Geotechnical Earthquake Engineering
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Module - 4
Lecture - 14
Strong Ground Motion (Contd…)

Let us start our today's lecture on this video course of geotechnical earthquake engineering. Let us look at the slide here. On this video course of geotechnical earthquake engineering for NPTEL, we are going through this module four on strong ground motion.

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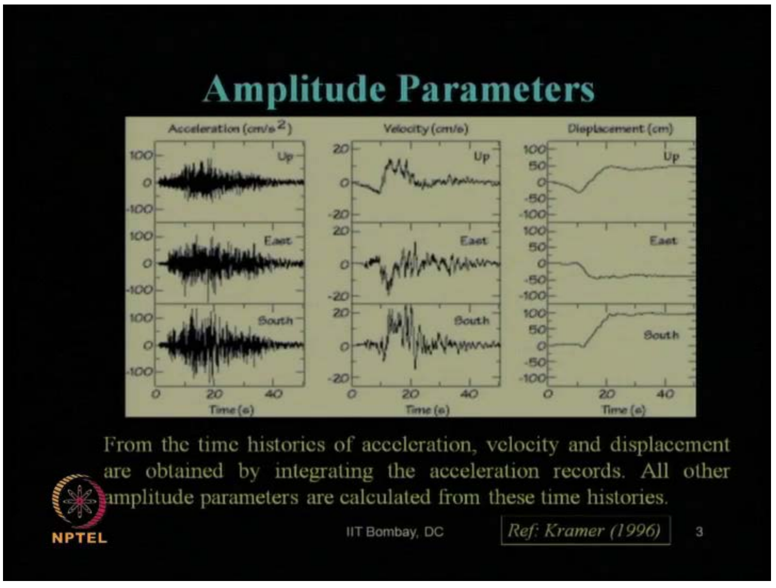

Let us do a quick recap what we have learnt in our previous lecture. Like we have learnt about what are the amplitude parameters of an earthquake; like it can be acceleration, velocity and displacement record with respect to time.

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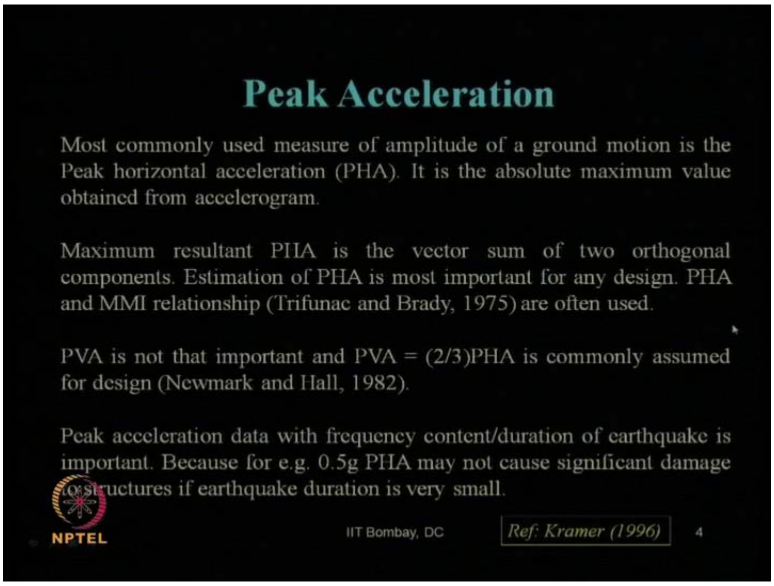

Then, how to define the peak value of acceleration, we have seen; that PHA – peak horizontal acceleration; and it can have two orthogonal components in north-south direction and east-west direction. And the maximum resultant value of that PHA, we can take by taking the vector sum of those two orthogonal PHA values. And, Trifunac and Brady in 1975 – they have proposed for the design, what is the value of PHA with respect to the relationship with the modified Mercalli's intensity scale – MMI scale. And generally, the PVA is not that important.

