-dimensional problem, the total, that is, static plus dynamic passive resistance can be obtained like this; which needs to optimized with respect to this angle of alpha.

Remember, in static case, we need to optimize it with respect to only this angle of chosen failure plane, alpha. But, in the case of dynamic problem, it is not only you have to optimize it with respect to alpha; but, you need to optimize this with respect to duration of earthquake also in terms of the frequency of earthquake. How it has been done? I will come to that now.

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The coefficient of seismic passive resistance (K_{pe}) is given by,


$$K_{pe} = \frac{1}{\tan \alpha} \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} - \frac{k_h}{2\pi^2 \tan \alpha} \left(\frac{TV_p}{H} \right) \\ \times \frac{\cos(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} \times m_1 - \frac{k_v}{2\pi^2 \tan \alpha} \left(\frac{TV_p}{H} \right) \\ \times \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} \times m_2$$

where

$$m_1 = 2\pi \cos 2\pi \left(\frac{t}{T} - \frac{H}{TV_p} \right) + \left(\frac{TV_p}{H} \right) \\ \times \left[\sin 2\pi \left(\frac{t}{T} - \frac{H}{TV_p} \right) - \sin 2\pi \left(\frac{t}{T} \right) \right] \quad \text{and} \\ m_2 = 2\pi \cos 2\pi \left(\frac{t}{T} - \frac{H}{TV_p} \right) + \left(\frac{TV_p}{H} \right) \\ \times \left[\sin 2\pi \left(\frac{t}{T} - \frac{H}{TV_p} \right) - \sin 2\pi \left(\frac{t}{T} \right) \right]$$

The seismic passive earth pressure distribution is given by,

$$p_{pe}(t) = \frac{dP_{pe}(t)}{dz} = \frac{\gamma z}{\tan \alpha} \frac{\sin(\alpha + \phi)}{\cos(\alpha + \delta + \phi)} \\ - \frac{k_h \gamma z}{\tan \alpha} \frac{\cos(\alpha + \phi)}{\cos(\alpha + \delta + \phi)} \sin \omega \left(t - \frac{z}{V_p} \right) \\ - \frac{k_v \gamma z}{\tan \alpha} \frac{\sin(\alpha + \phi)}{\cos(\alpha + \delta + \phi)} \sin \omega \left(t - \frac{z}{V_p} \right)$$

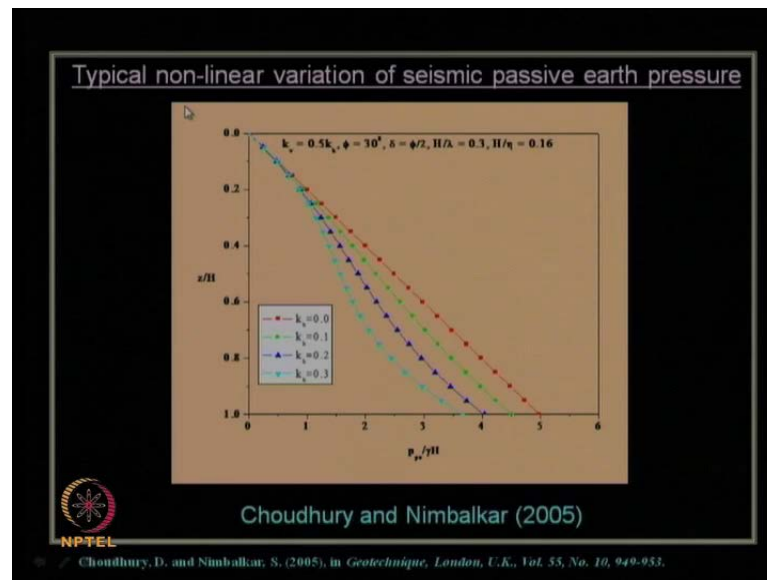
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Choudhury, D. and Nimbalkar, S. (2005), in *Geotechnique*, London, U.K., Vol. 55, No. 10, 949-953.

Let us see what is the coefficient of seismic passive resistance; coefficient of seismic passive resistance – which is nothing but the final design value, which designers will always look for when they are doing the design of any retaining wall. This is the closed form expression for $K_p e$. In this case, this m_1 and m_2 are expressed by this. You can see over here this parameter τ by capital T; this capital T is nothing but time period, which is related to the omega, that is, frequency as we all know; and, this τ nothing but duration of earthquake. So, we have to optimize this value of $K_p e$. What should be the design value of $K_p e$? As we know, for passive case, it should be the minimum value. So, minimum value of $K_p e$ needs to be obtained with respect to a combination of this alpha value and this τ by T value. So, that optimization has to be carried out. It is an optimization problem to obtain the minimum value of $K_p e$ or design value of $K_p e$; which was not present in the case of static problem. In that case, we have to minimize only with respect to this alpha value.

