Geotechnical Earthquake Engineering
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Module - 2
Lecture - 3
Basics of Vibration Theory

Let us start our today's lecture on this NPTEL video course on the subject Geotechnical Earthquake Engineering. Today is our lecture number three. We can look at the slide here. This is the title of the video course Geotechnical Earthquake Engineering which we are going through. Today is our lecture three. In the previous lecture we discussed about the module one which is the introduction to geotechnical earthquake engineering and we completed module one in our previous lecture. A quick recap, what we have learnt in our previous lecture.

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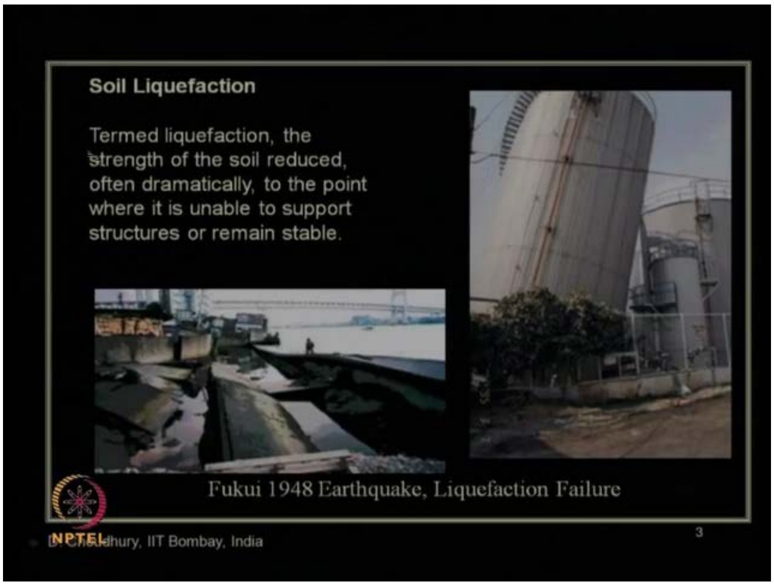

We talked about soil liquefaction related issues like, which is comes under the purview of geotechnical earthquake engineering, that is the term liquefaction where the strength of the soil is significantly reduced, often dramatically, due to the earthquake forces to the point where it is unable to support the structure or remain stable. In this condition of soil liquefaction what happens? The solid soil grains try to behave like a fluid material or it completely loses its shear strength. So, that is what during the liquefaction process, if some soil on which a foundation has been constructed for a super structure if that soil gets liquefied or it flows like a fluid or behaves like a fluid or watery material; obviously, one can easily understand that the total structure constructed on that soil cannot remain stable, but it has to collapse or tilt or could penetrate inside the soil strata.

Through this picture we have seen the case study or example of Japan Fukui earthquake of 1948 the liquefaction related failures. This portion shows the liquefied zone and after liquefaction, what happens? The secondary part or the immediate effect of soil liquefaction is that once the soil gets liquefied that fluidic material or watery material starts flowing from higher gradient to lower gradient, like the water flows as we know. That term is called lateral spreading. After liquefaction the immediate effect is the lateral spreading. And that spreading can occur even at a very, very gentle slope of even 0.5 degree slope also that kind of lateral spreading can occur. Through this picture, as we can see, this huge structure which was constructed on a foundation after the failure of the soil due to soil liquefaction, you can see the amount of tilting which occurred on this super structure. Finally, it totally collapsed or tilted like this.

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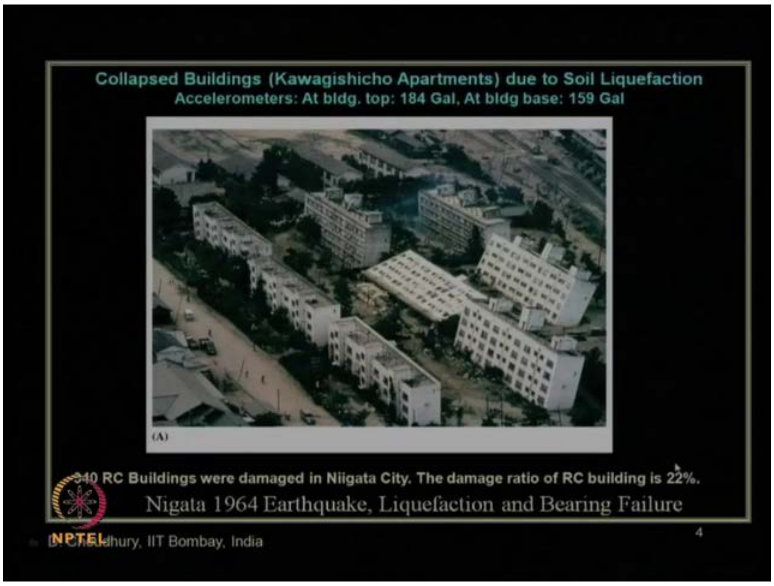

We have also seen in our previous lecture that the collapse of building due to the liquefaction failure or soil liquefaction. This is a common picture, as I have already mentioned in my previous lecture, which one can find out as the cover picture of the book written by Steven L Kramer. This is a very good example, I will say, where the geotechnical earthquake engineering or the soil liquefaction is important. You can see, this is the picture of the Nigata 1964 earthquake in Japan after the failure due to liquefaction and bearing failure.

Some of the houses in this row are standing as it is after the earthquake, but the same type of house in the near vicinity got collapsed like this or tilted like this, why? Because based on the local soil profile there can be a possibility of soil gets liquefied at certain pockets because of the soil data. That is the reason why it is not only necessary to design a correct super structure to withstand earthquake, but it is also very highly necessary to go through the dynamic soil properties at various locations of the proposed construction or structure at a particular site. So that, we can estimate the possibility or chances of soil liquefaction through a estimation of liquefaction hazards etcetera; which we will discuss of course, through this course of geotechnical earthquake engineering in due course of time.

This collapsed building of Kawagishicho Apartments due to the soil liquefaction has been shown over here. The accelerometer reading say, at the top of the building it is 184 gal whereas, at the base of the building 159 gal. And about 340 reinforce concrete buildings were damaged in the entire Nigata city after this Nigata 1964 earthquake. And the damage ratio of this reinforce concrete building is about 22 percent, you can see over here, compared to other Fukui earthquake which we have discussed in our previous lecture; where the damage was 100 percent; where there was non reinforced buildings etcetera.

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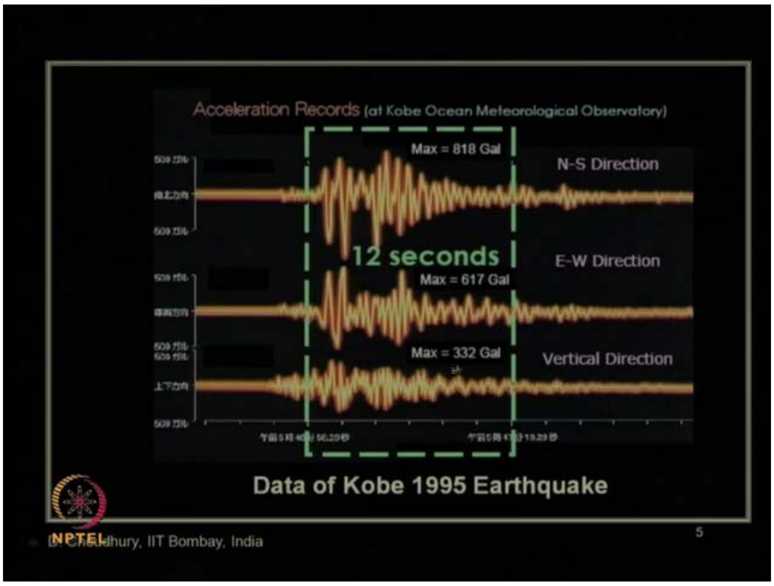

We have studied also in the previous lecture about the earthquake data of 1995 Kobe or Hanshin earthquake. This is the great Kobe earthquake; we have seen the acceleration record at the station of Kobe ocean meteorological observatory point. This acceleration record shows the horizontal acceleration in the North South direction, the other one is showing another horizontal acceleration in the East West direction and the third acceleration record shows the vertical direction acceleration record. So, acceleration versus time record; you can see over here, typically for about 12 seconds that major earthquake was occurring during that 1995 Kobe earthquake. With a maximum value or p g a in the horizontal direction, as 818 gal in the North South direction, say with 617 gal in the East West direction and 332 gal in the vertical direction at that location of that Kobe ocean meteorological observatory.

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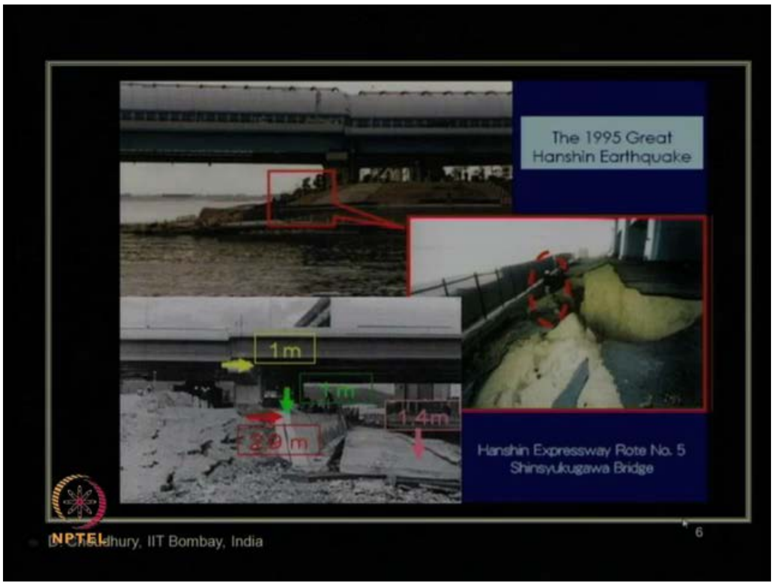

We have also discussed in the previous lecture about the geotechnical damages due to liquefaction and lateral spreading which ultimately damage the super structure, also after this 1995 Kobe earthquake in Japan or the great Hanshin earthquake. You can see in this picture, this is the bridge apartment. That apartment after the soil gets liquefied and after that lateral spreading has occurred, you can see the movement of this portion by horizontally about 2.9 meter and vertically about 1 meter. At this level the movement was in the super structure about 1 meter in the bridge and at this location the vertical movement was about 1.4 meter. If we see this bridge of Kobe after the damage, that is in the Hanshin Expressway Rote number 5, you can see, this is the typical height of a human being or a of a person. Compared to that the amount of vertical displacement and lateral displacement is much more than even a human height, that is why it is shown over here just to make a comparison to understand how much big settlement or big movement can occur after the liquefaction and that lateral spreading has occurred.

