

**Geotechnical Earthquake Engineering**  
**Prof. Deepankar Choudhury**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Bombay**

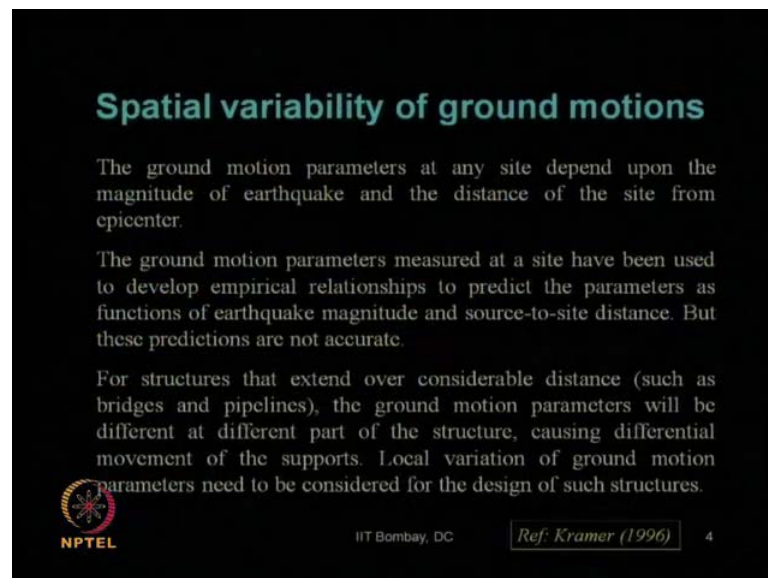
**Module - 4**

**Lecture - 16**

**Strong Ground Motion (Contd...)**

Let us start our today's lecture of this NPTEL video course on geotechnical earthquake engineering. Let us look at the slide here, in this video course on geotechnical earthquake engineering, we were going through module number four which is on strong ground motion, and we had started in the previous lecture about the attenuation relationship. So a very quick recap, what we have learnt in our previous lecture.

(Refer Slide Time: 00:57)




**Spatial variability of ground motions**

The ground motion parameters at any site depend upon the magnitude of earthquake and the distance of the site from epicenter.

The ground motion parameters measured at a site have been used to develop empirical relationships to predict the parameters as functions of earthquake magnitude and source-to-site distance. But these predictions are not accurate.

For structures that extend over considerable distance (such as bridges and pipelines), the ground motion parameters will be different at different part of the structure, causing differential movement of the supports. Local variation of ground motion parameters need to be considered for the design of such structures.

 NPTEL IIT Bombay, DC Ref: Kramer (1996) 4

We have discussed about the spatial variability of ground motions, that is how this ground motion vary from the hypo center point of earthquake or the epicentral location to a particular distance or a site, where we are planning any new construction or any new structure to be constructed. So, the ground motion parameters they vary in terms of magnitude, and in terms of distance for a site from the epicentral location.

So that variation that variability of ground motion we need to study, because that is most important for our design of any structure at a particular site, how they do vary with respect to magnitude and with respect to the site to source distance.

(Refer Slide Time: 01:44)

**Amplitude Parameters - Estimation**

Predictive relationships for parameters (like peak acceleration, peak velocity) which decrease with increase in distance are called attenuation relationships.

**Peak Acceleration**

Campbell (1981) developed attenuation relationship for mean PHA for sites within 50 km of fault rupture in magnitude 5.0 to 7.7 earthquakes:

$$\ln \text{PHA}(g) = -4.141 + 0.868M - 1.09 \ln [R + 0.0606 \exp(0.7M)]$$

Where  $M = M_L$  for magnitude  $< 6$  or  $M_s$  for magnitude  $> 6$ ,  $R$  is the closest distance to fault rupture in km.

Latest mostly used relationship in western North America is given  
Boore et al. (1993)

Ref: Kramer (1996)

NPTEL IIT Bombay, DC 5

So, in that connection we have mentioned what is known as attenuation relationship that when this predictive parameters, predictive relationship parameters like either peak acceleration, peak velocity intensity excreta, how they decrease with increase in the distance, those are nothing but the attenuation relationship. We have seen what is the peak acceleration attenuation relationship, the pioneering work proposed by Campbell in 1981 for the North California region for North America in the California region with PHA value can be obtained using this attenuation relationship for the magnitude range between 5 to 7.7 and the site to fault source should be within the 50 kilometer distance, where this  $M$  has to be used as local magnitude scale if it is within 6, if it is more than 6 then surface magnitude scale has to be used. And this  $R$  is nothing but the closest distance from the site to the fault rupture in the unit of kilometer.

(Refer Slide Time: 02:58)

### Amplitude Parameters - Estimation

Attenuation relationship in western North America is given by  
Boore et al. (1993)


*(From North American Earthquakes (magnitude 5-7.7) within 100 km of surface projection of fault)*

$$\text{Log PHA}(g) = b_1 + b_2(M_w - 6) + b_3(M_w - 6)^2 + b_4R + b_5 \log R + b_6 G_b + b_7 G_c$$

$R = (d^2 + h^2)^{1/2}$ ,  $d$  - closest distance to the surface projection of the fault in km.

|                            |                            |
|----------------------------|----------------------------|
| $G_b = 0$ for site class A | $G_c = 0$ for site class A |
| $G_b = 1$ for site class B | $G_c = 0$ for site class B |
| $G_b = 0$ for site class C | $G_c = 1$ for site class C |

Site classes are defined next slide on the basis of the avg.  $V_s$  in the upper 30 m).


IIT Bombay, DC
Ref: Kramer (1996)
6

Later on Boore et al in nineteen ninety three they proposed an advanced or modified attenuation relationship for the same peak horizontal acceleration for the western north America region. That is using the earthquake historical records of this region, they extended the distance up to 100 kilometer from the epicenter to the source point. And magnitude range they have used the same magnitude range, and this was their proposed attenuation relationship, where various parameters like R, G b, G c coefficients based on the site class.


(Refer Slide Time: 03:40)

### Definitions of Site Classes for Boore et al. (1993) Attenuation Relationship

| Site Class | $\bar{V}_s$ in Upper 30 m (100 ft) |
|------------|------------------------------------|
| A          | > 750 m/sec (2500 ft/sec)          |
| B          | 360–750 m/sec (1200–2500 ft/sec)   |
| C          | 180–360 m/sec (600–1200 ft/sec)    |

### Coefficients for Attenuation Relationships of Boore et al. (1993)

|        | Component |       |       |       |        |       |       |      | $\sigma_{\log \text{PHA}}$ |
|--------|-----------|-------|-------|-------|--------|-------|-------|------|----------------------------|
|        | $b_1$     | $b_2$ | $b_3$ | $b_4$ | $b_5$  | $b_6$ | $b_7$ | $h$  |                            |
| Random | -0.105    | 0.229 | 0.0   | 0.0   | -0.778 | 0.162 | 0.251 | 5.57 | 0.230                      |
| Larger | -0.038    | 0.216 | 0.0   | 0.0   | -0.777 | 0.158 | 0.254 | 5.48 | 0.205                      |


IIT Bombay, DC
7

Site class they have classified based on the average shear wave velocity in the top 30 meter from the ground surface based on different value or ranges of values like this given in this table. We have seen what such site class means it is a rocky site, this is the hard soil or steep soil, this is the soft soil site and various coefficients b 1, b 2, b 3 up to h have an their corresponding standard deviation values as proposed by Boore et al.

(Refer Slide Time: 04:07)

**Attenuation Relationship for peak horizontal rock acceleration by Toro et al., 1994 (for mid continent of North America)**

$$\ln \text{PHA (g)} = 2.2 + 0.81(M_w - 6) - 1.27 \ln R_m + 0.11 \max[\ln(R_m/100), 0] - 0.0021R_m$$

$$\sigma_{\ln \text{PHA}} = (\sigma_m^2 + \sigma_r^2)^{1/2}$$

Where  $R_m = (R^2 + 9.3^2)^{1/2}$ , R being closest horizontal distance to earthquake rupture (in km),  $\sigma_m = 0.36 + 0.07(M_w - 6)$ , and


- 0.54 for  $R < 5$  km
- $\sigma_r = 0.54 - 0.0227(R - 5)$  for  $5 \text{ km} \leq R \leq 20 \text{ km}$
- 0.2 for  $R > 20$  km

**Attenuation relationship for subduction zone (Youngs et al., 1988)**

$$\ln \text{PHA (g)} = 19.16 + 1.045M_w - 4.738 \ln [R + 205.5 \exp(0.0968M_w)] + 0.54 Z_i$$

$$\sigma_{\ln \text{PHA}} = 1.55 - 0.125M_w$$

R = closest distance to the zone of rupture in km and  $Z_i = 0$  for surface and 1 for intraslab events

 NPTEL IIT Bombay, DC 8

Next, we had seen in the previous lecture, what is the attenuation relationship for PHA proposed by Toro et al for the midcontinent of North America? So, if somebody wants to use the, what is the design acceleration etcetera from this attenuation relationship in California region, they should use the Boore et al equation and if somebody is planning to do a design in suppose the Texas region or Denver region, which is in the central or midcontinent of North America they should go for the Toro et al expression. So that is what we have to be very careful where this attenuation relationship has been developed for that is most important.

So, this equation where different values here also they have used the moment magnitude like Boore et al. And another attenuation relationship proposed only for the subduction zone by Young's et al in nineteen eighty eight is given by this equation. We had seen in the previous lecture.

(Refer Slide Time: 05:10)

**Peak Velocity Attenuation Relationships (Joyner and Boore, 1988)**  
 (for earthquake magnitudes 5-7.7)

$$\log \text{PIIV (cm/sec)} = j_1 + j_2(M-6) + j_3(M-6)^2 + j_4 \log R + j_5 R + j_6$$

Where PHV can be selected as randomly oriented or larger horizontal component  
 $R = (r_0^2 + j_7^2)^{1/2}$ , and  $r_0$  is the shortest distance (km) from the site to the vertical projection of the EQ fault rupture on the surface of the earth.

