Geotechnical Earthquake Engineering
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Module - 5
Lecture - 18
Wave Propagation (Contd…)

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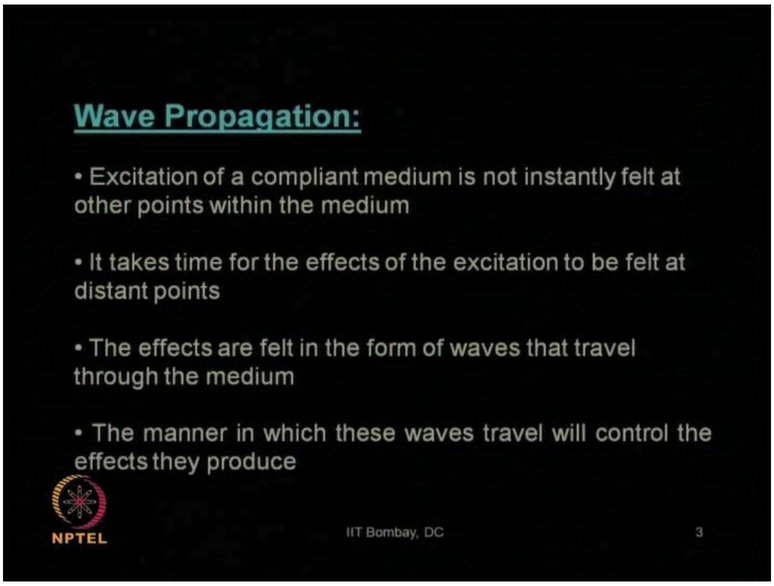

Let me start today's lecture on NPTEL video course Geotechnical Earthquake Engineering. In the previous lecture, we have started with module number five that is wave propagation. A quick recap what we had learnt in our previous lecture. We have seen, what is wave propagation? How waves are getting generated for any kind of excitation or vibration; and it travels through a particular media from one point to another point.

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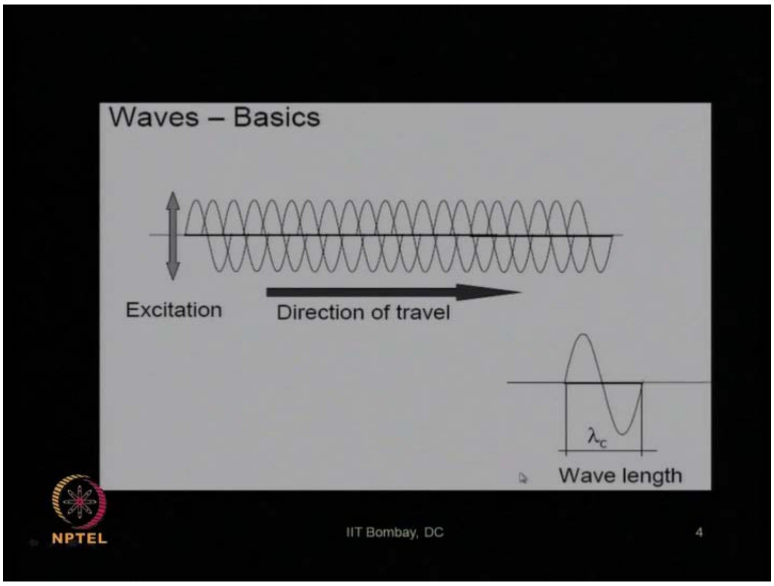

This is the basic of waves; that is the excitation occurs; then it travels in the form of wave in that particular media, so this is the direction of travel. And the definition of wavelength, we have seen in the one cycle of the wave required to complete the distance is wavelength over here.

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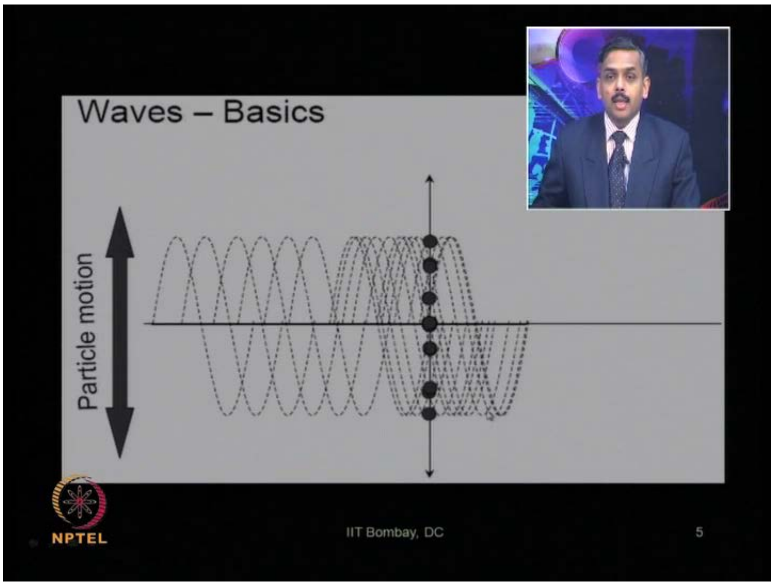

And the particle motion, we have also seen, it can be in different directions compared to the direction of the movement of the wave.

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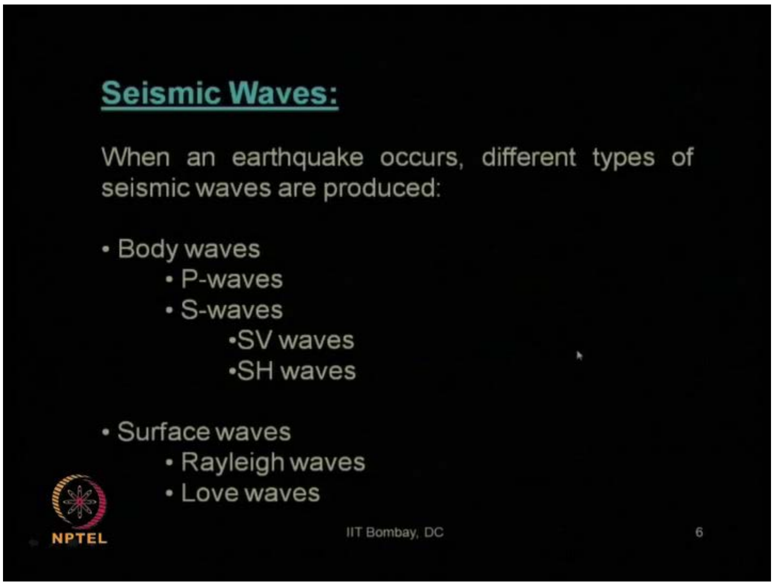

Also we have talked about what are the various types of seismic wave, that is when that excitation is due to the release of earthquake energy. What are the different types of waves? Two major types: body waves and surface wave. In the body wave also we have seen classification like primary wave and shear wave; within shear wave also we have seen two categories like SV wave and SH wave depending on the direction of movement of the particles with respect to the movement of the wave propagation. Also the surface wave, the sub classification we have seen: the Rayleigh wave and Love wave.

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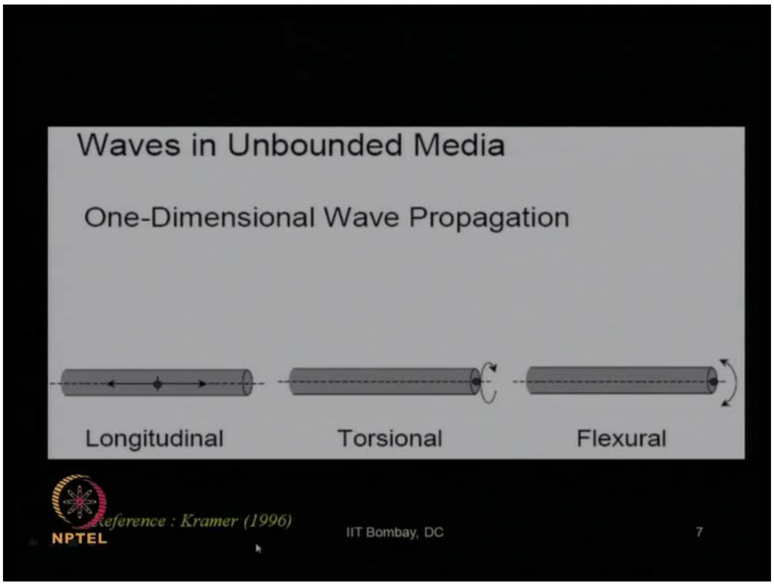

We started with the simplest case in the previous lecture that is the waves in unbounded media or the infinite media. In that case we have started one-dimensional wave propagation that is wave is propagating only in one dimension, say in x direction; in this direction only. Based on different types of propagation of the wave we have seen the three major sub classification: one is longitudinal wave, another torsional wave and another one is flexural wave, depending on the movement of the particles compared to the movement of that wave. So when wave is propagating in x direction or one dimension like this, when particles also get influenced or moved or travelled in the same direction we are calling it as longitudinal wave propagation; when particles are moving in this direction we call it as torsional wave propagation; and when particles are moving in this direction we call it as flexural wave propagation.