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We have also seen in our previous lecture that is the sand boiling or the soil liquefaction in a open ground. That is how the ground water rushing out to the surface after the liquefaction. This is after the soil gets liquefied at the shallow death. Entire water or muddy water, I will say, comes out or gushes out like this. So, sand blow in mud floods used for salt production in the southwest of Kandla port in Gujarat, this happens after the 2001 Bhuj earthquake in Gujarat.

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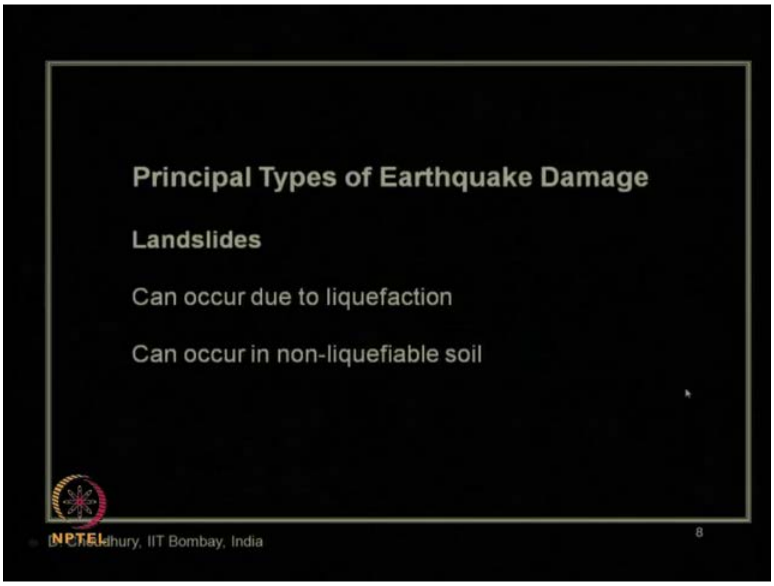

In the previous lecture we also discussed about the principle types of earthquake damages due to the landslides also. And we have mentioned that landslide can occur due to liquefaction, also it can occur in non liquefied soil. That is it can even occur in rocky strata or rocky slopes also where there is no chance of getting the material to get liquefied. Even steep soil or when there is no water present there is no chance of getting the material to get liquefied; even in that case also due to extra horizontal load or shaking due to the earthquake the slope or the instability problem of the slope may arise due to this earthquake. So one of the damage can be due to the liquefaction for the landslide to occur or the slope instability to occur, another can be even in the non liquefied soil due to the additional earthquake load. All this landslide problems or slope stability problem we will discuss later on in this course when we will discuss further about the design of slope and stability aspects under the earthquake condition.

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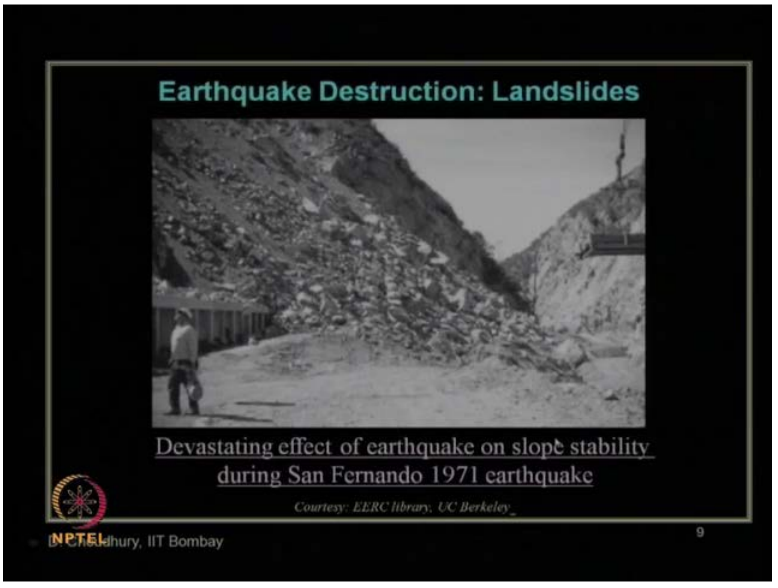

Through this picture we have shown that, what is the devastating effect of earthquake on the slope stability after the San Fernando earthquake of 1971; this is the landslide that occurred after the earthquake.

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We have also seen that earthquake destruction on the retaining structure, that is you can see, this is the portion of the retaining wall which withstood the earthquake of September 1999 Chi Chi earthquake which occurred in Taiwan. But some location, there was a failure of the retaining wall; you can combine it probably with slope stability also for this retaining wall, as you can see, this wall is constructed on a slope. This is the exacerbated view of this portion that is entire thing came down. So, retaining wall also got failed as well as the slope instability occurred, under the earthquake condition of this Chi Chi Taiwan earthquake.

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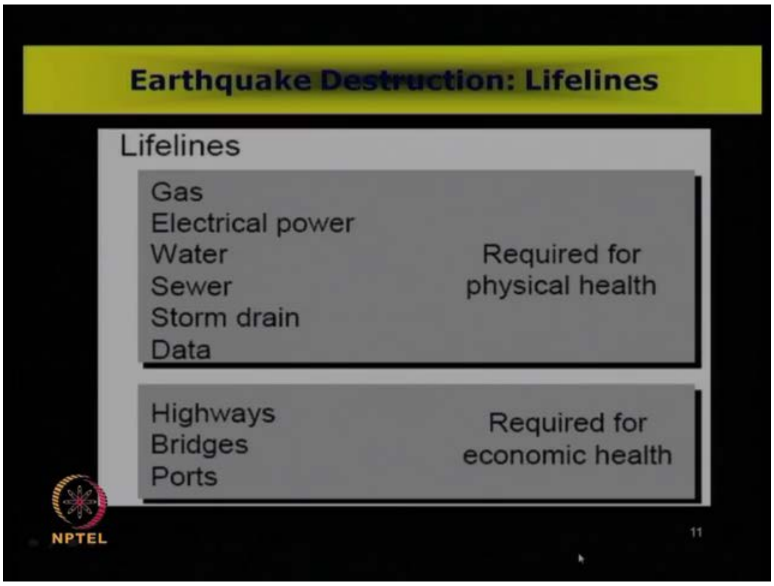

We have also seen in our previous lecture that earthquake destruction can make a disastrous effect on the lifelines also. And for that we have categorized the lifelines in two major categories like: one set of lifelines, we can call them, those are required or essential for the physical health of a human being or a mankind or the society rout to run and another set of lifelines which are necessary for the economic health of a country or of a city or of a civilization or of a society. These are the lifelines which are necessary for a physical health of human being like gas line, electric power line, water line, sewer line, storm drain line and data points.

Whereas, for the economic health of a city or of a country or of a society the lifelines are highways, bridges, ports, airports, etcetera. These structures when they get damaged during the earthquake the disaster or amount of damage is much more on the society because these are the basic lifelines. Why life lines? Because without this surviving or the normal behavior of a mankind or a society is not possible. That is why these are called the lifelines. If these lifelines are getting damaged during an earthquake then we can easily expect that there will be severe damage or severe loss in a particular locality. So, we have to make sure that no such earthquake destruction occurs mainly on these lifelines, which are required either for physical health of the human being or for the economic health of the society.

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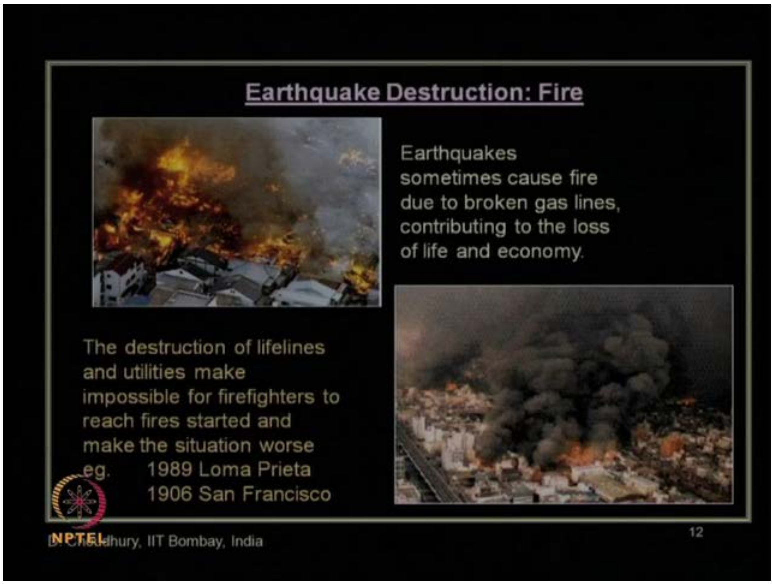

We have also seen the secondary effect of earthquake destruction through fire. In the previous lecture we have discussed it that during earthquake, what happens? Sometimes the gas lines or the pipe lines which carry the gas or oil etcetera or even the electrical lines, they get damaged or they get broken because they will be running says few meters to several kilometers also. Once these lifelines get damaged what happens? There is high probability or high chances that these gases coming out from this broken gas line or the pipeline, and this oil which are coming out or even the electrical lines which are getting disrupted, they can easily ignite or can catch fire in a huge locality after the earthquake.

So, along with the earthquake related damages in a area, this fire hazards are indirect effect, but very well connected to the earthquake damages and it propagates at a so faster rate that damages will be much more in that locality. The destruction of the lifelines and utilities make impossible for the fire fighters to reach the fires started and make the situation much worse. You can see the example in this picture like 1906 San Francisco earthquake in California and US. That is an example where not only due to earthquake, but more hazard or more damages occurred after the earthquake due to the fire related disasters. Also another example of fire related damages due to the earthquake destruction of broken gas line was Loma Prieta earthquake of 1989 in again California, in USA.

