Geotechnical Earthquake Engineering
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Module - 7
Lecture - 26
Seismic Hazard Analysis (Contd…)

Let us start our today's lecture for this NPTEL video course on geotechnical earthquake engineering. We were going through module number seven; that is seismic hazard analysis.

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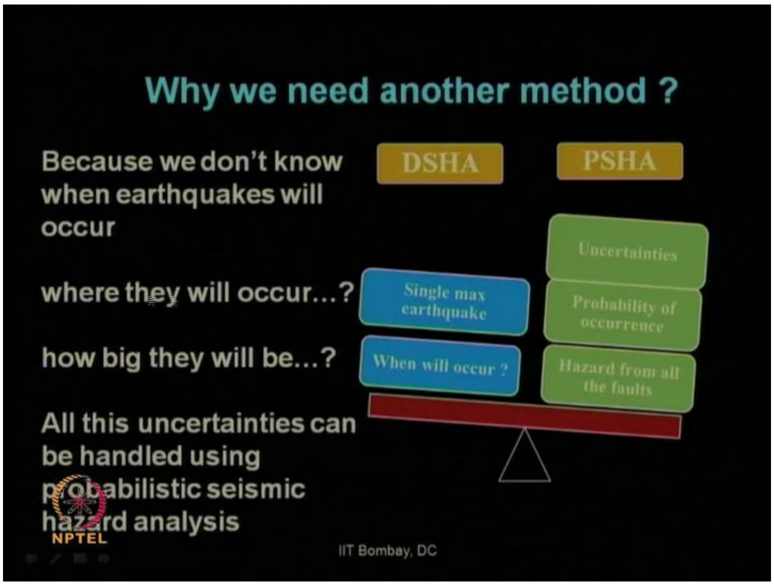

Let us have a quick recap, what we have learnt in our previous lecture. We have seen why deterministic seismic hazard analysis is not sufficient enough, and we need to go for another method or more realistic method, where all the uncertainties involved during the earthquake process can be taken care of. So, that is why, the PSHA or probabilistic seismic hazard analysis has been arrived at; where we can take care of uncertainty involved in the occurrence and the magnitude of these earthquakes, and also the probability of occurrence of all these hazards, which is not possible in the DSHA, because which consider only the single maximum earthquake for estimating the hazard parameter.

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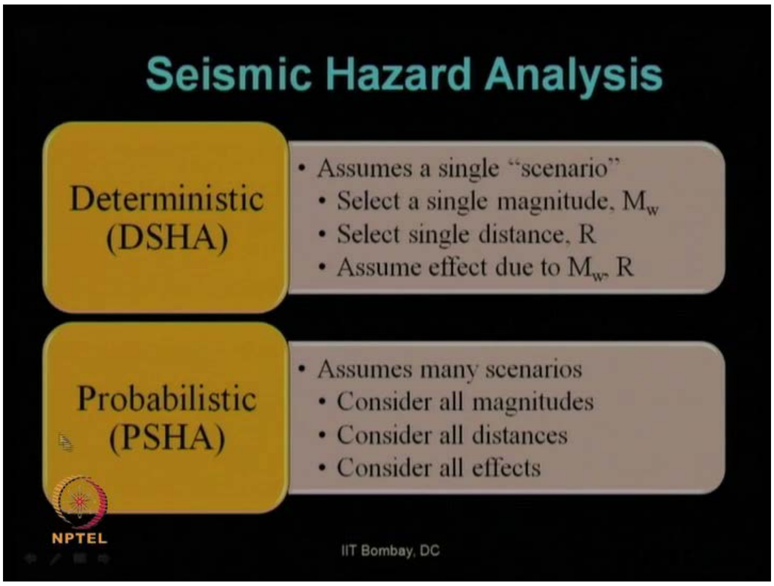

We have seen a comparison like this as well; that is, in DSHA, we consider the single scenario, which is the worst case scenario always. So, it is based on the single value of magnitude, which is M max, which is based on the single distance, which is R minimum. And the combination of these two maximum magnitude and minimum distance gives us the hazard parameter; whereas, in PSHA, we have seen that we need to consider the various scenarios like we should consider all the magnitudes; we should consider all distances and also all the effects to obtain that hazard parameters to consider the uncertainty involved in this process.

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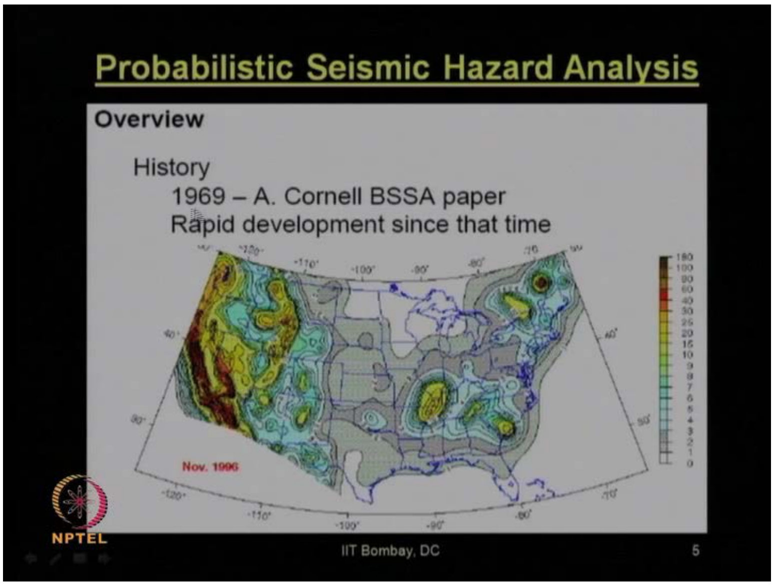

We have also mentioned in previous lecture that, for PSHA or probabilistic seismic hazard analysis, the pioneering work, the credit goes to Cornell. In 1969, this method was first established through this bulletin of seismological society of America paper BSSA paper.

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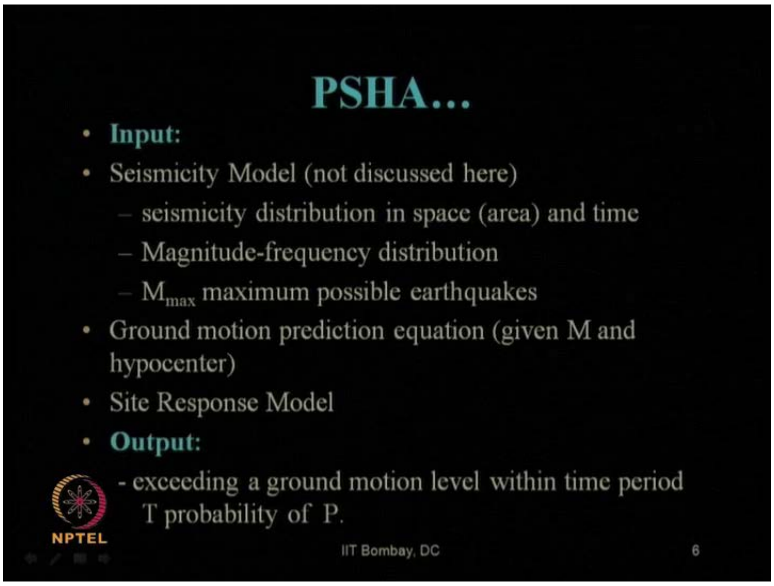

For PSHA, the basic inputs, which are required like seismicity model based on the seismicity distribution in space and time; magnitude-frequency distribution; and, the maximum possible earthquakes. Also, we have seen GMPEs or ground motion prediction equations, which are related to magnitude and hypocenter, which is nothing but the attenuation relationship using both this magnitude and hypocenter parameter and site response model. So, these are input values. In output, what we get? We get that particular hazard parameter, which we are trying to find out; that value exceeding a particular ground motion level within the time period of T with a probability of occurrence of some P.

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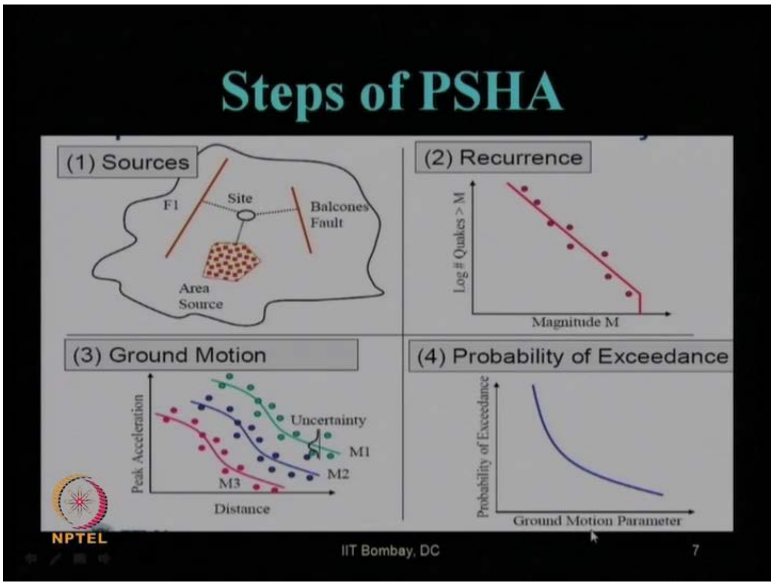

What are the various steps of PSHA? There are major four steps again. Like DSHA, we have seen also for PSHA four major steps. The differences are like here we have to identify first the sources or the boundary within which we will consider all the sources. Now, here not only the minimum distance we need to consider, but all the distances from site to source; and, the recurrence, that is, concurrence of an earthquake greater than a particular chosen magnitude by our engineers say 4.5 magnitude earthquake or 5 magnitude earthquake and their number of occurrences with respect to that magnitude we need to plot. Then, the peak acceleration, that is, the chosen hazard parameter versus distance based on various attenuation relationship for different sources needs to be plotted. And, within each of them, we need to find out the uncertainty based on the probability distribution function or the chances of probability of occurrence of a particular event within that source. Finally, we have to estimate what is the probability of exceedance of a particular event to occur at that site based on various GMPs.

