Geotechnical Earthquake Engineering

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Module - 5

Lecture - 17

Wave Propagation

Welcome to today's lecture of NPTEL video course on geotechnical earthquake engineering. Let us look at the slide here; for this video course geotechnical earthquake engineering, a quick recap of what we had learnt in our previous lecture. In the previous lecture, we were going through the module number 4, which is strong ground motion.

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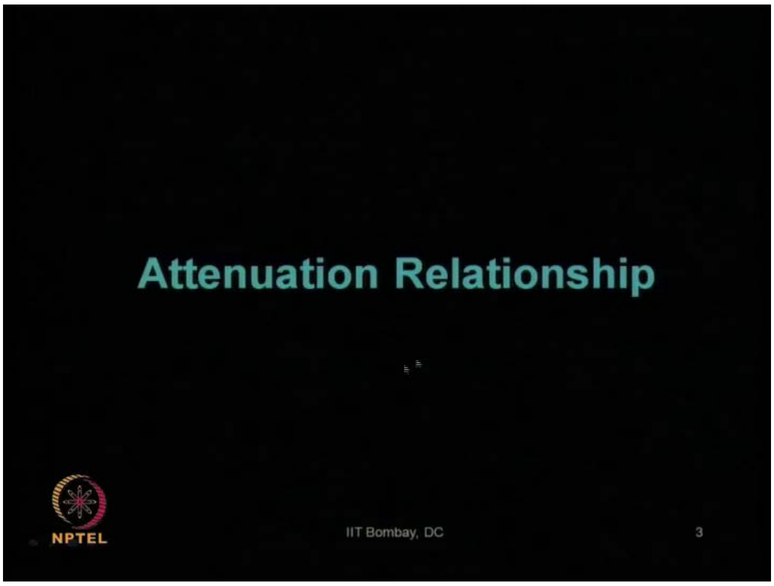

So, in that, let us see in the slide, we were talking about various attenuation relationships available. Earlier to that lecture, we discussed about the development of the attenuation relationship worldwide, mainly in California region, where it started with the work of Campbell and then Boore et al.

Further, we continued the attenuation relationship developed for India, because we mentioned that it is not correct, if somebody uses the attenuation relationship of an

Indian site for future designer construction with attenuation relationship available for other countries. They must use the local attenuation relationship to take care of the local fault characteristics, earthquake behavior, geology of the region, and soil behavior and many other local concerns. So, that is why it is always advisable to use the local attenuation relationship for the design, so that correct estimation for the earthquake hazard etcetera can be developed. In that connection, we have seen that not only a particular region of attenuation relationship is important.

Suppose, we have given the example earlier, for India, if somebody wants to find out the seismic hazard or attenuation relationship or want to do any earthquake resistant design at a particular site, say in Mumbai, they should not use the attenuation relationship available for Himalayan region or north east of India. They should use the attenuation relationship available for the peninsular India. In peninsular India, as we know, we can consider Mumbai city is placed in that region. So, that is why use of attenuation relationship for particular design problem and estimation of input data is very important and essential.

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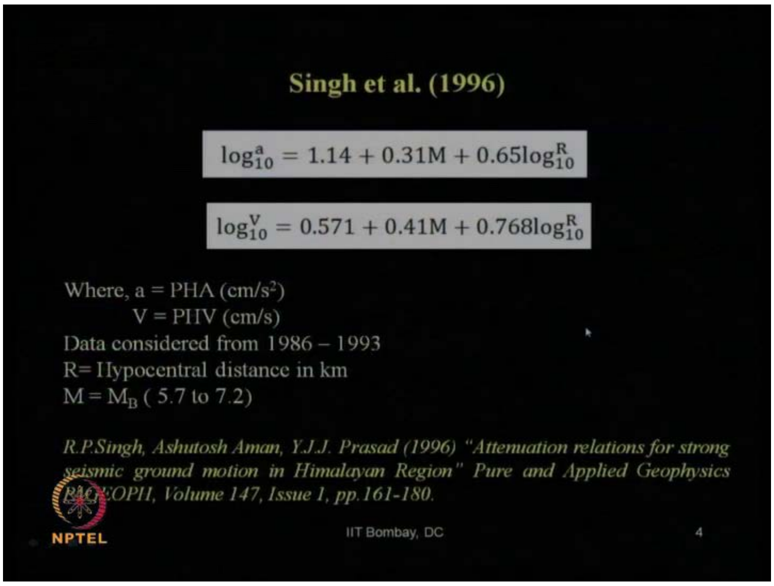

So, what we had seen earlier in the previous lecture is that various attenuation relationships, which are available in the literature that are given by Singh et all in 1996. I mentioned this is the work done by professor R.P.Singh and his research group from IIT, Kanpur.

So, this is the attenuation relationship, which can be used to estimate the peak horizontal acceleration. Units are given over here because, these are empirical relationships. So, that is why we have to be very very careful about what type of unit has been proposed while developing that empirical relationship. Another relationship for the peak horizontal velocity is also proposed, which is in centimeter per second. They considered the earthquake data for a period in between 1986 to 1993 and this was developed only for the Himalayan region.

So, these attenuation relationships for acceleration and velocity are valid only for Himalayan region and that to for the data considered within this period. What was the range of the magnitude for which this equation is applicable? They use the body wave magnitude in between 5.7 to 7.2. So, that is the range of the magnitude for which this proposed empirical relationships are valid. Also, in this equation, the r is mentioned as the hypo central distance in the kilometer unit.

So, knowing all these constraints, we should know the limitations of using this equation when somebody wants to further extend it or use it for the estimation of the attenuation relation for the strong ground seismic motion in the Himalayan region of India for PHA and PVA.

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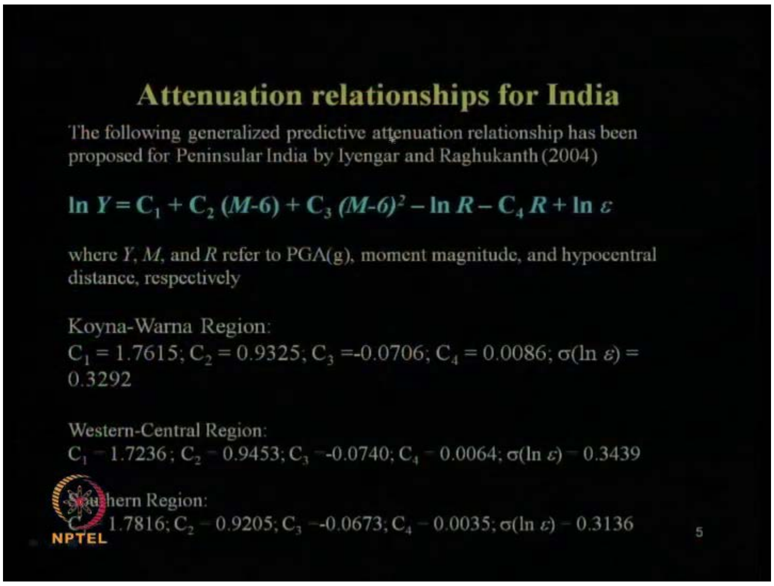

Then, we had seen in the previous lecture about the attenuation relationship proposed by Iyengar and Raghukanth in 2004. As I said, this is the work done by a professor Iyengar at IISC, Bangalore, with his PhD student at that time Dr.Raghukanth, who is now faculty at IIT, Madras. They developed the attenuation relationship for peninsular India. So, if somebody wants to do any earthquake related design, they have to use this equation and not the Himalayan region attenuation relationship or north east region etcetera or the Californian earthquake attenuation relationship for obvious reasons.

So, this is the basic form of the attenuation relationship, which they have proposed. As we know, this is the basic form as proposed by Boore et al while developing the attenuation relationship for California region. So, in this equation, Iyengar and Raghukanth mentioned this value of Y is nothing but, the PGA, that is peak ground acceleration in the G unit and m is the moment magnitude and r is the hypo central distances in kilometer unit. This C 1, C 2, C 3, and C 4 are the various coefficients which are different for different regions within the peninsular India also.

So, from the considered earthquake data, they have proposed for Koynawarna region. These are the use of various coefficients; whereas for the western central region, these are the values and for southern region, these are the values. So suppose, somebody is interested to do any earthquake analysis and design in Bangalore city, then they should use these coefficients. So like that, we have to be region specific and area specific, based on the proposed various available attenuation relationships.