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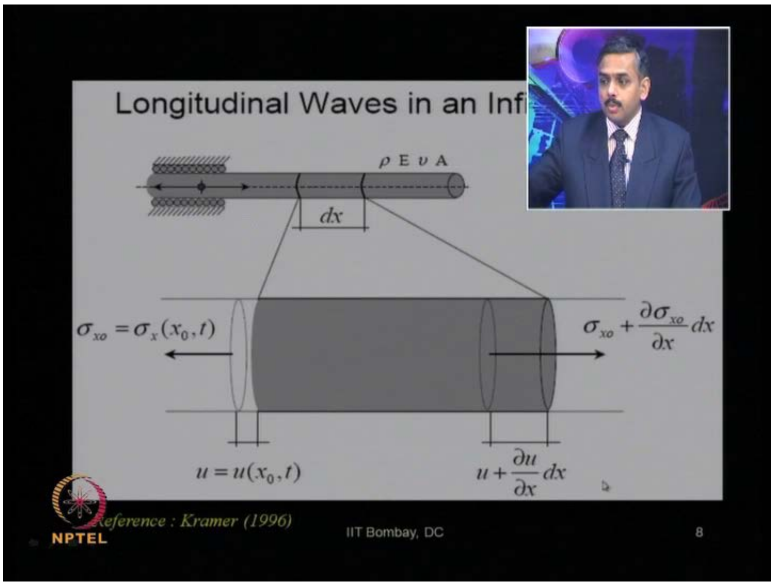

Then we have started in the previous lecture; we have derived this one, we can see in this derivation that is how this longitudinal wave in infinite rod they travels through; we have taken this infinite rod in the x direction, only one direction.

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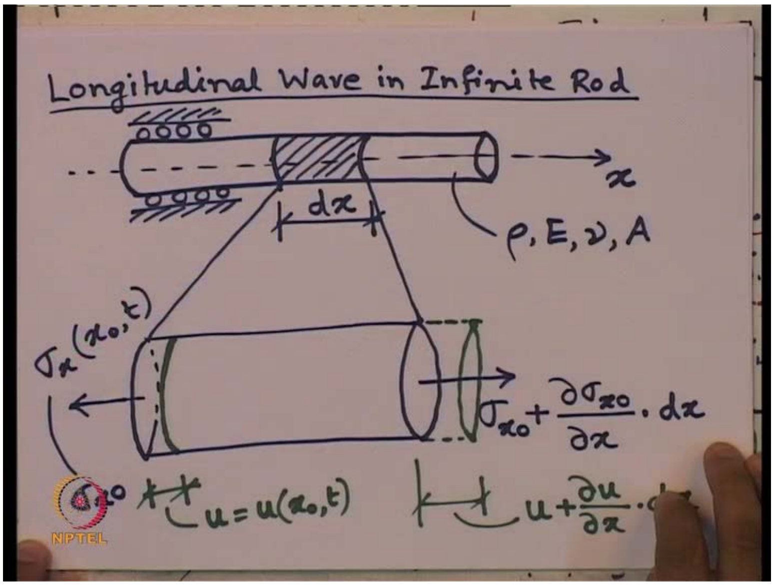

When we are talking about the longitudinal wave and small infinitesimal element or length in that infinite rod, we have considered of length dx. And various material property of the rod were given to us like density of the rod, young's modulus of the rod, Poisson's ratio of the rod and cross sectional area of the rod. If we exaggerate this infinitesimal small length d x, we will see there will be a difference of stresses at both the ends of the rod. Why this difference of stresses will occur? Because when wave is travelling through this media particles are getting excited; when particles are getting excited nothing but it is subjected to the inertia force. And that inertia force will cause some unbalanced force within the internal system of the body or rod which will balance that inertial force due to the travel of the wave through that media.

So, in both the ends if we consider this end as the co-ordinate x 0 or the starting point and time T the stress we can mention at sigma x, which is a function of x 0 and T as sigma x naught. And at this end we can say there is change of the stress over the length of the dx due to the travel of that wave in that media is sigma x naught plus that del sigma x naught by del x over the length of dx. That is the increment in the stress in that element due to the travel of the wave through the media. And corresponding to that what is the displacement at both the ends of this element, small element; at this end suppose due to the passage or due to travel of that wave, the movement of the particle say at that end is u which is a function of x naught and T once again; and at this point, let us say, there is an increment in the change of that function of u over the length d x is del u del x into d x, that is the displacement at this end.

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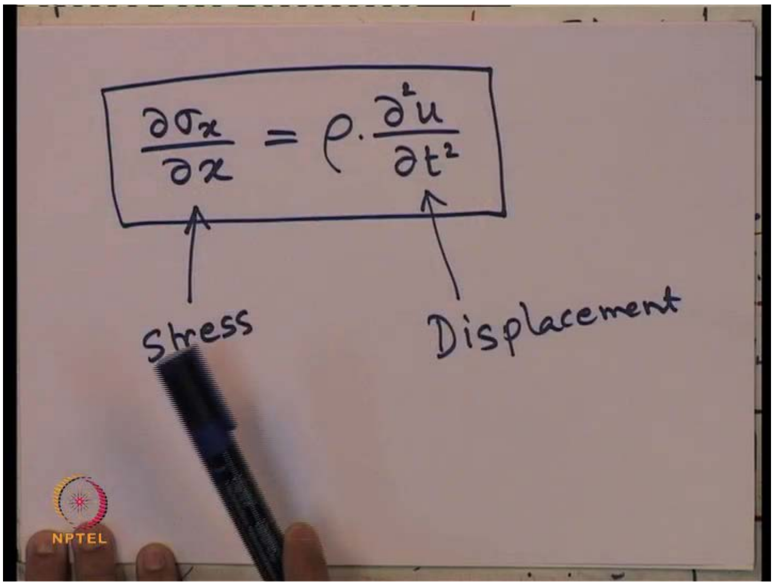

So knowing that, we have derived through the simple force equilibrium in that x direction that is equating the internal force; that is due to the stress balance whatever internal force occurred, with respect to the inertia force which is getting generated due to the travel of the excitation through that media in the form of wave propagation, we arrived at this equation which is the relationship between the stress and displacement like this.

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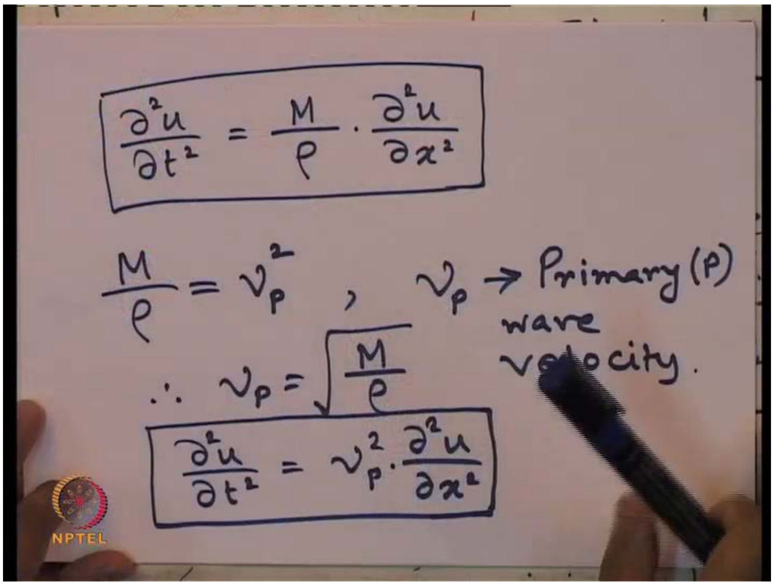

And after deriving that for further simplification what we have found out that is using further the stress strain relationship and strain versus displacement relationship we arrived at this expression. That is this is the acceleration; and this is the displacement which double derivative with respect to the x; which is through an operator M by rho, which operator M by rho is defined as the square of the primary wave velocity or p wave velocity v p. Our final governing equation for one-dimensional longitudinal wave propagation is expressed in this form that is del 2 u by del T square equals to v p square times del square u by del x square, now further let us start deriving in today's lecture.

