

**Geotechnical Earthquake Engineering**  
**Prof. Deepankar Choudhury**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Bombay**

**Module - 7**

**Lecture - 25**

**Seismic Hazard Analysis (Contd...)**

Let us start our today's lecture for this NPTEL video course on geotechnical earthquake engineering. We were going through the module number seven that is seismic hazard analysis. Let us have a quick recap, what we have learnt in our previous lecture.


(Refer Slide Time: 00:40)

**Deterministic Seismic Hazard Analysis**

**Maximum credible earthquake (MCE)**  
is the maximum earthquake that appears capable of occurring under the known tectonic framework.

**Maximum probable earthquake (MPE)**  
is the maximum historical earthquake and the maximum earthquake likely to occur in a 100-year interval.

Many DSHA used both the MCE and MPE for evaluating hazards.

 NPTEL IIT Bombay, DC 3

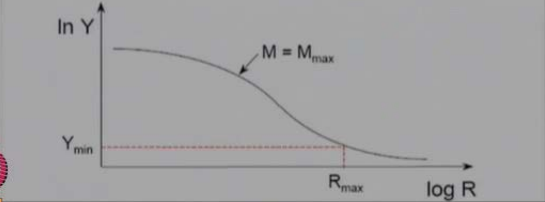
In the previous lecture, we talked about, what is deterministic seismic hazard analysis. And for that we have seen two definitions of earthquake like maximum credible earthquake – MCE, and maximum probable earthquake – MPE. And for this deterministic seismic hazard analysis or DSHA, we use both this MCE and MPE for evaluating the hazards.

(Refer Slide Time: 01:11)

**Deterministic Seismic Hazard Analysis**

***Identification and Characterization of all Sources***

- Estimate maximum magnitude that could be produced by any source in the vicinity of site.
- Find value of  $R_{\max}$  corresponding to  $M_{\max}$  at threshold value of parameter of interest,  $Y_{\min}$ .



IIT Bombay, DC

For obtaining the DSHA or doing the DSHA, first step is to identify and characterize all sources involved for a corresponding site, where we want to find out the seismic hazard in deterministically. So, we first estimate the maximum magnitude that could be prepared, that could be produced by any source in the near vicinity of the site. So, there can be several faults from the past historic data or the fault characteristics we can estimate what is the maximum value of  $M_w$  in the nearby region, which can occur for that site.

Then find out the value of  $R_{\max}$ , that is, maximum distance corresponding to that  $M_{\max}$  value at that threshold parameter of our interest. Threshold parameter of our interest means for the seismic hazard, which parameter we want to obtain. It can be peak ground acceleration; it can be peak horizontal velocity; it can be spectral acceleration – any of such kind of value. So, we have to determine, what is the threshold number, we are interested in; that is, this  $y_{\min}$  value we have to identify from our design perspective.

Say let us denote for example, for peak ground acceleration, 0.05 G is the minimum value, which we want to know. So, we have to see from various faults, etcetera, what is the attenuation for different values of magnitude – earthquake magnitude and that value of say PGA with respect to the distance. Now, once we know that variation, what we can do? From this plot of log-log plot of this, that particular hazard parameter, which we

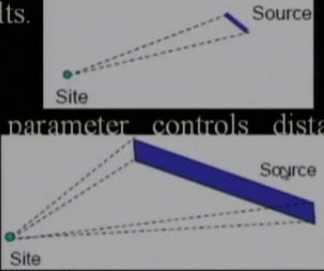
want to obtain – that versus R value we can plot. And based on our corresponding or the parameter of interest minimum value or threshold value, we find out from this curve, which is for the maximum magnitude. That is what we have to select – maximum magnitude. What is the distance? We will automatically get. So, that distance will give us the idea; that is, from a particular site, how far all the faults we are going to consider.

(Refer Slide Time: 03:46)

**Deterministic Seismic Hazard Analysis**

*Identification and Characterization of all Sources*

- **Characterize Geometry:**
  - Point Source : Constant source – site distance. volcanoes, distant short faults.
  - Linear Source : One parameter controls distance. Shallow distant faults.



NPTEL  
IIT Bombay, DC 5

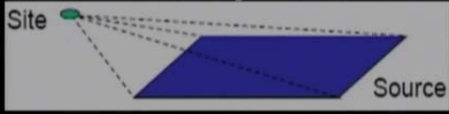
Then, next is once we find out that, up to which distance we will consider all the sites, then we have to characterize the geometry of all the sources. So, how to characterize that? There can be either point source, where from the site to the source, we will have a constant value or constant distance, because it is just similar to like a point. And for example, volcanoes, short distances, etcetera are such category. For linear source also, we use only one parameter that controls, because it is in one dimension; say if we consider about XYZ coordinate system, we can put it in just one coordinate system by orienting the axis in that direction.

(Refer Slide Time: 04:38)

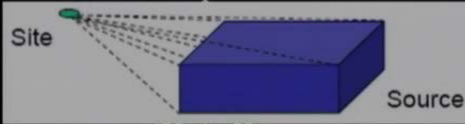
**Deterministic Seismic Hazard Analysis**

*Identification and Characterization of all Sources*

- **Characterize Geometry:**
  - Area Source : Two geometric parameters control distance. volcanoes, Constant depth crustal source.



- Volumetric Source : Three parameters controls distance.



NPTEL IIT Bombay, DC 6

Then, another can be area source, where two geometric parameters are required to identify or to characterize the source. In that case, if we have XYZ axis system, we request say X and Y system to identify the source. And another is called volumetric source, where we require all the three parameters, that is, X coordinate, Y coordinate and Z coordinate of a source from the site to identify the source. So, that is the different types of sources by which we need to characterize them.

(Refer Slide Time: 05:14)

**Deterministic Seismic Hazard Analysis**

Establish earthquake magnitude - typically  $M_{\max}$

- Empirical correlations
- Rupture length correlations
- Rupture area correlations
- Maximum surface displacement correlations
- “Theoretical” determination
- Slip rate correlations

NPTEL IIT Bombay, DC 7

Next, we have seen, establish the earthquake magnitude – typically the  $M_{max}$ . And for obtaining the  $M_{max}$ , as I have mentioned, we use the empirical correlations from the fault characteristics; like from rupture length correlations, rupture area correlations or the maximum surface displacement correlations or the theoretical determination and based on the slip rate correlations.

(Refer Slide Time: 05:42)

### Deterministic Seismic Hazard Analysis

- *Define Source – Site Distance: Must be consistent with predictive relationship. Should include finite fault effect.*
- *Select Source – Site Distance.*

Distance to surface projection of closest point on rupture surface

Epicentral distance

Distance to closest point on rupture surface

Hypocentral distance

NPTEL

IIT Bombay, DC

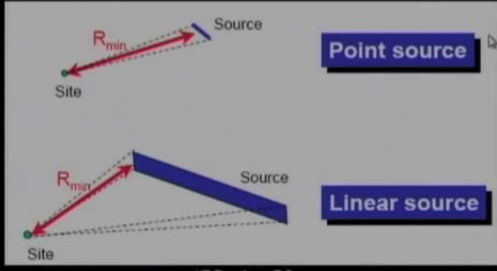
8

Then, we need to define what is known as source to site distance. And for defining that, what we select, always the minimum distance from the source to site. And it must be consistent with the predictive relationship; that is, whatever predictive relationship of attenuation we are going to use for that site, that corresponding distance we need to consider. So, there are several distances like it can be epicentral distance if we consider; from site to the epicenter the distance if we consider, that is known as epicentral distance. If we consider from site to hypocenter, it is hypocentral distance. We can consider the distance closest point of the rupture from the site or we can consider the surface projection of that closest point to the site. So, these are various distances. And any of this we can consider depending on which predictive relationship we are going to use.

(Refer Slide Time: 06:41)

### Deterministic Seismic Hazard Analysis

*Select source – site distance parameter:*  
Typically assume shortest source – site distance (“worst case” scenario).



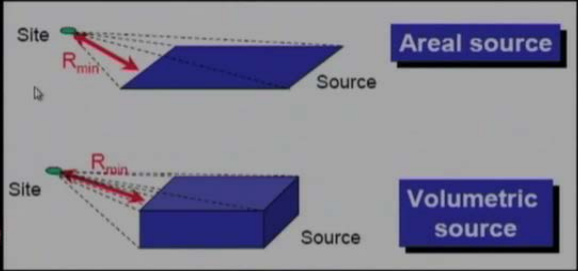
The diagram shows two scenarios for source-site distance. In the top scenario, a 'Point source' is represented by a blue dot, and a 'Site' is a green dot. A red arrow labeled  $R_{min}$  points from the site to the source. In the bottom scenario, a 'Linear source' is represented by a blue line segment, and a 'Site' is a green dot. A red arrow labeled  $R_{min}$  points from the site to the closest point on the linear source. The NPTEL logo is in the bottom left, and 'IIT Bombay, DC' and the number '9' are in the bottom right.

Then, we have seen that, for site to source distance parameter, typically, we need to consider the worst case scenario. Worst case scenario means the minimum distance from the site to source. So, if it is a point source, obviously, it is a single distance we do not have any control; that distance itself will be the minimum distance. If it is a linear distance, among the linear distance, obviously, the nearest point or minimum x distance we have to consider.

(Refer Slide Time: 07:08)

### Deterministic Seismic Hazard Analysis

*Select source – site distance parameter:*  
Typically assume shortest source – site distance (“worst case” scenario).



