Geotechnical Earthquake Engineering
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Module - 8
Lecture - 31
Site Response Analysis

Let us start our today's lecture on NPTEL video course geotechnical earthquake engineering. In the previous lecture we completed our module 7 which is seismic hazard analysis, so let us have a quick recap what we have learnt in our previous module. In the previous lecture while discussing module 7, we have gone through an example problem that is a case study on seismic hazard analysis for the state of Gujarat in India, this is a part of a Ph. D thesis of Dr Jaykumar Shukla at IIT Bombay.

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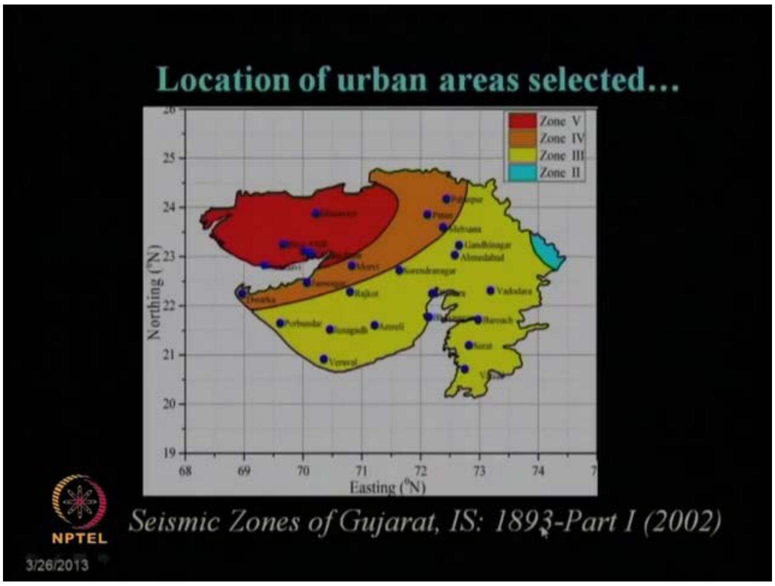

So, 25 urban cites or areas were selected for the development of seismic hazard analysis using both deterministic and probabilistic approach. This is the map of Gujarat state in India including the seismic zonation map as per our Indian seismic code IS 1893 part 1 of the year 2002 version. So, the Gujarat, it is the only state in India which is having all the four zones mentioned in the IS code, that is zone 2 the least vulnerable one and zone

5 the most vulnerable one, accordingly the color shades are showing over here. So, we have selected in starting form the most vulnerable zone that is zone 5 to zone 4 and 3 only the least vulnerable area zone 2 we have not considered in this work.

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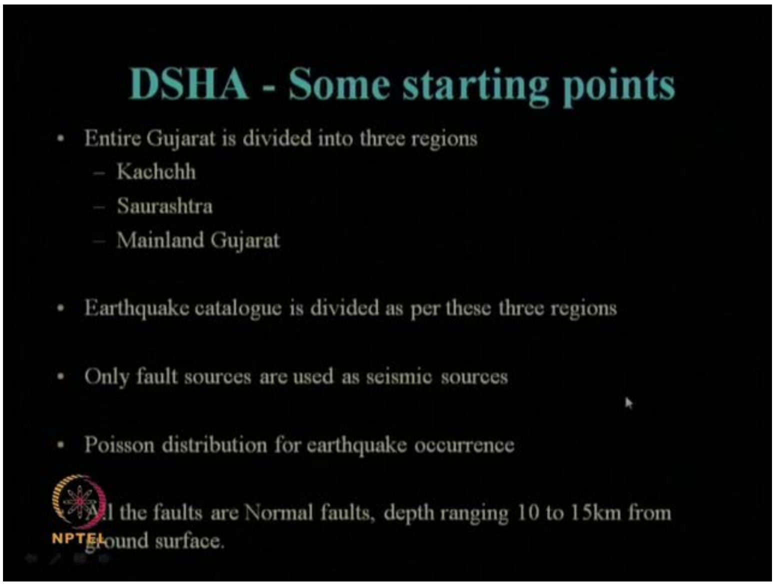

In the DSHA as I have already mentioned the entire Gujarat was divided to conduct the DSHA in three regions - Kachchh region, Saurashtra region and mainland Gujarat. For Kachchh region it is in seismic zone 5, that is the highest seismic hazard zone. Then Saurashtra region which is in zone 4 and then mainland Gujarat which is in zone 3. Earthquake catalog also was divided as per these three regions and only fault sources were considered for the seismic sources. Poisson's distribution of our earthquake occurrence was taken care of and one assumption has been made in this study that all faults are normal faults with depth ranging between 10 to 15 kilometer form the ground surface that is typically the shallow earthquake events.

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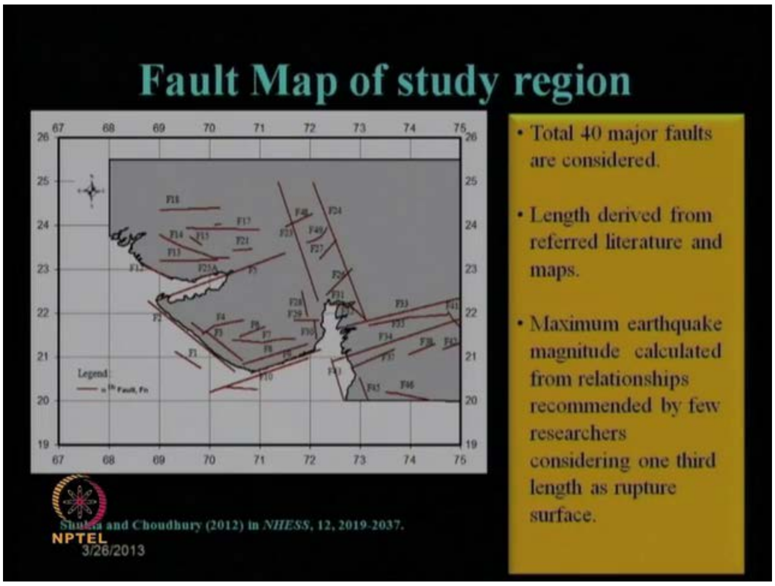

This is the fault map of entire Gujarat region and there are several numbers of faults, but only 40 major faults were considered in this analysis and they are length etcetera were obtained from the referred literature.

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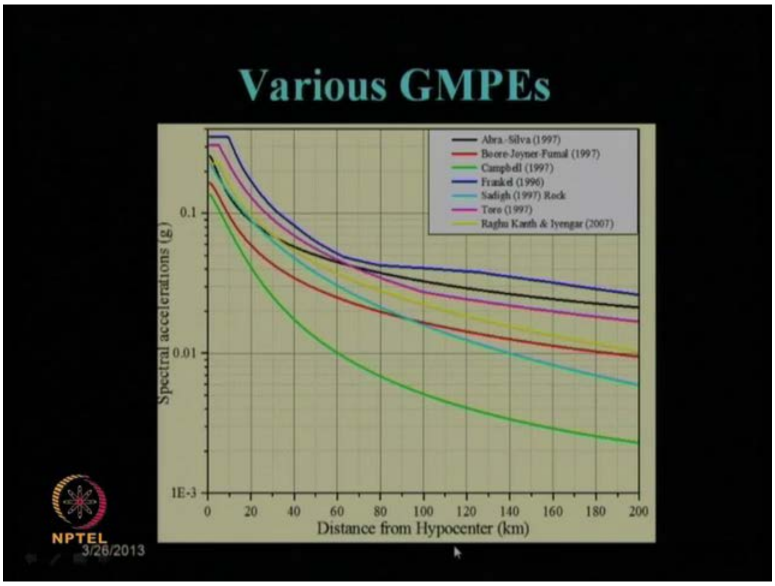

Then various ground motion prediction equations, total 7 ground motion prediction equations were used in this study starting from Abraham and Silva 1997, Boore et al 1997 to the Raghukanth and Iyenger 2007 g m p which is for the peninsular India.

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DSHA Results

Name of City/Urban Area	Location		Deterministic Seismic Scenarios (Controlling Fault/Magnitude-Distance pairs)					
			Short Period (0.2 sec) (two stored building)			Long Period (2 sec) (highrise buidlings)		
	N°	E°	Fault	M_w	Distance (km)	Fault	M_w	Distance (km)
Ahmedabad	23.030	72.580	F24	5.5	14.290	F17	7.0	171.240
Amreli	21.602	71.219	F6	5.5	15.500	F7	6.0	25.330
Anjar	23.112	70.023	F14	7.0	13.320	F14	7.0	13.320
Baroach	21.718	72.977	F33	5.5	7.190	F33	5.5	7.190
Bhavnagar	21.770	72.143	F30	5.5	8.000	F30	5.5	8.000
Bhuj	23.252	69.662	F14	7.0	14.500	F14	7.0	14.500
Dholavira	23.883	70.218	F17	7.0	5.000	F17	7.0	5.000
Dholera	22.248	72.195	F31	5.5	20.330	F28	6.0	33.030
Dwarka	22.241	68.966	F2	5.5	12.000	F13	7.0	89.240
Gandhidham	23.071	70.135	F14	7.0	15.960	F14	7.0	15.960
Gandhinagar	23.229	72.651	F26	5.5	31.570	F17	7.0	174.240
Jamnagar	22.466	70.066	F13	7.0	34.480	F13	7.0	34.480
Junagadh	21.518	70.456	F7	6.0	14.370	F7	6.0	14.370
Mandavi	22.833	69.346	F13	7.0	34.720	F13	7.0	34.720
Mehsana	23.598	72.380	F24	5.5	15.010	F17	7.0	145.610
Morvi	22.814	70.829	F14	7.0	59.090	F14	7.0	59.090
Palanpur	21.171	72.438	F24	5.5	10.250	F17	7.0	152.810
Patan	23.850	72.114	F49	5.5	15.600	F17	7.0	115.390
Porbundar	21.643	69.611	F2	5.5	11.640	F43	7.0	110.190
Rajkot	22.283	70.800	F13	7.0	83.280	F13	7.0	83.280
Surat	21.194	72.819	F34	5.5	17.760	F13	5.5	17.760
[illegible]	22.718	71.637	F25	6.0	30.570	F17	7.0	105.020
[illegible]	22.306	73.187	F24	5.5	22.190	F24	5.5	22.190
[illegible]	20.610	72.925	F37	5.5	19.000	F37	5.5	19.000
[illegible]	20.912	70.353	F2	5.5	12.000	F2	5.5	12.000

Shukla and Choudhury (2012) in *NHESS*, 12, 2019-2037.

Then the DSHA results were shown in this table. We have seen that it is not necessary whether short period or long period, that is in both the cases short period is typically referred to those buildings where it is a smaller number of heights of the building where as long period refers to those structures or those buildings where it is a high rise building. So adopting that what we have seen earlier in the previous lecture that few cities which are marked here in blue color like Ahmadabad, we can see the dominating fault for this deterministic seismic scenario are different when we are considering whether it is a short period effect or long period effect. That means the dominating fault for the low rise building is one where as it is another one for high rise building for the same Ahmadabad region. Whereas some cities or some places like Baroach, we have seen that the fault responsible for this short period and long period are same.

So, it determines that what type of buildings are getting damaged and which fault is responsible and what magnitude and what distance those are existing form the site can be obtained from this deterministic seismic hazard analysis.