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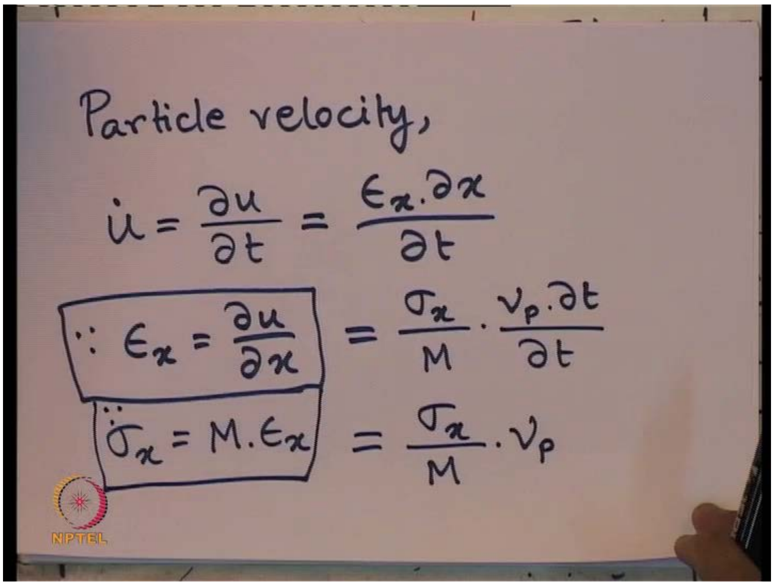

We are starting our today's lecture with the derivation on this longitudinal one dimensional wave propagation only, through the derivation of particle velocity. Because when the particle is moving we are interested to know how much the particle velocity will occur. Particle velocity, let us say, is u dot which is nothing but del u del T as I have already mentioned; for one dimensional wave propagation it is nothing but equals to d u d t. But when we will talk later on about three dimensional or the generalized case we have to take corresponding direction of displacement. And corresponding direction of displacement we have to differentiate that is why it is better to always write in the form of partial differential rather than a full differential.

So, del u del T which we can further simplify or further we can express it in terms of what is del u in terms of strain if we want to plot if we want to write it is nothing but this. Because earlier what we have seen how this? Because this strain, how it is defined? it is del u by del x. Knowing this I can put it like this. Which on further simplification that sigma x by m times Vp into del t by del t, this is on further simplification. How I am able to further simplify it, because I know the stress strain relationship. What is the stress strain relationship? Which is sigma x is connected through M times this epsilon x. And on further simplification of this del x you can put it like v p times del T in the x direction. So, what this will simplify further, this gives me sigma x by M times this V p.

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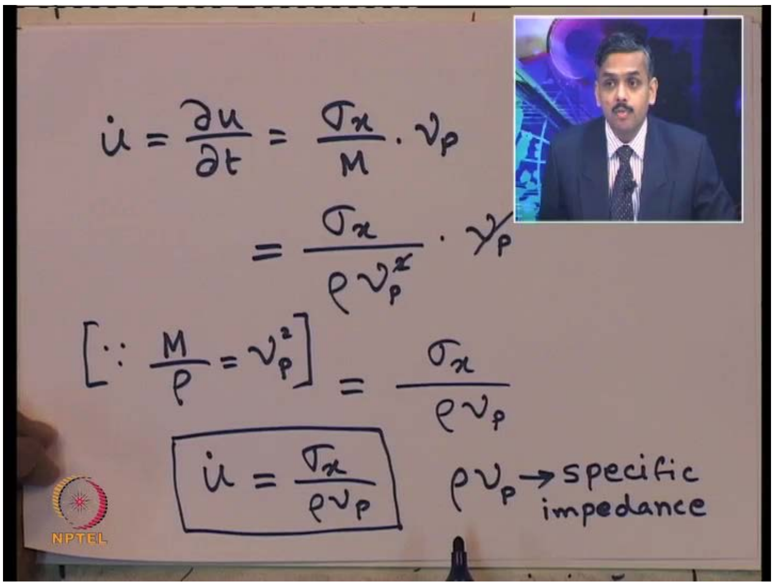

This is, because this relationship is known to us we simplify it further like this. Now let us further see how I can simplify it in a simple form. That particle velocity which we are talking about that is del u del T, now became sigma x by M times V p that is the up to which I have already derived in the previous slide. Now which I can further write as sigma x by rho times V p square, why? Because already we have seen that M by rho equals to V p square. Hence M that constraint modulus I can write it rho times V p square which on further simplification will give me, this goes out, so sigma x by rho times V p. This is an important relationship once again because it gives us the particle velocity due to the movement of a wave in this longitudinal direction.

So, this is the velocity of the particle which can be estimated using this relationship. And in this relationship, this parameter rho times V p is called specific impedance. What is defined as specific impedance? Specific impedance is nothing but the multiplication of the density of the material with the velocity. In this case it is a p wave velocity or primary wave velocity. That decides about the specific impedance we will see that this velocity depends on what type of wave it is propagating; when it is longitudinal wave that is p wave we will call it p wave or longitudinal wave velocity; if it is some other wave as we have seen the torsional wave also or shear wave then it will come as Vs. So, specific wave impedance in general it is the product of the material density with the velocity of a particular wave which is travelling through a particular media. It can be V p if it is a longitudinal wave, it can be Vs if it is a torsional wave.

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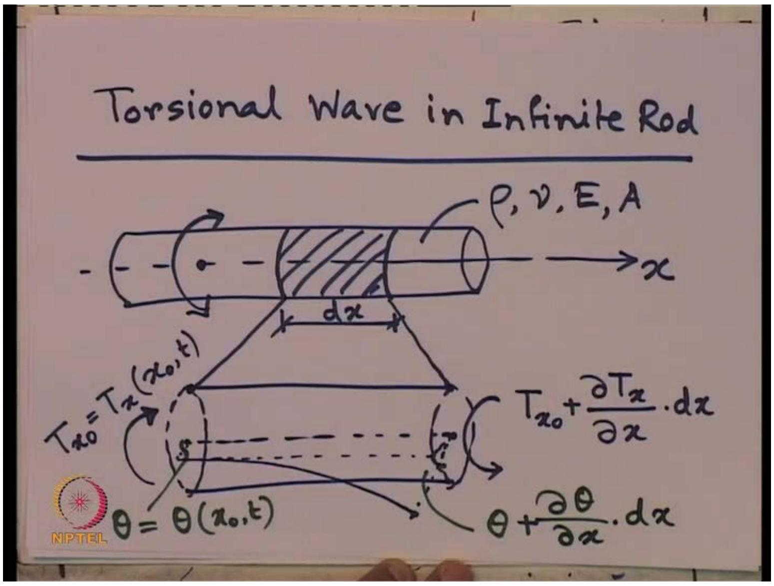

Now, let us come to the next type of wave which is the torsional wave. Now, let me start deriving for torsional wave equation, in infinite rod. We have already seen the derivation for the longitudinal wave, now we are coming to the next type of wave which is the torsional wave as we have mentioned. Let us take again the same infinite rod. And in this case this wave is moving in this direction x, but particles are moving in this direction that is it is getting twisted or torsioned. Here also let us take an infinitesimal small length of d x of that material and the material property should be given to us like rho density then mu poisson's ratio, E young's modulus and A is the cross sectional area. These are given input data.

Now, if I exaggerate this infinitesimal length once again, we can see here on this end and this end there will be, at this point if I take the central axis over here this central axis will get twisted or deformed like this; at this point it will move in this direction and further it will get inclined in this direction, we will explain that. This end we are talking about say the torsion is, say T x naught which is nothing but T x which is a function of that x naught and t. This point reflects this and this point is getting reflected at this end. There will be a torsion in the reverse direction to maintain the equilibrium. And this difference of torsion will occur because the wave is traveling through this media. That torsion is nothing but T x naught plus, what is the increment in the torsion d? del tx by d x over the length of that dx.