The diagram shows two scenarios for source-site distance. In the top scenario, an 'Areal source' is represented by a blue rectangular area, and a 'Site' is a green dot. A red arrow labeled  $R_{min}$  points from the site to the closest point on the areal source. In the bottom scenario, a 'Volumetric source' is represented by a blue rectangular volume, and a 'Site' is a green dot. A red arrow labeled  $R_{min}$  points from the site to the closest point on the volumetric source. The NPTEL logo is in the bottom left, and 'IIT Bombay, DC' and the number '10' are in the bottom right.

For area source also, the R minimum will be nothing but the minimum of the distance of that x and y coordinate. So, root over x square plus y square of any point of that area source needs to be considered minimum value of that for our worst case scenario. Similarly, for volumetric source, we need to consider root over x square plus y square plus z square – minimum of that will be the R minimum from the site to source.

(Refer Slide Time: 07:38)

**Deterministic Seismic Hazard Analysis**

**Select controlling earthquake**

- Decision based on ground motion parameters of greatest interest
- Consider all sources
- Assume  $M_{\max}$  occurs at  $R_{\min}$  for each source
- Compute ground motion parameters based on  $M_{\max}$  and  $R_{\min}$
- Determine critical values of ground motion parameters

NPTEL IIT Bombay, DC 11

Then, we need to consider the controlling earthquake; that is, how to identify or how to select the controlling earthquake, decision is based on the ground motion parameters of greatest interest. We consider all the sources involved within that R max distance and assume that M max occurs at R minimum for each sources. So, that is one assumption. Like in the previous slide, what we have seen, suppose this is the volumetric source, we have from site to source, this is the minimum distance – R minimum. But this site may be the maximum one. And it may happen for the source, the magnitude of occurrence of earthquake from the past history may be different from these two ends if the source length or source volume is much bigger compared to this distance. It is quite possible.

But even then in this deterministic process of seismic hazard analysis, we assume that, the same magnitude or same maximum magnitude occurs at every point. Hence, we consider M max also occurs at this minimum distance. That is what it means that, assume M max occurs at R minimum for each of the sources. Then compute the ground motion

parameters based on that  $M$  max and that  $R$  minimum and then determine the critical values of the ground motion parameters.

(Refer Slide Time: 09:06)

**Deterministic Seismic Hazard Analysis**

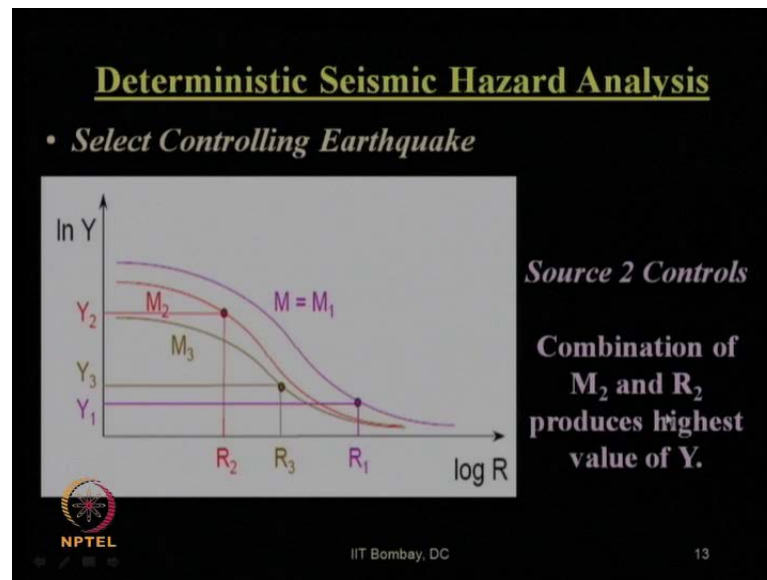
- *Select Controlling Earthquake*

The diagram shows a central yellow dot labeled 'Site'. Three sources are identified: Source 1 (a linear source with magnitude  $M_1$  and distance  $R_1$ ), Source 2 (an areal source with magnitude  $M_2$  and distance  $R_2$ ), and Source 3 (a linear source with magnitude  $M_3$  and distance  $R_3$ ). The NPTEL logo is in the bottom left, and 'IIT Bombay, DC' and '12' are in the bottom center and right respectively.

So, this is how we select the controlling earthquakes. Suppose from this site, we have three identified from that  $R$  max distance based on the  $M$  max data; these are the sources let us say – source 1, source 2 and source 3. So, among these, source 1 is nothing but a linear source; source 3 is also a linear source; whereas, source 2 is an areal source. Now, from this site, we need to find out the minimum distances to each of the sources. So, this  $R_1$  is the minimum distance between site to source 1;  $R_2$  is from site to source 2; and  $R_3$  is from site to source 3.



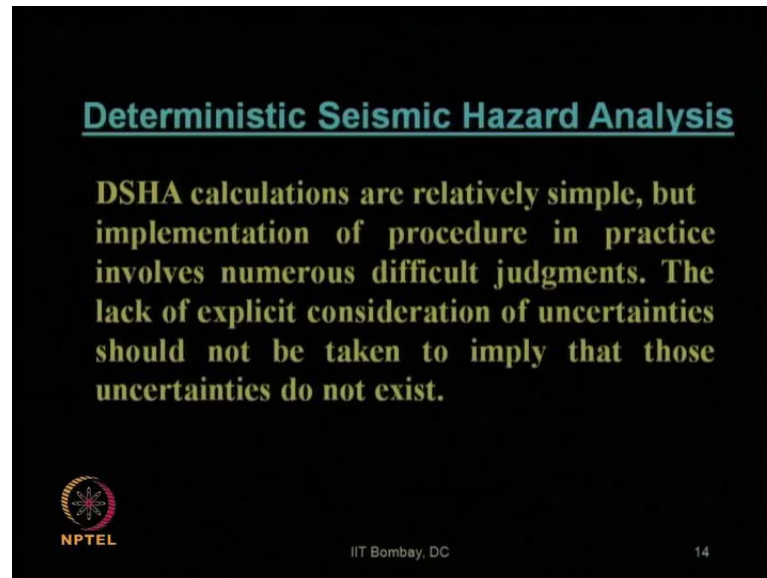
(Refer Slide Time: 09:46)



Now, once we find out those distances, what we need to do? We need to now plot our chosen parameter of our hazard analysis, which is of our interest. It can be PGA; it can be spectral acceleration; it can be PHV, anything. So, depending on the attenuation relationship, we know for each of the sources, what is based on that maximum magnitude for each source. For each source, this  $M_1$  is maximum magnitude for source 1;  $M_2$  is maximum magnitude for source 2;  $M_3$  is maximum magnitude for source 3. So, for each of them, we have this parameter versus distance relationship based on that attenuation relationship.


Now, what we can do, we have identified this  $R_1$ ,  $R_2$ ,  $R_3$  in the previous slide. So, we need to now find out this  $R_1$  and project it over here; then get the value of  $Y_1$ . Then similarly,  $R_2$  project it here; get the value of  $Y_2$ . Get the value of  $R_3$  over here; project it and get the value of  $Y_3$ . That way, we get the values of our hazard parameter of interest like  $Y_1$ ,  $Y_2$ ,  $Y_3$ . And among them, whichever gives the maximum value – that is nothing but our final result of seismic hazard. So, from this example, what we can see,  $Y_2$  is maximum. That automatically says that, source 2 controls the hazard for that site. So, for the chosen site in the previous slide, what we have seen, this site is controlled by this source 2, because  $Y_2$  that (( )) the hazard parameter is yielding as the maximum value at this source 2. So, combination of  $M_2$  and  $R_2$  produces the highest value of  $Y$ . So, that is the final result of our deterministic seismic hazard analysis.

(Refer Slide Time: 11:43)



**Deterministic Seismic Hazard Analysis**

**DSHA calculations are relatively simple, but implementation of procedure in practice involves numerous difficult judgments. The lack of explicit consideration of uncertainties should not be taken to imply that those uncertainties do not exist.**

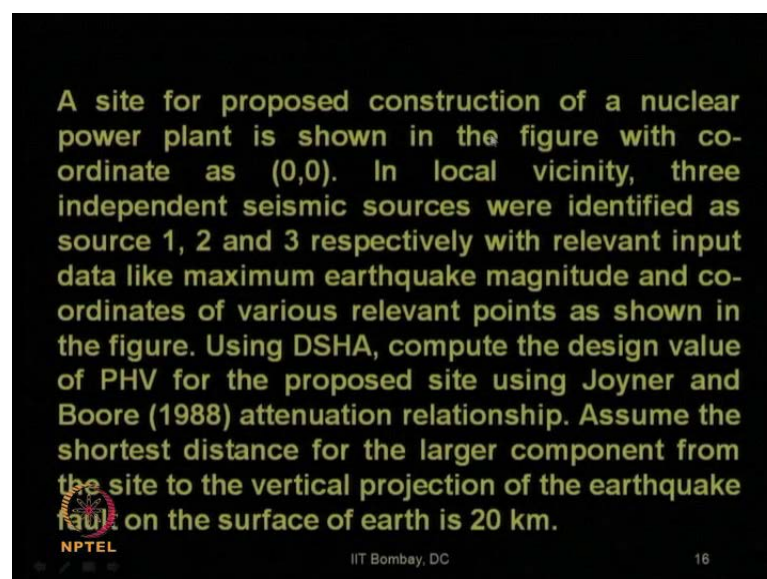
 NPTEL

IIT Bombay, DC


14

So, we have seen the various features of DSHA like DSHA calculations are relatively simple as we have seen. But implementation of procedure in practice – it involves numerous difficult judgments like whether it really occurs; whether that maximum earthquake occurs or not; whether that distance chosen minimum is justified or not. So many other certainties and uncertainties are involved. So, the lack of explicit consideration of uncertainties should not be taken to imply that those uncertainties do not exist. Actually, all the uncertainties exist, but we consider always the worst case scenario or the critical scenario for the deterministic seismic hazard analysis.