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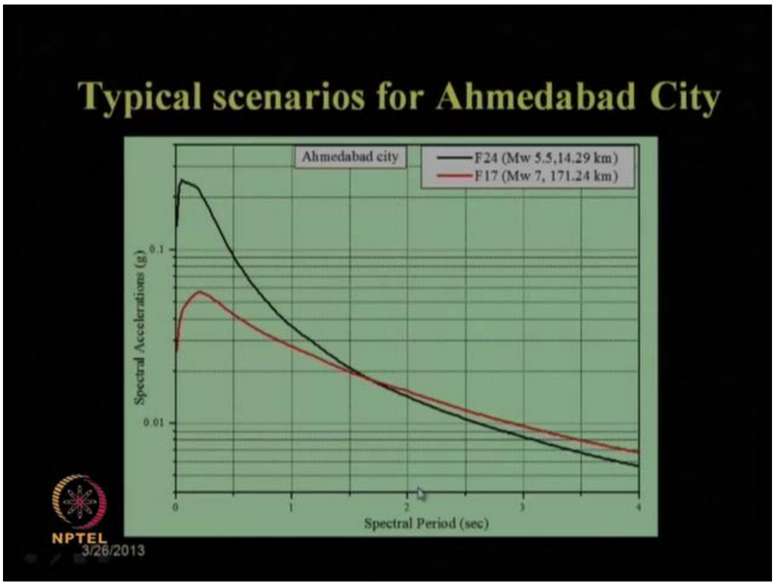

And so the typical scenario for Ahmadabad city has been elaborated over here, through this spectral acceleration verses spectral period plot as we have already mentioned this red color line shows the effect on the long period, where as this black color line shows the effect on the short period.

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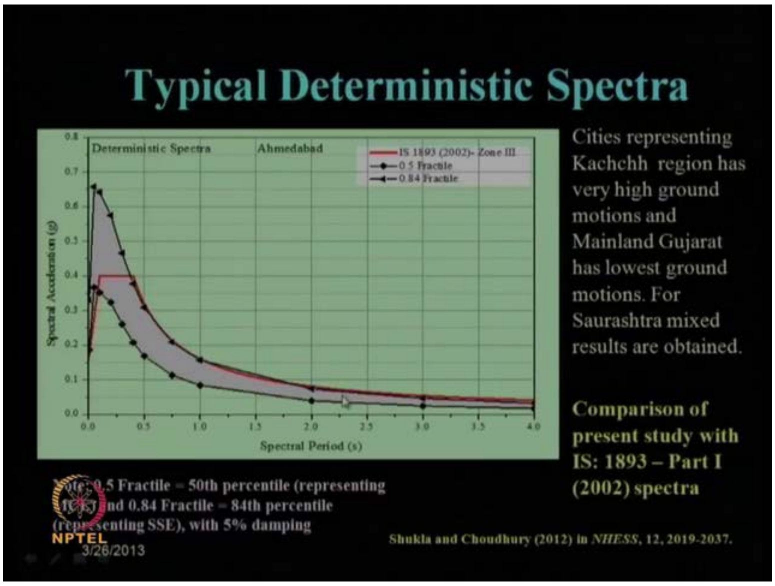

Typical deterministic spectra for the Ahmadabad city is compared again with the observations or the method proposed in the IS code, that is what is the spectral acceleration versus spectral period plot. We can see for zone 3 this is the typical plot as shown in the red color and we have compared that with respect to 0.5 Fractile and 0.85 Fractile. What are those Fractile I have already explained in the previous lecture and the details of this study can already be obtained which are available in the journal this one.

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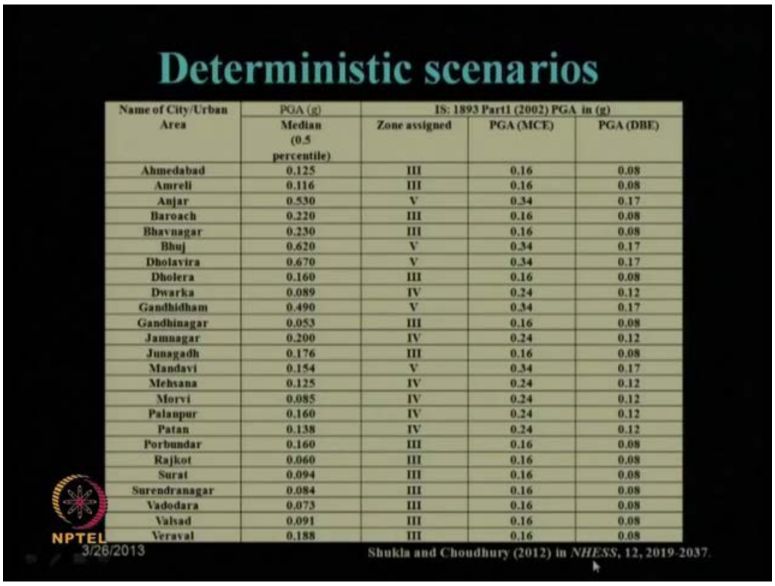

Name of City/Urban Area	PGA (g)	IS: 1893 Part1 (2002) PGA in (g)		
	Median (0.5 percentile)	Zone assigned	PGA (MCE)	PGA (DBE)
Ahmedabad	0.125	III	0.16	0.08
Amreli	0.116	III	0.16	0.08
Anjar	0.530	V	0.34	0.17
Baroach	0.220	III	0.16	0.08
Bhavnagar	0.230	III	0.16	0.08
Bhuj	0.620	V	0.34	0.17
Dholavira	0.670	V	0.34	0.17
Dholera	0.160	III	0.16	0.08
Dwarka	0.089	IV	0.24	0.12
Gandhidham	0.490	V	0.34	0.17
Gandhinagar	0.053	III	0.16	0.08
Jamnagar	0.200	IV	0.24	0.12
Junagadh	0.176	III	0.16	0.08
Mandavi	0.154	V	0.34	0.17
Mehsana	0.125	IV	0.24	0.12
Morvi	0.085	IV	0.24	0.12
Palanpur	0.160	IV	0.24	0.12
Patan	0.138	IV	0.24	0.12
Porbundar	0.160	III	0.16	0.08
Rajkot	0.060	III	0.16	0.08
Surat	0.094	III	0.16	0.08
Surendranagar	0.084	III	0.16	0.08
Vadodara	0.073	III	0.16	0.08
Valsad	0.091	III	0.16	0.08
Veraval	0.188	III	0.16	0.08

So, the deterministic scenario for each city is mentioned over here and those are compared with respect to IS value recommended PGA value which are as per the MCE and PGA value as per DBE, MCE maximum credible earthquake where as DBE is design basis earthquake which is typically half of the MCE.

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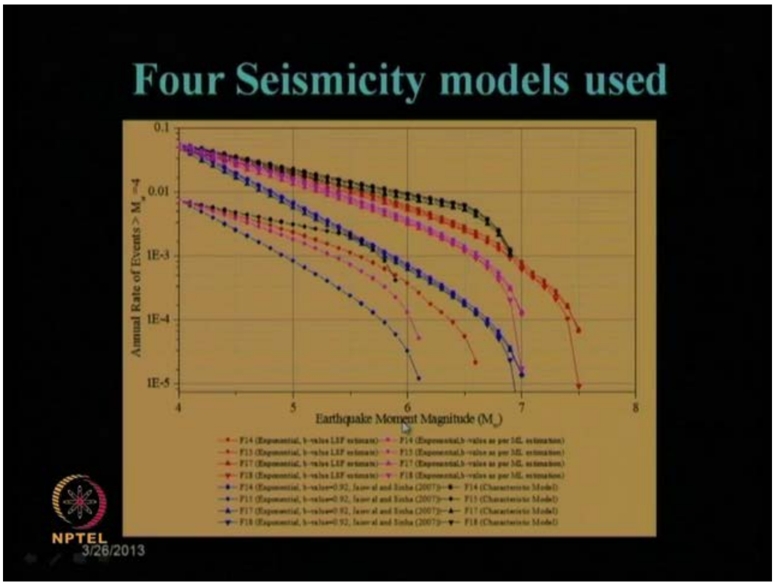

Then we have seen that probabilistic seismic hazard analysis for Gujarat also were carried out. For that four seismicity models were considered, one seismicity model is the exponential model using least square feet method, another is exponential model using maximum likelihood estimate, another exponential model using the b value proposed by Jaiswal and Sinha for entire India peninsular region and another one is the characteristics model.

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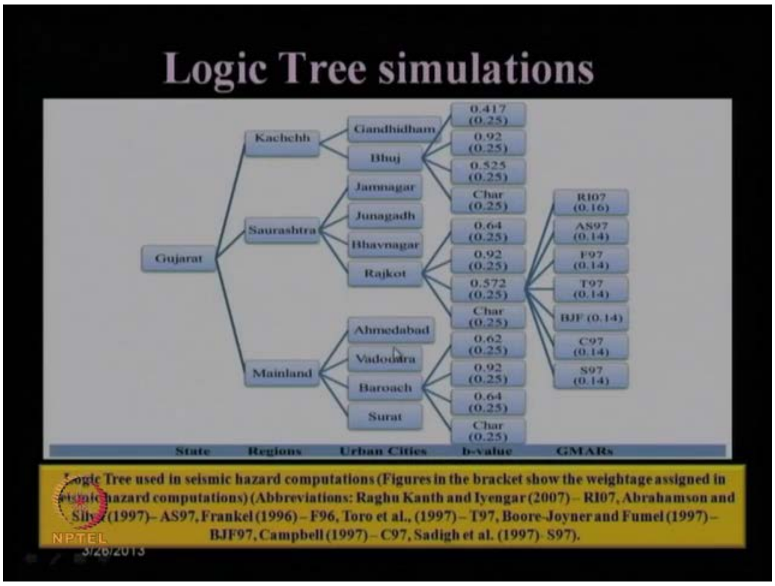

So, using this four seismicity model then arrived at the logic tree simulation for various cities. First Gujarat has been subdivided in three major regions and within each region we have taken several number of cities, only few are mentioned here. Actually all are taken care of and suitable weightage factors are given for each city corresponding to this four different models, that is the b value which has to be chosen finally, and all the seven attenuation relationship or all the seven ground motion prediction equations were considered to get the weight factor which has to be applied for the probabilistic seismic hazard calculations.

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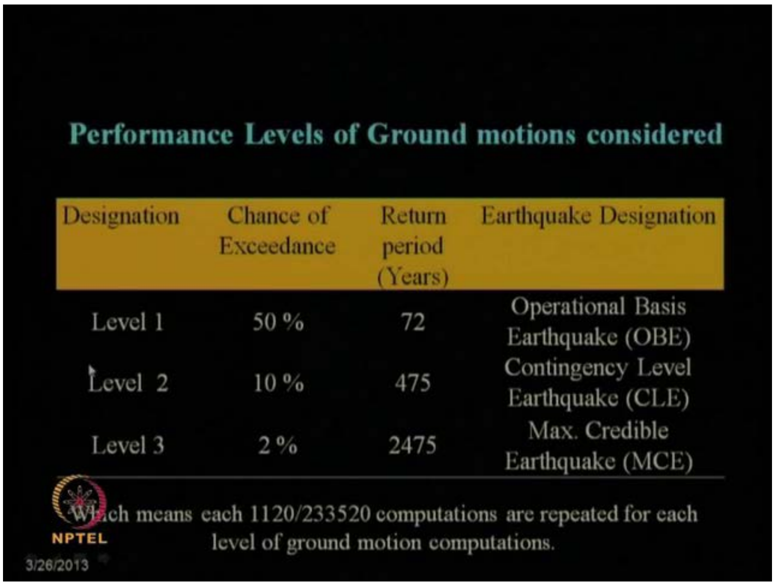

Then we have also seen that the designation of level 1 earthquake, level 2 and level 3 earthquake corresponds to the chance of exceedance in 50 percent, 10 percent and 2 percent in 50 years times, corresponding return periods also we can calculate easily like 72 years, 475 years and 2475 years which are mentioned like this, operational basis earthquake OBE, contingency level earthquake CLE and maximum credible earthquake MCE.