The coefficients  $j_i$  are given in the table below:  
 Coefficients after Joyner & Boore (1988) for PIIV Attenuation Relationship

| Component | $j_1$ | $j_2$ | $j_3$ | $j_4$ | $j_5$   | $j_6$ | $j_7$ | $\sigma_{\log \text{PHV}}$ |
|-----------|-------|-------|-------|-------|---------|-------|-------|----------------------------|
| Random    | 2.09  | 0.49  | 0.0   | -1.0  | -0.0026 | 0.17  | 4.0   | 0.33                       |
| Larger    | 2.17  | 0.49  | 0.0   | -1.0  | -0.0026 | 0.17  | 4.0   | 0.33                       |

NPTEL IIT Bombay, DC 9

Then the velocity attenuation relationship like peak horizontal velocity PHV can be obtained in the unit of centimeter per second as proposed by Joyner and Boore in nineteen eighty eight for earthquake magnitude between 5 to 7.7. And various parameters are given over here with their coefficients  $j_1, j_2, j_3$  up to  $j_7$  with the standard deviation value.

(Refer Slide Time: 05:37)

**Amplitude Parameters - Estimation**  
 A.Patwardhan et al. (1978):

$$\ln y = \ln A + B M_s + E \ln [R + d \exp(f M_s)]$$

Where,  $y$  in  $\text{cm/s}^2$ ,  $d=0.864$  and  $f = 0.463$

| Path                | A(for median) | A( for mean) | B     | E     |
|---------------------|---------------|--------------|-------|-------|
| Path A (rock)       | 157           | 186          | 1.04  | -1.90 |
| Path A(stiff soil)  | 191           | 224          | 0.823 | -1.56 |
| Path B (stiff soil) | 284           | 363          | 0.587 | -1.05 |

Path A: Shallow focus earthquakes (California, Japan, Nicaragua and India), 63 records  
 Path B: Subduction (Benioff) zone earthquakes (Japan & South America) 23 earthquakes,  $5.3 \leq M_s \leq 7.8$ , 32 records

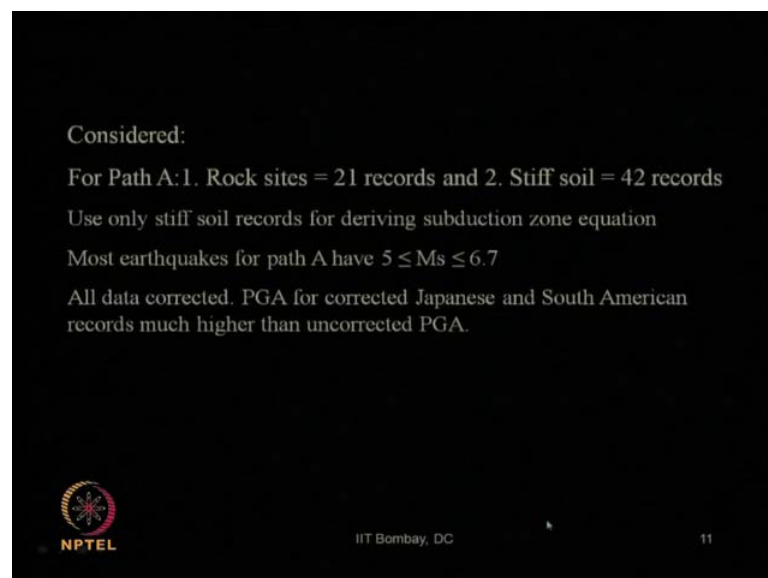
NPTEL IIT Bombay, DC 10

Then we had seen the attenuation relationship in terms of acceleration, horizontal acceleration in unit of centimeter per second square as proposed by Patwardhan et al in


nineteen seventy eight using the different types of earthquake; shallow earthquake from California, Japan, Nicaragua and India of total 63 records and for subduction zone earthquake using the Japan and South American earthquake of 23 earthquake including total 32 records.

So, for shallow earthquake they have used media two different types of media rocky strata and stiff soil strata, whereas for subduction zone earthquake they only got the data from the stiff soil region. So, that is why they have given different coefficients A B E etcetera in this equation has proposed in this table and chart.

(Refer Slide Time: 06:34)



Considered:  
For Path A: 1. Rock sites = 21 records and 2. Stiff soil = 42 records  
Use only stiff soil records for deriving subduction zone equation  
Most earthquakes for path A have  $5 \leq M_s \leq 6.7$   
All data corrected. PGA for corrected Japanese and South American records much higher than uncorrected PGA.

 NPTEL IIT Bombay, DC 11

(Refer Slide Time: 06:35)

**Aptikaev & Kopnichev (1980)**

$\log Ae = a_1M + a_2\log R + a_3$

| Ae (cm/s <sup>2</sup> ) | a <sub>1</sub> | a <sub>2</sub> | a <sub>3</sub> |
|-------------------------|----------------|----------------|----------------|
| ≥160                    | 0.28           | -0.8           | 1.70           |
| <160                    | 0.80           | -2.3           | 0.80           |

> PGA corresponds to S-wave  
 > Used five source mechanism categories (about 70 records, 59 earthquakes from W. N. America including Hawaii, Guatemala, Nicaragua, Chile, Peru, Argentina, Italy, Greece, Romania, central Asia, India and Japan):

1. Contraction faulting (uplift and thrust), about 16 earthquakes
2. Contraction faulting with strike-slip component, about 6 earthquakes

NPTEL IIT Bombay, DC 12

Then we had also discussed this equation of Aptikaev and Kopnichev in nineteen eighty as given for the attenuation relationship for the acceleration using the shear wave magnitude scale and they only proposed the different five types of faulting. So, they only classified for different types of faulting what are the different equations to be used.

(Refer Slide Time: 07:04)

3. Strike-slip, about 17 earthquakes
4. Strike-slip with dip-slip component, about 6 earthquakes
5. Dip-slip, about 9 earthquakes

Use these approximately 70 records to derive ratios of mean measured, A<sub>0</sub>, to predicted PGA, Ae, log(A<sub>0</sub>/Ae), and for ratios of mean horizontal to vertical PGA, logA<sub>h</sub>/A<sub>v</sub>, for each type of faulting. Use every earthquake with equal weight independent of number of records for each earthquake.

• Results are:

|                                    | Category 1       | Category 2      | Category 3       | Category 4      | Category 5       |
|------------------------------------|------------------|-----------------|------------------|-----------------|------------------|
| log A <sub>0</sub> /Ae             | 0.35 ± 0.13 (16) | 0.11 ± 0.17 (5) | 0.22 ± 0.08 (17) | 0.06 ± 0.13 (6) | -0.06 ± 0.20 (9) |
| log A <sub>h</sub> /A <sub>v</sub> | 0.32 ± 0.13 (12) | 0.32 ± 0.08 (5) | 0.27 ± 0.07 (12) | 0.18 ± 0.10 (5) | 0.17 ± 0.11 (5)  |

where ± gives 0.7 confidence intervals and number in brackets is number of earthquakes used.

NPTEL IIT Bombay, DC 13

Aptikaev, F., & Kopnichev, J. (1980). Correlation between seismic vibration parameters and type of faulting. *Proceedings of Seventh World Conference on Earthquake Engineering, Vol. 1*, 107-110.

So, these are the relationship proposed for five different categories of faulting like strike slip, strike slip with dip slip, dip slip fault, etcetera, etcetera. All are detailed here, total 70 records they had used and these are the values of log of a naught by A e and log of A

h by A v is horizontal A v is vertical and this is the paper which appeared in nineteen eighty in the seventh world conference proceedings of earthquake engineering.


(Refer Slide Time: 07:35)

**PML (1982)**

$$\ln(a) = C_1 + C_2 M + C_3 \ln[R + C_4 \exp(C_5 M)]$$

Where a is in g,  $C_1 = -1.17$ ,  $C_2 = 0.587$ ,  $C_3 = -1.26$ ,  $C_4 = 2.13$ ,  $C_5 = 0.25$   
and  $\sigma = 0.543$

Used data from Italy (6 records, 6 earthquakes), USA (18 records, earthquakes), Greece (13 records, 9 earthquakes), Iran (3 records, 3 earthquakes), Pakistan (3 records, 1 earthquake), Yugoslavia (3 records, 1 earthquake), USSR (1 record, 1 earthquake), Nicaragua (1 record, 1 earthquake), India (1 record, 1 earthquake) and Atlantic Ocean (1 record, 1 earthquake).

 P.M.L. 1982. *British earthquakes. Tech. rept. 115/82. Principia Mechanica Ltd., London. Not Reported in Ambraseys et al. (1992)*

NPTEL IIT Bombay, DC 14

Then we had also discussed about the P M L nineteen eighty two proposed based on the British earthquake technical report, which is reproduced by Ambraseys et al in nineteen ninety two. The attenuation relationship for acceleration once again in the unit of g using this expression. They used worldwide earthquake data from Italy, US, Greece, Pakistan, Yugoslavia , USSR in those days, Nicaragua, India and Atlantic ocean earthquake.


(Refer Slide Time: 08:06)

**Gaull (1988)**

$$\log \text{PGA} = [(a_1 \log R + a_2)/a_3](M_L - a_4) - a_5 \log R - a_6 R + a_7$$

where PGA is in  $\text{m/s}^2$ ,  $a_1 = 5$ ,  $a_2 = 3$ ,  $a_3 = 20$ ,  $a_4 = 6$ ,  $a_5 = 0.77$ ,  $a_6 = 0.0045$  and  $a_7 = 1.2$

- Considered earthquakes of magnitudes about 3 and most from distances below about 20 km
- Adds 4 near source ( $5 \leq R \leq 10\text{km}$ ) records from US, Indian and New Zealand earthquakes with magnitudes between 6.3 and 6.7 to supplement high magnitude range

 Gaull, B.A. (1988). *Attenuation of strong ground motion in space and time in southwest Western Australia. Proceedings of Ninth World Conference on Earthquake Engineering, Vol. II, pp. 361-366.*

NPTEL IIT Bombay, DC 15



Now, let us come to our today's lecture. So, today also we will continue further in that attenuation relationship proposed by various researchers across the world and we will also emphasize the attenuation relationships, which are valid for India and subcontinent. So, for Indian region also we will see that at different locations like northern India in Himalayan region, in northeast India, in the peninsular India different attenuation relationships are proposed. And we have to be very careful about using those equations because when we are planning to go for a design required attenuation relationship at a particular site needs to be used. Not that for Indian design we should use the attenuation relationship proposed by Boore et al, which is valid for the north west America.