(Refer Slide Time: 12:37)



**A site for proposed construction of a nuclear power plant is shown in the figure with coordinate as (0,0). In local vicinity, three independent seismic sources were identified as source 1, 2 and 3 respectively with relevant input data like maximum earthquake magnitude and coordinates of various relevant points as shown in the figure. Using DSHA, compute the design value of PHV for the proposed site using Joyner and Boore (1988) attenuation relationship. Assume the shortest distance for the larger component from the site to the vertical projection of the earthquake fault on the surface of earth is 20 km.**

 NPTEL

IIT Bombay, DC

16

Next, in the previous lecture, we have gone through one example problem. This was the example problem – one site has been proposed for the nuclear power plant construction. So, we need to find out the deterministic seismic hazard analysis, because we have seen, that is the seismic hazard analysis needs to be applied for important structure like nuclear power plant, dam, bridges, etcetera. So, in the figure, we will see later on that, this is the coordinate system 0, 0 for the site. And in the local vicinity, there are three sources: 1, 2, 3 with their respective coordinates. Using this, we need to find out the PHV value. PHV is the seismic hazard parameter, which we need to obtain through this process of DSHA using the attenuation relationship of Joyner and Boore. And also, what is the shortest distance for the larger component, that assumption and the distance is given to us; that R value.

(Refer Slide Time: 13:41)

Prof. D. Chandley  
Lec - 22, Date: 07/0/13

Soln

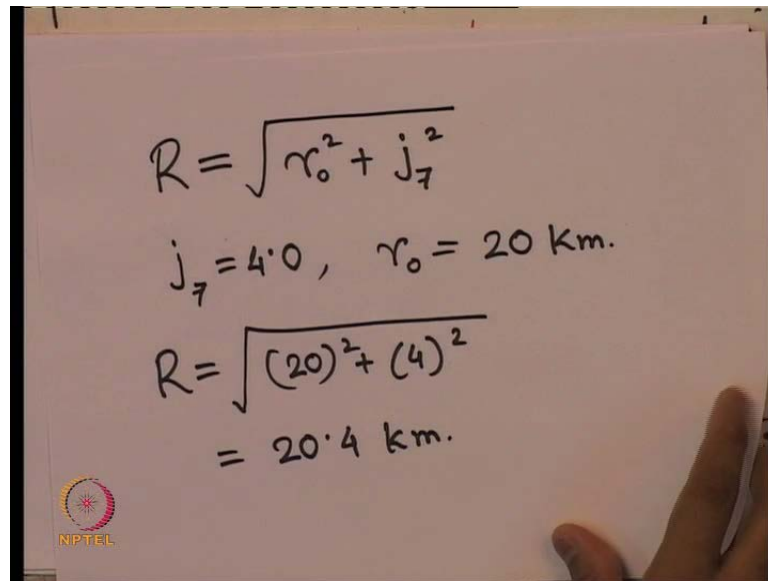
Source	Minimum Distance (km)	M <sub>max</sub>
1	$\sqrt{15^2 + 5^2}$ = 15.81	7.0
2	$\sqrt{30^2 + 20^2}$ = 36.058	6.0
3	> 15	6.5

← Controlling Source

DC/1

So, we have seen in our previous solution, this is the solution we have worked out that, for three different sources, first step is to identify what are the minimum distances. And we consider for each of them, M max actually occurs. So, from our given data, we find out these are the M max values given and these are minimum distance from source to site; and the combination, which gives us minimum distance with maximum magnitude – obviously, that will control the hazard parameter. So, this is the controlling earthquake for our example we have seen.

(Refer Slide Time: 14:18)

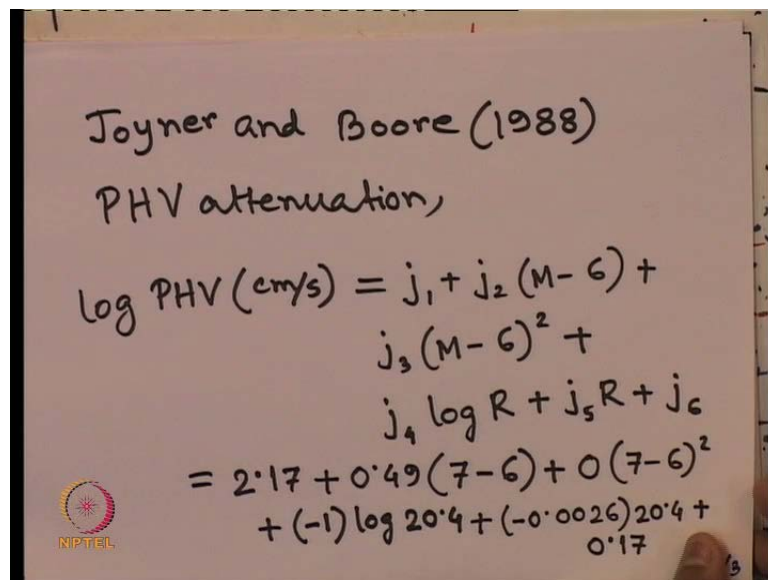


A photograph of a whiteboard with handwritten mathematical equations. The equations are:  $R = \sqrt{r_0^2 + j_7^2}$ ,  $j_7 = 4.0$ ,  $r_0 = 20 \text{ km.}$ ,  $R = \sqrt{(20)^2 + (4)^2}$ , and  $= 20.4 \text{ km.}$ . There is a small NPTEL logo in the bottom left corner of the whiteboard.

$$R = \sqrt{r_0^2 + j_7^2}$$
$$j_7 = 4.0, \quad r_0 = 20 \text{ km.}$$
$$R = \sqrt{(20)^2 + (4)^2}$$
$$= 20.4 \text{ km.}$$

Then, further, we have calculated the values of this R distance using this formula for Joyner and Boore equation.

(Refer Slide Time: 14:25)



A photograph of a whiteboard with handwritten text and equations. The text reads: "Joyner and Boore (1988)", "PHV attenuation,", and the equation:  $\log \text{PHV (cm/s)} = j_1 + j_2 (M-6) + j_3 (M-6)^2 + j_4 \log R + j_5 R + j_6$ . Below this, a numerical calculation is shown:  $= 2.17 + 0.49(7-6) + 0(7-6)^2 + (-1) \log 20.4 + (-0.0026) 20.4 + 0.17$ . There is a small NPTEL logo in the bottom left corner of the whiteboard.

Joyner and Boore (1988)  
PHV attenuation,

$$\log \text{PHV (cm/s)} = j_1 + j_2 (M-6) + j_3 (M-6)^2 + j_4 \log R + j_5 R + j_6$$
$$= 2.17 + 0.49(7-6) + 0(7-6)^2 + (-1) \log 20.4 + (-0.0026) 20.4 + 0.17$$

Where, this Joyner and Boore equation for PHV is given by this expression. All these coefficients are known to us for a computed R value.

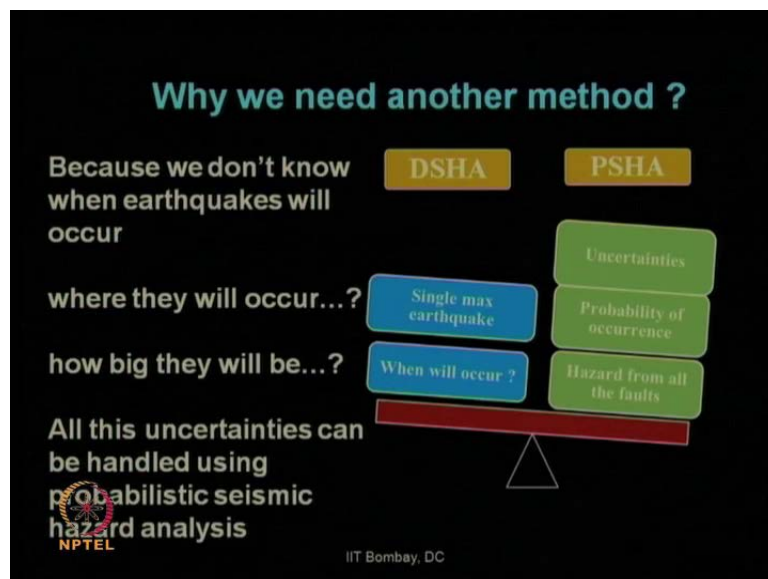
(Refer Slide Time: 14:35)

$$\log \text{PHV (cm/s)} = 1.46733$$
$$\text{PHV} = 29.33 \text{ cm/s}$$

Ans ..

Finally, we obtain that, the PHV for the site for the construction of nuclear power plant, the value of PHV or the design value is this much based on this deterministic seismic hazard analysis. So, with that, we have completed our previous lecture. Let us see how we can continue further in today's lecture. Now, we have discussed so far, the deterministic seismic hazard analysis in detail. What are the pitfalls?