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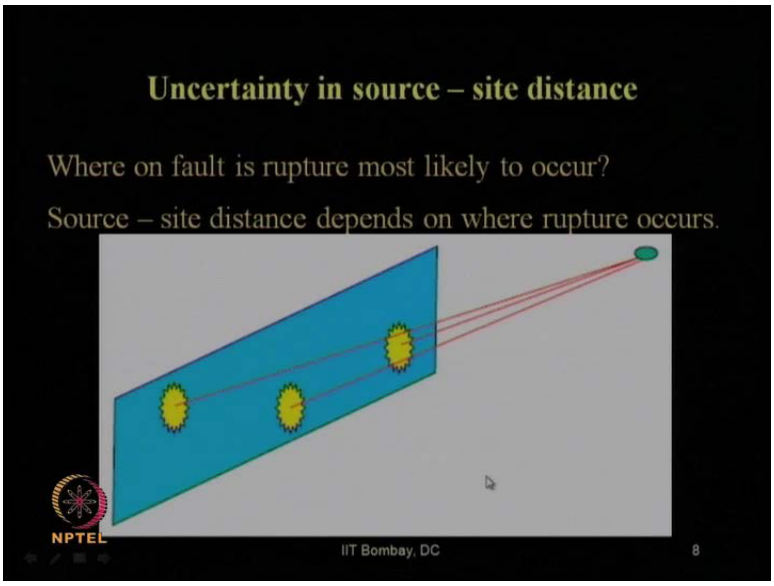

For that, the first step we have seen, identify the source. Once we identify the source, we are not sure that which point of the rupture will be more crucial; not each and every point as we have given equal weight age in the case of DSHA. In PSHA, we need to consider which are the more prone for the rupture.

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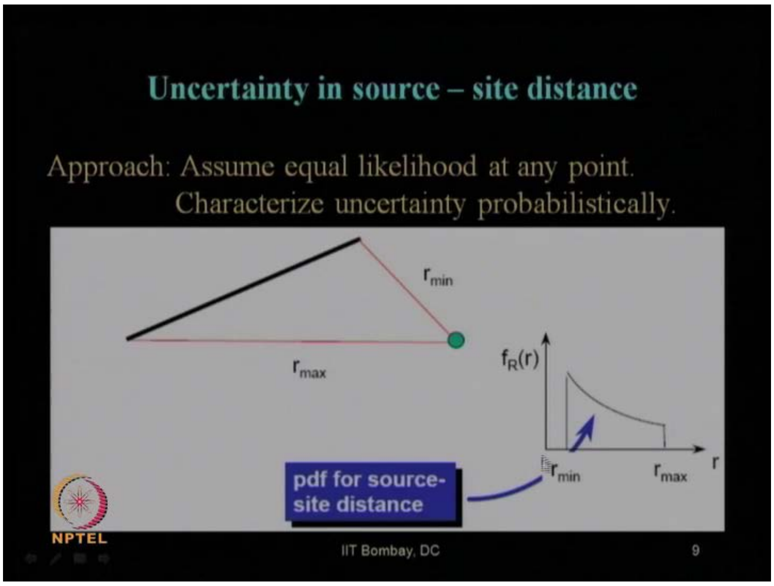

For that, we need to plot a probability distribution function using the histogram process; the center of each histogram based on the various distances from site to sources like this and plotting it in this fashion.

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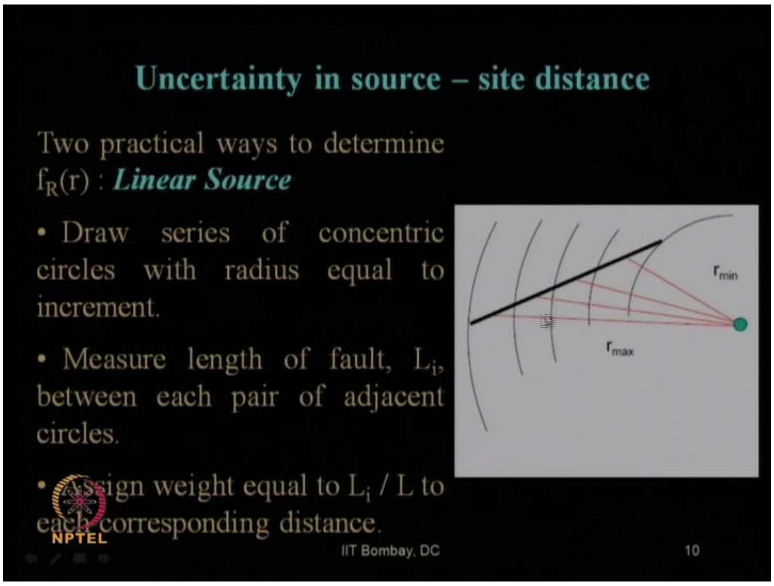

For that purpose, we can divide the source. If it is a linear source in this fashion, by drawing the concentric circles considering this site as the centre point, using equal increment of this radius, we can divide the entire source. And, from the center of each source, we can find out various distances and give or assign some weightage of these equal intervals L i by L.

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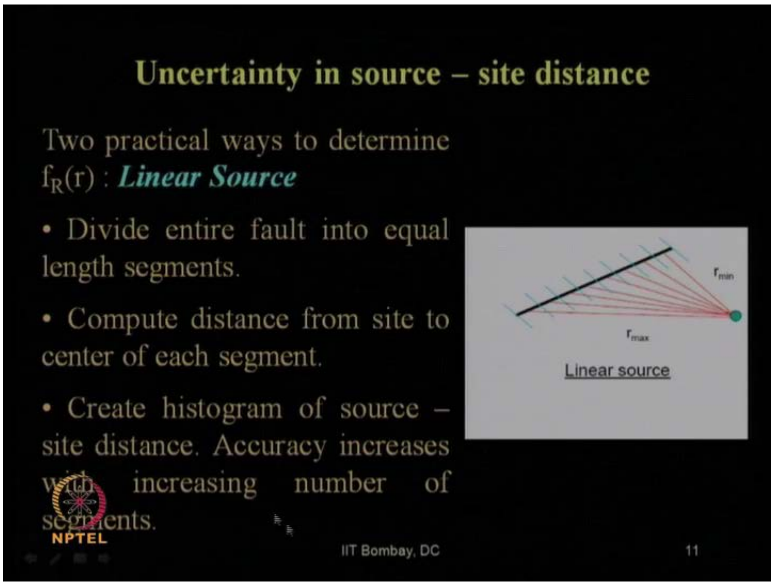

Similarly, by dividing the entire length into equal number of segments and giving the weightage to them, we can draw the histogram of source to site distance.

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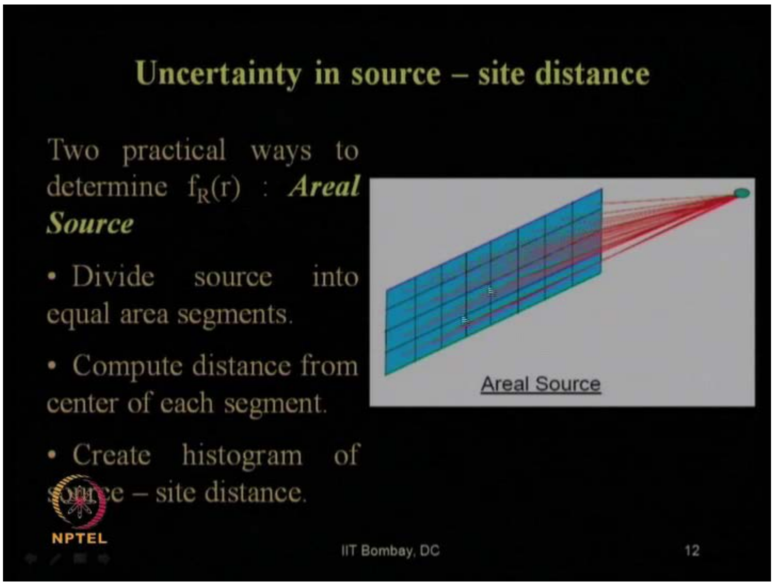

For areal source, we can divide the area into a number of smaller equal areas like this; and, we can find out various distances to their centers and plot that histogram.

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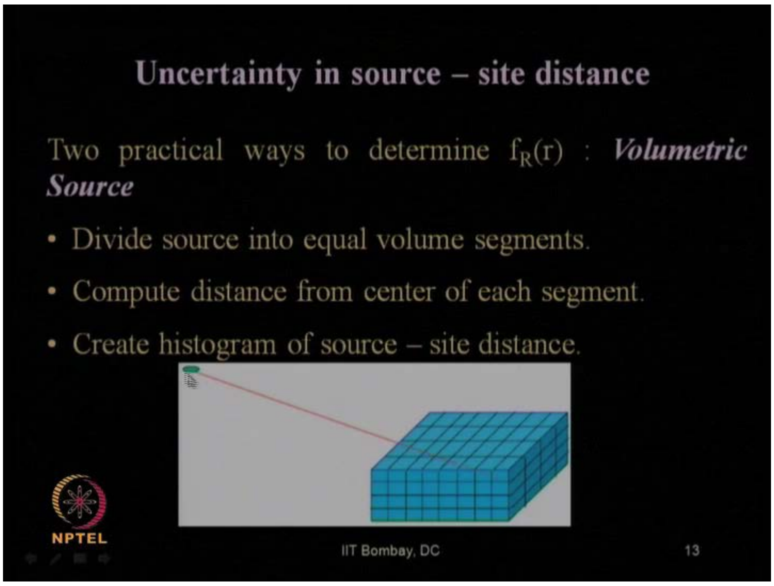

Similarly, for volumetric source also, we can divide the entire volume source into a number of equal small volumetric segments; and, we can find out various distances from center of each volume to the site and create the histogram of that source to site distance.

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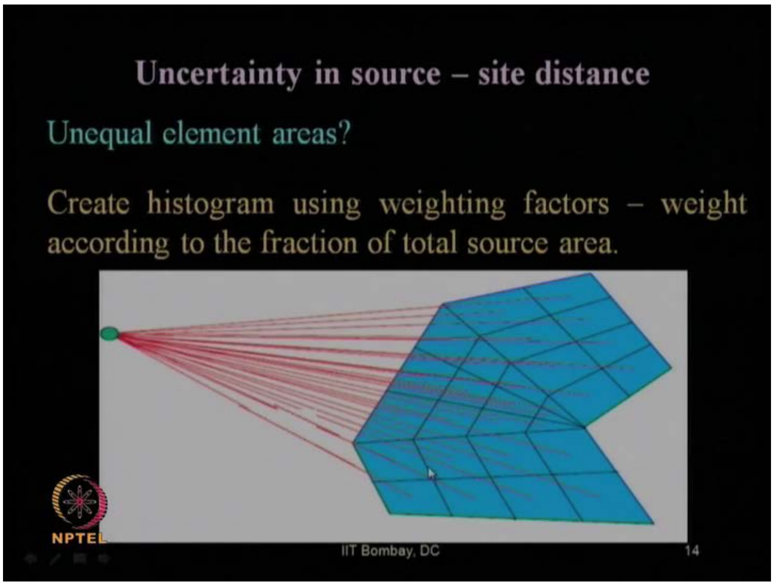

Suppose if we have unequal or random or haphazard shape of the source; in that case, we have to assign the weighting factor corresponding to different segments. And, that different weight factor we have to use to calculate the fraction of the total area, which is getting involved or we are plotting in the histogram.