Earlier, people use to think about as proposed by Newmark and Hall in 1982; but, in recent days, people say is better to estimate PVA also, that is, peak vertical acceleration. And, in absence of the data of PVA, as per suggestion by Newmark and Hall, 1982, PVA can be considered as two-third of the value of that PHA for the design purpose. And, peak acceleration data with frequency content and duration is more important, because we have seen that, it is not necessary that only that PHA value will be more important; it is also necessary how much is the frequency content and how much is the duration of an earthquake.

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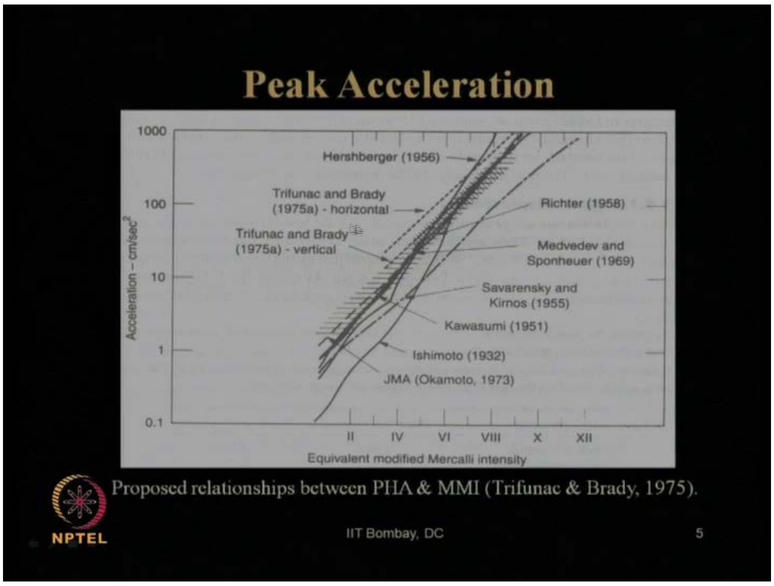

Then, we have seen that relationship between PHA versus MMI, and PVA versus MMI, so this is for the vertical; this for the horizontal as proposed by Trifunac and Brady in 1975 corresponding to various modified Mercalli intensity scale. So, if you know the MMI value of a particular earthquake at a site, using this relationship, you can get the value of PHA or PVA, which can be further used for the design.

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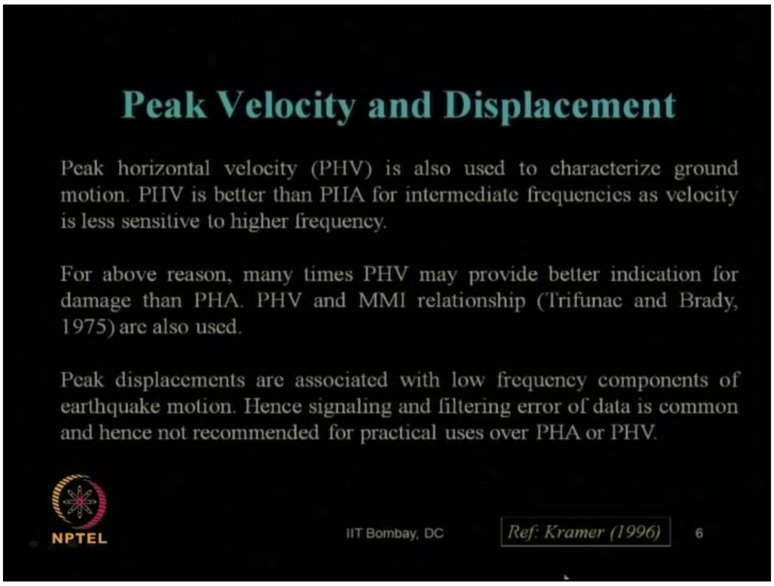

Then, we talked about the value of PHV that is peak horizontal velocity. And, we have mentioned that, PHV is a better estimation than PHA because of capturing the intermediate frequency also very well; whereas, the peak displacement or peak horizontal displacement and peak vertical displacement - though they are associated with the low frequency content, but because of the filtering error that is, after filtering out the noises, etcetera, it will be very difficult to get reliable results in terms of the peak displacement values. So, that is why, in most of our earthquake engineering design, either we use PHA, that is, peak horizontal acceleration or PHV – peak vertical acceleration.

