Geotechnical Earthquake Engineering
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Module - 8
Lecture - 32
Site Response Analysis (Contd…)

Let us start our today's lecture for this NPTEL video course on geotechnical earthquake engineering. We were going through our module number 8 which is site response analysis. Let us have a quick recap what we have learnt in our previous lecture on this module 8 that we have started learning on site response analysis.

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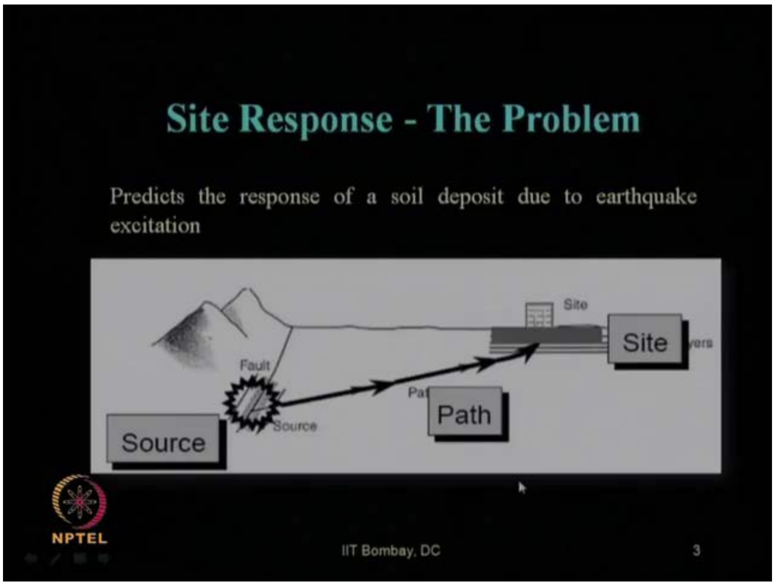

We have mentioned how the problem of site response is defined, that is it is the prediction of the response of a soil deposit due to the earthquake excitation. And in that process we have seen we have three major faces or phases, three major phases are source, path and site. Source of earthquake through which it travels, that path of earthquake and the site or the surficial layer close to ground surface when it travels, what will be the behavior. So, that site response or ground response we want to learn in detail.

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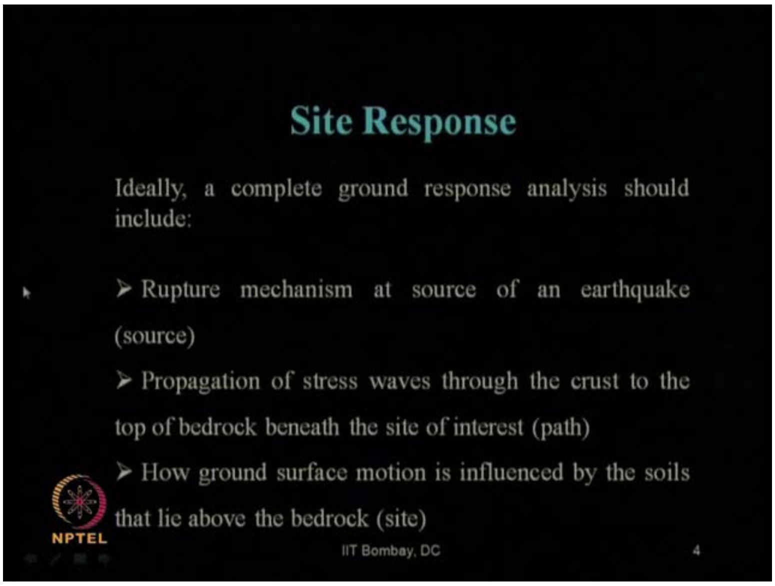

So, for that site response ideally speaking these are the three steps of source, path and site.

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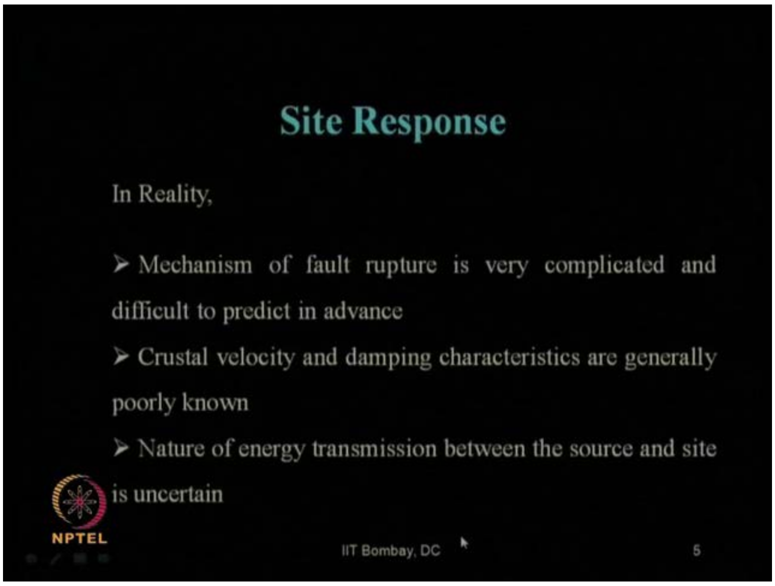

What we have mentioned over here is having the complicacy of in reality getting the complete information about the fault rupture; that is the complete information about source. Then complete information about crustal velocity and the damping characteristics which are not properly known, this is regarding the path, and then nature of energy transmission between that source and the site is uncertain.

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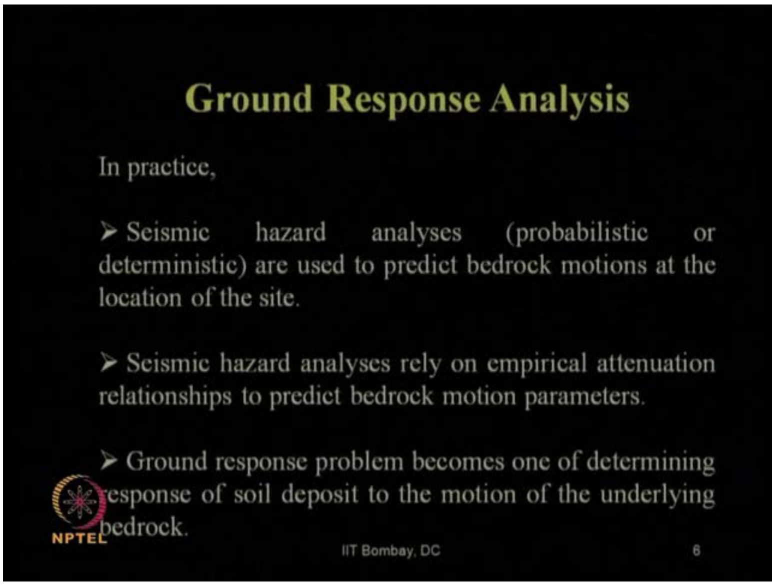

So, after knowing those real problems what we practice actually for the ground response or site response analysis are like doing seismic hazard analysis, either probabilistic or deterministic it can be and also using the bed rock motions, then seismic hazard analysis has to rely on the empirical relationship, attenuation relationship and ground response problem becomes one of determining response of soil deposit to the motion of the underlying bed rock.

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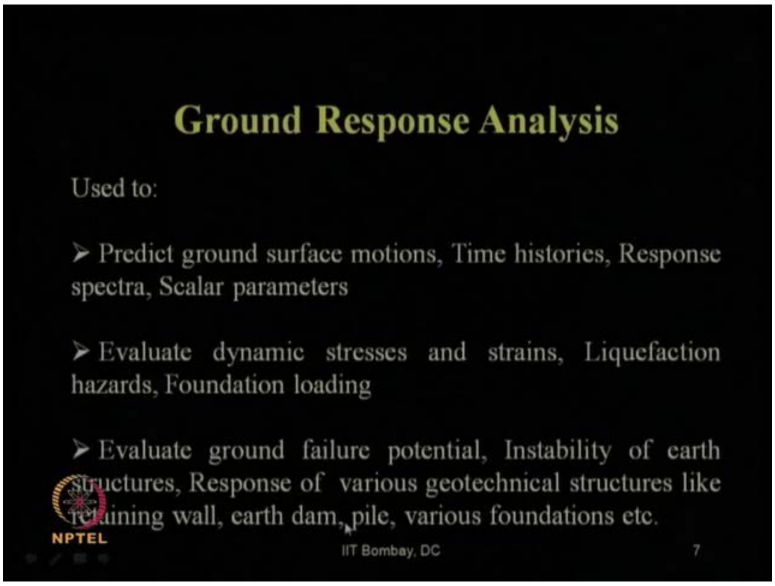

And these ground response analysis where these are used, these are actually extensively used to predict the ground motion at the ground surface, time history, response spectra, scalar parameters, evaluate the dynamic stress strain, liquefaction hazard, foundation loading due to the earthquake, then evaluate the ground failure potential, instability of earth structure, response of various geotechnical structures like retaining wall, earth dam, pile, various foundations on all these things when we want to study their seismic behavior we should first go through the ground response analysis. That is the very basic fundamental step.

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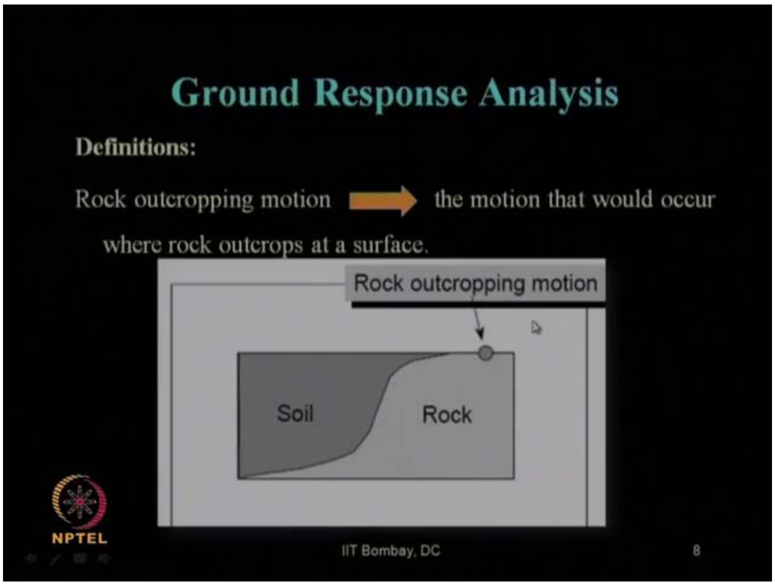

So, some of the definitions also we have learnt in our previous lecture like what is called as rock outcropping motion. When a rock surface exposed to the ground level or ground surface that place of ground motion, where it is known is known as rock outcropping motion.

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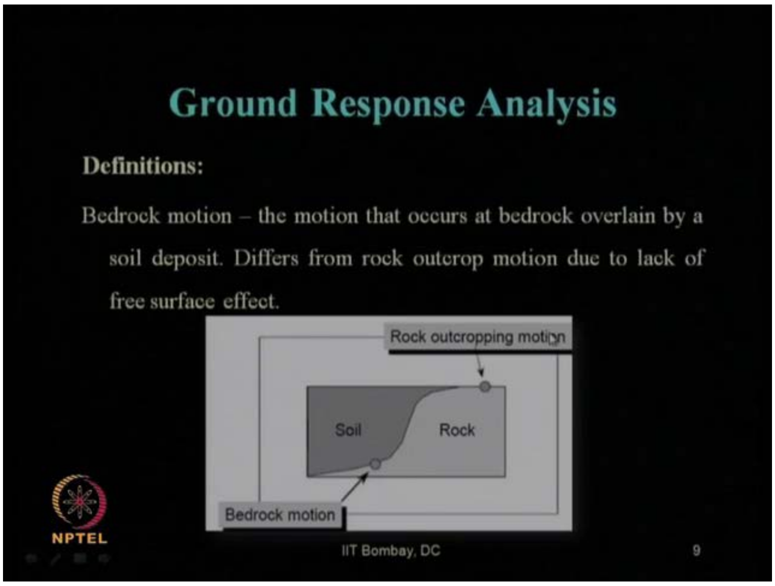

Then we have seen what is known as bed rock motion. When the bed rock is below the thick soil deposit or soil layer, in that case at the top of the rock surface before the soil layer starts where the earthquake motion or seismic motion if it is known, we call it as bed rock motion.

