

Geotechnical Earthquake Engineering
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Module - 4

Lecture - 15

Strong Ground Motion (Contd...)

Welcome to this NPTEL video course on Geotechnical Earthquake Engineering. Let us look at the slide here, on this video course of geotechnical earthquake engineering, we are going through the module 4 of this course, which is on Strong Ground Motion.

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Other Spectral Parameters


RMS acceleration : This is the parameter that includes the effects of amplitude and frequency, defined as

$$a_{rms} = \sqrt{\frac{1}{T_d} \int_0^{T_d} [a(t)]^2 dt}$$

Where $a(t)$ is the acceleration over the time domain and T_d is the duration of strong motion

AI - The Arias Intensity is a measure of the total energy at the recording station and is proportional to the sum of the squared acceleration. It is defined as

$$AI = \frac{\pi}{2g} \int_0^{T_d} [a(t)]^2 dt$$

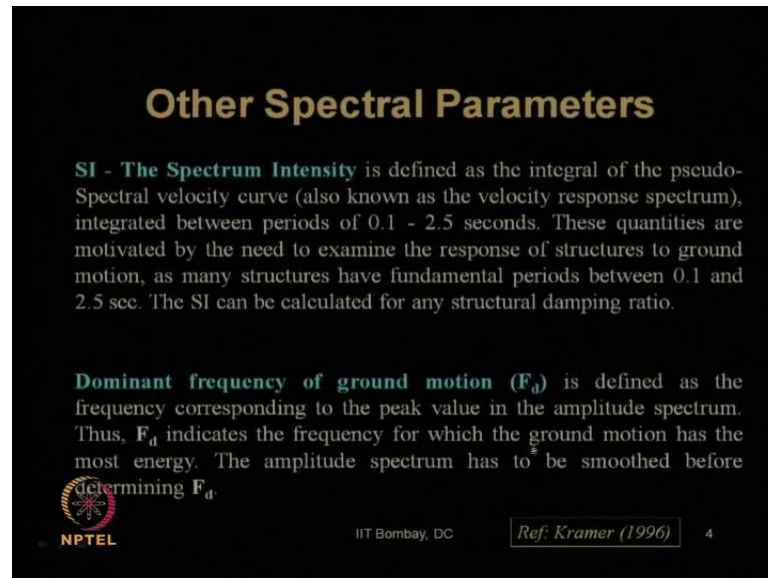
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Let us do a quick recap, what we have learnt in the our previous lecture. We have discussed about various spectral parameters, like what is called root mean square acceleration, how it can be estimated like a rms is nothing but root over as we find out mean and that is square. So, a whatever the is the acceleration time history that function square integrate it over 0 to T d dt divided by that T d, that is the duration.

And it is written over here, a t is the acceleration over the time domain and T d is the duration of the strong motion. Then we have seen what is called arias intensity and how

it can be measured it is nothing but the measure of the total energy which is coming out during an earthquake at the recording station; how it can be estimated that arias intensity A I can be estimated using this function.


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Other Spectral Parameters

SI - The Spectrum Intensity is defined as the integral of the pseudo-Spectral velocity curve (also known as the velocity response spectrum), integrated between periods of 0.1 - 2.5 seconds. These quantities are motivated by the need to examine the response of structures to ground motion, as many structures have fundamental periods between 0.1 and 2.5 sec. The SI can be calculated for any structural damping ratio.

Dominant frequency of ground motion (F_d) is defined as the frequency corresponding to the peak value in the amplitude spectrum. Thus, F_d indicates the frequency for which the ground motion has the most energy. The amplitude spectrum has to be smoothed before determining F_d .

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Then we had discussed, what is called spectrum intensity like it is defined as the integral of the pseudo spectral velocity curve. We have derived, what is called pseudo spectral velocity, pseudo acceleration curve, pseudo displacement curve all those things we seen in the previous lecture. So, also known as the velocity response spectrum, that has to be integrated between the periods of 0.1 seconds to 2.5 seconds.

So, we have seen also the reason what why this period has been chosen, because most of the damaging earthquakes are coming within this frequency range, if you inverse it you will get in hertz, what is the frequency? So, these quantities are motivated by the need to examine the response of the structure to ground motion as many structures are fundamental periods between this 0.1 to 2.5 second, as I have mentioned this is the basic reason.

So, this spectrum intensity it can be calculated for any structural damping ratio, then we have seen what is known as dominant frequency of ground motion, which is denoted as F_d . It is defined as the frequency which is corresponding to the peak value in the amplitude spectrum. So, this F_d indicates the frequency for which the ground motion

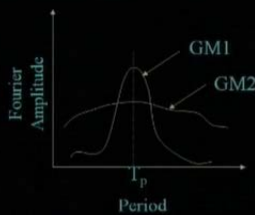
has the maximum energy. And the amplitude spectrum has to be smoothed before obtaining this F_d like what we do for the other spectrum curve also.

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Other Spectral Parameters

Predominant Period (T_p): Period of vibration corresponding to the maximum value of the Fourier amplitude spectrum. This parameter represents the frequency content of the motion. The predominant period for two different ground motions with different frequency contents can be same, making the estimation of frequency content crude.

Bandwidth (BW) - of the dominant frequency; measured where the amplitude falls to 0.707 ($1/\sqrt{2}$) of the amplitude of the dominant frequency. Again, this is based on a smoothed amplitude spectrum.



T_p is same for the two ground motions, though the frequency content is different

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Then we had also seen in our previous lecture, what is known as predominant period. Predominant period T_p through this picture we have seen it is not necessary that the Fourier amplitude of two ground motion need be need to be same to have same predominant period. They can have different Fourier amplitude that is maximum amplitude can be different in Fourier spectrum, but their predominant period can be same.

Predominant period is nothing but that period which corresponds to that maximum value of Fourier amplitude. So, period of the vibration corresponding to the maximum value of Fourier amplitude spectrum is known as predominant period. And this parameter represents the frequency content of the motion, the predominant period for two different ground motions with different frequency contents can be the same as shown in this picture, making the estimation of frequency content little crude.

