Geotechnical Earthquake Engineering

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Module - 5

Lecture - 20

Wave Propagation (Contd…)

Let us start our today's lecture on NPTEL video course geotechnical earthquake engineering. Let us look at this slide on this course geotechnical earthquake engineering, today we are taking lecture number 20 on this course. We were going through module 5 of this video course that is, wave propagation let us do a quick recap what we had learnt in our previous lecture.

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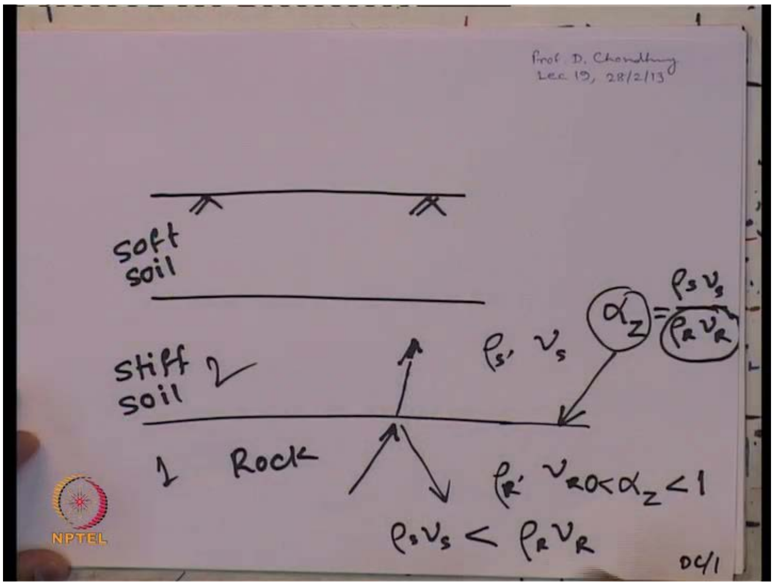

Let us look at these pages, where I had made derivations of various expressions like what is called specific impedance of a particular soil media or any material. So, when we have layered soil, which is practically available as we know. How the specific impudence ratio is defined as the density times the velocity of the seismic wave; it can be shear wave, it can be primary wave, depending on what type of wave velocity, we are considering. The product of that is nothing but the specific impedance of a particular layer.

So, when the seismic waves are traveling in this way from the bottommost layer to the topmost layer slowly. So, in that case, the specific impedance ratio between the two layers supposed the rock layer and the soil layer is given by this ratio, that is, the specific impedance of the soil layer divided by the specific impedance of rock layer. This comes like denominator will be the specific impedance of the, that particular layer where from the incident wave is coming from.

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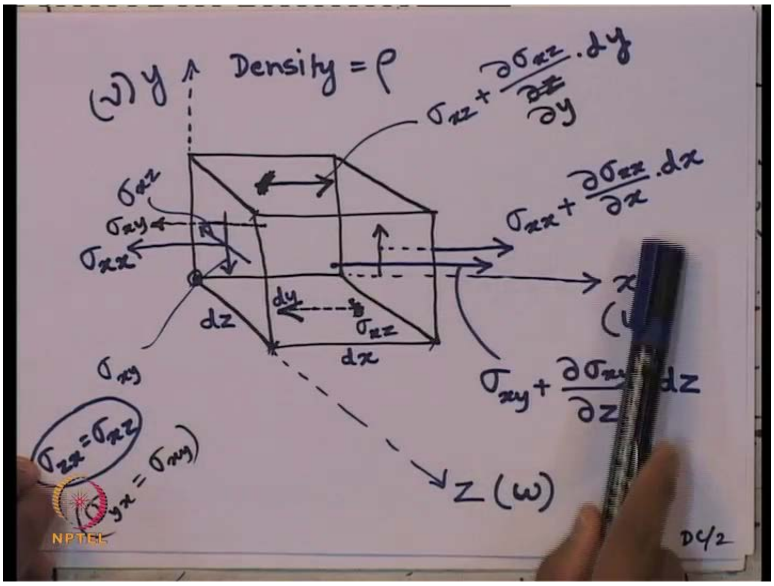

Then we had seen the derivation of three-dimensional wave propagation, for three-dimensional wave propagation in only x directional forces, we had seen initially, so that we will get the x directional wave equation.

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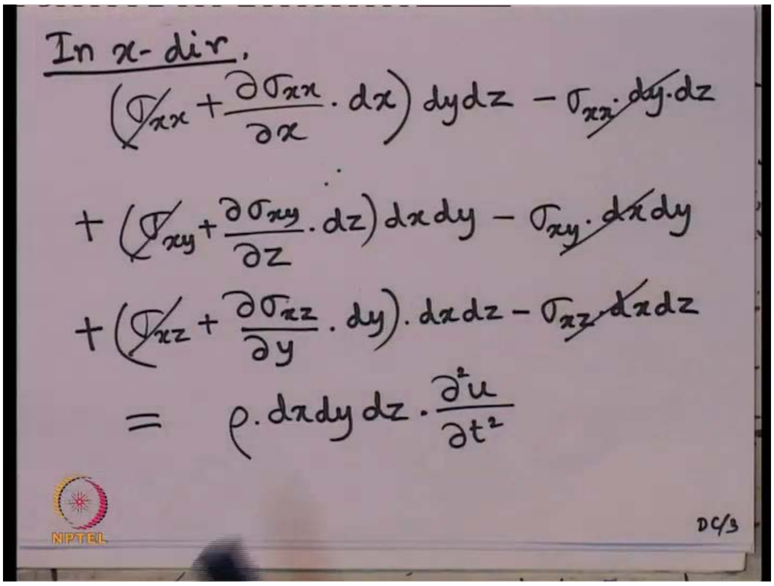

So, based on the equilibrium of all the forces, we arrived at this expression for the x direction of forces.

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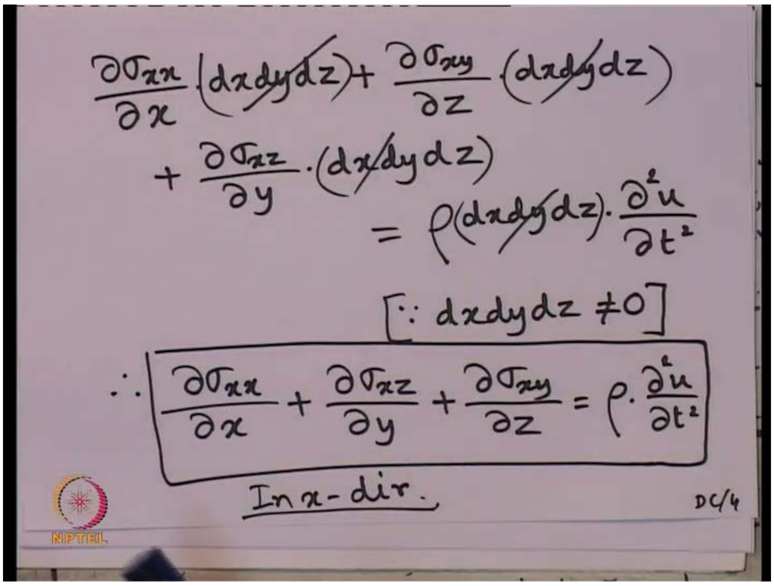

So, on simplification of that finally, we got this governing equation of motion in x direction for wave propagation, that is del sigma x x by del x plus del sigma x z by del y plus del sigma x y by del z equals to rho times del square u by del t square, where u is the displacement in the direction of x. So, this parameter indicates the acceleration in x direction; rho is the density of the material; sigma x x is the normal stress along the x direction sigma x z and sigma x y are the shear stresses.

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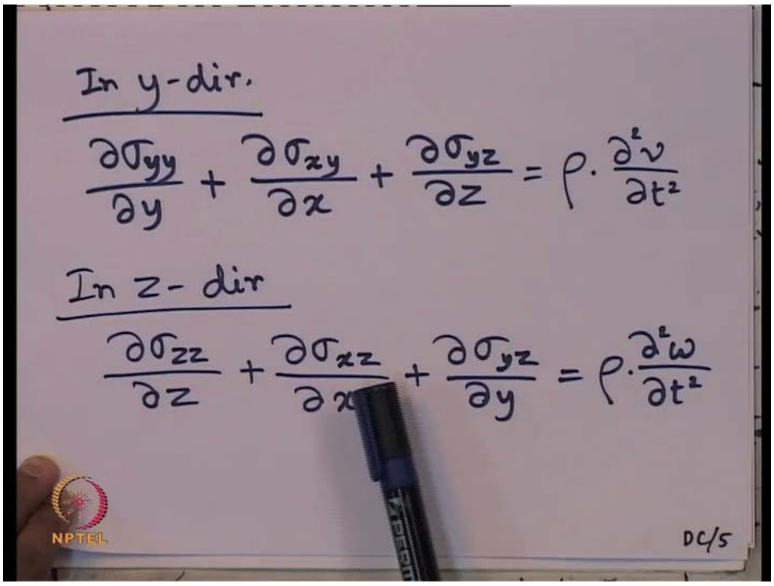

In the similar way we had seen that for other two direction in three dimension, we can also have this relationship, that is, in y direction the relationship is given by del sigma y y by del y plus del sigma x y by del x plus del sigma y z by del z equals to rho times del square v by del t square; where v is the displacement in the direction of y axis. So, this parameter indicates del square v by del t square is the acceleration in y direction and rho is the density of the material and sigma x sigma y y is the normal stress along y direction sigma x y and sigma y z are the shear stresses. Similarly, for z direction the governing equation of motion of wave propagation is del sigma z z by del z plus del sigma x z by del x plus del sigma y z by del y equals to rho times del square w by del t square, where w is the displacement along z direction.

Hence, this del square w by del t square indicates the acceleration in the z direction; again rho is the density of the material sigma z z is the normal stress along z direction sigma x z and sigma y z are shear stresses.

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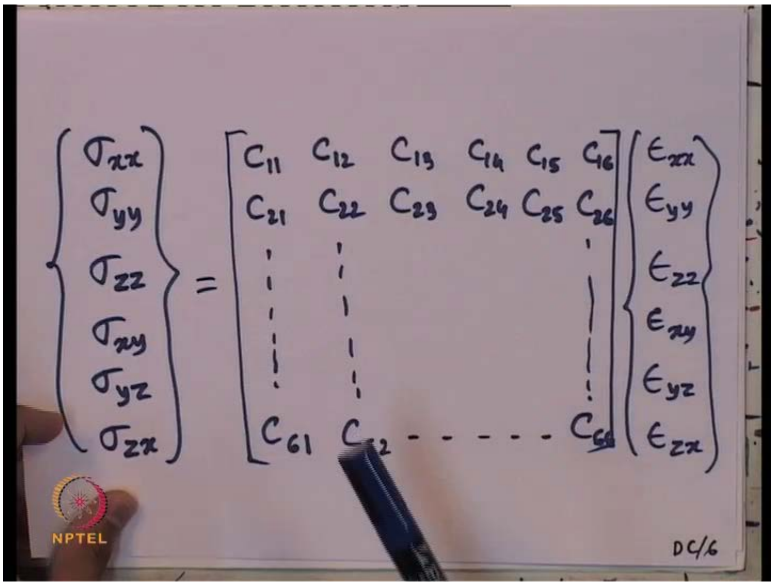

Now, we also have seen how we can express the stresses with respect to the corresponding strain through the modulus? As we know, stress strains are related through modulus. So, when we combine and write all the stress vectors and strain vectors through the modulus matrix like this the shape takes like this form, that is three normal stresses sigma x x sigma y z sigma y y and sigma z z and three shear stresses sigma x y sigma y z and sigma z x are related to the corresponding strains which are epsilon x x epsilon y y epsilon z z or normal stain and epsilon x y epsilon y z and epsilon z x are shear strain and these coefficients are nothing but the modulus which relates to this stress to strain vectors.

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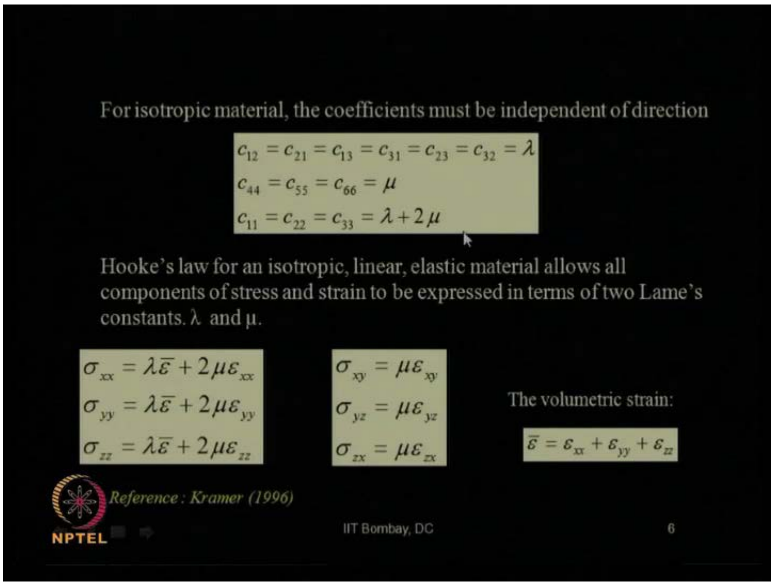

Now, coming to today's lecture let us look at the slide over here. So, for any isotropic material we know, what is isotopic material that is, it is having in all the directions similar property in x, y, z directions. The coefficients must be independent of the direction; that means, in the matrix, what we have seen? This parameter that is C 12 should be equals to C 21; similarly C 13 should be equals to C 31, C 23 should be equals to C 32 and that should be equals to lambda; what is lambda? Lambda is one of the Lame's constant we know, that stress strain relationships can be expressed in terms of two Lame's constants, which are lambda and mu; and other coefficients like C 44, C 55, C 66 these are mu whereas C 11, C 22, C 33 these are lambda plus 2 mu.