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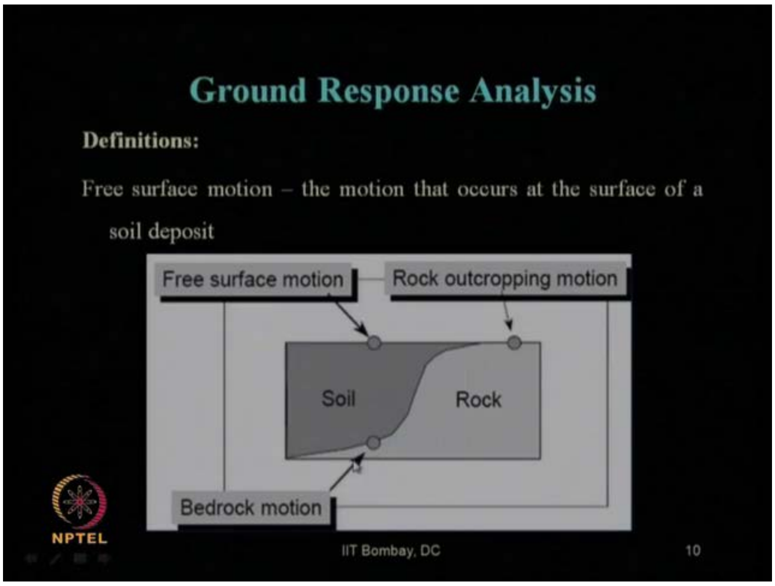

Then another terminology we have seen free surface motion which is nothing but the ground motion which is known at the free ground surface of the soil layer that is known as free surface motion.

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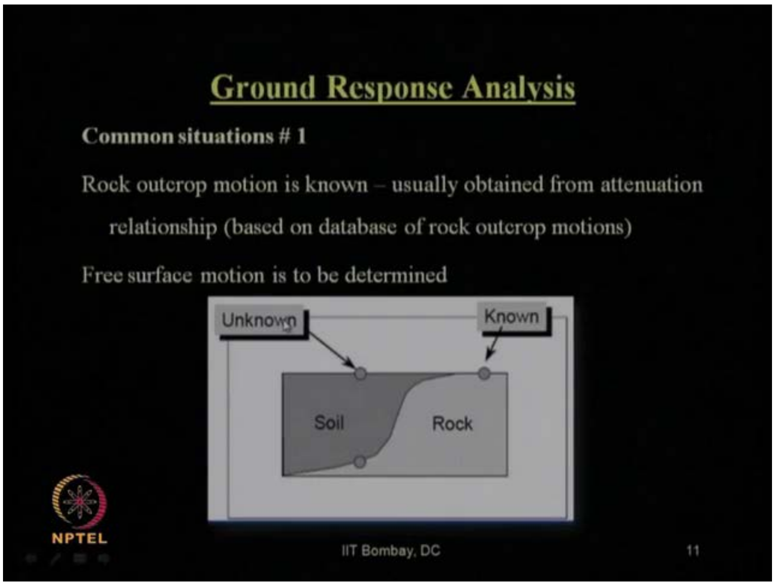

Then we have discussed in the previous lecture that common situation can arise like this. First situation can be say free surface motion is unknown, whereas rock outcrop motion is known. We need to find out these value, how to do that we will see.

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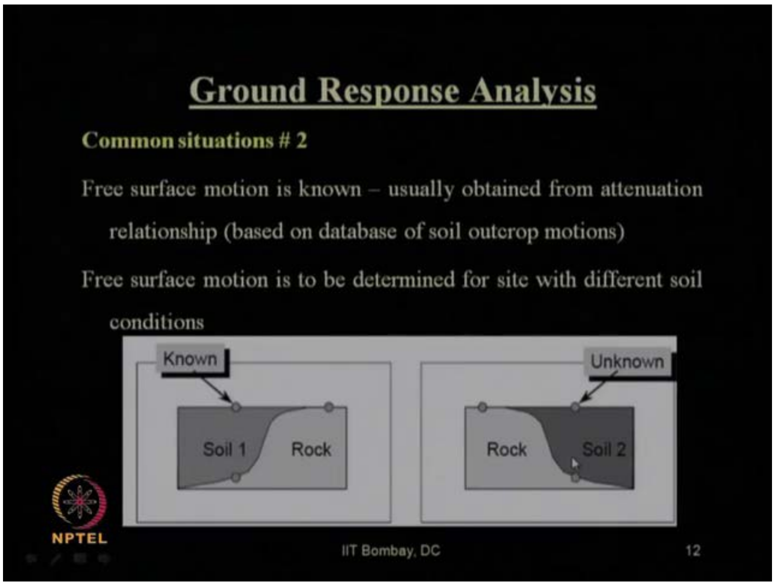

In the second common situation we have learnt let us say at free surface motion, it is known. We do not know at rock outcrop motion or we do not know the free surface motion at another soil site which is having the same common rock, so how to obtain those things we have seen.

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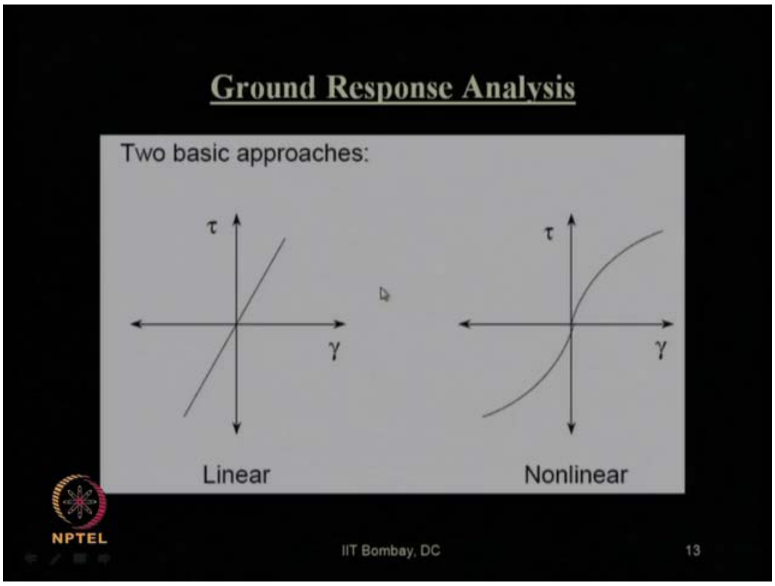

There are two basic approach to carry out the ground response analysis. One is known as linear ground response analysis, another is called non-linear ground response analysis depending on what type of behavior of the soil material we are considering for an earthquake load, when we are considering the shear stress versus shear strain relationship of the material linear. We call it linear ground response analysis. When we are considering it as non-linear we call it as non-linear ground response analysis. Within non-linear also we said there are two cases one is equivalent linear, another is absolutely non-linear. So, we have seen, we have discussed in this, we can look at this.

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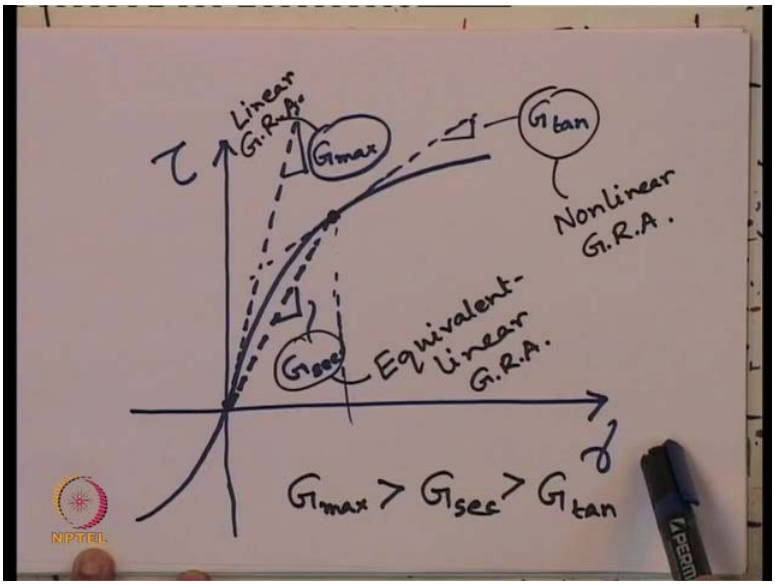

So, relationship of linear ground response analysis using G max, G tan for non-linear ground response analysis and in between them is equivalent linear ground response analysis using G secant modulus and this is how the shear modulus maximum one is obviously the maximum, greater than the secant shear modulus, greater than the tangent shear modulus, which we have also discussed in our dynamic properties module.

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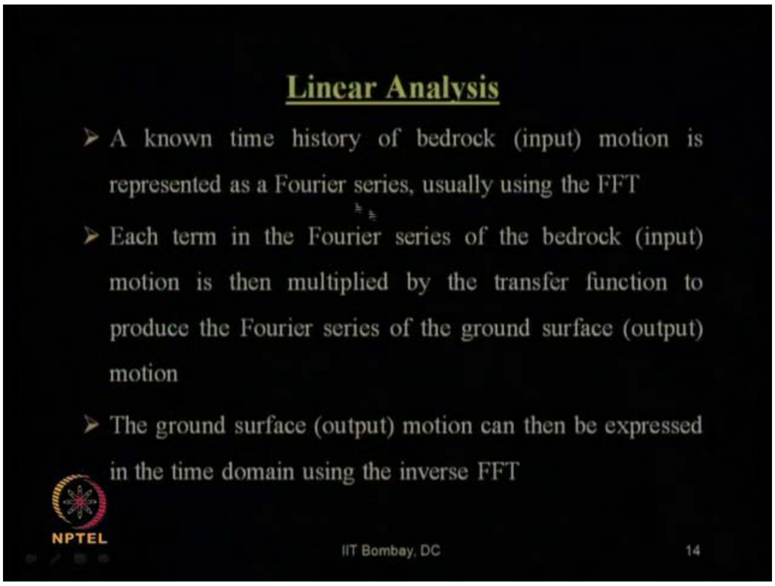

Then in the previous lecture we also learnt what is the step for doing the linear ground response analysis. First step is if we consider any known motion which we call as a input motion. Let us say bed rock motion is a known input motion. We can represent it by Fourier series and using first Fourier series transform then we can use a multiplier, which is known as transfer function. Transfer function is nothing but a multiplier to the input motion to get an output. That output can be, let us say at ground surface, when your bed rock motion is known again using the inverse of that first Fourier transform.

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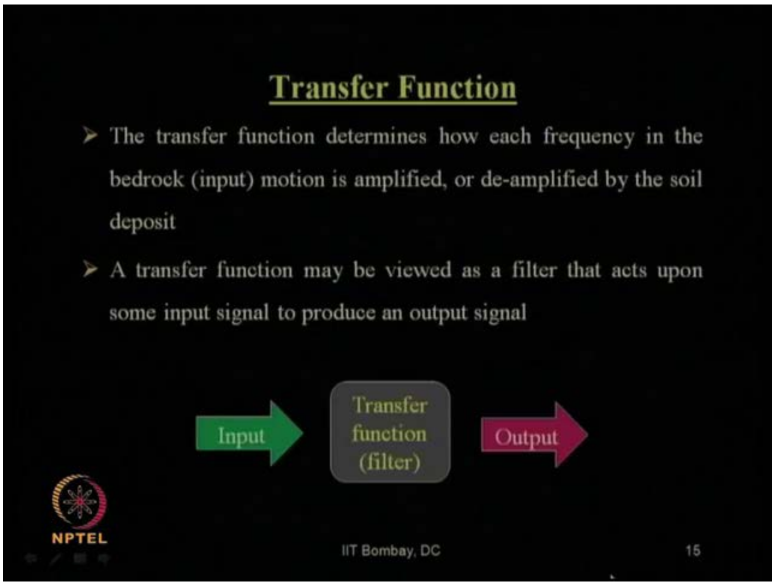

So, the transfer function it determines how the each frequency in the bed rock motion is getting amplified or it is getting de-amplified so that transfer function are multiplied decides whether the input motion increases, that means amplification or decreases that is de-amplification and the output value is coming out.

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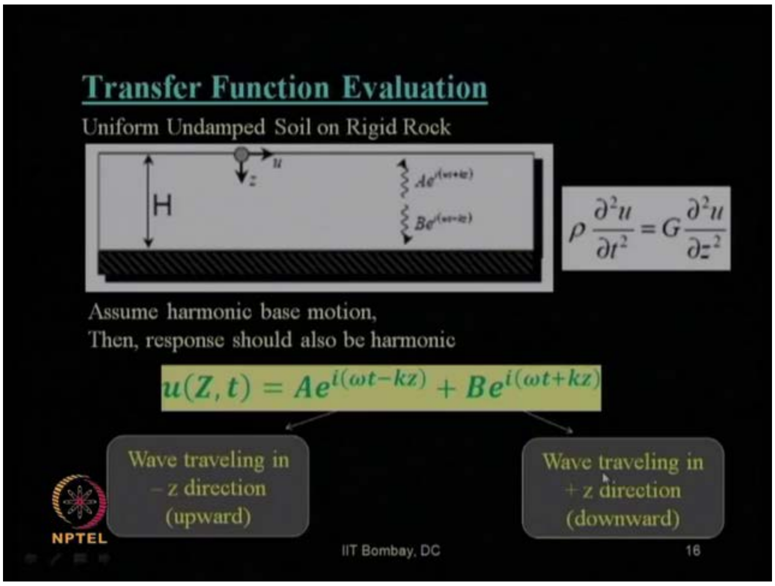

How to obtain the transfer function is mathematically evaluation also we started learning for the simplest condition of uniform soil layer when there is no layering of soil, homogeneous soil of thickness H, undamped condition no damping of soil property is considered it is though unrealistic, but for the first derivation we have considered it. Then we have seen for one dimensional wave propagation, what is the basic governing equation of motion for one dimension which we have discussed in the module on wave motion. In wave motion module we have considered this is the basic equation and if we have a harmonic base motion the solution or response of the displacement function can be expressed in this form.