We should not use that attenuation relationship for any design of Indian structures. So, having said so, let us see what are the other attenuation relationship worldwide developed like Gaulle in nineteen eighty eight proposed the attenuation relationship using the data points of south west western Australia. You can see the paper Gaulle nineteen eighty eight attenuation of strong ground motion in space and time in southwest western Australia. It published again in that proceeding of the ninth world conference of earthquake engineering in this volume and page number.

So, log of PGA that is peak ground acceleration can be estimated like  $a = 1 + \log R + \frac{2}{3}M - 4 - 5 \log R - 6 + 7$ , where this PGA will be in the unit of meter per second square. So, we should always consider this unit, these units are very important. You can see at different attenuation relationship we have different units. So a 1 is 5, a 2 is 3, a 3 is 20, a 4 is 6, a 5 is 0.77, a 6 is 0.0045, a 7 is 1.2. As we know these values are obtained based on how many data points they have taken for the prediction or proposing this empirical relationship.

So now after this eighty eight onwards, suppose if you take some more earthquake data points from that region your this coefficients are going to get changed; so considered earthquakes of magnitudes about 3, and most from the distance below about 20 kilometer. That is the range they have used and adds four near sources. Near sources means this R value that is from the site to the fault or hypo central distance within the 5 to 10 kilometers range records from the US, Indian and New Zealand earthquake with magnitude between 6.3 to 6.7 to supplement the high magnitude range, because in the

western Australia you hardly get any large magnitude of earthquake in the historical record.

So, that is why what they did they use the small earthquake values in and for high earthquake magnitude they have used other country data. Like New Zealand earthquake they experienced higher magnitude of earthquake data, Indian earthquake and U S data four near source data within this range that is, close to epicentral location. Those data they have combined with their shallow magnitude or low magnitude data and combinely they have proposed this expression. So, we cannot say that this is truly for the western Australia because they have punched here the other earthquake taken from the other regions, right.

(Refer Slide Time: 12:03)

**Singh et al. (1996)**

$$\log_{10}^a = 1.14 + 0.31M + 0.65\log_{10}^R$$
$$\log_{10}^V = 0.571 + 0.41M + 0.768\log_{10}^R$$

Where, a = PHA (cm/s<sup>2</sup>)  
V = PIIV (cm/s)  
Data considered from 1986 – 1993  
R= Hypocentral distance in km  
M = M<sub>B</sub> ( 5.7 to 7.2)

*R.P.Singh, Ashutosh Aman, Y.J.J. Prasad (1996) "Attenuation relations for strong seismic ground motion in Himalayan Region" Pure and Applied Geophysics JGP/AGU, Volume 147, Issue 1, pp.161-180.*

**NPTEL** IIT Bombay, DC 16

Now, coming to the pure Indian earthquake acceleration attenuation relationship and velocity attenuation relationship the pioneering work can be mentioned as proposed by the R. P. Singh and his research group from IIT Kanpur in nineteen ninety six.

So, in nineteen ninety six they proposed the attenuation relation for strong seismic ground motion in Himalayan region. So, they considered only the Himalayan region of earthquake till nineteen ninety six and this paper was appeared in Pure and Applied Geophysics Journal in this volume and issue number and page numbers are given over here. In this equation you remember earlier all the equations were log natural, here it is log to the base 10, it is the difference. a, a is the peak horizontal acceleration in the unit

of centimeter per second square. This can be calculated using this expression and another expression they have given for the velocity attenuation relationship.

So, for the velocity it is a PHV, peak horizontal velocity in the unit of centimeter per second. This is the values coefficients and M and R. So, what is R, R is nothing but the hypo central distance in the unit of kilometer. Hypo central distance means from your site to the hypo central, whatever is the distance in the kilometer unit. You have to put in this equation and what is the M, M is the magnitude scale, but which scale they have proposed MB, MB is that body wave magnitude scale they have proposed that is 5.7 to 7.2, between this range this equation is valid. So, this range of earthquake magnitude can be used further to calculate the acceleration and velocity attenuation with respect to distance from a specific source to a particular site of your concerned, where you are planning to construct your structure.

(Refer Slide Time: 14:16)

**Attenuation relationships for India**

The following generalized predictive attenuation relationship has been proposed for Peninsular India by Iyengar and Raghukanth (2004)


$$\ln Y = C_1 + C_2 (M-6) + C_3 (M-6)^2 - \ln R - C_4 R + \ln \varepsilon$$

where  $Y$ ,  $M$ , and  $R$  refer to PGA(g), moment magnitude, and hypocentral distance, respectively

**Koyna-Warna Region:**  
 $C_1 = 1.7615$ ;  $C_2 = 0.9325$ ;  $C_3 = -0.0706$ ;  $C_4 = 0.0086$ ;  $\sigma(\ln \varepsilon) = 0.3292$

**Western-Central Region:**  
 $C_1 = 1.7236$ ;  $C_2 = 0.9453$ ;  $C_3 = -0.0740$ ;  $C_4 = 0.0064$ ;  $\sigma(\ln \varepsilon) = 0.3439$

**Southern Region:**  
 $C_1 = 1.7816$ ;  $C_2 = 0.9205$ ;  $C_3 = -0.0673$ ;  $C_4 = 0.0035$ ;  $\sigma(\ln \varepsilon) = 0.3136$

 NPTEL 17

Another attenuation relationship for India, specifically it was proposed, it is also can be considered as a pioneering work because it is for the peninsular India. Peninsular India means the central Indian portion, central and little southern portion of the India. So, that generalized predictive attenuation relationship has been proposed for peninsular India by Iyengar and Raghukanth from IISC Bangalore professor Iyengar and his research group his research student Raghukanth who is now in IIT Madras, a faculty. In two thousand four in their paper they had proposed this attenuation relationship for peninsular India.

You can see this is the equation they had proposed natural log they have used  $Y$  equals to  $C_1 + C_2 \times M - 6 + C_3 \times (M - 6)^2 - \ln R - C_4 \times R + \ln \epsilon$ . So they have proposed if you see and look at the form of the equation, in the similar form of that proposed by Boore et al in nineteen ninety three right. The form of the equation only the coefficients and values will change this  $C_1$ ,  $C_2$ ,  $C_3$  depending on the collected historical earthquake in this peninsular region.

So, where this  $Y$ ,  $M$  and  $R$  refers to  $Y$  is nothing but peak ground acceleration in the unit of  $g$  you are getting  $M$  is the moment magnitude of earthquake and  $R$  is the hypo central distance in kilometer unit. Now they have proposed the different coefficient this  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$  and this  $\epsilon$  for different region. Like for Koynawarna region they proposed  $C_1$  equals to 1.7615,  $C_2$  as 0.9325,  $C_3$  as minus 0.0706,  $C_4$  as 0.0086 and sigma, that is standard deviation of that parameter of  $\ln \epsilon$  is 0.3292. Similarly, for western central region of India they proposed these are the coefficients and for the southern region they proposed this equation.

So, suppose somebody wants to use the attenuation relationship for further design of any structure in Bangalore region they should use this southern region coefficients, if somebody close to the Koyna region wants to do a design they should use these coefficient, somebody wants to do a design in the western central region of India they should use this coefficients for obtaining the acceleration attenuation relationship, as proposed by Iyengar proposed for peninsular India region.

(Refer Slide Time: 17:22)

**Attenuation relationships for India**

The following predictive attenuation relationship for peak vertical ground accelerations has been proposed for Himalayan Region of India by Sharma (2000).

$$\log(A) = -2.87 + 0.634 M - 1.16 \log(X + e^{0.62M})$$

where  $A$ ,  $M$ , and  $X$  refer to PGA(g), moment magnitude, and hypocentral distance, respectively.

The database consisting of 66 peak ground vertical accelerations from five earthquakes recorded by Strong Motion Arrays in India have been used to develop the relationship.

Vertical to horizontal acceleration ratio can be estimated based on hypocentral distance as,

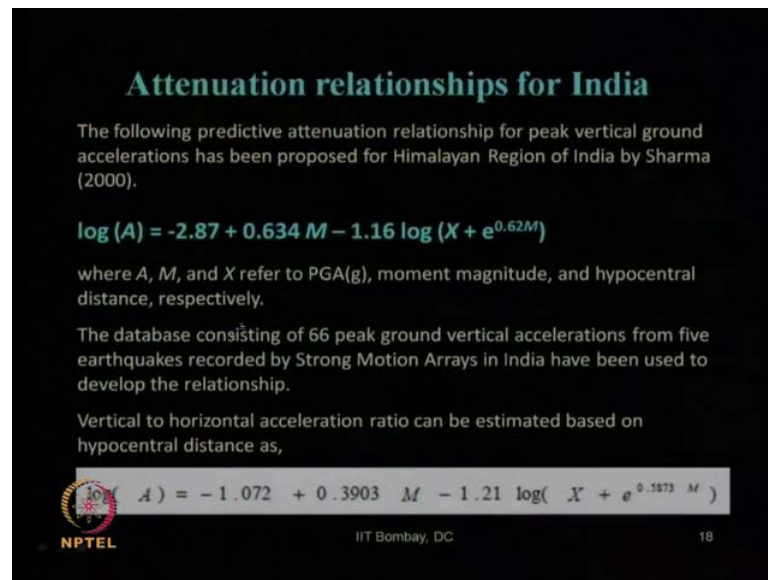
$$\log(A) = -1.072 + 0.3903 M - 1.21 \log(X + e^{0.1873 M})$$

NPTEL IIT Bombay, DC 18

Coming to some more attenuation relationship for India the following predictive attenuation relationship for peak vertical ground acceleration, this is vertical ground acceleration you remember not the horizontal has been proposed for Himalayan regions; so this specific to the Himalayan close to Himalayan region of earthquake of India by Sharma. This is a professor M. L. Sharma IIT Roorkee and this is research group they develop, they initially proposed for the horizontal peak ground acceleration also, but which I am referring here in this slide that is for the peak vertical ground acceleration. What is the equation they proposed, this is the simplest equation they proposed for determining the attenuation relationship of vertical seismic acceleration. In this equation this  $A$  is peak ground acceleration vertical, in the unit of  $g$  then  $M$  is moment magnitude of earth quake and this  $X$ , capital  $X$  refers to the hypo central distance in the unit kilometer.