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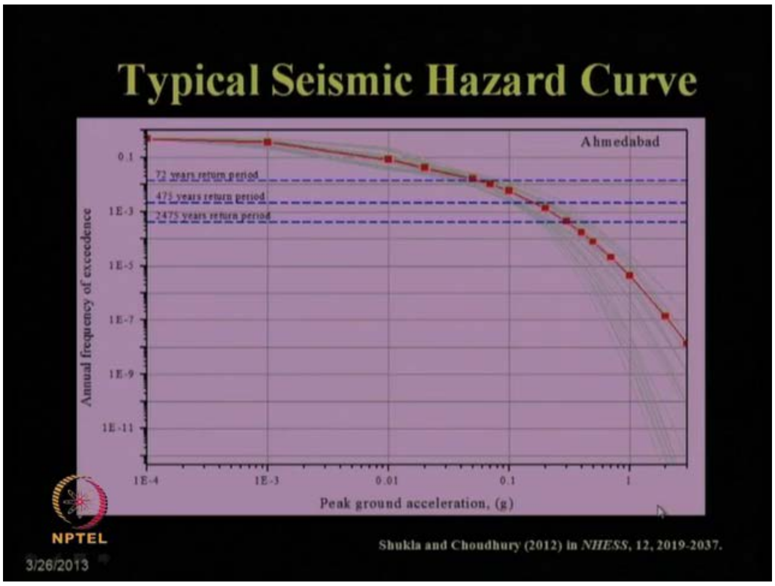

So, these are the result for each city that is typical seismic hazard curve. Using the probabilistic seismic hazard estimation one can easily obtained that 72 year return period, 475 and 2475 years return period.

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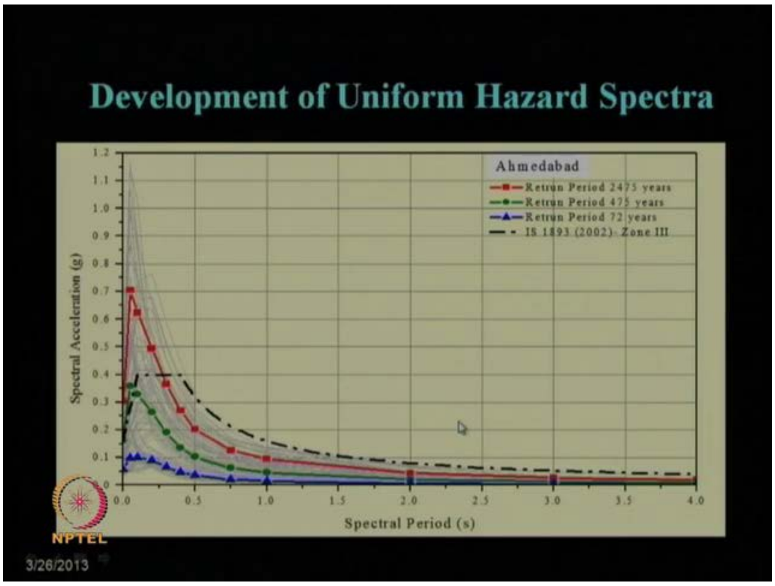

Then uniform hazard spectra has also been developed corresponding to different return periods and that has been compared with respect to IS codal provision which shown in the black line. So, you can see the IS values are pretty close at least the maximum value or the peak value corresponds to the return period of 475 years. Whereas if we want to design it for the critical structures with 2475 years of return period we need to use the much higher value of spectral acceleration for design, not the IS recommended value. So, a proper caution need to be considered when somebody is designing very important structures using the uniform hazard spectra as proposed in IS code and as can be obtained from a local seismic hazard analysis like this.

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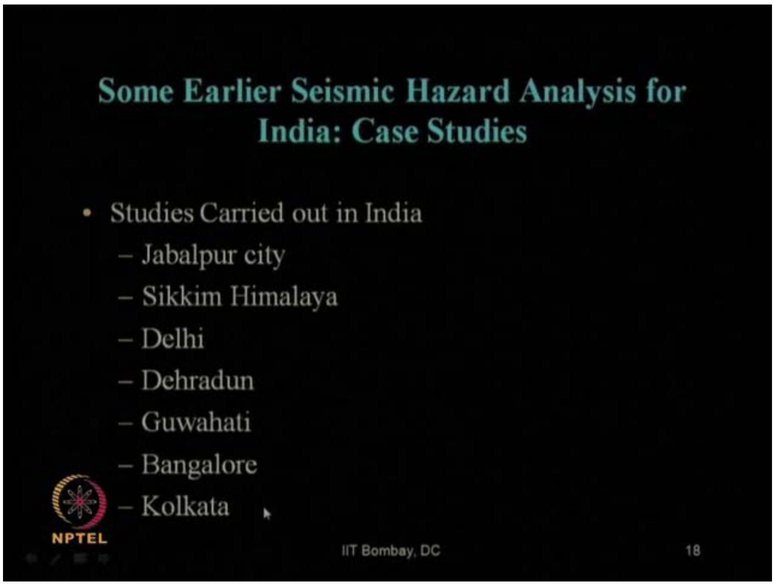

Then we had also mention that the previous researchers in India, they have also done the seismic hazard analysis like various case studies we discussed, case studies which are available for India for Jabalpur city, for Sikkim Himalaya, for Delhi, for Dehradun, for Guwahati, for Bangalore, for Kolkata. There are other cites as well, the seismic hazard analysis are coming up slowly slowly. So, like that it is a need of today that each and every important region should be having the detail analysis of this or results of this seismic hazard analysis using both deterministic as well as probabilistic seismic hazard analysis, because then only a important structure can be constructed as we have seen there is a severe mismatch of some particular return period value corresponds to IS recommended values of spectral acceleration for design.

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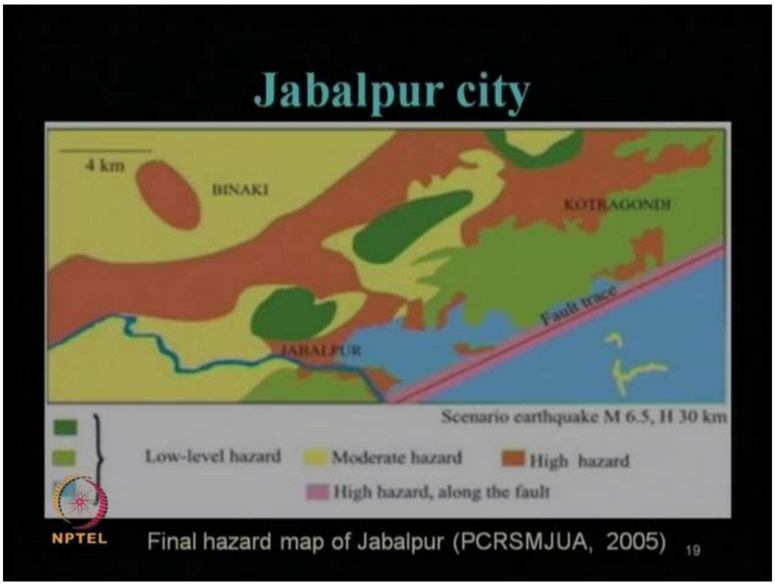

So, this is the seismic hazard map as for prepared for Jabalpur city as per this reference as you can see low hazard level, moderate hazard level, high hazard level and high hazard along with the fault trace.

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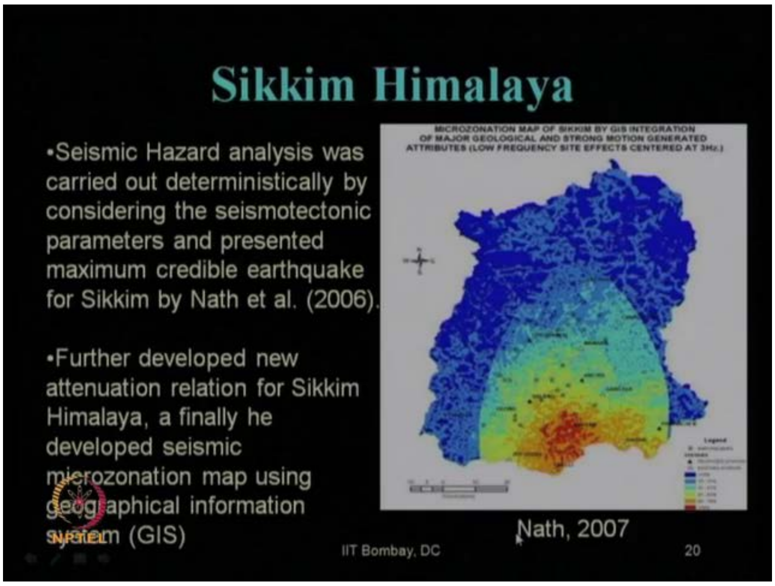

For Sikkim Himalaya also as given by this reference, the map of microzonation for the Sikkim has been shown in the GIS interface like this with different values of the intensity of earthquake accelerations.

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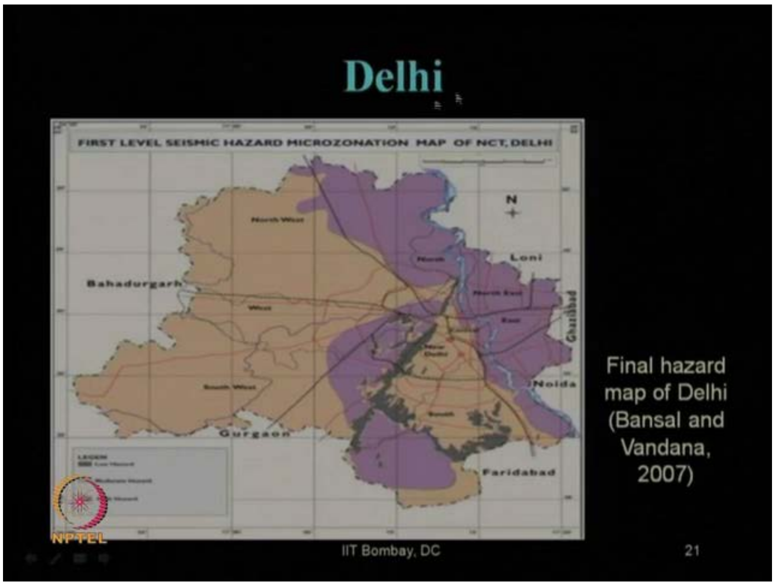

For Delhi also the final hazard map has been given by these references there are many other researchers also who has done or prepare the seismic hazard microzonation map for Delhi.