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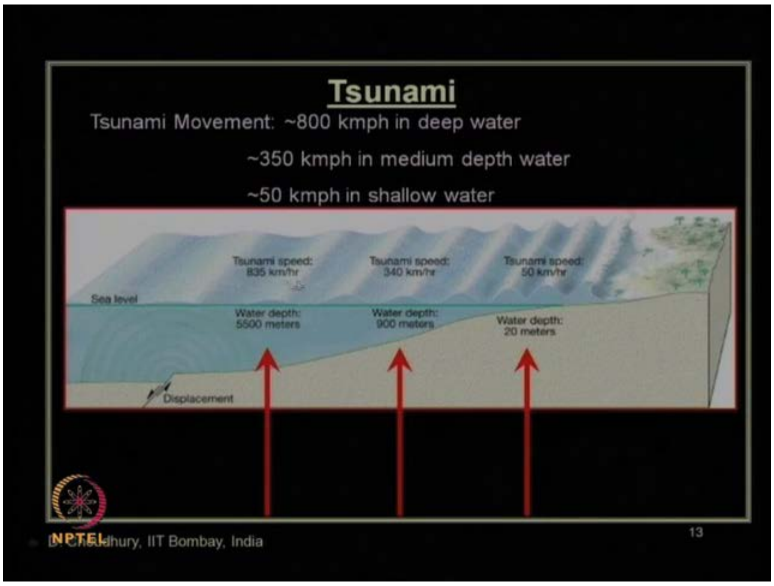

Another earthquake related events, we have already discussed in our previous lecture that is Tsunami, the harbor wave. The tsunami when it gets generated at the large water body or sea or ocean due to some vertical component of a displacement of the sea floor or the ocean bed, then what will happen? This wave will get generated because this standing water above this floor bed will get disturbed from its equilibrium position, due to this vertical component of the movement of this floor. And once it starts flowing, then the speed of the tsunami will be much higher at this deep sea location, but height of the wave will be much shallower. But as it approaches close towards the coast or towards the shore, you can see, it will increase the height of Tsunami, but speed will reduce drastically. That is why, this is the typical speed in deep water, this is the typical speed in medium depth water and this is the typical speed in shallow water. But height continuously rises because the total energy flux of Tsunami remains almost constant.

To maintain the energy flux which is nothing but proportional to the multiplication of this speed and the height, so one component decreases that speed means, automatically the height has to increase when the tsunami the reaches to the coast. As a result when the Tsunami wave strikes the coastal region or the shore it comes with very large amount of height, even 10 meters or much more than that, like we have already noticed into 2011 Tohoku earthquake. After Tohoku earthquake of Japan in March 11, 2011 there was a very big Tsunami of height more than 10 meter at some places, and that 10 meter high which is more than 3 storied building heights. That much high tsunami wave came and once it comes to the shore or coastal region; obviously, it devastates or entire region will be inundated and everything will be washout or flowed out.

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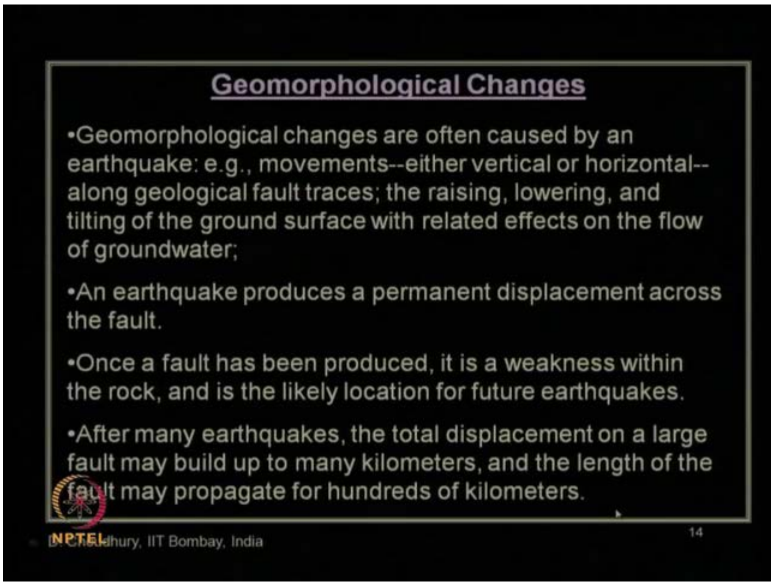

Due to earthquake, we have also seen that the various geomorphological changes occurred, we have discussed in our previous lecture like ground surface will either move vertically up or down, also horizontally they will move. Geological fault traces will form the raising and lowering and tilting of ground surface. Any changes of all this kinds can happen to the ground surface after an earthquake, and it also effect the flow of ground water that is in which direction it will flow. That is suppose a region was at higher gradient to lower gradient after that quick just reverse of that, that is the higher gradient settles so much that it become a lower gradient. So; obviously, the ground water flow will change its direction. Not only that the depth of ground water also will change because of the change of this ground surface due to earthquake. So, that is why all this geomorphological changes can occur after an earthquake. An earthquake produces a permanent displacement across the fault which, we have learnt in the previous lecture, and once a fault has been produced it remains as a weakness within the rock forever. That in no way one can remove that weakness in that rock. So, those are the points where future earthquakes can occur. And after many earthquake the total displacement on a large fault may build up to many kilometers, that may be the length of a fault. And the length of the fault can go hundreds of kilometers, so with that length of the fault.

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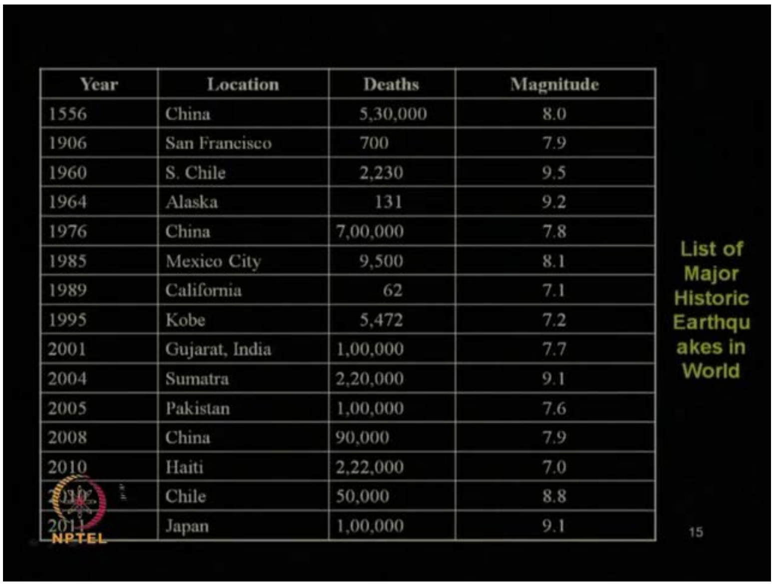

Year	Location	Deaths	Magnitude
1556	China	5,30,000	8.0
1906	San Francisco	700	7.9
1960	S. Chile	2,230	9.5
1964	Alaska	131	9.2
1976	China	7,00,000	7.8
1985	Mexico City	9,500	8.1
1989	California	62	7.1
1995	Kobe	5,472	7.2
2001	Gujarat, India	1,00,000	7.7
2004	Sumatra	2,20,000	9.1
2005	Pakistan	1,00,000	7.6
2008	China	90,000	7.9
2010	Haiti	2,22,000	7.0
2010	Chile	50,000	8.8
2011	Japan	1,00,000	9.1

We have seen list of major historic earthquakes, around the world there are several large earthquakes. Some of them are listed over here, you can see the latest one I have mentioned is 2011 Japan earthquake where number of deaths where more than a lakh as per official record and magnitude was 9.1.

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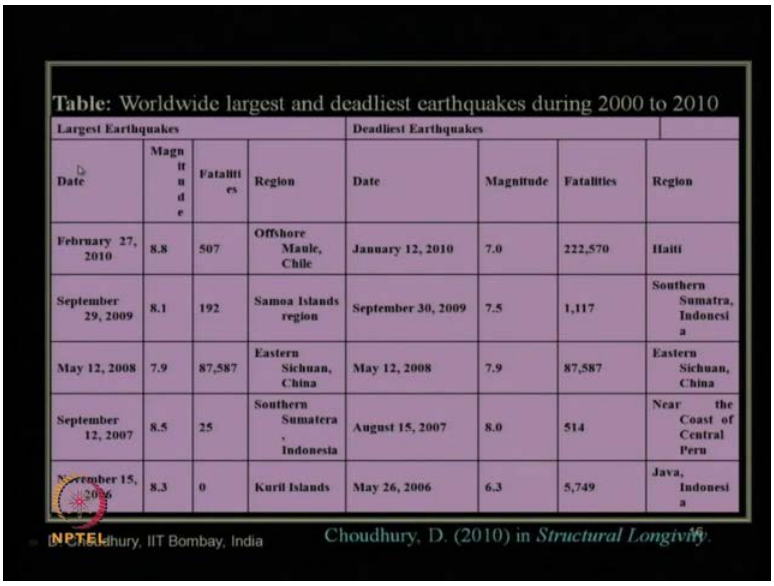

Table: Worldwide largest and deadliest earthquakes during 2000 to 2010

Largest Earthquakes				Deadliest Earthquakes			
Date	Magnitude	Fatalities	Region	Date	Magnitude	Fatalities	Region
February 27, 2010	8.8	507	Offshore Maule, Chile	January 12, 2010	7.0	222,570	Haiti
September 29, 2009	8.1	192	Samoa Islands region	September 30, 2009	7.5	1,117	Southern Sumatra, Indonesia
May 12, 2008	7.9	87,587	Eastern Sichuan, China	May 12, 2008	7.9	87,587	Eastern Sichuan, China
September 12, 2007	8.5	25	Southern Sumatera, Indonesia	August 15, 2007	8.0	514	Near the Coast of Central Peru
November 15, 2006	8.3	0	Kuril Islands	May 26, 2006	6.3	5,749	Java, Indonesia

Then we have discussed about the world’s largest and deadliest earthquake during the period of year 2000 to 2010. And all the details, as I have mentioned is available in this journal paper of journal Structural Longivity in 2010. It is not necessary that largest or maximum magnitude earthquake need to be the deadliest one, that is it cause the maximum killing of people as well as loss of life and loss of property. Why? Because it may happen, that sometimes, this large earthquake occurs at a place where number of people staying is much less or number of houses located there are much less or the number of houses constructed at that location are very well designed to withstand the large magnitude earthquake.