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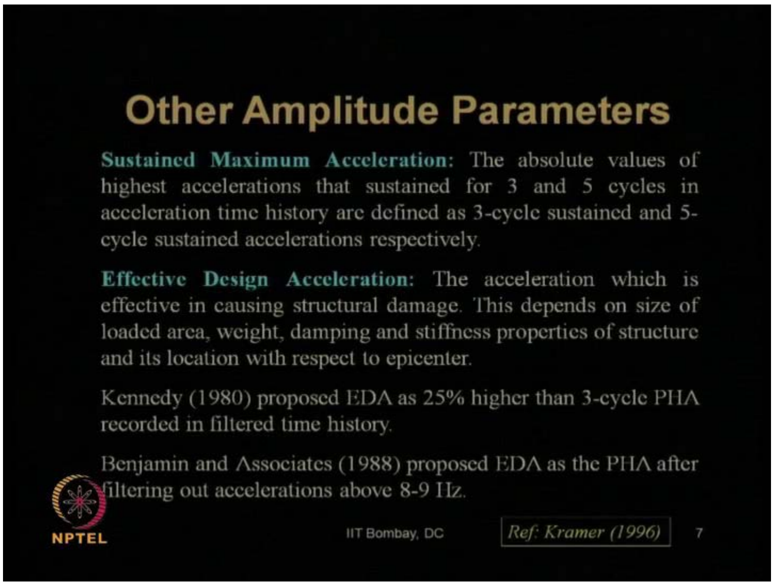

Then we talked about various other amplitude parameters like sustained maximum acceleration. We have seen, it can be a 3 cycles sustained or 5 cycles sustained depending on the requirement; that is, how much that highest value of acceleration is continuing through how many numbers of cycles. Then, we have seen for the design, what is known as effective design acceleration. And as per the proposal by Kennedy in 1980, that EDA can be considered as 1.25 times of that 3 cycles sustained maximum acceleration of PHA. And, Benjamin and Associates proposed that if you filter out all the higher frequencies; that is, above 9 hertz frequency if you filter out all accelerations, then whatever acceleration spectrum or acceleration response you get with respect to time, that acceleration time history, whatever PHA value, it gives; directly you can use that as the design or effective design acceleration.

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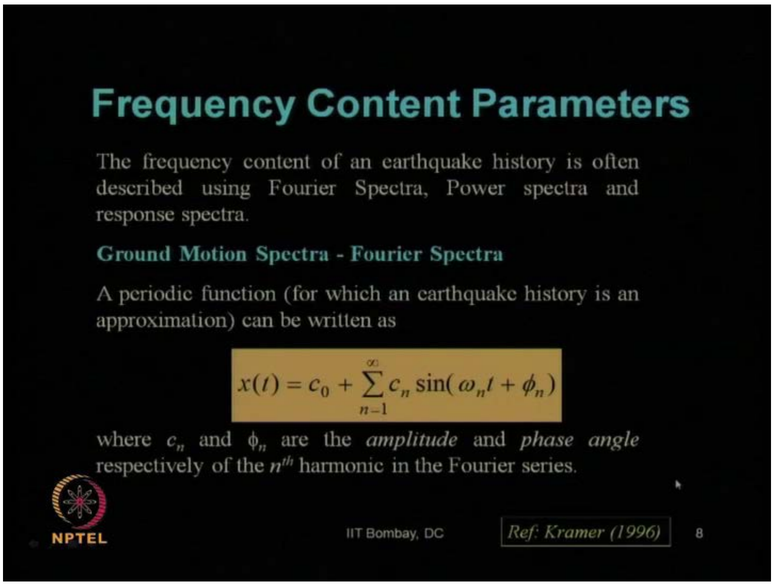

Then, to provide the frequency content parameter, we have seen either using Fourier spectrum or by power spectrum or by response spectrum, we can find out how much frequency content exists in a particular earthquake motion. Fourier spectrum is nothing but it is an assumed Fourier series of randomly behaved earthquake motion that we express in the form of a Fourier series through this periodic function. And, this value is given as the amplitude; and this value is given as the phase angle of – whether if you consider this as acceleration or displacement, accordingly, this value will correspond to that value of amplitude and phase.