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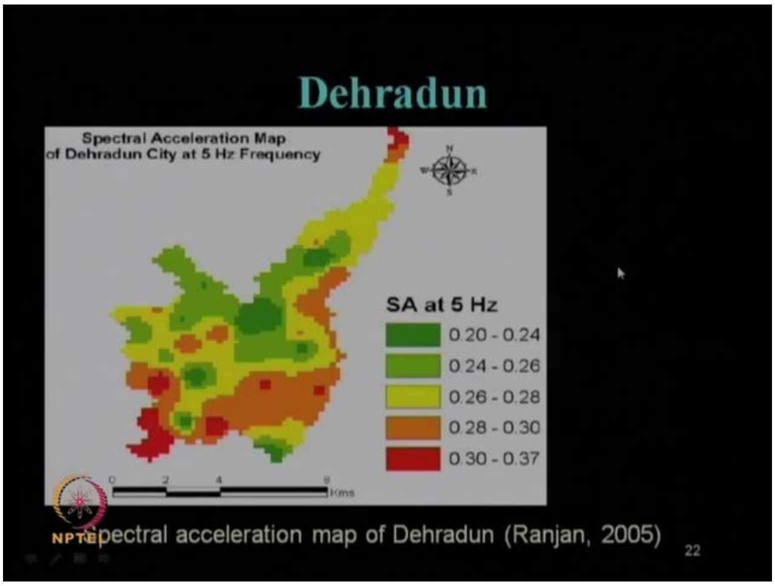

For Dehradun city also this is the reference, the spectral acceleration map at 5 hertz frequency level has been prepared, the hazard map. This is the hazard map seismic hazard map.

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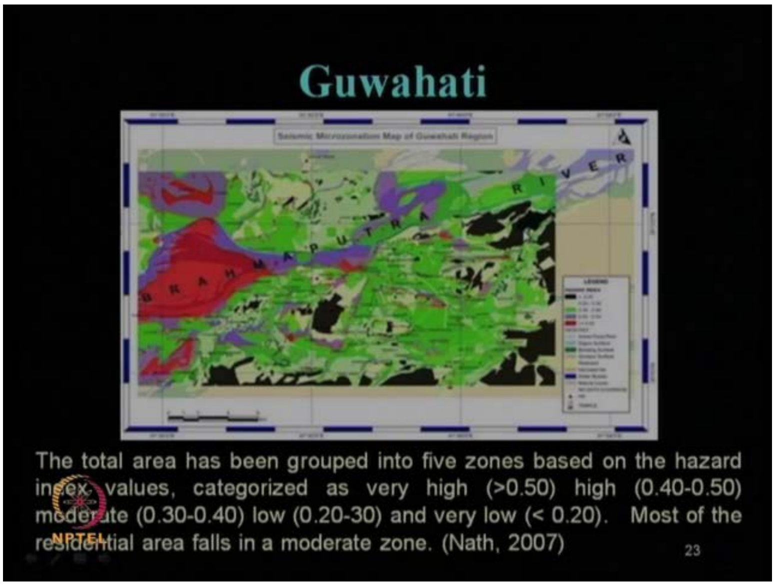

For Guwahati city also this is the reference. It has been sub divided into five major zones with very high, high, moderate, low and very low seismic hazard value with the index values as given over here.

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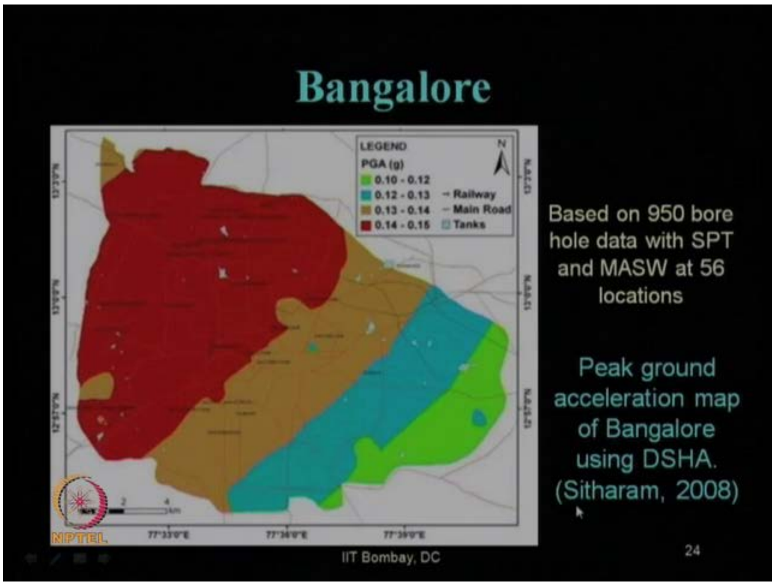

Then for Bangalore city also the seismic hazard map, peak ground acceleration map for entire Bangalore city was proposed by in this reference. You can see the values of PGA recommended as per different color codes.

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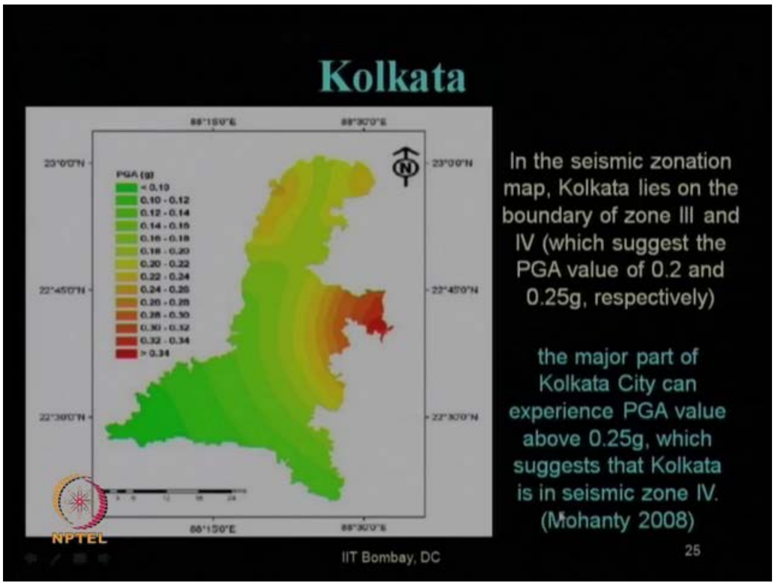

And for Kolkata city also the seismic zonation or seismic microzonation map was proposed and you can see here the variation of the PGA values of Kolkata city as proposed in this reference. So, with that in the previous lecture we completed our module seven on seismic hazard analysis. So, in today's lecture we are starting with next module. Our module eight is on sight response analysis. So, what is site response analysis? Let us start with the beginning.

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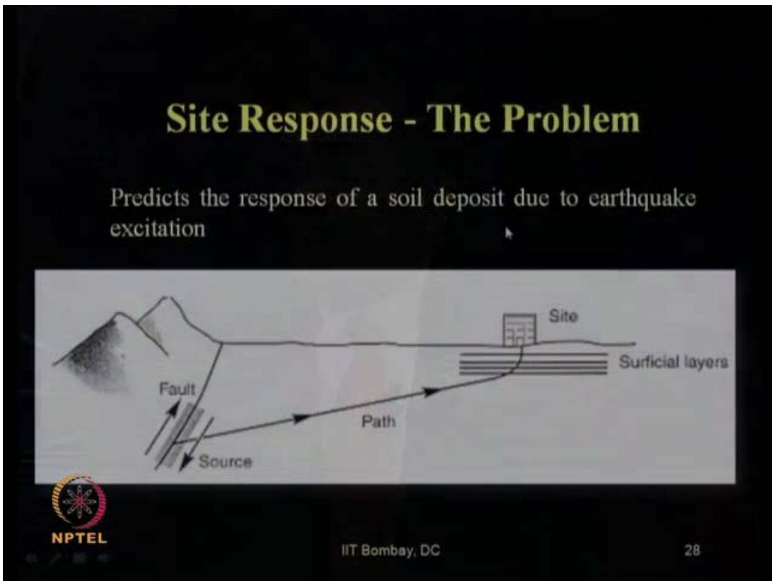

So, the problem of site response analysis is nothing but to predict the response of a soil deposit due to earthquake excitation. So, when earthquake occurs what are the various steps let us see through this figure. Typically there is movement in the fault and energy gets released. So, that is the point where we call it as a source of earthquake, then from the source when the energy get released, seismic waves get generated and they travel in all the directions.

So, once the seismic waves travels finally, the reach to the ground surface though the surficial layers close to ground surface. So, that part where we are going to construct any structure or any building or the site of our interest is nothing but our region where we are interested to know about the seismic effect which is occurring at a faraway place maybe or nearby place. So, that typical steps involved or regions involved you can see one is source, another is path through which this seismic waves are traveling and finally the site about which we are interested. So, in the site response analysis we want to know the response of this site corresponding to an earthquake occurring in nearby or far away which is occurring at a source through this medium of path.

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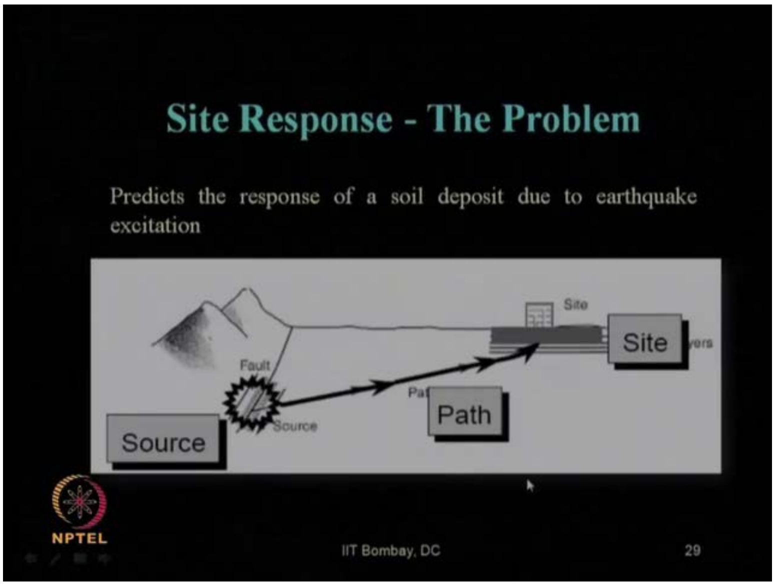

So, you can see again in this picture the same thing. This is the source it travels through the path and finally it comes closer to this surficial layers and when the seismic waves passes to the surficial layer, it will have some typical characteristics as we have discussed already in wave propagation theory, that is body waves. Then close to the ground surface they will form the surface waves and all those analysis, the site response analysis we need to find out to know that from the source to this site how much changes will occur in the amount of say acceleration, acceleration time history or any other dynamic information which we should know for doing the earthquake analysis. So, that is nothing but to take care of the local site response analysis.

So, site response analysis why it is so important to our geotechnical engineers because it is not that for all the sites the response will be same from though the source may be same. Suppose, this source from where again seismic waves are traveling in this direction also. Suppose, there are some hills, so rock rocky profile we can expect here. So, if it reaches here it will have a different type of site response than if it reaches here where suppose if we have a thick soil deposit or soft deposit.

So, it depends on what type of material you have close to a surficial layer close to the ground surface. Depending on that the impotency of the site specific response or a location specific ground response is necessary to analyze and to know for further design of structure at that particular site, because here whatever design philosophy you will adopt, the design philosophy will obviously need to change to another site where it is not a hilly terrain or rocky terrain, but it is a soft soil terrain.