Now, let us look at the corresponding; this is about the torsional force which I am considering or torsional stresses. Now corresponding the rotation or the displacement at this end let us say this rotation or this displacement is, say theta which is nothing but theta as a function of x naught and T at this end. Whereas, at this point this theta is; there is a change, as you can see, I have shown the central line over here, that central line got further twisted because there is a difference of the torque at both the end. So obviously, compared to this end whatever got initially twisted, this will get more twisted or further twisted. That additional twist, let us express it here theta plus del theta over the distance of d x. With that now let us see what further simplification we can do in this case.

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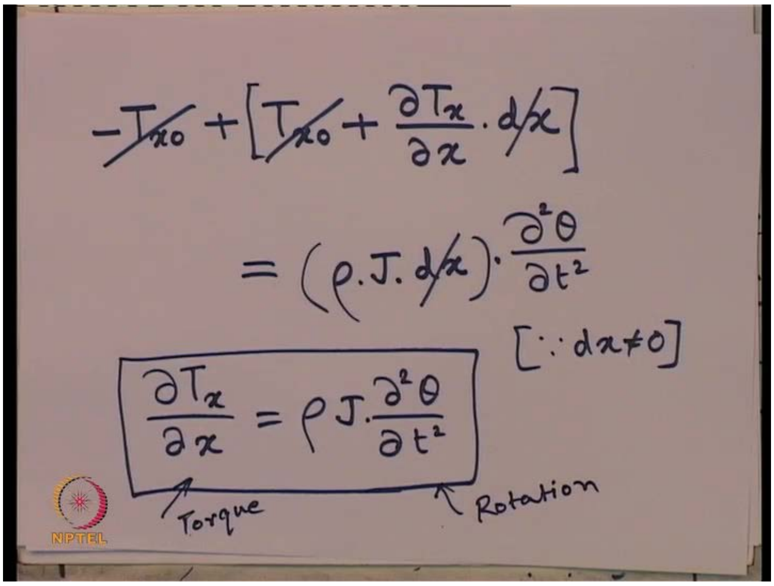

When we write this equation of motion further in the simplest form, similar to our longitudinal wave, what we can write over here? In this case this T x naught, let us say it was in the clockwise direction, plus this T x naught plus del T x by del x over the length of d x. This was in the clockwise direction; this was in the anticlockwise direction. Let us go back and look at here; this was clockwise, I am taking negative sign for this; and this was anticlockwise, I am taking positive sign for this. The net torque which is acting on this element is nothing but this one. That net torque is nothing but what is the inertia force acting on that element when the wave is passing through.

Now what is that inertia force? Inertia force in this case it will be; what is the amount of acceleration? First inertia force we will be multiplication of the mass times acceleration. In this case how much is the acceleration; how much is the displacement first of all, it is a rotational displacement; as we have seen, this rotational displacement is nothing but the theta. So, the acceleration will be double derivative with respect to time of this theta parameter. Let us write the acceleration first, del 2 theta by del T square is nothing but the rotational acceleration. And that rotational acceleration we can multiply it with respect to the mass. So, what we can do in this case, how it will change? It will change rho J times d x. Why in this case this is the corresponding mass, because it is a rotational acceleration. So, we have to take mass moment of inertia. How the mass moment of inertia will come into picture? J is nothing but area moment of inertia in the polar direction or the perpendicular direction through which the torque is applied that is above the x axis. So, J is the moment of inertia above this x axis. If I multiply that J with respect to d x I will get corresponding type of volume, in terms of the corresponding to area moment of inertia to mass moment of inertia when I want to convert it through this mass density rho.

That is why this operator will give me a mass moment of inertia times, the rotational acceleration will give me a total inertial force which will be nothing but the resultant torque which is acting on the system. Now let us simplify; this goes out, again I can cancel this with this, because this d x is non 0. What further I can simplify and write it, del T x del x is equals to rho J del 2 theta del t square. That is the relationship between torque and the rotational acceleration or it is a relationship between torque and rotational displacement. Like earlier for longitudinal wave what we had written stress displacement relationship, here also we can write this is a torque rotation relationship.

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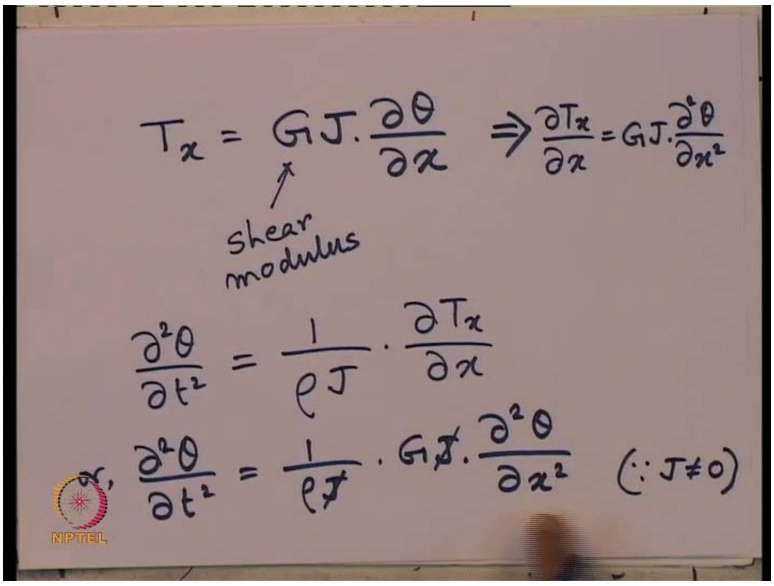

Through this torque rotation relationship let us see further how we can simplify this relationship. In this case that torque T or T x in this case let us say, can be always expressed by this; this is the common known form of torque versus rotation relationship. Let us see over here T x is equals to G J times del theta del x; what is G? G is called shear modulus. This is polar moment of inertia in terms of the area is concerned; this theta is the displacement. So del theta del x will give me a rotational strain. That rotational strain if you multiply with respect to this G and J you will get the torque. That is the common relationship between the torque and the rotation. If I use this concept further, in my previous equation, what simplification we can do?

We can write the simplified form del 2 theta del t square equals to 1 by rho J del T x by del x. Let us look at our previous equation. This was the equation; torque rotation relationship which I am simplifying now del 2 theta del t square equals to 1 by rho j del T x by del x. If I differentiate this T x further with respect to x, what I will get? From this I can write that del T x del x is nothing but G J times del 2 theta by del x square. That we can now put over here, which will give us del 2 theta del t square equals to 1 by rho J times, GJ times del 2 theta del x square. Now, this J gets cancelled because J is non 0; the polar moment of inertia cannot be 0 for this rod what we have seen.

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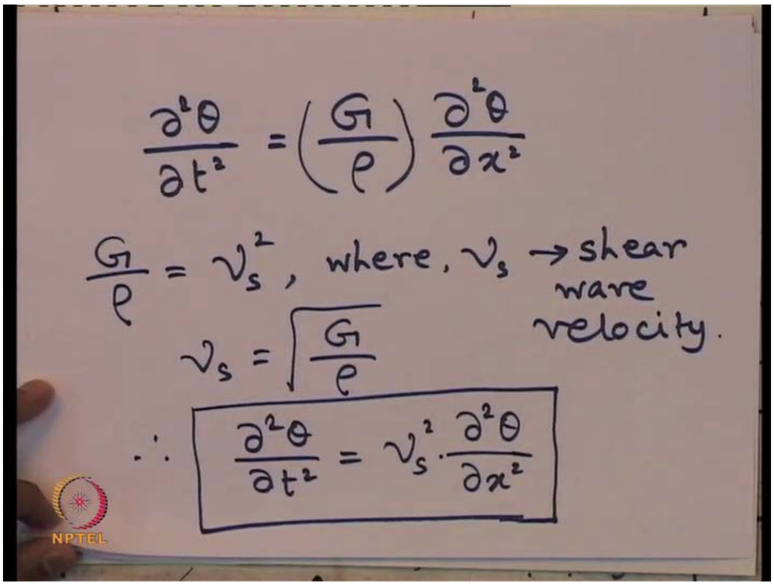

What is the simplified form? The simplified form of this equation now will be, let me write down, del 2 theta del t square equals to G by rho times del 2 theta del x square. Now this operator G by rho; this G by rho is defined as V s square, where this V s is called shear wave velocity. That means, V s is nothing but root over G by rho. What we can write that del 2 theta by del t square equals to V s square times del 2 theta del x square. This is our basic equation of motion for wave; what type of wave? Torsional wave which is passing through an infinite rod in one dimension. This equation of motion is the simplest form for the torsional wave. Now, if I combine this longitudinal wave and torsional wave what way we can write this form of equation?