So obviously, it will not cause a large amount of damage. Whereas, if another earthquake occurs at a another place which may not be a very high magnitude or largest magnitude of that year, but still it can occur or it can be considered as the deadliest earthquake, because it occurs in a very highly populated area or in an area where the buildings and structures are not properly designed to withstand the earthquake forces or non engineered constructions are there.

For example, you can see in this table, it is listed like in 2010 the largest earthquake is nothing but the Chile earthquake of February 2010 of magnitude 8.8. Whereas, the deadliest earthquake of 2010 is the Haiti earthquake which is having a much lower magnitude than this Chile earthquake, that it is about 7 magnitude, but number of fatalities was much, much more than that Chile earthquake and because of the damages occurred in that region of Haiti. Similarly, for other years also all the list has been given and we had discussed it in the previous lecture.

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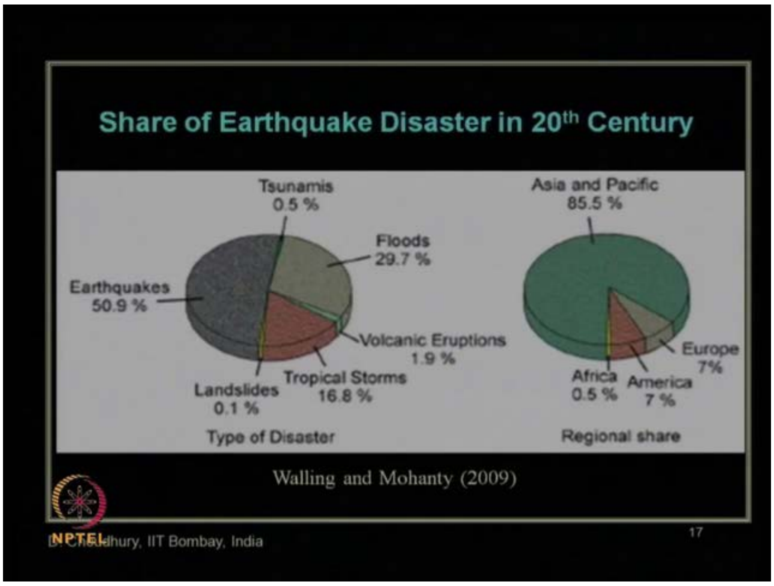

In the previous study we have also discussed about the share of earthquake disaster in the entire 20 th century, as it given by Walling and Mohanty in 2009. In this pie chart one can easily see, what is the need for doing the research in this area of earthquake engineering? Because you can see among all the disasters in the entire 20 th century the share or the portion of earthquake disaster is the maximum. It is more than 50 percent, it is about 51 percent among all other disasters worldwide. And second highest one is nothing but the flood related disasters, which is close to 30 percent. The third highest is the tropical storm which is about 17 percent, the fourth highest one is the volcanic eruption close to 2 percent, the fifth one is the Tsunami related disaster hazard and the sixth one is landslide related hazard.

Automatically this pie chart shows, for the entire century, of 20th century that earthquake takes maximum share of the disaster; which automatically says why a research, further research on this earthquake engineering very, very essential, and that to for which region? If we talk about the regional share of this disaster of earthquake, you can see over here, Asia and pacific region shared more than 85 percent share of that earthquake disaster. And entire Europe about 7 percent, entire America about 7 percent and Africa about just 0.5 percent. This is the share of the earthquake disaster as per various regions wise. So, it is very clear to us, that our country India being staying in this region of Asia and pacific we need to do more research on this earthquake engineering, on geotechnical earthquake engineering, in particular related to earthquake because of this reason.

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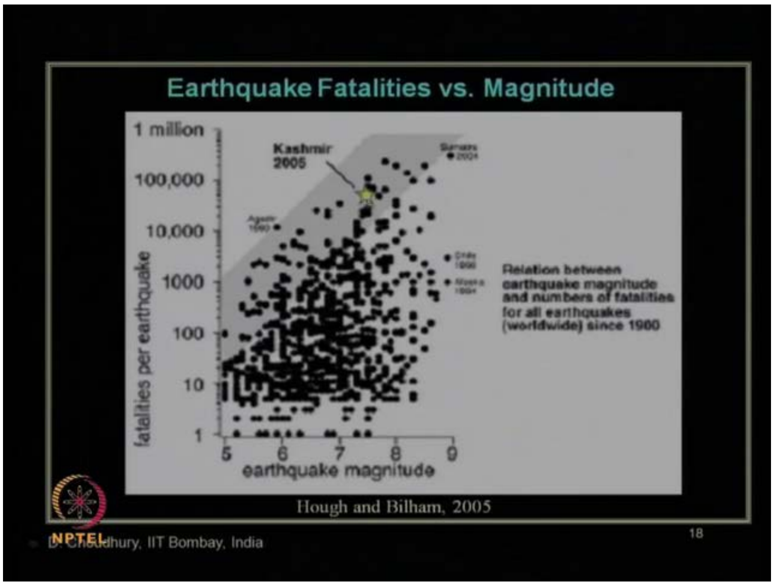

This slide we have discussed in the previous lecture, like earthquake related fatalities versus the magnitude, we can see over here Hough and Bilham in 2005. They have mentioned that how many fatalities of earthquake and earthquake magnitude; their relationship, worldwide relationships. Since the year 1900 they have given the data that is 1900 up to 2005, and plotted the results. It is quite expected that earthquake magnitude the higher means, number of fatalities will be higher. And lower earthquake magnitude will be giving lower number of fatalities. But you can see some reverse trained also.

That is suppose, let us look at this point, that is at higher earthquake magnitude, number of fatalities pretty low, why? As I have already mentioned, chances are may be this are higher, earthquake magnitude locations are not in a highly populated area, that is may not be there is huge number of people staying that is why the number of fatalities is less and properly engineered design. On the other hand, you can see, at small magnitude of earthquake large amount of fatalities occurred. Because this extreme can be considered as like, may be, these are the highly populated area. And a combination of them can be the constructions which were existing at that site where non engineered or not well designed to take care of, even a small magnitude or moderate magnitude of earthquake. So compare to that the Kashmir earthquake of 2005 falls under this category. That is the magnitude somewhere here and number of fatalities of somewhere here, it is pretty high in this scale. So, with that we have completed our previous module that is module number one in our previous lecture.

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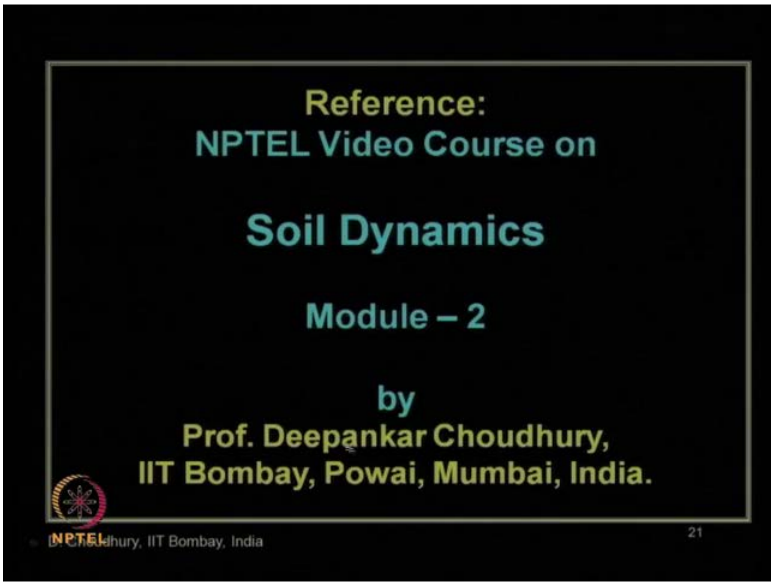

In today’s lecture let us start our next module that is module two which is basics of vibration theory. In this module two we will talk about the basics of vibration theory. But before starting this module two, let me give a reference for this module two that is basics of vibration theory. That reference is nothing but another video course of NPTEL on the topic soil dynamics. In the same NPTEL video course soil dynamics another module we have, module number two, which was given by me, that is I took that lecture of soil dynamics for the NPTEL video course. In that module two we have discussed thoroughly these basics of vibration theory.

So, in this course just I will recap some of those basics of vibration theory. Because for this advanced course on this geotechnical earthquake engineering it is not our main goal to understand the basics of simple vibration. To learn that simple basic fundamentals of vibration theory I will request all viewers to go through my another video course developed for NPTEL which is on soil dynamics. And module two of that course will give you the detail idea about the basic fundamentals of this vibration theory. So, I will just quickly go through, for this course of geotechnical earthquake engineering about the fundamentals of basics of vibration theory.