So, once you know  $M$  value  $X$  value at your particular location of your site, from your seismic zone map and other information like active fault close to that site, you can automatically calculate what will be your attenuated value of the peak ground acceleration, peak vertical ground acceleration using this expression for the Himalayan region of India. Suppose somebody want to construction in Roorkee region construction they should attenuation relationship to the obtain peak vertical ground acceleration, which was proposed in two thousand. This is the paper, which appear in again the world conference proceedings in two thousand by professor M. L. Sharma.

(Refer Slide Time: 19:26)



**Attenuation relationships for India**

The following predictive attenuation relationship for peak vertical ground accelerations has been proposed for Himalayan Region of India by Sharma (2000).

$$\log(A) = -2.87 + 0.634 M - 1.16 \log(X + e^{0.62M})$$

where  $A$ ,  $M$ , and  $X$  refer to PGA(g), moment magnitude, and hypocentral distance, respectively.

The database consisting of 66 peak ground vertical accelerations from five earthquakes recorded by Strong Motion Arrays in India have been used to develop the relationship.

Vertical to horizontal acceleration ratio can be estimated based on hypocentral distance as,

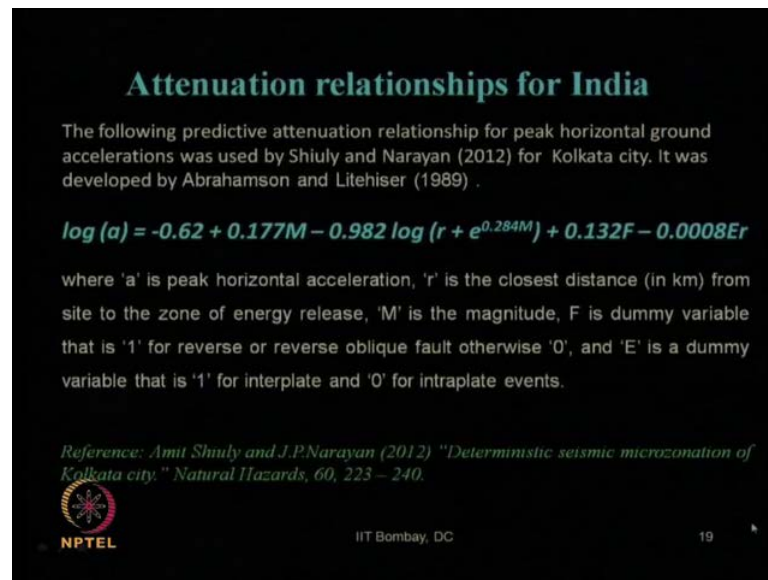
$$\log(A) = -1.072 + 0.3903 M - 1.21 \log(X + e^{0.1873 M})$$

NPTEL IIT Bombay, DC 18

So, how much database was consisted to derive this attenuation relationship? The database consisting 66 peak ground vertical acceleration from 5 earth quake record by strong motion arrays in India. So, only 5 earth quake were used where total 66 data of this peak vertical acceleration were recorded and those were used to develop this equation. So, somebody wants to comment on this equation obviously you can mention that from two thousand to this two thousand thirteen there is a chance of in change of this coefficients etcetera, when you want to obtain this log of A because between two thousand to two thousand thirteen whatever accelerations, whatever earth quake have occurred if those historical earth quake have experience or have the record of this peak vertical ground acceleration you can use those further for the updation of this equation.

So, which is possible. So, that is why as I said earlier this is the continuous process of evaluation of the research in this topic. And another equation Sharma et al had proposed in this paper like vertical to horizontal acceleration ratio. How it can be compared and can be estimated based on the hypo central distance. So, this is for the horizontal one they proposed this equation, which originally was proposed earlier than this two thousand.

(Refer Slide Time: 21:06)




**Attenuation relationships for India**

The following predictive attenuation relationship for peak horizontal ground accelerations was used by Shiuly and Narayan (2012) for Kolkata city. It was developed by Abrahamson and Litehiser (1989) .

$$\log (a) = -0.62 + 0.177M - 0.982 \log (r + e^{0.284M}) + 0.132F - 0.0008Er$$

where 'a' is peak horizontal acceleration, 'r' is the closest distance (in km) from site to the zone of energy release, 'M' is the magnitude, F is dummy variable that is '1' for reverse or reverse oblique fault otherwise '0', and 'E' is a dummy variable that is '1' for interplate and '0' for intraplate events.

Reference: Amit Shiuly and J.P.Narayan (2012) "Deterministic seismic microzonation of Kolkata city." *Natural Hazards*, 60, 223 – 240.

 NPTEL IIT Bombay, DC 19

Now, let us come to another attenuation relationship of India which is a very recent finding for Kolkata city. You can first see this reference of the paper by Shiuly and Narayan in two thousand twelve paper, deterministic seismic microzonation of Kolkata city. In which appeared in the Natural Hazards Journal this is the volume number page number. You should go through this paper this is again from IIT Roorkee earthquake engineering department. They developed the following predictive attenuation relationship for peak horizontal ground acceleration for Kolkata city. It was developed by Abrahamson and Litehiser in nineteen eighty nine based on their model of equation, not based on their data. They did not provide that the coefficients the coefficients proposed by Shluly and Narayan using the historical development of that region.

So, what they have proposed log of a equals to minus 0.62 plus 0.177M minus 0.982 log of r plus e power 0.284 M plus 0.132 F minus 0.0008 E r. In this case a is the peak horizontal acceleration in g unit, R is the closet distance in the kilometer unit from the site to the zone of energy release. That means site to the hypo center, right or focus of the earth quake m is the earth quake magnitude. F is a dummy variable. Why they have used this dummy variable?

To take care of different type of fault moment, they mentioned that use 1 the value of F you can use as 1, when it is a reverse fault or oblique fault, otherwise if it is not a reverse oblique fault, otherwise fault like normal fault etcetera you can use it value of f as 0. This

parameter this parameter vanishes. So, this parameter appears 0.132 comes when it is a reverse or oblique fault.

And E is again another dummy parameter Er, r is already the distance I said E is again another dummy parameter comes because of the, whether it is a inter plate earth quake or intra plate. That is between the plate boundary or within the plate, that is what it means. So, you have to use the value of E as 1 if is a inter plate earth quake, but if it a intra plate earth quake then put the value of E as 0, that is what they have proposed. Ok now, let us see other attenuation relationship developed for India. You can see in this slide.

(Refer Slide Time: 24:14)

### Attenuation relationships for India

The following predictive attenuation relationship for intensity has been proposed for India by Martin and Szeliga (2010):

$$\log N = a + b (I-2)$$

where  $N$  is the cumulative no. of observations per year and  $I$  is the intensity value.

| City      | a     | b     | Return Time (Years) for Intensity |     |     |
|-----------|-------|-------|-----------------------------------|-----|-----|
|           |       |       | V                                 | VI  | VII |
| Mumbai    | -0.81 | -0.27 | 42                                | 78  | 145 |
| Delhi     | -0.66 | -0.18 | 16                                | 24  | 36  |
| Bangalore | -1.07 | -0.28 | 81                                | 155 | 295 |
| Kolkata   | -0.72 | -0.14 | 14                                | 19  | 26  |
| Chennai   | -1.04 | -0.20 | 44                                | 69  | 110 |

Reference: Stacey Martin and Walter Szeliga (2010) : "A Catalog of Felt Intensity Data of Earthquakes in India from 1636 to 2009." *Bulletin of the Seismological Society of America*, 100( 2), 562-569.

NPTEL IIT Bombay, DC 20

First let us see the reference; this is Stacey Martin and Walter Szeliga in two thousand ten. Their paper which shows a catalog of felt intensity data for 570 earth quakes in India during the period sixteen thirty six to two thousand nine. So, they have taken all 570 recorded data between sixteen thirty six to two thousand nine. These are intensity based data, remember these are not magnitude scale data. This is a different attenuation relationship compare to the other what we have learnt so far. This paper was published in the journal Bulletin of Seismological Society of America, B S S A in short. We call this is the volume number of the journal, you should go through this paper because this gives you the idea of the intensity.

Because in the olden days it was very difficult to obtain the magnitude because in those days determination of the magnitude scale was not that sophisticated as what we have



today. So, in those days mostly it is based on the people's feelings or how they felt about a particular earth quake. That is why it is mostly on the intensity based data. So that is why the sixteen thirty six towards those ages you will get mostly intensity based data and they have used all the intensity based data of the 570 earth quake which are having record actually, between this time period sixteen thirty six to two thousand nine. And what they have proposed finally, that the predictive attenuation relationship for intensity.

So using this equation of course, you are getting the intensity because they have used the intensity data;  $\log N = a + bI - 2$ . So, what is this N? N is the cumulative number of observations per year and it is nothing but the intensity value. That is per year how many observed earthquake you can expect for a particular intensity and I is the value of the intensity and a and b are the coefficients. And what are those coefficients they proposed in this table. You can see for various cities they have proposed various coefficients and depending on what are the return time in terms of years for the intensity. That return times is nothing but, this N right this is the cumulative number of observations per year which automatically gives you the return period right.

So, using this expression you can finally, get the value of I knowing the value of N, a and b from this table suppose for Mumbai region you want to find out the intensity, you can based on this return period based on whether you want a intensity of six or intensity of five, intensity of six or intensity of seven you should use corresponding value of return period like this and corresponding value of coefficients a and b like this. If you want to do the estimation for this intensity based attenuation in Delhi region you should use these values. if you want to use for Bangalore region these values. If you want to use Kolkata region these values, for Chennai region these values. What you can look at here, see return time in years you can see over there for Delhi for Kolkata these are pretty low values, low values means they have more chances of occurrence.