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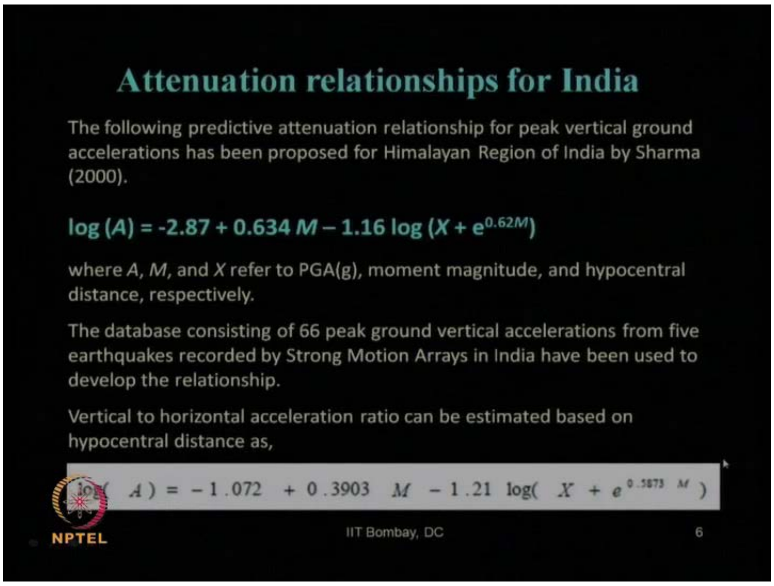

Then, we had also seen the work done by Sharma in 2000, where he proposed the attenuation relationship for peak vertical ground acceleration only for the Himalayan region. Before that, he worked for the horizontal peak ground acceleration also. He is a professor at IIT rookie in earth quick engineering department, professor M.L. Sharma. So, this is the basic form of the equation he had proposed, where a is nothing but the PGA. In this equation it is nothing but, the vertical component and m is the moment magnitude of earthquake and X refers to the hypo central distance in kilo meter unit. Now, how many data base or data points he has used to derive or arrive at his empirical relationship? 66 PGA values in the vertical directions he has used from the 5 recorded earthquake motions. Also with respect to the vertical to horizontal acceleration ratio, this is the equation proposed by him.

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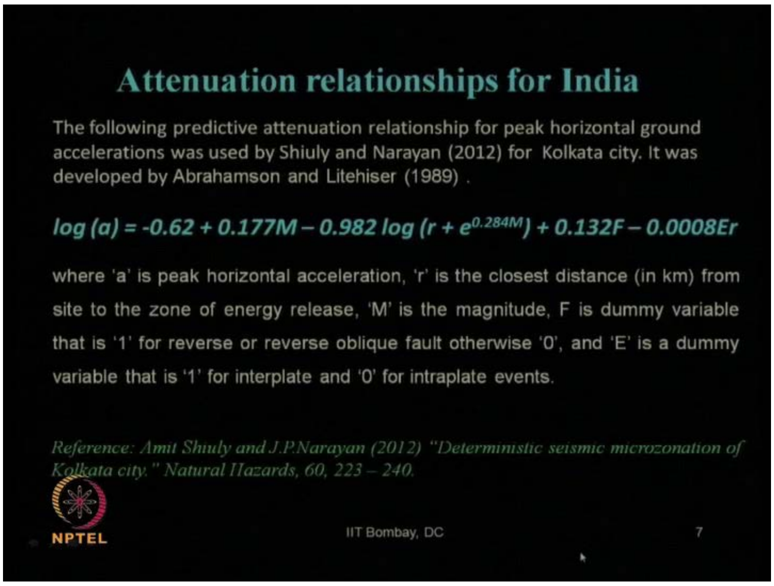

We had also seen from our previous lecture that, for Calcutta region, it is eastern part of India for Kolkata city, Shiuly and Narayan in 2012, have developed or proposed the attenuation relationship for peak horizontal ground acceleration given by this expression. Where, a is the peak horizontal acceleration, and r is the closest distance in kilometer unit from the site to the zone of energy release. That means, it is nothing but the hypo central distance. M is the magnitude and other parameters like few dummy parameters like F and E, they have proposed to take care of the type of the fault and the what type of plate events it refers to.

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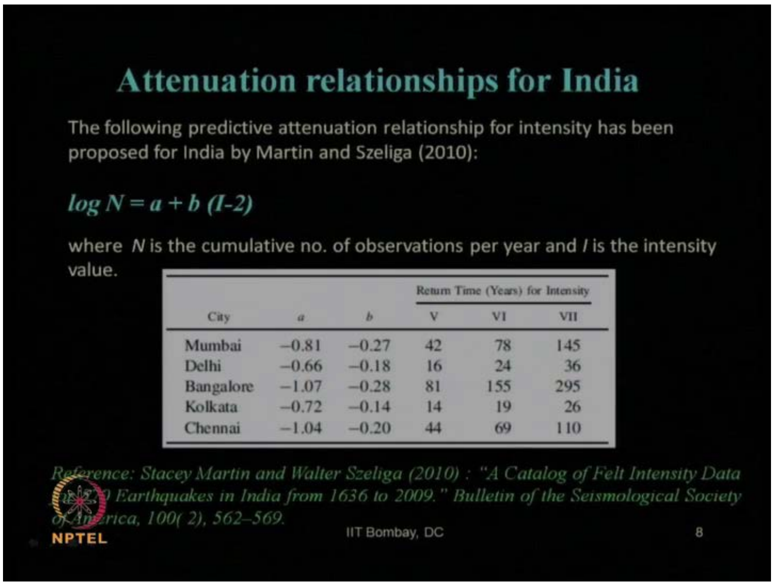

Other than this magnitude based attenuation relationships proposed for India, we had also seen in our previous lecture that some researchers have proposed attenuation relationship for intensity based calculations also. That is, how the intensity of earthquake attenuate with respect to the distance was proposed. As we had seen earlier, Martin and Szeliga in 2010 had proposed the intensity based attenuation relationship in this form of equation, where N is the cumulative number of observations per year and I is the intensity of earthquake which can be obtained. So, for various cities like Mumbai, Delhi, Bangalore, Kolkata, and Chennai, they had proposed various values of these coefficients a and b and what are the various numbers of return time period for the value of seismic intensity of 5, 6 and 7.

So, they considered the earthquake data points of about 570 earthquakes between the period of 1636 to 2009. I mentioned in my previous lecture that, in olden days, whatever earthquake data we have, historical earthquake data, those are mostly intensity based. That is, how the damages occurred at a particular location due to earthquake or how it was felt that time by the people who observed it or who felt that earthquake. But later on, in recent days, these are mostly magnitude based, which of course, we can correlate with respect to the intensity based scale.

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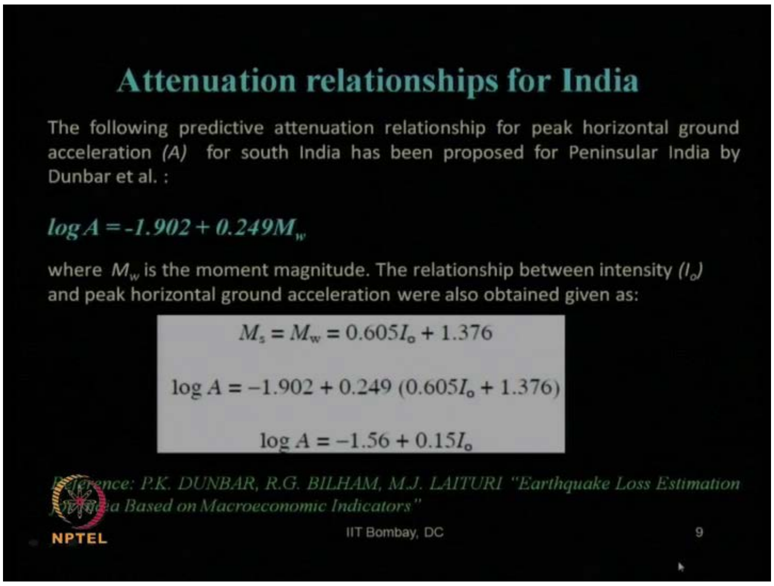

We had also seen the attenuation relationship proposed by Dunbar et al. This is the form of attenuation relationship for peak horizontal ground acceleration for South India only. So, this is applicable only for south India. That is, the attenuation relationship for peak horizontal acceleration in terms of moment magnitude. Further, they had proposed this moment magnitude in terms of the intensity based calculation. So in turn, they have given another equation, where one can get the attenuation relationship of acceleration with respect to the intensity based calculation.

But, what we have mentioned, what about the major limitation of this equation that, in this equation they did not propose any distance based. That is, how far is the earthquake energy release point, that is, the hypo central distance. It is another very important parameter which must come into any type of attenuation relationship. Because, the attenuation relationship means that particular parameter which we want to analyze how it is decreasing with the increase in the distance from the earthquake originated point or the hypo central point. So, that distance is a very important parameter, which is not reflecting in this equation.