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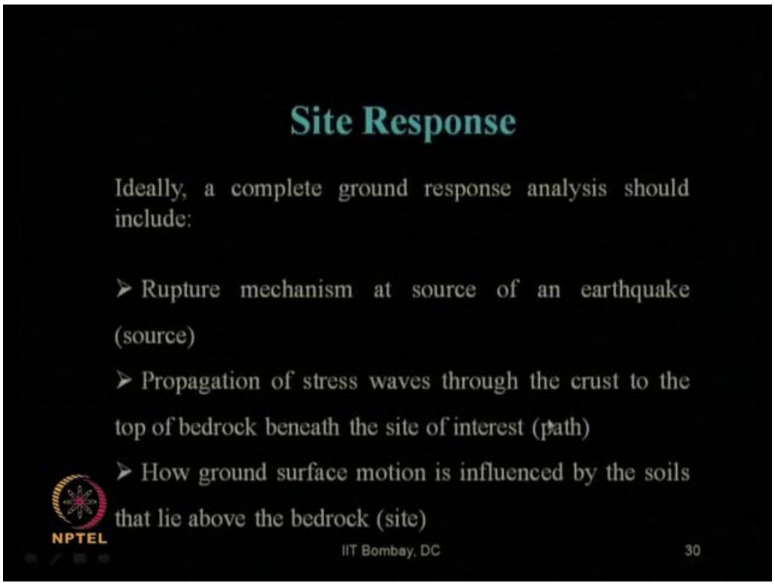

So, what are the steps in site response analysis. Ideally a complete ground response analysis should include, these are the three steps or three media one is rupture mechanism at the source of an earthquake should be known completely which we have already named as source in the previous figure which we have discussed. Next is propagation of this stress waves through the crust to the top of the bed rock beneath the site of interest, which we call as the path and next is how the ground surface motion is influenced by the soils that lie above the bed rock which we call as the site.

That means if we go back further to this previous picture, this is the first one source where earthquake energy gets released, then the stress waves or seismic waves travel through a path, then suppose this is the upper most rock layer when we are talking about the crustal layer. Say, this is the uppermost layer of the rock which is known as bed rock. Why the name bed rock? Above it there will be a soil layer. So, that is the call as the bed rock. Then after passing through the soil those seismic waves finally will affect the ground and the structures constructed in that ground. So, that is why we said source, path and site all these three should be considered ideally.

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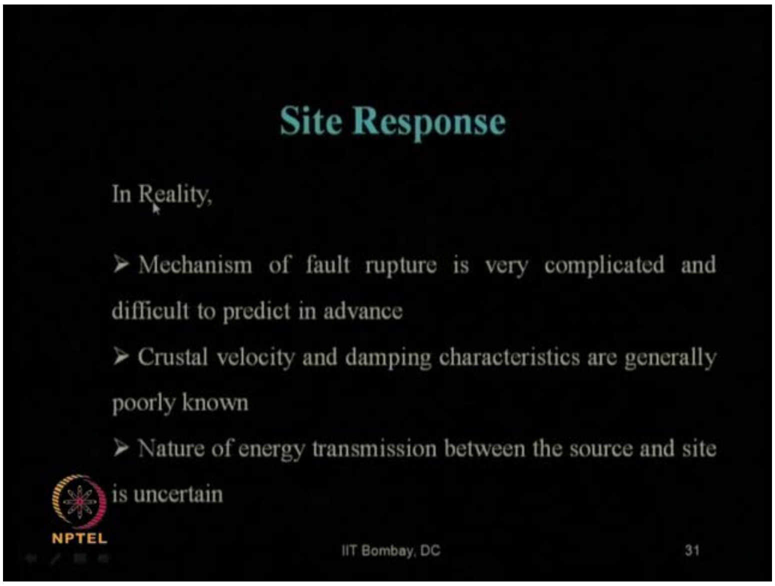

But in reality what we can see, when we start doing this site response analysis we will find these are the complicacy or the problem which will arise. So, in reality we will find mechanism of the fault rupture is very complicated and difficult to predict in advance which we all know, because you cannot predict in what way the fault is going to get ruptured. So, if the fault rupture details etcetera are not correctly available or correctly mapped or correctly known how we are going to have an analysis done for a future structure. So, to do the ground response analysis or site response analysis we should know the fault behavior and mechanism of fault movement, which obviously earthquake to earthquake, fault to fault everything varies. So, that is a complicated part in this analysis.

Second point that is source is a complicated source, second point is regarding the path the crustal velocity and the damping characteristics are generally poorly known as we have already discussed, if you remember in one of our previous module we discussed about the strength of the crustal plate like if it is a deep crust or shallow crust, whether it is oceanic plate or the continental plate depending on different values of the strength of the crustal plate we have mentioned, but those values are obtained not as an exact value with trial and errors the seismologist or geologist have arrived at those ranges are typical values. So, that typical values may differ obviously from case to case or site to site or crustal movement to crustal movement. And also this velocity of these seismic waves, already we have seen that what are the typical velocities of body waves like p waves and s wave in crust.

We have seen there is typical range given right for p wave we have mentioned it is 6 kilometer per second, but range we have mentioned it is 5 to 7 kilometer per second. Similarly, for shear wave also we mentioned it is typically 3 kilometer per second, but it can also vary in the range of 2.5 to 3.5 like that. So, that exact value of crustal velocity is also not known in many case or poorly known.

So, you can see here the problems where it is arising in reality when we are starting our site response and in the third step that is the nature of energy which is getting transmitted between the source and the site is totally uncertain and also the damping characteristics through which they are going through. So, these are the complexity of the problem when we are starting any site response analysis, but can we stop with that? No, we have to at least come up with some solution which will gives us a fairly rough estimate which should be good for starting a design at a site with having some data of site response or ground response analysis to take up a future work.

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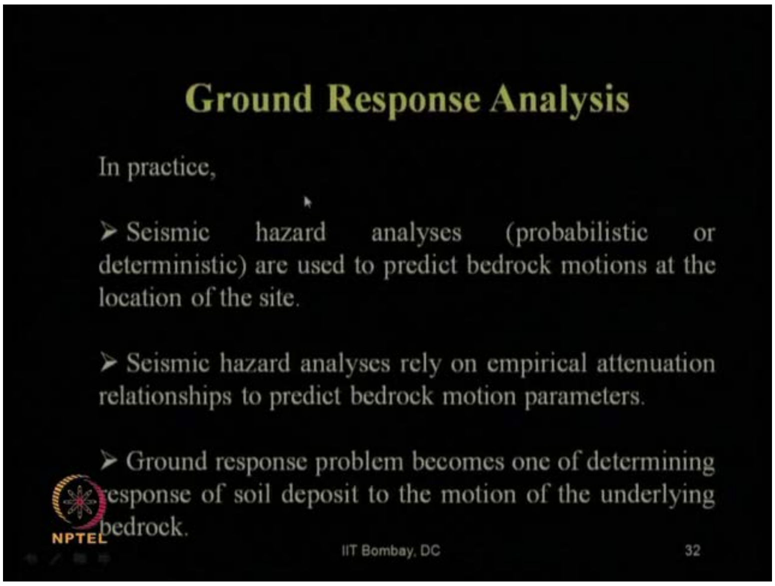

So, that is what for ground response analysis in practice what we do, we do the seismic hazard analysis. It can be probabilistic or deterministic or combination or both are used to predict the bed rock motions at the location of the site, that already we have discussed in our previous module in detail, the seismic hazard analysis how we are doing.

Then this seismic hazard analysis when we are doing that it rely on empirical attenuation relations that also we have seen because all the attenuation relationships are empirical based on the past earthquake data. If you change your data set you are going to arrive probably at a another attenuation relationship. So, to predict the bedrock motion parameter you have to rely on these things, that is what we use and ground response problem becomes one of the determining response of soil deposit to the motion of the underlying bed rock. That is what we adopt when we start any design process at a particular site.

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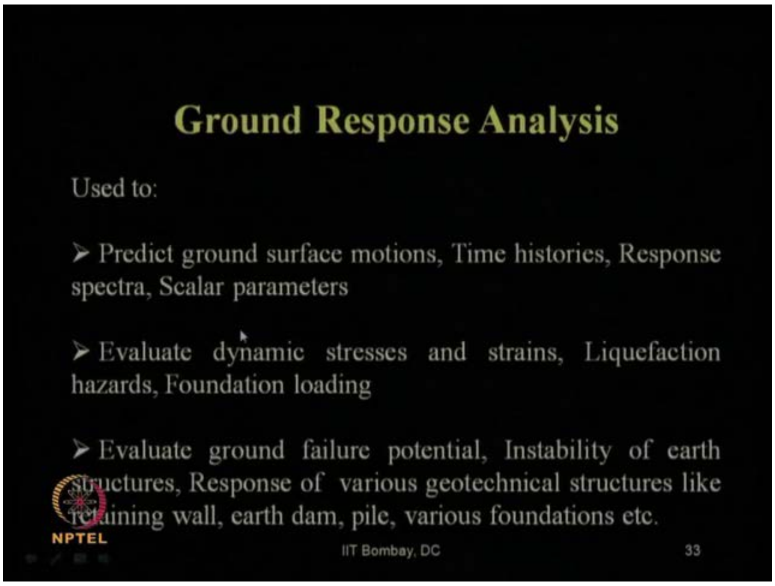

So, for that what we need to do? So, we used to predict the ground motion parameters, this ground response analysis those are used for predicting ground motion parameters, time histories, response spectra, scalar parameters etcetera. Ground response analysis also used to evaluate the dynamic stresses and strain in the soil, liquefaction hazard, foundation loading that is what are the extra loading are coming for say suppose a pile is getting constructed in a soil.

So, how much extra lateral and vertical loadings are coming from the seismicity of the event due to the effect of the local soil condition. That comes also from the analysis of ground response. That we will discuss in detail in one of our module later. Then evaluate the ground failure potential for doing that also you, we require ground response analysis. Also to do the instability of earth structure analysis, response of various geotechnical structures like retaining wall, earth dam, pile, various types of foundation, how they behave in a soil to know that we should have the ground response analysis available. The results of ground response analysis should be known to us.

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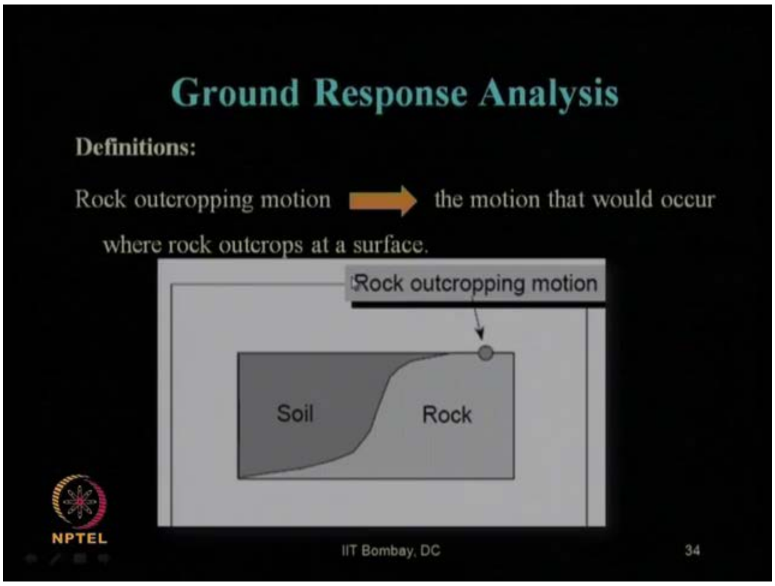

Let us go through few definitions which we will be using very often while going through this ground response analysis. First one is rock out cropping motion. What is rock out cropping motion? Let us look at this picture. Suppose, if we have a soil site like this and a rock site like this. So, typically rock will be below the soil which is quite obvious, but sometimes even rock surface also can become open on the ground surface, quite possible like hilly terrain as I said.