(Refer Slide Time: 15:05)



Let us see, we need another method. Why? Because we have seen none of the uncertainties involved in the process of seismic hazard analysis are considered in our

deterministic seismic hazard analysis, because we do not know when the earthquakes will occur. Like when we are considering for a particular source that, this is the maximum value of earthquake, which has occurred from the historical data. So, automatically, the question arises, how many years back we will go to identify that, which value of maximum magnitude we will select. We will go back 100 years? We will go back 1000 years? We will go back 150 years? We will go back 500 years? Nothing is specified. So, it depends on which earthquakes need to be selected. There is a lot of uncertainties; lot of question mark. Also, if you go suppose 200, 300 years back earthquake history data, many a times, many data may not be that reliable mostly based on the experience of the people, how they felt the earthquake, if it is recorded at all in that manner.

Also, where they will occur? That is, it is not necessary that, in a long length of a fault, it will occur the hypocenter at every point. Like what we considered in deterministic seismic hazard analysis that, equal amount of chances for the sources, so entire source length. Suppose if a fault is running for several hundreds of kilometer; in the deterministic seismic hazard analysis, we consider that, as if the future earthquakes are also prone to occur at any point equally.

But it is not so. It has to be based on certain mathematical calculation or on certain based on some probability that, at this point, there is a more probability of occurrence of that magnitude of earthquake rather than another point if it is running for several hundreds of kilometers, because when we are considering the distances as I said, in deterministic seismic hazard analysis, typically 250 kilometer or 300 kilometer of radius of distance from a particular site we considered. And among that, if a fault length itself is more than 100 kilometers, we have to be very selective that, what are the chances that, an occurrence of a particular magnitude in that fault at a particular location, how much chances are there? So, those uncertainties will require to be addressed. So, where they will occur is a big question mark.

Then, how big they are? That is their magnitude once again. As I said, we do not know that, how much magnitude may come and what is the probability of occurrence. It is not that. Suppose we are planning to design our structure say for 50 years or say for 100 years. So, unless we do a proper mathematical calculation based on the probability or chances of occurrence of a maximum value of earthquake at a particular source, it will be

very much unrealistic. Suppose chances of occurrence from the probability estimation or probability calculation if we found that, maximum value of earthquake at a particular source, chances of occurrence is say 1 in 200 years. And now we are designing say, the structure for a design life of 50 years.

So, we should not use that value, which is having an occurrence of once in 200 years, because that will be more unrealistic; rather than that, we should select such a value, which is having chances of occurrence or probability of occurrence once in 50 years time; that value of maximum earthquake we need to consider. So, that is why, how big of that magnitude we need to select based on the chances of occurrence also needs to be addressed. So, all these uncertainties can be handled using another type of seismic hazard analysis that is called probabilistic seismic hazard analysis. So, in short it is called PSHA – probabilistic seismic hazard analysis.


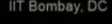

Like at the beginning, we mentioned seismic hazard analysis is categorized in two classes: one is DSHA – deterministic seismic hazard analysis; another is PSHA. So, you can see, this picture clearly shows this is the DSHA; this is PSHA. So, in DSHA, we have single maximum earthquake in our consideration. We do not consider all other earthquakes. Whatever the maximum value, that only, that is, worst case scenario we consider. And when will they occur – that also is the very big question mark to the community? That is, how far we will go? How many years we will go back? Whereas, PSHA – how it is advantageous? Why the fulcrum shows the inclination towards this; that is, automatically shows it is more logical. Why?

Let us see. It takes care of all the uncertainties involved in the process. We will go through now slowly when we will describe this PSHA. It takes care all these mentioned uncertainties what we discussed just now. Their probability of occurrence, that is, how much chances are there to occur that value of earthquake, that distance of earthquake, that fault of consideration, etcetera. And hazard is taken from all the faults, but not with equal values. It has to be taken with different magnitudes based on this probability or uncertainties involved. Like here in DSHA, what we have considered? Hazard is taken from all faults; but we have considered all faults are equally responsible. But it should not be, it cannot be. So, the PSHA considers all these uncertainties. That is the advantage of PSHA over this DSHA if we compare like this.

(Refer Slide Time: 21:45)

**Probabilistic Seismic Hazard Analysis (PSHA)**

- PSHA characterizes uncertainty in location, size, frequency and effects of earthquakes and combines all of them to compute probabilities of different levels of ground shaking
- **Goal:** to quantify the rate (or probability) of exceeding various ground-motion levels at a site, given all possible earthquakes
- **Traditionally:** Peak Ground Acceleration (PGA) used to quantify ground motion
- **Present time:** Response Spectral Acceleration (SA) preferred – expected SA in accordance to the natural frequency of the building

 NPTEL  IIT Bombay, DC  18

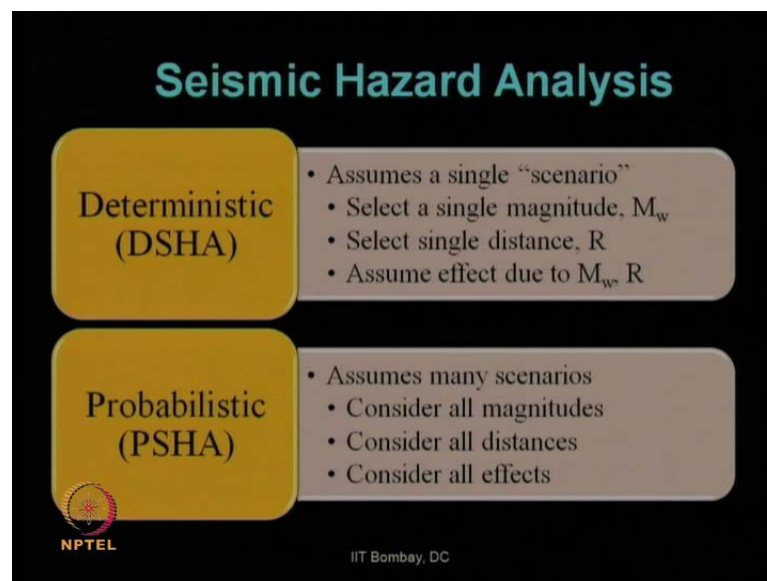
So, if we see the basic concepts of this probabilistic seismic hazard analysis or PSHA in short, PSHA characterizes uncertainty in location, uncertainty in size, uncertainty in frequency; frequency means the number of times of its occurrence and effects of earthquakes and combines all of them to compute this probabilities of different levels of ground shaking. So, whatever is the hazard parameter, whether PGA, PHV or spectral acceleration, that value, what are the chances or probabilities need to be computed mathematically based on considering uncertainties involved in the earthquake location, earthquake size, number of occurrences their effects – all of these together. So, the goal is to quantify the rate or the probability of exceeding various ground motion levels at a site given that, all possible earthquakes are taken into consideration. So, it is fully based on the probability calculation; that is, it should exceed a certain value, which is of our interest for the design.

And traditionally, the peak ground acceleration used to quantify the ground motion. But other than that, in present time, response spectral acceleration like SA also preferred, because expected SA is in accordance to the natural frequency of the building. We will discuss this little later that, how it is correlated, because as I have already defined you in previous one of the lecture, while deriving the relationship of spectral acceleration. Spectral acceleration with respect to time period of an assumed single degree of freedom system, we have already considered; and from that, we have seen, based on different time period of the system, we can identify, how much is the spectral acceleration. And



that value we can design for building. So, that is why, it is said, for a particular building or for a particular structure say bridge or anything, it will have a natural frequency. If we can calculate the natural frequency, we can convert it to natural period; from that period based on this spectral acceleration, we can use or we can get a value, which is a design value. That is why, the results of PSHA can be expressed either in the form of PGA or in the form of spectral acceleration. These are common outputs.

(Refer Slide Time: 24:35)

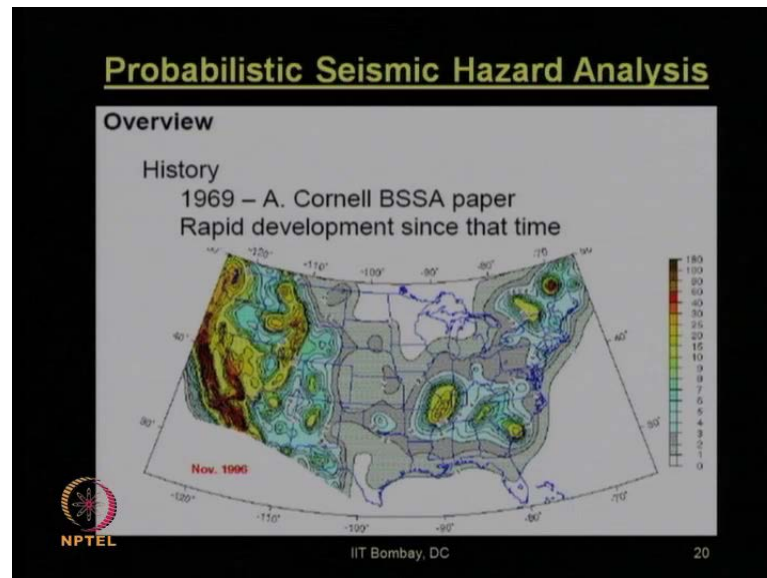


Once again, if we compare this DSHA and PSHA, like in DSHA, we have seen, we assume a single scenario. What is that single scenario? That is worst case scenario or the maximum case scenario. So, here we select a single magnitude  $M_w$ , which has to be the maximum magnitude. We select a single distance, which has to be a minimum distance. And we assume, the effect due to both this maximum magnitude and minimum distance for each sources to calculate the hazard parameter. Whereas, in PSHA, we assume many scenarios; like we consider all the magnitudes; not only the maximum, but also other magnitudes; that means, suppose in the several sources, among those sources, not only the sources, which produce maximum magnitude, but also the sources, which produce say lesser than maximum magnitude also needs to be considered.