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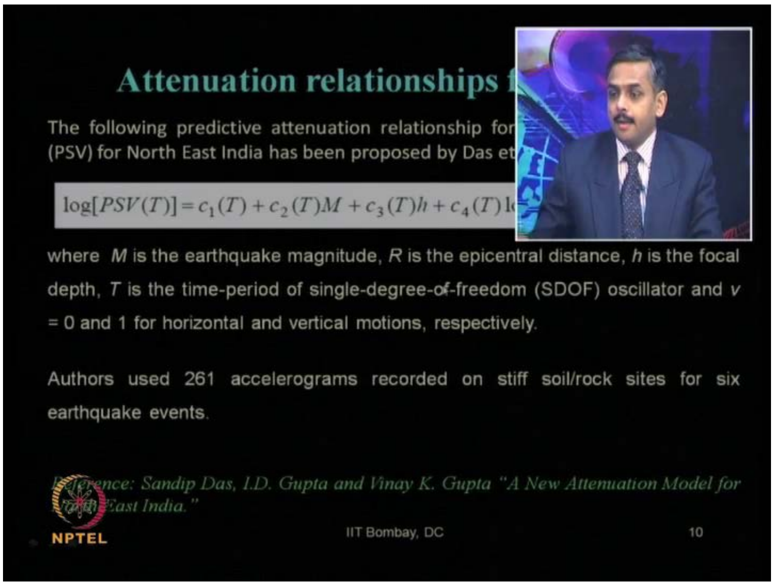

Then, we had seen in our previous lectures as proposed by Das et al, that this is the attenuation relationship for pseudo spectral velocity. We have already derived what is pseudo spectral velocity, complete derivation and its definition etcetera. They had proposed in terms of moment magnitude, epicentral distance, focal depth and the time period based on the single degree of freedom oscillator system, through which we actually arrive at any kind of spectral acceleration or spectral velocity response.

So, about 261 accelerogram recorded on stiff soil and rock sides for 6 earthquake events. That was the total data points, which was considered by them to arrive at this attenuation relationship for pseudo spectral velocity.

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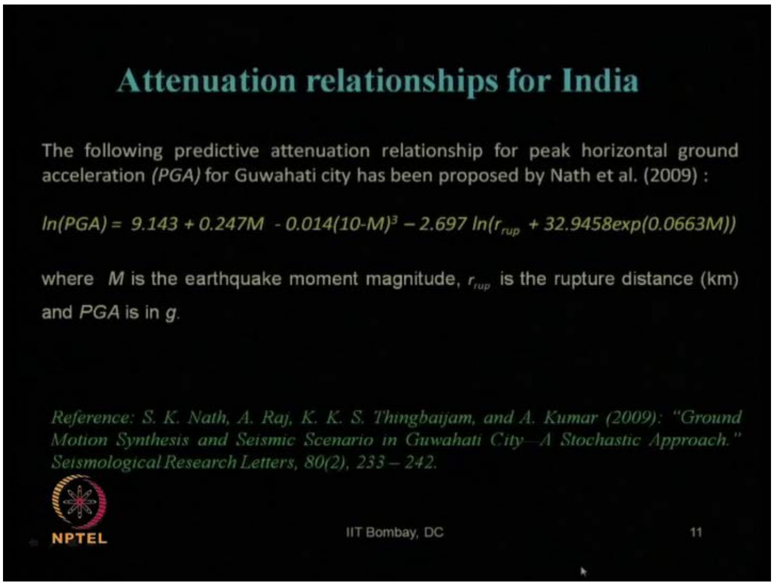

Then, we had also seen for India, the attenuation relationship proposed by Nath et al. As we can see in this slide in 2009, Nath et al proposed for peak horizontal ground acceleration attenuation relationship only for Guwahati city. So, they considered the earthquake response of Guwahati city and used the stochastic approach to arrive at this attenuation relationship, where M is the earthquake moment magnitude and r r u p is nothing but, the rupture distance in kilometer unit.

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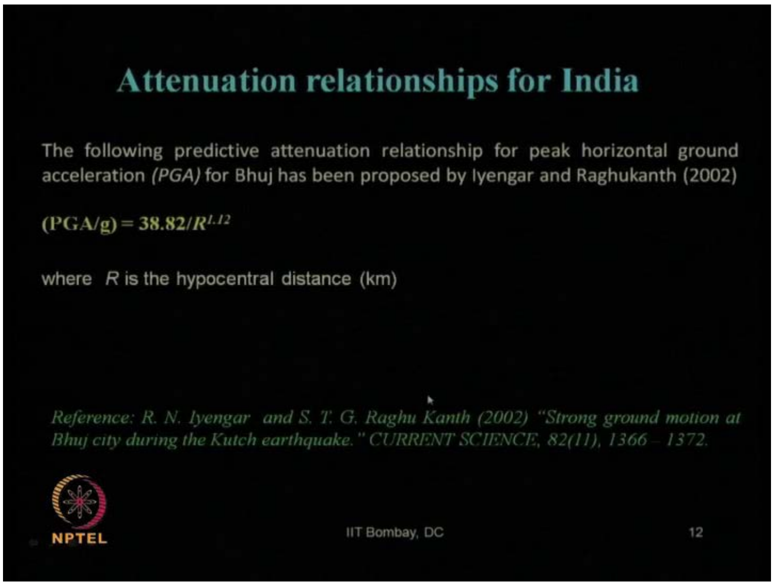

Also, further we had discussed in our previous lecture that, the attenuation relationship proposed for the peak horizontal ground acceleration proposed by Iyengar and Raghukant in 2002 is for only Bhuj city. So, this attenuation relationship is also very case specific. That is why you can see this is only for a particular earthquake. That is, for 2001 Bhuj earthquake, they had proposed this attenuation relationship. Hence, in this equation, if you look carefully, only the hypo central distance is a parameter or a function through which the attenuation relationship of PGA in the horizontal direction was proposed. There is no magnitude based calculation since it is developed for a particular magnitude of earthquake. So, that is why, obviously, there is no reason why that magnitude scale will come as a function in this proposed equation.

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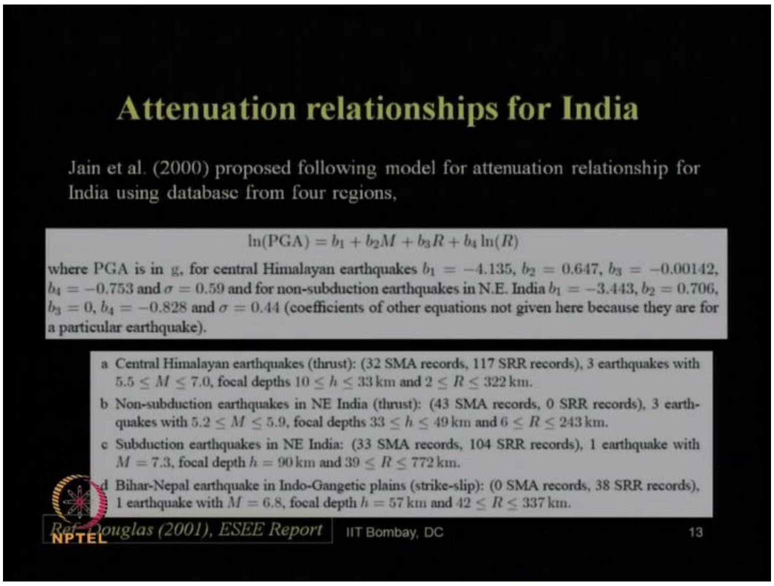

Then, we had also discussed the attenuation relationship proposed by Jain et al in 2000 for the peak horizontal earthquake acceleration. So, in this form of equations, various coefficients b 1, b 2, b 3, and b 4 were proposed for four basic regions. That is, central Himalayan region of earthquake with thrust type of fault, non subduction zone of earthquake in north east part of India with thrust type of fault, subduction earthquake in north India which is for only one specific earthquake and Bihar Nepal earthquake in Indo Gangetic plain for strike slip type of fault. That is also developed for only one specific earthquake.

So, other two regions were developed for three earthquakes with so many numbers of recorded data points. So accordingly, you can see over here, the coefficients b 1, b 2, b 3, and b 4 are different for central Himalayan region and for north east region. So, for these two cases, case a and case b, they had proposed various coefficients b 1, b 2, b 3, and b 4. Their values are of course different, but for c and d, they have not proposed or shown any coefficients over here because, those are case specific or only for a particular earthquake. So obviously, the variation there is not in terms of the earthquake magnitude is concerned.