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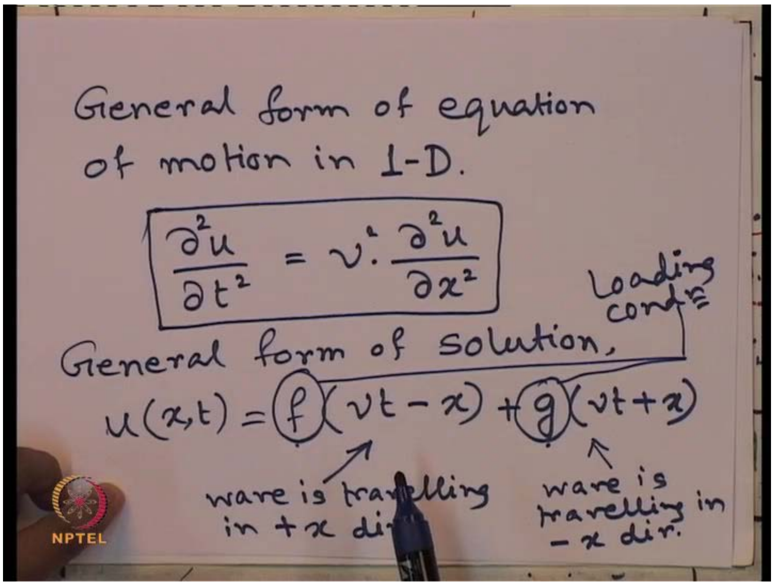

If I write down the general form of equation of motion in one dimension in 1-D, what we can write? It is del 2 u by del t square equals to v square times del 2 u by del x square. 1 dimension is that x dimension we have taken; and u can be the displacement for the case of longitudinal wave in x direction; and it can be theta in the case of torsional wave. V should be V p in the case of longitudinal wave; and it will be V s in the case of torsional wave. This also, u is the displacement for the case of longitudinal wave; and theta or rotation for the case of torsional wave. But basic form of equation is like this; that is you differentiate the displacement 2 times with respect to time which will give you the acceleration, which is related through this second differential of the displacement with respect to the space co-ordinate or the x dimension, which you have considered for the wave propagation. Through the operator, through the velocity of the media of the wave which is either p wave or longitudinal wave or the shear wave depending on what type of displacement you are considering.

So, what should be the general form? Let us see now the general form of solution of this equation. General form of solution of this equation can be expressed as u as a function of x and t can be written as some function of that V times t minus that direction x plus another function g times vt plus x. In this case, in this function this solution is when wave is travelling in positive x direction, then this is the solution we will get; that is velocity times t, it will get you the distance parameter. It will be a function of the distance parameter because when you are getting the solution of this second order differential in terms of u you are integrating it 2 times, you will get the solution in terms of the distance. That distance function will come as a function of this.

And this portion is nothing but when wave is travelling in negative of x direction. We have seen what is positive x direction and what is our negative x direction? And this form of f and g depends on the form of, the type of loading. This function f and this function g, they depend on loading condition. With that loading condition the different types of solution we will get for this governing equation of motion which we have arrived over here. We will now see next, that what will be the solution, if we suppose say a particular type, say a harmonic type of loading; simple harmonic loading next we will take to find out what will be the general form of the equation or general form of the solution for harmonic loading. That is why I said this functions g and f they will be dependent on the loading form. We will take a sinusoidal wave as if it is passing through a media we will see what will be the complete form of solution for an harmonic loading or for a sinusoidal function.

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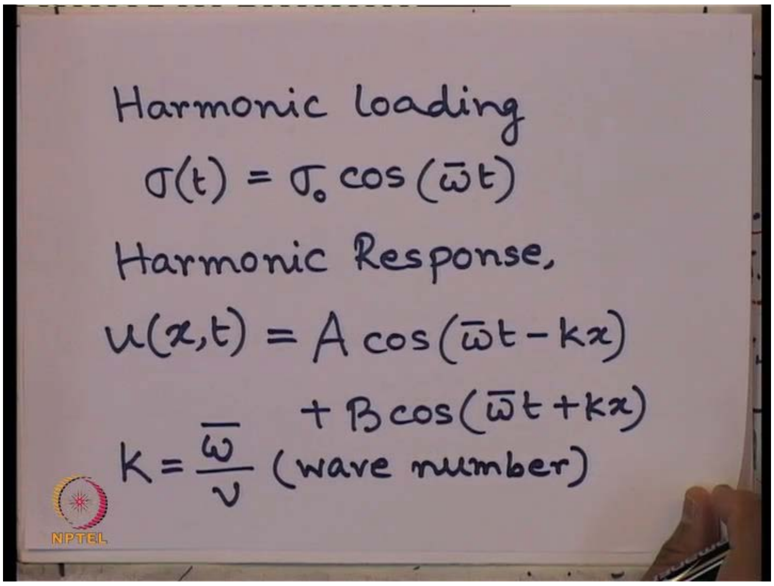

Now, let us see what will be the solution of this governing equation of motion for the case of harmonic loading. When we have some harmonic loading; that is suppose our sigma t is expressed in the form of sigma not cosine of, say omega bar times t. This is a form of harmonic loading; in that case the harmonic response can be represented as u of x t; that is the solution which we have seen in the previous lecture, what will be the form of the solution? For harmonic response the solution will be some function A cosine of omega bar t minus k x plus another constant cosine of omega bar T plus k x. That we have already seen; this one is for wave which is travelling in the positive x direction; and this is for the wave travelling in the negative x direction. And where from we will get these two constants A and B in the solution, from the boundary conditions; that is when the boundary conditions are known in that case using initial condition and boundary condition; we will see various cases now. Then we can estimate these constants A and B.

Now in this case, what is k? k is nothing but it is expressed as this circular frequency omega bar divided by V; V is nothing but the velocity. Let us see, we have already seen this form of general form of solution in this slide. When we have written this velocity if we express that circular frequency with respect to the velocity of the wave travelling in the media that ratio we can express in the parameter K which is known as wave number. This is called wave number and another parameter we can introduce over here; we will mention it now, that lambda.

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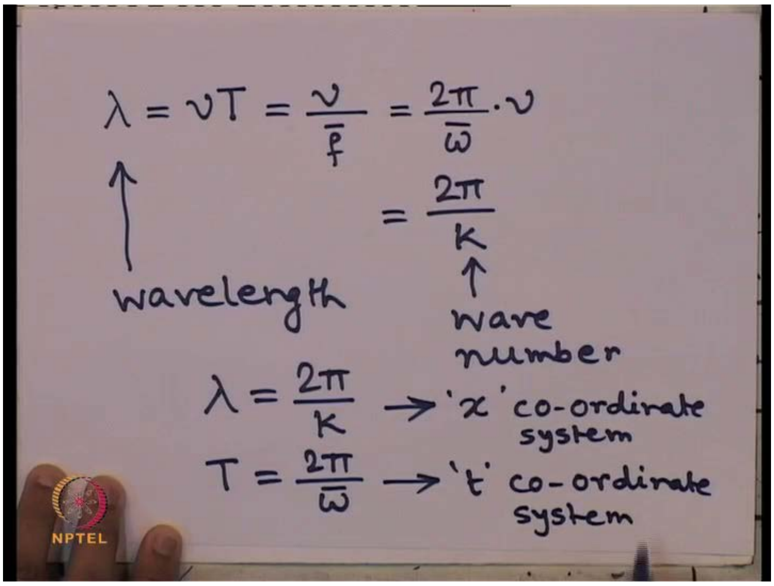

What is lambda? It is nothing but that velocity of the wave multiplied with the time period capital T, which we can write it as V by f that is the frequency. Time period we can always write by 1 by f , which is nothing but 2 pi by omega bar times that V which is on simplification 2 pi by K, because K is nothing but this omega bar by V. This lambda parameter is called wave length. This is wave number; and this is wave length. What we can see that lambda equals to 2 pi by K and T equals to 2 pi by omega. This relationship is, that is wavelength with respect to wave number it is in the x or displacement coordinate system; and this time period versus circular frequency, this relationship it is in t coordinate system or the time coordinate system. Now, putting these values over here what we can get on further simplification from this solution.