So, if we talk about our Mumbai region, in the Malabar hill region obviously it is a rocky site because rock pops up where as in the Mumbai itself the JNPT site or the coastal region, it will be thick soil site, much below than that we will probably get a bed rock. So, this type of soil and rock combination or characteristics of place wherever we get, the rock out cropping motion means the ground motion which would occur when the rock out crops at the surface, that is the rock comes on the ground surface. So, rock out cropping motion means whichever ground motion we are getting at the ground surface provided that it is a rocky ground surface, that we call as rock out cropping motion.

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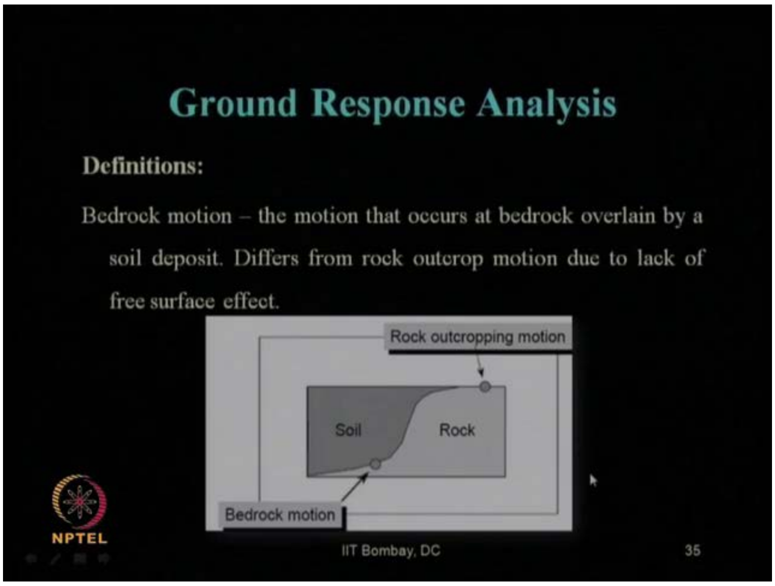

Next terminology is bed rock motion. What is bed rock motion? In the same picture now if we look at the soil site, this soil is finally exposed on the ground surface like this, but below the soil there is a bed rock, you can see over here. So, the bed rock motion means the motion that occurs at bed rock overlain by a soil deposit. So, it differs from the rock out crop motion due to lack of free surface effect. What does it mean that is the ground motion or the earthquake motion which is occurring at this level, at the interface of the rock and soil like this is called as bed rock motion, because this rock is not exposed to ground surface, it is underlain below a soil deposit.

So, this motion and this motion even during a same earthquake at that place will be different. Why it should be different? We have already learnt in the module of wave theory or wave mechanics that depending on the boundary conditions whether it is a free surface or a non free surface, the stresses, displacement of the waves will be different. So, that is why the bed rock motion at this place and rock out cropping motion at this place for the same earthquake because of same waves will be different. So, that is why this terminology bed rock motion is different from rock out cropping motion. So, rock out cropping motion has to be opened at the ground surface, but bed rock motion means it is never open to the ground surface.

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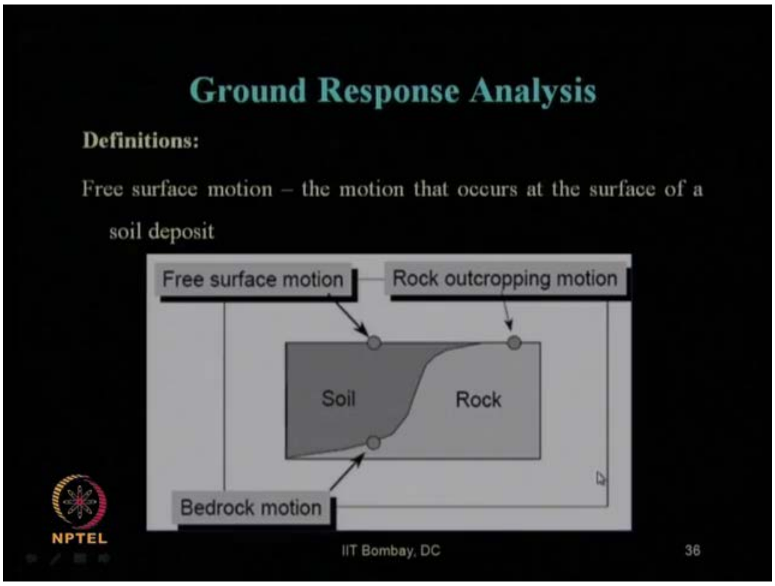

Free surface motion what is called free surface motion it is the motion that occurs at the surface of a soil deposit. So, in the same picture now we have added this point where soil is opened up in the ground surface when we talk about the seismic motion or earthquake motion at that point that is known as free surface motion. So, these are the various points which we have defined rock out cropping motion, bed rock motion and free surface motion. Now, we should see through the ground response analysis how we are getting these values. So, what are the possibilities can arise let us see the common situation.

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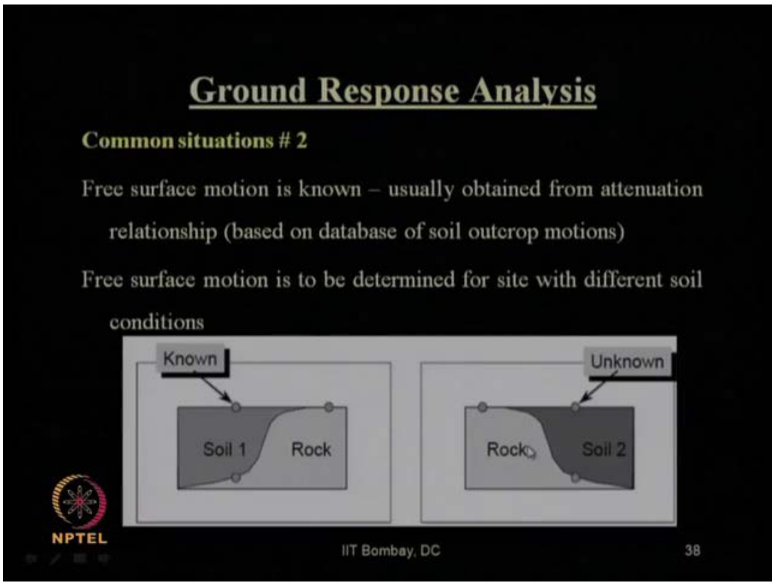

So, common situation number 1. First, let us discuss one of the possible situation where rock out crop motion is known. Let us say this value rock out crop ground motion is known. How it will be known to us, because you may have your seismograph etcetera located at this place, using that you can record it. So, usually it is obtained from the attenuation relationship also you can get that, based on the data base of rock out cropping motion.

So, this value may be known. What we need to do? Suppose, from that site in another nearby distance you need to find out what is free surface motion which is at the ground surface of a soil deposit. So, this is known and this is unknown we need to find out. As this is known this is also unknown let us say, we need to first find out the value over here and then the value there, we will see how.

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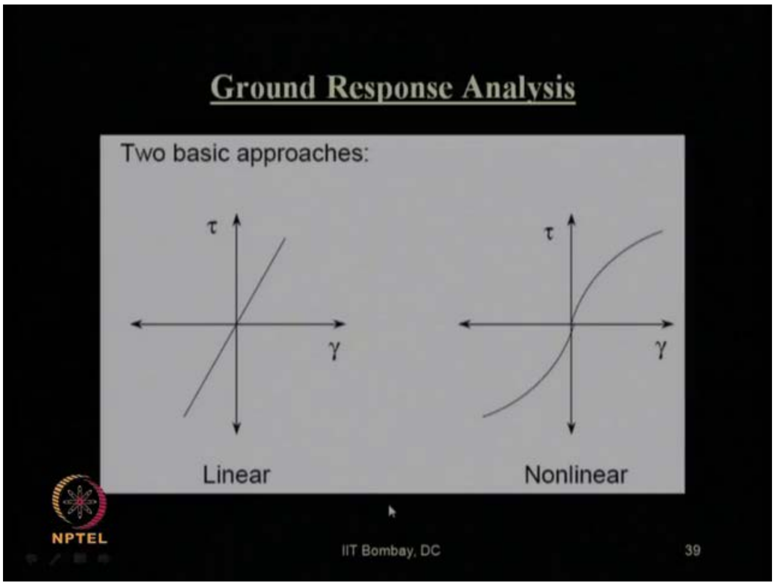

Next, common situation number 2; in that case let us say this free surface motion is known. Let us say we had a seismograph or some data source where we got this point or may be using the attenuation relationship for a soil out cropping motion like various attenuation relationship will be available for soil out crop motion also. Suppose, this value is known and this is unknown. So, free surface is to be determined for site with difference soil condition. Let us say the problem in this case what it mentions at one soil site it is known, we do not know the rock out cropping motion, we do not know the bed rock motion and there is another soil site, soil site number 2 in close vicinity or a nearby distance within few kilometers even, but that value of outcrop motions, soil out crop motions or the free surface motion is unknown. So, for that site we need to obtain it. Suppose if you have the data set available within India let us say at a soil site in Delhi, it is known. Then close to Delhi say in Gurgaon in another site you want to find out what is the ground response value or the free surface motion value at another site, then you need to go through this ground response analysis. So, that is the advantage of doing ground response analysis to get the ground motion or earthquake motion or seismic motion at different points.

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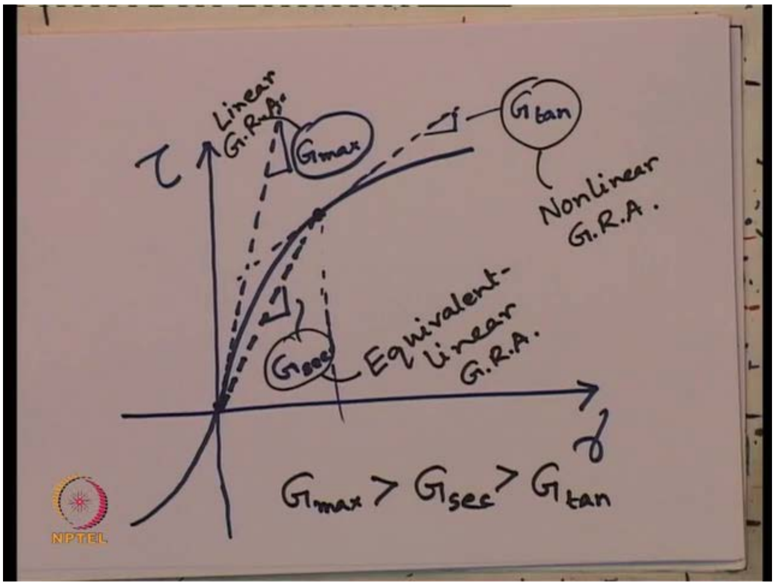

So, to do that, to carry out the ground response analysis typically, we have two basic approaches. First approach is called the linear approach and the second approach is called non-linear approach. In the linear approach what we consider why we call it as linear approach because in that case the shear stress versus shear strain behavior of the soil material though which the motion or through which the earthquake waves are passing through are considered or assumed to behave linearly like this.