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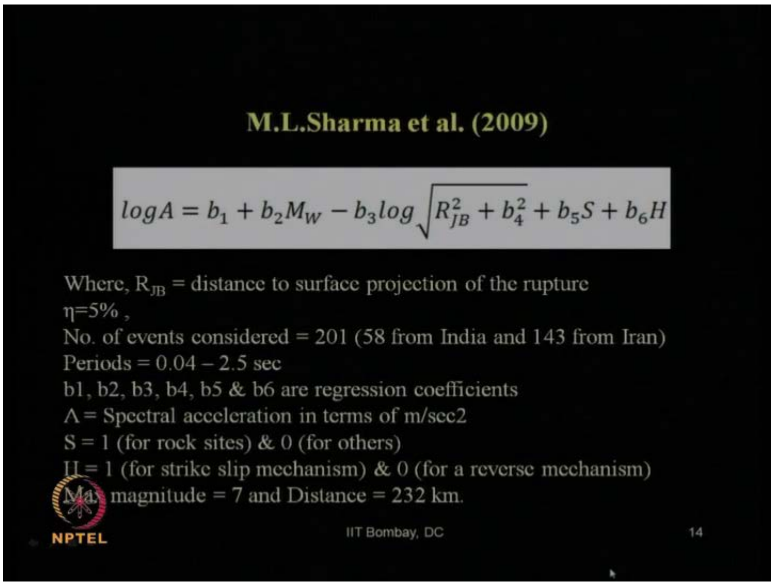

Then, we had also seen how Sharma et al in 2009 proposed the attenuation relationship. But, we cannot say that the attenuation relationship can be used for India because, their data points or the events that they considered to develop this empirical attenuation relationship for acceleration, were taken partly from India and partly from Iran actually, more from Iran and less from India. So obviously, this attenuation relationship cannot be said that it is completely the Indian attenuation relationship or should be used only for India.

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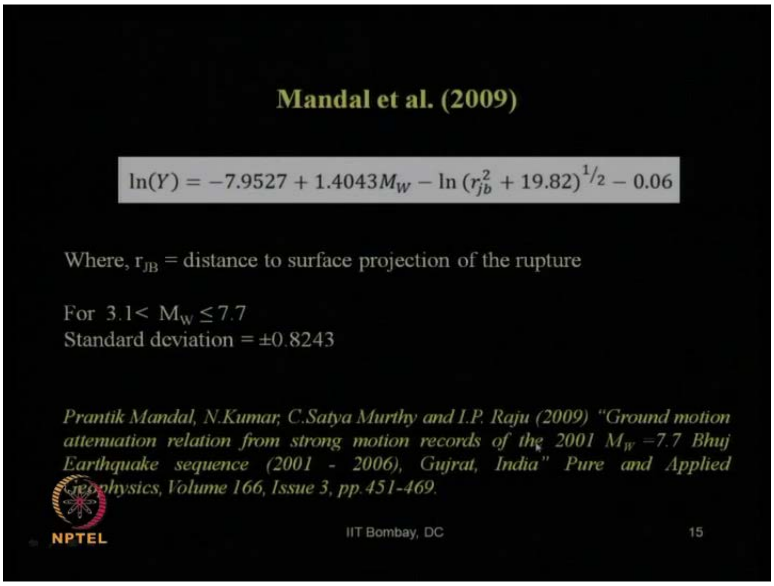

Also, we had seen the attenuation relationship proposed by Mandal et al in 2009. Prantik Mandal and the research group as mentioned over here. So, this is for Bhuj region of earthquake in between 2001 to 2006. So, that is the period they considered all the magnitude of earthquake, and their values were within the range of 3.1 to 7.7. The value r j B means distance to the surface projection of the rupture point and this is the proposed equation of attenuation relationship for the earthquake acceleration PGA.

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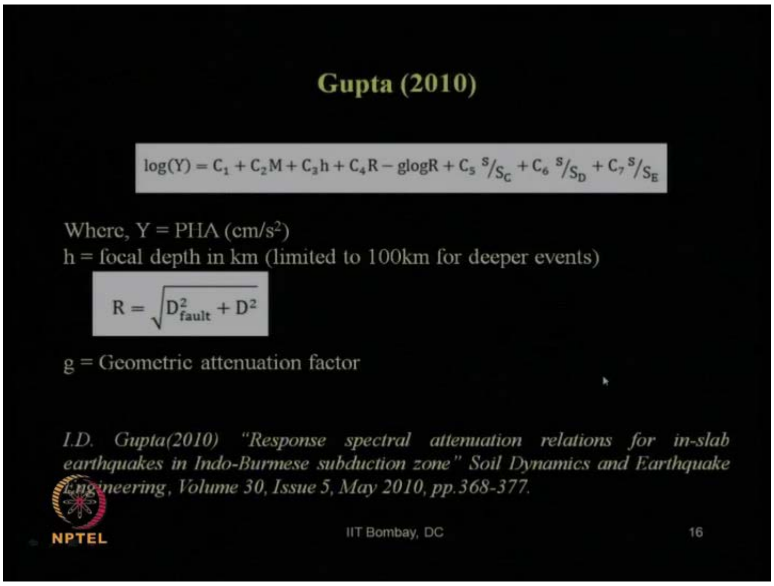

Also, we had seen the proposed attenuation relationship of Gupta in 2010 for PHA using this empirical relationship, where h is the focal depth and g is the geometric attenuation factor over here. So, this is the paper from which this information has been taken for Indo-Burmese subduction zone of earthquake in trust slab earthquake events.

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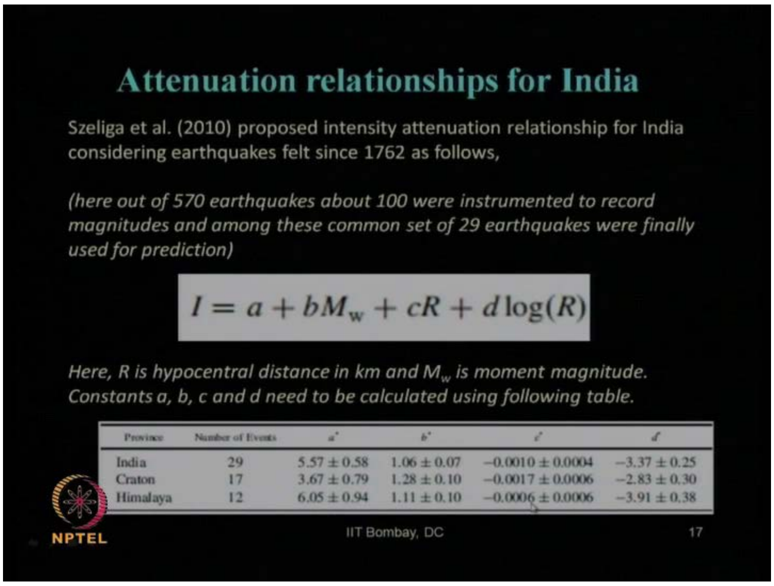

Then, we had also seen that attenuation relationship in terms of intensity. That is, already we obtained two intensity based attenuation relationships. One was directly intensity based and there was no effect of the magnitude. We mentioned on that, because it is intensity based, but there was no effect on the hypo central distance also mentioned in that intensity scale. That is why we said that was a drawback.

Now, let us see what Szeliga et al in 2010 had proposed for intensity based attenuation relationship. This is the proposed equation, where you can see it is a function of the magnitude as well as the distance. So, this is, I will say a better equation when compared to the previous intensity based attenuation relationship. They had considered all the earthquakes felt since 1762 up to 2009. Nearly 570 earthquakes, where about 100 of them were instrumented earthquakes. That is, recent earthquakes, whereas the remaining other earthquakes were non instrumented. That is why, those were mostly intensity based, based on the amount of damages occurred and how it was felt.

A set of 29 earthquakes were finally used for this development of the empirical relationship of the prediction, where R is the hypo central distance in kilometer unit, M w is moment magnitude and a, b, c, and d are the various constants, which can be obtained from this chart as proposed by them for different regions. They had proposed for entire India, for creation region and for Himalayan region. How many number of events they considered to develop or to arrive at those coefficients that is also mentioned. As I told you earlier, this is an ever evolving or changing process. Because, after 2010, suppose if somebody is interested to arrive at this attenuation relationship or want to modify it further up to today's date, say 2013, up to 2013 they have to take up whatever earthquake data is available for this region. Then automatically, if the number of events increases, these coefficients and their standard deviations etcetera will change. So, those who will be watching this video mode of NPTEL, say after 5 years or 10 years, they will understand that these equations are valid on those date only.

There is quite a good number of expected proposed attenuation relationships will probably come up by that time in another 5 years time or 10 years duration. So, this is an ever changing or ever evolving process. Wherever any major earthquake occurs, if you incorporate those earthquake instrumented data points in your attenuation relationship or in the prediction of attenuation relationship, these values of coefficients and corresponding standard deviation will keep on changing. So hence, the form of the equation will remain the same, but, these coefficients etcetera needs to be updated with more records of earthquake data.