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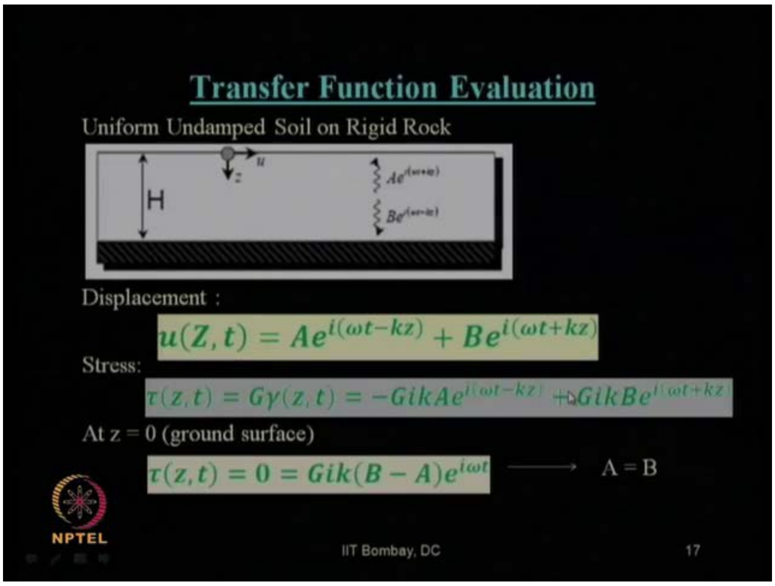

If displacement is expressed in this form then shear stress can be obtained in this pattern because shear strain we can get by differentiating this displacement function in terms of Z and multiply with respect to the shear modulus, we will get the shear stress. The condition, boundary condition which is known to us that will help us to get this values of the constants. So, that boundary condition one which is known to us is at ground surface, stresses should be 0. So, using that we got A and B should be equal.

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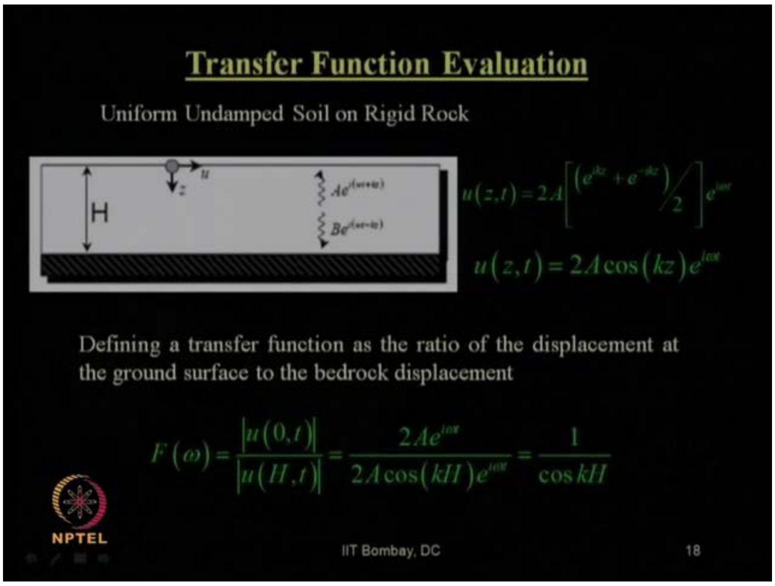

If A and B are equal we have seen the equation boils down to this form. If it is so we can write it in this fashion and the transfer function is defined as the ratio of the displacement at ground surface to that at the bed rock level. So, that is how transfer function is represented here as the ratio of modulus of the displacement at ground surface by modulus of the displacement at bed rock level below our thickness of soil layer of H which on simplification we have seen, it come out to be 1 by cosine of k H.

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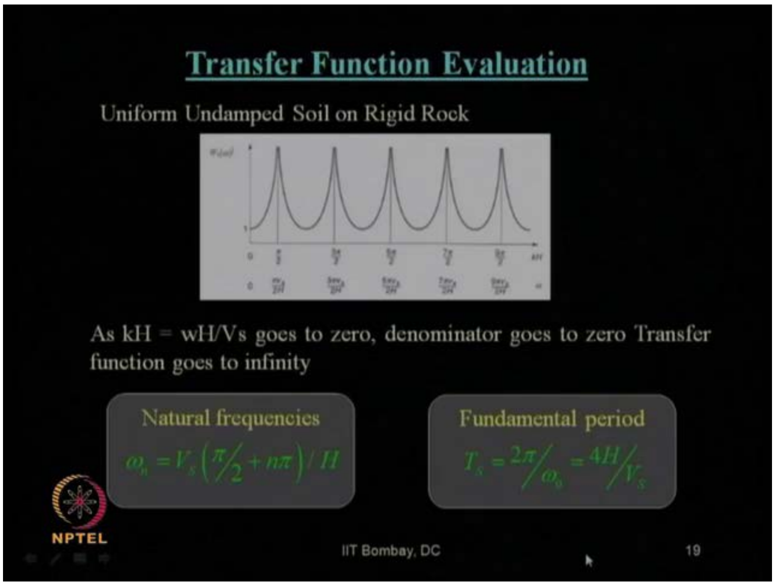

So, with that we have seen uniform undamped soil on rigid rock. They will follow this type of pattern that is if we follow the transfer function versus the k H plot, it will follow this pattern putting different values of k H. And as this k H goes to 0 you can see k H, these value goes to 0, the denominator goes to 0 and transfer function goes to infinity. It should be k H goes to pi by 2. Then the transfer function goes to infinity. Please correct it, k H goes to pi by 2 at that case transfer function goes to infinity. At k H equals to 0 transfer function is 1, at k H equals to 0 transfer function is 1.

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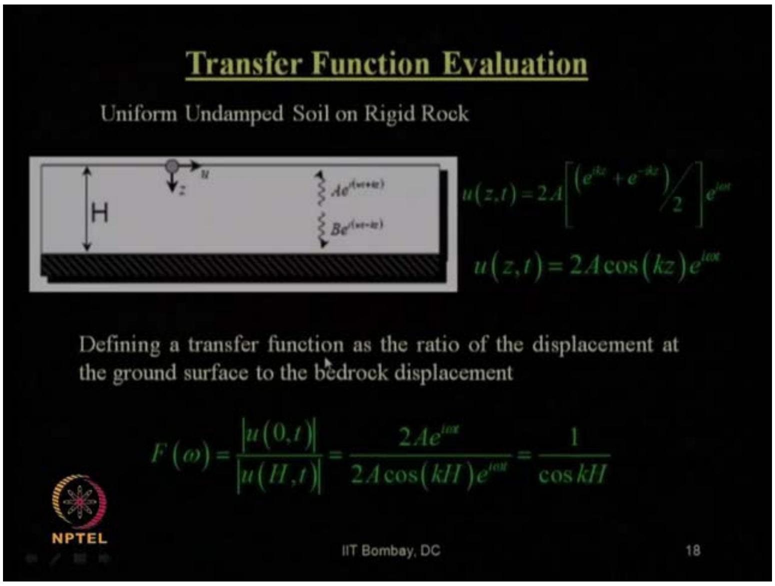

As we can see in this equation k H 0 means cos 0 means 1 so transfer function is 1, but k H is pi by 2 means transfer function goes to infinity.

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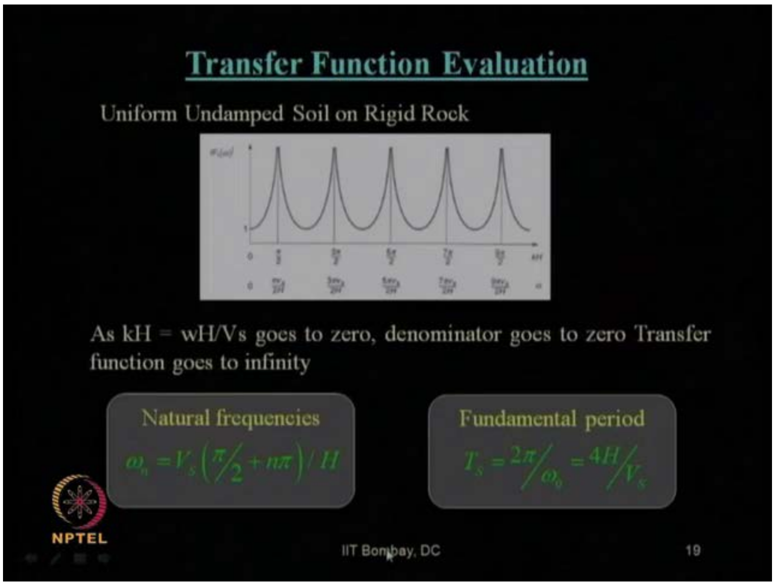

So, natural frequencies we can write in this form omega n is n numbers of natural frequency we can get here, V s times pi by 2 plus n pi, n is the integer starting from 0 1 2 3 like that by H. So, fundamental period, what is fundamental period? It is nothing but that period which corresponds to fundamental frequency, fundamental natural frequency and what is fundamental natural frequency, it is the least among the all natural frequency and that is omega 0, that is if we put n value equal to 0 we get that fundamental natural frequency omega 0 as pi V s by 2 H. So, by putting omega 0 as pi V s by 2 H the value of fundamental period we get at 4 H by V s, that already we have used earlier in one of the module and also in soil dynamics course that this is the way how to obtain the fundamental period of any soil layer which is having a thickness of H and having a shear wave velocity of V s. So, with that we completed in our previous lecture.

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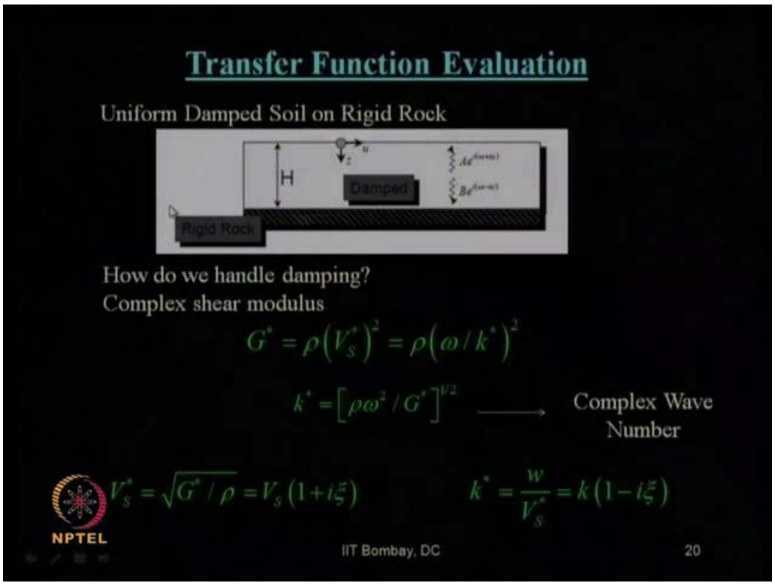

Now, let us start our today's lecture. Now, let use come to more realistic case of damped soil that is instead of considering undamped soil profile, let us now take damped soil layer, but still we are considering uniform or homogeneous soil layer of thickness H, but damped. So, how to obtain the transfer function for that type of soil, let us see. So, how to handle that damping? In that case we will have a complex shear modulus. How it is defined the complex shear modulus G star, it is nothing but rho times V s star square because we know shear modulus and shear wave velocity they are correlated by this G equals to rho V s square. Remember this value of G we will get is G max because we are talking about linear ground response analysis.

So, that can be expressed as what is V s? V s we can express as omega by k where k is a wave number. Now in this case it is k star which is the complex wave number. So, k star by rearranging this equation we will get as rho omega square by G star under root of that fine so that is complex wave number. So, V s star is nothing but root of G star by rho which we can express as V s times 1 plus i eta, this eta is nothing but the damping ratio. So, that is how we mention. Let us use it in the form of a complex number. So, k star which is defined as omega by V s star can be written as k times 1 minus i omega 1 minus i eta. So, that is complex wave number compare to the wave number which we have discussed for the undamped case.