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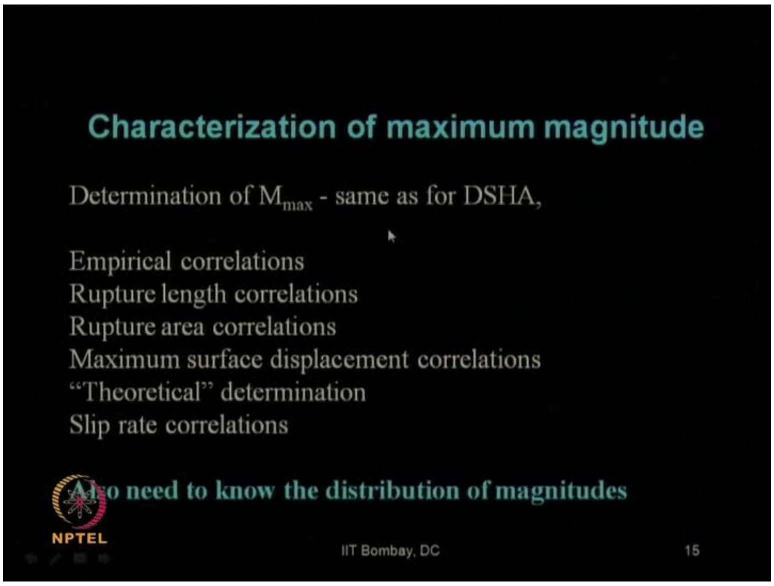

Whereas, for obtaining the maximum magnitude, it is the same methodology, is used as we consider for DSHA also; like either using empirical correlations based on rupture length, rapture area, maximum surface displacement; or, based on theoretical determination, based on seismic moment criteria or the slip rate correlations, we can find out what are the maximum value of M.

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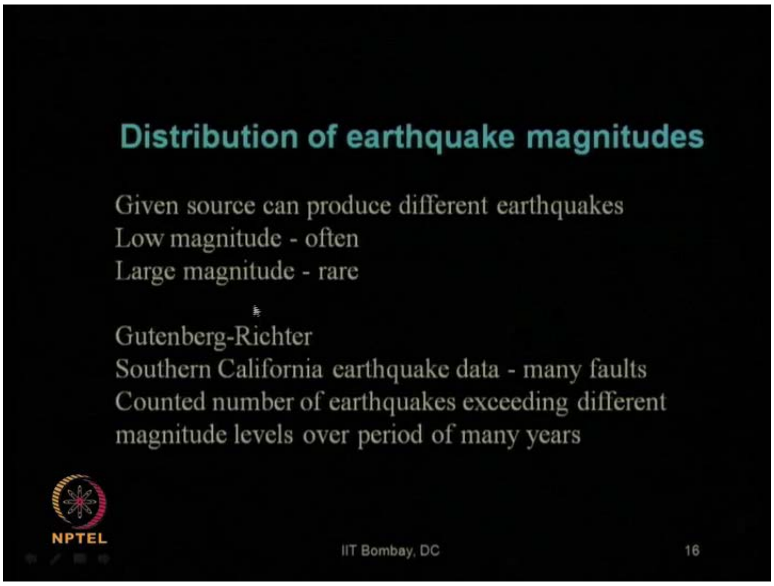

Now; obviously, at given source, at one particular source, several magnitude of earthquake may occur in the first and chances of occurrence of low magnitudes are quite often or frequent; whereas, large magnitude occurrence of earthquake chances are very rare or less frequent. So, using that, Gutenberg and Richter proposed that we can plot these number of occurrences of earthquake with respect to their magnitudes.

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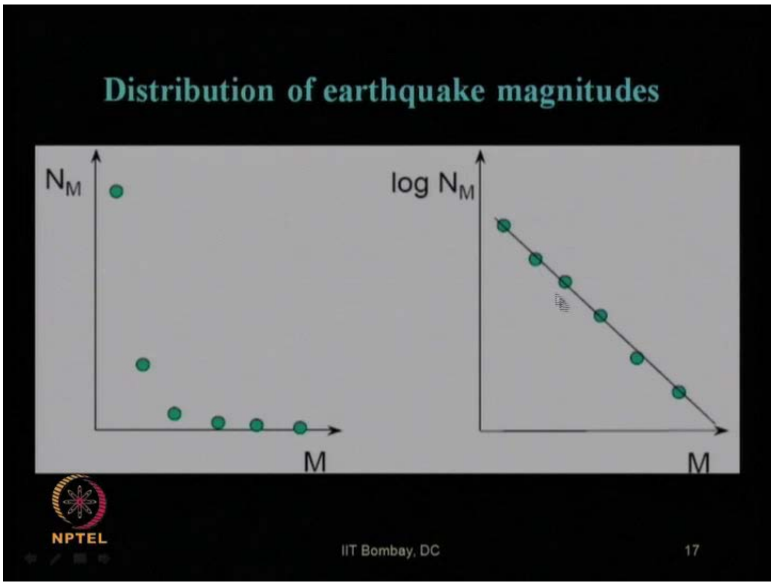

In this fashion, like y-axis can be number of occurrence of a particular magnitude of M versus that magnitude M. Suppose we are considering for our design say magnitude more than 5; let us say, we are interested; all the magnitude more than 5 we need to plot and their number of occurrences. So, obviously, less magnitude of earthquake will occur more number of times and higher magnitude of earthquake occurs less number of times. So, typically, a behavior from historical data we will find, which is kind of an exponential like this in a normal plot of that number of occurrence versus magnitude. So, if we plot it in the semi-log plot like log of number of occurrence of that magnitude of earthquake versus magnitude, it will be kind of a linear relationship like this.

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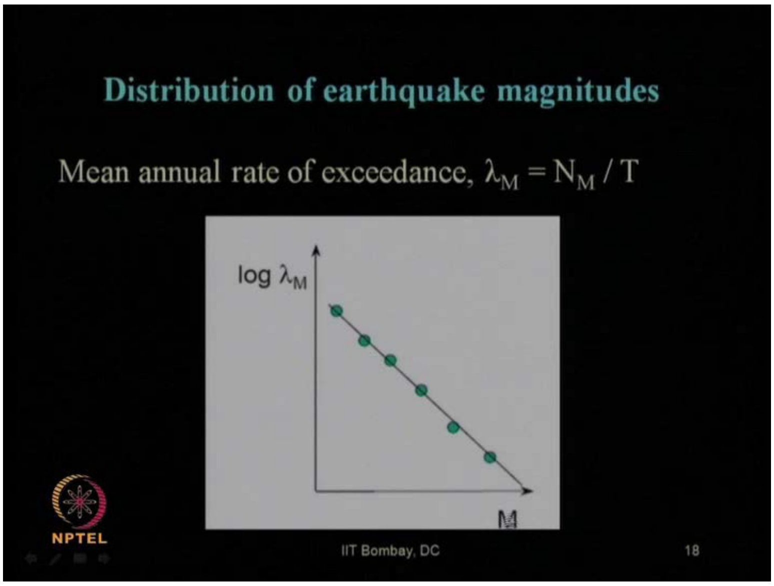

The same thing we can plot it in this fashion considering the mean annual rate of exceedance, which is nothing but lambda M, which is defined as that number of occurrence of earthquake divided by that over the time period T, which is considered, which will give of course per year basis, which is nothing but mean annual rate. So, that log of lambda M versus M will give us the same trend, same relationship – the linear relationship like this.

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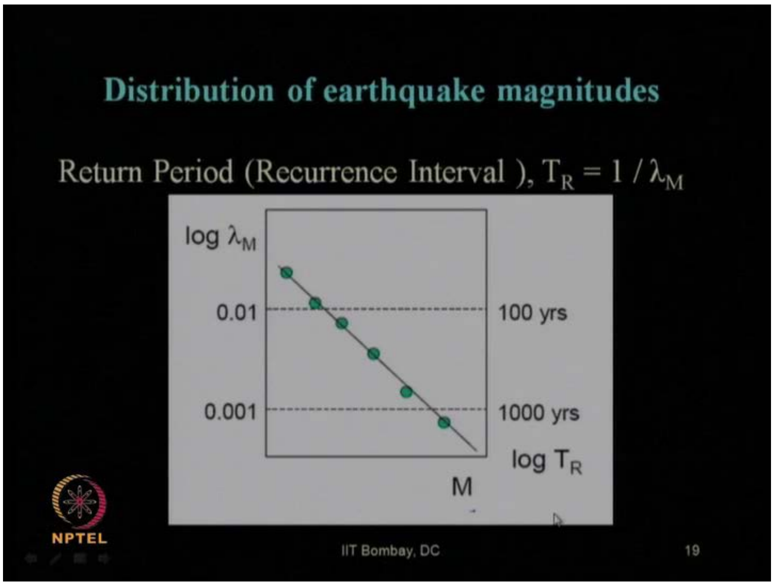

Then, we have defined another parameter – return period or recurrence level, which is T R, which is nothing but inverse of the mean annual rate of exceedance of that event. So, 1 by lambda M will give us… So, for example, log of lambda M, value of 0.01 indicates T R value of corresponding to 100 years, because 1 by 0.01 is nothing but 100 years; that means, mean rate of exceedance of a particular earthquake say magnitude 5 is 0.01 at a particular source. For the same source, in 100 years, that 5 magnitude will occur once is the same word telling in the different way. So, this is the same plot, same y-axis in different directions increasing; that is, lambda M increasing vertically up, T R increasing vertically down is shown over here.