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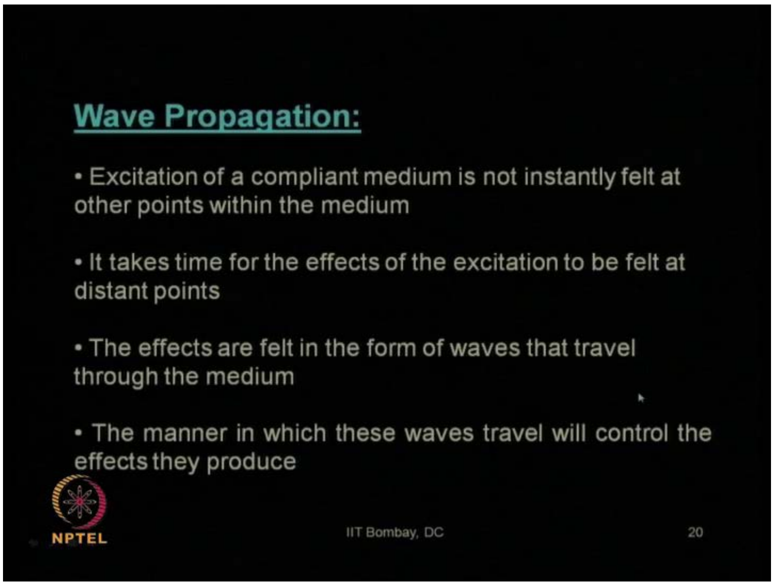

So with that, in the previous lecture, we completed our module number 4. So, in today's lecture, we will start with our next module. Let us see in the slide, we are starting today's lecture with module number 5, which is wave propagation. So, in this module number 5, let us see what is wave propagation? Let us look at the slide. In this module 5, so what is wave propagation? It is excitation of a compliant medium is not instantly felt at other points within the medium. Because, whenever any vibration or any excitation occurs in any medium, say soil or rock or any other material in a medium, obviously, that excitation or that vibration will not be immediately felt within the entire media at other points.

Even in water also, we have seen that, when you through a stone in the water, waves etcetera starts forming. So, when you are throwing the stone in the water, waves are forming. So, it takes little bit of time to propagate that wave. It is not that instantly when you are dropping that stone in the water immediately the waves will be felt at the entire media. It takes little bit of time for those waves to travel from your source point, where you created that excitation or vibration, from that point to another point in that media or other point. So, slowly it moves further in that medium. So, that is what is mentioned in the slide over here. It takes time for the effects of the excitation or vibration to be felt at other distance points, which is known to all of us. The effects are felt in the form of waves that travel through the media. So, as we know, these waves etcetera will form and slowly they will travel and through that wave, this excitation or this vibration will be felt from one point, where that disturbance has been created to any other point in that media.

So, waves actually carry that excitation from source point to other points in that particular media. So, the manner in which these waves travel will control the effects they produced. So, how they carry these behavior or the excitation in a particular region or in the particular material? That we will see in this wave propagation. That is, how this wave propagates or travels or carries these disturbances or excitation from one point to the another point, when they are traveling through a particular media

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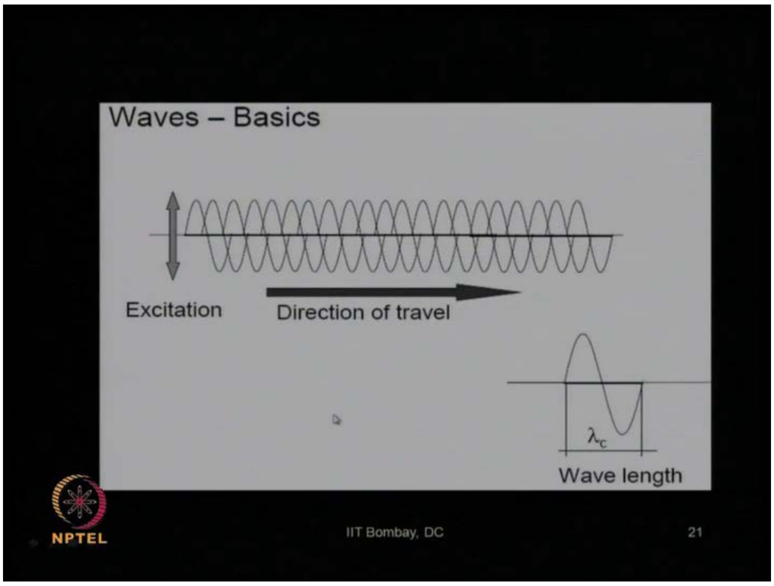

Now, let us come to the basics of this wave. That is, when any excitation takes place at a particular point, what happens? Finally, the waves travel in this direction. So, this is the direction of travel. So, these are the formation of the waves. Waves get formed and they travel like this. So, this excitation, finally from this point it reaches to another point. Of course, it travels in various directions. We have shown here the basic of this travel of this excitation from one point to another point through these waves. So, what is the wave length in the distance unit? If we say, to complete one cycle whatever is the distance, that is nothing but, the wave length. Generally, wave length is represented in this form of lambda or lambda c.

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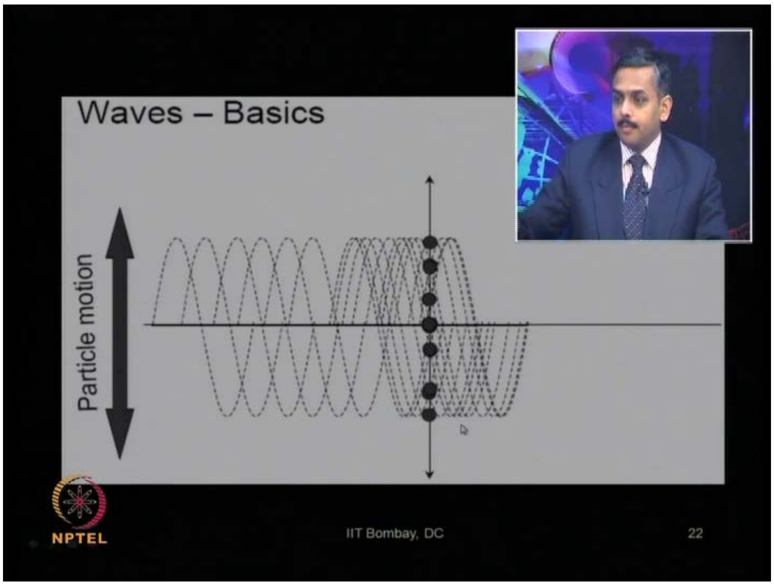

Now, how the particles move? So, when waves travel, waves travel like this. So finally, the direction of travel of the waves is in this direction. But, when they travel through a particular media, the particles of that media also gets excited because the waves traveling. These are nothing but they are carrying the excitation through that media. So, while carrying that excitation, what is happening? These particles start jumping or moving like this. So, this is the movement of the particle. So, it can be either in this direction or it can excite in this direction also, depending on what type of wave it is traveling through.

So, identifying that particle motion is a different aspect than the wave propagation. That is why it is shown separately over here. But, wave propagation is the cause and particle excitation or particle motion is the result.

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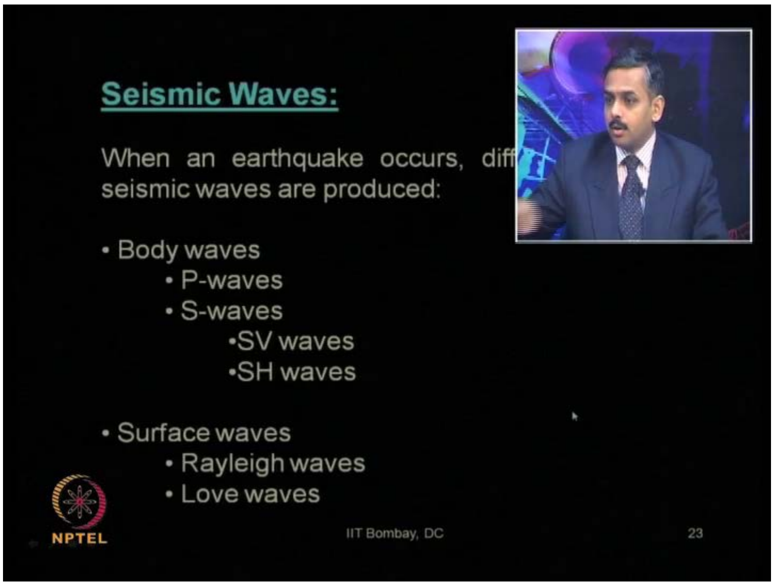

So, when we are talking about the seismic waves, that is when that excitation comes from and release of energy due to earthquake. So obviously, here also that excitation needs to be carried from one source point, where the energy gets released to another point through these waves. So, what are these waves due to the earthquake? These are nothing but, the seismic waves. So, when an earthquake occurs, different types of seismic waves are produced. As we have seen in our previous lecture, we have discussed thoroughly the various types of earthquake waves and we have also seen how those wave mechanics is used to estimate the epicenter of the earthquake.