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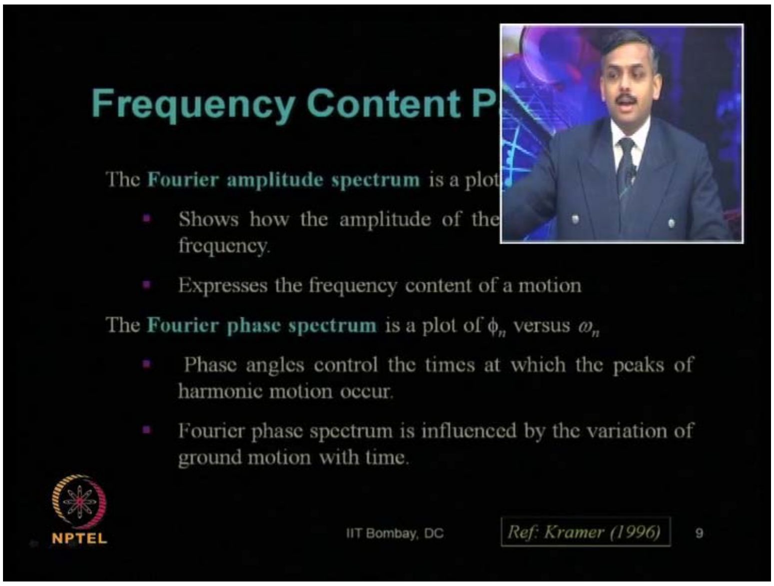

We have seen what is known as Fourier amplitude spectrum, which is nothing but a plot of that amplitude versus frequency. And also, we have seen Fourier phase spectrum, which is nothing but a plot of phase versus frequency.

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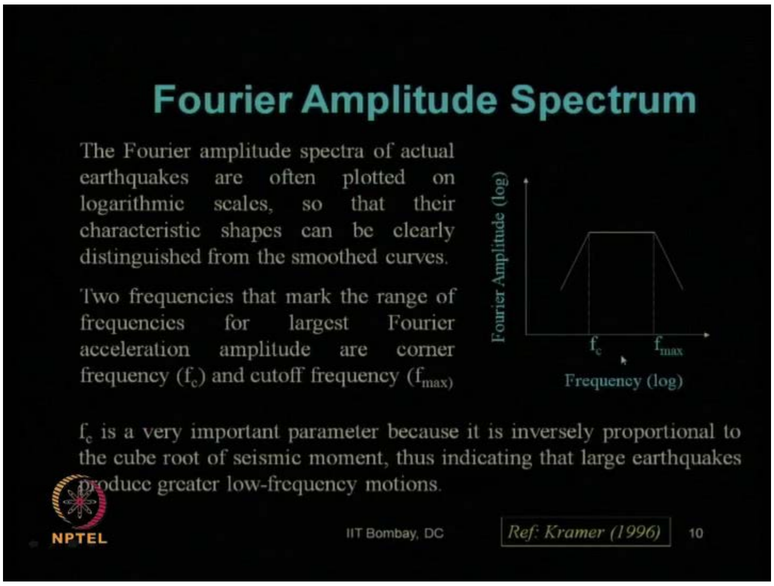

This is the typical plot of Fourier amplitude spectrum – Fourier amplitude in log scale in y-axis and frequency in the log scale in the x-axis. (( )) what is the definition of the corner frequency and the cut-off frequency; that is, the minimum value of the frequency at which that maximum amplitude or maximum value of that Fourier displacement or acceleration, whatever you take as a response, that will start. And, cut-off frequency is up to that maximum frequency up to which it will continue.