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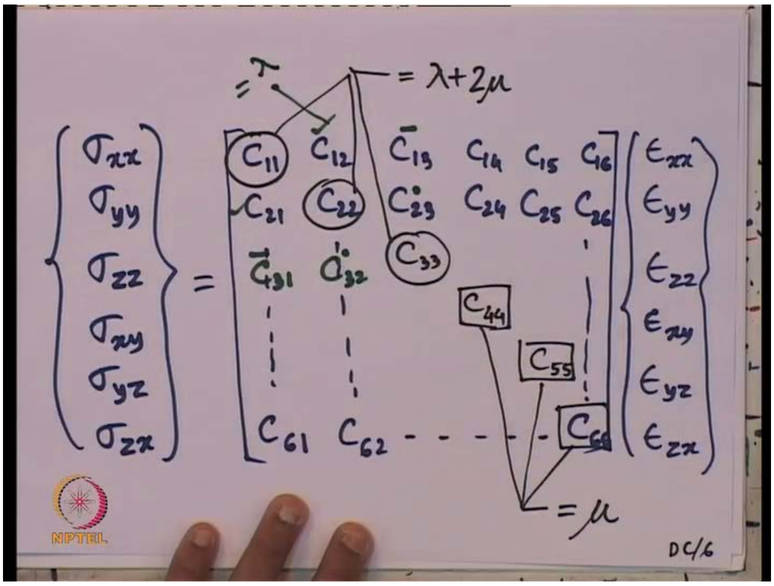

So, knowing this what we can write down over here that this C 11, C 22 and C 33 from here these three will be equals to lambda plus 2 mu and other diagonal elements C 44 C 55 C 66, that is, this one, this one and this one these will be equals to mu and remaining this C 12 and C 21 they are equal, then C 13 and C 31 are equal, then C 23 and C 32 they are equal and their values all these green color which I have highlighted; their values are equals to lambda.

So, similarly the other parameters are also related. For isotropic material Hook's law for an isotropic linear elastic material allows all components of stress and strain to be expressed in terms of these two Lame's constant. So, hence on simplification if we further want to write this sigma x x in terms of epsilon x x and other strain terms it will take this form; sigma y y will take this form, sigma z z will take this form. So, these are the normal stresses and the shear stresses sigma y x y sigma y z and sigma z x will take this form, where as this epsilon bar is nothing but it is called the volumetric strain which is nothing but summation of three normal strains that is epsilon x x plus epsilon y y and epsilon z z.

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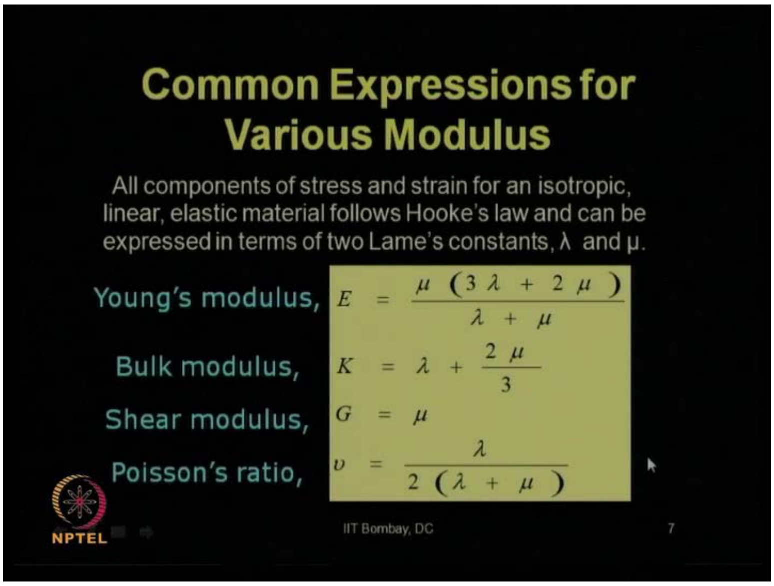

Now, these are the common expressions for various modulus, which we used in our mechanics related problems. So, all components of stress and stain for an isotropic linear elastic material which follows Hook's law can be expressed in terms of this two Lame's constant as we have already mentioned lambda and mu.

So, using those two Lame's constant we can express the young's modulus; which is expressed typically using this symbol E is given by mu times 3 times of lambda plus 2 times of mu by lambda plus mu. Bulk modulus K, that is, expressed as lambda plus 2 times of mu by 3. Shear modulus G that is nothing but this mu itself and Poisson's ratio mu is given by, Poisson's ratio mu is given by lambda by 2 times lambda plus mu. So, these are the common expressions or common modulus and Poisson's ratio which is expressed in terms of Lame's constant which is very useful for our mechanics oriented any problem which we are handling. Like for our soil mechanics also we handle these constants.

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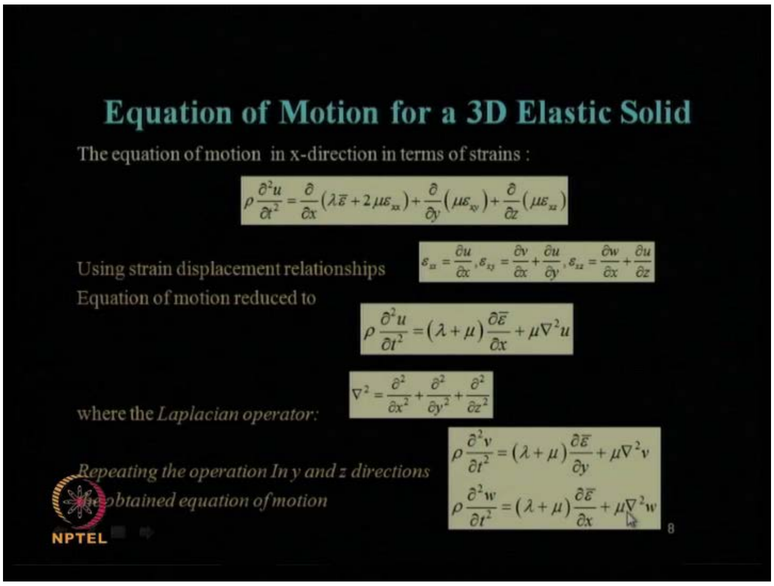

Now, let us see how we can further solve the equation of motion for a three dimensional elastic solid. Already the basic governing equation of motion we have derived. So, let us take only in one direction, say first let us consider only the x directional equation. So, what was the x directional equation that was rho times del square u by del t square. Let us go back and look at the equation once again in this slide.

So, this rho times del square u by del t square equals to this parameter. Now let us express all these stresses that is normal stress and shear stresses in terms of strain and those modulus or through the Lame's constant. So, what we can write let us see in this slide now. So, that sigma x x becomes lambda times epsilon bar plus 2 times mu times epsilon x x, that we have already seen. This is the expression for sigma x x similarly for sigma y z and z x we can use this relationship which we can put in that equation. So, the equation takes a form of this one. Okay.

Now, once we get the relationship in terms of strains now we have to look how we can expressed this strain in terms of displacement, that is, what we did for one dimensional equation also that is while solving the one directional equation we did the similar procedure. So, for three dimensions we are following the similar procedure that is using the strain versus displacement relationship, how they are related let us look at it.

So, normal strain in x direction epsilon x x is nothing but del u del x; where u is the displacement in x direction and the shear strains are related like epsilon x y will be noting but, del v del x plus del u del y and epsilon x z will be del w del x plus del u del z. So, using this relationship in this equation once again that equation reduces to this form.

So, once it reduces to this form we can simply use this Laplacian operator that grad square what we called this grad square is nothing but del square by del x square plus del square by del y square plus del square by del z square. So, basically this equation is now of the form rho times del square u by del t square equals to lambda plus mu times del epsilon bar by del x; where epsilon bar is volumetric strain plus mu times del square u by del x square plus del square u by del y square plus del square u by del z square, that is the total form of the equation.

Similarly, in the other two directions that is in y direction and z direction also we can obtain similar expression like rho times del square v by del t square will take the form, lambda plus mu times del epsilon bar by del y plus mu times grad square v, now it will change to v for y direction and for z direction it will be rho times del square w by del t square equals to lambda plus mu into del epsilon bar by del x plus mu times grad square times w.

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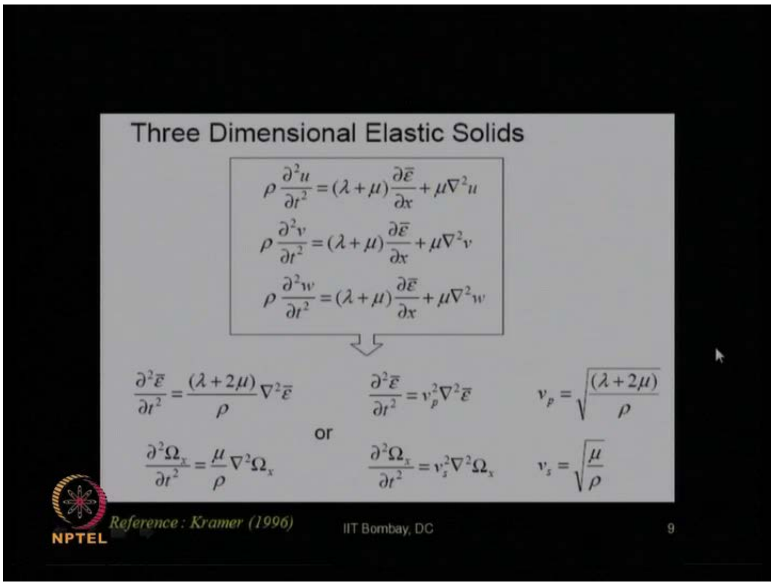

So, these are the three-dimensional equations, which we have seen just now.

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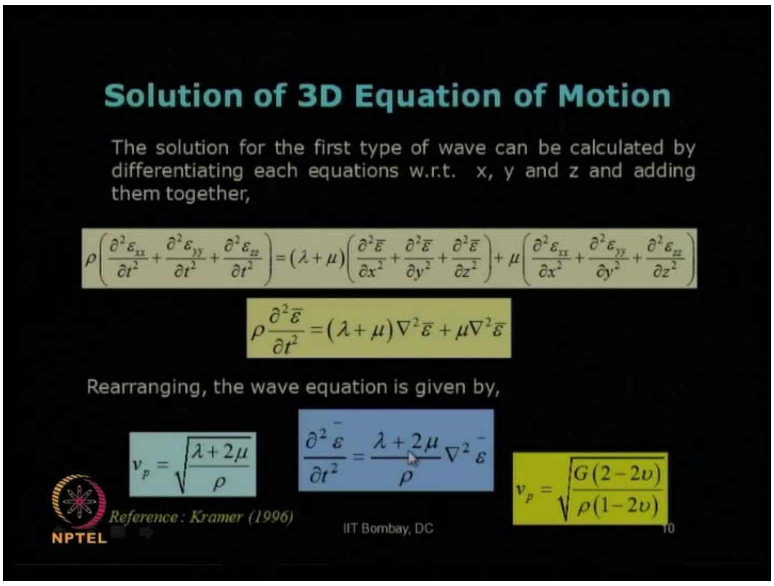

Now, how to get the solution of that we should look into that, that is, the solution for three dimensional wave equation. The solution for the first type of wave it can be calculated by differentiating each equation with respect to x y and z and then we can add them together. So, by doing so what we can get finally, that rho times del square epsilon x x by del t square plus del square epsilon y y by del t square plus del square epsilon z z by del t square will take the form lambda plus mu times. This we have take the summation of that plus mu times this one. Which on further simplification we can write this is nothing but mu times Laplacian operator that grad square times epsilon bar because this sigma x x plus sigma y y plus sigma z z is nothing but epsilon bar that is volumetric strain.

So, rearranging the wave equation, we can write down that this can be expressed as lambda plus 2 mu by rho grad square epsilon bar; where in this case the V p can be expressed as root over lambda plus 2 mu by rho and this on further simplification can be given with this form.

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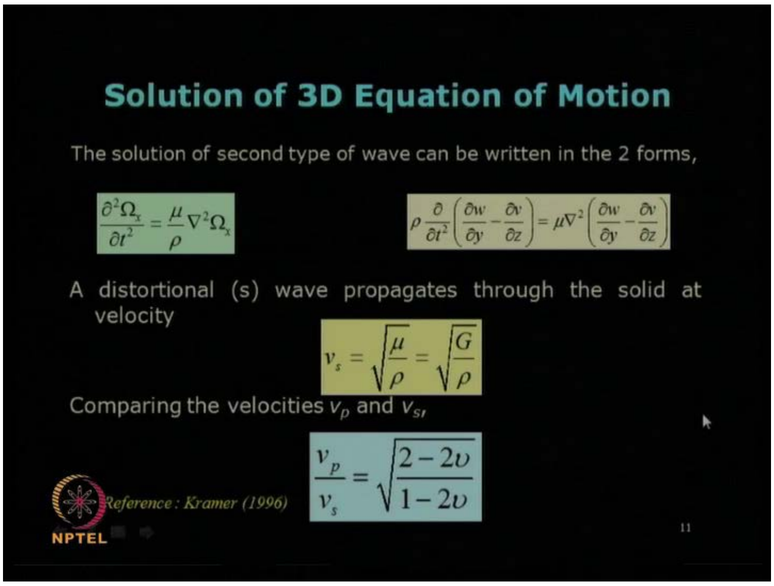

And the solution of the second type of wave, that is, for the tortional wave what is the first type and second type. In this first type we have taken the longitudinal wave. So, that is why all the strain related to longitudinal direction. Next case we are considering second type of wave which is the tortional in nature. When we are talking about tortional wave, we have to take the corresponding tortional strain. So, that is why we have use the relationship of the tortional strain relationship in this equation which will give us finally, the distortional wave velocity which is expressed as V s which is nothing but root over mu by rho. So, that is nothing but root over G by rho because we know Lame’s constant mu equals to G.