So, when we are talking about wave propagation, let me come back to the classification of seismic waves. There are two major classifications or two types of seismic waves. They are body waves and surface waves. Now, within body waves, again we have sub classifications like P waves and S wave. P wave is nothing but, as you already know, primary wave or compressional wave or P wave and S is secondary wave or shear wave. S wave also can be further sub classified into two types of waves. That is, one is called S V waves and another is called S H waves. What are these things? That is, S waves in vertical direction and S waves in horizontal direction. So, those are the two components of S wave, by which S wave can be further classified.

Surface waves, when we are talking about the sub classification of surface waves, there are two types of surface wave. One is Rayleigh wave and another is Love wave. How their movement and motions etcetera occurs?

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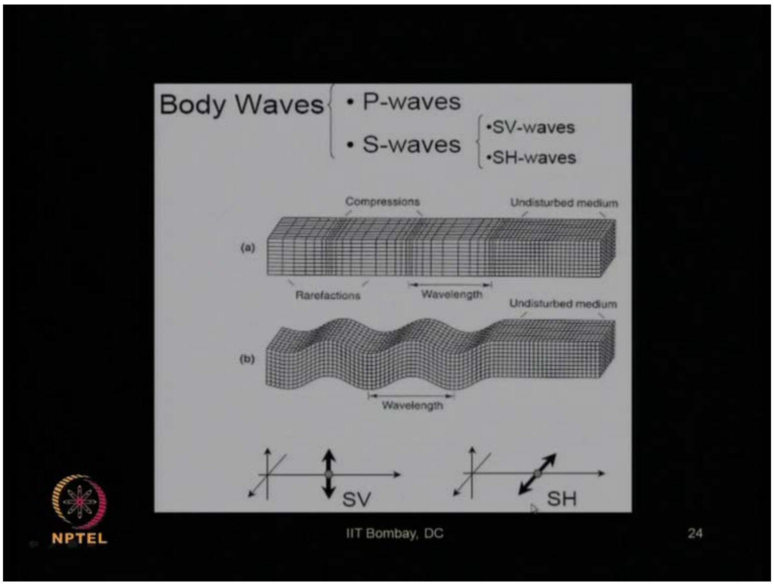

Let us look at this slide. Though we have discussed earlier in module 3, for this course, again let me go through that further. So, body waves like P wave and S wave. S wave can be further classified into S V waves and S H wave. For P wave, this figure A, you can see over here. So, when it travels through a media, what happens? There will be compression and expansion successively in that media.

That is, some portion of that media will be expanded like this and some portion of the media will be compressed like this, when the wave travels through that media. This shows the picture of the undisturbed media and that creates a wave length. That is, one cycle is between one compression to another expansion or starting of another compression. So, that demarcates the wavelength in the case of P wave. So, compressional wave or P wave or primary wave, when it travels through the media, the particles also gets excited in the same direction of the motion of the wave.

Picture B should be looked into for the S wave. You can see from picture B that this is the undisturbed media, and this is the direction of the movement of the wave. But, in this case, particle excite in the perpendicular direction to that direction of movement of the wave. So, that is why, this kind wavy form will get formed. As I said, there are two components of this S V wave. One is S vertical component where particle will move in this perpendicular direction to the direction of the movement of the wave and another is H, that is in the horizontal direction of this one. This one will be horizontal movement of the particle compared to the direction of the propagation of the wave.

So, you can see, so far whatever earthquake excitations or accelerations we talked about, that is horizontal acceleration and vertical acceleration, majority of them will be caused by horizontal acceleration. Horizontal acceleration will be caused by this S H and the vertical will be caused by either this S V or the P wave that we will further discuss because, that depends on the direction of propagation of your wave. So, this S H and S V will create the perpendicular movement compared to the direction of the wave; whereas in P wave, it is in the same direction of the particle movement as well as the wave movement.

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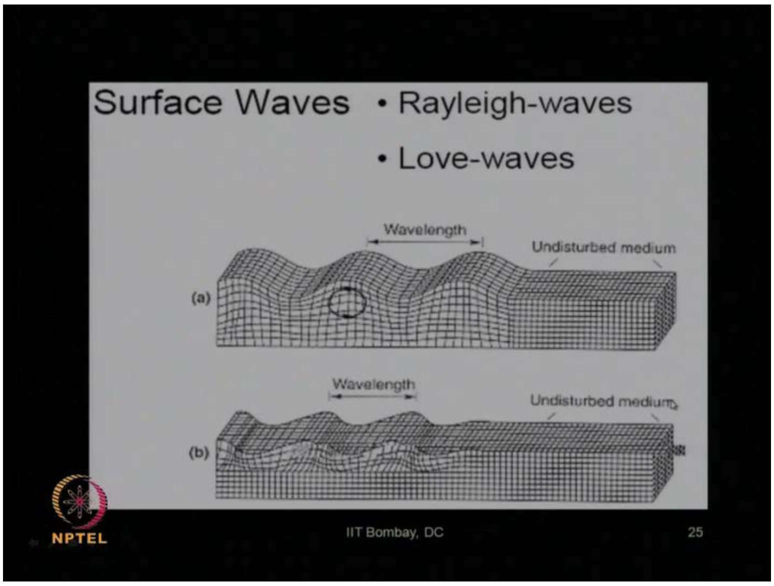

For Surface wave like Rayleigh wave and Love wave, we have seen that these are the behavior. Picture a is for Rayleigh wave and picture b is for love wave. This type of rotary motion that you can see over here for this surface wave, this is the undisturbed media and this is the disturbed media when waves are passing through. So, this also creates particle movement and particle movement is in this fashion compared to the direction of the propagation of the wave.

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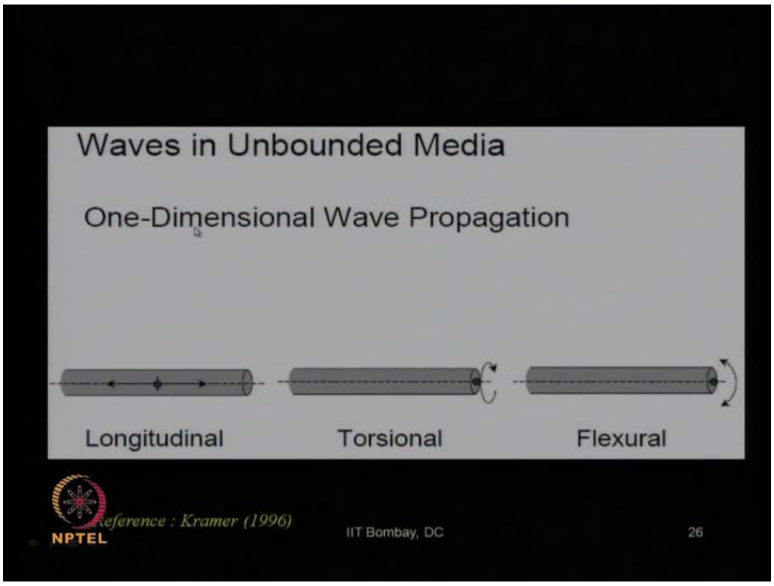

Now, let us discuss the derivations and various types of waves when they travel through a particular media. So first, let us try with the simplest case of waves in unbounded media. That is the media is non bounded in both the directions. It is infinity. We can consider the media, whatever the media or whatever the material we are considering, both the direction is extending to infinity. So, that is why unbounded. There is no boundary. So, for that let us first talk about one dimensional wave propagation in this unbounded media. If we want to see the types or categories of that one dimensional waves in unbounded media, they can be classified into three categories. There are called Longitudinal wave, Torsional wave and Flexural wave.

What are these waves and how they are different? That is, when a wave is traveling through this one dimensional, one dimensional means only in this direction and only in this dimension. Say x, we are considering the travel of the wave or the direction of movement of the wave. Now, how the particles get excited? That decides what type of wave we are considering for longitudinal one dimensional wave.

Wave movement direction is in this direction. Particle also gets excited or particle motion is also in the same direction. So, that is why based on the particle motion, it is called longitudinal motion or longitudinal one dimensional wave propagation. Whereas for the second case, when we call it torsional? When wave propagates in this direction, that is in this x axis direction, but the particles get disturbed or moves in this direction, that is in this fashion. That is in the torsional way or in the perpendicular way to the axis of this x.