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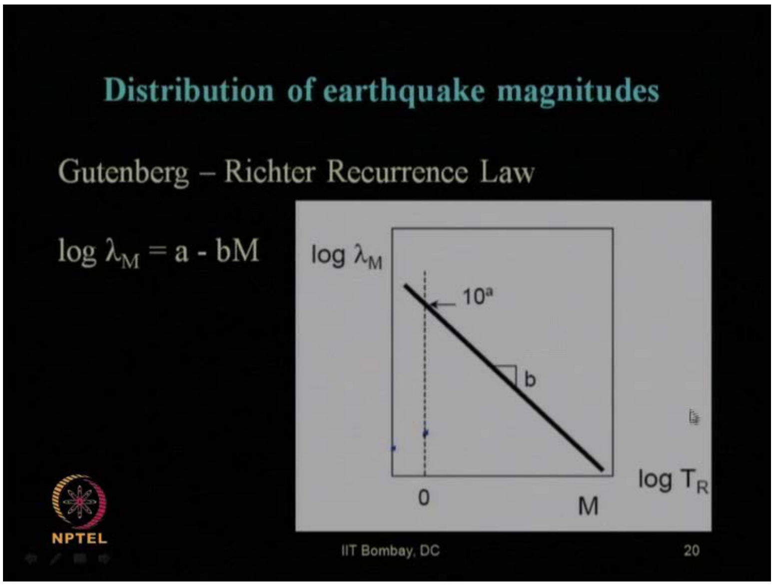

Now, Gutenberg and Richter proposed this law; that is why it is called Gutenberg-Richter recurrence law; that is, how many times that earthquake is going to occur; its written period or recurrence is given or known as Gutenberg-Richter recurrence law based on this distribution, which is represented by this simple form of equation – log of lambda M equals to a minus b M. These coefficients a and b need to be obtained from the collected historical earthquake for a particular location or particular region. So, based on that, for each and every region in the earth, we can find out Gutenberg-Richter recurrence law. Like for India, for Gujarat region, we have obtained this Gutenberg-Richter recurrence law. We will see that example later on. So, this a parameter and b parameter needs to be obtained based on local seismic events historical collected data. And, this is the plot of log of lambda M versus M. And, 10 to the power a denotes corresponding to magnitude 0; that is, the intercept here; and, b is the slop of this line.. So, with that, we completed our previous lecture.

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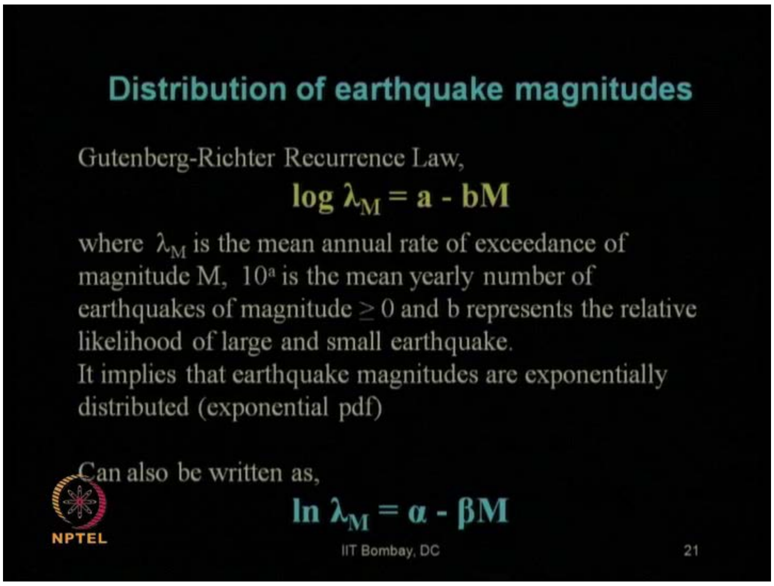

We are continuing in our today's lecture now. Today, what we can see over here, once we learnt the Gutenberg-Richter recurrence law, which is given in this form; in this equation, lambda M is defined as the mean annual rate of exceedance of that magnitude M. And 10 to the power a is the mean yearly number of earthquakes of magnitude greater than equals to 0, because it is the intercept in that corresponding to M equals to 0. And, b represents the relative likelihood of large and small earthquake, because as we can see, this gives the slope. These slopes automatically if it is the stiffer, that means that occurrence of large earthquake are much lesser; if it is a flatter, then chances are more. So, that is why it gives… It represents the relative likelihood occurrence of large or small earthquake.

And, it implies that earthquake magnitudes are exponentially distributed. As I have already mentioned, because if you plot this linear relationship, you will get in semi-log in this fashion only if the original distribution in normal scale is something like this exponential. So, what we can do, the same equation Gutenberg-Richter recurrence law – another way to write it as l n. This is natural log. This is log to the base 10; and, this is natural log of lambda M equals to alpha minus beta M. Just we are changing the scale from log to the base 10 to natural log.

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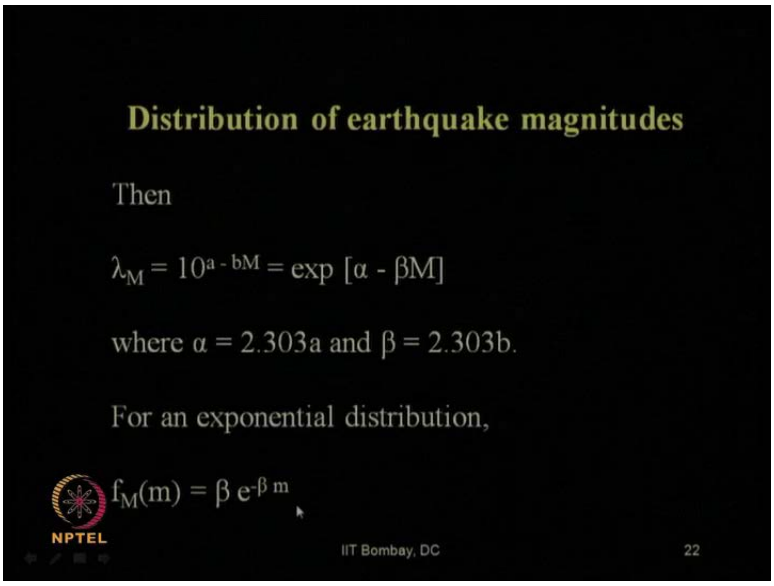

Then, what we can say? Lambda M from first equation is 10 to the power a minus b M when we are using this, because this is log 10. So, lambda M equals to 10 to the power a minus b M, which can be expressed as equals to exponential of alpha minus beta M because, here from this relation, we can write lambda M equals to e to the power alpha minus beta m. That is what I have written. Hence, if you see the similarity, what we can write? Alpha is nothing but 2.303 times a; that l n comes into picture. l n versus log to the base 10 – that relationship, that number comes into picture. And, beta is nothing but 2.303 of b. So, for an exponential distribution, this distribution can be represented as f of m equals to beta e to the power minus beta m. That will be the relationship.

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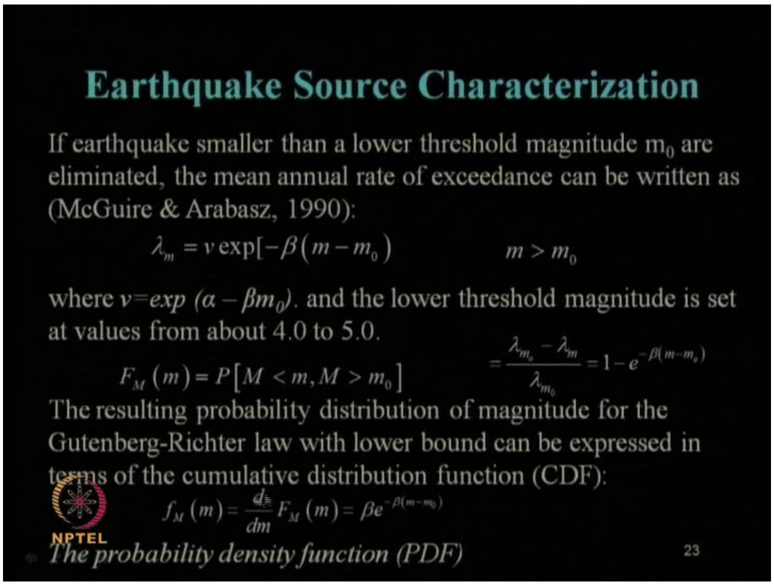

Now, let us see earthquake source characterization – how to characterize this. If the earthquake smaller than a lower threshold magnitude m naught are eliminated, then the mean annual rate of exceedance can be written as proposed by McGuire and Arabasz. In 1990, this expression was proposed. That lambda m, which is nothing but the mean annual rate of exceedance of that event can be proposed as mu times e to the power minus beta times m minus m naught; that is, instead of minus beta m, now, we are using minus beta times m minus m naught; where, this m has to be greater than m naught. So, what is m naught?

Let me explain it little clear way. It says it is the lower threshold magnitude; that means, m naught is that value of the earthquake magnitude below which we are not concerned about; that is, as an engineer, suppose we are doing the hazard analysis; say, we are not bothered about the earthquake, which are having magnitude say less than 4.5, because it hardly damages our civil structures. See if it is so, then that 4.5 will be the minimum threshold magnitude. You are interested more than 4.5 whatever earthquake has occurred. So, why you will then take for your Gutenberg-Richter relationship estimation, all the earthquake magnitude say 3, 3.5, 4. Why we should take that? We should not take. So, that is why, it is mentioned over here. You can select a threshold lower parameter of magnitude m naught based on the design criteria; and, you can neglect all those values of earthquake, which are less than that m naught. So, you are going to consider only those earthquake, which are more than m naught. And, for that, you identify what is the annual rate of exceedance, because this annual rate of exceedance may change automatically if you delete some of the data points from your historical data set below of certain magnitude; where, this mu is nothing but e to the power alpha minus beta m naught. That we have already seen in terms of exponential.

And, the lower threshold magnitude is set at values from about 4 to 5. As I have already said, it depends on the designers or it depends on the importance of the structure, etcetera. Suppose if it is a residential building not that so important thing, you can select even a higher magnitude. If it is very important structure say bridge say dam, etcetera; where, we should go for deterministic seismic hazard analysis; but, parallely, you can go for probabilistic seismic hazard analysis also in a logical way, but to select a threshold parameter say 4; like that. So, this depends on the engineers choice, engineers experience.