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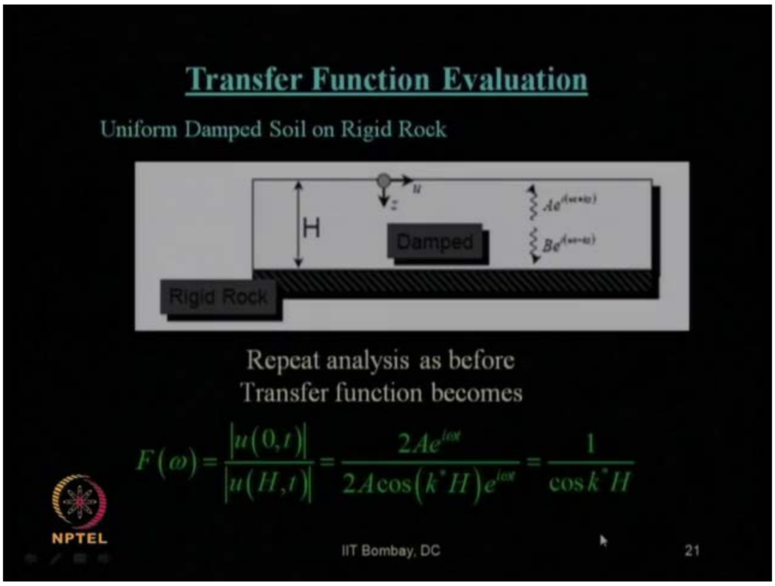

Now, knowing this what we can do? We can repeat the analysis like before what we have done for undamped case and the transfer function which defines as the same ratio at ground surface to at bed rock level. What will be the change? u of 0 t remain same 2 A e to the power i omega t, isn’t it because z become 0 there, but at depth H that is at bed rock level it becomes k star H instead of k H which on simplification we will get 1 by cos of k star H instead of k H what we got for undamped case. So, the only change is k become k star, from undamped to damped case.

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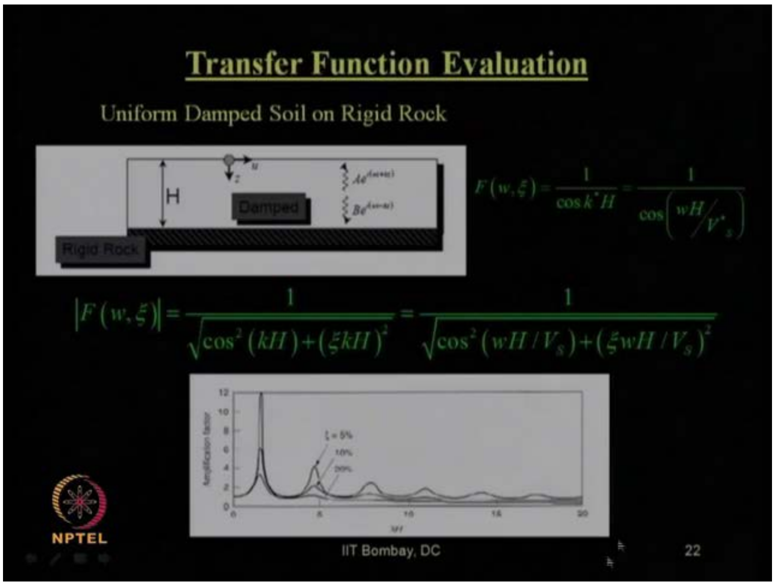

So, with that now if we want to plot the variation of this transfer function what we will get this transfer function now is a function of not only the frequency, but it is a function of damping ratio also which is involved in the k star which is involved through this V s star. We can express it like this. So, how we can express this transfer function that we have seen in the fundamentals or basics of vibration theory. If you recall the module on vibration theory or if you go though my other video lecture on soil dynamics where we have already discussed about dynamic magnification factor, if you go through those lectures that module 2 of soil dynamics lecture or module 2 of this lecture, you will find we can express these transfer function now in this format 1 by root over cosine square of k H plus eta k H whole square which if you want to express in terms of V s can be 1 by root over cosine square omega H by V s plus eta omega H by V s whole square.

Now, for different values of eta you can see in this picture. Now, we have plotted, these are the values of transfer function or amplification factor by the value k H. Why we mention the transfer function can also be called as amplification factor. Please note it down.

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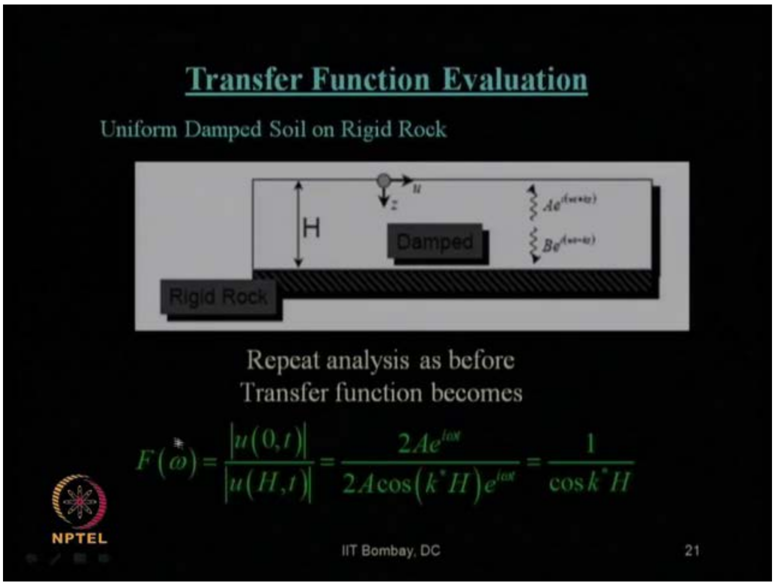

Transfer function can be mentioned as amplification factor also because if we go back to the definition of it, it is the ratio of displacement at ground surface to displacement at bed rock level. That means how much it gets amplified, that is why the transfer function itself gives you the value of amplification ratio.

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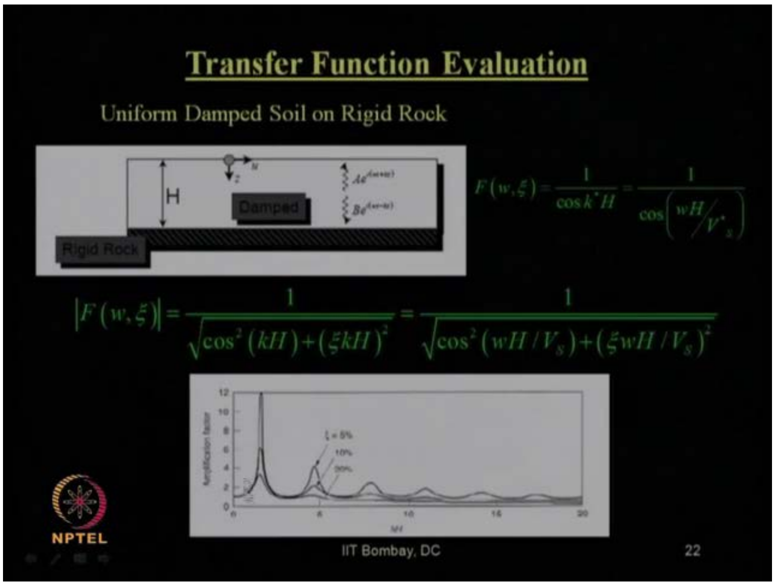

So, that is why the amplification factor at different values of k H, k H 0 means again it will become 1, this transfer function, but at different values of eta that damping ratio, the different peaks you will get at different values of k H. That means it never goes to infinity for a positive value of eta. It goes to infinity only if eta equals to 0 that is when there is no damping present.

So, that is why the profile will become now like this and if the damping ratio of the material, damping ratio of the soil layer increases form 5 percent to 20 percent through 10 percent there will be decrease in the peak values of this amplification factor at different values of k H. Can you see that? So, it depends very much on the damping ratio of the soil material, that what value of amplification you will get. And also at what value of k H you are computing it, that is what value of H by V s you are having for your particular site.

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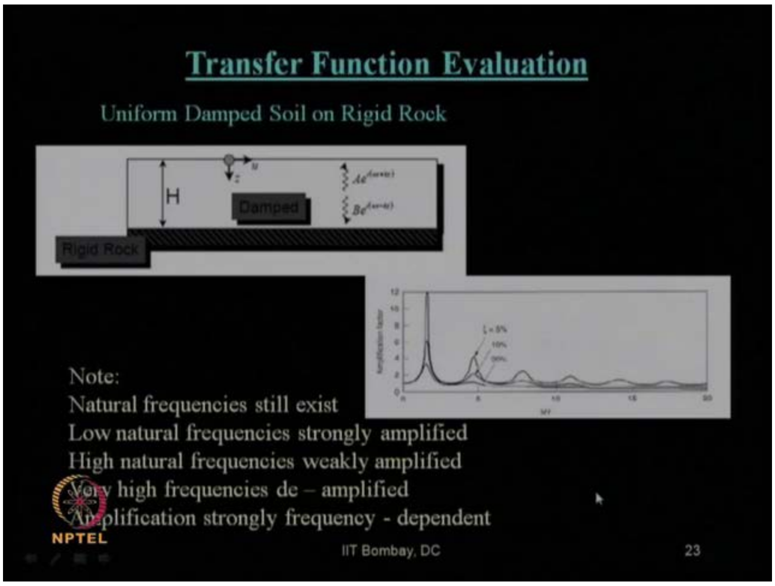

So, for uniform damped soil on rigid block already we get the solution like this, we can note that natural frequency still exist because we know the, as per the definition of natural frequency whether it is damped or undamped natural frequency remains same. So, natural frequency still exist, low natural frequencies they get strongly amplified. What does it mean if we look at the natural frequencies, lower values get strongly amplified than at the higher values. Higher values of k H obviously will give you higher values of omega n's.

That means lower natural frequency get strongly amplified, whereas higher values of natural frequencies are weakly amplified. So, if you have a higher and higher natural frequency the amplification will be lesser. So, this is one very important observation one should made here because we have already discussed that earthquake can have different ranges of frequency. When earthquake is coming and hitting at a particular site, we have seen between 1 hertz to 2.5 Hertz various frequencies it can have.

So, if you have a lower range of earthquake frequencies then chances of getting that frequency amplified is more than if you have a higher frequency of earthquake. We will see later on few examples that wherever there are high frequency earthquake, chances of amplification is lesser in that way it is helpful in a sense that frequency is large for the earthquake motion, but problem related to amplification or soft soil condition is lesser compared to if you have a low frequency earthquake that will have a more chance or more amplification.

Where as you can see beyond a certain value of k H there is no amplification, there is inverts of it that is de-amplification, that is instead of increasing it above 1 it will be below 1, that transfer function will be less than 1, where that condition occurs at very high frequency it gets de-amplified. That means if you have any earthquake, excitation, frequency say very high value of the frequency range, in that case you can probably expect that the ground surface motion, the frequency what you are getting that will get de-amplified. Whatever displacement you will get, whatever acceleration you will get that will get de-amplified or lesser than what you have at the bed rock level.

So which is good for a site where we want to construct some structure. De-amplification is always better because it is reducing the effect of earthquake when it comes to the ground surface or close to the over structure compared to long depth or deep below the bed rock level. So, amplification strongly frequency dependent which is quite clear from this result, amplification is very much dependent on this frequency.

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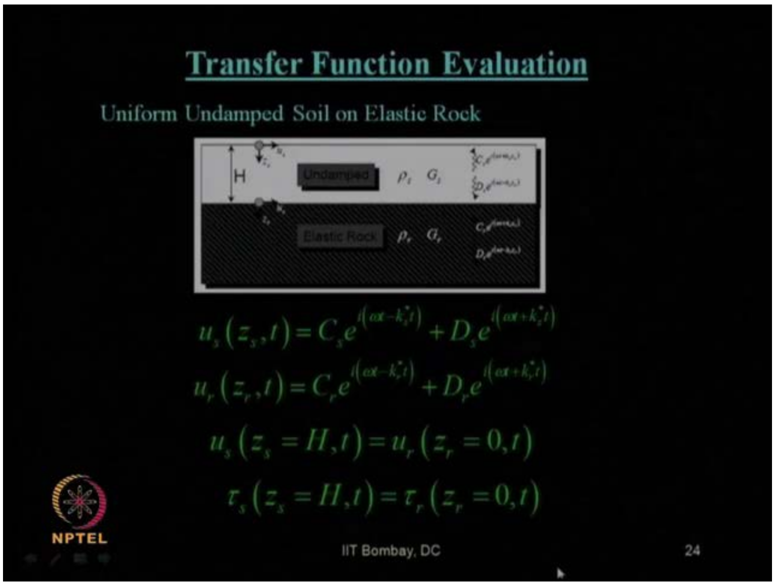

Now, let us come to a case where uniformed undamped soil we are considering on a elastic rock, on an elastic rock instead of rigid rock. That is now the soil condition and rock condition is something like this, the soil condition is homogeneous soil, uniform soil, undamped the basic case first we are analyzing with the soil density rho s and soil shear modulus G s, one dimensional wave propagation again we are considering and this is the limit of or access system of u s and z s. Whereas for the rock, elastic rock, the rock density is rho r, rock shear modulus is G r and this is the one dimensional wave propagation in the rock and the axis system or the coordinate system for the rock profile u r and z r as shown in this picture. So, what will be the solution for soil layer and rock layer for any wave propagation and their displacement function? The displacement function for soil profile will be something like this u s is a function of z s and t will be C s e to the power i omega t minus k star t plus D s e to the power i omega t plus k star t. Whereas for rock it will be in terms of z r and t so corresponding coefficients can be let us say C r and D r which we need to find out form the boundary conditions.