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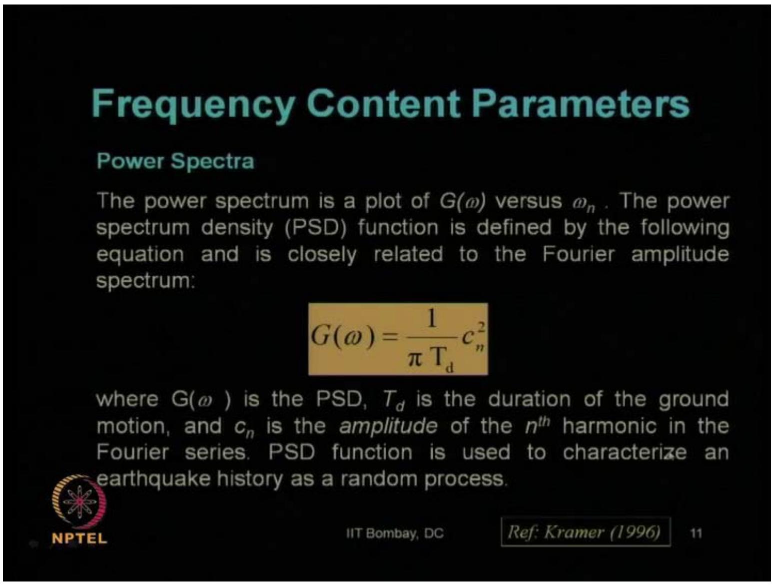

Then, we have seen another mathematical expression of this earthquake motion through power spectrum. The advantage of this, we have discussed that, it can consider the random process through using this power spectrum density function like this.

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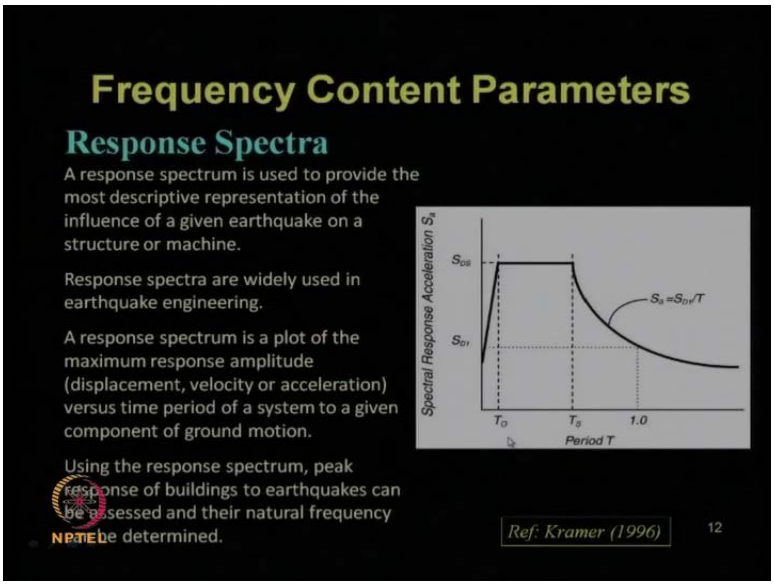

Then, we have discussed, the most widely used frequency content parameter through the response spectrum. As we have mentioned, response spectrum can be acceleration response spectrum or displacement response spectrum or velocity response spectrum.

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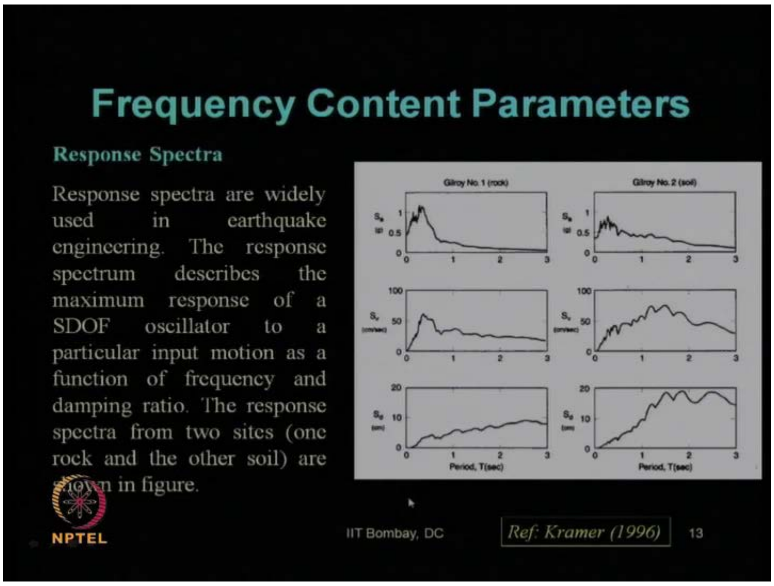

That we haven see for various rocky site and soil site, acceleration, velocity and displacement response spectrum. How this response spectrum is obtained? Through the single degree of freedom – mass-spring-dashpot model by applying a particular input motion corresponding to a particular exciting frequency and the damping ratio. We will derive it today in this lecture.