Then we have seen what is known as bandwidth, bandwidth BW of a dominant frequency it is measured where the amplitude falls to $1/\sqrt{2}$ of the maximum amplitude. So, that is known as the dominant frequency bandwidth again this is the based on the smoothed amplitude spectrum.

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Other Spectral Parameters

Central Frequency: Power spectral density function can be used to estimate statistical properties of ground motion. The n^{th} spectral moment and central frequency (Ω) is given by,

$$\lambda_n = \int_0^{\omega_s} \omega^n G(\omega) d\omega$$
$$\Omega = \sqrt{\frac{\lambda_2}{\lambda_0}}$$

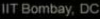

Central frequency is used to calculate theoretical median peak acceleration as follows,

$$U_{\max} = \sqrt{2\lambda_0 \ln\left(2.8 \frac{\Omega T_d}{2\pi}\right)}$$

Shape Factor – It indicates the dispersion of the power spectral density function about the central frequency,

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}}$$

It lies between 0 and 1, higher value indicates larger bandwidth.

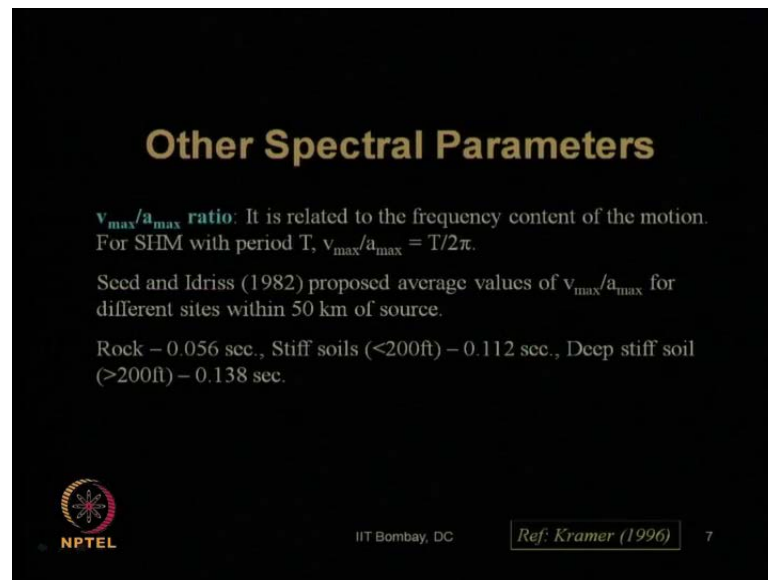


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Other spectral parameters which we had discussed in previous lecture like central frequency, how to estimate the central frequency we have seen we have used the P S D that is Power Spectral Density function. And using the power spectral density function this g ω this central frequency can be calculated this capital ω in this fashion in terms of λ_2 and λ_0 .

So, this central frequency it is further used to calculate the theoretical median peak acceleration using this expression. And the shape factor can also be estimated, it indicates the dispersion of the power spectral density function about its central frequency this one. So, it can be estimated like this which lies between the value of either 0 or 1 and higher the value that is close towards one indicates the higher or larger bandwidth.

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Other Spectral Parameters

v_{\max}/a_{\max} ratio: It is related to the frequency content of the motion. For SHM with period T , $v_{\max}/a_{\max} = T/2\pi$.

Seed and Idriss (1982) proposed average values of v_{\max}/a_{\max} for different sites within 50 km of source.

Rock – 0.056 sec., Stiff soils (<200ft) – 0.112 sec., Deep stiff soil (>200ft) – 0.138 sec.

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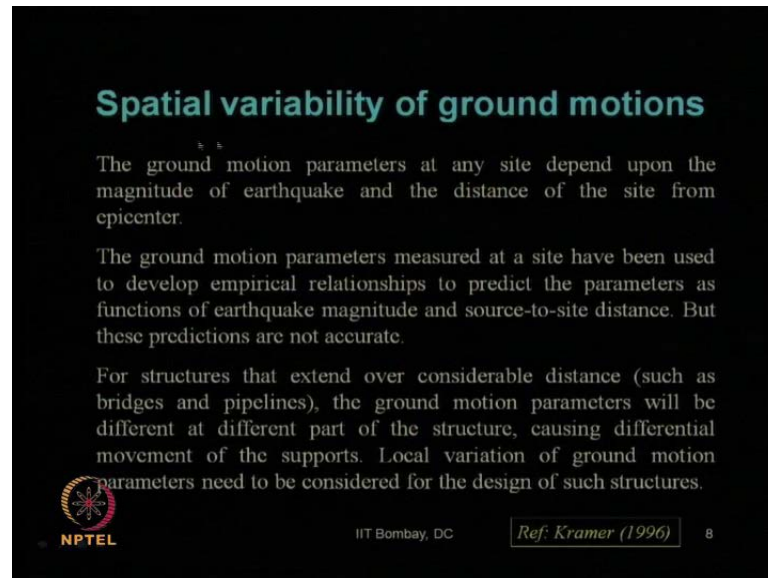
Then we had also mentioned about the others under the umbrella of other spectral parameters, the importance of parameter like v_{\max} by a_{\max} ratio. So, that v_{\max} by a_{\max} ratio we have also related it through the derivation in the previous lecture, that how this for a simple harmonic motion, if it is applied to a single degree of freedom system with period T .

Then we can write that v_{\max} by a_{\max} can be represented as that time period T by 2π that derivation also we have seen. Then Seed and Idriss in 1982 proposed the average values of this v_{\max} by a_{\max} for different sites within 50 kilometers of the source; that is the sites, which are located within 50 kilometers of the epicenter those only can be considered, and these values can be used in that case only.

So, for rocky site they mentioned it is the value is about 0.056 seconds; these are typical ranges remember these are not the fixed value, it can change a little bit depending on the several site conditions. Stiff soils that is stiff soils within the depth of 200 feet means, stiff soil is available at a shallower depth, in that case the typical range of values of that v_{\max} by a_{\max} is 0.112 seconds, and for deep stiff soil that is when the stiff soil is appearing at a large depth that is beyond 200 feet in that case the value will be 0.138 seconds.

That means, it is typically a loose soil or soft soil you have closed to the ground surface. So, I have we have seen as we move from the stiffer to a softer media that v_{max} by a v_{max} ratio keep on increasing.