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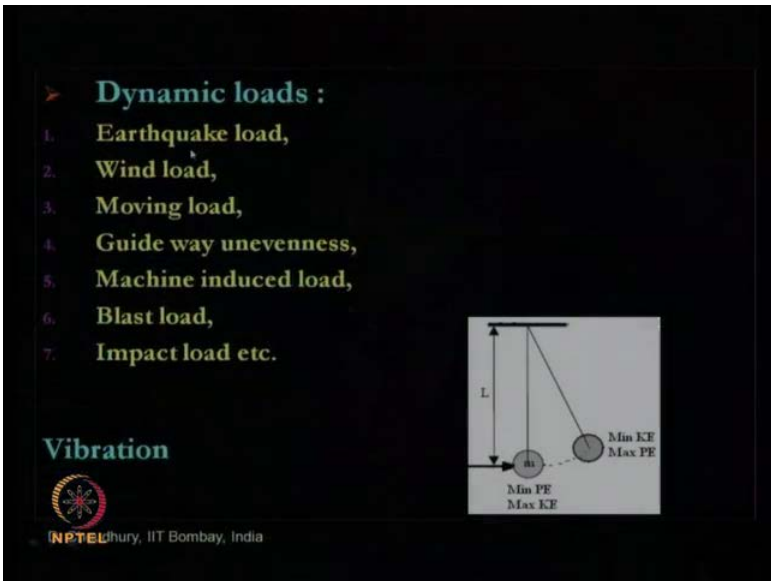

We have discussed what is called, first of all, the dynamic load? Dynamic load is nothing but a load which varies with respect to time first of all. That is the difference between a static load and the dynamic load. Now, if it varies only with respect to time can we call it a dynamic load? No. Another requirement is that, that time varying load which is acting on a body or on a system should create a vibration. What is a vibration? Vibration, we can say technically, it is nothing but a continuous exchange of potential energy to kinetic energy and vice versa, for a body which is subjected to this time varying load or the dynamic load. Unless the dynamic load creates a vibration and creates some sort of inertia in the body we cannot say that time varying load as a dynamic load. What are the examples of dynamic load? The very first example of dynamic load can be the earthquake load, which we are discussing in this course in detail. Because in earthquake loading, these loads are varying with respect to time as well as they create a body to vibrate. That is going through from potential energy to kinetic energy and continuous changing of that potential energy to kinetic energy and vice versa. And inertia will get attached too, due to that application of that time varying load or earthquake load.

Similarly, wind load is another dynamic load which also varies with respect to time, create vibration in a body and create inertia. Moving load also is a dynamic load which varies with respect to time. In addition to that it creates a vibration in the system guide way unevenness that is due to the unevenness of the pavement or run way etcetera it creates a dynamic load on the vehicle which is running or passing through that guide way or pavement or run way etcetera. That is the time varying load as well as it creates vibration on the system. Then, Machine induced load. That is also dynamic load because in the machineries we have the time dependent load as well as it creates a vibration in the system. Blast load is also dynamic load because they also vary with respect to time in addition to that it creates a vibration in the system. Impact load is also a dynamic load because it also varies with respect to time and creates a vibration.

But there are few time dependent load that is which varies with respect to time, but still those are not dynamic load. For example, suppose the construction or excavation period like, in the starting of excavation and end of excavation there is a change of load due to taking out of the material from the ground with respect to time. With respect to time the load has changed, but it is not a dynamic load because it does not create any vibration in the system. Another example is, there is water tank which is suppose filled up in the morning and at the end of the day the water tank is empty. There is a change of load in the system with respect to time, but still it is not a dynamic load, why? Because that time varying load does not create any vibration. Again in a classroom in the morning when the class starts at say 8 o clock in the morning, it was full with the students and teacher, but at the end of the day say when at 5 o clock the class ends there is nobody in the classroom. With respect to time there is a change in the load or time varying load, but that does not create any vibration in the system. So, those type of time varying load, as we know, are called live load. Those are nothing but another form of static load, but those are not at all dynamic load.

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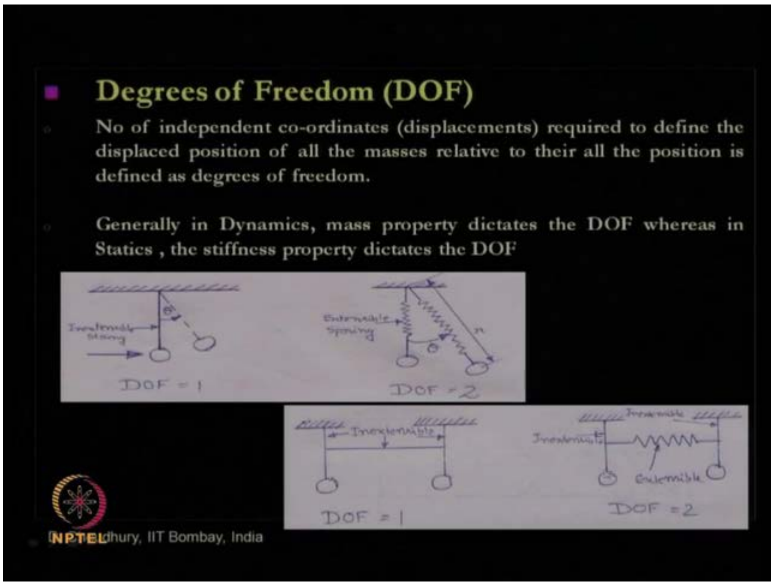

We have learnt in our another course of soil dynamics, what is known as degrees of freedom? This is the very basic or fundamental things to be identified in a process while modelling or analyzing any structure subjected to earthquake loading. Degree of freedom is nothing but number of independent coordinates or number of independent displacement. Coordinates can be defined either in terms of displacement or in terms of velocity or any other suitable parameter. Number of that independent coordinates which are required to define the displaced position or a new position of all the masses relative to their original position is defined as the degrees of freedom. Generally in the dynamics or in the problem related to the dynamic load, mass is the property which dictates about the degrees of freedom. Because the newly displaced position of the mass has to be identified and number of that minimum coordinate or independent coordinate will decide about the degrees of freedom; whereas, in the static problem the stiffness property of the system will dictate about the degrees of freedom.

If we want to see few examples of the degrees of freedom, you can see the simple example of this pendulum. This is an inextensible string, massless; and this is the bob with mass m. If we apply a dynamic load or tap it, it will start vibrating like this. It is having a degrees of freedom1, because if we identify this angle theta by using this coordinate system, this theta only, we can represent the position or displace location of this simple pendulum bob at any point of time, at any location. So, this theta is good enough to mention about the location of this displaced mass. Theta, a variation with respect to time; theta t can be considered as a degree of freedom for this system which is sufficient enough. Thus is the independent coordinate which is good enough to tell about its location we do not need this l, because l remains constant being, this as inextensible string. That is why the degree of freedom, when this problem is 1.

But if we replace this inextensible string with the extensible string or a spring like this, in that case the degree of freedom of the same problem will change to 2, why? In that case we require minimum 2 coordinates, one is theta coordinate another is this distance l or r whatever we say. That is because with respect to time, this theta and r both will change and we should know both this theta t and r t at different time, to mention about it displace position of this mass at different time. So, that is why these two are the minimum number of coordinates or independent coordinates because they are not dependent. That is decides as the degrees of freedom of this system, under the dynamic condition.

Similarly this example here we have 2 extends in extensible string which cannot extend all, or inextensible string and 2 masses are connected through this. Here also degrees of freedom 1, because only one angle like theta, if it makes with the dynamic load; the other one to maintain the equilibrium or stability of the system it also has to make theta. So, that theta is good enough to express its displaced position at any point of time for the entire system. That is the reason why the degree of freedom for this problem is 1. But if I change that connected rod from inextensible to 1 extensible string or a spring like this then the problem change to degrees of freedom 2, but still I am using this ties or this rods are inextensible. So; obviously, here will be one theta say theta 1; here will be another theta, theta 2. Because there will be a displacement at this length. Why two degrees of freedom? Because this theta 1 and theta 2, they are related through this length. It is not degrees of freedom 3, but it is 2. Because independent coordinate or minimum coordinate required to specify that displaced position at any point of time is this 2.

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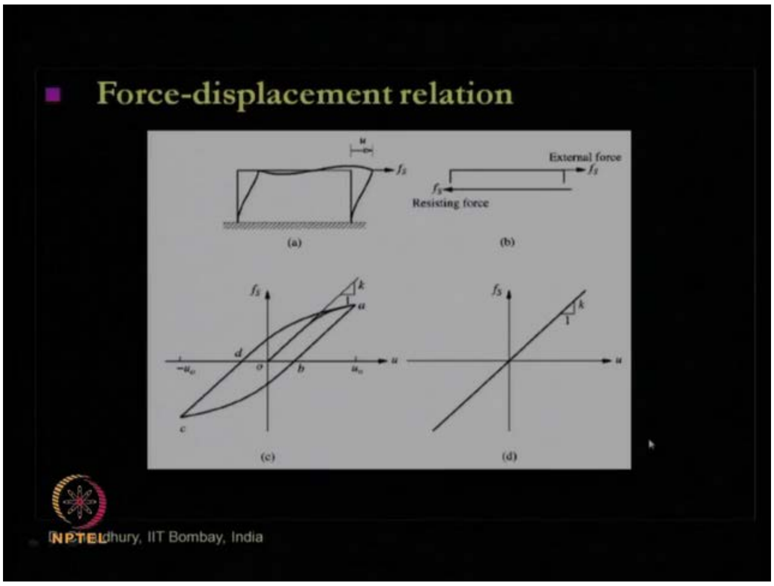

Then Force-displacement relationship, we know, when there is a spring force, the external force, the resisting force can be calculated like this. So, displacement versus spring force can be related through this pattern; that is it goes, then unloading, then again reloading. If we take the linear component of that force versus displacement or behavior of a spring or of a string or of a system we can say this k is called stiffness constant.

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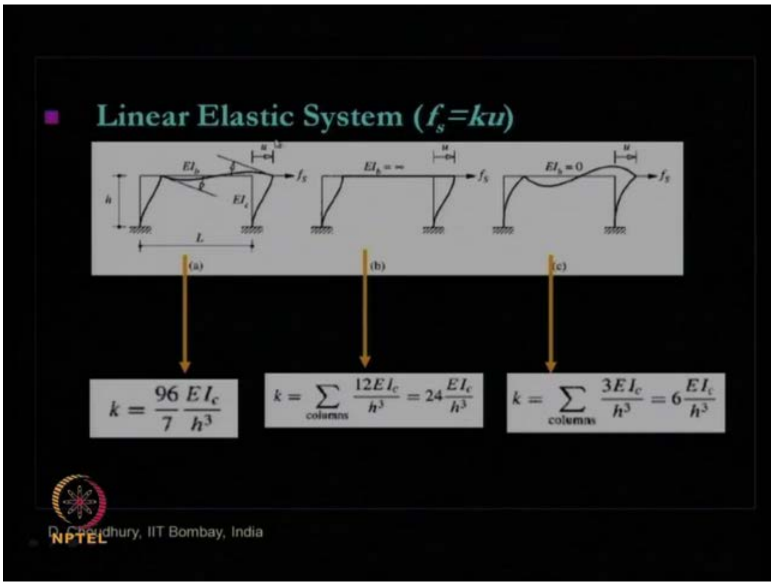

For a linear elastic system, as we have just seen, this spring force can be mentioned as f s equals to k times u; where k is the stiffness or spring constant times, u is the displacement. For different types of structure, depending on their boundary condition we can have different values of the stiffness. Like for this both fixed end structure single, single bay structure like this where we have the E I of column and E I of beam mentioned over here. This is the height of the column, this is the length of the beam, this is the displacement. For linear elastic spring system k can be written in this form that is the column only contributes to this spring constant for this problem. If we have E I value of the beam as infinity, then the displaced position will look like this. Then the value of k will change to the sum of the individual stiffness of the columns which is nothing but 12 E I by h cube, and there are two columns, that is why, it will become 24 E I by h cube E I of column. And if we have the beam stiffness is 0, that is E I of beam is 0, in that case the k will be sum of the individual stiffness of the column which is nothing but 3 E I by h cube for this problem. So, for two columns it will become 6 E I by h cube.