Consider all the distances. It is not necessarily only the minimum distance need to be considered. We have to consider from the site to source all the possible distances. And we have to give certain weightage to them. We will see that very soon. And consider all

the effects, that is, distance, magnitude – all these combined effect needs to be considered for this PSHA analysis.

(Refer Slide Time: 26:03)




Now, when this probabilistic seismic hazard analysis started; in the history, if we see the literature, the overview of literature, we can find out that, 1969 is the year when Researcher Cornell – he published the output of this probabilistic seismic hazard analysis in bulletin of Seismological Society of America. It is in short, we call BSSA – that paper. And since that 1969, there is a rapid development of that area throughout the world; that is, now, people are using this PSHA more than DSHA. So, 1969 is the pioneering year when this PSHA had started; and it was started by Cornell. And credit goes to Researcher Cornell.

(Refer Slide Time: 26:58)

**PSHA...**

- **Input:**
  - Seismicity Model (not discussed here)
    - seismicity distribution in space (area) and time
    - Magnitude-frequency distribution
    - $M_{\max}$  maximum possible earthquakes
  - Ground motion prediction equation (given  $M$  and hypocenter)
  - Site Response Model
- **Output:**
  - exceeding a ground motion level within time period  $T$
  - probability of  $P$ .

 NPTEL IIT Bombay, DC 21

What are the various inputs for this PSHA, which are necessary? Like input is seismicity model. Now, we are not discussing it here. We will discuss it later on, what are the various seismicity models, etcetera. When we will take an example problem for a particular area, that time we will consider various seismicity models. Like what they discuss in the seismicity model – how the seismicity is distributed in the space, that is, in the entire area and how it is distributed over the time; that means, from the various collected historical seismic data at different different times, how we can obtain a particular mathematical model, which will give us the occurrence of that seismic event in future time, because as we know, we cannot predict earthquake so far. So, we do not know in future, 10 years later, 20 years later, 50 years later or during the life span or design life of our building, which we are going to construct at a site, what will be the future occurrence of earthquake. So, how we can find it out? We find it out based on this seismicity model based on the collected historical earthquake data in that area and with respect to time. That is why, various seismicity models are available, various mathematical models are available, which we will discuss little later.

Now, magnitude and frequency distribution – magnitude means among this seismicity distribution, what are the values of their seismic events; that is, what are the magnitude of the seismic events. And frequency means the number of occurrences; that is, say from a source, we got maximum magnitude, various magnitudes of earthquake say magnitude 4, 4.5, 5; there are several numbers. But there are only hardly any one or two magnitudes

of say 7, 7.5 – that higher ranges. So, how those values are distributed needs to be identified. Among them, we have to find out what is  $M_{max}$ , that is, maximum possible earthquake for that fault or for that source. Remember here, when we are computing this maximum possible earthquake based on the seismicity model, you know, the chances of occurrence or chances of recurrence of a particular earthquake magnitude in the design life of the structure. What I was mentioning, which is not possible in DSHA; but in PSHA, we can do that.

Suppose for a particular site, we found out using seismicity model, that chances of occurrence of magnitude of earthquake say 8 is once in 200 years. But the design life of the structure is just 50 years. So, chances of occurrence of maximum earthquake in that 50 years, we need to find out, not that in 200 years. So, we should not design the structure based on that  $M$  value of 8; we should calculate it and find out what is the magnitude. It can be 7; it can be 6.5; it can be 7.5 based on how the seismicity values are distributed, how this magnitude versus frequency is distributed, and so many other mathematical terms. So, ground motion prediction equation is another input parameter, which gives the data on this magnitude with respect to hypocenter. So, basically, this ground motion prediction equation in short, we call them as GMPE; as you can see, the first letter of all these words. So, GMPE is nothing but those attenuation relationships. For a particular site, you see here the relationship has to be with respect to the both magnitude and hypocenter. So, whatever attenuation relationships are involved with the relationship with respect to magnitude and hypocenter needs considered, not the other relationship, where hypocenter is neglected or magnitude is neglected like that way.

Then, which site response model we are going to use? That also needs to be identified or needs to be chosen as an input parameter for this PSHA. Then our final output will be (Refer Slide Time: 31:35) exceeding a ground motion level within a time period of  $T$  with the probability of  $P$ ; that is, this is our final output. In the case of DSHA, what was our final output? What is the controlling earthquake? Which one gives us the maximum value of hazard parameter. But in this case, not the maximum value, we have to select that for a particular chosen time period, what is the probability of occurrence of exceedance. Suppose we are interested to know that, probability of occurrence say more than 70 percent chance of occurrence of a particular magnitude of earthquake in the design life of 50 years is that much. So, based on that, we have to compute the output; it

is not just maximum value. So, this is another specialty of this PSHA; that is, we have a control on this based on the probability of occurrence. We obviously will not design based on the 100 percent probability.

(Refer Slide Time: 32:37)

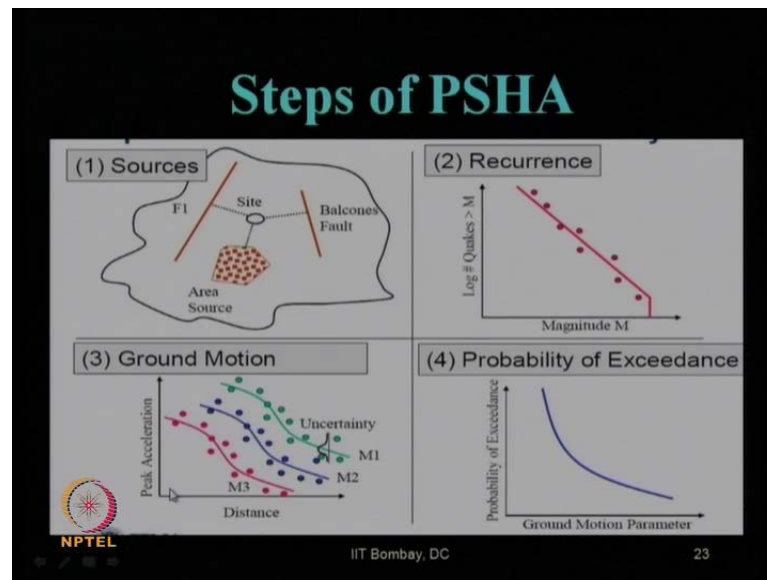
**PSHA Consists of four primary steps**

1. Identification and characterization of all sources
2. Characterization of seismicity of each source
3. Determination of motions from each source
4. Probabilistic calculations

NPTEL IIT Bombay, DC 22

PSHA – that consists of again four primary steps. Like DSHA also, here in case of PSHA, we can have major four steps to obtain the hazard parameter. What are those steps? First step is same, that is, identification and characterization of all sources. Then second phase is characterization of seismicity of each source. Then third step is determination of motions from each source. And the fourth phase is probabilistic calculations based on all these input data.

(Refer Slide Time: 33:13)



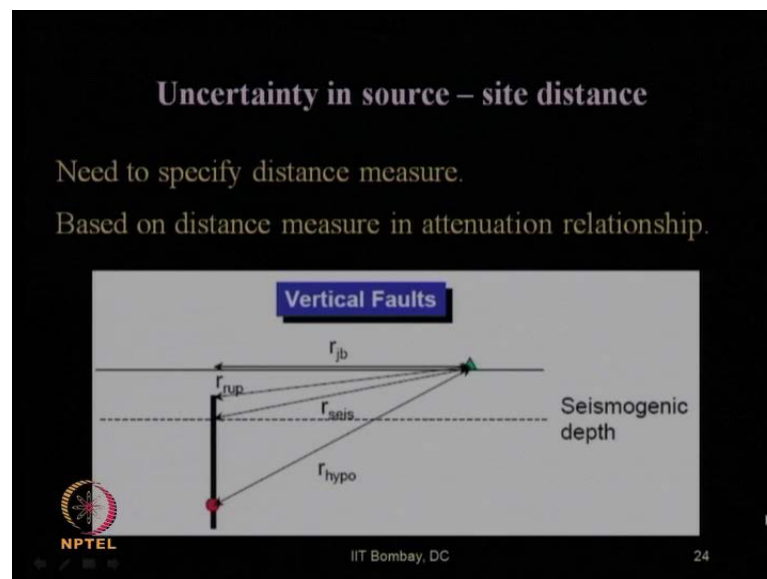
See if we see graphically, through a picture, these are the four steps as it is involved. You can see over here. First step is to identify the source. So, for that, again from the site, you can find out the zone; that is, up to which boundary you should consider the faults. In this case also, you can probably apply the same logic what we use for DSHA; that is, the maximum magnitude with maximum distance based on the threshold parameter of our interest. So, based on that, you can first identify the boundary. But once you identify the boundary, your all sources are coming within that. Suppose these are the various sources. In that case, it is not the minimum distance now; you have to take all the distances now. That is the difference from the DSHA to PSHA. How to take all the distances and etcetera? We will discuss very soon.