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Spatial variability of ground motions

The ground motion parameters at any site depend upon the magnitude of earthquake and the distance of the site from epicenter.

The ground motion parameters measured at a site have been used to develop empirical relationships to predict the parameters as functions of earthquake magnitude and source-to-site distance. But these predictions are not accurate.

For structures that extend over considerable distance (such as bridges and pipelines), the ground motion parameters will be different at different part of the structure, causing differential movement of the supports. Local variation of ground motion parameters need to be considered for the design of such structures.

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Now, let us start our today's lecture, in today's lecture we will start first with this spatial variability of this ground motion. As we know that this earthquake motion or ground motion they vary spatially; spatially means in the horizontal direction when it travels from one place to another place, then they vary a lot; now how this ground motion will vary spatially let us see.

So, the ground motion parameter at any site depends upon the magnitude of earthquake which is of course, known to us. And the distance of the site from the epicenter like if you move further and further way from the epicenter; obviously, this ground motion parameters will keep on decreasing. And if you are closed to the epicenter that ground motion parameter will be large.

Now, how to estimate that large or small value of ground motion with respect to distance, that we will see and also with respect to magnitude. The ground motion parameters measured at a site have been used to develop empirical relationship to predict the parameters as functions of earthquake magnitude and source to site distance, but this prediction are not accurate remember.

So, what we are going to now discuss that how this ground motion parameters, various ground motion parameters like say acceleration. This seismic acceleration how it varies from the epicentral location to any particular site of your interest, why this very important? Suppose from our historical earthquake data we know where are the active faults are located?

Now, from those active fault location, your planning to construct say a very high raise building at a particular site. Now, you are interested that at your site what will be the predicted or estimated value of the seismic acceleration, in case any earthquake comes in the near vicinity. And near vicinity means you will always consider those existing active fault locations.

Now from those existing active fault locations how far is your site is of a concern; obviously, at the fault location from historical data of the earthquake, whatever values of the acceleration you got. You are not expecting the same value of acceleration to be considered at your site, which is say little far away from that actual fault, because you are not going to say construct your building on the fault.

Say, in that case you should know how this acceleration value will change, when it when we will consider that value from the fault region to our site of interest. So, how this decrease of the acceleration from the epicentral location or the active fault location will take place that depends on various recorded previous data, as many as earthquake data are available for a particular site, you will be able to have a better prediction.

That is why we mentioned over here these are developed relation empirically, how this empirical relation have been developed using earlier earthquake history data. So, obviously, these predictions cannot be called as accurate, because in future earthquake you never know that the same site, the same fault may experience a much larger value of the earthquake, which probably it has not faced in historically.

So, these relationships we call them as attenuation relationship, so what is attenuation? That is with variation of the distance the value which changes that is called attenuate. So, whether it can be an acceleration, it can be a velocity, it can be a displacement, whatever be your ground motion parameters, which we are interested for our design. So, say we are interested about the seismic acceleration.

So, that seismic acceleration how they are changing with variation of the distance from an fault region to site of our construct, where we are going to construct certain structure. So, that distance through that distance, how that acceleration going to change that is called attenuation of that acceleration.

So, how to estimate that attenuation relationship for acceleration, for velocity, for intensity, various parameters, various ground motion parameters that we need to lean here. So, that is the reason as I said, we can use this developed empirical relations to predict further for our design of these ground motion parameters depending on the earthquake magnitude. And depending on that source to the site distance, but these relationships are not accurate, because as on when you have any new earthquake you have to update this equations.

Because, obviously you got more data to consider to upgrade your empirical relationship which is fully based on the available historical data. So, obviously, if your available data increases with time at a particular site due to future and other earthquakes, then you need update your empirical relationships also. So, that is the reason you will see the attenuation relationships are keep on developing and it is a always a hot topic of research among the researches that is to develop the correct.

Or I will say close to correct attenuation relationships in terms of say acceleration, in terms of say velocity, in terms of say intensity, whatever the ground motion parameter you want to select. But that is not a like a constant value or it does not remain static as the other problems in our book they remain static, but it needs a continuous updation with respect to time.

So, that is the reason why we should keep on going through the letters to journal papers, conference papers to see the update on this attenuation relationship. Even what I am teaching you today may not be valid those attenuation relationship for that particular site after say 5 years or 10 years. Because by that time suppose some new earthquake accrued at the close vicinity of that particular region, then automatically those attenuation relationships are also going to get effected they are going to get changed.

So, for the structures you can see here for the structures that extend over considerable distance, like such as the bridges or pipelines that is structures, which extends for several kilometers like pipelines etcetera travels from kilometers to kilometers. This ground

motion parameters will be different at different parts of the structure. So, for suppose you are going to design the support system for this pipelines, which are extending for several kilometers as these are extending for several kilometers at one end of the pipeline to another end of the pipeline.

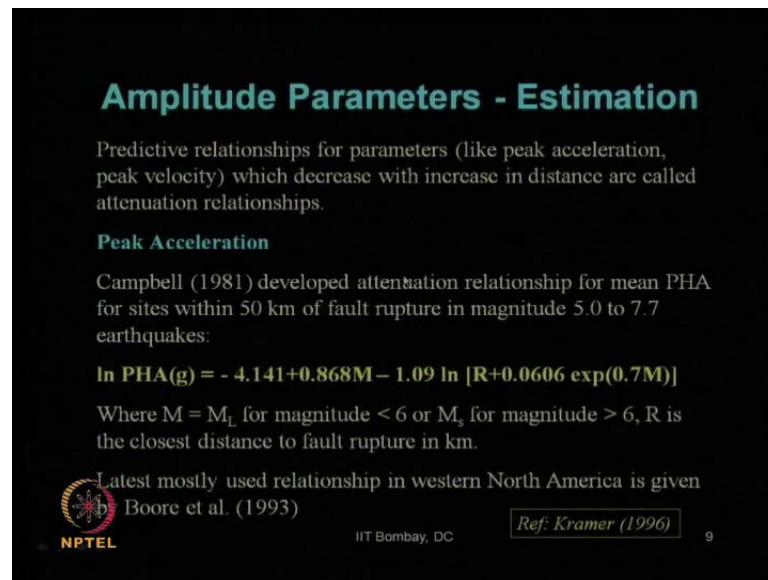
You may not have the same ground motion parameters which should be used for design, because by that amount of distance of ground motion parameters will travel it will definitely attenuate. So, for that pipeline you need to consider the variation of that ground motion parameter at different locations suitably, clear. So, even for the same structure you may need to consider that is why it is mentioned different ground motion parameters at different parts of the structure while designing.