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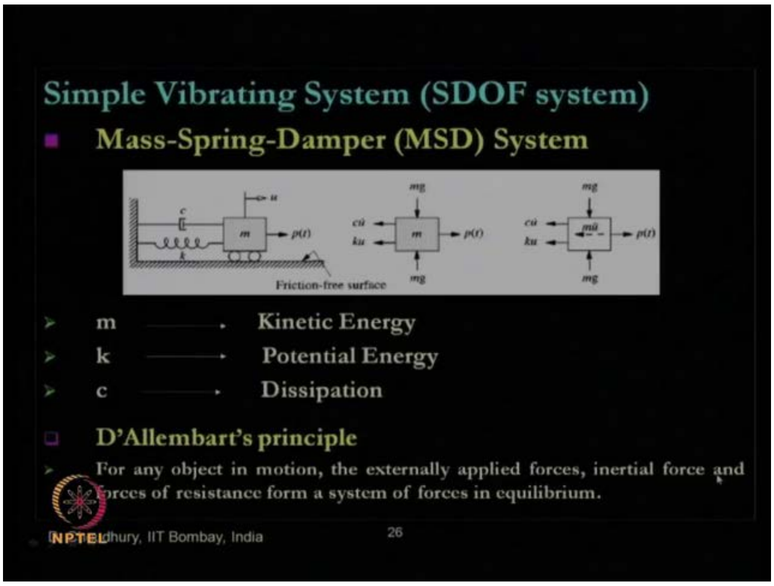

We had come to the simplest form of a vibration system that is simple vibrating system which is nothing but a single degree of freedom system. We have learnt, what is called degrees of freedom? Now single degree means it will have only one degree of freedom, and how to represent that? The simplest form of the simple vibrating system is to mention through a Mass-Spring-Damper system or we call it in short as MSD. That is mass spring and damper MSD model or mass spring port model or damper model which is represented through this picture. That is mass damper with the damping constant c, k as the stiffness, the spring constant; and u is that only degrees of freedom which with varies with respect to time.

And using this displacement or this coordinate we can identify the position of this structure at any point of time; and this p of t is the dynamically applied load of the system. This is the dynamic load; the load which varies with respect to time as well as it creates a vibration on the system. So, this is the fiction free surface and rollers, the weight that the static load will get automatically balanced to this roller reactions or support. We are only discussing here the dynamic component of the forces present. If we draw the free body diagram we can see on the mass, what are the force acting? This is the self weight of the mass m g, and this is the reaction coming from through this two rollers which will balance this. So, statically it will remain in equilibrium and the forces of resistance will be c times u dot is nothing but the velocity k times u is nothing (u is the displacement) but the spring force and c times u dot is nothing but the damper force. So, damper force is nothing but damping constant times the velocity and p t is the externally applied dynamic load.

Inside the mass there will be a inertia component getting developed. So, inertia force will be m times u double dot u. Double dot is nothing but the acceleration which is produced on the body. That creates mass times acceleration is nothing but inertia force as we know some simple Newton’s law. So, the mass represents the kinetic energy part, spring represents the potential energy part. As I have already mentioned the vibration is nothing but continuous change of kinetic to potential energy and vice versa. So, these two parameters are essential to represent any vibrating system or to analyze any dynamic system. The minimum basic two parameters we require is mass and spring.

Why this damper or dissipater comes into picture, because if the vibrating system starts vibrating; obviously, it will not vibrate forever. There will be some dissipating forces or dissipating energy which are nothing but in the form of sound energy, heat energy and so on. Those dissipation or loss of energy can be represented through this damper or through this c. That dissipation is also necessary which is produced to this damper or dashpot another name we use. Hence this mass spring and dashpot create a simple vibrating system of a single degree of freedom system as shown over here; otherwise mass and spring also can create a simple vibrating system if we do not consider any loss of energy.

Now, what is D Allembart's principle? We always use D Allembart's principle to maintain the equilibrium of the state or that the dynamic loading condition. It says that for any object which is in motion the externally applied forces say it is p of t here, inertial force it is m u double dot here, and the forces of resistance that is c u dot and k u, because when this p of t is applied on the system it tries to pull it in this direction at one instant. In that point, this damper force will try to put it or keep it in the original position and the spring force also will try to put it or keep it the system in the original position. That is why these are the direction of this forces. As well as the inertia force, inertia of the body also will try to keep it in the original position to maintain the equilibrium. That is why all these are forces of resistance. This is damper force and this is the spring force. So, all this forces will maintain a system of forces which will remain in equilibrium that automatically says that m u double dot plus c u dot plus k u should be equals to p of t. So, that comes from this application of D Allembart's principle.

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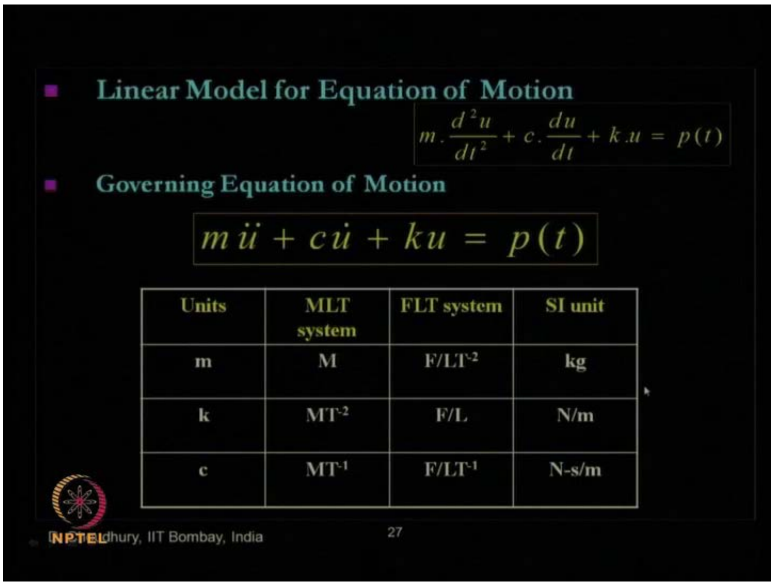

Hence, if we write the linear model for the equation, what we can do? The behavior of that m, we have to make sure that the mass is not changing with respect to time. If mass changes with respect to time we cannot write it simply like this because in that case mass also will be a variable which changes with respect to time. For example like a rocket when we are designing we cannot write it in this easy format or simple format. Similarly for a damper or dashpot if it is linear, then it will behave like this. That is the velocity versus the force, damper force versus velocity will follow a linear line passing through the origin with the constant of that line as c which is nothing but the damping constant. It is valid only for a linear dashpot or linear model, if we want to consider say design of wind tunnel or chimney etcetera there we cannot use this simple model, because they are it is we had non-linear variation of that c or the damper force with respect to the velocity. This is only for the linear model. Similarly, we have seen for linear spring only we can write this k times u as the spring force; otherwise it will be a non-linear variation as we have already seen in one of the picture like this.

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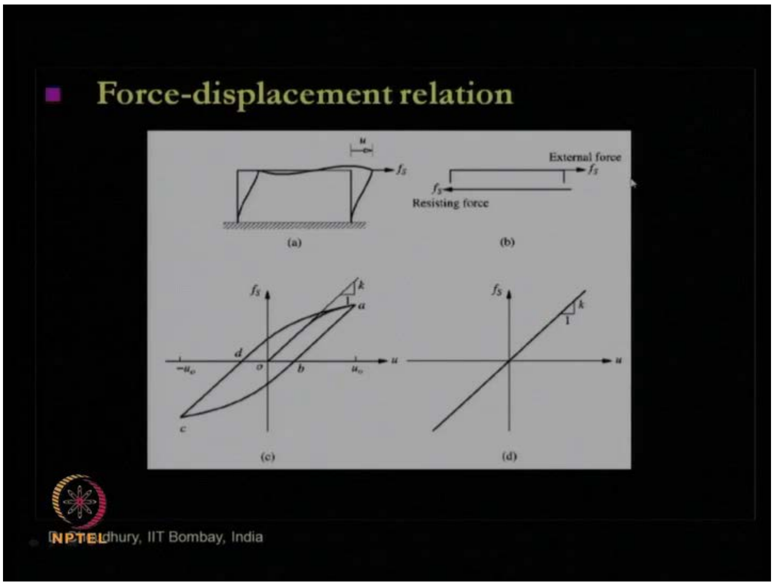

If somebody wants to consider this variation of the spring force versus displacement, this should be non-linear variation. But it has been idealized by this linear variation of the spring constant, with respect to this k. This is the linear variation passing through the origin. That is the reason why we mention over here that linear model for the equation of motion can be simplified like this, it can be written in this form. It is called the governing equation of motion for a linear system, for a single degree of freedom system of a simple vibrating system which is nothing but as I said, m u double dot plus u dot plus k u equals to p of t. And the corresponding units which are used for this m parameter k parameter and c parameter are in MLT system. In FLT system and in SI unit commonly used; and for mass we use k g, for stiffness we use newton per meter, for damper or damping constant we use newton second per meter units for the SI unit.