The next phase is recurrence of that earthquake for that site, which is influencing. What is that recurrence? That is, in y-axis, you have to plot number of, this is number of earthquakes exceeding some certain value. Suppose you may not be interested to all the earthquake magnitudes. As I have already mentioned earlier, therefore, civil engineering construction we are typically interested of earthquake magnitude of say 4.5 or 5. So, above that magnitude, whatever earthquake occurs, let us say, we consider only those number of earthquakes. And this is the magnitude, so magnitude versus number of earthquakes. Suppose magnitude of 5 occurs say 100 times in our chosen domain; say magnitude of 6.5 occurs say only 10 times. So, this is in log scale; this is magnitude. We

can plot like this. So, that will give us the distribution or the recurrence of that earthquake for that particular point or particular site.

Then, we can have the ground motion parameters; that is based on the attenuation relationship – peak acceleration versus distance. In this case, we will have the magnitude-based relationship. We know magnitude and distance relation  $M$  versus  $R$  for a particular chosen attenuation parameter. Now, within each of them, for every source – source 1, 2, 3, we have to select the percentage of probability of occurrence of particular event within each of them. Then we can arrive at that probability of exceedance of a particular value of this peak acceleration based on our ground motion parameter. That will be our final result. So, anyway, we will discuss it in further. So, it will be further clear. Also, we will discuss through example problem or applications in our subsequent lectures.

(Refer Slide Time: 36:18)



Now, first step is to identify the source to site distance. So, uncertainty involved in the source to site distance we need to first look in to. So, need to specify the distance measure; based on the distance measure in attenuation relationship. Like if we have the vertical fault like this and this is the site of our interest, we can either have hypocentral distance, seismologic distance, because this is the seismologic depth; this is the ruptured distance; and this is the epicentral distance. So, any of these particularly we have to use based on our corresponding attenuation relationship.

(Refer Slide Time: 36:57)

### Uncertainty in source – site distance

Need to specify distance measure.  
Based on distance measure in attenuation relationship.

**Dipping Faults**

NPTEL IIT Bombay, DC 25

Similarly, for dipping faults, we can have seismogenic distance, ruptured distance, hypocentral distance; epicentral is of course the same point (( )) coincides with the site. This is another way to compute.

(Refer Slide Time: 37:15)

### Uncertainty in source – site distance

Where on fault is rupture most likely to occur?  
Source – site distance depends on where rupture occurs.

NPTEL IIT Bombay, DC 26

Now, where on this fault is rupture most likely to occur? See fault is not a single point unless it is a point source. It will have certain dimension. Now, among that entire length or area or volume of the fault, whatever we consider, which are the points where the chances of occurrence of rupture or the energy emission during the earthquake is going



to be more? That probability we need to find out. So, you can see over there this is the site for which you are going to find out that hazard parameter. And say this is our fault; say this is the areal fault like this. There can be this point maybe having more chances of opening or rupture; this point maybe having more chances; may not be this point. So, why to give equal weightage to all the points like we have given in DSHA?

We have actually taken from this point to minimum distance. Considering that, this maximum rupture will occur at all points equally. But there is no reason why it should occur at all points equally. So, we need to find out that rupture, where it is most likely to occur. That probability we have to compute. Source to site distance depends on where that rupture occurs. Suppose if we are taking the hypocentral distance; then it is very much important even for the epicentral distance as well, because if the fault length is for several hundreds of kilometers, obviously, epicentral distance will also will change; even the nearest point to the rupture also will change. So, all these distance parameters will change.

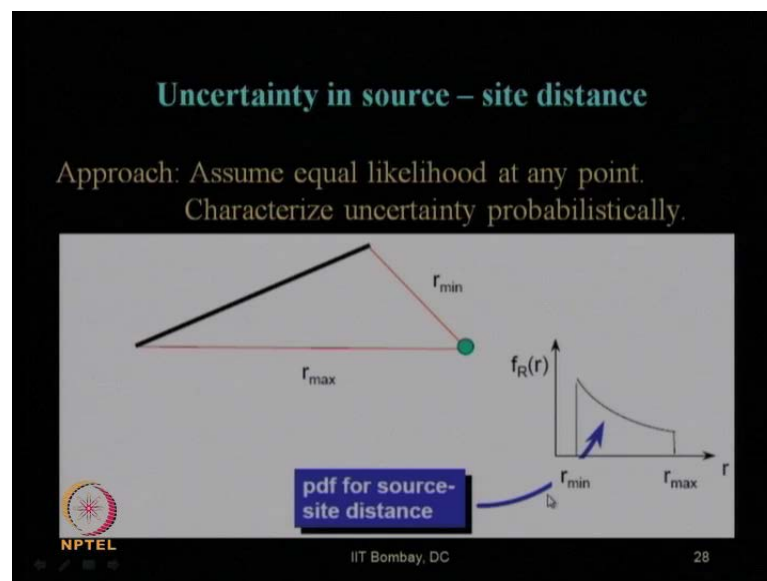
(Refer Slide Time: 39:04)

The slide features a black background with white and yellow text. At the top, the title "Uncertainty in source – site distance" is written in a light blue font. Below the title, the text "Where on fault is rupture most likely to occur? **We don't know.**" is displayed in white, with "We don't know." in bold yellow. Underneath, it says "Source – site distance depends on where rupture occurs." in white. The central part of the slide contains a 3D diagram of a rectangular fault grid, colored in shades of blue and green. From the right side of the grid, several red lines radiate outwards to a single point on the right, representing the source-to-site distance from various points on the fault. In the bottom left corner, there is a circular logo with a star-like pattern and the text "NPTEL" below it. In the bottom center, it says "IIT Bombay, DC" and in the bottom right corner, the number "27" is visible.

So, now, where on the fault, rupture most likely to occur? At the initial instance, our answer is we do not know. We do not know the result, that is, where the rupture is going to occur. Then how we can find out the probability of occurrence of that rupture at various points of that fault? So, what we need to do? We need to divide the entire fault whether it is a length or whether it is an area into small number of segments. And for

each segments, for each number of small portions within that fault from the center of that fault portion or subdivided area, you find out what is the distance from this site. Did you get my point? Look at here. These are all the distances measured to the center of each of these smaller elements within that entire fault; that is, entire fault you subdivide it into number of elements, because automatically, then your distance from site to the center of each of the element will change.

(Refer Slide Time: 40:24)



So, approach is like this. So, assume the equal likelihood at any point. Let us say, initially, let us give the equal likelihood, that is, equal chances of occurrence at each and every point. So, characterize the uncertainty based on the probabilistic way. How we are going to do? Let us see. Suppose this is our fault length. From the site, obviously, we will have various distances say this is  $r$  minimum and this is  $r$  maximum. Now, we have say the variation of that parameter based on the function of this  $r$  will have this kind of variation, that is,  $r$  minimum chances of occurrence more with  $r$  maximum chances of occurrence less. So, this is nothing but... This probabilistic distribution is nothing but probabilistic distribution function PDF – probability distribution function. So, PDF for source to site distance we need to first draw. That is what it says; that is, based on each and every distance... What we have seen here, based on each and every distance from minimum to maximum, you need to find out what are their chances of occurrence or what are the probabilities of occurrence of a particular chosen value of your concern. It

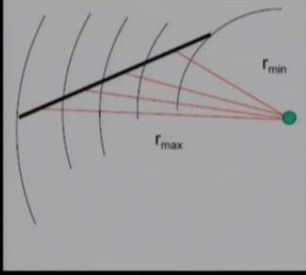
can be magnitude; it can be acceleration, etcetera, various input value. That probability distribution you need to find out.

(Refer Slide Time: 42:01)

**Uncertainty in source – site distance**

Two practical ways to determine  $f_R(r)$  : **Linear Source**

- Draw series of concentric circles with radius equal to increment.
- Measure length of fault,  $L_i$ , between each pair of adjacent circles.
- Assign weight equal to  $L_i / L$  to each corresponding distance.



IIT Bombay, DC 29

So, how to do this? Let us see. There are two practical ways to determine this probability distribution function of  $f$  of  $R$  as a function of  $r$ . Suppose for the case of linear source; let us take the example first for the linear source; that is, the fault or sources like linear dimension like this. So, we can have various values of  $r$  say this is  $r$  minimum and this is the  $r$  maximum. What we have done here, considering this site as the center of a circle, draw the series of concentric circle with radius of equal increment. Did you get my point? First, you draw a circle, which will touch the source. Then you do certain amount of increment in the radius; draw another circle. It will cut the source at another point. Do another equal increment of the radius. It will cut at another point. Another equal value of increment of radius – it will cut another point and further another point, so that entire of the source is subdivided.

Now, for each of the subdivided zone, you choose the center of each of the smaller zone. Like this is the center of this zone; this is the center of this zone; this is the center of this zone; this is the center of this zone; like that. So, from that each center points of that subdivided source to this site, you find out distance. So, this is one distance; this is another distance; this is another distance; this is another distance. Among them, one is minimum; another is maximum; and others are in between. So, measure the length of the

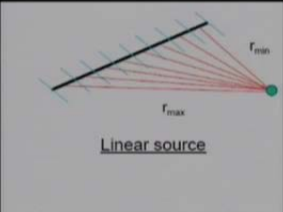
fault  $L_i$ . This is the length of the fault, which you have subdivided between each pair of the adjacent circles. That is what I said. Now, assign weight equal to  $L_i$  by  $L$  to each of the corresponding distance; that is, if you want to put equal weightage, what you can do? What is the weightage of this portion of the fault? That is nothing but say, this is  $L_1$ ; this is  $L_2$ ; this is  $L_3$ ; this is  $L_4$ , so  $L_1$  by total  $L$ . What is the weightage for this one?  $L_2$  by total  $L$ . What is the weightage for this one?  $L_3$  by total  $L$ . What is the weightage for this one?  $L_4$  by total  $L$ . Now, if you know some weightage factor, that also you can add over here; this is considering the equal weightage.