Because, it causing the differential movement of the support, unless you consider that if you suppose designing it with maximum value or some very high value. First of all it give you an uneconomic design it may be very safe, but why you should do an uneconomic design for such a long structure, which is extending for a longer span even for bridges also.

So, that is not advisable, what is advisable if you can find out proper ground motion parameters at different locations of this structures use them properly or suitably to design various parts of this structure. And local variation of the ground motion parameters need to be considered for the design of sub structure like local other variation depended on the material property etcetera needs to be considered when we are designing such structure at that site.

So, we will see first of all how this ground motion parameters vary spatially, that is in horizontal direction on the ground. So, that variation will change our input values for design, also later on we will see the site response analysis in our another module, which will guide us that is at a particular site depending on the presence of a particular material how this design criteria will keep on changing. So, let us see now that amplitude parameter, estimation of that amplitude parameters let us look at here.

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Amplitude Parameters - Estimation

Predictive relationships for parameters (like peak acceleration, peak velocity) which decrease with increase in distance are called attenuation relationships.

Peak Acceleration

Campbell (1981) developed attenuation relationship for mean PHA for sites within 50 km of fault rupture in magnitude 5.0 to 7.7 earthquakes:

$$\ln \text{PHA}(g) = -4.141 + 0.868M - 1.09 \ln [R + 0.0606 \exp(0.7M)]$$

Where $M = M_L$ for magnitude < 6 or M_s for magnitude > 6 , R is the closest distance to fault rupture in km.

Latest mostly used relationship in western North America is given by Boore et al. (1993)

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This predictive relationships for the parameters as I have mentioned just now our attenuation relationships. So, this predictive relationships for parameters like peak acceleration, peak velocity which decrease with increase in the distance from the source. Obviously, they will decrease, why they will decrease? Because they travel through a particular distance, so energy dissipated and with time and distance they will; obviously, decrease.

So, that peak acceleration and peak velocity will keep on decreasing as we travel for the distances from the source. So, with increase in the distance, this predictive relationships of the parameters will also decrease with increase in distance, those are called attenuation relationships, that is what I was telling. So, suppose if it is in terms of acceleration, we will call it as acceleration attenuation relationships if it is in terms of velocity, velocity attenuation relationships and so on.

So, let us see when that attenuation relationships started in our geotechnical earthquake engineering or the seismology topic, for peak acceleration the pioneering work was done by Campbell in 1981. So, Campbell first developed scientific attenuation relationship for mean value of P H A that is Peak Horizontal Acceleration, for peak horizontal acceleration Campbell first developed that acceleration attenuation relationships for the sites, which are located within 50 kilometer of the fault rupture.

So, these parameters are very important one should know that is the application of those attenuation relationships, which are empirically in nature what are the conditions of using those equations. So, these equation proposed by Campbell is valid for when your's concern site of interest is within 50 kilometer of the fault rupture and the magnitude of earthquake should be between 5 to 7.7. That means, in developing these expression of attenuation relationship of acceleration, he used those earthquake only, which had magnitude between 5 to 7.7.

And those earthquake he has considered and those sites he has considered which are within the 50 kilometer distance from the epicentral location or hypo-central location or from fault rupture. So, remember this data is mostly from the California region and North American region data. So, that is another point one should remember that this attenuation relationships is location specific or country specific or site specific also.

So, what is the final equation attenuation relationship for PHA that peak horizontal acceleration Campbell has proposed this is the equation. That \ln that is Natural Log PHA in terms of g is equals to minus 4.141 plus 0.868 M , where M is the magnitude minus 1.09, natural log of R plus 0.0606 exponential that is e to the power 0.7 M .

So, in this case Campbell mentioned this M means M_L that is the Local Magnitude for the magnitude below the value of 6. And this M equals to M_s that is surface wave magnitude for value of magnitude more than 6. So, first of all this is valid for magnitude between 5 to 7.7 and within that 5 to 7.7 also Campbell mentioned from 5 to 6, if it is there then use the local magnitude scale, if it between 6 to 7.7 use the surface wave magnitude scale. And this R is the closest distance to the fault rupture in the unit kilometer as we know the empirical relationships are unit biased.

So, here also we have to be careful about which unit should used, so R should be in kilometer, so that is the closest distance or the shortest distance between the fault and your site of concern, where your planning estimate. So, suppose at a particular site when you are planning to design any structure, construct any structure you want to know how much will be the PHA at that site using the Campbell equation how you can estimate that.

Suppose you have some information that there is a change of magnitude of occurrence of earthquake of this much, say a particular value at that site. So, M you can consider, R

should be known from your site to the closest fault location, which is active fault. So, R is also known to you, so once R and M these 2 values are known you can put in this equation of Campbell and you will get the value of PHA, that should be used for that site when you are planning to go for a design.

So, this is the peak acceleration attenuation relationship as proposed by Campbell. And latest mostly used relationship in the Western North America, Western North America means, basically the California region that is given by Boore et al in 1993.

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Amplitude Parameters - Estimation

Attenuation relationship in western North America is given by
Boore et al. (1993)

(From North American Earthquakes (magnitude 5-7.7) within 100 km of surface projection of fault)

$$\text{Log PHA}(g) = b_1 + b_2(M_w - 6) + b_3(M_w - 6)^2 + b_4R + b_5 \log R + b_6 G_b + b_7 G_c$$

$R = (d^2 + h^2)^{1/2}$, d - closest distance to the surface projection of the fault in km.

$G_b = 0$ for site class A	$G_c = 0$ for site class A
$G_b = 1$ for site class B	$G_c = 0$ for site class B
$G_b = 0$ for site class C	$G_c = 1$ for site class C

Site classes are defined next slide on the basis of the avg. V_s in the upper 30 m).