So, if it behaves linearly, if you take that linear behavior of shear stress versus shear strain during the earthquake motion at a particular site, then you can call it as linear ground response analysis, another analysis is called a non-linear, where you have the shear stress versus shear stain response is completely non-linear like this, it is not linear. So, if you take the non-linearity involved in the soil material behavior under the dynamic load or earthquake load due to this shear stress and shear strain, corresponding shear strain behavior in the soil material non-linearly then it is called non-linear ground response analysis. And as we have learnt in the module dynamic soil properties what we can mention over here that there will be another typical case.

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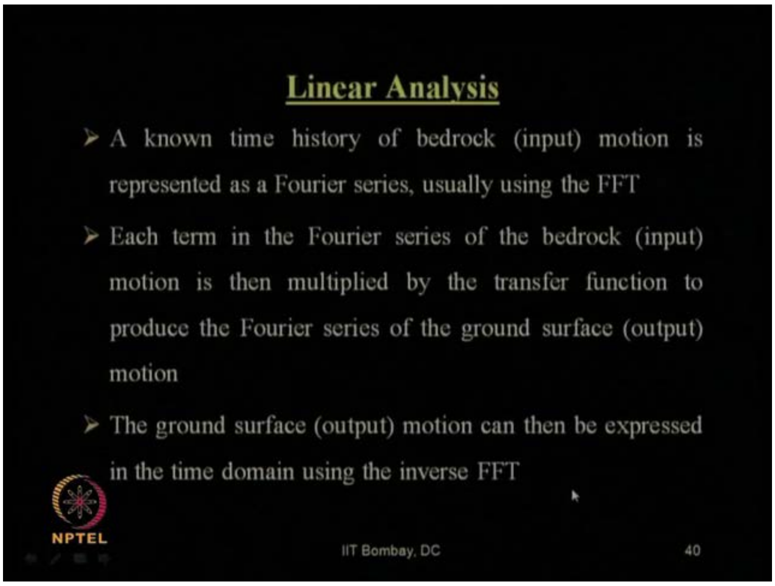

What is that typical case? I can plot it here shear stress versus shear strain, the behavior let us say non-linear behavior like this for a soil material, under dynamic loading or earthquake loading condition. As we have discussed in our previous, one of the previous module on dynamic soil properties, we can consider it at linear like this, initially linear. We can consider it like this and slope of this shear stress versus shear strain is nothing but the shear modulus. So, this case we got G max, maximum shear modulus or initial tangent modulus.

At any point, at any value of the shear strain whatever value of G we get that we call as tangent shear modulus and if we join any point of shear strain with respect to this initial point by a linear line like this, the slope of it can be considered as G secant that is secant shear modulus. So, this one is called when we are using this G max that is known as linear analysis. So, this we are using for linear ground response analysis GRA, this G tan we used for non-linear ground response analysis and this G sec secant modulus, G sec we use for equivalent linear ground response analysis.

Why it is called equivalent linear because we have taken the stress strain behavior at a point on the curve of non-linear behavior of the soil material, but we have joined that point with the initial value through a linear line or a straight line, that is why it is named as equivalent linear.

So, as we can see from this always G max will be higher than G secant should be higher than G tangent. So, maximum shear modulus as the name suggests it is always maximum one than secant shear modulus than tangent shear modulus. This is the, these are the values or ranges which one need to consider when we are trying to do the ground response analysis. So, if we look at here as I have already mentioned here two basic approach linear ground response analysis and non-linear ground response analysis and within non-linear we have another sub division which is called equivalent linear ground response analysis.

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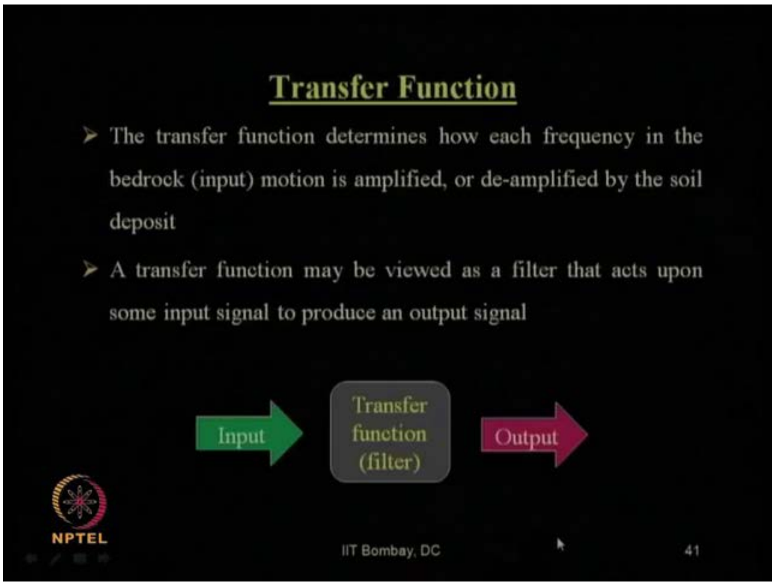

Now, in the linear analysis linear ground response analysis first a known time history of bed rock, bed rock motion let us say the is considered as a input motion. It is represented as a Fourier series. Let us express the time history, let us say it is an acceleration time history. So, that acceleration time history or the bed rock motion or it can be a displacement versus time history. It depends what you are interested in. So, that you can express as a function of Fourier series mathematically you can express in terms of Fourier series usually by using the first Fourier transfer. So, each term in the Fourier series of the bed rock that which is named as input motion is then multiplied by a factor which is called a transfer function.

So, what is transfer function? It is nothing but a multiplier, it is nothing but a factor by which we should multiply this input motion, so that it produces the Fourier series at the ground surface or output motion, that is free motion, that is if we do not know the free motion or output or ground surface motion then from bed rock motion we can get it like this. And then once you get the ground surface output motion, it can also be expressed in that time domain using the inverse of that first Fourier transfer.

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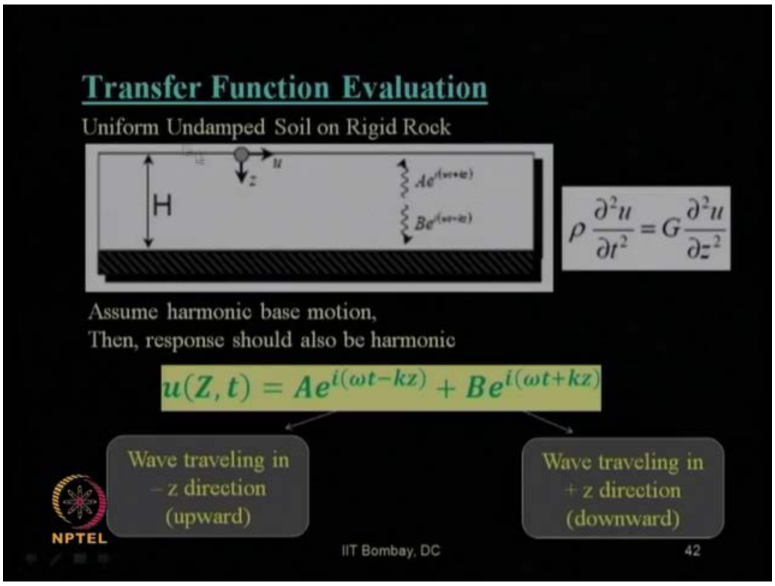

So, what is that transfer function? As I said it is a multiplier, it is nothing but a filter. The transfer function it determines how each frequency in the bed rock input motion is getting amplified or getting de amplified by the soil deposit. So, a transfer function it can be viewed as a filter that acts on the some input signal to produce an output signal. That means you have an input motion using this multiplier or transfer function or filter you will get an output motion. That is if you have the input or known value, what should be your input data always the known one.

So, it can be your bed rock motion using transfer function you will get free surface motion or suppose if you know the free surface motion and bed rock motion is unknown in that case your input will be free surface motion. You have to use the transfer function in such a way that your output will be the bed rock motion. Now, if I go back to one of those problem that is say soil site 1 this free surface motion is known, but at soil site 2 the free surface is unknown. So, what we should do? This is known, so we should find out the bed rock motion here first using the transfer function and if it is the same rock, same profile going on, and then another soil profile is coming then that bed rock motion we can use to obtain the free surface motion at second soil site using another transfer function. So, this transfer function depends on what we will see very soon.

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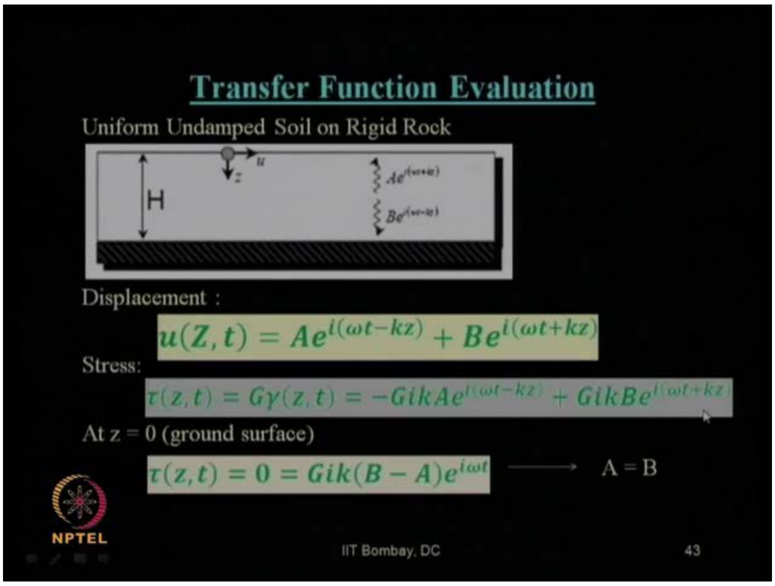

So, let us now evaluate this transfer functions. That is how to get that multiplier. Let us take first the simplest example that is how this derivation of the multiplier is obtained, let us consider an uniform undamped soil on a rigid rock. So, the boundary condition is this is a rigid rock, on top of it there is a soil layer of thickness H and the dimensions from the ground surface are measured in this axis system, u is the displacement in this direction, z is the depth axis.

Now, when waves are moving what we have seen earlier from the wave motion, equation of wave motions for uniform and undamped soil the equation basic governing equation was in one dimensional wave remember we have considering one dimensional wave. This was the equation, do you remember, if you recap the previous one of the previous module in wave motion we discussed, we derived actually where from this equation is coming.

So, if we talk about the shear wave velocity then the relationship was rho del square u by del t square equals to g times del square u by del z square. That was the equation we have seen earlier and if we consider the harmonic base motion, then the response also should be harmonic. So, if the response is harmonic we have also seen what will be the form of the solution, form of the solution or the displacement function as a function of Z and t can be written in this form A e to the power i omega t minus k z plus B e to the power i omega t plus k z where A and B are the constants which need to be determined based on the boundary conditions. And omega is the circular frequency of the excitation and k is the wave number. So, this portion signifies the wave traveling in minus z direction which is nothing but moving upward like this because our positive z direction is in this way. Whereas this part gives us the solution for wave traveling in downward direction or plus z direction. So, this is the solution in the harmonic based motion.