So, intensity finally what you will get using this equation will be much higher than Mumbai and Bangalore. Where Mumbai and Bangalore you are having large values of this return time for the intensity, clear. So finally, you can use these values to obtain a particular intensity at a particular return period of your value at a particular city.

(Refer Slide Time: 28:33)

**Attenuation relationships for India**

The following predictive attenuation relationship for peak horizontal ground acceleration ( $A$ ) for south India has been proposed for Peninsular India by Dunbar et al. :

$$\log A = -1.902 + 0.249M_w$$

where  $M_w$  is the moment magnitude. The relationship between intensity ( $I_o$ ) and peak horizontal ground acceleration were also obtained given as:

$$M_s = M_w = 0.605I_o + 1.376$$
$$\log A = -1.902 + 0.249(0.605I_o + 1.376)$$
$$\log A = -1.56 + 0.15I_o$$

Reference: P.K. DUNBAR, R.G. BILHAM, M.J. LAITURI "Earthquake Loss Estimation in India Based on Macroeconomic Indicators"

NPTEL IIT Bombay, DC 21

Now, let us come to another attenuation relationship of India which was proposed by Dunbar et al. Dunbar et al this earthquake loss estimation for India based on macroeconomic indicators, they proposed this equation of attenuation relationship for peak horizontal ground acceleration  $A$  for the south India.

So, this equation is valid only for the southern India and it has been proposed for peninsular India that is southern part of the peninsular India you can say. So, this equation gives you acceleration value  $M_w$  is in moment magnitude and the relationship between intensity, that is intensity of earthquake and the peak horizontal ground acceleration were also given by this expressions. That is suppose somebody is having intensity scale how to convert it to magnitude scale of  $M_w$  or  $M_s$  using this equation for this region of earthquake. Whatever data they have used and then if you use the intensity based equation the same equation, if you put in terms of  $M_w$  just change to this one, right. So, on simplification the same equation in terms of intensity will be like this.

So what does it mean, suppose in southern India you are trying to find out what is the acceleration attenuation, attenuated value of acceleration at a particular site you should only know the intensity or you should only know the magnitude. Then you will get the what is the estimated value of the attenuation relationship, but remember the drawback or limitation of this equation. These are based on only the magnitude parameter or the

size of the earthquake not based on the distance, which is also very important. So that distance criteria is not considered, so that is the major drawback or limitation of using this equation. Now, let us come to another attenuation relationship let us look at here.

(Refer Slide Time: 30:52)

**Attenuation relationships for India**

The following predictive attenuation relationship for pseudo spectral velocity (PSV) for North East India has been proposed by Das et al. :

$$\log[PSV(T)] = c_1(T) + c_2(T)M + c_3(T)h + c_4(T) \log(\sqrt{R^2 + h^2}) + c_5(T)v$$

where  $M$  is the earthquake magnitude,  $R$  is the epicentral distance,  $h$  is the focal depth,  $T$  is the time-period of single-degree-of-freedom (SDOF) oscillator and  $v = 0$  and  $1$  for horizontal and vertical motions, respectively.

Authors used 261 accelerograms recorded on stiff soil/rock sites for six earthquake events.

Reference: Sandip Das, I.D. Gupta and Vinay K. Gupta "A New Attenuation Model for North East India."

NPTEL IIT Bombay, DC 22

So, the following predictive attenuation relationship for pseudo spectral velocity P S V for north for east India has been proposed by Sas et al. So, this is the paper of Das, Gupta and Gupta this is done by IIT Kanpur research group professor Vinay K Gupta and his student and I. D. Gupta from i m d. So, this is the paper new attenuation model for north east India. So this is as I was mentioning suppose somebody wants to design in the Guwahati city they should use this kind of attenuation relationship.

So, region specific attenuation relationship use of that is very essential you can see for north east India the pseudo spectral velocity, this is the spectral velocity remember it is not the peak value of the velocity, it is not the acceleration also. This is a spectral velocity spectral, velocity we have seen how it is idealized compare to or based on the single degree of freedom mass print dashpot model.

So, where this  $M$  value is the earth quake  $R$  value is the epicentral distance,  $h$  is the focal depth and  $T$  is the time period of a single degree of freedom system oscillator, this  $T$  and this value of  $v$  is equals to  $0$  and  $1$ , whether it is a horizontal or vertical. So for horizontal motion it will be  $0$  for vertical motion it will be  $1$ . Then you will get corresponding value of pseudo spectral velocity in horizontal direction and vertical direction. So, they used

261 accelerogram recorded data on the stiff soil rocky site from the 6 earthquake events in this region of north east India.

(Refer Slide Time: 32:47)

| Period $T$ | $c_1'(T)$ | $c_2'(T)$ | $c_3'(T)$ | $c_4'(T)$ | $c_5'(T)$ |
|------------|-----------|-----------|-----------|-----------|-----------|
| 0.040      | -0.5402   | 0.3140    | 0.0039    | -0.9001   | -0.4251   |
| 0.048      | -0.4277   | 0.3097    | 0.0038    | -0.8873   | -0.4164   |
| 0.055      | -0.3065   | 0.3042    | 0.0039    | -0.8854   | -0.4107   |
| 0.065      | -0.1401   | 0.2974    | 0.0039    | -0.8854   | -0.4055   |
| 0.080      | 0.0614    | 0.2913    | 0.0040    | -0.8845   | -0.4057   |
| 0.095      | 0.2108    | 0.2878    | 0.0040    | -0.8854   | -0.4157   |
| 0.110      | 0.3427    | 0.2852    | 0.0042    | -0.9009   | -0.4334   |
| 0.130      | 0.4989    | 0.2849    | 0.0044    | -0.9326   | -0.4607   |
| 0.150      | 0.6054    | 0.2912    | 0.0046    | -0.9684   | -0.4896   |
| 0.180      | 0.6374    | 0.3101    | 0.0047    | -1.0019   | -0.5288   |
| 0.220      | 0.5375    | 0.3301    | 0.0046    | -0.9870   | -0.5578   |
| 0.260      | 0.4110    | 0.3421    | 0.0046    | -0.9472   | -0.5716   |
| 0.300      | 0.2716    | 0.3498    | 0.0045    | -0.8965   | -0.5741   |
| 0.360      | 0.0446    | 0.3608    | 0.0044    | -0.8207   | -0.5639   |
| 0.420      | -0.1583   | 0.3738    | 0.0043    | -0.7682   | -0.5479   |
| 0.500      | -0.2913   | 0.3912    | 0.0040    | -0.7505   | -0.5364   |
| 0.600      | -0.3369   | 0.4145    | 0.0032    | -0.7672   | -0.5375   |
| 0.700      | -0.4101   | 0.4418    | 0.0024    | -0.7797   | -0.5397   |
| 0.850      | -0.6807   | 0.4854    | 0.0011    | -0.7384   | -0.5378   |
| 0.950      | -1.1532   | 0.5225    | -0.0002   | -0.5955   | -0.5285   |

Reference: Das et al. IIT Bombay, DC 23

And these are the various values of those C 1, C 2, C 3 etcetera based on different values of the period. These values of the period are typical range of period for a simple harmonic oscillator and within the practical range of our structural periods, as I have already mentioned earlier.

(Refer Slide Time: 33:08)

### Attenuation relationships for India

The following predictive attenuation relationship for peak horizontal ground acceleration (PGA) for Guwahati city has been proposed by Nath et al. (2009) :

$$\ln(PGA) = 9.143 + 0.247M - 0.014(10-M)^3 - 2.697 \ln(r_{rup} + 32.9458 \exp(0.0663M))$$

where  $M$  is the earthquake moment magnitude,  $r_{rup}$  is the rupture distance (km) and PGA is in  $g$ .

Reference: S. K. Nath, A. Raj, K. K. S. Thingbaijam, and A. Kumar (2009): "Ground Motion Synthesis and Seismic Scenario in Guwahati City - A Stochastic Approach." *Seismological Research Letters*, 80(2), 233 – 242.

NPTEL IIT Bombay, DC 24

Now, coming to another attenuation relationship for India; let us see the following predictive attenuation relationship which is proposed for peak horizontal ground acceleration.

(Refer Slide Time: 33:25)


**Attenuation relationships for India**

The following predictive attenuation relationship for peak horizontal ground acceleration (*PGA*) for Guwahati city has been proposed by Nath et al. (2009) :

$$\ln(PGA) = 9.143 + 0.247M - 0.014(10-M)^3 - 2.697 \ln(r_{rup} + 32.9458 \exp(0.0663M))$$

where *M* is the earthquake moment magnitude, *r<sub>rup</sub>* is the rupture distance (km) and *PGA* is in *g*.

Reference: S. K. Nath, A. Raj, K. K. S. Thingbaijam, and A. Kumar (2009): "Ground Motion Synthesis and Seismic Scenario in Guwahati City - A Stochastic Approach." *Seismological Research Letters*, 80(2), 233 - 242.

 NPTEL IIT Bombay, DC 24

That is PGA in the horizontal direction for Guwahati city only, remember this attenuation relationship first of all it is for peak horizontal ground acceleration and it is only for the Guwahati city. That was the proposed by Nath et al in two thousand nine. Professor Nath is from IIT Kharagpur geology geosciences department.

So, his research group proposed this attenuation relationship for a particular city based on the collected historical earthquake data of that region. So, what is the proposed relationship, that natural log of PGA equals to 9.143 plus 0.247 M minus 0.014 times 10 minus M whole cube minus 2.697 natural log of r rupture plus 32.9458 e to the power 0.0663 M. In this case m they have mentioned it is a earthquake moment magnitude that is M W and r r u p is nothing but the rupture distance. Rupture distance that is from the site to your focal focus point or the hypo central point in kilometer unit and PGA this value what you are getting in terms of g.