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So, let us see now the equation or an attenuation relationship, which is proposed by that Boore et al 1993 for the California region earthquake. So, let us look at here in the slide, attenuation relationship in Western North America, which is given by Boore et al 1993. So, this was mentioned as latest as far as this book of Kramer is concerned which was published first in 1996.

So, remember in the previous slide I said that this is the latest, latest means as on 1996, but if you take today's time of 2013, it is not the latest there are several other attenuation relationships have come up after this Boore et al 1993 also, so remember that carefully. So, Boore et al also considered the magnitude of earthquake between 5 to 7.7 by the way, why this magnitude scale has been considered by all the researchers as you can see, because minimum magnitude of 5 is responsible for starting of any structural damage as we have discussed earlier in our lecture.

So, 5 onwards is mostly of our concern for our civil engineering design and why the upper limit of 7.7, because above 7.7 also earthquakes are available, but those are rare in nature. And of course, you cannot design your structure for your earthquake magnitude say 9 or 9.5 that very high, then your structure will be extremely uneconomic in terms of design. Because chances are probability of occurrence that of that very high value of earthquake magnitude is very, very low.

So, you cannot use that very high value and make all your construction or the cost of construction you can raise abruptly; that is the reason; typically they have considered this range of magnitude 5 to 7.7 for deriving all these attenuation relationship. And what is the advantage of Boore's equation, Boore considered Boore et al they considered the distance up to 100 kilometer from the surface projection of the fault.

So, Campbell considered only those earthquake for deriving that empirical relationships within 50 kilometer of the fault region. And Boore at all they considered within 100 kilometers, so they have taken more earthquake data; that is what it shows. And of course, from 1981 to 1993 whatever earthquake occurs they have considered those earthquake data as well.

So, that is why if you want to use suppose you have given a choice between Campbell equation and Boore's equation, it is always advisable to go for using this Boore et al equation; because it is more updated than the Campbell equation. But why then we are learning we are learning because this is the step wise development in this area. So, that is the reason you should know the older predictive relationships also based on, which one can further do a research work and study and then apply the latest attenuation relationship.

So, now, let us see what is that Boore et al's attenuation relationships given here, like \log of PHA of g is equals to some coefficients Boore et al have mentioned; b_1 plus b_2 times M_w minus 6 plus b_3 times M_w minus 6 whole square plus b_4 times R plus b_5 times \log of R plus b_6 times G plus b_7 times G c. Now, what are these parameters first of all m_w we all know it is the moment magnitude of earthquake.

So, unlike Campbell's equation where local magnitude and surface magnitude were used, Boore et al use the more correct technically magnitude, which is the moment magnitude. So, M_w they have used this is another advantage or another progress from the Campbell

equations to Boore's equation. And what is that value of capital R, capital R is calculated as $\sqrt{d^2 + h^2}$, where this small d is the closest distance to the surface projection of the fault in kilometer and h is the depth.

So, this the closest distance and depth, so that way what you are getting the R the resultant distance can you see that. So, this is the closest distance this is the depth, so you are getting a resultant distance from that fault location to your sight of concern. And this $b_1, b_2, b_3, b_4, b_5, b_6, b_7$ these are the some coefficients which is given in the next slide. And this G_b and G_c are another few coefficients which are based on different site what are those let us see.

The value of G_b is 0 for a site class A what is this is it site class various type of soil site has been classified into different categories like A, B, C etcetera based on their average shear wave velocity value at those site. On first 30 meter or top 30 meter that is from the ground level to up to 30 meter of depth what is value average value of shear wave velocity based on that this site classed were found A, B, C we will see in the next slide.

So, that G_b value should be taken as 0 in this equation, if it is for site class A and G_c also has to be taken for 0 if it is site class A. G_b has to be taken as 1 if it is site class B, G_c has to be taken as 0 if it site class B, G_b has to be taken as 0 for site class C and G_c has to be taken as 1 if it is a site class C.


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Definitions of Site Classes for Boore et al. (1993) Attenuation Relationship

Site Class	\bar{v}_s in Upper 30 m (100 ft)
A	> 750 m/sec (2500 ft/sec)
B	360–750 m/sec (1200–2500 ft/sec)
C	180–360 m/sec (600–1200 ft/sec)

Coefficients for Attenuation Relationships of Boore et al. (1993)

	Component								
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	h	$\sigma_{\log PHA}$
Random	-0.105	0.229	0.0	0.0	-0.778	0.162	0.251	5.57	0.230
Larger	-0.038	0.216	0.0	0.0	-0.777	0.158	0.254	5.48	0.205


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So, these are the various definitions of site classes as was proposed by Boore et al in 1993 for using in this attenuation relationship. So, that site class A means that upper 30 meter v_s value shear wave velocity value 30 meter means close to about 100 feet from the ground surface. That shear wave velocity should be greater than 750 meter per second, which typically means it is a very hard soil it can be a rock.

So, site class B means the v_s value will be within the range of 360 to 750 meter per second, so this a very stiff soil or hard soil. And site class C means the average upper 30 meter v_s value should be within 180 meter per second to 360 meter per second, which is for the soft soil.

Now, what are those coefficients to be used for using this Boore et al expression of attenuation relationship of 1993, these are the coefficients b_1 , b_2 , b_3 , b_4 , b_5 , b_6 , b_7 . And what are the values of h also should be considered in that equation they have proposed, like if you want to consider the random earthquake motion, these are the values you should use.

And if you want to use the larger value or the higher value of earthquake motion for your design, then you should use this values of b_1 , b_2 , b_3 , b_4 , b_6 , b_7 and h value. And what is this sigma log of PHA these values, this values shows the typical standard deviation. Because when Boore et al proposed this equation remember, based on some collected historical earthquake data point. Now, obviously this equation is not passing through all the data points as we do in the empirical relationship. Suppose we have various points we find out the best fit.

So, in the same way they have proposed this is the kind of a best fit curve, through all the recorded or observed data from the historical earthquake within this magnitude and within this distance. But obviously there will be some scattering from the actual value to this predicted equation and that will have some kind of standard deviation. So, what is that standard deviation one should know, like when we are proposing any empirical relationship this standard deviation if those are high. Obviously, you have a bad correlation if standard deviation is very low, then you have a good correlation.