(Refer Slide Time: 44:46)

**Uncertainty in source – site distance**

Two practical ways to determine  $f_R(r)$  : **Linear Source**

- Divide entire fault into equal length segments.
- Compute distance from site to center of each segment.
- Create histogram of source – site distance. Accuracy increases with increasing number of segments.



The diagram shows a horizontal line representing a linear source. A point on the right represents the site. Several lines radiate from the site to different points along the source. The distance to the closest point is labeled  $r_{min}$  and the distance to the farthest point is labeled  $r_{max}$ . The text 'Linear source' is written below the line.

IIT Bombay, DC 30

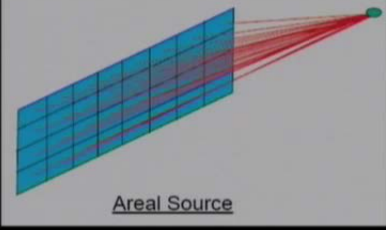
Now, another way to divide the source is same for linear source; instead of drawing that concentric circles of equal radial increments, you can subdivide it into equal number of lengths of segments; that is also possible. So, that is what it says, divide the entire fault into equal length segments. Now, compute distance from site to center of each of the segment; the same ways, center of each of the segments to site, those distances are obtained. Now, once you get that, what you need to do next? Next is create the histogram of this source to site distance using those weightage factor that  $L_i$  by  $L$ . So, accuracy increases with increasing number of segments; that is quite obvious. If you divide it into more number of segments, you will get a more refined results or better results.

(Refer Slide Time: 45:49)

### Uncertainty in source – site distance

Two practical ways to determine  $f_R(r)$  : **Areal Source**

- Divide source into equal area segments.
- Compute distance from center of each segment.
- Create histogram of source – site distance.



Areal Source

NPTEL IIT Bombay, DC 31

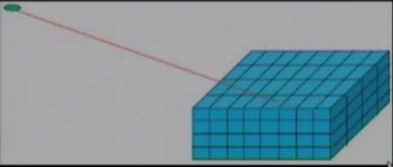
Similarly, for areal source, what we do? The areal source we have to subdivide into smaller areas of equal area segments like this and compute the distance from the center of each source to that site. And finally, you create again that histogram of source to site distance.

(Refer Slide Time: 46:13)

### Uncertainty in source – site distance

Two practical ways to determine  $f_R(r)$  : **Volumetric Source**

- Divide source into equal volume segments.
- Compute distance from center of each segment.
- Create histogram of source – site distance.



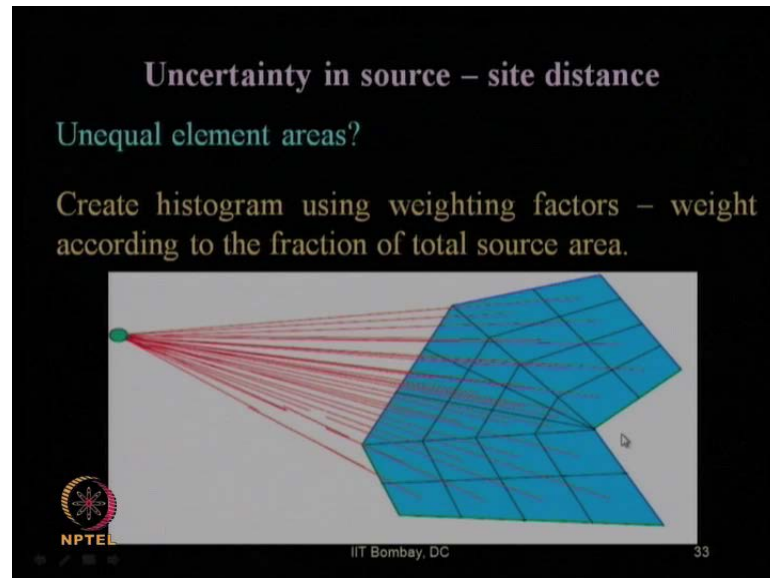
Volumetric Source

NPTEL IIT Bombay, DC 32

Similarly, for volumetric source also, we divide the source into equal volume of segments. So, each small brick elements are nothing but individual smaller segments equally distributed for the entire source. And compute the distance from the center of

each segment to that site. And then create the histogram of the source to site distance based on the given weightage; that is, individual volume say  $V_i$  –  $V_i$  to the total volume. For area, it will be individual area say  $A_i$ ;  $A_i$  by total area.

(Refer Slide Time: 46: 57)



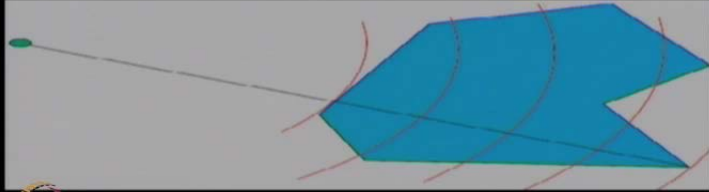
So, giving that, now, if we have unequal element, what we need to do? Suppose the source can be some haphazard like this. Then there is no way you can go for equal area of element. Then there is no other choice, but to go for unequal element areas. So, in that case, create the histogram using that weighing factor. Like earlier, we have given equal area, equal length, equal volume, because thinking that it will have equal weightage. But if you have some irregular shape of the source; in that case, you are suppose not able to divide it equally, you may have unequal area; in that case, you use the weighing factor; that is, suppose this  $A_1$  by  $A$  is something different than  $A_2$  by  $A$ . Earlier it was equal. Now, it is different. So, that weight factor you should consider to be a fraction of the total area when you are attaching or when you are plotting that histogram. So, you add that area with respect to that weightage factor when you are drawing the histogram for that various distances; that is, at different distances, you are getting some histogram of weight age, so that you need to plot.


(Refer Slide Time: 48:25)

**Uncertainty in source – site distance**

Quick visualization of pdf ?

Use concentric circle approach – let's “see” basic shape of pdf quickly.



 NPTEL

IIT Bombay, DC

34


Now, quick visualization of that probability distribution function or PDF – use of concentric circle approach. Let us see the basic shape of that PDF quickly. This is again another irregular one. So, you can have the concentric circle like this and give the subsequent different weightage factor.

(Refer Slide Time: 48:49)

**Characterization of maximum magnitude**

Determination of  $M_{\max}$  - same as for DSHA,

- Empirical correlations
- Rupture length correlations
- Rupture area correlations
- Maximum surface displacement correlations
- “Theoretical” determination
- Slip rate correlations

 **Also need to know the distribution of magnitudes**

IIT Bombay, DC

35

Now, characterization of this maximum magnitude – determination of  $M_{\max}$  is same as we have done for the case of DSHA; that is, either you use this empirical correlations like Wells and Coppersmith equation we have seen; based on rupture length or rupture

area or maximum surface displacement correlation or various theoretical determination based on seismic moment, etcetera; or, the slip rate correlations that we have seen.


Now, also we need to know the distribution of this magnitude. In the DSHA, we were not bothered about to know how this magnitude is distributed, because we have chosen only the maximum value and then we have further proceeded with that calculation. But here we are now interested to know how this maximum magnitude for various sources they are distributed.

(Refer Slide Time: 49:41)

**Distribution of earthquake magnitudes**

Given source can produce different earthquakes  
Low magnitude - often  
Large magnitude - rare

Gutenberg-Richter  
Southern California earthquake data - many faults  
Counted number of earthquakes exceeding different magnitude levels over period of many years

 NPTEL

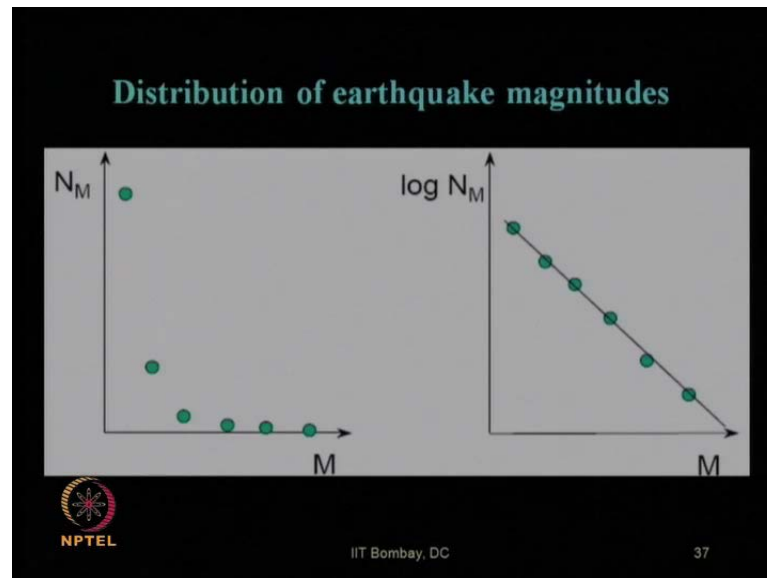
IIT Bombay, DC 36

Given source can produce different earthquakes. For example, generally, the low magnitude will be occurring very often; whereas, large magnitude will occur very rarely for a particular site; which is quite obvious. Say a magnitude of 4.5 or 5 or 5.5 occurs so many times; whereas, magnitude of 7.5 in the same source is occurring very few times. So, that number of occurrences of earthquake we need to find out. For that, Gutenberg-Richter relationship we need to use. We will see that very soon. Like for example, Southern California earthquake data – many faults are involved; counted number of earthquakes exceeding different magnitude levels over a period of many years; that means, what is the different magnitude level? As I said, it depends on the basic design consideration; like it depends on engineer's decision that, whether we should consider 4.5 magnitude and above as earthquake for our consideration or 5. So, it is up to you when you are doing the design. And over how many years of periods, whether you need



to consider 100 year of period or 200 years of period of earthquake data? Like that So; obviously, if you have authentic data; more the data, better is the prediction.