Now, if we compare this torsional or shear wave velocity expression with the primary wave velocity expression V p because V p we have put the expression for lambda and mu we got in terms of G and nu that is Poisson’s ratio. This shear modulus density and Poisson’s ratio, if we compare this expression of V p with respect to V s what we can see that, ratio of that primary or longitudinal wave velocity to the secondary or shear wave velocity can be expressed as root over 2 minus 2 mu by 1 minus 2 nu which is nothing but a function of Poisson’s ratio only; that means, the ratio of primary wave velocity to secondary wave velocity or P-wave velocity to S-wave or shear wave velocity is a function of Poisson’s ratio of the material only.

So, suppose from any test we obtained for a particular soil or for a particular material, the shear wave velocity knowing the Poisson's ratio of that material we can also able to estimate the primary wave velocity or vice versa. Suppose, we estimate the primary wave velocity we can get the shear wave velocity also.

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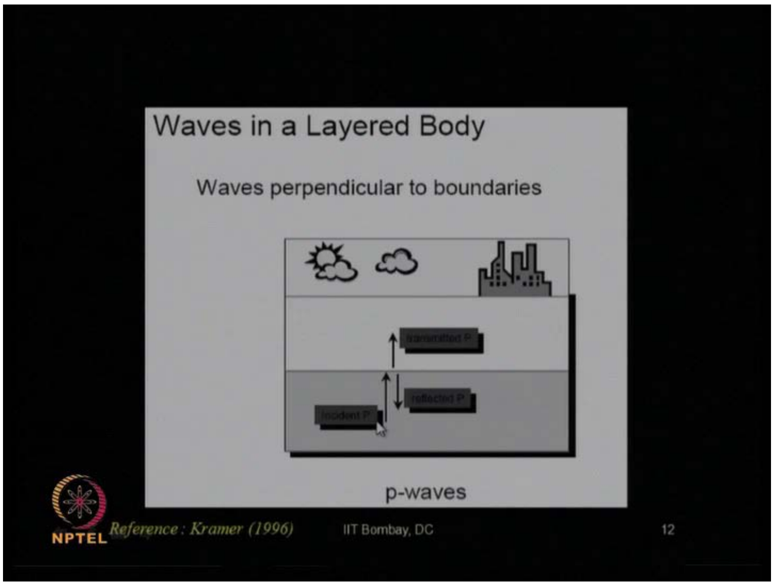

Now let us talk about waves travelling in the layer body. Now, waves perpendicular to the boundaries; obviously, they will also get transmitted as a perpendicular wave, isn't it? Because incident wave comes back as a reflected wave transmitted wave also perpendicular like this. So, this is for P-wave, that is, incident P-wave will have components like reflected P-wave and transmitted P-wave.

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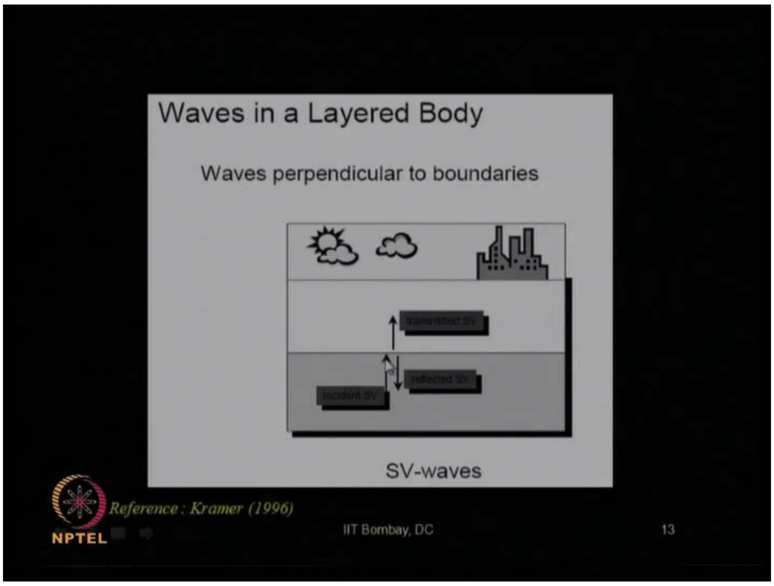

But if we have SV-waves, that is, shear wave in vertical directions waves in perpendicular to the boundaries that is vertical SV incident wave will have same reflected SV wave and transmitted SV waves.

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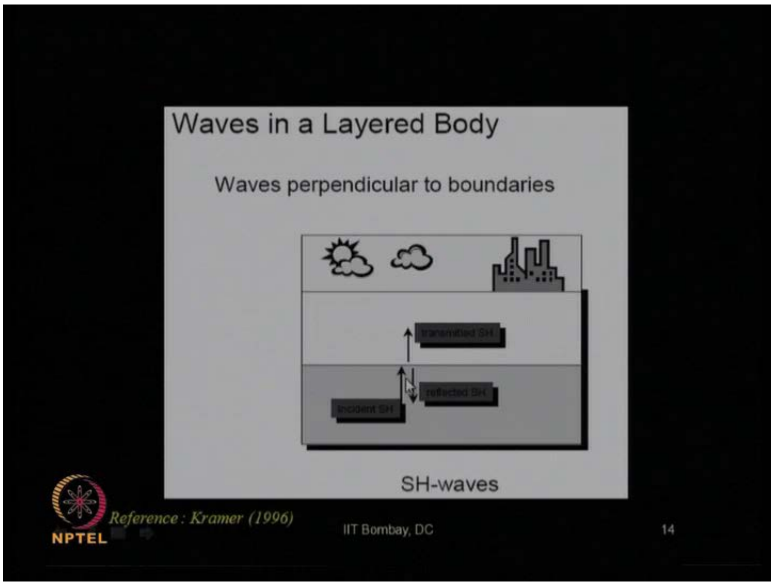

And SH-wave vertical incident SH-wave will have reflected SH-wave and transmitted SH-wave. So, these three cases are true only when we are talking about vertical incident wave. Remember this is for only vertical incident wave.

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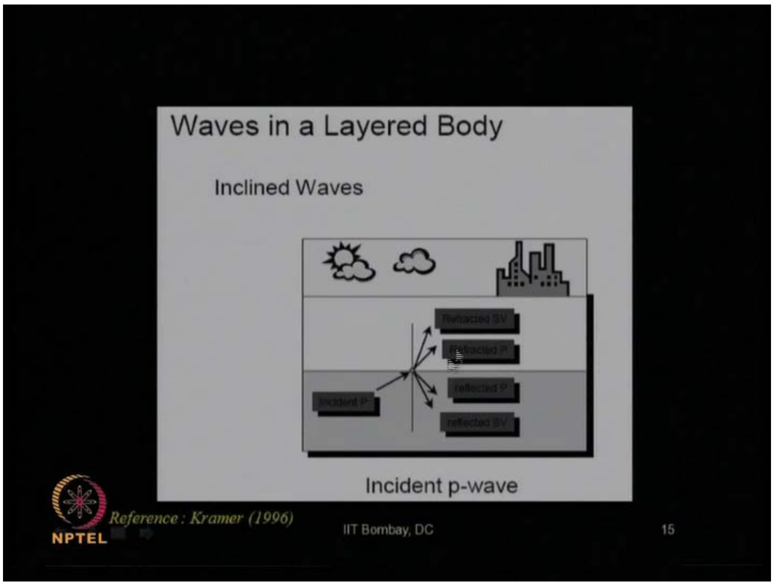

Now, if we have inclined waves, which is always possible because when any earthquake occurs at hypocenter waves travels in all the directions seismic wave. So, it is not necessary that wave will come as an incident wave to a material boundary as a vertical wave but, it can come also as a incident wave, like this. So, when incident P-waves intersects between the two materials some portion will get reflected back in the same material and some will get refracted back or transmitted back in the other material.

So, what are the possible formation of other waves, like incident P-wave will generate reflected P-wave and transmitted P-wave of course but, in addition to that they generate also reflected SV-wave and refracted or transmitted SV-wave; that means, P-wave when it is inclined, that is, the incident P-wave is inclined and meet at the boundary of a two materials like this, they not only remain as P-wave they also create the SV-waves in both the media.

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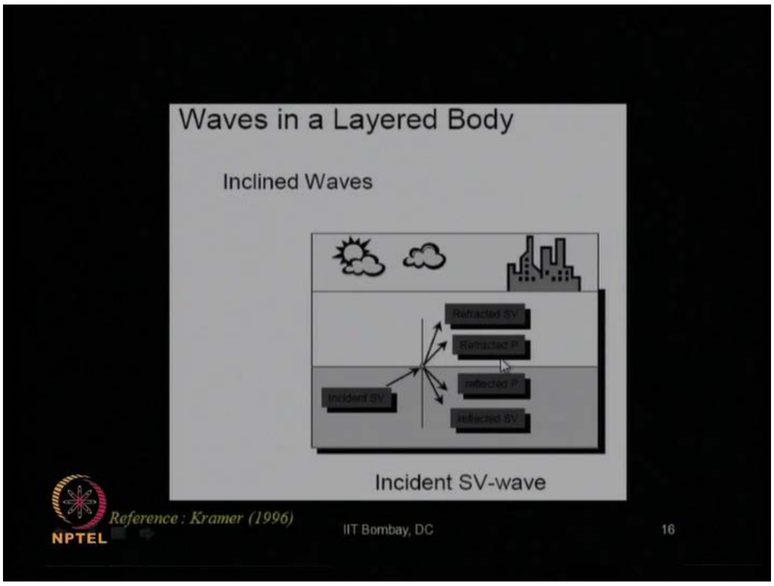

And when we have incident SV-wave as inclined one similarly, it generates same reflected SV wave and transmitted SV-wave, but in addition they generate reflected P wave and transmitted or refracted P wave also.

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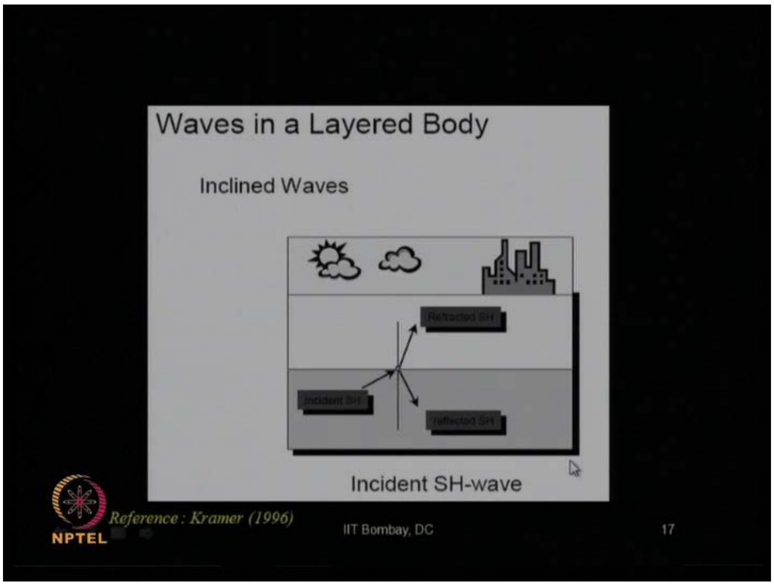

Whereas when inclined SH-wave as an incident wave intersects between the two material they become only reflected SH-wave and refracted or transmitted SH-wave there is no formation of P-wave. Why this is? Because the reason you can see very clearly why for SV and P they are getting correlated or generated because it is in the vertical direction and P also excites in the vertical direction. So, wave propagation direction is in the vertical directions as well as the particle movement also occurs in this direction though it is a tortional one, but whereas, in SH-wave it is in the horizontal direction so, that cannot generate a P-wave.

So, that is the reason why SV-wave can be can generate P-wave or vice versa, P-wave can generate SV-wave for an inclined wave but, not the SH-wave though it is an inclined one but, for vertical one they will remain as a pure P-wave or pure SV-wave or pure SH-wave.

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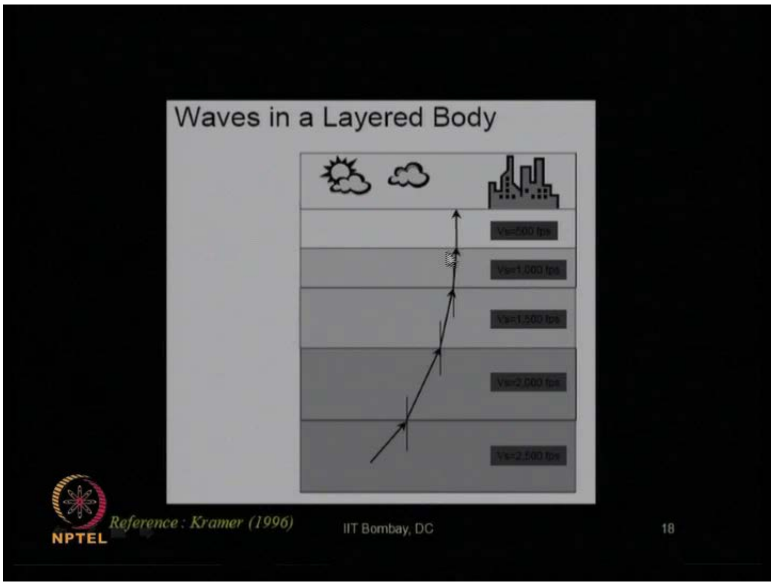

Now, in our geotechnical earthquake engineering problem, you will see later on most of the time when we consider for our foundation or any other analysis of the design in the soil media, that is, close to the ground surface and our range of depth typically between about; 20 meter, 30 meter 35 meter up to that range not too far or bellow the ground surface. So, most of the time in our analysis what we consider all these seismic wave as nearly vertical or vertical seismic waves; whether it is a shear wave, whether it is a primary wave we consider them as a vertical in nature not an inclined one.