Now, this F M – that can be expressed as probability of occurrence of a particular magnitude of earthquake, which is less than of a magnitude m at that site given that magnitude must be more than that threshold value of earthquake. What does it mean? We need to find out that probability distribution function. For that, it is nothing but the probability of occurrence of an event, which is having for example, let us say, this magnitude say 8 earthquake. We are going to design our site for a future structure, which has to withstand earthquake magnitude let us say maximum value of 8. And, our threshold value let us say it is 4.5. So, it will be that probability of occurrence of magnitude, which will be less than 8; but, that magnitude has to be more than that 4.5. That is what it says. We are clear about this probability. So, as you can see, obviously, to study this probabilistic seismic hazard analysis, it is assumed that you know the basic concepts of probability. This is one important thing I want to highlight over here. Knowledge of basic probability distribution everything is necessary for this course for this chapter to understand. So, I will request all the viewers, those who do not have the basic background of probability, may go to any standard probability distribution book and may learn very basics of the probability distribution; and then, can consult this video lecture of this particular topic on probabilistic seismic hazard analysis, which is necessary.

So, now, by defining that, what way we can write this equation? We can write it in this form, that is, lambda of m naught minus that lambda of m by lambda of m naught, which is nothing but 1 minus e to the power minus beta m minus m naught. So, the resulting probability distribution of magnitude for the Gutenberg-Richter law with the lower bound – this lower bound m naught – can be expressed in terms of cumulative distribution function, which in short, we call as CDF. Cumulative distribution function is nothing but the first derivative of this distribution function with respect to that chosen magnitude. So, d of dm by F of M of m – this function; if you differentiate, you should get this expression, that is, beta e to the power minus beta m minus m naught, because if you differentiate this one with respect to m, that is what we should get. So, this is known as cumulative distribution function for the probability distribution function.

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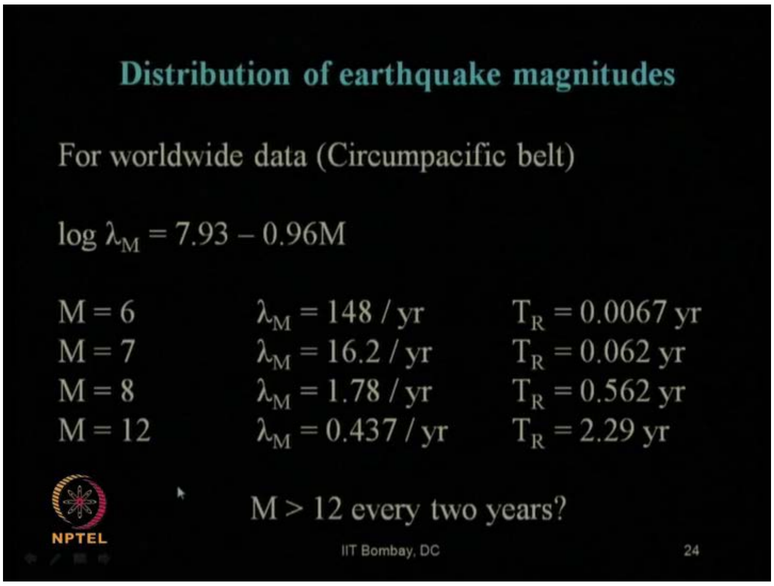

Now, using that concept, let us take one example problem. Suppose for the worldwide seismic data, based on that, taking care of the circumpacific belt region of former earthquake, previous researchers had proposed the Gutenberg-Richter relation with the values of a and b like this; that is, log of lambda M equals to 7.93 minus 0.96 M. This is the a value 7.93 and b value of 0.96 for this circumpacific belt based on the worldwide data. Now, if we want to apply this Gutenberg-Richter recurrence law with this known value of a and b, let us find out various magnitude of earthquake and their mean annual rate of exceedance and the year. So, if you plot M equals to 6 in this equation, what value of lambda M we will get? It is coming 148 per year. So, T R is coming 0.0067 year. This is written period. If you put M value of 7, then lambda M is coming 16.2. We can just calculate this very easily. T R will come as 0.062. If you put M equals to 8 lambda M is coming 1.78 per year and T R is coming 0.562 year; that means, 8 magnitude of earthquake will have a chance of occurrence in half a year. That is what it means.

If we put M equals to 12 just for the sake of analytical computation; in this equation, if we put M value as 12, what we can see? The lambda m values – you will get some value by putting in this equation; and, T R value you are getting 2.29 year; that means, what does it signify? It signifies the magnitude of 12 or more will occur every 2 year interval. But, is it true? No, never; it cannot occur like that. Suppose if I put M equals to 15, I will get another value say something will come. Does that mean magnitude of 15 will come in that number of period of interval? No; then, what to do. So, we will see that how we need to consider or remodify this upper limit or upper value of the magnitude in this Gutenberg-Richter equation. So, this equation what we have talked about? Here we have talked about the lower threshold, lower magnitude that is based on our interest on design parameter that is above which the problem to our structures starts coming in. Based on that, we have to select that minimum value or threshold value of magnitude.

Now, also, we have to select the maximum value, because otherwise, it will give us this kind of unrealistic data; that unrealistic data obviously, is not having any meaning in the calculation of this probabilistic seismic hazard or recurrence period. It does not give us any data, because if I use M 20, which will never occur – magnitude of 20; but, still it will give some value. So, we have to know where we should stop; where is the upper limit to stop in this Gutenberg-Richter recurrence law, so that we have both lower threshold and upper threshold to continue further for the calculation of this written period or the mean annual exceedance.

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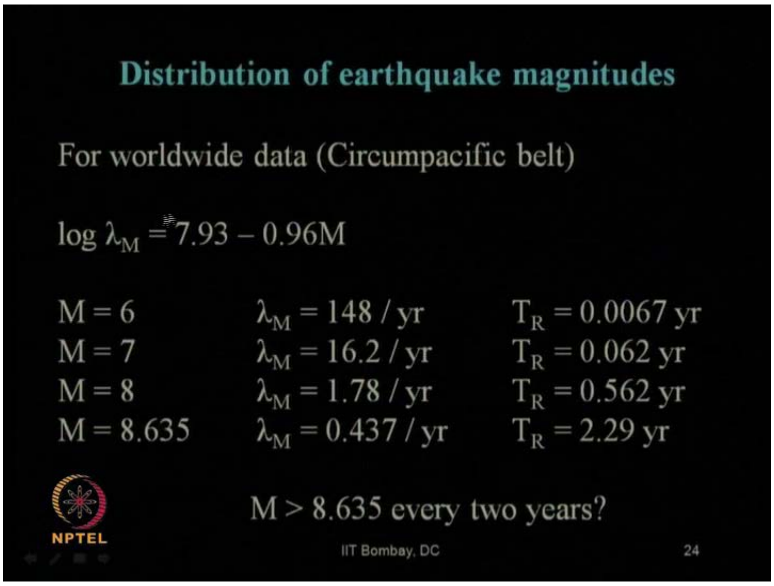

So, for worldwide data like for circumpacific belt, the Gutenberg-Richter equation or recurrence law has been proposed like this that log of lambda M equals to 7.93 minus 0.96 M. So, how we can use this equation? Like for selected value of the magnitude of earthquake say M equals to 6, lambda M you can easily calculate using this equation – this Gutenberg Richter recurrence law; you will get the lambda M. And correspondingly, you will get the T R value, which is nothing but inverse of this lambda M. Similarly, for other magnitudes also, you can calculate these values.

Now, you can see from this equation. Suppose if we use M equals to 8.635 in this equation; the lambda M value comes to be 0.437 per year, which gives us T R value of 2.29 year. So, what does it mean? Is it means that magnitude of greater than 8.635 will occur every 2 years? So, let us see what is the applicability or ranges of applications of this magnitude. Earlier we have mentioned about that, while developing this Gutenberg-Richter recurrence law, we have taken a minimum threshold value about which we are interested. Now, we will see up to which magnitude this Gutenberg-Richter recurrence law is applicable.

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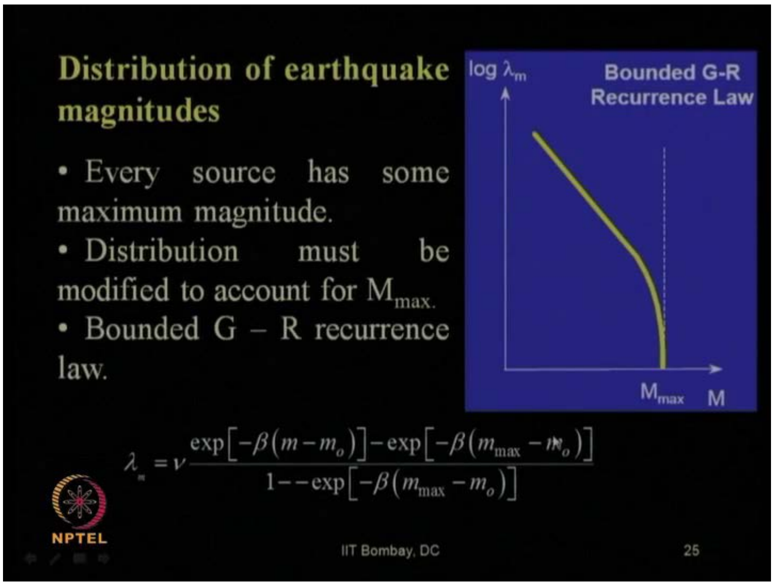

So, now, let us look at this distribution of earthquake magnitude data; that is, again that y-axis is log of lambda M and x-axis is M. Now, every source has some maximum magnitude when we are arriving at the Gutenberg-Richter recurrence law. So, for each of the sources, we know about the maximum magnitude. So, distribution must be modified to account for this maximum magnitude; that is, we should not use that proposed Gutenberg-Richter recurrence law for any magnitude, because that magnitude may not occur in that region from the possible sources for that particular site for which we are analyzing. So, we have to take care the maximum magnitude, which each of the source is already recorded or having the data. So, that is why, it says distribution must be modified to account for this maximum magnitude. And, bounded Gutenberg-Richter recurrence law hence comes into picture. Why it is called bounded? Because we are fixing that law at the maximum value of say M max, which is responsible for a particular site.

So, instead of going this equation linearly increasing like this at any value of M, we have to bound it at some maximum value, so that beyond that value, this equation is not applicable. So, that is the applicability of the maximum magnitude. Lower value – of course, we have earlier itself put a boundary for the lower or M minimum or M threshold; that is based on what value we are interested for our seismic hazard analysis and so on. So, the bounded Gutenberg-Richter recurrence law will look like something like this. So, lambda M now will take the shape of mu times e to the power minus beta times m minus m naught minus e to the power of minus beta of m max minus m naught divided by 1 minus x e to the power minus beta times m max minus m naught. So, in this case, what we can see? This term has been introduced. Earlier, this m naught was present; m naught is nothing but the minimum value or threshold value of magnitude about which we are interested; it can be 3.5; it can be 4; it can be 4.5 depending on designers choice or engineers choice. Now, this M max has been introduced, which has to be taken care of and correspondingly for any value of M. Whatever value of M you are interested to find out the lambda M, that you can use this equation .