Now in this case what boundary condition we need to use, let us see. u s at the value of z s equals to H that is the soil layer displacement at depth H should be equal to the displacement at depth of 0 as far as the rock layer is concerned because displacement compatibility has to be maintained. So, that means u s at this level and u r at this level should be equal. That is what is mentioned over here. Another boundary condition we can write it like this, stresses the shear stress at this level form soil and shear stress form this rock layer at this point should also be equal for stress compatibility. So, this is the condition.

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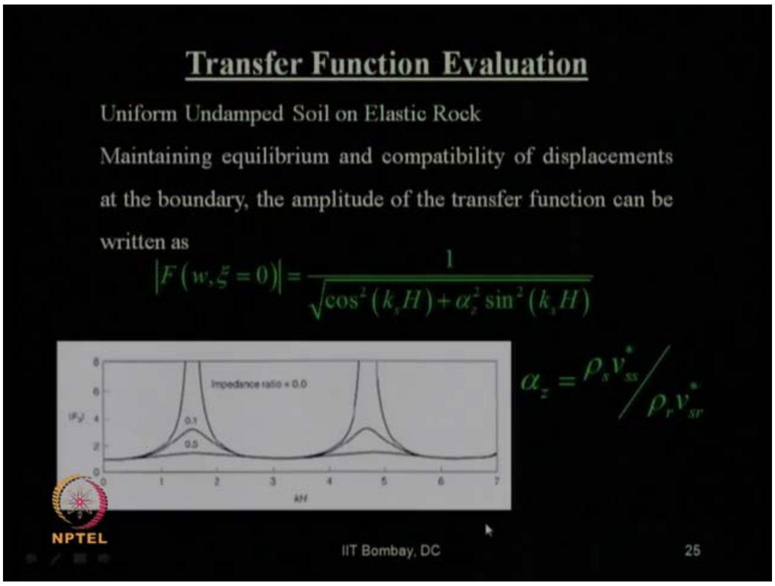

With that uniform undamped soil on elastic rock maintaining the equilibrium and compatibility of displacement at the boundary, the amplitude of the transfer function now can be written as this way F of omega, now eta equals to 0 because we are considering undamped case. If it is damped then it will become non 0 as we have already discussed.0

It can be expressed in this form 1 by root over cosine square k s H plus alpha z square sine square k s H where this alpha z is nothing but specific impedance ratio. Already we have learnt this parameter in the previous, one of the previous module which is nothing but ratio of specific impedance of upper layer to the lower layer where form the wave is coming. So, the specific impedance ratio alpha z will be rho s by times V s that is shear wave velocity in soil star divided by rho r, that is density of rock by V s r that is shear wave velocity in rock media star.

See if we plot how the transfer function will look like for different values of k H. Again for k H equals to 0 it should be 1 as we can see form this equation, whatever be the value of this ratio alpha z, but for other values of impedance ratio it will have some finite value even though the k H value reaches, let us say pi by 2, it will not go to infinity. It will go to infinity only if the impedance ratio is 0. Otherwise if this term is non 0 still you will get some value. So, obviously you will get a finite value that is why depending on impedance ratio between the 2 layers that is soil layer and rock layer, if impedance ratio increases your amplification decreases.

Can you see that? It is impedance ratio 0 means infinity, if it is 0.1 peak is here, if it is 0.5 peak is here. So, what will be the impedance ratio for a soil to rock, obviously it will be less than 1 always because rock will have much higher value of specific impedance than soil. So, it is always less than 1, but you can see as the value reduces from 0 to 0.1 to 0.5 there is a significant decrease in the transfer function or the amplification ratio which is another important finding when we want to talk about the behavior of different soil layers over a rock layer.

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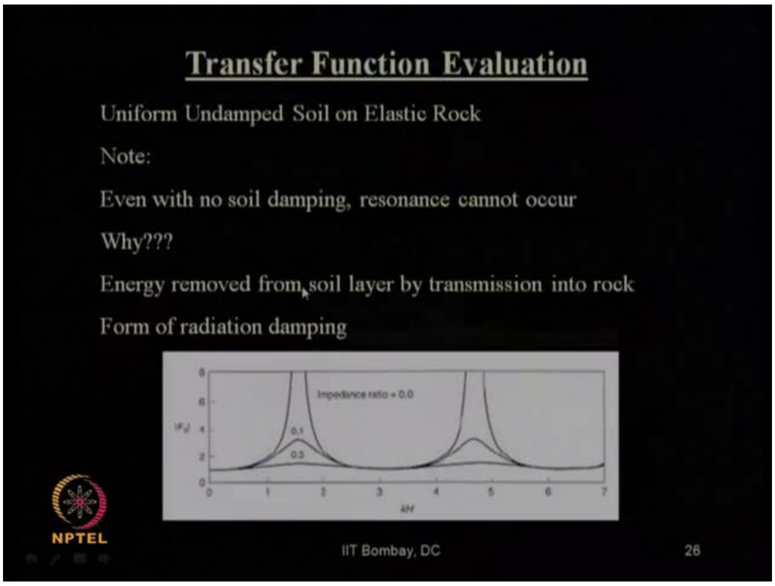

Now, for this once again we can see this uniform undamped soil on rock layer even with no soil damping, that is remember we have not considered any soil damping. We considered undamped soil media, still resonance cannot occur, still this resonance is not occurring. We are getting some maximum value, but still it is not going to infinity that is what it means. Why it is so because energy removed from soil layer by transmission into the rock in the form of radiation damping. That is in the first case when we considered uniform undamped soil layer over a rigid rock, we did not consider the radiation damping.

Now when we are considering the rock layer elastic properties of the rock, here you are considering that effect of radiation damping which helps to reduce this amplification or reduce this transfer function value. That means even if you do not consider the viscous damping or material damping of the soil layer, there is some other damping which is radiation damping, this we have mentioned earlier also in one of the module that we have 2 damping criteria, one is material damping or viscous damping another is radiation damping. So, here you can see the influence of both. In this case only we are showing the influence of radiation damping and when we will consider the damped soil we can see the influence of both radiation damping as well as viscous damping or material damping.

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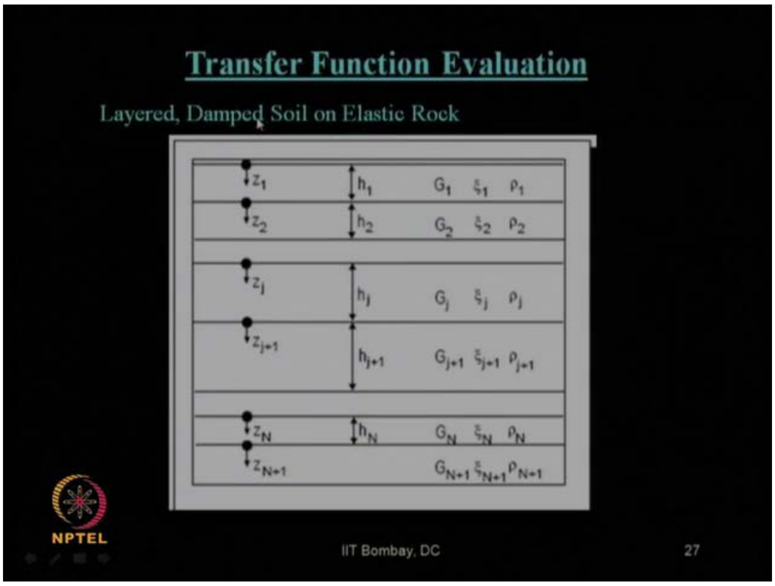

Now, for transfer function evaluation in a layered soil that is let us consider the most generalized case of layered soil and damped soil on an elastic rock. So, that type of layer what we can do when we are for doing the ground response analysis for that type of layered soil, we will have different layered thickness of soil like h 1, h 2 like that up to say h n and h j is any j eth layer, h j plus 1 is any j plus one eth layer and for individual layer the material properties which are necessary for our ground response analysis are like shear modulus variation damping and density.

Then z we are measuring for each layer from the starting of that layer, that means for layer 1 we are starting measuring z form the ground surface, for layer 2 we are starting measuring z from the interface of layer 1 and layer 2 like that all the z have been shown over here. So, now how to estimate the transfer function for any layer, say any j eth layer, we will look at the next slide.

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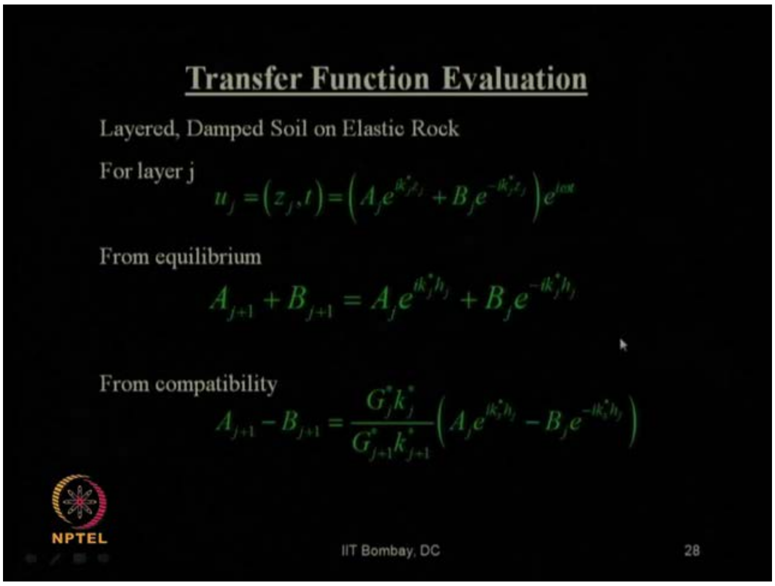

So, for any layer j the displacement function, that is u j as a function of z j and t should be expressed like this because that solution for the displacement we have already seen in terms of coordinate and time. Now, for equilibrium what we can say that whatever be the displacement function equation for j eth layer and j plus one eth layer they must be compatible. That means if I go back to this slide, that is from the j eth layer whatever displacement we are getting at this point and whatever displacement we are getting form this j plus one eth layer at this point must be equal. That means displacement compatibility has to be maintained and also equilibrium has to be maintained. That is the stress condition also calculated for this j eth layer at this depth of h j and the stress condition at j plus one eth layer at this level of 0 at the starting point of this j plus one eth layer should be equal. So, the equilibrium and the compatibility will give us these 2 relationships by using which we can get these constants.

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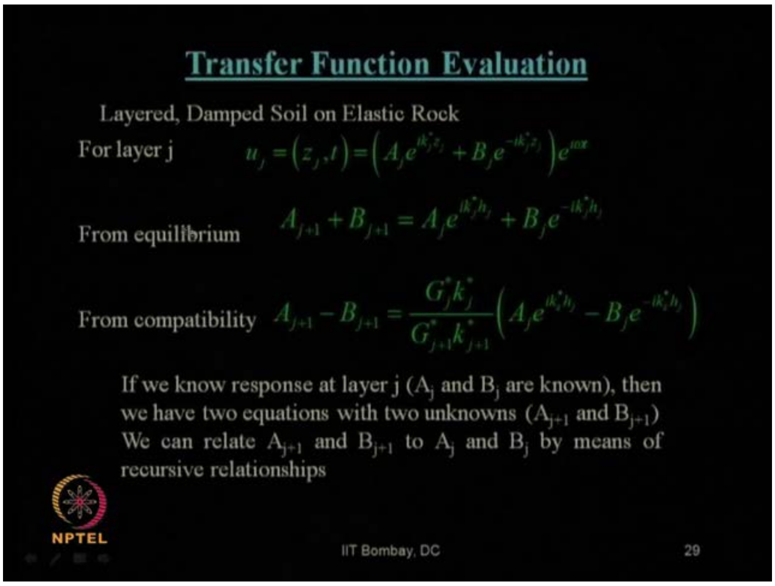

Now, let us look at further if we know the response at layer j, then what will be known to us like A j and B j will be known to us. How this response at layer j will be known because let us look at this slide once again.