And this is the paper where you will get the details you should go through this paper like Nath et al two thousand nine, ground motion synthesis and seismic scenario in Guwahati city as stochastic approach, it was published in the Seismological Research Letter. We call in short as SRL. This is the volume number and this is the page numbers.

(Refer Slide Time: 35:19)


**Attenuation relationships for India**

The following predictive attenuation relationship for peak horizontal ground acceleration (*PGA*) for Bhuj has been proposed by Iyengar and Raghukanth (2002)

$$(PGA/g) = 38.82/R^{1.12}$$

where *R* is the hypocentral distance (km)

Reference: R. N. Iyengar and S. T. G. Raghukanth (2002) "Strong ground motion at Bhuj city during the Kutch earthquake." *CURRENT SCIENCE*, 82(11), 1366 - 1372.

 NPTEL IIT Bombay, DC 25

Now let us see another attenuation relationship which was proposed for peak horizontal ground acceleration for P G A, for Bhuj region only. That is in Gujarat, Bhuj region after that two thousand one Bhuj earthquake Iyengar and Raghukanth the professor Iyengar from ISSC Bangalore and his Phd student those days proposed this equation in two thousand two. You can see the reference of that paper this Iyengar and Raghukanth two thousand two, strong ground motion at Bhuj city during the Kutch earthquake.

It was published in the journal Current Science, this is the volume number this is the page number. So how they have proposed? They have proposed that PGA that is peak ground acceleration in horizontal direction, they mentioned it clearly in the paper, in terms of g. That can be calculated as 38.82 divided by R two the power 1.12. Where this R is the hypo central distance in the unit of kilometer and remember in this case it is irrespective of magnitude, it is they have propped only the distance based. So, that is a major limitation of this equation because they consider only one scenario earthquake which is the Bhuj earthquake or that Kutch region earth quake.

So, that is why based on whatever the distance changes based on the change of distance at various locations in Gujarat and that vicinity location in Gujarat that facility, how the PGA value for a design of any structure will change that is what their problem statement or their goal to obtain. That is in the Bhuj region suppose if an earth quake similar to of magnitude of Bhuj two thousand one earth quake occurs how much value of PGA

attuned value of PGA in horizontal direction should be used based on this distance, hypo central distance from the site to the source can be used or can be estimated for design of civil structures.

(Refer Slide Time: 37:42)


**Attenuation relationships for India**

Jain et al. (2000) proposed following model for attenuation relationship for India using database from four regions,

$$\ln(\text{PGA}) = b_1 + b_2M + b_3R + b_4 \ln(R)$$

where PGA is in g, for central Himalayan earthquakes  $b_1 = -4.135$ ,  $b_2 = 0.647$ ,  $b_3 = -0.00142$ ,  $b_4 = -0.753$  and  $\sigma = 0.59$  and for non-subduction earthquakes in N.E. India  $b_1 = -3.443$ ,  $b_2 = 0.706$ ,  $b_3 = 0$ ,  $b_4 = -0.828$  and  $\sigma = 0.44$  (coefficients of other equations not given here because they are for a particular earthquake).

- a Central Himalayan earthquakes (thrust): (32 SMA records, 117 SRR records), 3 earthquakes with  $5.5 \leq M \leq 7.0$ , focal depths  $10 \leq h \leq 33$  km and  $2 \leq R \leq 322$  km.
- b Non-subduction earthquakes in NE India (thrust): (43 SMA records, 0 SRR records), 3 earthquakes with  $5.2 \leq M \leq 5.9$ , focal depths  $33 \leq h \leq 49$  km and  $6 \leq R \leq 243$  km.
- c Subduction earthquakes in NE India: (33 SMA records, 104 SRR records), 1 earthquake with  $M = 7.3$ , focal depth  $h = 90$  km and  $39 \leq R \leq 772$  km.
- d Bihar-Nepal earthquake in Indo-Gangetic plains (strike-slip): (0 SMA records, 38 SRR records), 1 earthquake with  $M = 6.8$ , focal depth  $h = 57$  km and  $42 \leq R \leq 337$  km.


 Ref: Douglas (2001), ESEE Report IIT Bombay, DC 26

Now, coming to next attenuation relationship for India as proposed by Jain et al two thousand; this research was done by professor S. K. Jain at IIT Kanpur and research group in two thousand. Now he is director IIT Gandhi Nagar. So, their research group proposed the following model for attenuation relationship for India using the database from four region. So, we will see, what are those four regions? Let us first look at the attenuation relationship. So this is the attenuation relationship natural log of PGA is equals to b 1 plus b 2 times M plus b 3 times R plus b 4 times natural log of R, so where this PGA is in g. So as I say earlier whatever value your getting using this equation that will be suppose 0.236 g like that. So, for central Himalayan earth quake he has proposed this values of quotients b 1, b 2, b 3, b 4 like this like b 1 is minus 4.135 b 2 is 0.647 b 3 is minus 0.00142 b 4 is minus 0.753 and the sigma value is 0.59 for that whatever earth quake they have used and for non subduction earth quakes in north east India.

So, this was for the central Himalayan earthquake, means suppose the Uttaranchal or Uttarakhand region, that is the central Himalayan region and for the north east India. That is for Assam and other places, there values or coefficients they have proposed different coefficients you can see over here. So, the coefficients of other equations not

given, because they are for a particular earthquake; that is for a single earthquake you have used these two are the generalized equation.

So how they have divided the database into four regions, so these are the four regions a, b, c, d as you can see they have divided in four regions like central Himalayan earthquake, those are thrust type. How many record they had? 32 S M A records and 117 S R R records from three earthquakes with the magnitude between the 5.5 to 7. So that is the magnitude range they have taken and for that earth quake focal depths were between 10 to 33 kilometer.

So, which are essentially shallow earth quake as we know and that hypo central distance capital R was reported to be between 2 to 322 kilometer that is, those earth quake data points which they have used within this range of hypo central distance. The second category or second region they have considered is none subduction earth quake in the north east India. In the north east India the thrust type of earth quake again thrust type false moment 43 S M A records they have used and no S R R records for that. Again 3 earth quake data they have used within the range of values between 5.2 to 5.9, which is very marginal range I will say.

So focal depth they have used between 33 to 49 kilometer which is again a shallow earth quake and the value of R that is hypo central distance they have used between 6 to 243 kilometer; whereas category sea or region sea they sub divided or considered as subduction earth quake in the north east India. For that they had taken 33 S M A record and 104 S R R records from one particular earth quick. That is why it is mentioned over here that other region equations are developed based on a single earth quake, particular earth quake.

So cannot be generalized, unless you have more number of earth quake you cannot generalized. So, for one particular earth quake of magnitude 7.3 only with focal depth of h equals to 90 kilometers. So, we can say it is something about intermediate earth quake, not a pure shallow earth quake and R value that is hypo central was between 39 to 722 kilometer at different recording stations for the same earth quake. And the forth category or forth region they considered in their study like Bihar, Nepal earth quake in indo gangetic plain where the strike slip type of fault moment had occurred. These are thrust fault moment, this is strike type of fault moment here 0 SMA records, but 38 SRR



records they used for one particular earthquake. Again for magnitude of 6.8 and focal depth  $h$  was 57 kilometer, which is again a shallow earth quake and that hypo central distance was between 42 to 337 kilometer for different recording stations. So, these are the input data based on which they had proposed this equation.

So, when we are planning to use this equation, first of all we have to be specific for which region we want to use. So, suppose for central Himalayan region to one if we want to find out the PGA value we should use this coefficients, if we want to use for north east India we should use this coefficients and we should remember that this are valid for which range of magnitude range. Beyond that we should not use this equation for prediction of the peak ground acceleration attenuation relationship.

(Refer Slide Time: 43:44)

**M.L.Sharma et al. (2009)**

$$\log A = b_1 + b_2 M_W - b_3 \log \sqrt{R_{JB}^2 + b_4^2} + b_5 S + b_6 H$$

Where,  $R_{JB}$  = distance to surface projection of the rupture  
 $\eta = 5\%$ ,  
 No. of events considered = 201 (58 from India and 143 from Iran)  
 Periods = 0.04 – 2.5 sec  
 $b_1, b_2, b_3, b_4, b_5$  &  $b_6$  are regression coefficients  
 $A$  = Spectral acceleration in terms of  $m/sec^2$   
 $S = 1$  (for rock sites) &  $0$  (for others)  
 $H = 1$  (for strike slip mechanism) &  $0$  (for a reverse mechanism)  
 magnitude = 7 and Distance = 232 km.

NPTEL IIT Bombay, DC 27

Coming to another attenuation relationship as proposed by M. L. Sharma et al in two thousand nine, again from I I T Roorkee earthquake engineering department. This is the equation they have proposed  $\log$  of  $A$  equals to  $b_1$  plus  $b_2$  times  $M_W$  minus  $b_3$  times  $\log$  of root over  $R_{JB}$  square plus  $b_4$  square plus  $b_5$  times  $S$  plus  $b_6$  times  $H$ . Where this  $R_{JB}$  is the distance to surface projection of the rupture with damping ratio consideration of 5 percent, number of events, how many events they considered. Total 201, 58 from India and 143 from Iran.

So, we cannot say it is valid only for India it is a punched equation proposed using different earth quake from different regions and all the coefficients etcetera. Magnitude are given over here.

(Refer Slide Time: 44:33)

**Mandal et al. (2009)**

$$\ln(Y) = -7.9527 + 1.4043M_w - \ln(r_{jb}^2 + 19.82)^{1/2} - 0.06$$

Where,  $r_{jb}$  = distance to surface projection of the rupture

For  $3.1 < M_w \leq 7.7$   
Standard deviation =  $\pm 0.8243$

*Prantik Mandal, N.Kumar, C.Satya Murthy and I.P. Raju (2009) "Ground motion attenuation relation from strong motion records of the 2001  $M_w = 7.7$  Bhuj Earthquake sequence (2001 - 2006), Gujrat, India" Pure and Applied Geophysics, Volume 166, Issue 3, pp.451-469.*

**NPTEL** IIT Bombay, DC 28

Another researches, another group of researches Mandal et al in two thousand nine, this is the paper you can see. This group of researchers they published ground motion attenuation relation from strong motion records of the two thousand one M W 7.7 Bhuj earth quake sequence. So two thousand one to two thousand six data they have collected post Bhuj earth quake.