Similar, to what we want to predict through the r square value that regression coefficient, if r square value is very high means good correlation if r square value is very low, then bad correlation. Similar, way the standard deviation if it is very low, then it is good

means less variation and if standard deviation is very high, then it is a poor relationship. So, that is why they have mentioned automatically as this is based on some recorded data these are the values of that variation of that log of PHA, which can be calculated from that given equation.

This is the standard deviation values, if you use the random earthquake motion and this is the value if you use the larger values of the earthquake. So, obviously suppose somebody want to use the same equation, that is they can keep the same format of the equation, but add some more data points from 1993 to this present day of 2013 20 years. Earthquake data at the same site, same location West and North America that is California region this standard deviation will change is it clear. So, to have a better standard deviation may be you have to predict or change this coefficients little bit here and there.

That is the way how people do the research in this area of developing attenuation relationship for a particular site based on the historical collected data, but for that you should know, the entire place geology and earthquake data completely.

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Attenuation Relationship for peak horizontal rock acceleration by Toro et al., 1994 (for mid continent of North America)

$$\ln \text{PHA (g)} = 2.2 + 0.81(M_w - 6) - 1.27 \ln R_m + 0.11 \max[\ln(R_m/100), 0] - 0.0021R_m$$

$$\sigma_{\ln \text{PHA}} = (\sigma_m^2 + \sigma_r^2)^{1/2}$$


Where $R_m = (R^2 + 9.3^2)^{1/2}$, R being closest horizontal distance to earthquake rupture (in km), $\sigma_m = 0.36 + 0.07(M_w - 6)$, and

- 0.54 for $R < 5$ km
- $\sigma_r = 0.54 - 0.0227(R - 5)$ for $5 \text{ km} \leq R \leq 20$ km
- 0.2 for $R > 20$ km

Attenuation relationship for subduction zone (Youngs et al., 1988)

$$\ln \text{PHA (g)} = 19.16 + 1.045M_w - 4.738 \ln [R + 205.5 \exp(0.0968M_w)] + 0.54 Z_1$$

$\sigma_{\ln \text{PHA}} = 1.55 - 0.125M_w$, R = closest distance to the zone of rupture in km and $Z_1 = 0$ for surface and 1 for intraslab events



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Now, let us come to the next attenuation relationship for peak horizontal rock acceleration, which was proposed by this attenuation relationship for peak horizontal rock acceleration by Toro et al in 1994, that is after Boore et al. So, this Toro et al they proposed for mid content of North America, mid content of North America means

middle portion of the US like Texas etcetera; those places can be considered as the middle portion nor the western coast neither the eastern portion.

So, midcontinent of North America the equation proposed by Toro et al is \ln natural log of PHA in terms of g you will get whatever value; that means, by calculating this, whatever value you are getting. Suppose you are getting 0.316 that means, it is 0.316 g that is what it means is given by this equation. Where in this equation, this sigma can be estimated using this further this expression that is the variation if in magnitude and variation in the distance.

And this R_m in this equation what R_m you need to use is nothing but $R^2 + 9.3^2$ square under root, where this R is nothing but the closest horizontal distance to the earthquake rupture in the kilometer unit. That is from your site to that closest fault rupture location and this sigma m value can be considered as $0.36 + 0.0015 M_w - 6$.

So, in this case also they have use the M_w scale that is moment magnitude scale and sigma R for various values of R it is given over here. Then another attenuation relationship for this peak horizontal acceleration for the subduction zone we have already learnt what is subduction zone earlier it is proposed by Young's et al in 1988 that is a it is the previous one to this. This is the expression which they have proposed, this is the empirical relation for attenuation relationship.

And in this case they obtain the value of sigma \ln PHA can be calculated using this, where in this equation M_w is the again moment magnitude and R is nothing but the closest distance to the zone of rupture in the kilometer unit. And this Z_t can be considered as 0 if it is interface event and it can be considered as one if it is a intraslab event for a subduction zone. Now, let us see the other attenuation relationship, suppose the velocity attenuation relationship.

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Peak Velocity Attenuation Relationships (Joyner and Boore, 1988)
(for earthquake magnitudes 5-7.7)

$$\log \text{PHV (cm/sec)} = j_1 + j_2(M-6) + j_3(M-6)^2 + j_4 \log R + j_5 R + j_6$$

Where PHV can be selected as randomly oriented or larger horizontal component
 $R = (r_0^2 + j_7^2)^{1/2}$, and r_0 is the shortest distance (km) from the site to the vertical projection of the EQ fault rupture on the surface of the earth.

The coefficients j_i are given in the table below:
Coefficients after Joyner & Boore (1988) for PHV Attenuation Relationship

Component	j_1	j_2	j_3	j_4	j_5	j_6	j_7	$\sigma_{\log \text{PHV}}$
Random	2.09	0.49	0.0	-1.0	-0.0026	0.17	4.0	0.33
Larger	2.17	0.49	0.0	-1.0	-0.0026	0.17	4.0	0.33

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Let us look at the slide here this is showing the peak velocity attenuation relationship which is proposed by Joyner and Boore in 1988 for the earthquake magnitude again within the range 5 to 7.7 that is the magnitude scale they considered. And this is they have proposed empirical relationship of velocity attenuation, peak horizontal velocity PHV in log. Whatever value you will get from this it will be in the unit of centimeter per second as I said empirical relations are unit bias, so we should be careful about the unit.

So, the value which you will get by using this equation it will give you value of PHV in centimeters per second. And some coefficients j_1 plus j_2 times M minus 6 plus j_3 times M minus 6 whole square plus j_4 times log of R plus j_5 times R plus j_6 . Where this PHV can be selected as randomly oriented or larger horizontal component as it has been done for the PHA also by Boore et al in the similar fashion.

And this value of the R is calculated like r naught square plus this j_7 another another coefficient j_7 square under root this r naught is the shortest distance in the kilometer unit from the site to the vertical projection of the earthquake fault rupture on the surface of the earth. So, that is the shortest distance if you take a vertical component of a particular site to a particular fault location.