What is the basic reason for that let us now understand it through this picture? At a large depth where through earthquake hypocenter or focus, the energy gets dissipated so, seismic waves are getting formed, let us say some of them will remain as vertical so; obviously, they will keep on traveling as a vertical wave till to the ground surface but, some of them will be definitely inclined one like this, but typical soil strata in practice what we will get typically we will get from softer layer to stiffer layer. If it is so, what we can have typical ranges of shear wave velocities will be of increasing or ascending order if we go deeper and deeper from the ground surface.

That indicates if an incident wave with a large inclined or inclination angle incident like this, the refracted or transmitted wave in the next layer which is the softer one compare to this layer will be more towards a vertical direction. Why? because these waves also follows the Snell's law and we know as per Snell's law it is sine of I divided by V remains constant that is sine of angle of inclination of incident wave of refracted wave by velocity of the media or velocity of the wave in that media remains constant.

So; obviously, as the velocity decreases as we go in a softer layer and softer layer. So; obviously, this angle also should get decreased to remain it constant, right? That is the reason as we go for a stiffer or bottom most strata to close to ground surface or a softer strata, the rays keep on moving towards the vertical and close to the ground surface, they will almost become vertical. So, that is the reason why in our geotechnical earthquake engineering problem for our civil engineering designs etcetera, we consider close to the ground surface, all the waves seismic waves are vertical in nature. We do not consider the inclined waves for design considerations because then it will be too complicated not only that it does not have any such reason why we should take non vertical seismic waves because this phenomenon is known to us.

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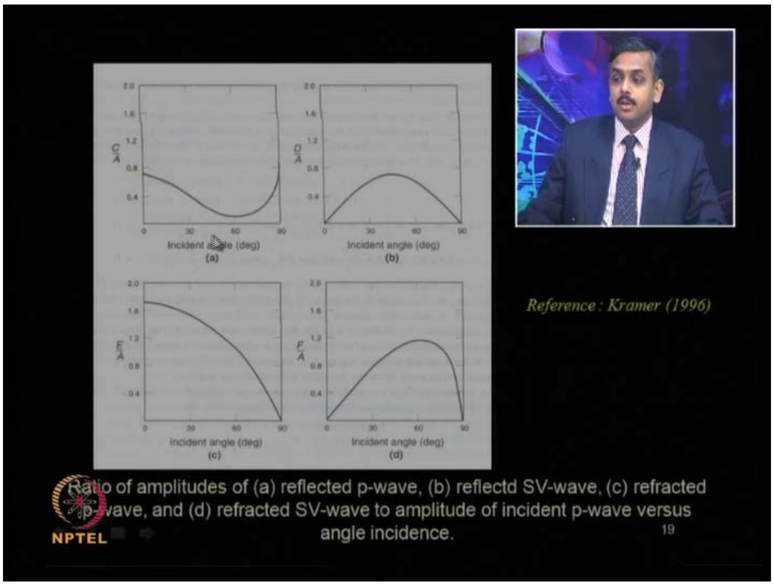

Now, let us look at this slide, this shows what is the ratio of amplitude of the; reflected P-wave, reflected SV-wave, refracted P-wave and refracted SV-wave to the amplitude of incident P-wave versus the angle of incidence. So, this x axis in all of this figures a b c and d shows the incident angle in degree and vertical axis, that is, y axis shows the ratio of the for figure (a) it is the reflected P-wave by the incident P-wave, the amplitude of those two wave. You can see they decrease with increase in incident angle up to an angle of say about 60 degree then they keep on increasing, that is, the typical behavior has been obtained. Whereas, for next one that is reflected SV-waves to the incident P-wave it increases up to an angle of say about its 45 degree and then keep on decreasing.

Similarly, figure (c) shows the ratio of the amplitude of refracted P-wave to the incident P-wave, you can see it keep on decreasing with increase in the incident angle and the last figure that is figure (d) shows the refracted SV-wave to the amplitude of the incident P-wave you can see over here as the incident angle increases typically up to about 60 degree it increases then further it decreases. So, what does it mean you can correlate it very easily, the reflected P-wave and refracted P-wave they have these two behavior, where as reflected SV-wave and refracted SV-wave they have these two behavior with respect to the angle of incident. These allows us to take how much wave amplitude we should consider for a particular incident angle if it all we are interested to consider the inclined waves at a large depth.

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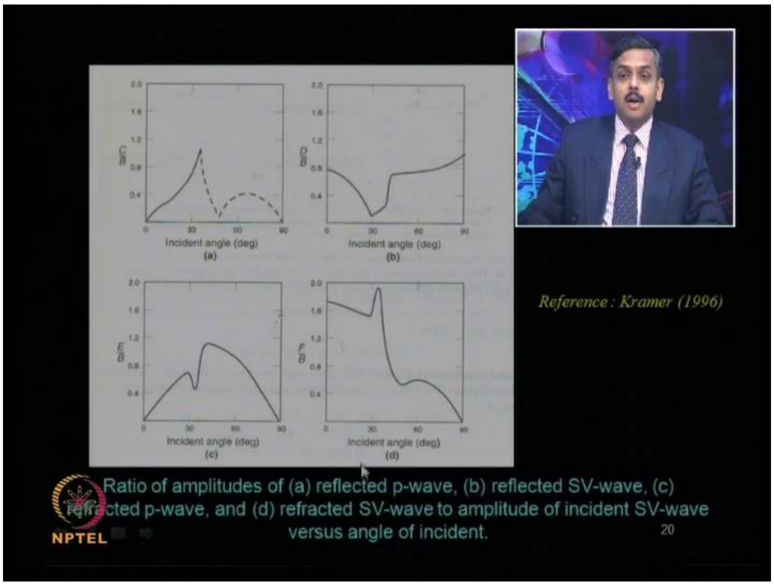

This is another picture which shows the ratios of amplitude of; reflected P-wave, reflected SV-wave, then refracted P-wave and refracted SV-wave to the incident SV-wave versus the angle of incidence.

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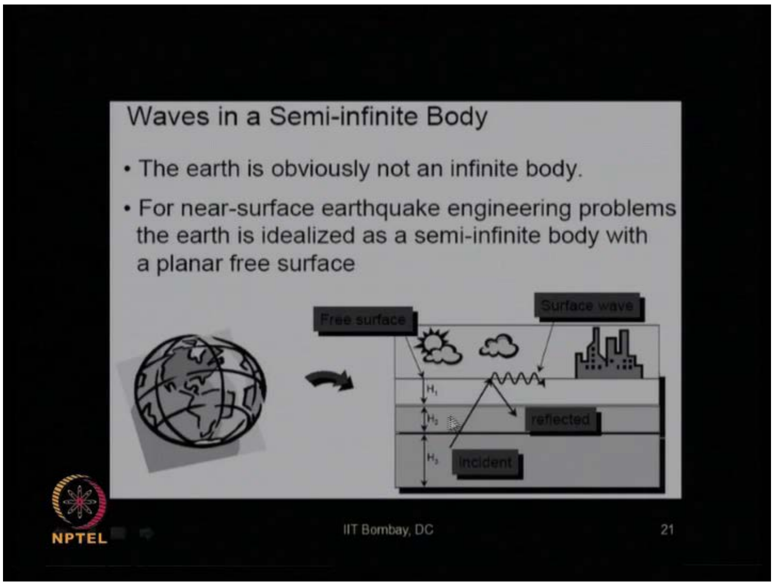

Now, let us talk about waves in semi infinite body. The all the derivations which we have derived so far the basic governing equations and the solution for them those are for infinite body, whether it is a one dimensional wave equation or three dimensional wave equation what we have derived those are for infinite body but, in reality in earth we cannot say it is an infinite body. Earth is a semi infinite body. So, let us look at the slide over here the earth is; obviously, not an infinite body because when your seismic waves are starting and propagating from the hypocenter or the focus of the earthquake it travels and slowly it comes to the ground surface, once it reaches to the ground surface it comes to a boundary.

So, we have to consider the stress boundary conditions instead of a considering the infinite body. So, it is a semi infinite body as we know always we consider in soil mechanics in geotechnical engineering problems it is a semi infinite body. For near surface earthquake engineering problems the earth is idealized as semi infinite body with a planar free surface. So, for shallow earthquake or near surface earthquake we generally consider that this boundary or ground surface as a planar free surface, right? and that is the semi infinite body that is what we consider.

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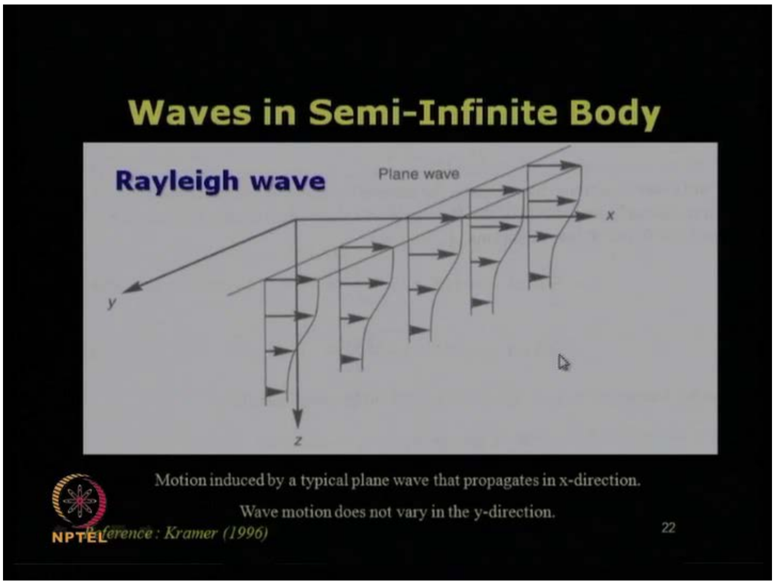

Hence, there will be another form of seismic wave which we have already discussed which are surface waves, that is, when the seismic waves reaches close to the ground surface they will be forming another form of waves. They can be classified as surface wave among them I have already mentioned that Rayleigh wave is one of them and another one is love wave. Let us look at the behavior of those surface waves which generates only in semi infinite body, remember these surface waves will not generate in the infinite body which we had considered earlier. This generates because they have a free boundary, free planar boundary. So, if we have a planar boundary like this, what earth is assumed to have for a shallow earthquake, in near earthquake source, then for near earthquake we will get the surface waves generated in this fashion.

So, Rayleigh wave how it will look like if we consider in three dimension say x direction is this one, y direction is this one and z direction is this one, the wave propagation for the surface wave only, I am talking about surface wave, this is the typical plane behavior of the surface wave. That is it will have the directional values in x direction and in z direction. So, in x direction they have magnitude and in z direction they are decreasing with respect to depth. As the name suggest it is the surface wave; obviously, it is expected its value will be maximum at the ground surface and it will reduced drastically as we go deeper and deeper because then it is nothing but a body wave, right?

But, if you look at this behavior there is no component in the y direction can you see that it is in the x z plane only. So, we can use this so this is the motion induced by a typical plane wave that propagates in the s direction. If we consider the wave is propagating in x direction then wave motion does not vary in the y direction. So, if we take in the y direction then we have variation in y and z only. There will be no component in the x direction that is what it means for the surface type Rayleigh wave.

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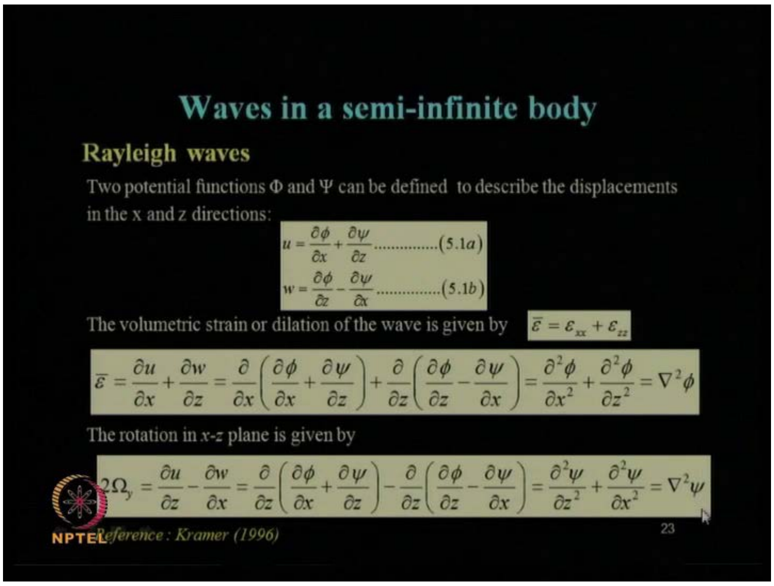

So, let us now see how this Rayleigh wave velocity we can derive or we can express the solution for the Rayleigh wave, knowing this behavior of the Rayleigh wave that it will be only in one planar direction. So, two potential functions let us define as phi and psi can be defined to describe the displacement in x and z direction. So, these two potential functions describes, describes the displacement function in x direction and psi express the displacement function in z direction. There is nothing in y direction as we have mention because it is propagating in the x direction. So, we have corresponding displacement as u and w, v is not present here. So, u can be expressed as del phi by del x plus del psi by del z and w can be expressed as del phi by del z minus del psi by del x.