Now, how to get the seismic passive earth pressure distribution over the entire depth? This is one advantage of this pseudo-dynamic method. Like in pseudo-static method, earlier, as we have mentioned, in most of the cases, we were getting only the total value of the earth pressure except for one method, where we have mentioned about that method of slices; otherwise, we were getting the total value; we were not getting the distribution. But, in this case of pseudo-dynamic method, as we have already taken this infinitesimally small slices, so, at each level of the depth, you are getting the design value of this $P_p e$. So, automatically, that gives you the distribution; or, it can be mathematically expressed in this form. This is the variation of seismic earth pressure distribution with respect to depth. So, once you know the distribution, automatically, you know the point of application of the total passive resistance as well; which will help you to put the reinforcement in the wall section when you are designing the wall. So, all these details are available in this journal paper of Geotechnique by Choudhury and Nimbalkar, 2005.

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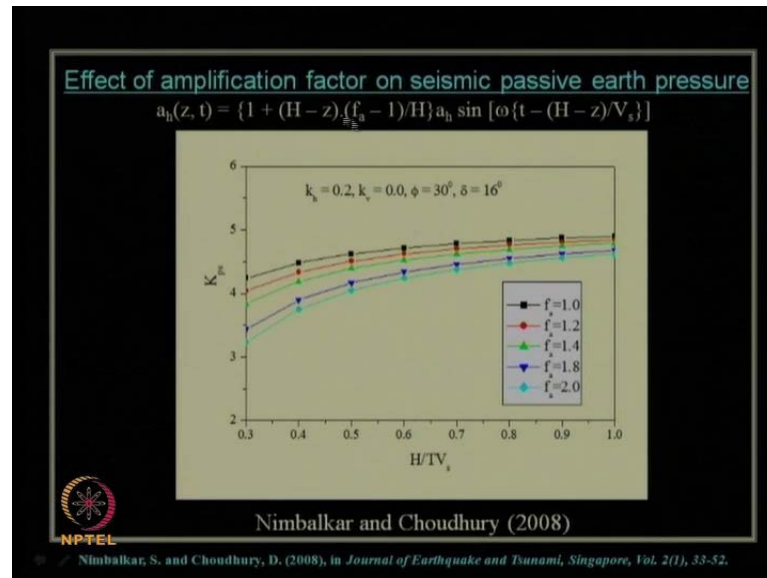


Now, let us see, how the results are varying as far as the proposed pseudo-dynamic method is concerned. So, for the seismic passive resistance, you can clearly see now, the seismic earth pressure distribution is fully non-linear. Earlier, in all the pseudo-static method, we were getting linear earth pressure distribution, which should not be the case, because we know, due to seismicity, the earth pressure cannot be hydrostatic or the triangular distribution; it cannot be a linear variation; it will be a non-linear process. So, that non-linearity has been also captured in this pseudo-dynamic method. So, these are all the advantages of pseudo-dynamic method in addition to that what I have mentioned in the first slide of this pseudo-dynamic approach.

You can see the results. This red line shows the value when k_h equals to 0; that means, under the static condition, because k_v we have considered for 0.5 times k_h . So, if k_h is 0, k_v is also 0. So, this red line shows the passive earth pressure under static condition. And, as the seismicity increases, as we know, the direction of the seismic acceleration will act in both the directions, whether it is horizontal or vertical; but, we have to take the critical value for the design. And, what will be the critical value for passive case? It will be the minimum one. And, minimum one is nothing but it should reduce with increase in the seismicity. So, that is what, from the static case of this red line, as you can see, as the seismicity increases from k_h value – 0 to 0.1, 0.2, 0.3 g, this value of passive earth pressure distribution under earthquake condition is also reducing. So, those are design values. And, as the seismicity increases, the non-linearity of this variation is also