So, what are this coefficients j_i 's all these j_i that is $j_1, j_2, j_3, j_4, j_5, j_6, j_7$ all are given over here for both random and larger with the value of sigma log of PHP. So,

coefficients after Joyner and Boore for PHV attenuation relationships are given over here, this also can be found out in the book of Kramer 1996.

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Amplitude Parameters - Estimation

A.Patwardhan et al. (1978):

$$\ln y = \ln A + B M_s + E \ln [R + d \exp(f M_s)]$$

Where, y in cm/s^2 , $d=0.864$ and $f = 0.463$

Path	A(for median)	A(for mean)	B	E
Path A (rock)	157	186	1.04	-1.90
Path A(stiff soil)	191	224	0.823	-1.56
Path B (stiff soil)	284	363	0.587	-1.05

Path A: Shallow focus earthquakes (California, Japan, Nicaragua and India), 63 records

Path B: Subduction (Benioff) zone earthquakes (Japan & South America) 23 earthquakes, $5.3 \leq M_s \leq 7.8$, 32 records

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Now, coming to other attenuation relationships like amplitude parameters estimation as given by Patwardhan et al in 1978 \ln of y what is y ? Y is nothing but PHA that is the peak horizontal acceleration in the unit centimeter per second square is can be calculated as \ln plus A plus $B M_s$ plus E times \ln of R plus d times e to the power $f M_s$. So, in this case d is 0.864 and f is a coefficient 0.463 and for various path of travel that is whether it is a rocky path like path a or stiff soli or another soft soil.

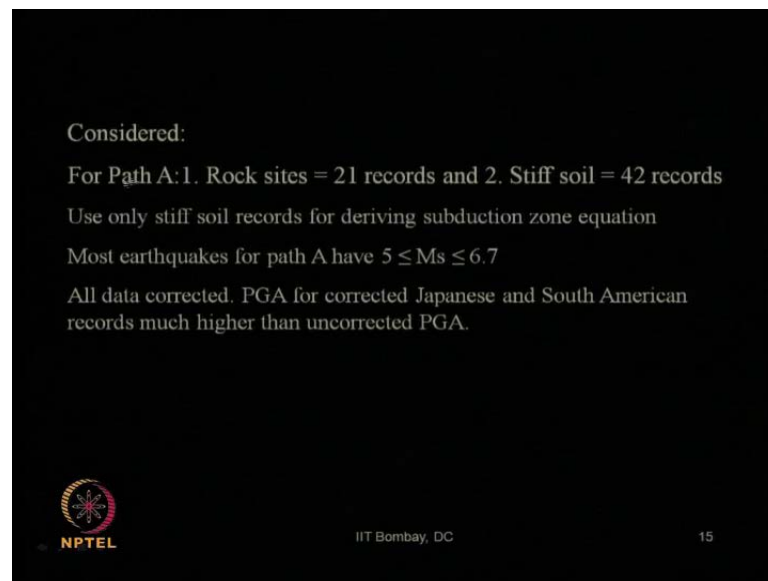
You have various values of this parameters A , the median value as well as the mean value and the parameters B and E those values are given over here. So, for path a what it was considered shallow focus earthquake and Patwardhan et all considered 63 records of the earthquake for developing this empirical relationship. And where from the earthquake had taken shallow earthquakes from California region from Japan from Nicaragua and from India.

These four places they have total 63 earthquake record, which they have used to propose this attenuation relationship for acceleration. And for the path B is for the subduction zone earthquake for which they have taken 23 earthquake record from Japan and south America within the value of this M_s that is surface wave's magnitude between 5.3 to

7.8. And from 23 earthquake total 32 records were observed that is some of the earthquake they have more records at different stations that is what it means.

So, one is for the shallow earthquake another is the subduction zone earthquake that is the difference between the two path. So, within shallow earthquake if it travels through rocky site or stiff soil, this two values has to be used and for the subduction zone if it is traveling through stiff soil, then this value has to be used is it clear.

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Considered:

For Path A: 1. Rock sites = 21 records and 2. Stiff soil = 42 records

Use only stiff soil records for deriving subduction zone equation

Most earthquakes for path A have $5 \leq M_s \leq 6.7$

All data corrected. PGA for corrected Japanese and South American records much higher than uncorrected PGA.

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Now, for they considered that Patwardhan et al for path A rock sites 21 records stiff soil 42 records that is why the equation they have given both for rock y site and stiff soil site. And use only the stiff soil records for deriving the subduction zone that is why for path B we have seen only for the stiff soil it is available. And for most of the earthquakes for path A have M_s value between 5 to 6.7 and all data have been corrected that is the raw data has to be corrected. And that PGA for corrected Japanese and south American records are much higher than the uncorrected PGA value that is what the Patwardhan et al they proposed.

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Aptikaev & Kopnichev (1980)


$\log Ae = a_1M + a_2\log R + a_3$


Ae (cm/s ²)	a_1	a_2	a_3
≥ 160	0.28	-0.8	1.70
< 160	0.80	-2.3	0.80

➤ PGA corresponds to S-wave

➤ Used five source mechanism categories (about 70 records, 59 earthquakes from W. N. America including Hawaii, Guatemala, Nicaragua, Chile, Peru, Argentina, Italy, Greece, Romania, central Asia, India and Japan):

1. Contraction faulting (uplift and thrust), about 16 earthquakes

 Contraction faulting with strike-slip component, about 6 earthquakes

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Now, coming to another attenuation relationship given by Aptikaev and Kopnichev in 1980 the equation proposed is \log of A_e is equals to $a_1 M$ plus $a_2 \log$ of R plus a_3 . Where A_e is nothing but acceleration in the unit of centimeter per second square and a_1 , a_2 and a_3 are the coefficients and what they have mentioned if the acceleration value, which you are getting by using this equation is more than 160 centimeter per second square you should use a_1 , a_2 , a_3 this 3 coefficients. Or if it is less than 160 centimeter per second square you should use these values of a_1 , a_2 , a_3 .