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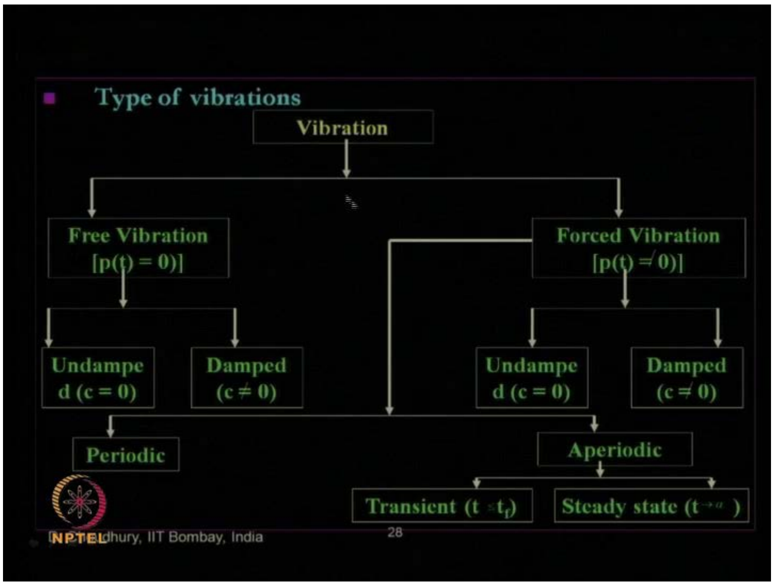

Then about the various types of vibration; Vibrations can be categorized into two major components: one is called free vibration, what is called free vibration? when that externally applied dynamic load that is the p of t is 0; then why that vibration occurs? Because that vibration occurs due to the initial displacement and initial velocity, initial tapping or initial application of a force is necessary to create a free vibration. that is why either the initial displacement or the initial velocity or the combination of them should be non zero when we are considering a free vibration. Because there is no other externally applied dynamic load acting on the system; it is trying to vibrate freely. So, that p t should be 0 in that equation that is called free vibration; otherwise if that p t is non zero, if some non zero value or some dynamic load is externally acting on the system, then we call it as forced vibration.

Within free vibration we have sub classification as undamped that is when there is no damper c is 0 that is, we are not considering any dissipation of energy or any loss of energy, that is called undamped free vibration system. And damped free vibration system is that when we are considering that damping is non zero that is damping is present. Similarly force vibration also can be sub classified as undamped forced vibration when we do not consider any loss of energy and damped forced vibration when we are considering loss of energy through the damper. Another way of sub classifying the force vibration is called periodic and aperiodic. What is periodic forced vibration? If the behaviour of the vibration or the load which is acting on the system is repeating after certain intervals that is the same behaviour is repeating after certain interval then we call it as periodic. And what is aperiodic? When it is not repeating after a certain interval then we call it as aperiodic or another word we used for this is random.

So, within aperiodic or random again we have two sub classification: one is called transient type aperiodic force vibration, when the time for which the dynamic load is acting is for a finite time duration. For example, the earthquake loads. Our earthquake loading problem we have to discuss as a transient aperiodic forced vibration because here the application of that load acts for a finite time. Whereas, the steady state where the application of that load acts infinity of the time that can be considered for example, the wind load as aperiodic statistic forced vibration.

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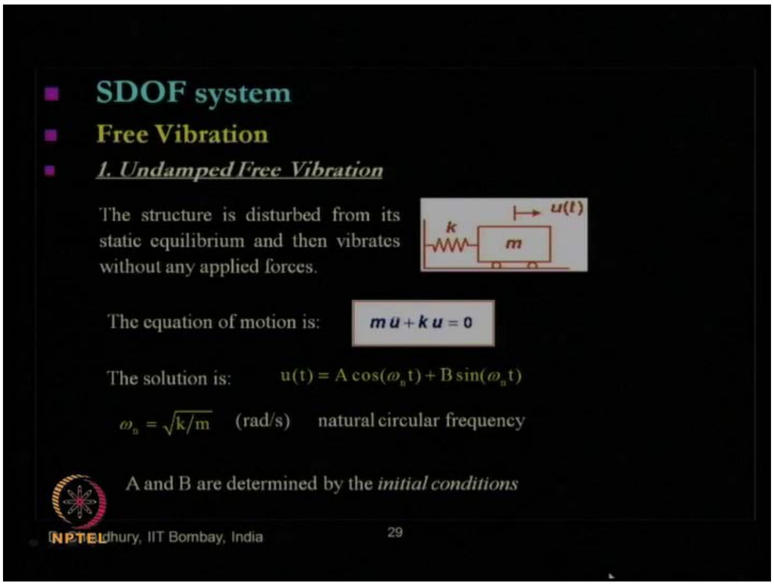

Then we have also seen in our soil dynamics course for this module two, what is the solution for the free vibration under the simplest condition of undamped free vibration. The basic equation will boils down to m u double dot plus k u equals 0, because it is not present as well as the damper is not present. The solution of this second order differential equation is simple; u of t equals to A cosine of omega n t plus B times sin of omega n t. This omega n is nothing but it is called the natural circular frequency which is expressed as root over. k by m k is the stiffness mass and n is the mass it is expressed in the unit of radiant per second which is natural frequency in the circular nature; and a and b are the constants which are determined from the initial condition, as I said initial displacement and initial velocity must be non zero for the free vibration. So, from the initial condition that is at when t equals to 0, there will be some displacement or some velocity or both should be present, from that condition we can get that constants a and b.

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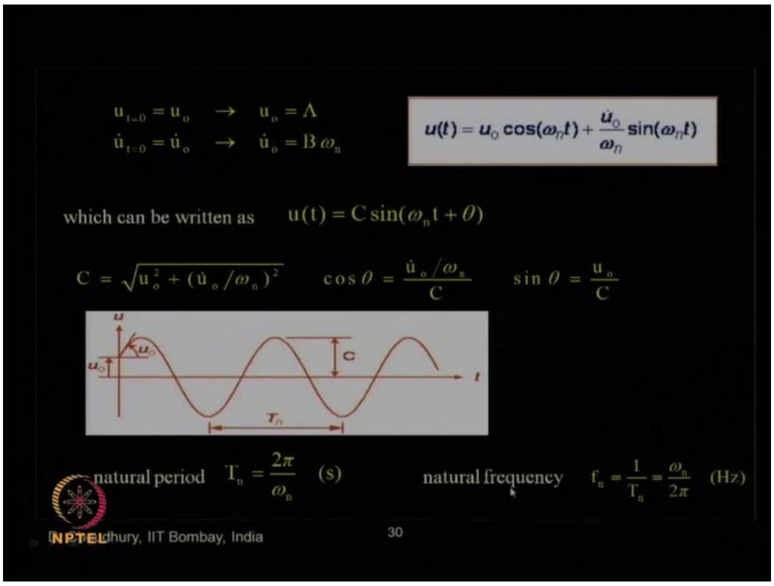

So, using those initial displacement say at t equals to 0 as u naught and initial displacement velocity as u naught dot we can get the constants of A and B and this will be the complete solution, and how to know the behavior? That is u of t with respect to variation of the time. This is the behaviour at t equals to 0, this is the initial displacement the slope at starting gives us the initial velocity and this is called t n, this is called C which is nothing but the amplitude of motion. This amplitude of displacement can be obtained like this. And natural frequency can be obtained f n equals to omega n by 2 pi in the unit of hertz and the natural period can be calculated as 2 pi by omega n in the unit of second.

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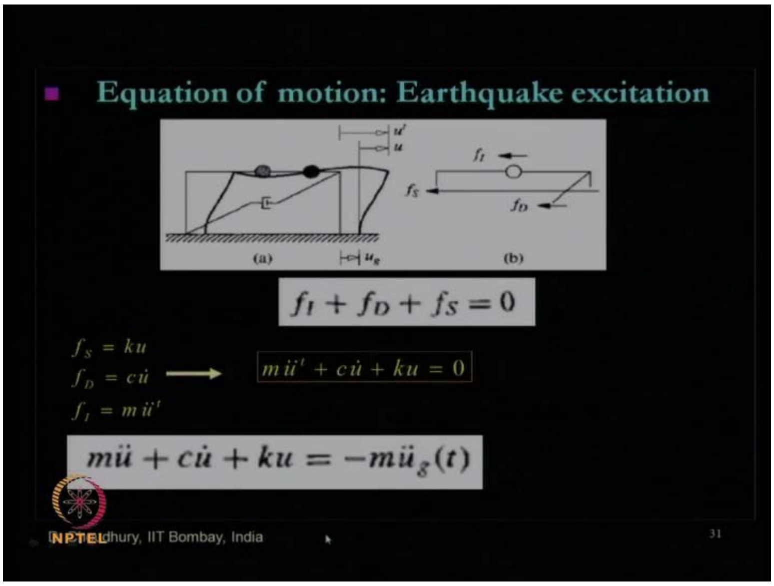

Then for the earthquake excitation, how the equation changes? This is the ground displacement, this is the total displacement at the top for a simple single degree of freedom system concentrated mass over here, this is the damper and this is the relative displacement at the upper portion. We can write down the inertia force, damper force and string force equals to 0. And it has to be put mass is related to this total displacement over here; that means, the inertia force will be related with the total acceleration; and for the damper and spring it will be the relative displacement. So, relative velocity and relative displacement has to be considered which automatically gives us the final equation of motion for a linear system like this for an earthquake excitation.

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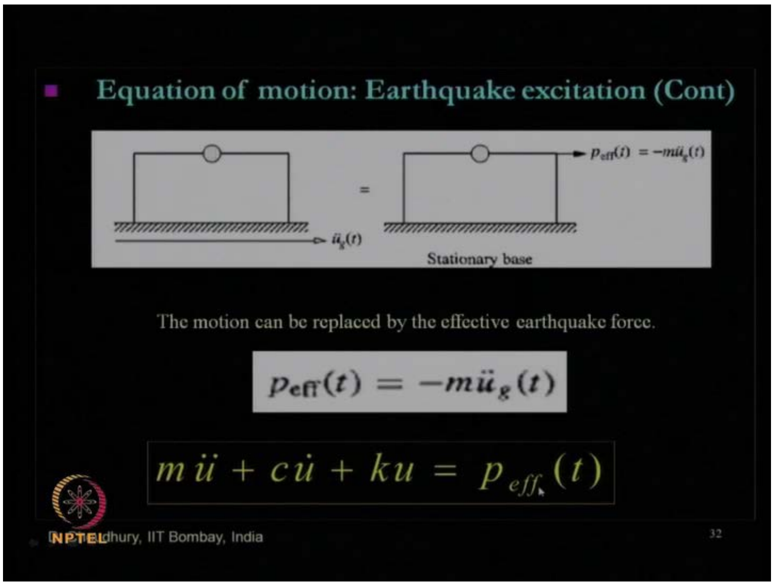

In this case the effective dynamic load or externally applied dynamic load is nothing but this minus m u g double dot times t; u g double dot t is nothing but the ground acceleration during an earthquake. If you multiply with respect to the mass of the system with the negative sign you will get the amount of dynamic load, which is acting on the system due to the earthquake excitation.