During and after Bhuj earthquake Gujarat, India. It has been published in the journal Pure and Applied Geophysics, this volume issue number page number. You can see this attenuation relationship gives the acceleration again and this r g b in this equation is nothing but distance to the surface projection of the rupture in kilometer unit and M W moment magnitude value is valid between this range and standard deviation is this much.

(Refer Slide Time: 45:27)

**Gupta (2010)**


$$\log(Y) = C_1 + C_2M + C_3h + C_4R - g \log R + C_5 \frac{S}{S_C} + C_6 \frac{S}{S_D} + C_7 \frac{S}{S_E}$$

Where, Y = PHA (cm/s<sup>2</sup>)  
h = focal depth in km (limited to 100km for deeper events)

$$R = \sqrt{D_{\text{fault}}^2 + D^2}$$

g = Geometric attenuation factor

*I.D. Gupta(2010) "Response spectral attenuation relations for in-slab earthquakes in Indo-Burmese subduction zone" Soil Dynamics and Earthquake Engineering, Volume 30, Issue 5, May 2010, pp.368-377.*

 NPTEL IIT Bombay, DC 29

Another attenuation relationship for acceleration this Y is PHA peak horizontal acceleration in centimeter per centimeter per second square unit. As proposed by Gupta in two thousand ten. This I. D. Gupta in two thousand ten this is the title of the paper response spectral attenuation relation for in slab earthquakes in indo Burmese subduction zone so this is a indo Burmese subduction zone they have considered the earthquake. Which is published in Soil Dynamics and Earth Quake Engineering Journal, these are the details of the journal issue. This h in this equation, this h is the focal depth in the kilometer unit and has to be limited within 100 kilometer. For the deeper events you have to use this equation.

(Refer Slide Time: 46:15)

### Attenuation relationships for India


Szeliga et al. (2010) proposed intensity attenuation relationship for India considering earthquakes felt since 1762 as follows,

*(here out of 570 earthquakes about 100 were instrumented to record magnitudes and among these common set of 29 earthquakes were finally used for prediction)*

$$I = a + bM_w + cR + d \log(R)$$

*Here, R is hypocentral distance in km and  $M_w$  is moment magnitude. Constants a, b, c and d need to be calculated using following table.*

| Province | Number of Events | a'          | b'          | c'               | d'           |
|----------|------------------|-------------|-------------|------------------|--------------|
| India    | 29               | 5.57 ± 0.58 | 1.06 ± 0.07 | -0.0010 ± 0.0004 | -3.37 ± 0.25 |
| Craton   | 17               | 3.67 ± 0.79 | 1.28 ± 0.10 | -0.0017 ± 0.0006 | -2.83 ± 0.30 |
| Himalaya | 12               | 6.05 ± 0.94 | 1.11 ± 0.10 | -0.0006 ± 0.0006 | -3.91 ± 0.38 |

 NPTEL IIT Bombay, DC 30

Other equation like Szeliga et al had proposed the in two thousand ten this is the intensity based equation. Once again consider the earthquake felt attenuation relationship since seventeen sixty two, seventeen hundred and sixty two to two thousand nine they have taken, but as I said the older earthquake only the intensity scale will be available.

So, intensity based again 570 data point, 100 instrumented record. That is about 570 earthquake point 500 were instrumented which are in the recent days. So, this is the proposed intensity based attenuation relationship. So, for different region like entire India how many events, for Craton region, for Himalayan region how many events and these are the coefficients a, b, c, d and R is the hypo central distance in kilometer M W is the moment magnitude can be used.

(Refer Slide Time: 47:12)

### Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

Ground Motion Prediction Equation (GMPE) can be expressed commonly as,

$$\ln Y = f_{mag} + f_{dis} + f_{fs} + f_{sm} + f_{st} + f_{hw} + B$$

where the magnitude term is given by the expression

$$f_{mag} = \begin{cases} c_0 + c_1 M; & M \leq 5.5 \\ c_0 + c_1 M + c_2 (M - 5.5); & 5.5 < M \leq 6.5 \\ c_0 + c_1 M + c_2 (M - 5.5) + c_3 (M - 6.5); & M > 6.5 \end{cases}$$

the distance term is given by the expression


$$f_{dis} = (c_4 + c_5 M) \ln(\sqrt{R_{RUP}^2 + c_6^2})$$

the style-of-faulting (fault mechanism) term is given by the expressions

$$f_{fs} = c_7 F_{fs} + c_8 F_{NM}$$

$$f_{st} = \begin{cases} Z_{TDR}; & Z_{TDR} < 1 \\ 1; & Z_{TDR} \geq 1 \end{cases}$$

the hanging-wall term is given by the expressions



31

Now, functional form of attenuation relationship as proposed by Bozorgnia et al in two thousand ten; this is the ground motion prediction equation, ground motion prediction equation which are nothing but an attenuation relationship equation, which can be standardized and commonly expressed in this form. We have already seen this, right? Several researches are proposed attenuation relationship or ground motion prediction. Why another name? It is called G M P E because in this way you can predict what can be a ground motion at a particular site. So, that is why G M P E is a common term which people use in our earthquake engineering problem and these are the various efficient f magnitude, f distance.

So, on this various term this attenuation value of whether acceleration or intensity or velocity it depends on. That is the magnitude term based on different scale of magnitude. Distance term how far it is from the source to site, then style of faulting term that is what type of fault. We have seen fault based attenuation relationship earlier. So, this is the fault based attenuation, then hanging wall term. Next one oblique strike slip all these hanging wall type of term.

(Refer Slide Time: 48:36)

### Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

$$f_{\text{site}} = c_1 f_{\text{site},R} f_{\text{site},M} f_{\text{site},Z} f_{\text{site},\delta}$$

$$f_{\text{site},R} = \begin{cases} 1; & R_{JB} = 0 \\ \frac{[\max(R_{RUP}, \sqrt{R_{JB}^2 + 1}) - R_{JB}]/\max(R_{RUP}, \sqrt{R_{JB}^2 + 1})}{(R_{RUP} - R_{JB})/R_{RUP}}; & R_{JB} > 0, Z_{TOR} < 1 \\ & R_{JB} > 0, Z_{TOR} \geq 1 \end{cases}$$

$$f_{\text{site},M} = \begin{cases} 0; & M \leq 6.0 \\ 2(M - 6.0); & 6.0 < M < 6.5 \\ 1; & M \geq 6.5 \end{cases}$$

$$f_{\text{site},Z} = \begin{cases} 0; & Z_{TOR} \geq 20 \\ (20 - Z_{TOR})/20; & 0 \leq Z_{TOR} < 20 \end{cases}$$

$$f_{\text{site},\delta} = \begin{cases} 1; & \delta \leq 70 \\ (90 - \delta)/20; & \delta > 70 \end{cases}$$

NPTEL IIT Bombay, DC 32

So, these are various functions and shallow site response start.

(Refer Slide Time: 48:38)

### Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

the shallow site response term is given by the expression

$$f_{\text{site}} = \begin{cases} c_{10} \ln\left(\frac{V_{230}}{k_1}\right) + k_2 \left[ \ln\left[A_{1100} + c\left(\frac{V_{230}}{k_1}\right)^n\right] - \ln[A_{1100} + c] \right]; & V_{230} < k_1 \\ (c_{10} + k_2 n) \ln\left(\frac{V_{230}}{k_1}\right); & k_1 \leq V_{230} < 1100 \\ (c_{10} + k_2 n) \ln\left(\frac{1100}{k_1}\right); & V_{230} \geq 1100 \end{cases}$$

and the basin (sediment depth) response term is given by the expression

$$f_{\text{sed}} = \begin{cases} c_{11}(Z_{2.5} - 1); & Z_{2.5} < 1 \\ 0; & 1 \leq Z_{2.5} \leq 3 \\ c_{12} k_3 e^{-0.75 Z_{2.5}} [1 - e^{-0.25(Z_{2.5} - 3)}]; & Z_{2.5} > 3 \end{cases}$$

NPTEL IIT Bombay, DC 33


That is whether it is a shallow earth quake type, intermediate or deep earth quake type. Then basin response term, that is the sediment or soft basing depth how much? So, based on that, this equation can be effected.

(Refer Slide Time: 48:56)

### Functional Form of Attenuation Relationship (Bozorgnia et al. 2010)

where  $Y$  is the estimate of the geometric mean horizontal component of "seismic coefficient"  $C_Y = F_Y / W$  for an inelastic SDF system;  $M$  is moment magnitude;  $R_{RUP}$  is the closest distance to the coseismic rupture plane (km);  $R_{JB}$  is the closest distance to the surface projection of the coseismic rupture plane (km);  $F_{RI}$  is an indicator variable representing reverse and reverse-oblique faulting,  $F_{NM}$  is an indicator variable representing normal and normal-oblique faulting,  $Z_{TOR}$  is the depth to the top of the coseismic rupture plane (km);  $\delta$  is the dip of the rupture plane ( $^\circ$ );  $V_{S,30}$  is the time-averaged shear-wave velocity in the top 30 m of the site profile (m/s);  $A_{1100}$  is the median estimate of PGA (in units of "g") on a reference rock outcrop  $V_{S,30} = 1100$  m/s;  $Z_{2.5}$  is the depth to the 2.5 km/s shear-wave velocity horizon, typically referred to as basin or sediment depth (km); and  $\epsilon$  is a random error term with zero mean and total standard deviation  $\sigma_{ln Y}$  given by the equation,

$$\sigma_{ln Y} = \sqrt{\sigma^2 + \tau^2}$$

  $\sigma$  is the intra-event, or within-earthquake, standard deviation and  $\tau$  is the inter-event, or between-earthquake, standard deviation

IIT Bombay, DC 34

So, based on all these influence parameter one can estimate what is the proposed value of PGA as given by Bozorgnia et al. So, this is the functional form means this is a generalized form, so if somebody wants to predict or propose a very good empirical relationship of attenuation relation or G M P, the best attenuation relationship will be to use all these terms, that is magnitude, distance, fault, hanging faults, site, basin everything. Then only we can say it is a complete attenuation relationship or complete G M P E or best G M P E.