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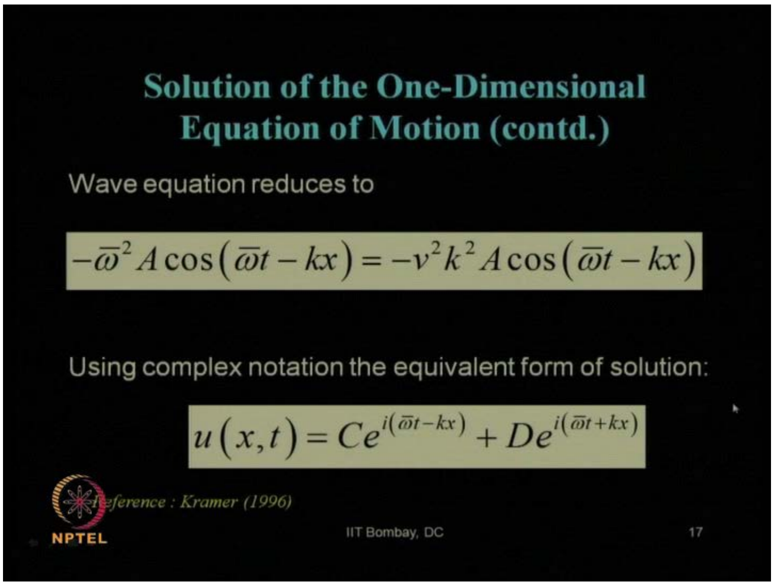

We can see now in the slide using this relationship; let us look at the slide over here. The wave equation further reduces to this minus omega bar square A cosine of omega bar t minus k x will be equals to minus v square k square A cosine of omega bar t minus k x. Now, this solution further using the complex notation the equivalent form of solution can be of this form; that is u of x t can be expressed as C that is another constant will come e to the power i omega bar t minus k x plus D another constant times e to the power i omega bar t plus k x. This is the complex number notation in terms of complex number, as we know that cosine sine functions etcetera, we can express very easily in this format. The same thing has been done. Only the suitable coefficient will get changed or altered like this.

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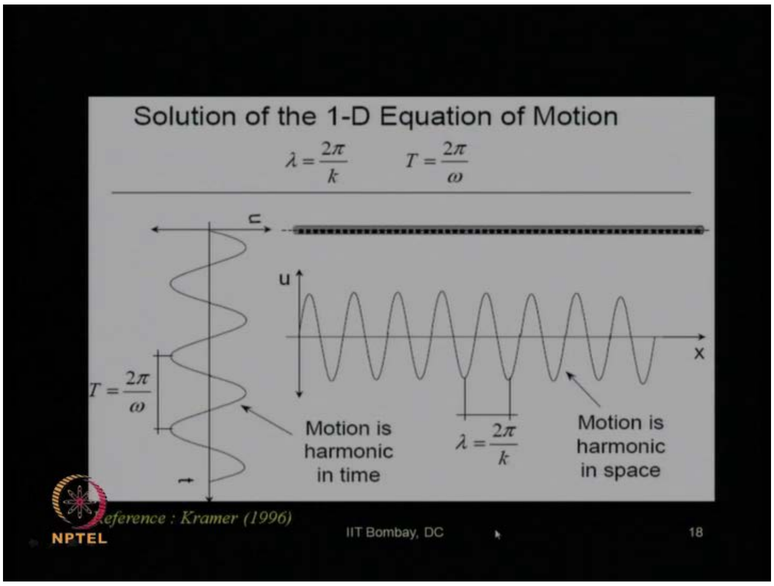

Now, what I was discussing to you, just now the equivalence of harmonic response in time and in space scale. Let us look at the solution of that one-dimensional equation of motion. This is the relationship in space in x coordinate system and this is the relationship in T or time coordinate system; equivalence between wavelength and wave number equivalence between time period and circular frequency. If you look at this plot, when you plot the response of that u that is the displacement in x direction; this is the axis, and this axis is T axis that is time axis we have the harmonic response in this manner. Harmonic response will be something of this form, where this time period T is nothing but define like this equals to 2 pi by omega.

Whereas if the same solution; if you want to plot it in the x direction that is we know the solution of u of x t. In this plot I have represented the solution u of T. And u of x, now I am plotting over here, what is shown over here, suppose if you want to plot the variation of u with respect to x. Harmonic response will be something like this because the solution which we have seen just now that is harmonic in nature both in time scale as well as in space coordinate system. That is why the response will be something like this where this is the definition of lambda that is the wavelength is equals to 2 pi by k, k is the wave number. That is the relationship or equivalence we can say between the time coordinate system and the space coordinate system; that is once we want to know the combined effect we should know the combined behavior like this, in time coordinate system as well as in space coordinate system.

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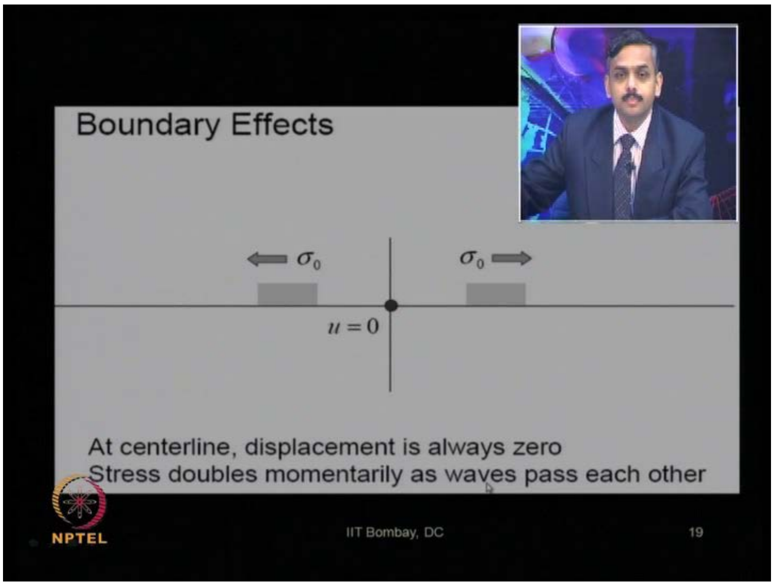

Now, let us come to the various boundary effects that is how to estimate these constants depending on various boundary conditions. Let us look at the slide over here. This slide says, at the center line, suppose we have a boundary like this, at the center line the displacement is always 0, what happens? If the displacement is 0, in this way the stresses get doubles momentarily as the waves passes each other that is two waves they are passing each other; this wave is travelling from here to here; this wave is travelling from this side to this side. At the intersection, at the centerline, what happens? the displacement u becomes 0, but this stresses that amplitude of stresses sigma naught they gets doubled.

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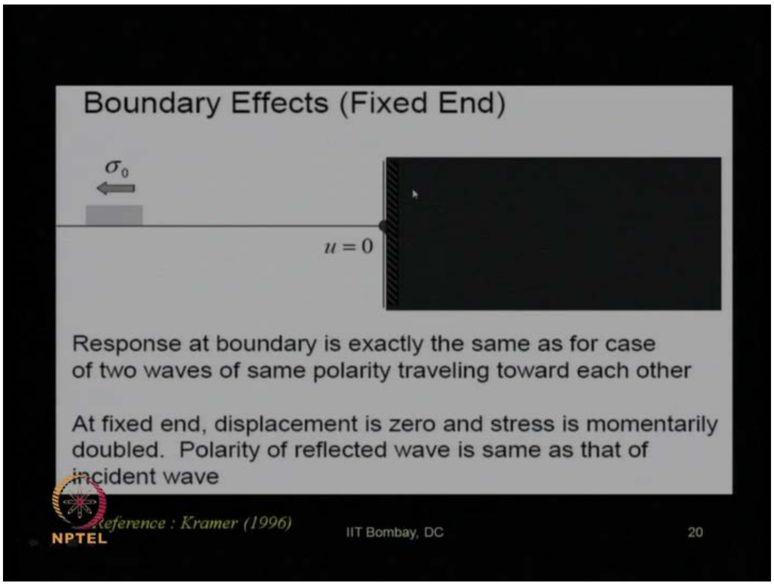

Another boundary effect, let us see when we have the fixed end; suppose wave is travelling and this is the fixed end of a body or that infinite rod which we are considering for this one-dimensional wave propagation, what will be the response at the boundary? It is exactly the same as the case of two waves of same polarity travelling towards each other. That is the last slide or previous slide, but we have seen the same effect will come at the free fixed end also that is u will be 0, but at the fixed end this displacement is 0 that is always the boundary condition. Because whatever it happens at the fixed end there should not be any displacement; that is the criteria for a fixed end.