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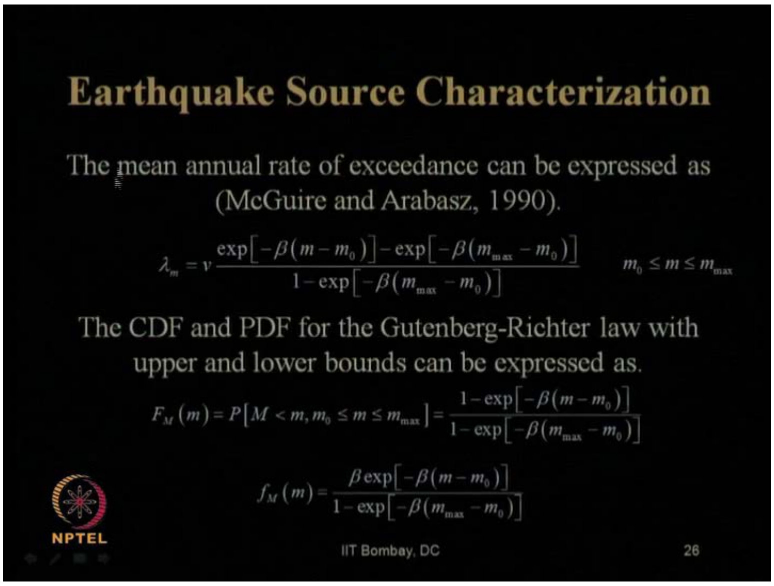

Now, let us further proceed to earthquake source characterization through this value of mean annual rate of exceedance, that is, lambda m, which is expressed now as per the McGuire and Arabasz, 1990 equation as given by this expression. As I have just now mentioned, in this case, what is the range of applicability of this magnitude m? That should be within this m max and m naught; that is what it means. So, this equation is applicable for this range of m; m is earthquake magnitude. So, the cumulative distribution function or probability distribution function for the Gutenberg-Richter law with the upper bound and lower bounds like these two: this is the upper bound; this is lower bound – can be expressed as like F of M of occurrence of magnitude M will be probability of occurrence of any magnitude M, which is less than that particular value, which you select for the design say small m, where this small m should lie between the maximum value and the threshold value or minimum value. So, that will be expressed as

1 minus e to the power minus beta times m minus m naught by 1 minus e to the power minus beta m max minus m naught. So, this F M of m is expressed in this fashion.

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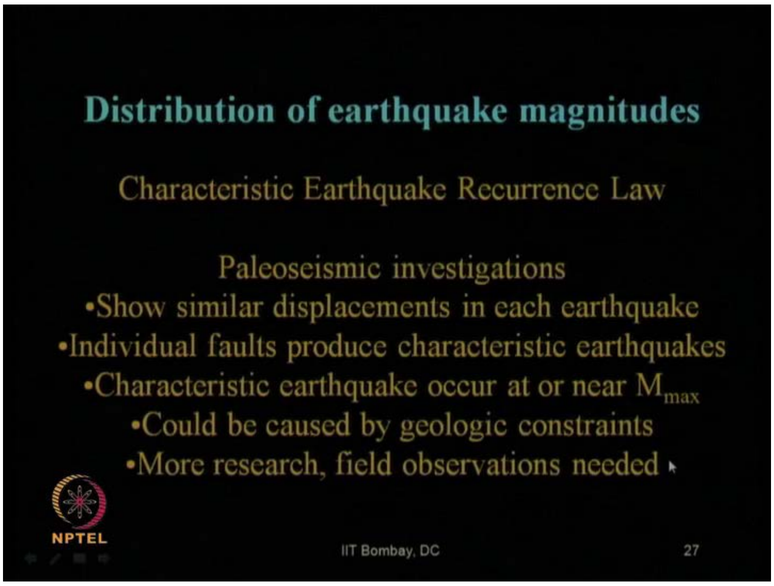

Now, if we talk about the distribution of earthquake magnitudes like what are the characteristics of earthquake recurrence law, we can see that paleoseismic investigations – all the investigations are taken into care of while developing any earthquake recurrence law. So, show similar displacement in each earthquake; individual faults produce characteristic earthquake; characteristic earthquake occur at or near M max; and, could be caused by geologic constraints; and, more research and more field observations are required; that is, today, what earthquake recurrence law we are proposing? That is not absolutely true forever; that means, that recurrence law is bound to get changed with time. So, in this year 2013, whatever earthquake recurrence law, whatever we are getting for various areas based on the historical earthquake data collected; that is obviously, bound to change with due course of time as many more earthquake records are or statistics are recorded and taken care of while proposing that earthquake recurrence law.

So, we should remember that, there is always an open-end scope of research in this problem; that is, for any particular area, earthquake recurrence law or Gutenberg earthquake recurrence law can keep on changing with time. So, whatever today we are getting, maybe (( )) small period of time say 1 year or 2 year. But, it may change over the time span of say 5 years or 10 years when many more earthquake data have been collected over the period of time, which are having a probability to change that earthquake recurrence law of Gutenberg-Richter.

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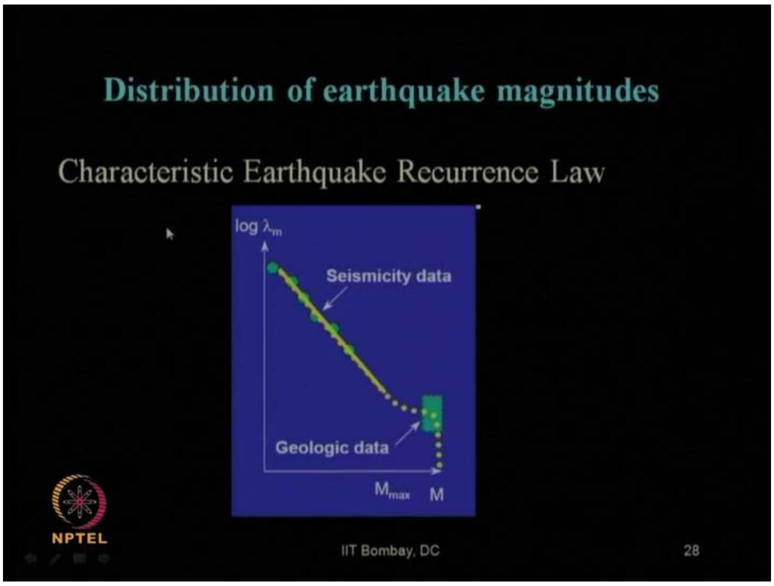

Now, let us look at this characteristic earthquake recurrence law, what we have mentioned. These are various seismicity data, which we are collecting from paleoseismic data points, whatever is recorded or instrumented or visualized or mentioned in several sources. So, based on that, you can get this information. And, from the geologic data of a site also, you can get some information. Suppose from the fault characteristics, etcetera, we have seen how we can estimate the magnitude. So, based on that, you can have all these data points when you are going to propose any earthquake recurrence law. And, you have to fix a boundary of M max whatever for that particular site or for that particular region you have already information with you.

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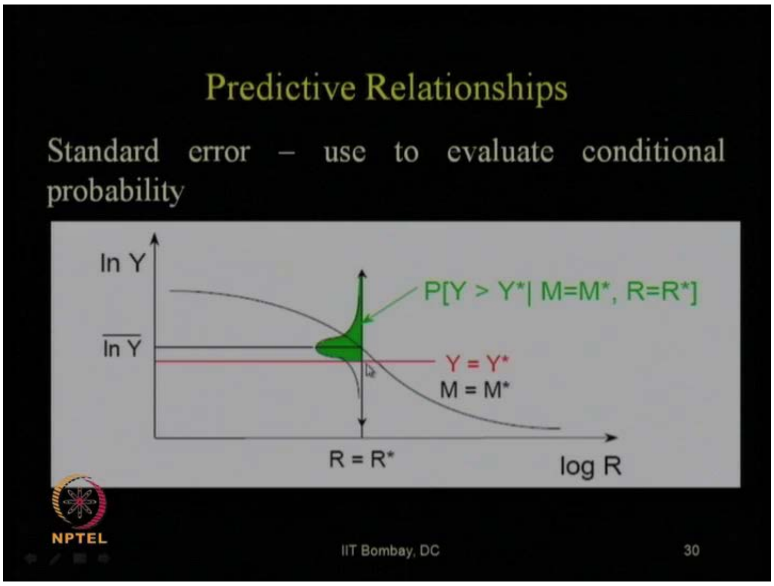

Now, let us talk about the predictive relationships like standard error. It is used to evaluate the conditional probability. How we can find out the conditional probability? Suppose this is the probability distribution function for any particular parameter; say, as we have already mentioned, this is kind of attenuation relationship we proposed in this form like log of that parameter; it can be peak ground acceleration; it can be spectral acceleration; it can be peak horizontal velocity; various parameters, which you are interested about. These are dependent on the log of R value – the distance; as the distance increases, they are going to decrease; that is nothing but attenuation.

In this, at a particular value of R equals to say R star at which you are interested; and, let us say this curve is designed for or this curve is obtained for a magnitude M equals to M star; like for any relationship we have seen, we generally use M equals to M max in deterministic seismic hazard analysis. So, like that. And, this R we use for deterministic seismic hazard analysis as R mean. So, for a particular value of that curve of at M equals to M star for a chosen parameter with attenuation relationship, now, if you select a value of R; along that R, if you find out the probability distribution function – this is conditional probability – that is, probability of occurrence of that parameter; it can be PGA; it can be spectral acceleration; as I said, whatever you choose. So, probability of occurrence of that parameter greater than some particular value; say you are interested to know that probability of occurrence of PGA of say more than 0.3 G, where it occurs. So, that is what it says.

So, probability of occurrence of PGA greater than some particular value for a given M equals to M star and R equals to R star; that is what it shows. So, what does it mean? If the probability distribution is something like this, with the mean value of l n Y bar like this; because we know this peak is nothing but mean value of the probability distribution. So, probability of occurrence of greater than that Y star… So, we have to find out the value of Y star from here; let us say this is the value of Y star. So, this is the line, which we are having Y equals to Y star. So, more than that, whatever probability we have – the shaded area; this green color shaded area will give us value of that probability. Similarly, suppose if our chosen value is less than the mean value; then, obviously, Y equals to Y star will be somewhere here; and the probability – the same probability will be the shaded area – green shaded area over here; same thing. So, these are nothing but the use of this conditional probability.