(Refer Slide Time: 49:39)

### Fourier Amplitude Spectra

If  $f_{max}$  is assumed constant for a given geographic region (15 Hz and 20 Hz are typical values for Western & Eastern North America respectively), then the spectra for different quakes are functions of seismic moments  $M_0$  and  $f_c$  which can be related (Brune, 1970 & 1971) thus:

$$f_c = 4.9 \times 10^6 v_s \left( \frac{\Delta\sigma}{M_0} \right)^{1/3}$$

Where  $v_s$  is in km/sec,  $M_0$  is in dyne-cm, and  $\Delta\sigma$  is referred to as stress parameter or stress drop in bars. Values of 50 bars and 100 bars are common for stress parameters of Western & Eastern North America respectively.

 IIT Bombay, DC Ref: Kramer (1996) 35


Now, let us come to the Fourier amplitude spectra. Once again like earlier we have seen in Fourier amplitude spectra two values like corner frequency and the maximum frequency. So if  $f_{max}$  is assumed as constant for a given geographic region, for geographic site this  $f_{max}$  maximum value is constant. Say suppose it can be 15 hertz or 20 hertz as reported by western and eastern north America respectively, like for western north America it is 15 hertz for eastern part it is 20 hertz that  $f_{max}$  value. And this  $f_c$  then can be calculated using the seismic moment value in this fashion as proposed by Brune in nineteen seventy and then nineteen seventy one, where this  $v_s$  is in kilometer to the crustal velocity of the shear wave. And  $M_{naught}$  is in the unit of dyne centimeter and this  $\Delta\sigma$  refer to the stress parameter or stress drop in the unit of bar. That they mention that use 50 bar and 100 bar for western and eastern North America. So, using this you can find out what is the cut off frequency for Fourier series response spectra of amplitude, right. Then another important parameter is the ratio of  $v_{max}$  by  $a_{max}$ .

(Refer Slide Time: 50:08)

**Ratio  $v_{max}/a_{max}$**

This ratio is proportional to the magnitude and distance dependencies proposed by McGuire (1978) as shown in the table below:

| Site Conditions | Magnitude Dependence | Distance Dependence |
|-----------------|----------------------|---------------------|
| Rock sites      | $e^{0.40M}$          | $R^{0.12}$          |
| Soil sites      | $e^{0.15M}$          | $R^{0.23}$          |


IIT Bombay, DC
Ref: Kramer (1996)
36

So, ratio of  $v_{max}$  by  $a_{max}$  this ratio is the proportional to the magnitude and distance dependency proposed by McGuire in nineteen seventy eight as shown in the table below. That is depending on site condition whether it is rocky site or soil site magnitude dependency will be a function of  $e$  to the power 0.4 times  $M$  and for soil site it will be  $e$  to the power 0.15  $M$ .  $M$  is the magnitude of earthquake and distance dependency will be  $R$  to the power 0.12 and  $R$  to the power 0.23 based on rocky site or soil site.




So, using this suppose for a known earth quake, historical earthquake you know the magnitude of earth quake, now from that earth quake location point at another particular site where you are interested to estimate what will be my parameters of that  $v_{max}$  by a  $a_{max}$  in future you can estimate very easily. How you will estimate? For the older earth quake you should know the value of what is  $v_{max}$  and  $a_{max}$  at that particular site whether it is a rock site or soil site, based on that that will be known to you, that magnitude will be known to you. The distance within which you are using that is data point should be known to you. Now for different soil site or rock site based on their distance and the design value of earth quake, which you want to consider for the design based on that let us look at the slide here.

(Refer Slide Time: 52:41)

**Ratio  $v_{max}/a_{max}$**

This ratio is proportional to the magnitude and distance dependencies proposed by McGuire (1978) as shown in the table below:

| Site Conditions | Magnitude Dependence | Distance Dependence |
|-----------------|----------------------|---------------------|
| Rock sites      | $e^{0.40M}$          | $R^{0.12}$          |
| Soil sites      | $e^{0.15M}$          | $R^{0.23}$          |


IIT Bombay, DC
Ref: Kramer (1996)
36

It will be simply propositional to this magnitude dependency and the distance dependency so using a simple problem which is worked out in Kramar nineteen ninety six book you can understand this in a simpler way.

(Refer Slide Time: 52:57)


## Estimation of other parameters

### RMS Acceleration

Hanks and McGuire (1981) used a database of California earthquake of local magnitude 4.0 to 7.0 to develop an attenuation relationship for RMS acceleration for hypocentral distances between 10 and 100 km,

$$a_{rms} = 0.119 \frac{\sqrt{f_{max} / f_c}}{R}$$

where  $f_c$  is the corner frequency,  $f_{max}$  is the cutoff frequency, and  $R$  is in kilometer.



IIT Bombay, DC

Ref: Kramer (1996) 37


Then estimation of other spectral parameters I have mentioned the definition earlier. Now, how to estimate that I am mentioning here this root mean square acceleration can be computed using the equation proposed by Hanks and McGuire in nineteen eighty one this is valid for the Californian earth quake between the range of 4 to 7. And the hypo central distance between 10 to 100 kilometer. Then only this proposed empirical relationship is valid that is A R M S can be calculated as 0.199 times root over  $f_{max}$  by  $f_c$  by  $R$ . This  $f_c$  is corner frequency you know how to calculate corner frequency for this region because I have already mentioned this in the previous slide.  $f_{max}$  is the cut off frequency that is also known for this region western part of America and eastern part of north America it is known,  $R$  is in kilometer based on what distance you want to find out this R M S acceleration can be estimated, right.

(Refer Slide Time: 54:12)

Kavazanjian et al. (1985) used the definition of duration proposed by Vanmarcke and Lai (1980) with a database of 83 strong motion records from 18 different earthquakes to obtain

$$a_{rms} = 0.472 + 0.268M_w + 0.129 \log \left( \frac{0.966}{R^2} + \frac{0.255}{R} \right) - 0.1167R$$

where  $R$  is the distance to the closest point of rupture on the fault. The database was restricted to  $M_w > 5$ ,  $R < 110$  km (68 mi), rupture depths less than 30 km (19 mi), and soil thicknesses greater than 10 m (33 ft).



IIT Bombay, DC

Ref: Kramer (1996) 38

Then then Kavazanjian et al in nineteen ninety five use the definition of the duration proposed by these researches with a data base of 83 strong motions form 18 different earthquake and proposed another empirical relation of computing this root mean square acceleration, using this equation. Where  $R$  is the distance to the closest point of the rupture on the fault and the data base was restricted to  $M_w$  value more than 5. That is below 5 it should not be used and  $R$  that is the hypo central distance should be within 110 kilometer and rupture depth was less than thirty kilometer and the soil thickness will be greater than the 10 meter. Based on all these conditions these researches proposed the attenuation proposed the empirical relation to estimate the root mean square acceleration.


(Refer Slide Time: 55:04)

**Arias Intensity**

Campbell and Duke (1974) used data from California earthquakes to predict the variation of Arias intensity within 15 to 110 km (9 to 68 mi) of magnitude 4.5 to 8.5 events.

$$I_a (m / sec) = 313 \frac{e^{M_s(0.33M_s - 1.47)}}{R^{3.79}} S$$

|         |                |                                |
|---------|----------------|--------------------------------|
| where S | $0.57R^{0.46}$ | for basement rock              |
|         | $1.02R^{0.51}$ | for sedimentary rock           |
|         | $0.37R^{0.81}$ | for alluvium $\leq$ 60ft thick |
|         | $0.65R^{0.74}$ | for alluvium $>$ 60ft thick    |

 and R is the distance from the center of the energy release in kilometers.

IIT Bombay, DC Ref: Kramer (1996) 39

Further to estimate the arias intensity empirically Campbell and Duke in 1974 using the Californian earth quake data between the distance of 15 to 110 kilometer and between the magnitude range of 4.5 to 8.5 scale, they proposed using those historical data points. This is the empirical relation to estimate the arias intensity  $I_a$  in the unit of meter per second, using this relation  $313 \frac{e^{M_s(0.33M_s - 1.47)}}{R^{3.79}} S$ .

This S is dependent on the type of material in the site local soil type that is for basement rock S can be calculated using this expression, for sedimentary rock this expression, for alluvial soil within 60 feet of thickness this equation and alluvial soil of thickness more than 60 feet this expression, where this R is the distance between the center of energy released and the site of your concern in the unit of kilometer so basically the hypo central distance.


(Refer Slide Time: 56:28)

Wilson (1993) analyzed strong motion records from California to develop an attenuation relation which, using the Arias intensity definition of equation which can be expressed as

$$\log I_a (m/sec) = M_w - 2 \log R - kR - 3.990 + 0.365(1 - P)$$

where  $R = \sqrt{D^2 + h^2}$

$D$  is the minimum horizontal distance to the vertical projection of the fault plane,  $h$  is a correction factor (with a default value of 7.5 km (4.7 mi)),  $k$  is a coefficient of inelastic absorption (with default value of zero), and  $P$  is the exceedance probability.



NPTEL

IIT Bombay, DC

Ref: Kramer (1996) 40

Then Wilson in 1993 analyzed the strong ground motion record from California region once again, and develop the attenuation relationship, which can be used further to estimate the arias intensity using this equation. This equation is proposed by Wilson in 1993 in this equation  $D$  is the minimum horizontal distance to the vertical projection of the fault plane and  $h$ , this  $h$  is the correction factor with the default value of 7.5 kilometer and  $k$  is the coefficient in this case,  $k$  is a coefficient of inelastic absorption with default value of 0 and  $P$  is the exceedance probability. That is how much exceedance of an earthquake probability you want to use to estimate that arias intensity that was used by Wilson in nineteen ninety three. So with this, we have come to the end of our module number four, we will start again in the next lecture with our next module, module five.