So, if the displacement is zero, what happens to the stresses? They come and go back. Stresses is momentarily doubled, the stress becomes doubled. Polarity of the reflected wave is the same as that of incident wave. How the stresses get double at the fixed end? suppose it is having an amplitude of sigma naught, it comes, travels, comes to the fixed end that is an incident wave that is which comes and falls on this boundary or fixed end then get reflected back, with the same magnitude or same amplitude of stress. That is why at this fixed end what happens? It gets doubled, because if you see at the solution of sigma or u if you put u equals to 0 at this boundary for a wave travelling in a positive x direction and negative x direction, if you sum them up the stresses, it gets doubled; the amplitude at this boundary where u equals to 0. Because incident wave completely comes back and goes back as a reflected wave.

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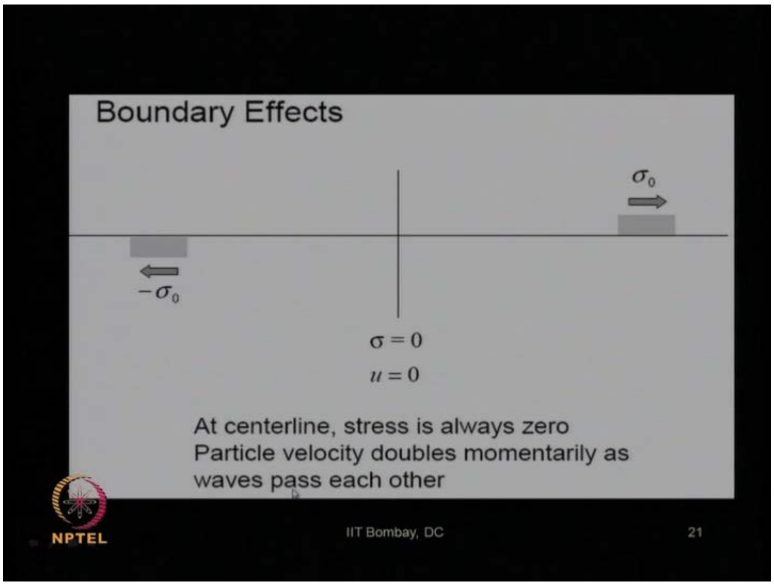

Let us see another boundary effect suppose there is a center line; where say, the stress has to be always 0, what will happen? Particle velocity doubles momentarily as the waves pass each other. If suppose there is the center line where the stress condition has to be 0 and displacement also has to be 0; for that condition the velocity particle velocity doubles up, we will see where these different types of boundary exists in practice. Another boundary effect is the free end.

Let us see what happens in the free end. In the free end response at the boundary is exactly the same as the case of two waves of opposite polarity travelling towards each other, these opposite polarity not of same polarity; when same polarity travels the different thing happens we have discussed. So, at the free end what happens? Stress has to be 0; at free end stress release will be there. Stress cannot be there. So, stress has to be 0, but displacement is momentarily doubled. Polarity of the reflected wave is opposite of that incident wave; we will see these things through mathematical expression also and through understanding of practical examples.

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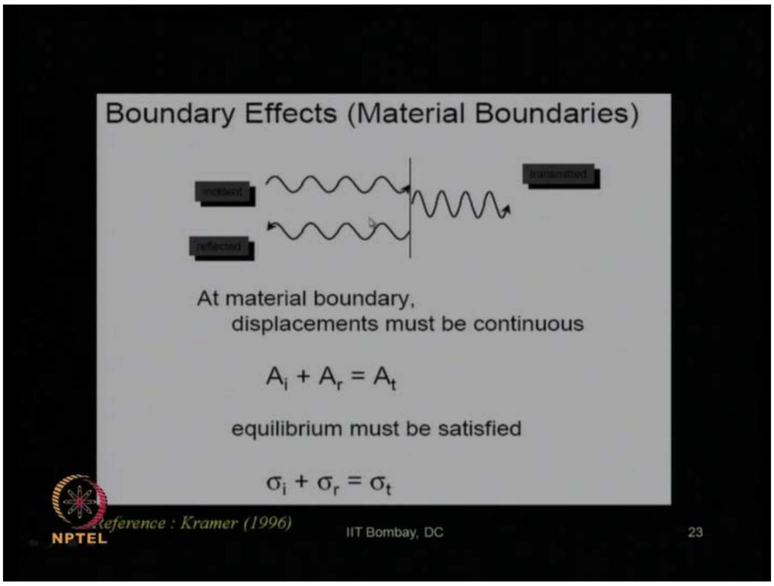

Now, let us explain it further the material boundary effect. Suppose we have two materials say, this is material one; this is material two. There is the boundary between two material time. Many a time it happens in our geotechnical engineering or geotechnical earthquake engineering, when we discuss there can be say, a rock and a soil; or even within a soil say dense sand or loose sand or stiff clay or soft clay or sand clay several such combinations of two materials can happen in practice. What happens? When the wave travels in such kind of different material and reaches the boundary, what happens to the wave? This the original wave which we call as incident wave; suppose this incident wave is coming from this material 1 and when it hits the boundary of the 2 material some of the wave will get transmitted in the second material and some of them will get reflected back in the old material or same material.

Eventually what are the waves we are handling at this boundary? One is incident wave in the main or material 1; one is transmitted wave or refracted wave in the material 2 and another is reflected wave in the same material 1 where from the incident wave comes from. To maintain the compatibility at this material boundary, what are the conditions to be satisfied? At material boundary displacements must be continuous; there is the displacement compatibility. At this boundary the displacement compatibility has to be maintained and stresses must be in the equilibrium. These are the two conditions always we have to apply at any boundary. So, if the displacement compatibility has to be there what we can write whatever amplitude of incident wave A i and whatever amplitude of this reflected wave A r summation of that should be equals to the amplitude of the transmitted wave. Then only we can maintain the displacement compatibility in this boundary between two materials.

Again if we want to maintain the equilibrium of the forces or equilibrium of stress, what condition should be satisfied at this boundary? That is the stress amplitude due to the incident wave and the stress amplitude due to the reflected wave; there algebraic sum should be equals to the stress amplitude due to the transmitted wave. Then only we will say that at this boundary the stress equilibrium is also satisfied. These are the two basic conditions we always need to satisfy the boundary, one is stress equilibrium and displacement compatibility.

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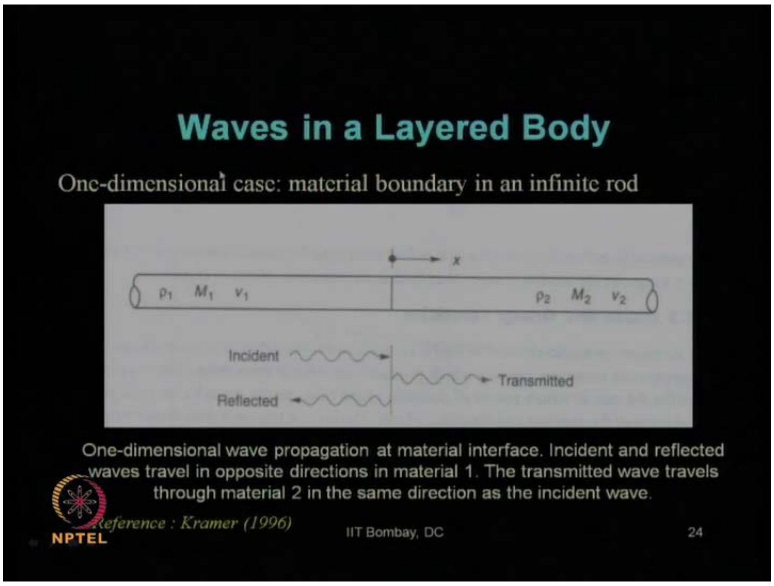

After satisfying that what we can see? When waves travel in a layered body; and still remember we are talking about one-dimensional wave only. We are talking about one dimensional wave propagation in a layered media. For one-dimensional case material boundary in an infinite rod is represented through this picture. We have material 1 as we have discussed just now and this is material 2. Suppose our x co-ordinate we are defining from this boundary onwards; this our positive x; this is our negative x; this boundary demarcates as the point of x equals to 0; this is the incident wave from material 1; this is the transmitted wave in material 2; this is the reflected wave in material 1 again.