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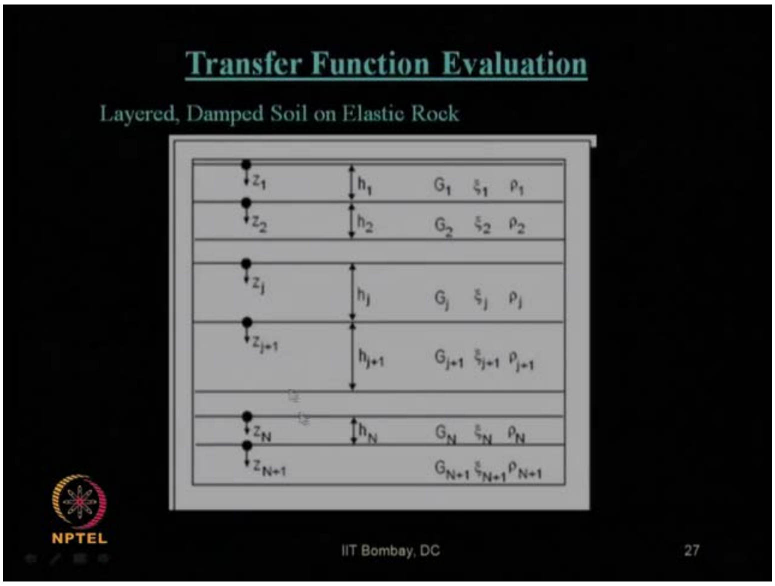

Suppose, what are the possibilities in which way we will know the response of j eth layer. One is if you consider the free boundary the stress condition should be known. Another criteria is when the bed rock motion you are considering, if you know the bed rock motion property form that bed rock motion acceleration time history you will get the displacement time history also. So, that will give another boundary value known. So, using those boundary values which are known what one can get, one can get the other two unknowns for any other layer and it is a successive process, like for one layer to another layer you can transfer this equation like you can write down. Suppose if you have n number of layers or n plus 1 number of layers for each layer you will have 1 equation that will lead you to n plus 1 equations. And then starting from any of the two boundaries either the ground surface or at the bed rock motion which is known to you, you can get the other intermediate layers, this constant values. Is it clear?

So, how we can do this exercise obviously for a multilayer system this exercise need to be done using a computer program and the commonly used computer program to do this ground response analysis are shake s h a k e, that is a very well known computer program which can do this ground response analysis for layered soil like this, another computer program is known as deep soil d e e p s o i l. We will come to the application of those software very soon when we will talk about the case studies.

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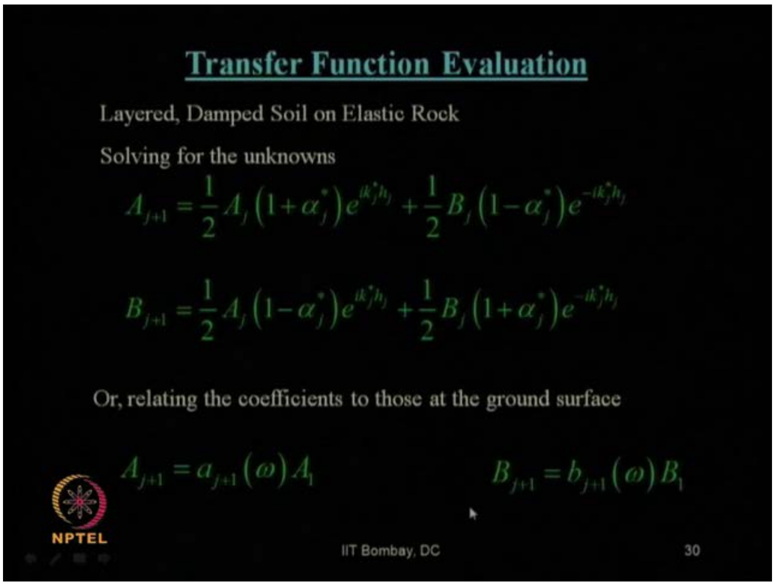

Now, solving for this unknowns what we can get, we can express this A j plus 1 and B j plus 1 in terms of A j and B j which we said you should be known form one of the boundary conditions. Now or relating the coefficients to those at the ground surface as I have mentioned, at ground surface this coefficients should be known, expressed in this format.

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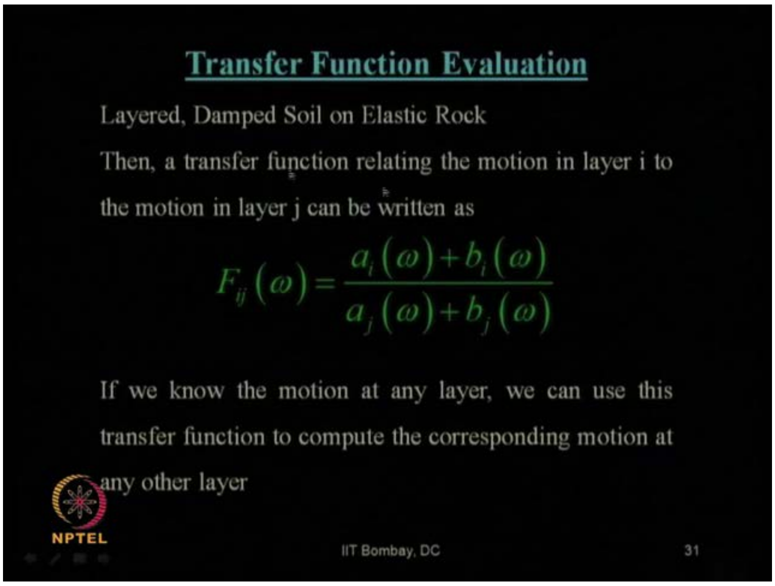

So finally, a transfer function relating the motion in layer i to the motion in layer j can be written in this form, like earlier transfer function we were expressing F of omega right and we said for damped layer it will be F of omega and eta, that is a function of natural frequency and function of damping ratio of the material. Now, if we want to generalize this transfer function from any layer i to any other layer j, that is earlier transfer function definition we have used for subsequent two layer, right. Layer 1 to 2 like that, but here we are referring to transfer function from layer i to j so in that case the equation can be written in this form a i omega plus b i omega by a j omega plus b j omega where these are individual displacement functions of that particular layer. That is the definition of transfer function still remain same it is nothing but the ratio of the displacement between the two layers.

So, it is multiplayer or the factor by which we need to multiply the known displacement to obtain the unknown displacement of another layer. So, if we know the motion at any layer we can use this transfer function to compute the corresponding motion at any other layer, that is what it means.

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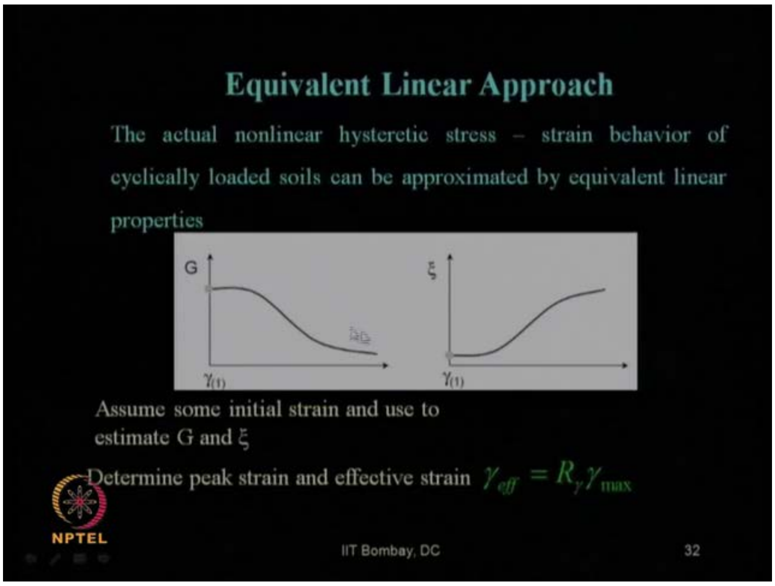

So, now let us see how this equivalent linear approach of this ground response analysis is carried out. So, the actual non-linear hysteretic stress strain behavior of a cyclically loaded soil can be approximated by its equivalent linear properties.

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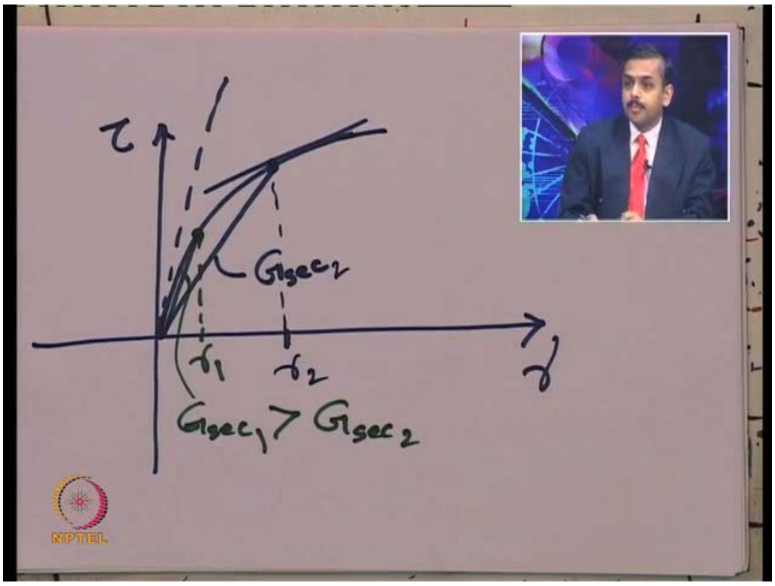

Like as we have already discussed earlier that for defining the equivalent linear analysis, the cyclically loaded soil we consider the stress strain behavior of this, this is the G max, this one is G sec and this one is G tan, right that we have already seen. Now, we are using this slope G secant modulus which gives us the equivalent linear approach. So, what it means that for a G value which is changing with respect to your chosen value of cyclic strain, as you can see if this point is here G sec is this much, if your, if I change this point to say gamma value of this one, say this is gamma 1 and this is gamma 2 that is at two different cyclic strain level, we are considering. What we can see here the G sec value will be this much.

So, obviously the G sec 1 is higher than G sec 2, right, is it clear. So, what way we can express this in the previous one of the earlier module on dynamic soil properties what we have discussed.

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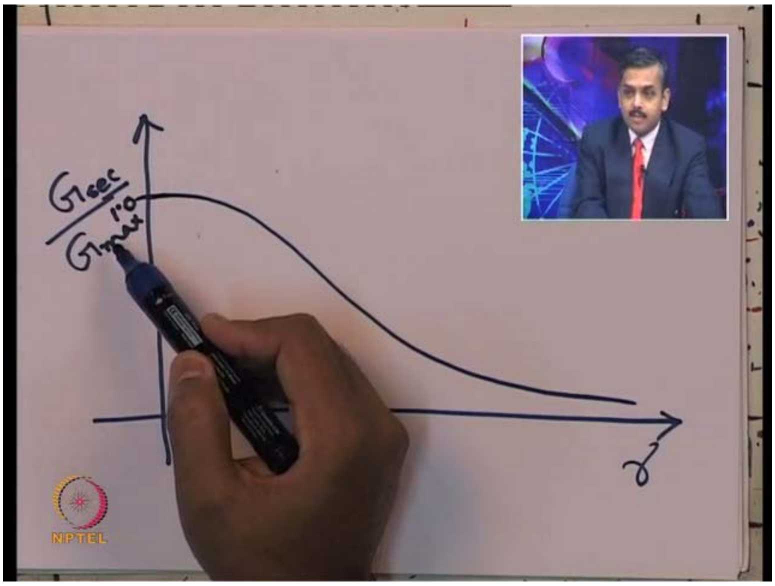

Let us recap it that when we plot this cyclic share strain and this ratio of G by G max in this case we actually talk about G sec by G max, what we have seen it will be following some trend like this, that we have mentioned as modulus reduction curve, right where it will start this will obviously start at the value of 1. Why 1 because at low cyclic shear strain obviously G sec will be nothing but G max.

So, if I look at here, this is our G max. So, at very low stain this G sec and G max is nothing but same. So, that is why their ratio will be 1 at low cyclic shear strain and as the shear strain increases, there will be a decrease in this ratio below 1 as you can see from this G max is always maximum. G max is greater than this one G sec one greater than G sec 2. So, as we are increasing our gamma from this initial value to gamma 1 then gamma 2 obviously this ratio of G sec by G max is keep on decreasing and that curve will look like this. This is modulus reduction curve.