So, through these potential functions of the displacement with respect to the direction space coordinate, the displacement functions can be expressed in this format. Hence, the volumetric strain or the dilation of the wave in this case takes the form of epsilon bar is nothing but epsilon x x plus epsilon z z because there is no y y component, right? And that epsilon bar can be further written as knowing this expressions of relationship between epsilon x x through u, we can express in this form.

So, which is nothing but epsilon bar volumetric strain equals to this grad square phi and the rotation in this x z plane. Why the rotation component comes into picture? Because this Rayleigh wave is nothing but it is a part of a shear wave. So, it creates a rotation about the point. So, that is why this will be the rotational component which can be expressed in this grad square by psi.

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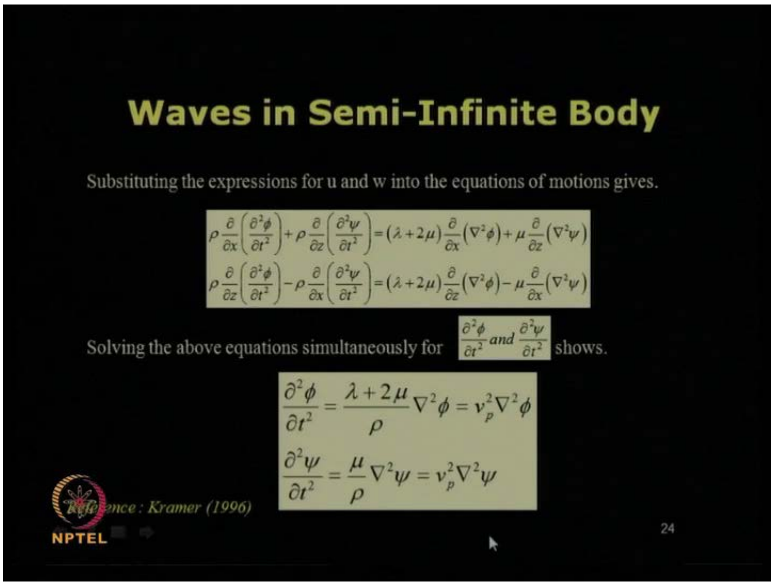

Substituting the expressions for u and w in the governing equation of motion, which we have we can get the solution in this form in terms of v p over here.

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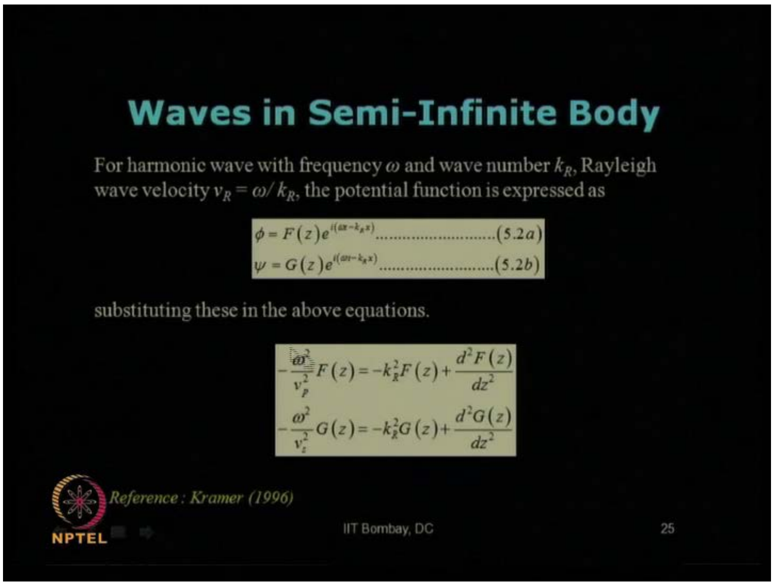

And on further simplification, that is, if we express for the harmonic wave with the frequency of omega circular frequency and if we denote the wave number as k R; k R because for Rayleigh wave, k is the wave number we have already seen earlier. So, Rayleigh wave velocity can be, wave velocity is expressed as velocity equals to 2 omega by k that we have seen earlier. So, in this case Rayleigh wave velocity can be expressed as omega by k R.

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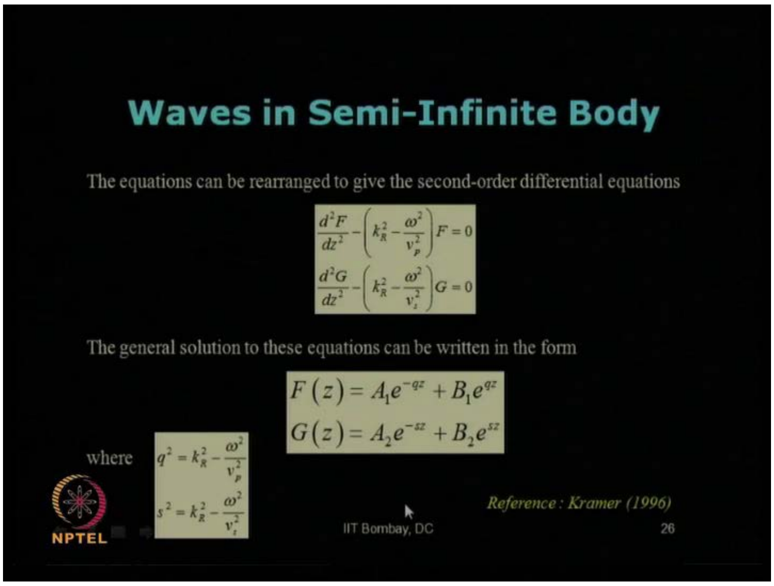

So, how we can express that in this function, substituting in this equation and on further simplification we will get using these parameters shown in this figure, in this slide.

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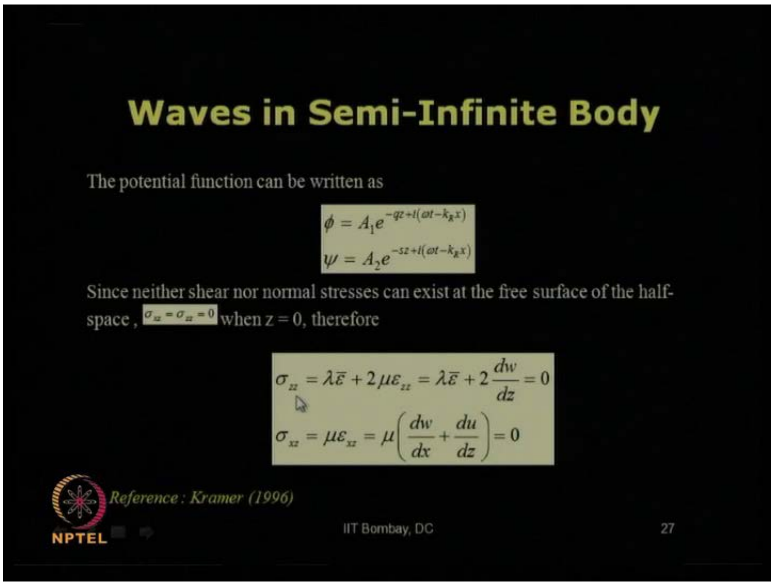

And the potential functions in this form we will get these equations for the sigma z z and sigma x z in two directions. Why we are putting it to 0? Because they will become zero at the ground surface, this is the boundary condition which is known for an elastic half-space. Am I right? This boundary condition is known because there will it should not be any stress at the free planar surface.

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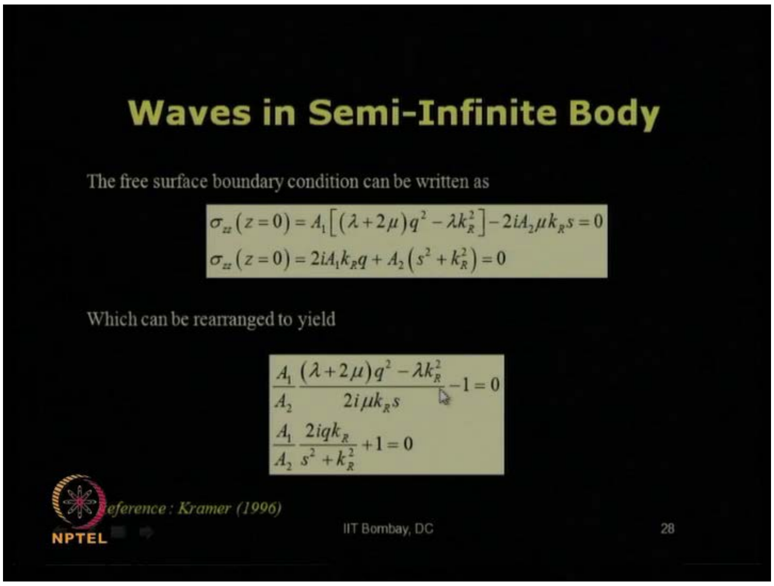

So, that is why if you put at the free planar surface at z equals to 0, this sigma z z 0 and sigma x z should be 0, using that that is what at z equals to 0 we are putting this expression. On further simplification of this equation one can get the relationship of this one.

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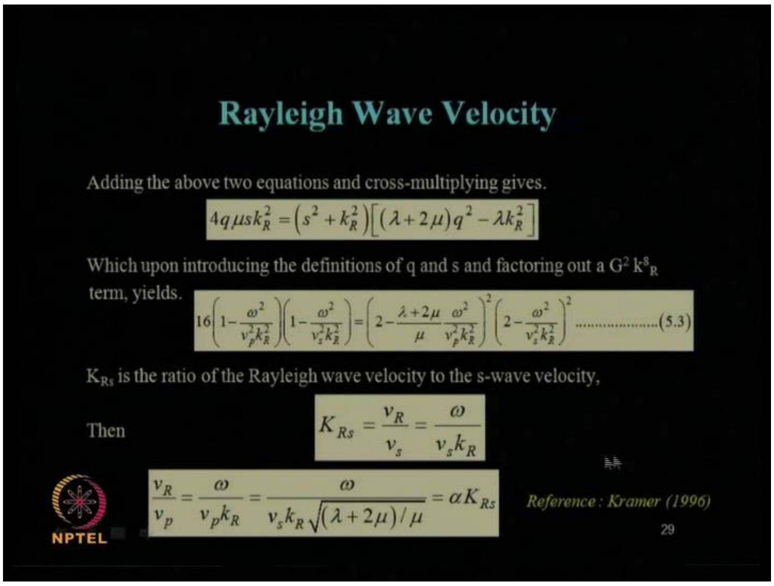

If we use this k R s with respect to the, it is the parameter which is ratio of the Rayleigh wave velocity to the shear wave velocity or S-wave velocity, that is, v R by V s we are expressing through this parameter capital k R s which is nothing but omega by k R is the v R and V s. Okay? So, if you use this expression then what should be the ratio of v R by V p; V R by v p will be omega by k R times V p. Now, V p we can expressed in terms of V s, using their relationship V p to V s, which we use should noting but alpha times k R s. What is alpha? That alpha we have already obtained over there, the ratio function of that V p by V s.

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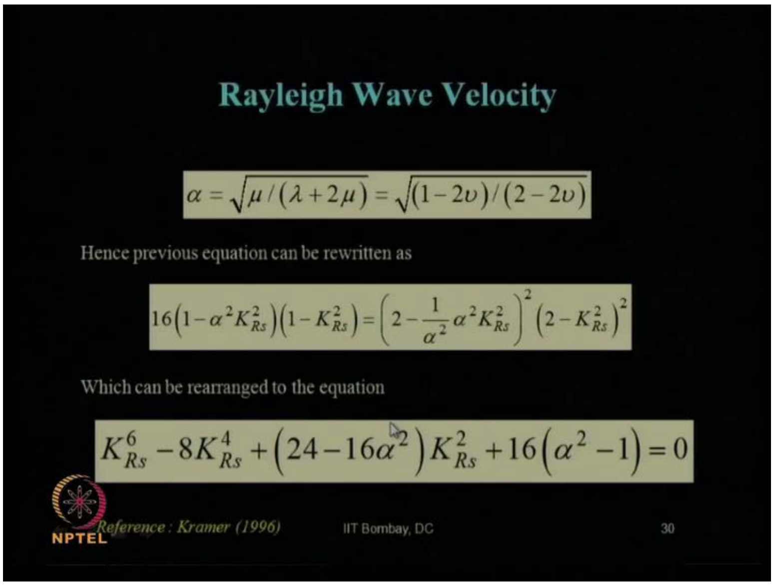

So, this is the alpha expression, root over 1 minus 2 nu by 2 minus 2 nu, this is the function of Poisson's ratio only. So, this alpha parameter is relating the V p to V s that already we have seen. Now, we are applying the same alpha parameter for relating the Rayleigh wave velocity to the shear wave velocity. So, using this parameter, putting it in this expression and on further simplification, this equation will take a form like this, which is further rearranged. And this is the final form of equation, which is very important for us to know, because this equation one has to use to obtain the Rayleigh wave velocity for any particular seismic wave motion in a semi infinite body or elastic half space.