So, that is called as torsional one dimensional wave propagation. That is, when the particle movement will be of this pattern. What is the flexural wave movement in one dimensional? In this case also, wave propagates in this direction, but the particle, they move in this direction. That is the direction of a bending. So the difference, as we know in longitudinal, it excites in this direction. Particle in torsional, it excites in this direction and that is the torsional movement of the particles and in bending or in flexural, the particle move in this direction. So, that is what is shown over here in these pictures. This is the flexural one dimensional wave propagation when wave is moving in this direction, but, the particle moves in this fashion. So, these are the three major categories of one dimensional wave propagation one can have.

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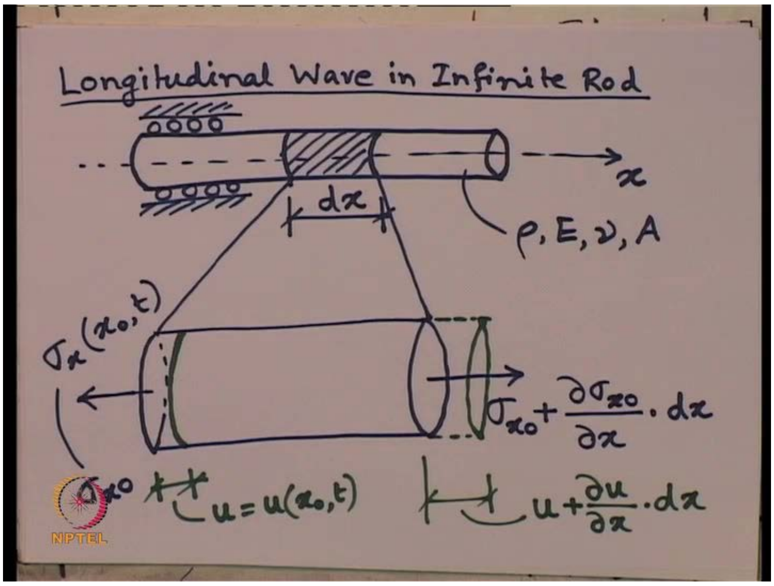

Now, let us come to the derivation of this one dimensional wave propagation. Let us start with the longitudinal wave in an infinite rod. So, let us draw here and derive it as we are considering one dimensional wave equation. So, let us say this is our x direction and in this one dimension only, we are considering our wave propagation. The boundary condition, we can say is like this. So that, we make sure that it is in the one dimension only. Let us say it is an infinite rod. So, we are talking about longitudinal wave in infinite rod.

So, let us see how it behaves? What are the various material property? We can consider like, say rho is the density of that material, e is young's modulus of that material, say nu is the poisson's ratio of the material and capital A is the cross sectional area of this rod. So, these are the various input parameters which we can consider. Now, let us take within this infinite rod, a small infinitesimally small element of length d x. So, we are considering this shaded region. We have taken an infinitesimal portion of the rod for consideration. So in that, we are now trying to find out how the variation of the stresses will be at these two ends of this infinitesimal small rod of d x length. So, what we can say here? This is one end and this is the other end you can see over here. At this side, let us say, the stress can be expressed as sigma x at a distance a x naught and at a time say t.

So, we are considering that point and that point and that point we have taken over here. So, that point coordinate, let us say it is x naught and at a particular time say t, the stress is expressed in this format. Let us say that, we are further reducing to sigma x naught at this face. At this end, let us say, the change in the stress in the x direction is this. Initial stress sigma x naught plus there is a change over this length of d x. So, this over this length of d x, it has changed by del epsilon del sigma x naught by del x into this, over this distance of d x.

So, why this change of stress has occurred? Because, we considered wave is passing through this media. So, this is the direction of the travel of the wave. For that, the particles are getting excited in that direction, because we have taken the longitudinal wave propagation. Because of that, what will happen? Let us show here, through some other color, say this point has come to this point, and this green one is the new position because, particle has moved. Particle has displaced. What made the particle to move? That wave which is propagating through that. So, that excitation has made the particle to move by this distance. That distance, let us say is u. So, that u is nothing but, let us say u at the distance at the coordinate x naught at a particular time say t. At this end, this end also has moved by a certain distance. Let us say the new location is this one. The particle has moved by this distance. So, how much distance it has moved? Over a distance of d x, its variation of this movement u will be u plus that del u del x over the distance of d x.

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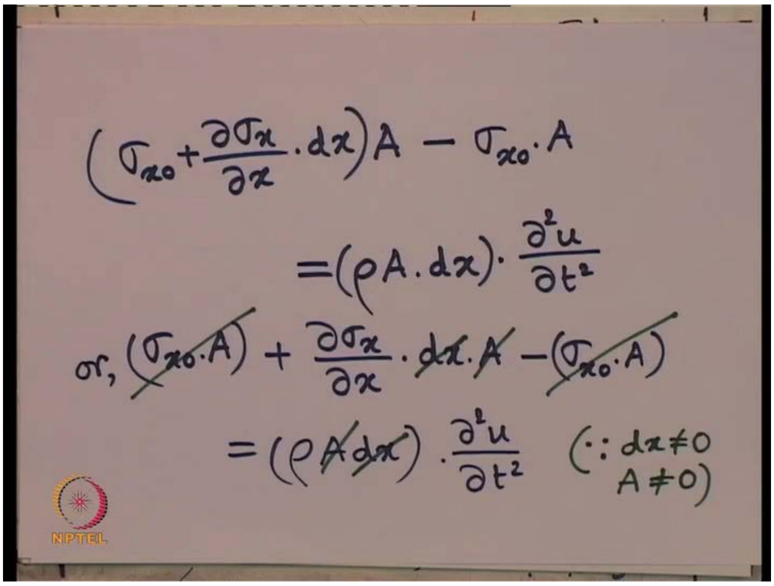

So now, if I use this one, what I can further say is that, the change in the behavior of this stress we will derive it now. It will be coming from, let us see over here. The next derivation, that is, what is the change in the stress at both the ends. What we obtained here, the difference of force is nothing but, stress sigma x naught plus del sigma x del x over the distance of d x. That stress variation, if I multiply with the cross sectional area that will give me a force.

So, force in the direction of the propagation of the wave. What is the force in the other direction? That is, what is the net force acting on that element? It is nothing but, minus sigma x naught times that cross sectional area A. Let me put this back over here again. You can see here. So, I have stress at this point and in this direction and stress at this point and in this direction. So, what will be the resultant force acting on this element? This stress times cross sectional area will give me force in this direction and this stress times cross sectional area in this direction, will give me force in this direction

So the net force, which is acting on this small element is nothing but, this one. Now, what this force? It is nothing but, the inertia force, which is acting on the system. Why this inertia force is acting? It is because of the excitation. Because of the waver traveling through that media, the particles get excited. Once the particle gets excited, as we have already seen, look at here. It has moved by this distance at two ends of that infinitesimal small portion. So, this portion has moved by this amount and this portion has moved by this amount.

So first, we should know what is the acceleration? So, what is the acceleration? Acceleration is nothing but, del 2 u by del t square. That is the acceleration because, u is the displacement. As we have already taken, if you differentiate it twice with time, you will get the acceleration at that particular direction. Why I am using del instead of d? In this one dimensional case, I can use d also. But when we take multiple directions, later on we will take the three dimension in the generalized case, we will see it is dependent on a particular direction which we are considering.

So, that is why it is the del partial differential not the complete differential of d. So, that is why it is always better to use this del from beginning itself. Now, this is the acceleration. Now, what is mass of that element? The mass of that element is nothing but, density is given to us and cross sectional area is given to us. The distance which we are considering is also known; so area times distance is nothing but volume and times density will give you the mass. So, mass times acceleration is nothing but, your inertia force. That inertia force is nothing but, which is causing that stress difference within that element.

Now, is it clear why that stress difference is between the two ends of the infinitesimal rod is occurring, that is this stress difference between two ends. So, that is because of this inertia. So now, if I further simplify it, what we can get from here? If we simplify this one further, you can see this sigma x naught times A and sigma x naught times A gets canceled from both the sides. d x d x also gets canceled. So, let me simplify it. So, it will be easier to understand sigma x naught times A plus del sigma x del x into d x into A minus sigma x naught times A equals to rho A d x times del 2 u by del t square. Now, in this case, we can easily cancel this. What else we can do? The cancelation, you can see on the both the sides, there is d x and d x and a. Why we cancel because, these are non zero parameters. You have to always remember because, d x is also a non 0 and cross sectional area is also non 0. Only then from both the sides, we can cancel these two parameters.