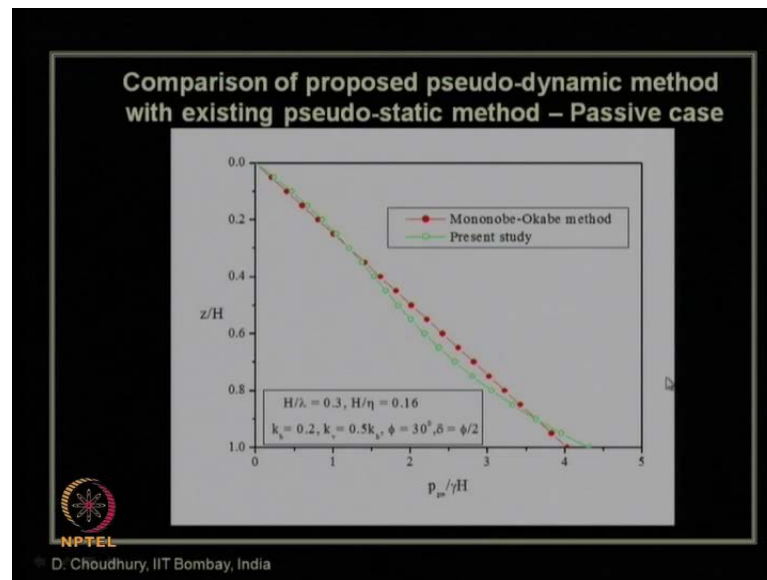
increasing. So, that automatically shows that, point of application, which was in the static case at one-third from the base of the wall is no longer at one-third, but it is below one-third for the case of passive earth pressure.

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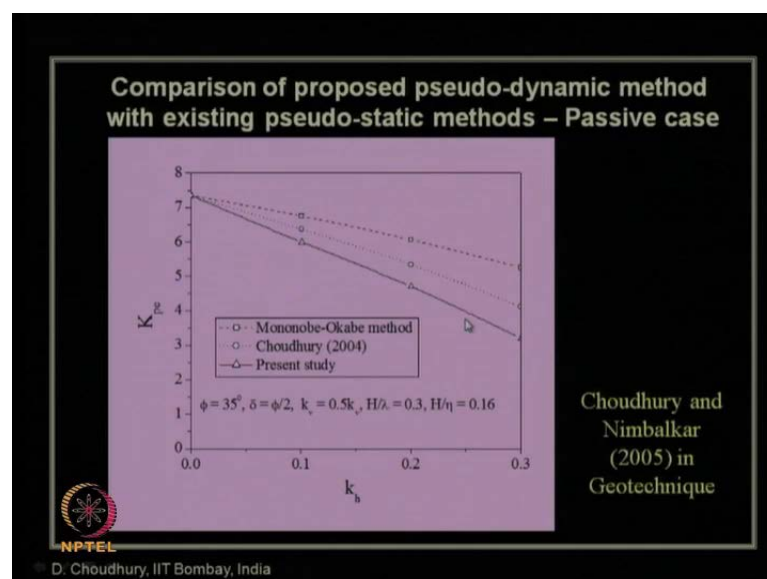
Some more results. When we are talking about the effect of soil amplification; how this amplification affects the design value of this earth pressure. Let us see, the seismic passive earth pressure coefficient; when there is no amplification; that means, f_a equals to 1; f_a equals to 1 means no amplification factor. That time, depending on this value of H by $T V_s$, you can see, this is the variation of K_{pe} for no amplification case. But, if there is an amplification in the soil; if suppose the soil gets amplified by 1.4 times or 2 times; then, in that case, the design value of K_{pe} is still decreasing. What does it mean? That means, the design value should be much lower than what it is the case for the non-amplified soil. So, this criticality also can be captured in the case of pseudo-dynamic approach, which is not possible in the case of pseudo-static approach. So, all these are advantages of pseudo-dynamic method.