Why it is called conditional probability? Because this is true for a given condition that, probability of occurrence of this event greater than Y star for a given condition that M should be equals to M star and R should be at R star. Suppose we select different R; instead of R star, let us say it is R prime. So, the curve will come over here. So, corresponding probability distribution function will be something different. Then, the probability value also will be different for another given R value. Why it is called conditional probability? Similarly, for M equals to M star; that is, whatever value of M you are using to arrive at this attenuation relationship or you are using for this attenuation relationship should be used. So, that condition is nothing but to be used for obtaining this conditional probability.

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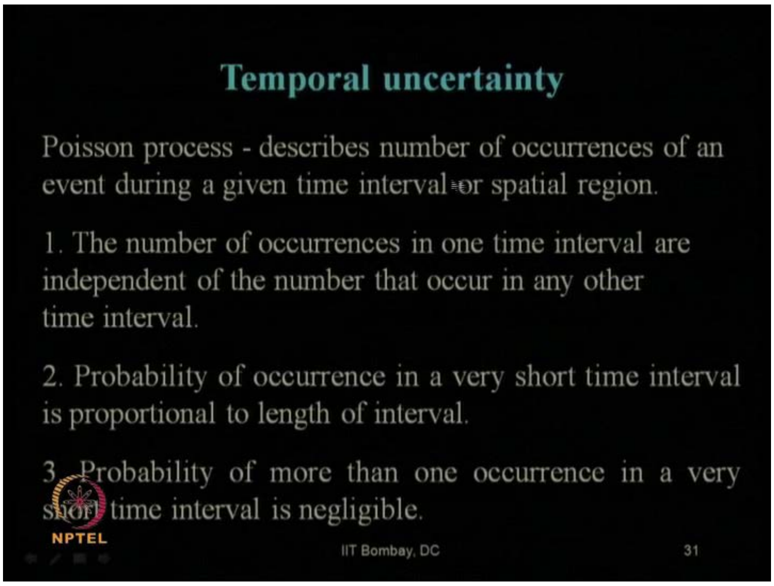

Now, let us look at these points of temporal uncertainty. So, what is temporal uncertainty? In the Poisson process of probability distribution, it describes number of occurrences of an event during a given time interval or special region. So, what does it mean? The number of occurrences in one time interval are independent of the number that occur in any other time interval. And, probability of occurrence in a very short time interval is proportional to the length of the interval. So, that means automatically, if you have a longer interval of any earthquake, if you have a more probability; can you see how we relate also the duration of earthquake through this Poisson's probability distribution function? And, probability of more than one occurrence in a very short time interval is negligible; which is quite obvious. So, we will see what is Poisson's distribution now.

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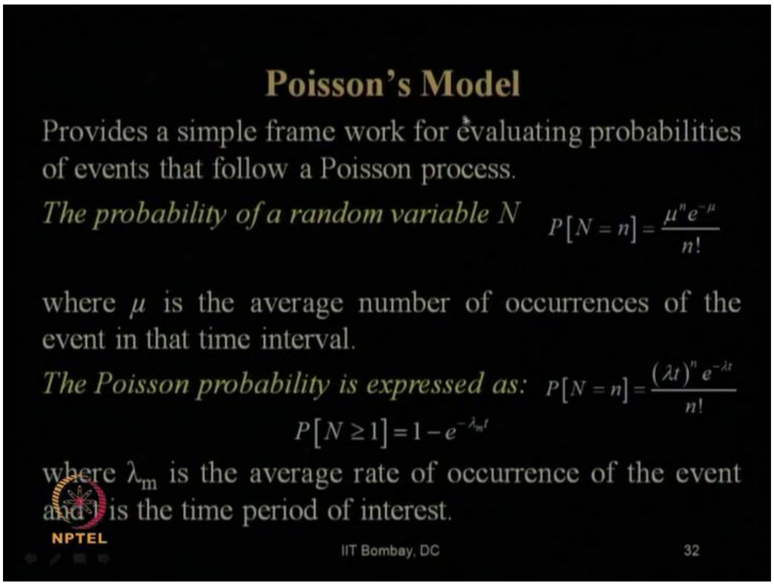

Poisson's model of probability distribution, which is well-known function; as I have already mentioned for this course, you should have a basic knowledge of the probability. So, once again, I am repeating it; I am not going to discuss detail of this model, where from it came, etcetera; you are advice to look at any standard book of statistics or probability for the details. So, let us see how the applications of this model in our geotechnical earthquake engineering or earthquake engineering we make use of. So, this Poisson's model provides a simple frame work for evaluating the probabilities of any event that follow a Poisson process. And, it has been observed by several researchers that most of the earthquake records – they follow a typical Poisson's process or Poisson's distribution. So, that is why in earthquake engineering, we use a Poisson's model, which describes the occurrence of an event of earthquake, the number of events of an earthquake in a more realistic manner.

The probability of a random variable – any random variable n, is expressed like this through the Poisson's model like this; P – probability of occurrence of that event N equals to number of times small n equals to can be computed as mu times to the power n e to the power minus mu by n factorial; where, this mu is called the average number of occurrence of that event for a given time interval. So, if we express it – the Poisson's probability in this fashion, how we can write it… because why we are changing this mu to lambda t; we will explain it very soon, because this is nothing but average number of occurrence. Look at here this definition of Poisson's model, which is nothing but… In our earthquake engineering, when we want to correlate, it is nothing but if we have the average rate of occurrence of an event of earthquake – lambda m – already we have used this. So, that times the time period t, which you are selecting for the computation; that will give you nothing but the average number of occurrence of that event. So, that is why, the Poisson's process as I have already mentioned, it best represents the earthquake magnitudes and their number of occurrences in this fashion. So, that probability can be simplified for our earthquake engineering in this fashion that probability of occurrence of an event capital N of number of times small n is expressed as lambda t times to the power n e to the power minus lambda t by n factorial.

So, the same expression if you use for suppose probability; what is the probability of occurrence of a particular event? What is the particular event? It can be say occurrence of a magnitude of more than say 7.5. So, that is what. Any event greater than equals to 1; greater than equals to 1 means that event will occur minimum once or more than that. That will be computed as 1 minus e to the power minus lambda t or lambda m t. Why? How you can obtain it? What is this probability of occurrence of n greater than equals to one? That is nothing but 1; 1 is always the 100 percent or full probability; 1 minus probability of occurrence of less than 1, which is nothing but you can say 0. So, that if you put over here, you will get this expression. How you are getting it? Because in that case, you are providing n equals to 0. So, 0 factorial becomes 1 to the power 0 is nothing but 1. So, you are left with e to the power minus lambda t. Can you see that? So, that lambda t we are now using that definition lambda m. In earthquake engineering, already we have mentioned, lambda m is nothing but average rate of occurrence of the event; and, t is the time period of our interest for which we are going to compute.

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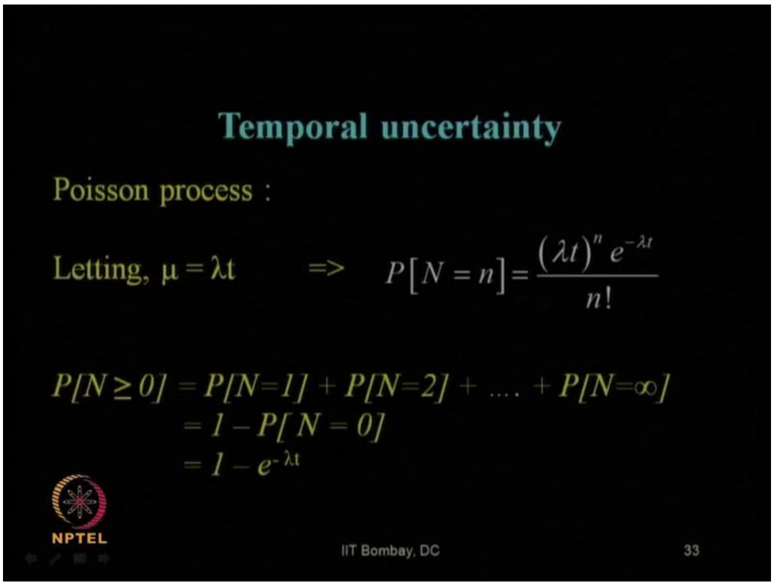

So, the Poisson's process considering this mu equals to lambda t is given by this. So, probability of occurrence… Suppose a particular event, that is, we are interested; let us say an occurrence of an earthquake of magnitude say more than 6.5 greater than equals to 0 times; that is, it will definitely occur. That is what it means. So, that will be nothing but probability of occurrence of this plus this plus this, which can be again obtained like this.

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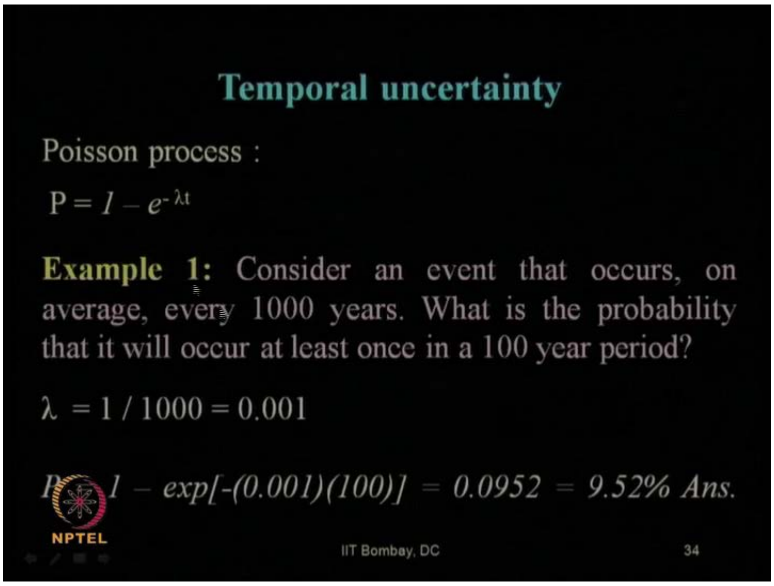

Now, let us see few examples of application of this Poisson's process, where it will occur at least once. This is the equation for probability of occurrence for an event with… At least once we have already derived that. So, if suppose a problem is given to us like, consider an event that occurs on average every 1000 years; let us say one earthquake of magnitude 7.5 occurs at your region every 1000 years from your recorded data let us say. So, the question is, what is the probability that it will occur at least once in the period of coming 100 years period? So, how we can calculate this? In this case, lambda value, which is the mean exceedance will be nothing but 1 by 1000, because it is occurring once in 1000 years. So, mean occurrence of exceedance is 1 by 1000, which is 0.001.