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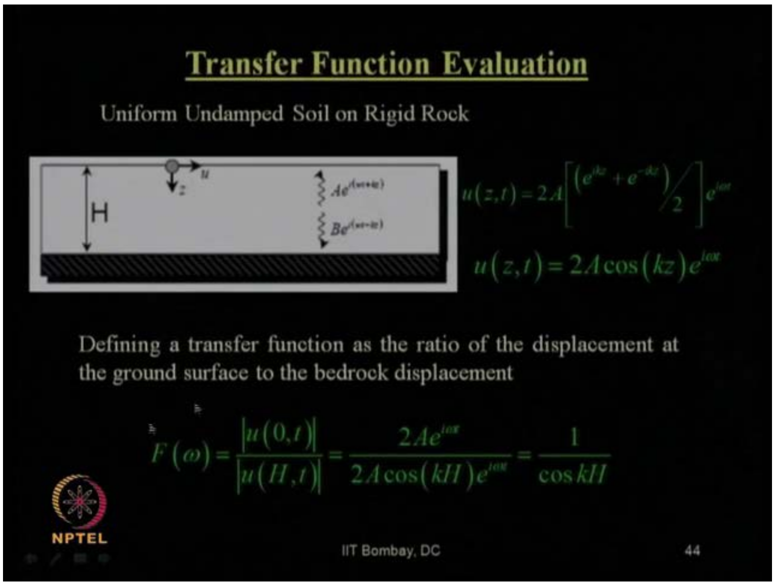

Now, the same problem if we want to find out the displacement and stresses and then apply the boundary conditions to obtain those unknown constants A and B, let us see what we can get. So, the displacement is given already to us in this form. What will be the stresses? Stresses is nothing but stress can be expressed as a function of z and t again which is nothing but shear modulus times shear strain. So, shear stress is nothing but shear modulus by shear stain. How we can obtain the shear strain? We can get it from the displacement function. So, that we are obtaining by differentiating it with respect to Z, if you differentiate the displacement with respect to Z what comes out from here you differentiate, so minus k will come out with the i here plus k will come out with an i.

At ground surface at this place boundary is condition is known. What is known boundary condition that stress should be 0 at ground surface, is it not? If the stress is 0, for z equals to 0, let us put in this solution this tau of z of t equals to 0. If you put that in this equation we will get the solution like this.

And this will gives us value of 0 only if A and B are equal to each other. Isn’t it why because t can be any non 0 value also, G is non 0 value, all these others are non 0. So, it will produce the result of 0 only if B and A are equal. So, we got one step for this solution.

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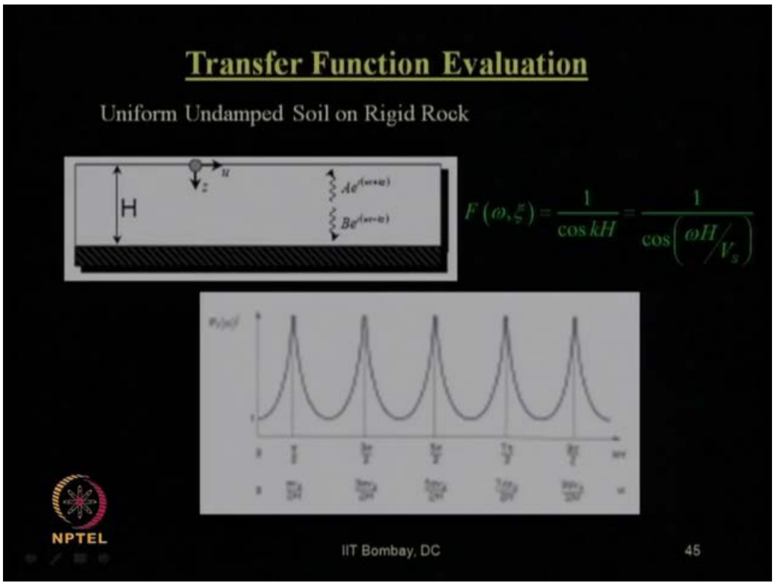

Now knowing A and B are equal how we can express the solution further that u of z t can be now expressed as two times of A because A and B are equal e to the power i k z plus e to the power minus i k z by 2 into e to the power i omega t, is it clear? Where from it is coming, if we go back we can express it in terms of sine and cosine which actually you will get in this form. So, that on simplification you can write it as 2 A cosine of what is this one? Cosine of k z e to the power i omega t.

So, defining a transfer function, how we define the transfer function let us see. It is nothing but the ratio of the displacement at ground surface to the bed rock displacement, that is transfer function if we write it as f omega, let us see here f omega is mentioned as a transfer function which is nothing but mathematically we are expressing it as ratio of displacement at ground surface by displacement at bed rock level, so u of 0 t, because z is 0 at ground surface by u of H of t modulus of that. So, what is u of 0 t, if you put z equals to 0, what you will get form this equation? We will get 2 A to the power, 2 a e to the power i omega t and if you put z equals to H at bed rock here, what you will get? We will get two a cosine of k H times e to the power i omega t. So, final output what is the transfer function 1 by cosine of k H. That means if your bed rock motion here it is known to get the motion at ground surface you have to use this multiplier of 1 by cosine of k H where k is a wave number and H is the thickness of soil layer. How we are using this, but remember this is for undamped. So, we will slowly go to the real condition of damped soil also.

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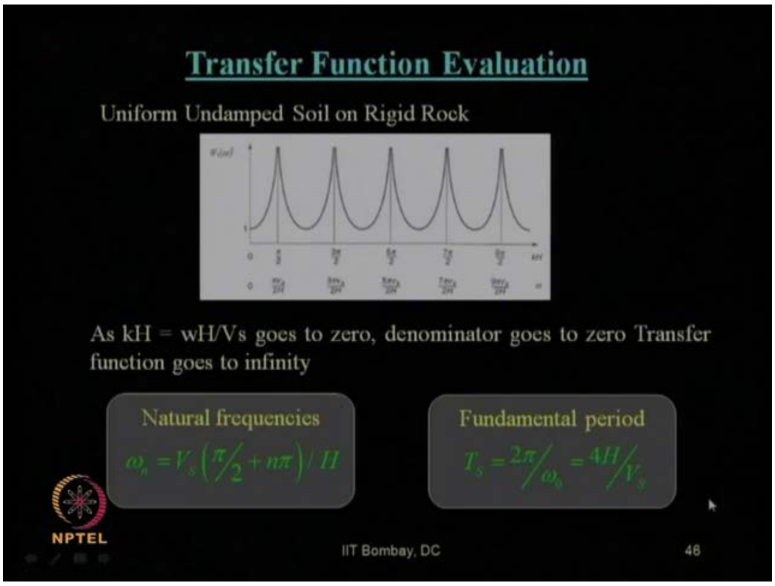

So, how this response will look like? Let us see, this F is a function of omega and eta. Why because 1 by cosine of k H, this is a function of depending on for undamped it is not a function of eta, eta is damping ratio, it is function of only omega because your wave number is dependent on that or correlated with that, that we have seen already and the thickness of the soil layer, which we can express as if you use the expression of k is nothing but omega by V s, isn't it that is what we have defined earlier that wave number is defined as omega by V s. So, transfer function for undamped case will be 1 by cosine of omega H by V s. So, if we plot that transfer function for various values of k H or for various values of this omega, we will get a plot like this. So, if we plot against k H, initially at k H values of equals to 0 it should be 1, if k H is 0 it will transfer function will be 1, isn't it. If k H value is equals to pi by 2 what should be the value of transfer function cos of pi by 2, it will become a infinity again at k H equals to pi what should be the value? Again it will reach 1 because remember we have defined it as mod of this. So, that is why always it is positive like this. If k H is 3 pi by 2 then again it goes to infinity, so on you will get a plot of transfer function for various values of k H in this fashion.

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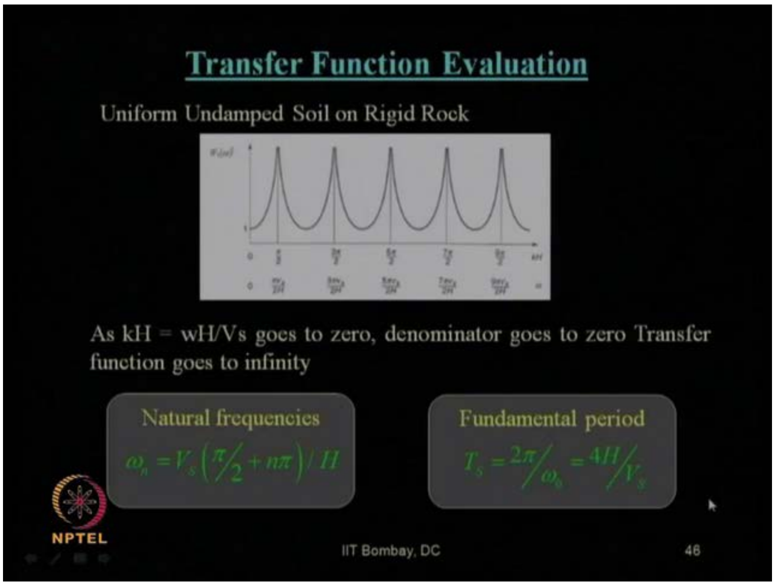

So, for uniform soil and undamped soil, those two are the basic conditions, remember soil needs to be homogeneous soil or uniform soil layer and undamped, then only this results will hold good, that is transfer function will behave like with respect to k H in this fashion or in terms of omega, if we plot we can express always omega in terms of that V s and H, now as k H which is equals to omega H by V s goes to 0, the denominator goes to 0 that means transfer function goes to infinity that we have already seen. What does it mean? It means natural frequencies we will get like this V s pi by 2 plus n pi where n is an integer 1,2,3,4 like that by H.

So, fundamental period what it can be? Fundamental period is nothing but 2 pi by omega naught. So, natural frequency, how many natural frequency, we are getting in a soil layer which is having the ground motion passing through the layer, there will be n numbers of natural frequency. You remember, you can look here the solution gives you n numbers of natural frequency. Among these natural frequency, we call fundamental natural frequency is that natural frequency which will have the least value, as I have already discussed in my other course on soil dynamics video course that fundamental natural frequency is the least among those natural frequencies. So, omega naught for n equals to 0 will give you obviously the least value of the natural frequency. So, that can be considered as omega naught. So, corresponding to that what will be your fundamental period? Period is nothing but t n equals to two pi by omega n so n numbers of [vocalize-noise] period you will get, but among them the fundamental period is that period which corresponds to fundamental natural frequency, that is why it is written 2 pi by omega naught.

So, omega naught what is the value of omega naught? If you put n equals to 0 here, it is nothing but pi V s by 2 H. So, T s value becomes two pi by pi v s by 2 H, so that 2 H comes up so it becomes 4 pi H by pi V s. So, pi and pi gets cancelled. So, finally, we get T s equals to 4 H by V s.

Now, can you see what earlier we mentioned as a thumb rule that typically we use how to obtain the fundamental period of any soil site this is the way we can calculate the fundament period of any soil site that is 4 H by V s because remember when we are talking about fundamental period, it does not depend on whether it is damped or undamped, like the natural frequency when we defined it is that frequency which is having under the free vibration undamped, that gives us the value of omega. So, corresponding to that where we are getting here the T s value that 4 H by V s is the fundamental period of any soil site with H as the thickness of the soil layer and V s as the shear wave velocity of the soil layer. So, these two parameters you should know to compute the fundamental period of any soil. Now, with this we have come to the end of today’s lecture, we will continue further in the next lecture.