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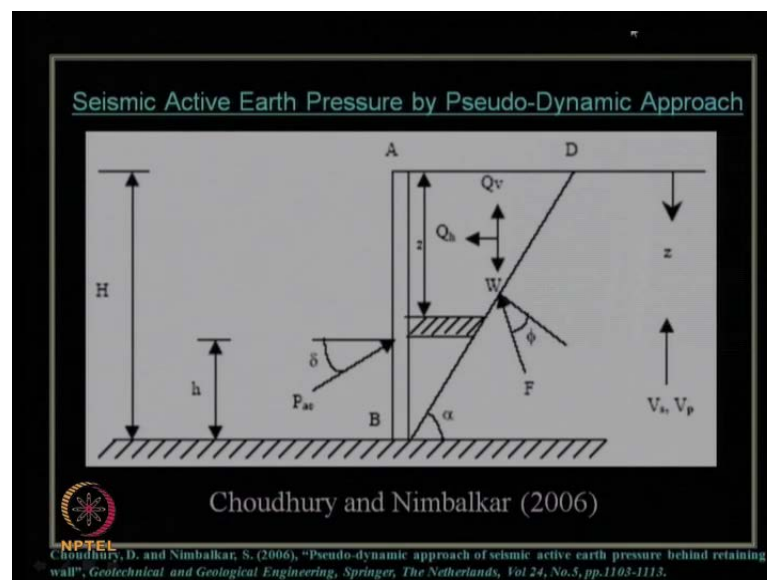
This is another comparison. You can see, for this case of k_h equals to 0.2 and k_v equals to half of k_h with this value of ϕ and δ , Mononobe-Okabe method, which is pseudo-static method – that will give a linear variation or hydrostatic distribution, straight line variation like this – the red colored; whereas the pseudo-dynamic method in the passive case, it is non-linear. And, this is the non-linear variation. You can always say, the total value of the passive earth pressure there is, that can be said almost similar for this particular chosen set of data value. But, the passive earth pressure coefficient values may vary, which is necessary for the design.

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Let us look at these values, which is also available in this journal of Geotechnique by Choudhury and Nimbalkar, 2005. This passive earth pressure coefficient, the design value – this dotted line is for Mononobe-Okabe with these rectangular boxes; then, this dotted line with circles are proposed by Choudhury in 2004. And, the triangular one is the present study means this pseudo-dynamic approach. You can see, the design value K_{pe} is minimum, what we are obtaining in the pseudo-dynamic method compared to the conventional pseudo-static method. So, what we can say, the critical value or the most desirable design value is proposed by pseudo-dynamic model, not the pseudo static model for the passive state of earth pressure; which is cleared from this picture also.

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Now, let us come to the next case of seismic active earth pressure and applying this new pseudo-dynamic method. So, for the case of seismic active earth pressure on the retaining wall, this is the line diagram; this is the retaining wall; planar failure surface is assumed again. This is the active state of earth pressure. So, that is why, this is the position of the active earth pressure – seismic active earth pressure P_{ae} at an angle δ with respect to normal to the wall; and, W is the weight of this failure wedge; Q_h is the seismic horizontal inertia force; Q_v is the seismic vertical inertia force. Remember, they act in both the directions when have to find out the critical direction of it by doing an optimization of analysis. And, F is the soil reaction. So, here also, this infinitesimal small element has been considered at a depth of z of thickness dz ; z is measured from the ground surface; and, V_s and V_p are the shear wave velocity and primary wave velocity

of the earthquake excitation. The details are available in the publication of Dr Sanjay Nimbalkar's Ph.D. thesis work; that is, Choudhury and Nimbalkar, 2006 in this Geotechnical and Geological Engineering – an international journal published by Springer; this is the volume number, page number, etcetera.

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$a_h(z, t) = a_h \sin [\omega \{t - (H - z)/V_s\}]$ and $a_v(z, t) = a_v \sin [\omega \{t - (H - z)/V_p\}]$
 where ω = angular frequency; t = time elapsed; V_s = shear wave velocity;
 V_p = primary wave velocity.

$$Q_h(t) = \int_0^H m(z) a_h(z, t) dz = \frac{\lambda \gamma a_h}{4 \pi^2 g \tan \alpha} [2 \pi H \cos \omega \zeta + \lambda (\sin \omega \zeta - \sin \omega t)]$$

where, $\lambda = TV_s$ is the wavelength of the vertically propagating shear wave and $\zeta = t - H/V_s$.

$$Q_v(t) = \int_0^H m(z) a_v(z, t) dz = \frac{\eta \gamma a_v}{4 \pi^2 g \tan \alpha} [2 \pi H \cos \omega \psi + \lambda (\sin \omega \psi - \sin \omega t)]$$

where, $\eta = TV_p$ is the wavelength of the vertically propagating primary wave and $\psi = t - H/V_p$.

The total (static plus dynamic) active thrust is given by,

$$P_{ae}(t) = \frac{W \sin(\alpha - \phi) + Q_h(t) \cos(\alpha - \phi) - Q_v(t) \sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)}$$

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What we can see? The similar approach has been adopted. In this case, it is the active state. So, the seismic inertia force in both horizontal and vertical direction is obtained in the similar manner. But, when you are doing the equilibrium and finding out the P a e, you have to do the proper analysis of all the forces involved and their corresponding directions, etcetera to get the design value of this. And, once you get the expression for P a e, next step is to do the optimization once again.