So, K R s as we know this is the ratio of Rayleigh wave to shear wave velocity. So, in terms of K R s you have to solve this equation, alpha should be known to you form a given material property through the Poisson's ration. So, in this equation what is unknown, only unknown is in K R s. This K R s parameter, this is the sixth order equation which by trial and error or using Newton Rapson's method etcetera, one can solve it very easily. So, once you get the value of K R s known, then what you can use, you can use this shear wave velocity which is known to you, you can use it to compute your v R value.

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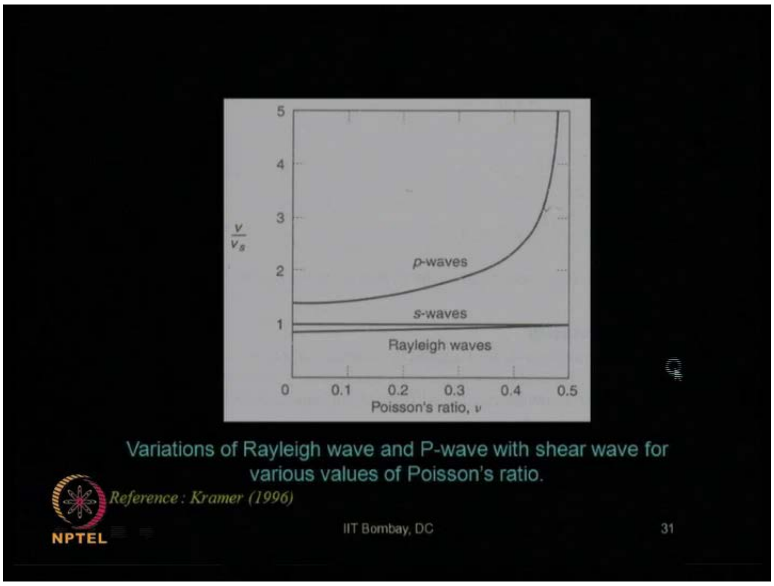

So, that is the way v R is calculated. So, if we see what is the variation of that Rayleigh wave velocity with respect to P-wave, shear wave and Poisson's ratio? Say x axis shows the variation of Poisson's ratio nu from 0 to 0.5 and S-wave is taken as the reference plane, that is, S with respect to S wave we are computing everything. So, that is why this y axis shows the ratio of any particular velocity to that shear wave velocity. So, shear wave to shear wave; obviously, it will be always one, that is, the reference frame for us. So, that is the line one for any Poisson's ratio.

Now, if we talk about P-wave velocity how we can find out this relationship that V p by V s relationship we know this is nothing but alpha right, that is, root over 1 minus 2 nu by 2 by 2 nu. So, putting the corresponding values of nu you can get this equation, you will see that if we put in this equation nu equals to 0.5 it will not give us a correct value, it will give an infinite value. So, it is giving an infinite value at nu equals to 0.5.

So, this relationship is not valid for Poisson's ratio of 0.5, that is, for soft saturated clay you should not use this relationship because it comes from basic mechanics, pure mechanics. Whereas it will not be valid for Poisson's ratio value of 0.5. In that case you need to find out individual value of V p and V s remember that where as Rayleigh wave velocity you can find it out very easily, that is, ratio of Rayleigh wave to shear wave velocity is close to one when it reaches Poisson’s ratio of 0.5 but, for lower Poisson’s ratio it is about 0.9 times of shear wave velocity, that is what we at the beginning itself when we talked about various seismic wave we mention, Rayleigh wave velocity is the form of shear wave velocity and its velocity is almost 90 percent or above for any values of Poisson’s ratio, which is satisfied through this relationship also through mathematical proof.

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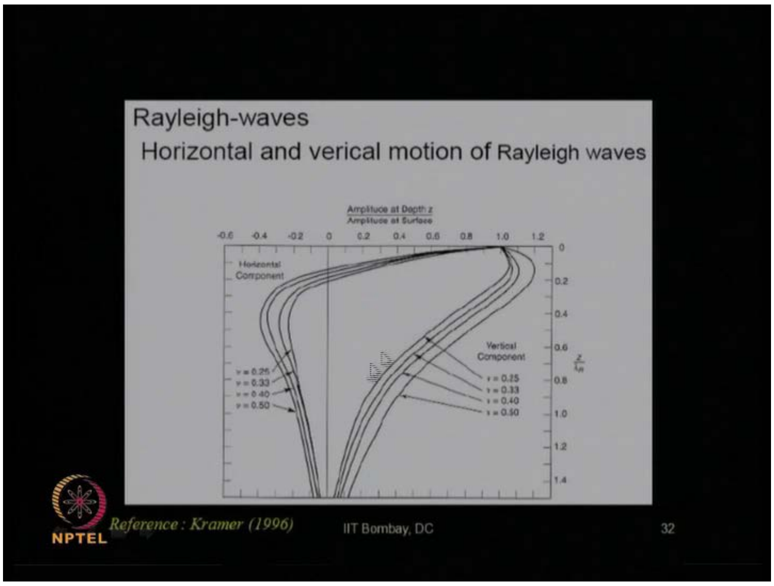

This is the variation of the Rayleigh wave amplitude corresponding to horizontal and vertical motion of Rayleigh wave. You can see this is the vertical component one at the ground surface and close to ground surface it is little higher than one but, it drastically reduces as you go deeper and deeper, similarly for the horizontal component also.

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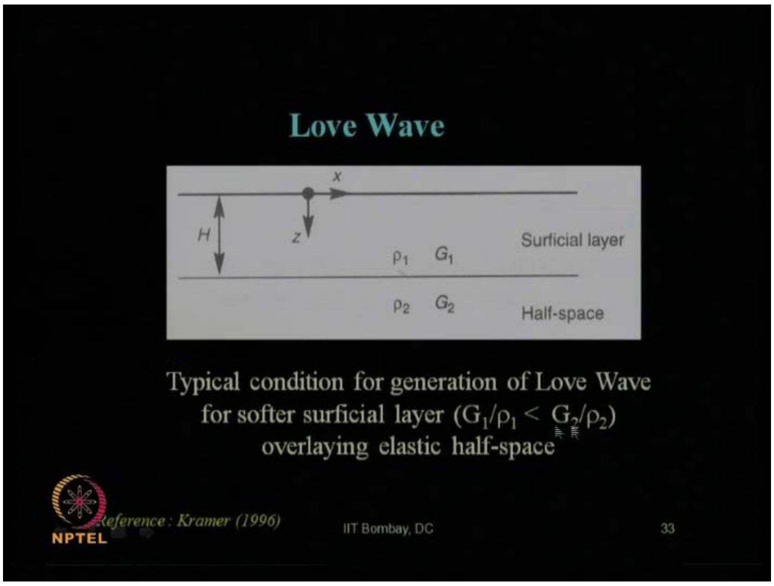

Coming to the next form of the surface wave which is love wave it is also surface wave. So, it is valid only for semi infinite body like earths ground surface, we have a free surface over here, we have x z plane over here and say this is the surficial layer of thickness of H, which is a finite small thickness compare to the other half-space elastic, half-space. And these are the material property let us say density is rho 1 and shear modulus is G 1 for surficial layer for half space material it is rho 2 and G 2.

So, that typical condition when this love wave will get generated for softer surficial layer; that means, this G 1 by rho 1 ratio should be much lower than this G 2 by rho 2. Remember, what is that G 1 by rho 1 ratio that is nothing but shear wave velocity square that automatically shows the shear wave velocity of the surficial layer should be much lesser than the shear wave velocity of the elastic half-space, then only the love wave will get generated otherwise not. So, if you have a surficial layer which is a stiffer material compare to your lower layer, the love wave will not get generated. So, remember about the condition for which love wave get generated.

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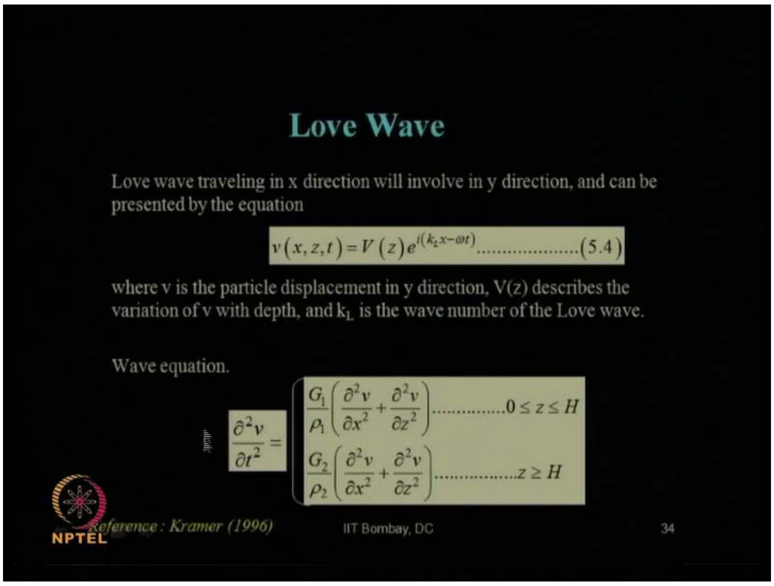

So, love wave traveling in the x direction will involve in the y direction and can be represented using the expression in this equation and the wave equation can be expressed in this form, when z lies between this surficial layer and beyond surficial layer for elastic half-sphere this is the expression in the y direction. Remember, this is the y direction wave equation.

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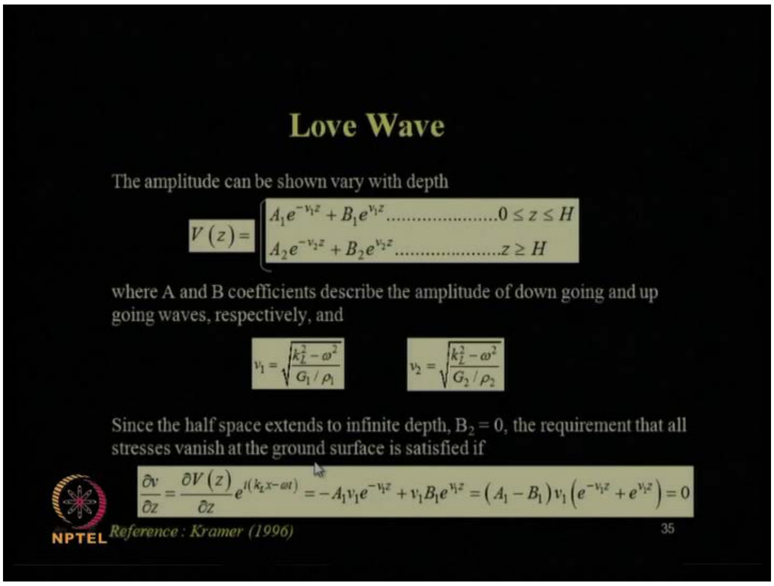

So, amplitude can be shown which varies with respect to depth using these functions. Now, these constant A and B can be obtained using the again, the ground conditions or the boundary conditions, that is, the stress condition should be 0 at ground level or boundary level. So, since the elastic half-space extends to infinite depth so, B 2 parameter should be equals to 0, the requirement of the stresses vanishes at the ground surface. So, that gives us stress condition at ground surface will be always to 0.

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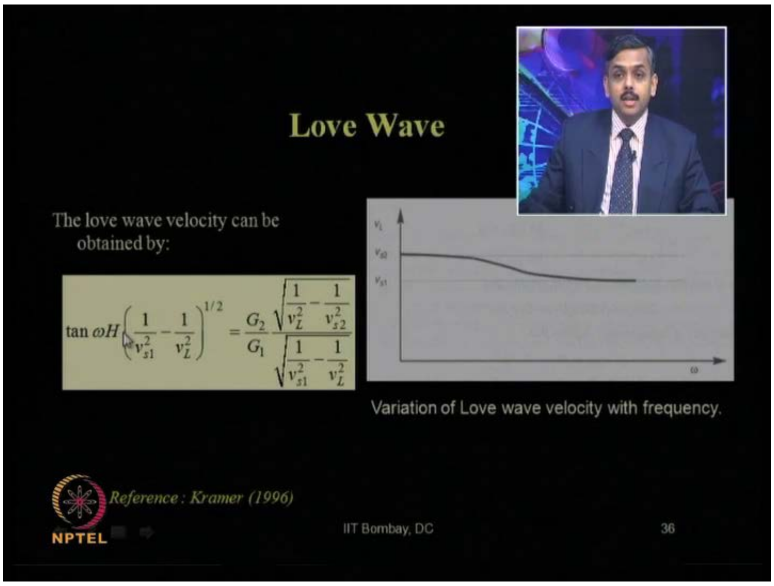

So, putting that in this expression finally, this simple expression can be shown for the love wave velocity. So, these v L is nothing but love wave velocity V s 1 is the shear wave velocity in the top surficial layer and G 2 G 1 are the shear modulus in elastic half-space and surficial layer and this V s 2 is the nothing but the shear wave velocity in the half-space or the second layer.

So, this is the variation with respect to the frequency, you can see if you want to estimate the love wave velocity, if it condition satisfied then only what wave we can formulated, the material property of the elastic half space and surface shear layers should be known to us, that is, G 2 and G 1 should be known to us, then shear wave velocity of the elastic half space and the surficial layer should be known to us. By knowing all these parameters for a particular frequency of any a particular earthquake and for a known thickness of your surficial layer, you can get the value of v L. All other parameters are known to this, in this equation.