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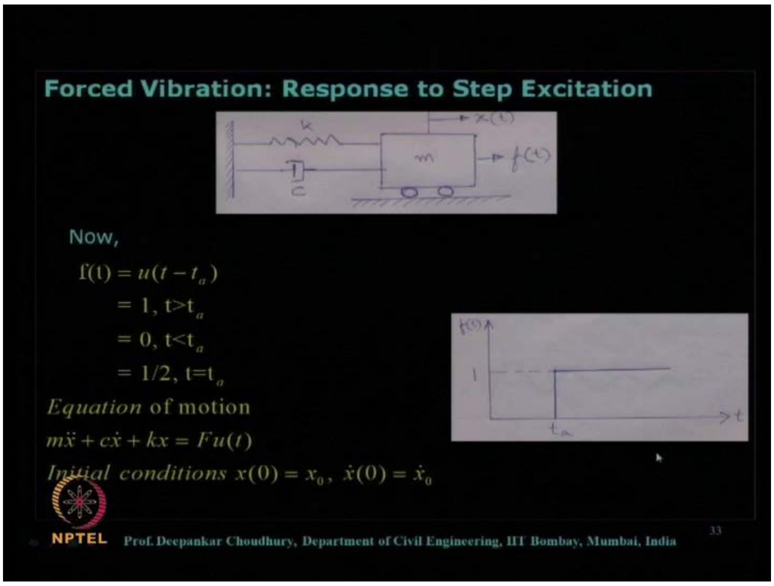

Forced vibration: Response to Step Excitation; how to get a solution for this? Suppose this is the single degree of freedom system, force f t is working; suppose this f t is of a step force in nature like a unit force starting at a time t a and then continuing forever. This f t can be expressed by u of t minus t a it is 1; when t is greater than t a it is zero; when t is less than t a at the point of t a it can be taken as average. So, equation of motion for any other force of f we can express in terms of this unit load.

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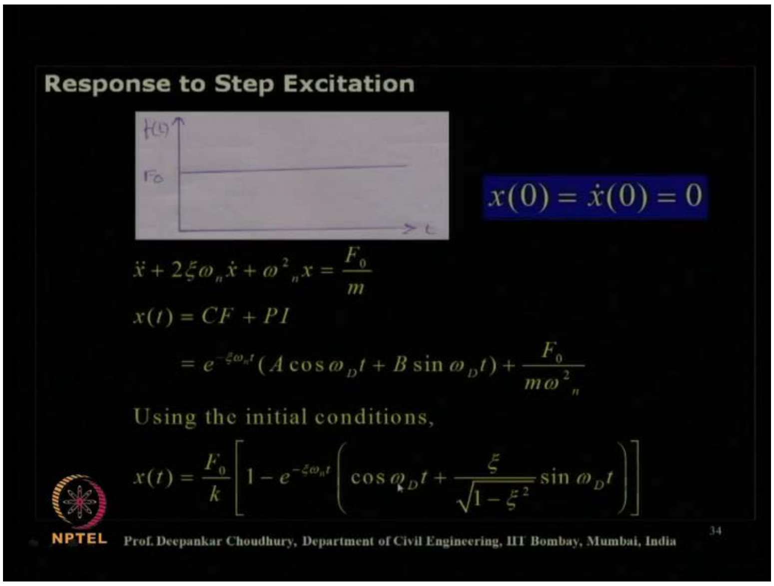

So, f times u of t than initial conditions we can use, that will give us the solution. In this fashion, this second order differential equation will have two parts of solution one is complimentary function and particular integral for determination of complementary function as we know we have to put 0 over here, and then solve this equation which we have done as a free vibration case and with this we have to find out the particular integral. This is the combined solution of the system under this forced vibration.

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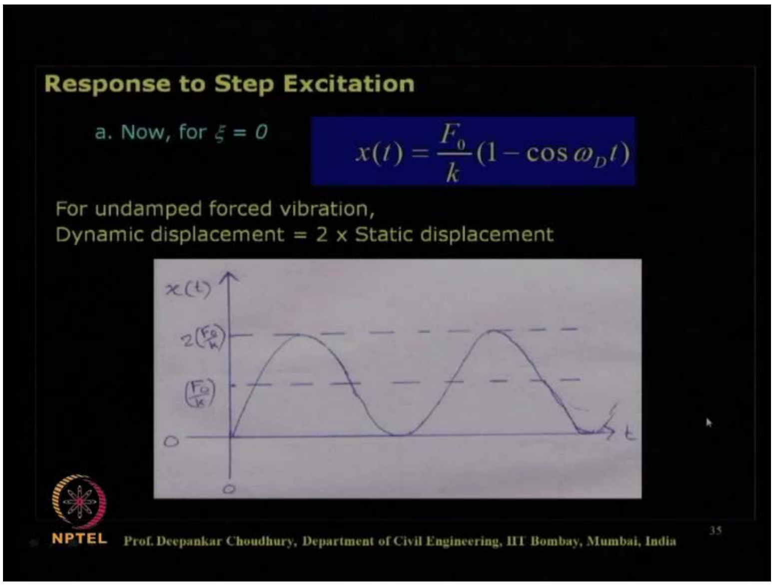

When there is no damper or damping force is present, this will be the behavior. You can see over here, this is the displacement with respect to time and this is the starting point at initial 00 condition, as we have taken over here initial displacement and initial velocity both are 0. For forced vibration we can take both initial conditions as 0, but not for free vibration. So, the static displacement is nothing but that f naught by k. This is the amount of static displacement and you can see the maximum value of dynamic displacement comes out to be 2 times of that static displacement. So, dynamic displacement is much more than static displacement this is another reason why the dynamic load needs to be studied properly.

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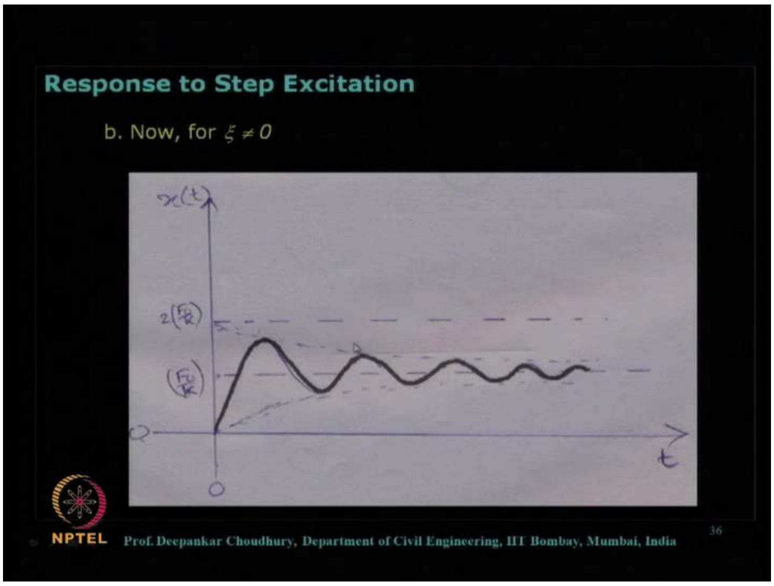

And if the damper is present then the behavior will be the damping force will come into picture, and with respect to time it will dissipate the energy.

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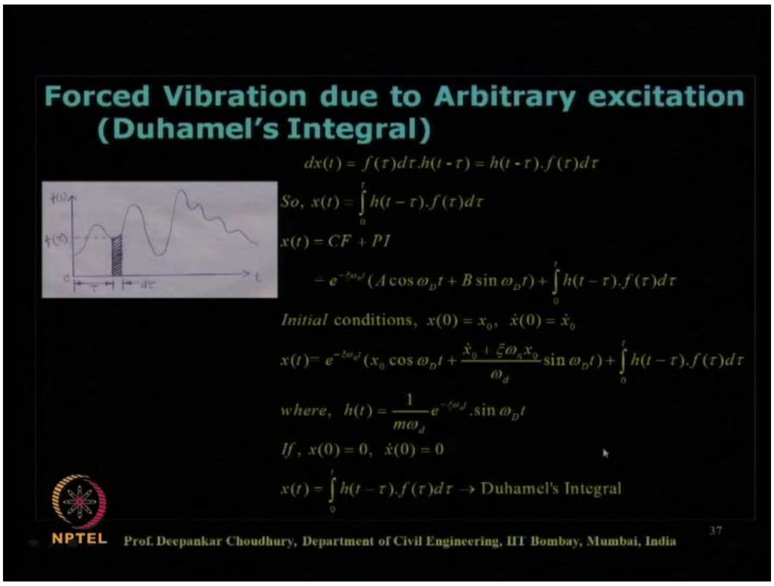

For any type of Arbitrary excitation, like for any random motion which is the case for our earthquake engineering, suppose the force applied, dynamic force applied behaves with respect to time, something like this, which is random in nature. We can take a very small infinite similar portion, at a particular time; after spending of particular time at a very small interval of d tau. So, we can express that load in this form which further can be used to solve for this complementary function and particular integral, complementary function we can easily solve by putting 0 to the right hand side of the basic governing equation, but for particular integral we need to put this loading condition. And how to get the solution? We get the solution of the particular integral by applying the Duhamel's Integral of mathematics in this format which we can go through again in our soil dynamics course of NPTEL video course. So, with this we have come to the end of our module two, as we can see in this slide, it is ends our module two, and we will start our next lecture with module three.