(Refer Slide Time: 51:10)

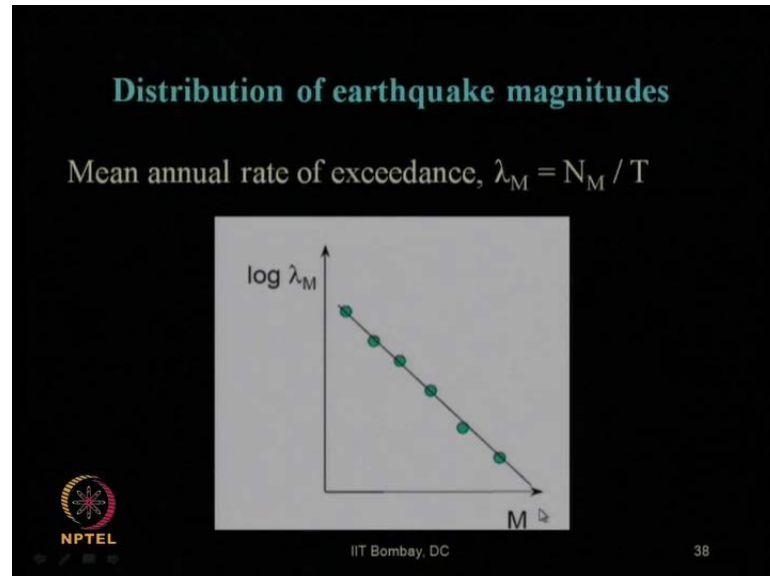


Now, with that, what we can do? Once you know, how many number of earthquakes and what are their magnitudes occurred, you can easily plot them like this. So, y-axis – suppose it is a number of occurrence of earthquake and  $M$  is the magnitude of earthquake. Say you are interested to know about magnitude of earthquake more than 4; then in this plot, you will only plot those earthquakes from the past historical data, which are having magnitudes starting from 4; obviously, the smaller magnitude of earthquake – as we said, they will occur more number of times than higher magnitude of earthquake – they will occur very less number of time or rarely. So, that is why, the points will be distributed something like this. So, if it is a normal scale, it will be some kind of exponential relation like this. Can you see? If you collect the historical earthquake data for any region, it will typically give this kind of shape in the normal plot – normal axis system.

Now, if the same relationship if you want to plot in log scale for the y-axis, but normal scale for the x-axis; that is, magnitude is still in normal scale, but y-axis, that is, number of their occurrences in log scale, it will give some relationship. All these points will be scattered like this; which will follow a certain kind of linear relationship in this semi-log plot. That is nothing but what Gutenberg and Richter had proposed. This is called

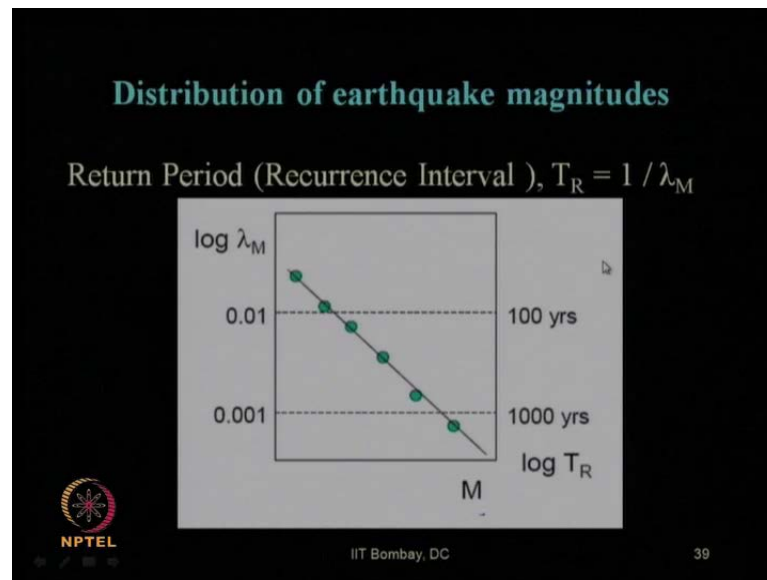
Gutenberg-Richter recurrence law; recurrence means how many times that magnitude will occur. So, we will see that soon.

(Refer Slide Time: 53:11)



This data – what it gives us? From the previous plot, we can redraw it in another fashion. What way we can plot it? We can plot the mean annual rate of exceedance, what we denote it as say lambda M. What is lambda M? This is nothing but  $N_M$ ; that is, the number of events of that magnitude  $M$  divided by  $T$ ; that is, for how many years you have chosen it. So, it will automatically give mean annual rate; that is, per year basis it will give the value. So, the same distribution if we plot now – log of that lambda M versus  $M$ , the same pattern will follow, because obviously, you have divided only with respect to the number of years what you have chosen for that number of events, which you have selected. So, this is called mean annual rate. Annual rate means per year. So, that is what we have obtained – of exceedance of that magnitude  $M$ .

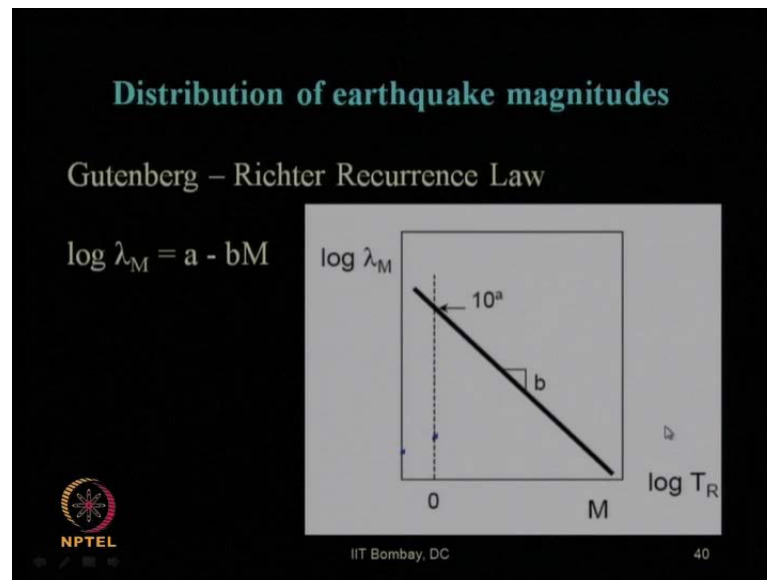
(Refer Slide Time: 54:29)



Now, next definition is return period. It is also called recurrence interval; that is, in what interval that magnitude or more will occur? So, how we can find it out? That return period or recurrence interval  $T_R$  is nothing but inverse of that mean annual rate of exceedance. This is showing the number of occurrence per year basis. So, if we inverse it, we will get, that magnitude will occur in how many years time. So, that gives us the return period or recurrence interval  $T_R$  like this. So, the same plot of that log of lambda  $M$  versus  $M$  in this fashion. If we select the y-axis on the other side as log of this return period  $T_R$ , this will be in the increasing fashion like this; the other side will be in increasing fashion in the other direction, because they are inversely proportional. Can you see that which is quite obvious. So, the plot will remain same; the values, etcetera; that is, lambda  $M$ ...

Let us take an example. Lambda  $M$  value of 0.01 will indicate  $T_R$  of 100 years, because 1 by 0.01 is nothing but 100 years. What does it mean? That is, mean exceedance – per year exceedance – annual rate of exceedance of a particular event; say magnitude of 5 earthquake occurrence is 0.01; that means, in other words, the same thing that, in 100 years time, that magnitude of 5 earthquake will get repeated. How they are correlated? Similarly, suppose lambda  $M$  of 0.001 will mean its return period is 1000 years.

(Refer Slide Time: 56:47)



So, what I had already mentioned, this is called Gutenberg-Richter recurrence law; that is, the plot of this log of lambda M versus magnitude M; that is in y-axis, you will have log of lambda M; x-axis will be your normal axis of magnitude M. And the other side you can put log of T R, because anyway it is same the other way round. So, what you can see over here, M value at 0, that is magnitude when it is 0, because it is a normal scale, we can have a 0 value. What corresponds to this curve, where it reaches, that is nothing but the intercept on this. That let us take as 10 to the power a; and the slope of this line, which is in the decreasing trend like this from here... The reason also we have mentioned, because smaller magnitude of earthquake occurs more number of times; higher magnitude of earthquakes occurs less number of times. So, that way, this gives the slope of b. So, typical Gutenberg-Richter recurrence law; that is how they return, what is their chances of occurrence of that magnitude of earthquake is given by this relation, that log of lambda M equals to a minus b M. That is the equation as you can see from this value. So, with this we have come to the end of today's lecture, we will continue further in next lecture.