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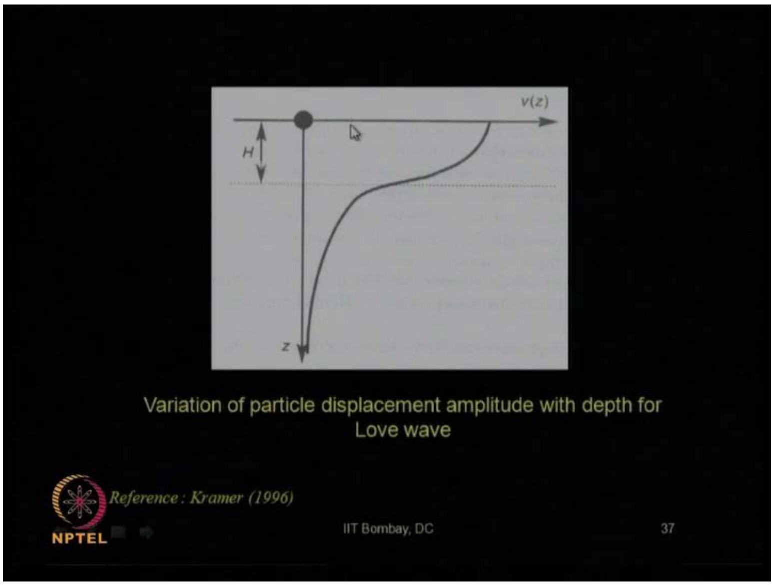

So, this is the variation of particle displacement amplitude with respect to depth for a love wave, you can see only for that surficial layer only that particle velocity will be significant. Beyond that it significantly diminishes or gradually goes to 0 as the depth increases. So; obviously, it is effect is seen only at a few shallow depth finite depth in the close to ground surface.

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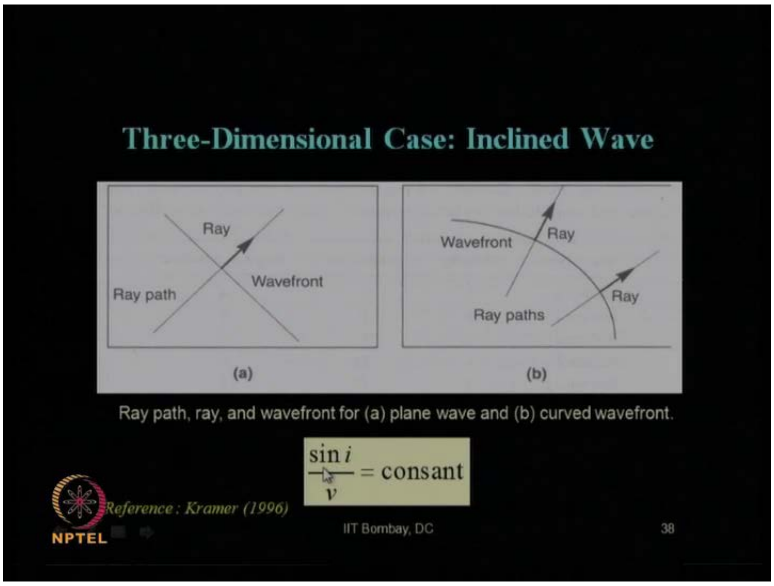

Now, when we talk about three dimensional case of inclined wave, ray path is going in this way wave front, we will have this rays getting, that is, the ray path ray and wave front for a plane wave and this is for a curve wave front but, whatever be the case as I have just few minutes back mentioned the Snell's law has to be valid for wave, that is, the basic condition as we know, that shows sine of i by v should be equals to constant.

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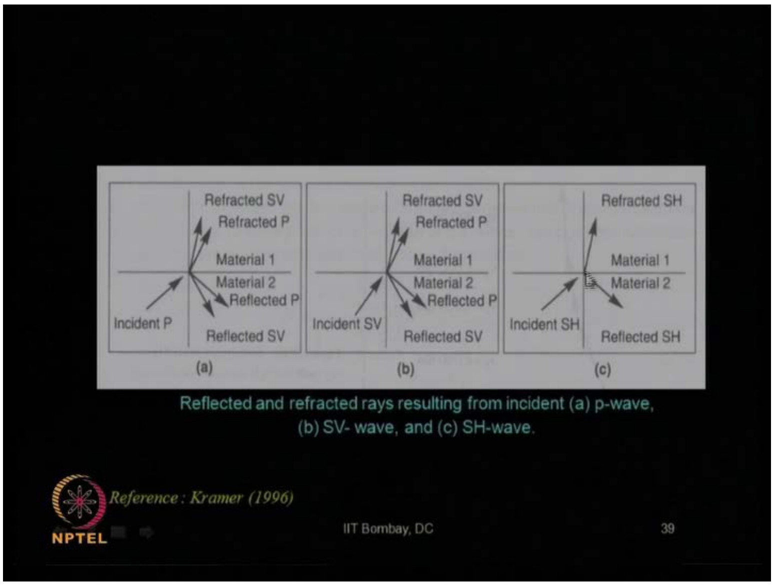

So, this one already we have seen the incident P-wave inclined one will generate P and SV-wave. Similarly, SV-wave incident inclined will generate P and SV-wave whereas, incident SH-wave inclined will generate only SH-wave.

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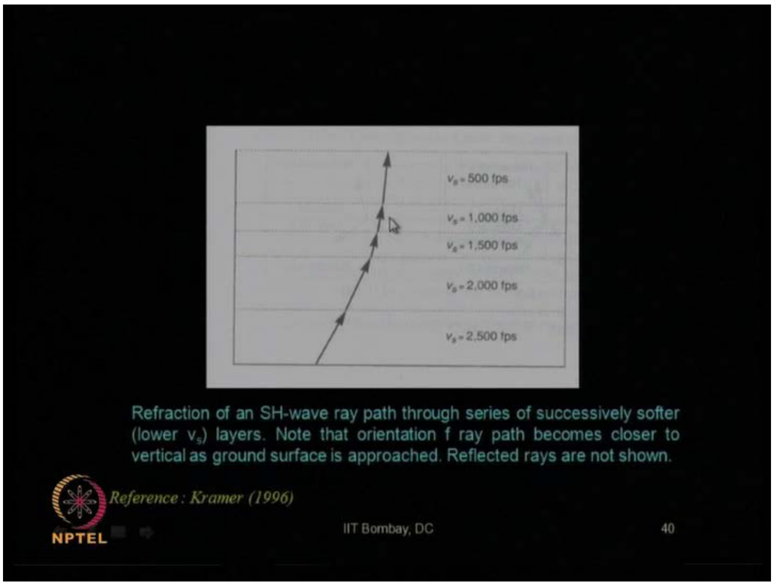

And also we have seen why we take the vertical component of the seismic wave close to the ground surface.

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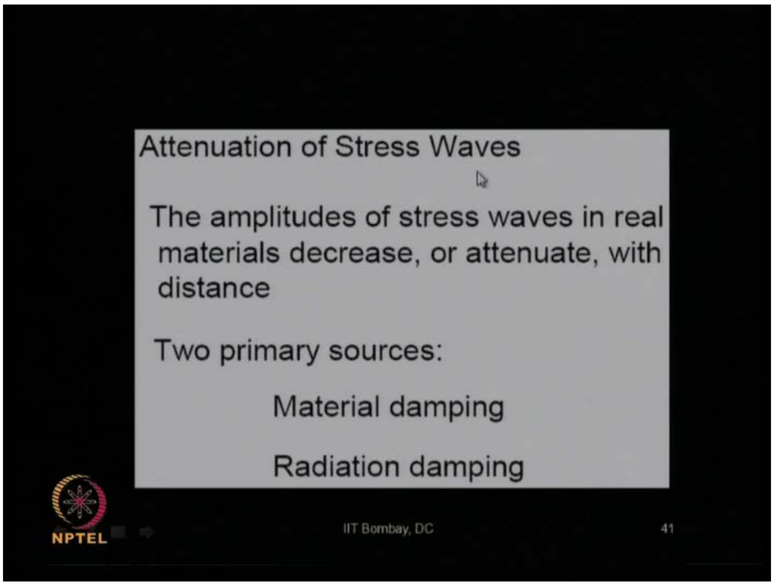

Now, when we talk about the attenuation of stress waves, when we are using the attenuations of various stresses, the amplitude of the stress waves in real material those things decrease or attenuate with the distance. What does it mean; that means that there are two primary sources of attenuation because of material damping and radiation damping. What does it mean by material damping? As we know every material in the earth is having some viscous damping, another terminology which we use correspond to this material damping is viscous damping, that is, each material is having some damping constant, that is, if some wave travels through that after some time it will damp in, right?

There will be a loss of energy, if some vibration occurs in that material in that media there will be a loss of energy in terms of sound or heat energy, etcetera, that we have already discussed in basic fundamentals of vibration theory. So, damping constant is always present for any material. That is the reason we call viscous damping is present because of that viscous damping what this stress amplitude of the wave will do, they will attenuate.

Attenuate means as the distance from the source of that seismic wave occurs that is the hypocenter or focus of the hypocenter or focus of the seismic wave or the earthquake from that point to another point of our interest, that is, at the site of our interest when we are considering a particular seismic wave we should consider this attenuation due to the material damping or viscous damping because; obviously, the amplitude of wave will get reduced drastically when it travels through a particular distance.

And what is the next source of that attenuation or reduction in that amplitude of that stress wave that is called radiation damping; radiation damping is nothing but when some waves get radiated from the its source point; obviously, with respect to its direction of movement at the distance increases it will get radiated or many number of raise will get generated, then automatically reduces the original amplitude of the wave which gets generated at the hypocenter or the focus of the earthquake.

So, that automatically shows that at our site of interest when we are considering particular seismic wave, what value of the stress we should consider it should be related to this attenuation, it should not be at the hypocenter whatever, the wave amplitude is getting generated the same amplitude we should not use in this case.

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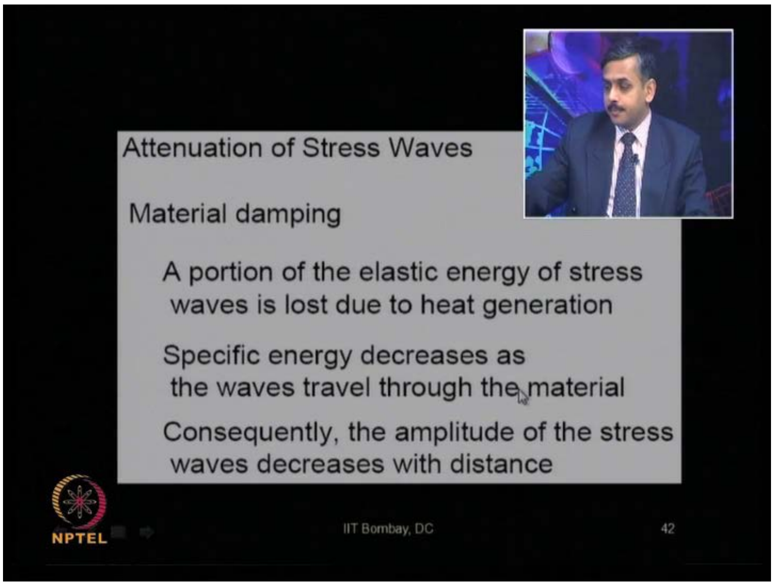

So, as we have mentioned the material damping or viscous damping a portion of the elastic energy of stress waves is lost due to heat generation. So, that is the reason of viscous damping for any material as we know. So, the specific energy decreases as the waves travel through the material in this manner. So, consequently the amplitude of the stress waves also decreases with the distance, that is, what it happens in this case.

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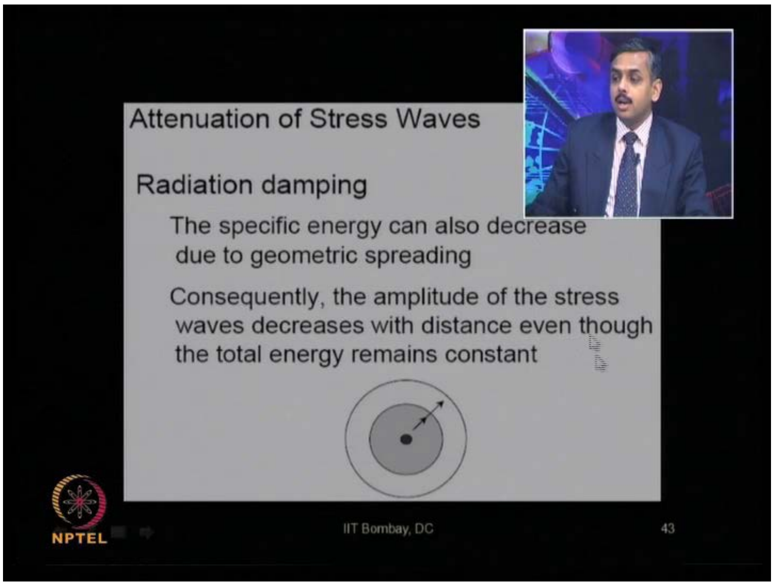

Where as in the next case of attenuation which we named as radiation damping, this is the picture through which we are showing the radiation damping. Suppose, this is the source this black point is the source through which the earthquake energy gets released, when energy gets released all the waves are getting generated and travels in all the direction. So, the specific energy can also decrease due to geometric spreading. So, this is because of the geometry, that is, as furthest point as you go from this point source there will be a decrease in the amplitude of that stress, that is, because of the distance related issue.