So, what are the basic properties of material 1 rho 1 is the density of the material 1, M 1 is the constraint modulus of material one, and v 1 is the velocity of wave. Suppose if it is a longitudinal wave it will be p wave v p 1 in this media and similarly for second media the corresponding parameters are rho 2, M 2, v 2 respectively. This shows one dimensional wave propagation at material interface; incident and reflected waves travels in the opposite direction which is quite obvious, suppose incident wave comes in this direction reflected wave has to go back in this direction that will help us to write the solution of these equations later on. Because we know dependent on positive x direction and negative x direction our equation form will change. So, the transmitted wave travels through material 2 in the same direction as the incident wave.

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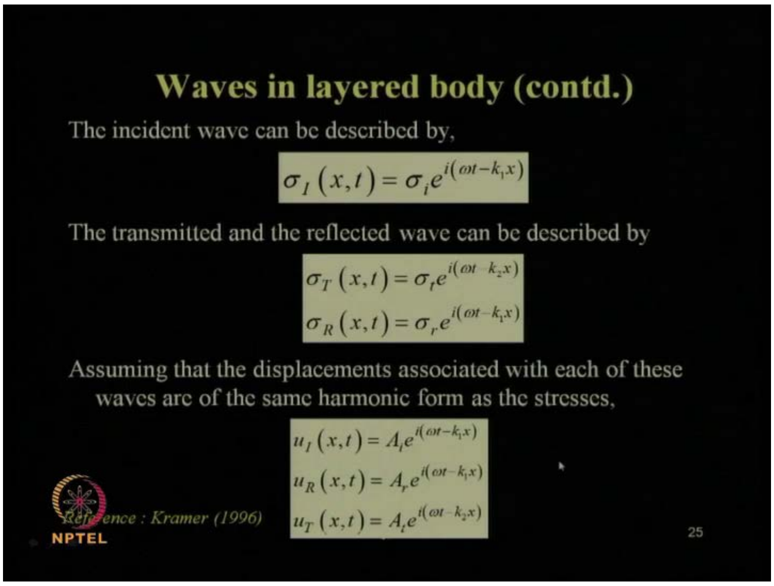

What we can see for the incident wave? Suppose if we describe it in the form of a complex number or say we are representing it in the form of a trigonometric function or in a harmonic fashion. That stress of the incident wave sigma I which is a function of x of t that we are representing as sigma i is the amplitude e to the power i omega t minus k one x, why minus? Because it is traveling in the positive x direction; and why k 1 we have used, it is in the material 1. It corresponds to properties of material 1. That will be nothing but omega 1 by v 1, whereas for the transmitted wave and reflected wave how we can describe these things. Transmitted wave sigma T x of t can be represented as sigma t e to the power i omega t minus k 2 x. So, this is for the material 2. Whereas, sigma R x of t. we are using the reflected wave. So, that is why as we are mentioning it is the reflected wave already the direction we have taken care of in this reflected, because we have already mentioned it is going in the opposite direction of incident wave. This sigma R already takes care of the direction. That is why we are further using minus k 1 x not as a plus k 1 x.

Now, assuming that the displacements associated with each of these waves; that is incident, transmitted and reflected waves are of the same harmonic form as that of stresses; that is displacements also we are representing in the harmonic fashion with u of that is incident wave displacement function u of x t of I is written as A i, A i is nothing but the amplitude of incident wave expressed in this fashion. u R x t that is the reflected wave displacement function is A r that is the amplitude of the reflected wave with e to the power i omega t minus k 1 x and for the transmitted wave u of T x of t is A t; that is the amplitude of transmitted wave e to the power i omega t minus k 2 x, this is in the material 2 that is why k 2.

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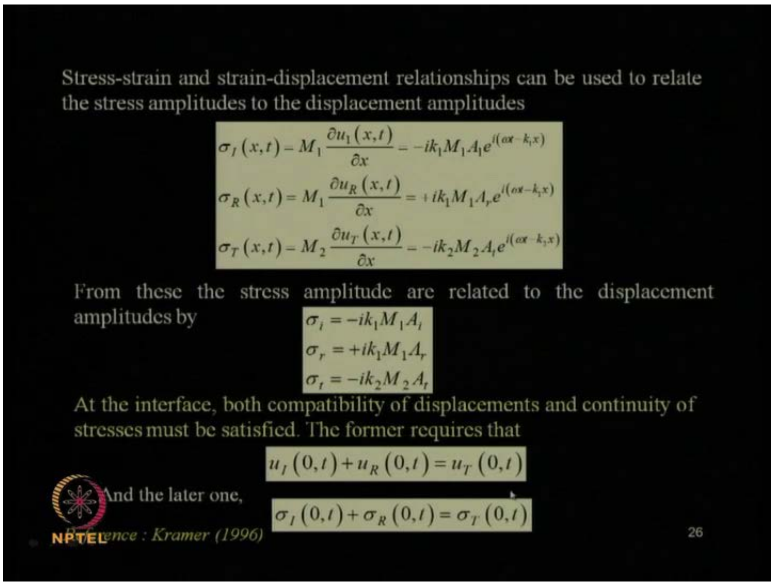

Now, what further we can write; that is now let us apply the stress-strain and strain-displacement relationships to relate this stress amplitude to the displacement amplitude that is stress strain relationship let us now apply. What we can get? This stress for incident wave that is nothing but stress can be written as modulus times strain. That is what we have written this is the modulus; which modulus we have to use for the incident wave, modulus of material 1; that is why M 1. What is the strain in that incident wave, that is nothing but what was our u i, that we have to differentiate with respect to x that will give us the strain for the incident wave. On further simplification of this expression of u of I, this is u of I, minus i k 1 M 1 A i e to the power i omega t minus k 1 x. So, just putting this expression of u I like this; and if you do a partial derivative of this with respect to x, x only you are doing the derivation not with respect to time. It will give us this component; because this k 1 comes out. So, i times k 1, that is why minus i k 1 came out M 1 was already there, A I was the amplitude; this is A I. Remember this is A I; this is M 1; this is k 1; this is A I and this function.

Now, when we are talking about the reflected wave; for reflected wave what we can express? It is again the stress is equal to the modulus times strain, modulus in the reflected wave media is M1. Now, reflected wave displacement function you have to differentiate that partial differentiation with respect to x. Now, as I said that reflected wave already takes care of the negative direction, so when we are using this minus k 1 x it comes out here minus i k 1, but there is an automatic negative sign, because it is going in the opposite direction. So, that makes it plus, is it clear. Why it become plus, though it is coming from here minus i k 1, because this reflected wave is in the opposite direction of this incident wave. That is why it has to be with opposite sign of this which is hidden in this reflected wave amplitude. Now for the transmitted wave sigma T x of T is nothing but modulus times strain. Now what is the modulus for transmitted wave? That is the modulus of the second material M 2; and the partial differentiation of that u of t with respect to x which gives us this value minus i k 2. So, k 2 minus i k 2 comes from here M 2 times A t times e to the power i omega t minus k 2 x.

From these the stress amplitude and the related displacement amplitudes can be written as; so stress amplitude what we had seen in the previous slide, so these are the stress amplitude; for incident wave, transmitted wave and reflected wave. And what are the displacement amplitudes? These are the displacement amplitudes form this stress strain relationship; this, this and this. That is what it is written, that is the correlation between stress amplitude with respect to displacement amplitude for incident wave, for reflected wave and for transmitted wave. After getting this correlation what we need to do? Now we have to use the boundary condition; that is at the interface of two material 1 and 2; both the compatibility of displacement and the continuity of stresses has to be satisfied.

So, what does it mean; that means, that whatever is u of i; that is incident wave displacement at x equals to 0, because that is the co-ordinate of the boundary at x equals to 0 of any time plus that reflected wave displacement u of R at same x equals to 0 at any time equals to, should be equal to the displacement of the transmitted wave at x equals to 0 of time. So, that displacement boundary condition has to be satisfied. Similarly the stress boundary condition also has to be satisfied at x equals to 0 point, because that is the boundary. So, x equals to 0 that incident wave stress plus reflected wave stress should be equals to the transmitted wave stress. So, with this we have come to the end of today's lecture, we will continue further in the next lecture.