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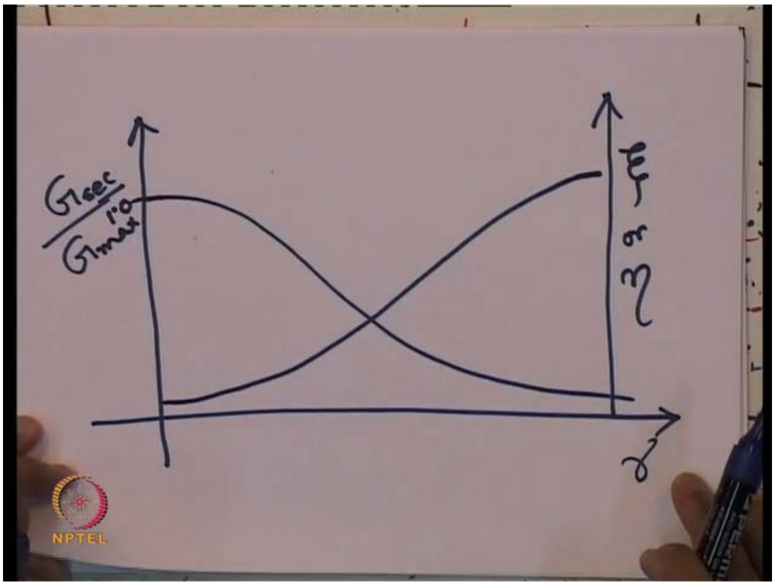

Similarly the same behavior if I want to plot on this side, the damping ratio this or sometime we use this expression also eta, if I want to plot the damping ratio it is having a reverse strain that means at low cyclic shear strain, it will have low damping ratio and as the damping ratio increases or I should say as the cyclic shear strain increases damping ratio also increases. So, this we have also discussed earlier in our dynamic soil properties that module.

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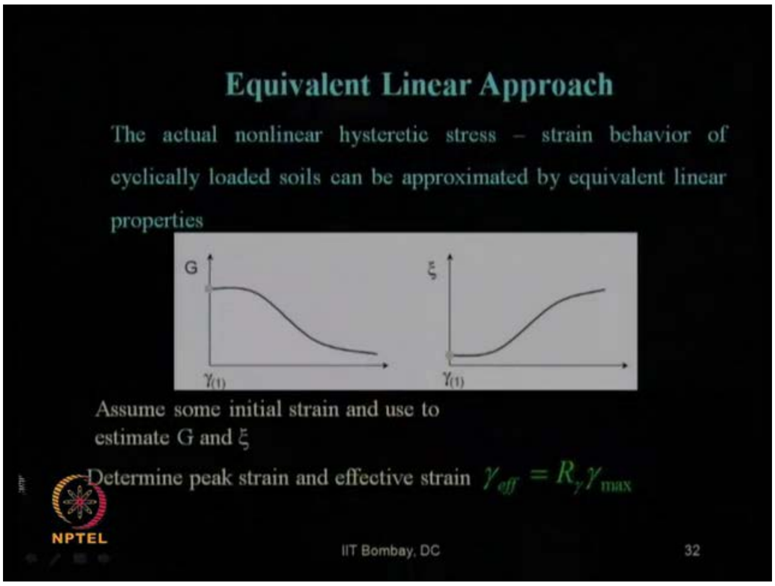

Now, let use look at this slide over here which shows this equivalent linear property so when we are talking about this axis G, it is basically not G it is the ratio of G by G max versus gamma. We will have different values of gamma starting form gamma 1 initial to different value gamma 2, gamma 3, gamma 4 wherever you want to do this analysis of ground response.

Similarly the damping ratio as I said it increases as in there is an increase in the cyclic shear strain. So, how to start this analysis now that is the question like we do not know at which value of cyclic shear strain, this material property is actually existing so we have to find out that value of gamma 1 where all the results converge. What does it mean it is a trial and error procedure in that computer program which it can do by starting with some initial strain. So, we always start with some initial value of strain, very small value of strain you can select as a first step or sometimes we call it as a reference strain by considering that one can estimate the corresponding value of G sec and eta, that is corresponding value of G and corresponding value of damping ratio for that particular strain level, low strain level. So, now using that one need to find out determine the peak strain and effective strain also.

Peak strain means what is the maximum value of cyclic strain the material or the soil can reach under a particular earthquake motion. So that gamma max and this an operator R gamma which will give you then effective strain gamma effective. I will show that how we are applying in the practical example or actual example.

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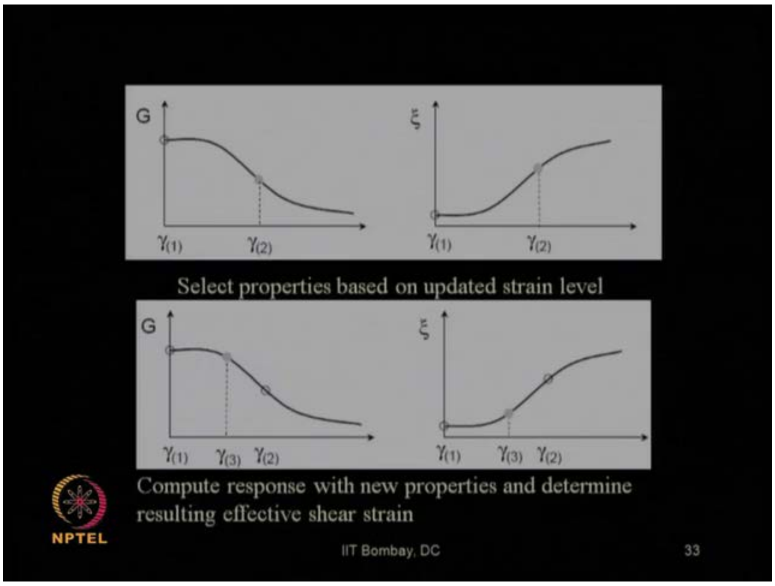

Now, when we are selecting this gamma 1 corresponding G value we are getting, next trial what we need to do by using this initial values we got some result. Now, by using those results of G and eta you will finally get another shear stain. Now, that shear strain obviously will not match with your initial chosen shear strain.

So one is assumed shear strain another is obtained shear strain. So, what you need to do? You need to go for trial and error procedure. Now in the next level you increase your shear strain gamma 2 and take that corresponding value of G and eta at gamma 2 level and do the repeat analysis until this value converges. So, that is what it says, select properties based on the updated strain level gamma 2, in next level compute the response with the new properties and determine the resulting effective shear strain and finally wherever the results converge that will give you the final effective strain and the final values of G and eta. So, it is an iterative procedure or trial and error procedure.

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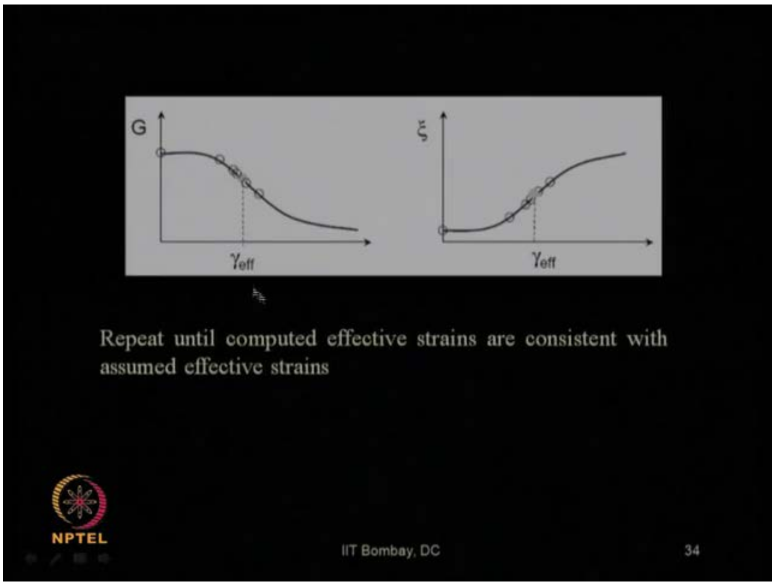

So, this where it converges see repeat until the computed effective stains are consistent with the assumed effective stress. So, that final value where the results converge that is nothing but you gamma effective. And that value of G and eta is our final value which is responsible for that particular ground response analysis subjected to a particular input motion. Remember it depends on your input motion also, we will see that very soon in an example.

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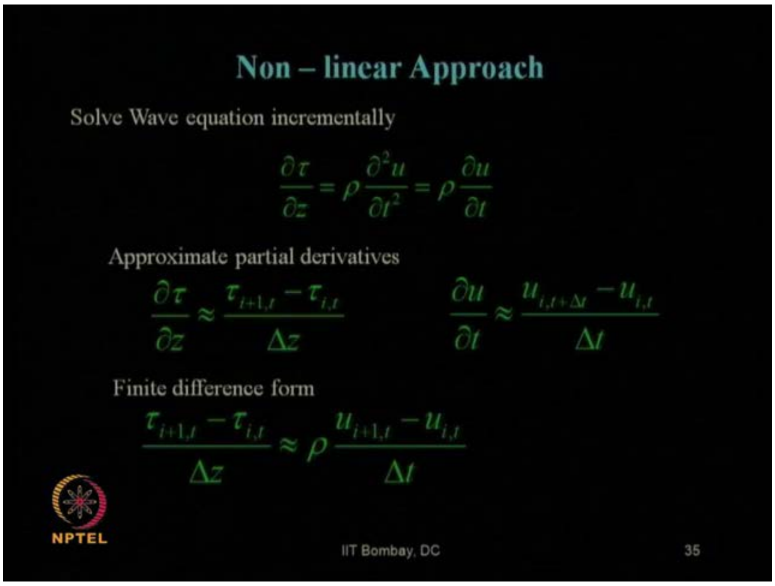

That is what about equivalent linear approach. Now, let us come to non-linear approach. In non-linear approach what is the difference? In this case you are no longer going to use that G secant, you remember now you need to use the G tangent modulus. So, to do that first thing we should start with solve the wave equation incrementally, that is in the relationship of this what we have already learnt here tau versus gamma relationship.

We have to study now incrementally this small, small, small segments we have to study so that incrementally you have to solve the wave equation as shown over here like d tau by d z is equals to rho times d del 2 u by del t square equals to rho times del u del t based on the displacement function approximate partial derivatives you can express in this form that is incremental form, that is we are going to now start doing these differential process of wave equations in numerical way by representing them in different numbers of segments. That is i plus one eth segment and i eth segment over a small interval of delta z and delta t.

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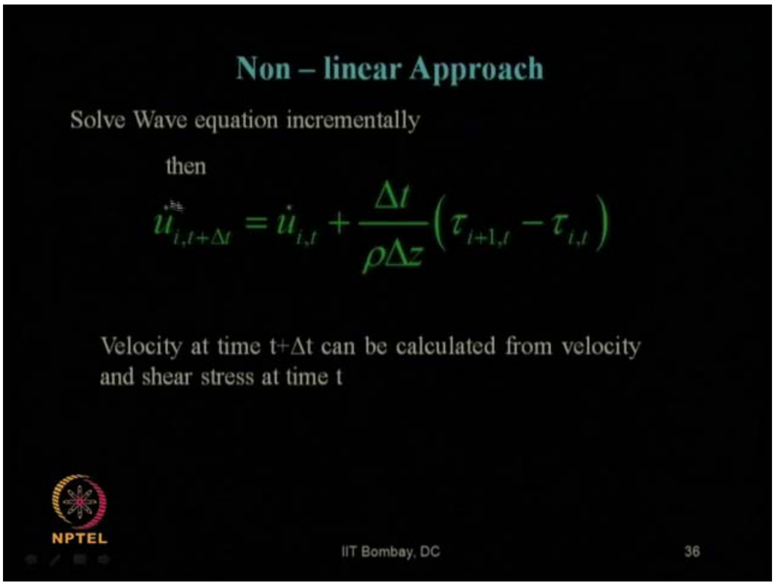

So, then you can get the velocity function also expressed like this.

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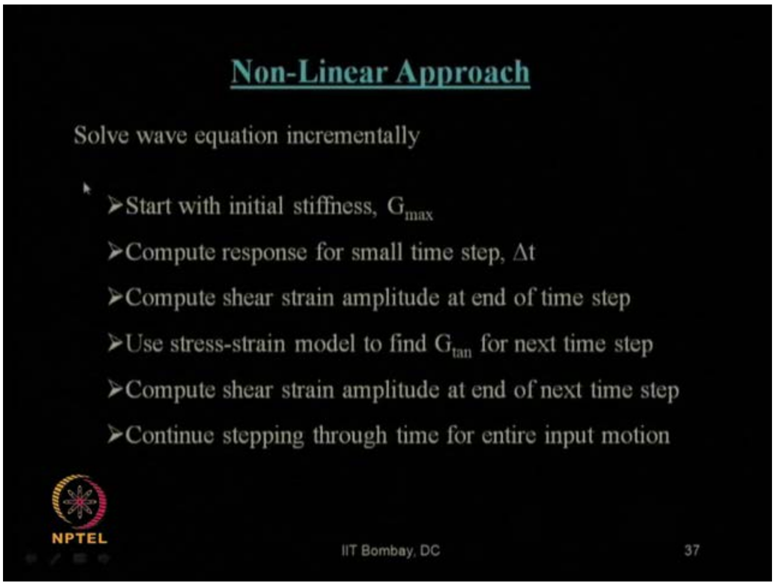

Now, what are the steps let us see. Solve the wave equation incrementally. To do that we have to start with the initial stiffness G max because remember this G max is nothing but it is also a tangent modulus. It is called initial tangent modulus as we already learnt in our dynamic soil properties.