So, the previous one was the material property related issue or viscous property of the material and this next one is related to the distance related issue. So, the distance related part will remain same what will vary for a particular location or particular site from the source point of earthquake is the material damping or viscous damping and of course, this radiation damping also will play a significant role of this stress amplitude reduction. When we will talk about the distance it travels from the source point to a point of our site of interest. So, consequently the amplitude of the stress waves decreases with increase in distance even though the total energy remains constant. Total energy remains constant but, it gets distributed over a larger area that is the reason, you can see initially it was it was emerged from this small black dot then in next phase it is in this grey area circular, then this white area of circular. So, as its spreads over larger and larger distance;

obviously, its amplitude will keep on decreasing though the total energy remains constant.

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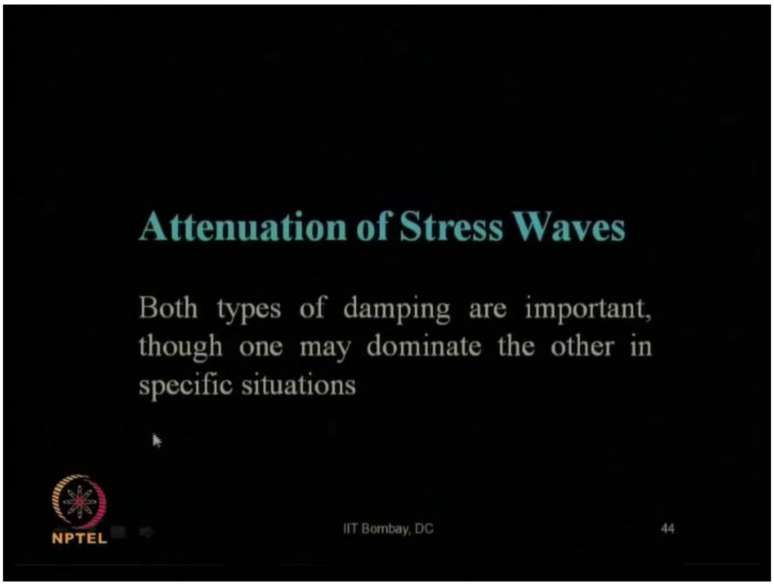

So, that attenuation of stress waves we have seen both types of damping are important, that is, whether it is a viscous damping or a radiation damping. So, in our seismic analysis in the geotechnical earthquake engineering analysis we will see later on that. When we talk about any analysis many a times researchers consider the viscous damping only or the material damping only. Many a times researches forget about to take the advantage of the radiation damping when the earthquake analysis is done, if the fault source or the hypocenter of an earthquake, past earthquake history form past earthquake history it is known to us we can utilize that for further estimation of the amplitude of stresses, which will help us to calculate the displacement also.

We have seen how these stress then displacement is co related right through the wave equations and their expression, already we have seen those things. So, both types of damping that is material or viscous damping and radiation damping are important, though one of them may dominate over the other in the case of specific situation. As we know suppose, if it a near source earthquake that is when your site of concern is close to the earthquake epicenter or the fault region you should not get much of an advantage from the radiation damping, you may get little advantage may be from material damping only but, if it is far away from the source even though suppose the material damping is not that major but, radiation damping also will contribute as a major parameter which can be considered in our analysis.

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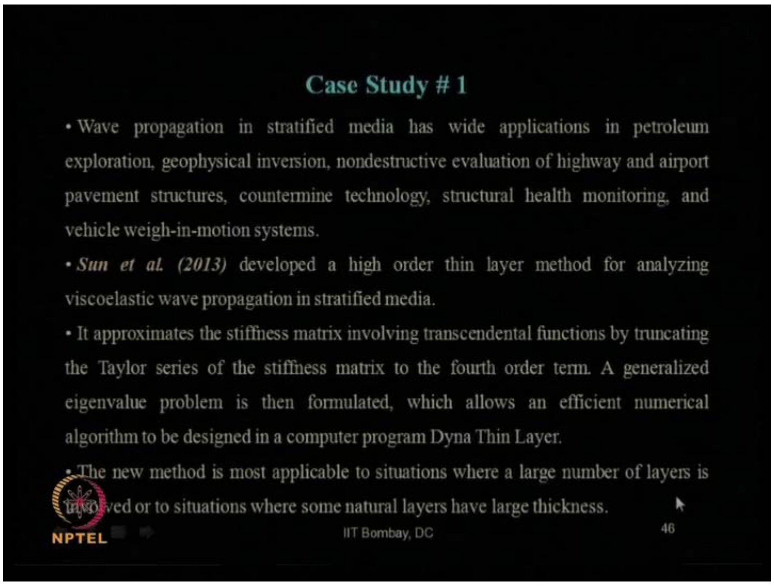

Now, let us quickly go through case study in this case. First case study wave propagation in stratified media has wide applications in petroleum exploration, geophysical inversion non destructive evaluation of highway and airport pavement structures, countermine technology, structural health monitoring and vehicle weight in motion systems. So, all these areas this wave propagation theory can be applied.

So, we are using one case study which is proposed by Sun et al in this 2013 itself, which is developed a high ordered thin layer method for analyzing visco elastic wave propagation in a stratified media. So, what they mention that it approximates the stiffness matrix involving the transcendental functions by truncating the Taylor series of the stiffness matrix to the forth order term a generalized Eigen value problem is then formulated which allows the efficient numerical algorithm to be design for in a computer program, that is what it is used in their paper.

The new method is most applicable to the situations where a large number of layers is involved or to the situations where some natural layers have large thickness. So, this methodology which is proposed by Sun et al is applicable for more layered soil.

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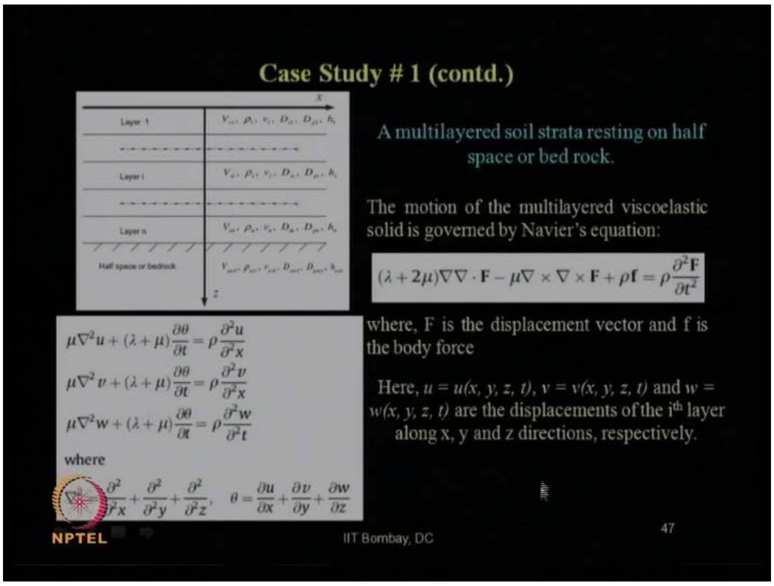

This is the case for many of our typical sites, as we will see later on also. Suppose, it is having several numbers of soil layers like this, layer one layer i th, n th layer and then we have the half-space or the bed rock elastic half-space. These are the various layers material property. So, a multilayered soil strata resting on the elastic half-space or a on a bed rock, the motion of the multilayer viscoelastic solid is governed by the Navier's equation in this form, that is, what they have mentioned the individual layer equation is given by this format in the three dimension as already we have seen in x direction, y direction and z direction. So, where this combined equation this F is the displacement vector and small f is the body forces.

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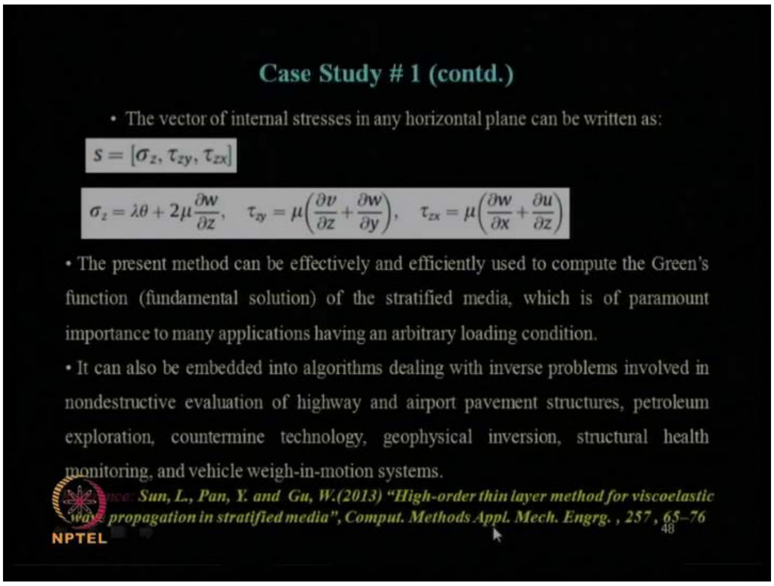

So, what they mention finally, the vector of the internal stresses in any horizontal plane can be written as a function of this stresses; normal stress and shear stresses. The present method can be effectively and efficiently used to compute the Green's function of a stratified media which is of paramount importance to many applications having an arbitrary loading condition and it can also be embedded into algorithms dealing with inverse problems involve with non destructive evaluation of highway and airport pavement structures, petroleum exploration etcetera. So, reference for this paper Sun et al 2013 this paper which appeared in the journal computer methods and applications in mechanical engineering this volume 257, these are the page number.

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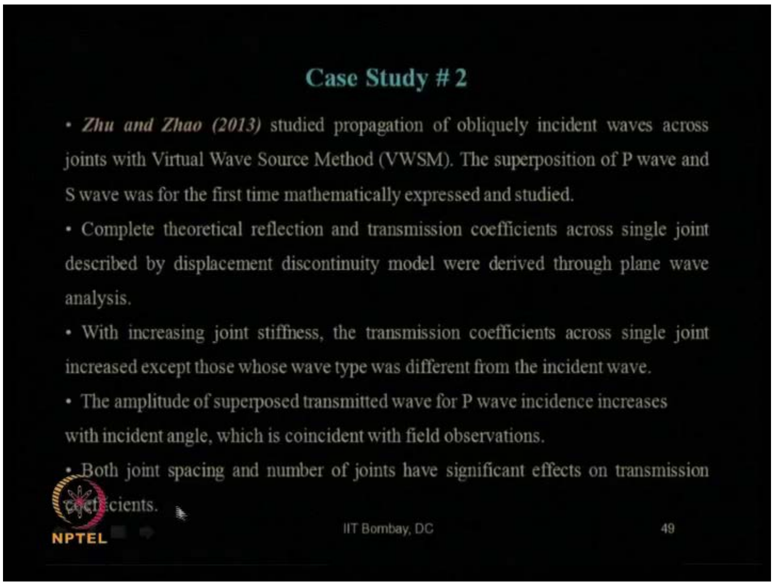

Another case study, which is proposed by Zhu and Zhao again in this 2013, they studied the propagation of obliquely incident waves, that is, if suppose any incident wave comes in oblique direction not as a vertical one across a joint, form a joint, when it is radiating suppose it comes as an inclined or oblique incident wave with the virtual wave source method they have proposed the superposition of P-wave and S-wave for the first the time they did it mathematically and expressed in this study. What they mentioned? The complete theoretical reflection and transmission coefficients across the single joint described by displacement discontinuity model; with increase in joint stiffness that transmission coefficients across the single joint increased, except those whose wave type was different from the incident wave.

The amplitude of superposed transmitted wave of P-wave incidents increases with incident angle which is coincident with the field observations also; and both the joint spacing of the fault and number of joints of the fault have significant effect on this transmission coefficient, that is how much P-wave and S-wave will get transmitted those are factors of those number of joints.

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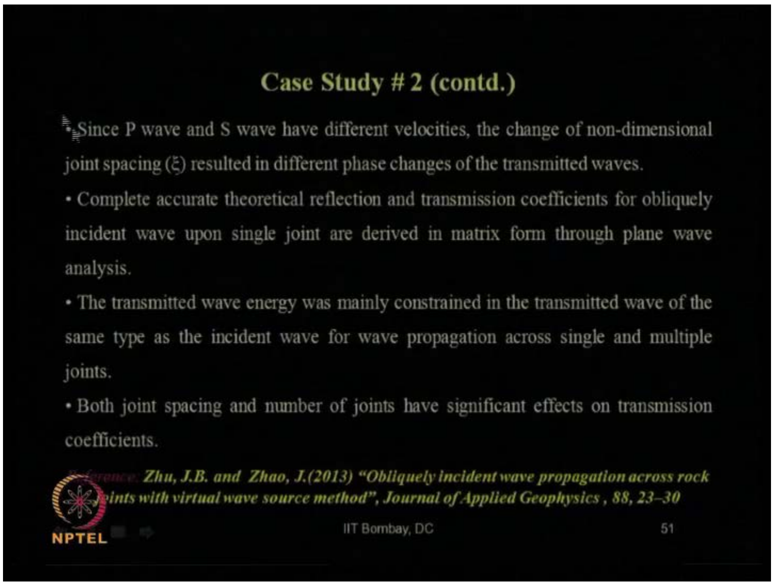

These show the P-wave incidence, S-wave incidence through this equation. They mentioned that since P-wave and S-wave had different velocities, the change of this non-dimensional joint spacing resulted in different phase changes of the transmitted waves; and the transmitted wave energy was mainly constrained in the transmitted wave of the same type of incident wave for wave propagation across single and multiple joints. For joint spacing, the number of joint has significant effect on the transmission coefficients.

The details about this study can be obtained in this paper by Zhu and Zhao in 2013, which appeared in the journal of applied geophysics of volume 88, these are the page numbers. So, with this we have come to the end of module 5. Let us look at the slide here this ends our module 5, which is wave propagation of this geotechnical earthquake engineering NPTEL video course.