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The seismic active earth pressure coefficient, K_{ae} is defined as

$$K_{ae} = \frac{1}{\tan \alpha \cos(\delta + \phi - \alpha)} + \frac{k_1}{2\pi^2 \tan \alpha} \left(\frac{TV_1}{H} \right) \times \frac{\cos(\alpha - \phi)}{\cos(\delta + \phi - \alpha)} \times m_1 + \frac{k_2}{2\pi^2 \tan \alpha} \left(\frac{TV_2}{H} \right) \times \frac{\sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)} \times m_2$$

where,

$$m_1 = \left[2\pi \cos 2\pi \left(\frac{t}{T} - \frac{H}{TV_1} \right) + \left(\frac{TV_1}{H} \right) \left(\sin 2\pi \left(\frac{t}{T} - \frac{H}{TV_1} \right) - \sin 2\pi \left(\frac{t}{T} \right) \right) \right]$$

$$m_2 = \left[2\pi \cos 2\pi \left(\frac{t}{T} - \frac{H}{TV_2} \right) + \left(\frac{TV_2}{H} \right) \left(\sin 2\pi \left(\frac{t}{T} - \frac{H}{TV_2} \right) - \sin 2\pi \left(\frac{t}{T} \right) \right) \right]$$

The seismic active earth pressure distribution is given by,

$$p_{ae}(z) = \frac{\partial P_{ae}(z)}{\partial z} = \frac{\gamma z \sin(\alpha - \phi)}{\tan \alpha \cos(\delta + \phi - \alpha)} + \frac{k_1 \gamma z \cos(\alpha - \phi)}{\tan \alpha \cos(\delta + \phi - \alpha)} \sin \left[w \left(\frac{t - z}{TV_1} \right) \right] + \frac{k_2 \gamma z \sin(\alpha - \phi)}{\tan \alpha \cos(\delta + \phi - \alpha)} \sin \left[w \left(\frac{t - z}{TV_2} \right) \right]$$

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And, this is the expression – closed form expression or the design expression, which designers will like to use always. This is the K_{ae} expression with m_1 and m_2 values over here. You can see, here also, the optimization requires with respect to the chosen alpha angle and this t by T ratio. And, here again, we are getting the advantage of seismic active earth pressure distribution – the variation of it with respect to wall; which gives us what the point of application of the total seismic active earth pressure will act; that also will give us the idea.

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Typical non-linear variation of seismic active earth pressure

Choudhury and Nimbalkar (2006), in *Geotechnical and Geological Engineering*, 24(5), 1103-1113.

$$a_h(z, t) = [1 + (H - z) \cdot (f_1 - 1)] a_g \sin [\omega(t - (H - z) \cdot V_s)]$$

Effect of soil amplification on seismic active earth pressure

Nimbalkar and Choudhury (2008), in *Journal of Earthquake and Tsunami*, 2(1), 33-52.

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This is the result you can see over here. This red line is the result for the static case. And, as the seismicity increases, as we know for the active state that, design value or critical value should increase, because in active case, we maximize the design value. So, that is why as the seismicity increases, the design value increases compared to the static one. And also, as the seismicity increases, the variation of the active earth pressure distribution with respect to depth, it is highly non-linear as you can see over here. What it can be said? The point of application of the total seismic active thrust, which is one-third from the base of the height of the wall; and, static case is no longer at one third, but it shifts upward than one-third. In passive case, it moves downward; in active case, it moves upward. So, that gives us the exact position, where and what amount of reinforcement in the reinforced soil wall when we are trying to construct and design a reinforced soil wall; when we are trying to construct RCC rigid retaining wall, where the reinforcement has to be provided. That is clear from these results; which is not available in the pseudo-static approach.

Again, the variation of this amplification factor can be seen over here; like when there is no amplification – this black line, this is the value of K_a – the design value without amplification. But, when the soil gets amplified during the earthquake process, that material property also can be taken in this pseudo-dynamic approach. And, you can see, the design value of K_a is increasing. Can you see? That automatically says that, we need to take the higher value of seismic active earth pressure when we are designing our wall in a soil, which is going to get amplified subjected to some earthquake motion. So, these details are available in the publication, the effect of amplification, etcetera on the seismic active earth pressure; publication of Nimbalkar and Choudhury in 2008 in the journal of Earthquake and Tsunami; this is the volume number and page number.

With this, we have come to the end of today's lecture. We will continue further in our next lecture.