So, this is kind of a trial and error procedure that is first suppose you use these values a_1 , a_2 , a_3 and got the value of A_e say less than 160. Then you should switch over to these values of a_1 , a_2 , a_3 and check whether still it is coming within 160. So, PGA corresponds to the surface wave and that is the magnitude which you need to use here the surface wave magnitude. And they use the five source mechanism categories that about 70 records from the 59 earthquakes from West North America they have taken again West North America means like California region, which is one of the major earthquake region as we know.

Including the Hawaii Island and Guatemala, Nicaragua, Chile, Peru, Argentina, Italy, Greece, Romania, central Asia, India and Japan that is all the major earthquake places all over the world they have considered. Total of 59 earthquake from which they had 70 earthquake records, which they have used to proposed this equation. So, the contracting

faulting that is uplift the trust about 16 earthquake they have used and contraction faulting with strike slip component about 6 earthquake.

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3. Strike-slip, about 17 earthquakes
 4. Strike-slip with dip-slip component, about 6 earthquakes
 5. Dip-slip, about 9 earthquakes

Use these approximately 70 records to derive ratios of mean measured, A_0 , to predicted PGA, A_e , $\log(A_0/A_e)$, and for ratios of mean horizontal to vertical PGA, $\log A_h/A_v$, for each type of faulting. Use every earthquake with equal weight independent of number of records for each earthquake.

• Results are:

	Category 1	Category 2	Category 3	Category 4	Category 5
$\log A_0/A_e$	0.35 ± 0.13 (16)	0.11 ± 0.17 (5)	0.22 ± 0.08 (17)	0.06 ± 0.13 (6)	-0.06 ± 0.20 (9)
$\log A_h/A_v$	0.32 ± 0.13 (12)	0.32 ± 0.08 (5)	0.27 ± 0.07 (12)	0.18 ± 0.10 (5)	0.17 ± 0.11 (5)

where \pm gives 0.7 confidence intervals and number in brackets is number of earthquakes used.

Aptikaev, F., & Kopnichev, J. (1980). Correlation between seismic vibration parameters and type of faulting. *Proceedings of Seventh World Conference on Earthquake Engineering, Vol. 1*, 107-110.

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Then strike slip type of earthquake from 17 strike slip with dip slip component 6 earthquake and dip slip earthquake about 9. So, they used these approximately 70 records as I have mentioned to derive the ratio of mean measured. So, this A_0 to be predicted PGA A_e with respect this ratio for the ratio of mean horizontal to vertical peak ground acceleration. And this value for each type of faulting use every earthquake with equal weight that is all these earthquake different five different categories of earthquake what is mentioned over here.

They provide equal weight age to all the independent event of number of records for each earthquake. And what are the results they proposed that \log of this A_0 by A_e and \log of A_h by A_v , A_h by A_v is horizontal to vertical peak values. So, for 5 different categories category 1, 2, 3, 4, 5 individually also they have given this values, that is this mean value with plus minus means this is at the standard deviation.

These are the variations and these are the mean values of this ratio of logs. So, where this plus minus gives 0.7 confidence interval and number in brackets shows the number of earthquake used that already mentioned over here right; these are the number of earthquakes those are used. So, this the reference from where this information has been taken you can see correlation between seismic vibration parameters and type of faulting.

Because, they have classified it with respect to different types of faulting, remember the other attenuation relationship they do not classify the type of fault. Whereas, here they have taken various types of faulting and based on that they have proposed different equations or different values of this ratio of $\ln a$ by $A_v A_n$ by A_e after computing A_e from your basic common equation is it clear.

So, this paper was published in proceedings of the seventh world conference on earthquake engineering in volume one these are the page numbers. So, this world conference of earthquake engineering occurs as you know every 4 years interval. So, recently the 15th world conference on earthquake engineering took place in the year 2012 in Lisbon, Portugal. So, the next one that is the 16th world conference on earthquake engineering we call it WCEE - World Conference on Earthquake Engineering that will be in 2060.


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PML (1982)

$$\ln(a) = C_1 + C_2 M + C_3 \ln[R + C_4 \exp(C_5 M)]$$

Where a is in g , $C_1 = -1.17$, $C_2 = 0.587$, $C_3 = -1.26$, $C_4 = 2.13$, $C_5 = 0.25$
and $\sigma = 0.543$

Used data from Italy (6 records, 6 earthquakes), USA (18 records, earthquakes), Greece (13 records, 9 earthquakes), Iran (3 records, 3 earthquakes), Pakistan (3 records, 1 earthquake), Yugoslavia (3 records, 1 earthquake), USSR (1 record, 1 earthquake), Nicaragua (1 record, 1 earthquake), India (1 record, 1 earthquake) and Atlantic Ocean (1 record, 1 earthquake).

 P.M.L., 1982. *British earthquakes. Tech. rept. 115/82. Principia Mechanical Ltd., London. Not Reported in Ambraseys et al. (1992)*

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Another attenuation relationship let us look at here, PML in 1982; PML 1982 this can be obtained from this British earthquakes technical report, number 115 by 82 principia mechanical limited London and reported by... So, this further reported by Ambraseys et al in 1992, that is this report was reproduced in Ambraseys et al report or Ambraseys et al paper. So, how they mention it can be calculated this is $\ln a$; a is the acceleration in the unit of g that is again whatever value you will get like 0.216 that will be 0.216 g

equal to $C_1 + C_2 M + C_3 \ln R + C_4 e^{C_5 M}$.

So, in this case the values of this C_1 is this minus 1.17, C_2 is 0.587, C_3 is minus 1.26, C_4 is 2.13, C_5 is 0.25 and the value of that sigma that is standard deviation while proposing this equation is 0.543. And what are the data earthquake data they used, remember they used the earthquake data from Italy 6 earthquakes, 6 records from USA, 18 earthquake, 18 records from Greece, 9 earthquake 13 records; that means, some earthquake are having multiple records at different distances, Iran earthquake 3 earthquake 3 records, Pakistan earthquake 1 earthquake 3 records, Yugoslavia earthquake 1 earthquake 3 records, USSR in those days because 1982 you can see. So, 1 earthquake 1 record Nicaragua 1 earthquake 1 record, India 1 earthquake 1 record and Atlantic ocean 1 earthquake 1 record. So, these are data points or data set they had used to propose this equation. So, with this we have come to the end of today's lecture, we will continue further with our discussion in our next lecture.