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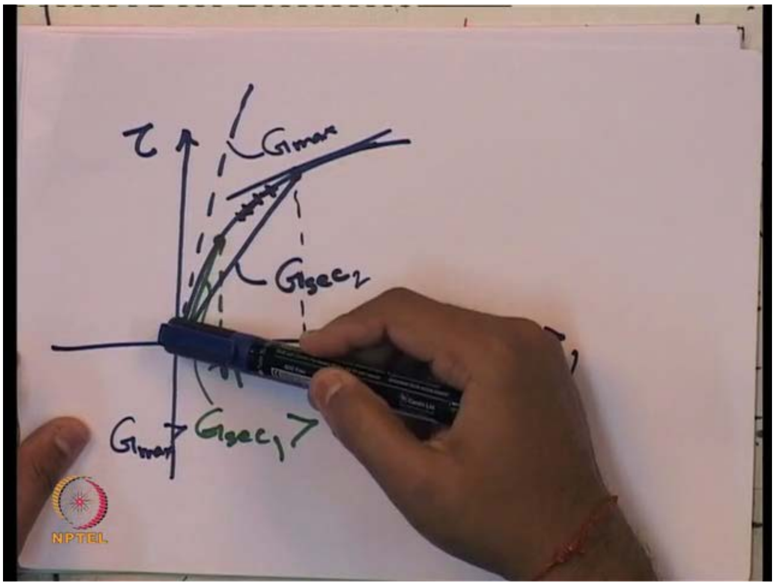

This G max is nothing but it is also an tangent, but it is initial tangent. So, that is why our analysis will start from this initial point so from here where we have to take this G max. So, that is what we are doing over here. Start with initial stiffness G max, then compute the response for small times step delta t, that incremental time step. Compute the shear strain amplitude at end of each time step. Use that stress strain model to find G tan for next time step. Remember at next time steps when we are getting another strain level now instead of talking G max it should follow the exact this variation of tau versus gamma incrementally.

So, we are now getting individual G tan for each individual increment. So that is what use the stress strain model to find that G tan for next time step, compute the shear stain amplitude at end of next time step and continue this step through time for entire input motion, that is the entire input motion of earthquake whatever you are using for that entire time you divide it into number of small time step and repeat it the process for each time step.

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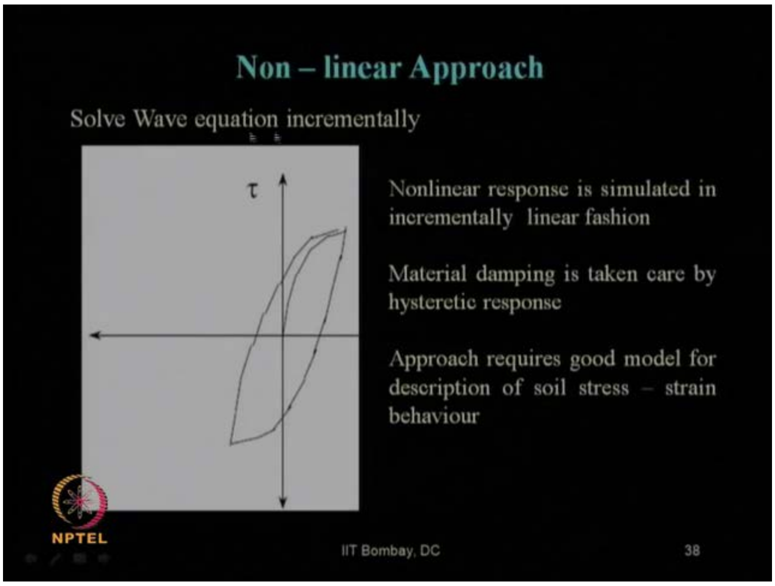

Solve this wave equation incrementally what you will get finally, the tau versus gamma relationship that is stress versus strain, shear stress shear strain relationship you will get something like this, that is from the incremental one. That will give you non-linear responses simulated in incremental linear fashion that is piecewise small, small, small, small linear, but when you join them you are getting non-linear fashion. So, material damping is taken care of by the hysteretic response. That hysteretic response if you take this obviously your material damping is well taken care of and approach requires good model for description of the soil stress strain behavior. That is when you are doing this you need to use a proper soil model.

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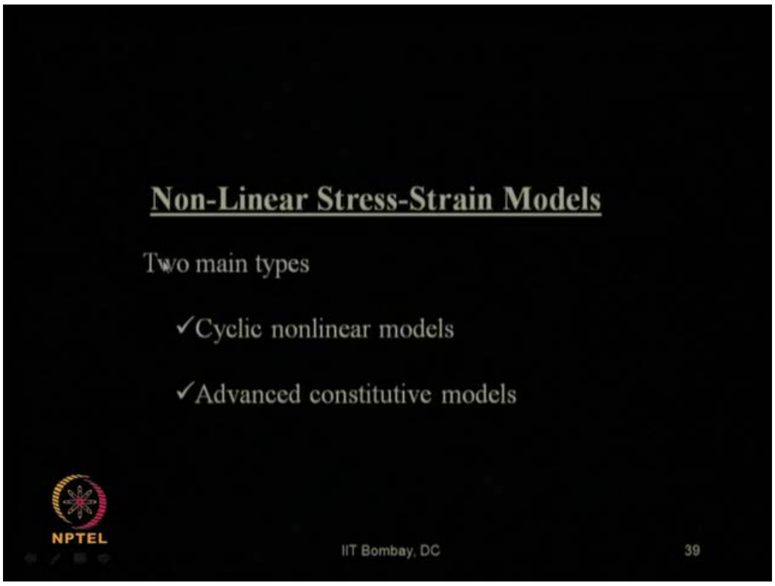

Now to do that there are two major soil models, non-linear stress strain soil models which are in use for this ground response analysis, one is known as cyclic non-linear model and another is called advanced constitutive model. Now, what is cyclic non-linear model?

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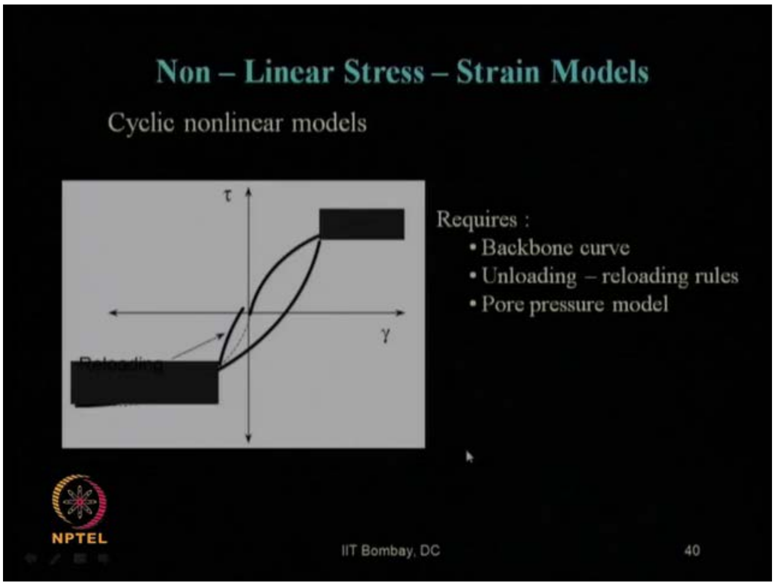

In cyclic non-linear model this is the reloading part, this is the loading part. You can see what are the things you require, you require this back bone curve, this is nothing but the back bone curve. Now unloading reloading rule must be followed here and pore pressure model can be modeled in this cyclic non-linear model. That is known as from your suppose if you are carrying out in the laboratory cyclic tri axial test, you will get for your soil this kind of behavior based on your number of loading and unloading. So, that curve will give you nothing but the back bone curve and loading unloading data also you will get that will also come along with the pore pressure model, that model can be used as one of the stress strain model when you are starting your ground response analysis. So, that is cyclic non-linear model, that is the simpler one.

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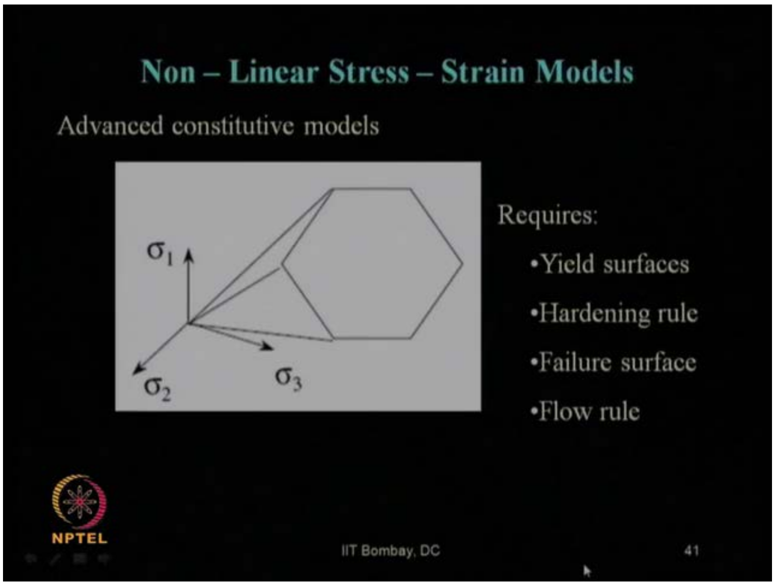

Now, a higher model is called the advance constitutive model. When you are using advance constitutive model suppose if we have this three orthogonal axis direction sigma 1, sigma 2, sigma 3 plain individual principal strain plain, we require the yield surface. How the yield surface is getting formed in this constitutive model. Now, strain hardening rule should be known, failure surface should be known and the flow rule should be known. So, these are the characteristics of any constitutive model and when you are doing the non-linear stress strain cyclic shear stress versus cyclic shear strain model you have to use this advance constitutive model. There are various advance constitutive model which are available in this standard deep soil or shakes software.

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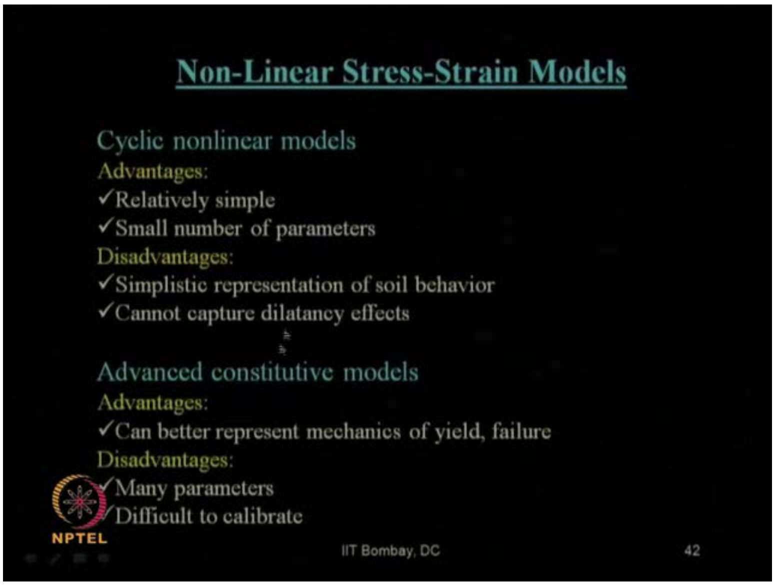

Now, a comparative study between this cyclic non-linear model and advance constitutive model, as I said the advantage of cyclic non-linear model is, it is relatively a simpler model because you can easily get that how the behavior of stress strain non-linearly based on your cyclic tri axial test or cyclic other lab test result. And it is small number of parameters are involved in that model, whereas the disadvantages of this models are simplistic representation of the soil behavior, why simplistic? Because here you are not aware about what flow rule is taken care of while stress hardening rule is taken care of all these issues and cannot capture the dilatancy effect that is another drawback of this cyclic non-linear model.

Whereas in advance constitutive model advantage is it can better represent the mechanics of the yield and the failure all together, but the disadvantages obviously several parameters are involved in this advanced constitutive model, and it is very difficult to calibrate to a particular soil profile using this advanced constitutive model. So, once you are going for advanced constitutive model in your ground response analysis, obviously it is expected to give a better result, but it will lead to an complex analysis of your ground response study. So, with this we have come to the end of today’s lecture, we will continue further in our next lecture.