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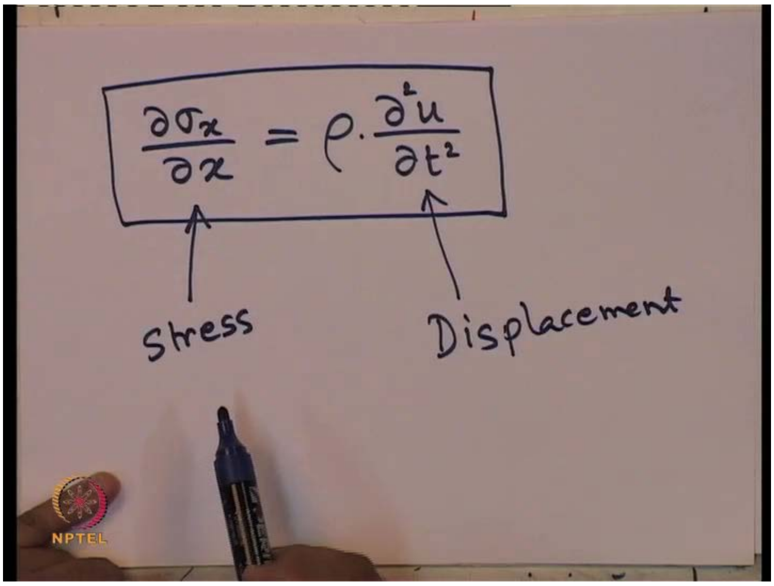

So after canceling that, what finally we are getting? Let us see. We will plot it here and we will further simplify that. del sigma x del x equals to rho times del 2 u by del t square. So, that is the basic governing equation for the one dimensional wave propagation in infinitesimal media. This is the basic equation. Now, we need to solve this.

So, this is the formation of the equation. That is, in this parameter, this sigma x dependency, this is the stress dependency and this is the displacement dependency. So, this is displacement dependency. So, how stress and displacement is connected? When a wave is traveling through a media that we can see from here, because from this simplification, we got it over here; you can see this equals to rho times del 2 u by del t square. Now, let us see if we can simplify further.

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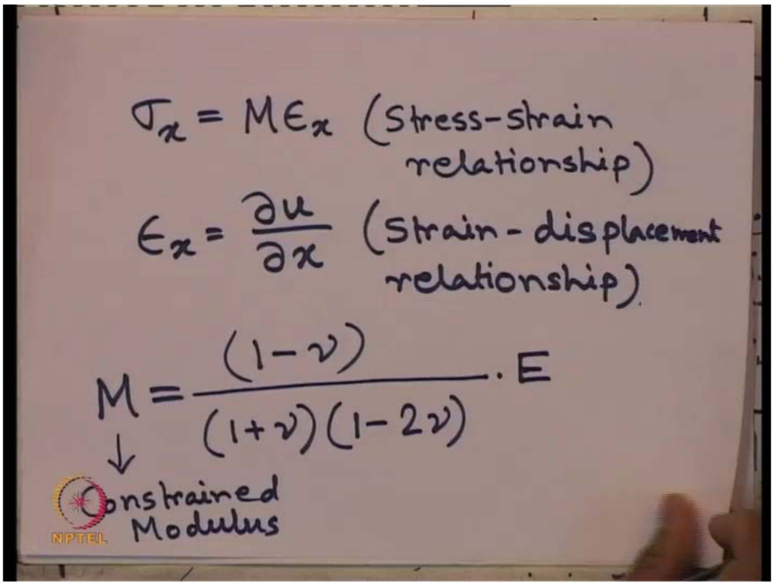

So, let us look at this derivation once again. Now, what are the other relationships known to us? We know that stress can be expressed as some modulus times strain. So, this is stress strain relationship. This we are getting from stress strain relationship. That is, always we know stress is related to strain through some modulus.

We can also express this strain in terms of displacement. So, that will be, in this case del u by del x because, u is the displacement. So, this is the strain displacement relationship. Now, this strain displacement relationship, in this case, when we are talking about one dimensional wave propagation, longitudinal wave propagation, this modulus which I have mentioned over here, m can be written in this form. Let me write it down because, this is already covered in basic solid mechanics course. So, I am not going to cover that aspects.

This stress with strain relationship through modulus, using any solid mechanics book or the NPTEL course on solid mechanics, we can have this relationship when we are talking about the longitudinal wave propagation. So, this is called constrained modulus and mu is called the poisson's ratio of the media and E is the young's modulus of the media. So, constrain modulus can be expressed in this form, which is connected through this stress strain relationship in this one dimensional wave propagation.

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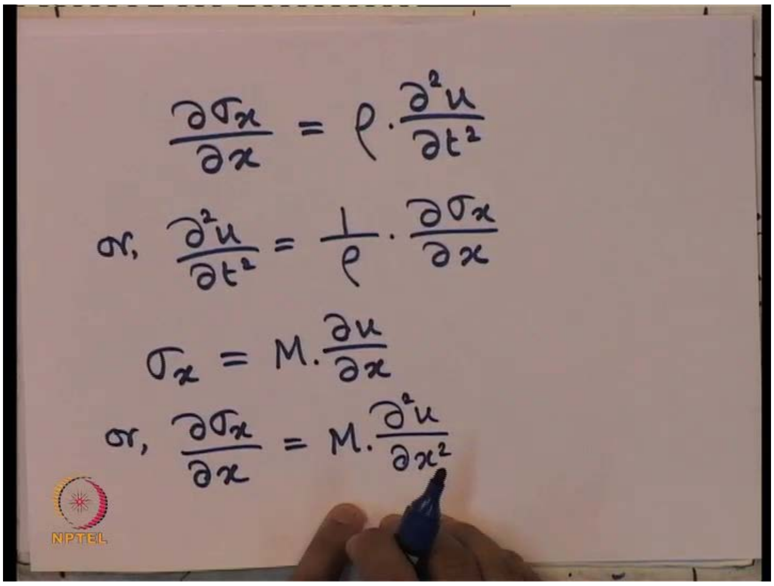

So, with this relation, what further simplification we can do? Let us see the derivation again. So, we can write the basic equation that we already got. That is, del sigma x del x equals to rho times del 2 u by del t square. That we had already derived. Now, in this case, we can simplify it further in this way or del 2 u del t square equals to 1 by rho del sigma x by del x.

Now, what is del sigma x del x? If you look at the previous relationship of stress strain and strain through displacement, strain is related like this, and which is related to stress like this. So, I can put it like this that stress can be expressed as m times, that constrain modulus times strain which is nothing but del u by del x. Or I can write del sigma x by del x is nothing but, that constrain modulus times del square u by del x square. I am differentiating it with x only. So, with that expression, if I now put it in this equation , what I will get on further simplification is that this equation will now take the form of del 2 u del t square equals to m by rho del square u by del x square

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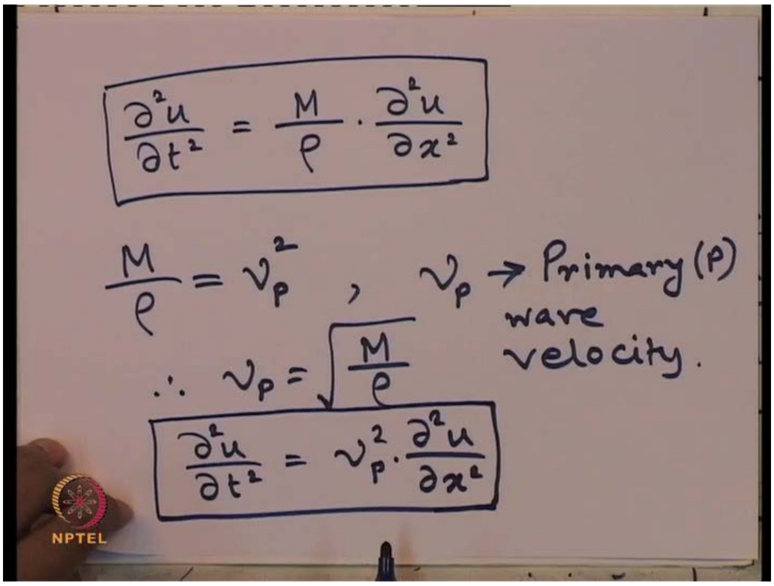

So, this is the further simplified version of that wave equation in one dimensional case for longitudinal wave. Now, this parameter m by rho is expressed as V P square. We will discuss that very soon. This V P is known as primary or P wave velocity. So, we can write V P equals to root over m by rho, where M is constrain modulus and rho is the density of the material. So with that, what we can write? del 2 u by del t square equals to V P square times del 2 u by del x square. So, that is the basic governing equation for longitudinal wave traveling in one dimension. So with this, we have come to the end of today's lecture. We will continue further in our next lecture.