So, what will be the probability of occurrence of at least once? At least once means 1 minus probability of occurrence of 0 times. That we have already seen. So, 1 minus e to the power minus of lambda is 0.001. And, what is our chosen time period? That is 100 years; over the 100 years period, how many times it will occur? So, if we simplify this, we will get the value as 0.0952; which is nothing but 9.52 percent is the probability. So, that is what it says that occurrence of that particular event in 100 years period at least once is only 9.52 percent. That is what we always express the probability as we know.

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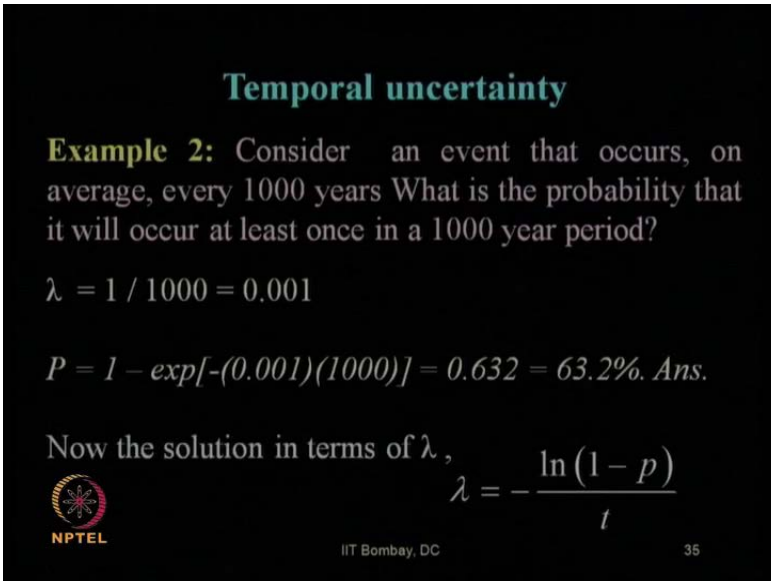

Let us take another example – second example – example number 2; that consider an event that occurs on average every 1000 years like the same event what we have discussed in example 1. What is the probability that it will occur at least one in that 1000 years period; instead of 100, now, we are computing 1000 years period. So, the laymen – what they will say directly? Because the given event is, it is occurring once in 1000 years. So, laymen those who are not knowing about the concept of probability, they will simply say it is already given that it is occurring once in 1000 years. So, if you are asking the question, what is the probability of occurrence that it will occur at least once in 1000 years, should be 1, that is, 100 percent; but, it is not so. So, let us see what probability says us. So, let us look at the calculation over here. So, in this case again, lambda is 1 by 1000, which is 0.001 mean exceedance. Now, our given value of t is 1000. So, if we put that probability of occurrence of at least once, which is nothing but 1 minus e to the power minus lambda t; if you simplify this, the value is coming 0.632, which is nothing but 63.2 percent; that means, probability of occurrence of that event at least once in 1000 years is not 100 percent, but 63.2 percent.

Now, if we change this form of solution or if we express this probability function in terms of lambda, then that is the probability of occurrence at least once. We are talking about that only. So, let us reshuffle it and express it in terms of lambda. How it should look like? The lambda should be expressed as lambda equals to minus log of 1 minus p by t, because the p was 1 minus e to the power minus lambda t. So, that is why, this minus lambda t e to the power of that; p equals to 1 minus this.

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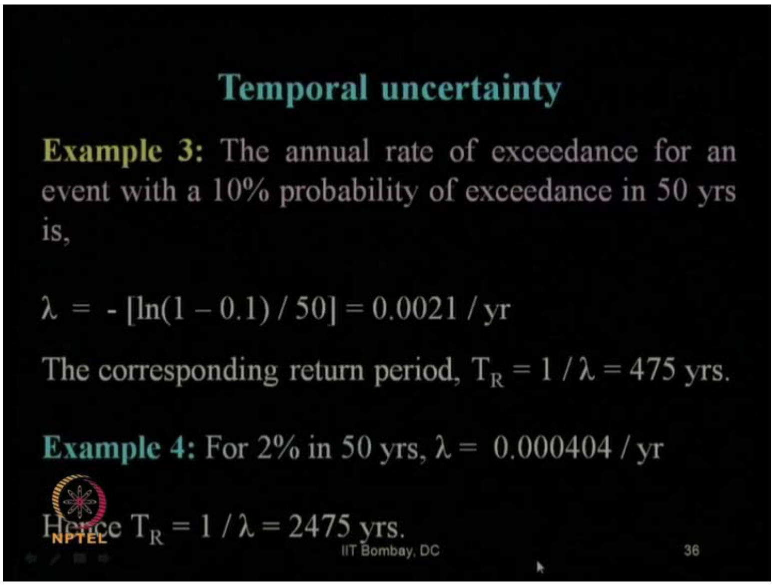

Now, let us take another example from the term of lambda; that is what we are interested now. So, the example 3 says let us say, the annual rate of exceedance for an event with the 10 percent probability of exceedance in 50 years time has to be computed; that means, a particular event let us say magnitude of 6.5 will occur with a probability of 10 percent in 50 years time. So, what is the annual rate of exceedance we need to find out. So, if we put in that expression of lambda, which is nothing but at least once. So, minus l n 1 minus probability. So, probability is 10 percent, that is, 0.1 divided by t; t is 50 years. So, it comes out to be 0.0021 per year. So, that is the mean annual rate of exceedance. So, what is corresponding return period? Corresponding return period is nothing but 1 by lambda; inverse of that, which is 475 years. Can you see that?

Let us see another example – example 4; for the same event, that is, the annual rate of exceedance we need to find out and the corresponding return period for an event with 2 percent of probability of exceedance in 50 years time. So, if it is 2 percent, this probability value will be 0.02 in this equation. This will be 50 only. If you put that, lambda value comes out to be this much; which if you inverse, T R value comes out to be 2475 years. Why I am showing these calculations? Because later on, you will see, these are very important values as most of the countries seismic design code also talks about these values; that is, we are interested to know that an event – the annual rate of exceedance of an event, which occurs with the 10 percent probability in 50 years time. So, for that, the return period is 475 years.

And, another event, which will occur with 2 percent probability in 50 years time for which the return period is 2475. These are very important for various types of design. Later on also, we will speak about this; that is, suppose for any structure of medium importance, we generally use this time of return period – 475 years, which is pretty common. And, for very important structures, we use typically the return period of this one – 2475 with 2 percent probability in 50 years of time. Others we use 10 percent probability in 50 years of time. So, like that.

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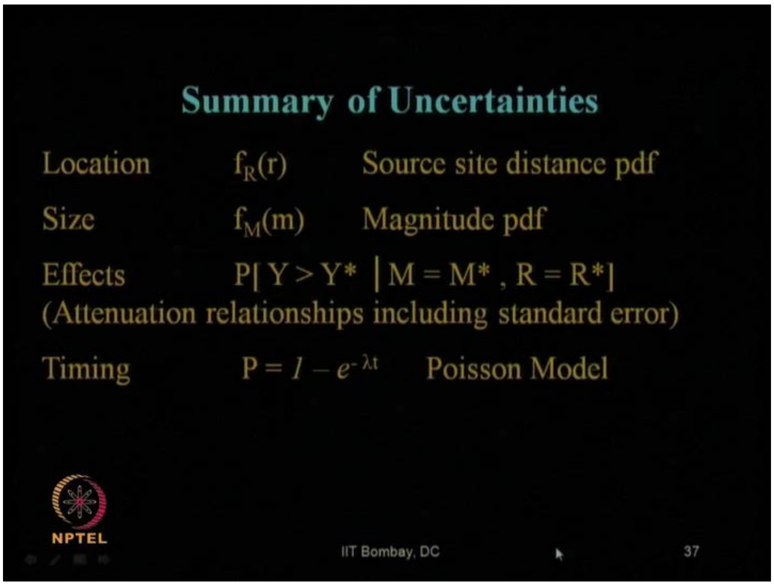

Now, if we look at the summary of all these uncertainties, what are the various uncertainties we have now discussed already? We have discussed about the location uncertainty, that is, source to site distance; that we expressed in terms of probability distribution function, in terms of f R of r. Like we have seen the histograms of various distances and we have divided and given various weightage to them, already we have seen. Next is size – uncertainty related to size of earthquake; that is, the magnitude probability distribution function. That magnitude probability distribution function also we have seen, is it not? Like for upper bound, lower bound of magnitude, Gutenberg-Richter recurrence law or earthquake recurrence law – all those things we have seen. And corresponding probability distribution function also we have discussed.

Third uncertainty is the effect. What is the effect? It is nothing but the attenuation relationship, which includes the standard error involved in the attenuation relationship. Why that standard error coming into picture? When we talked about in our one of the previous modules, each of the attenuation relationship is having some amount of standard error when they have been established in terms of empirical relations. So, those including those standard errors we have to find out what is the probability of occurrence of a particular event greater than some value about which we are interested to for a given value of this distance and for a given value of this magnitude. So, this is the conditional probability distribution considering the attenuation effect and the standard error. So, this is taking care of third uncertainty.

And, what is the forth uncertainty? Forth uncertainty is the timing. How we can consider that forth uncertainty? Through the use of this Poisson's model; like how this time factor will effect and their number of occurrences of course; that also we can take care of through this Poisson's model. So, in that fashion, all the uncertainties we need to look into this probabilistic seismic hazard analysis like location-related uncertainty, size-related uncertainty, effect-related uncertainty and timing-related uncertainty. So, considering all these uncertainty, further we can go ahead. And we have to now see, how we should take care of the combined effect of these uncertainty; because remember, none of these are independent events; they are also dependent to each other; some of them. So, how their conditional dependency of one event to another will affect our final probability consideration; that we need to now take care of. So, these are individual uncertainties. Now, we will see the combined behavior of the uncertainty. So, with this, we have come to the end of today's lecture. We will continue further in our next lecture.