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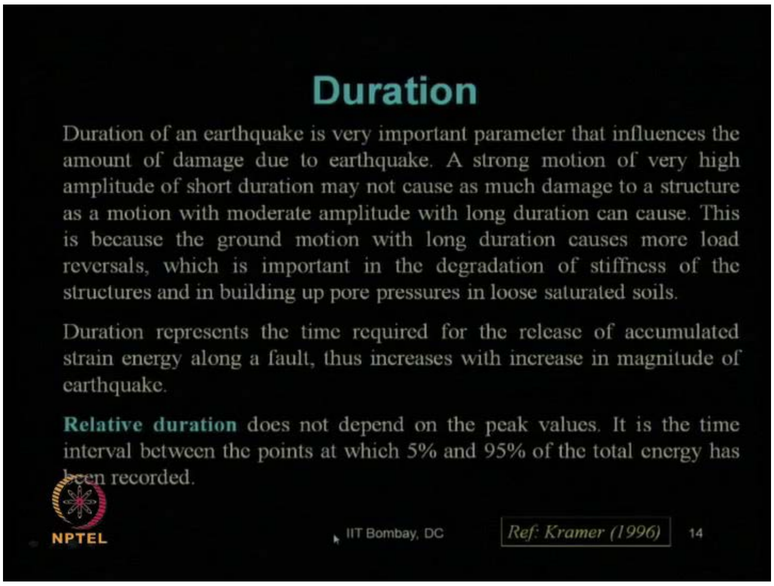

Also, we have discussed about the duration. In the duration, we have seen, it is nothing but the time required to release the entire strain energy during an earthquake process;

whereas, the relative duration is nothing but it is the time between that 5 percent to 95 percent release of the energy.

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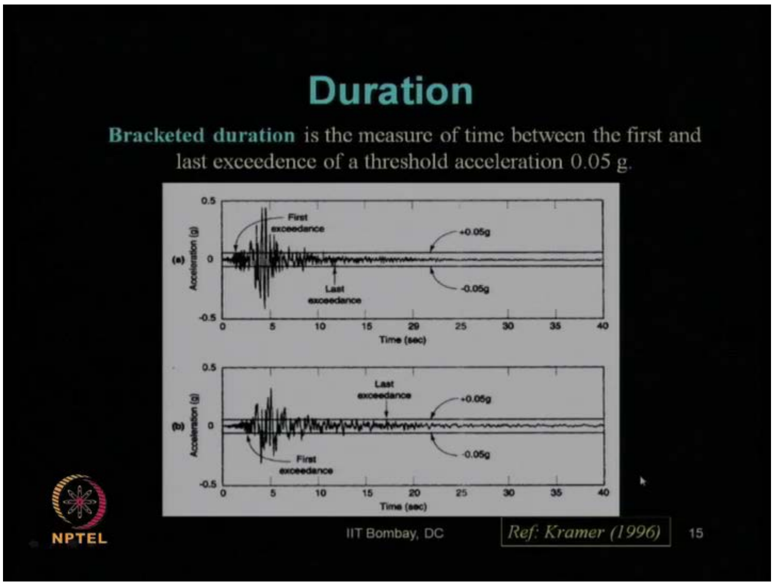

And, for engineering design, we have seen, another important duration parameter is nothing but bracketed duration, which considers the time duration, where earthquake acceleration is more than that minimum value or threshold value of acceleration, which causes damage to structure, which is 0.05 g. So, that is the time or duration, which is known as the bracketed duration. With this, we had completed our previous lecture.

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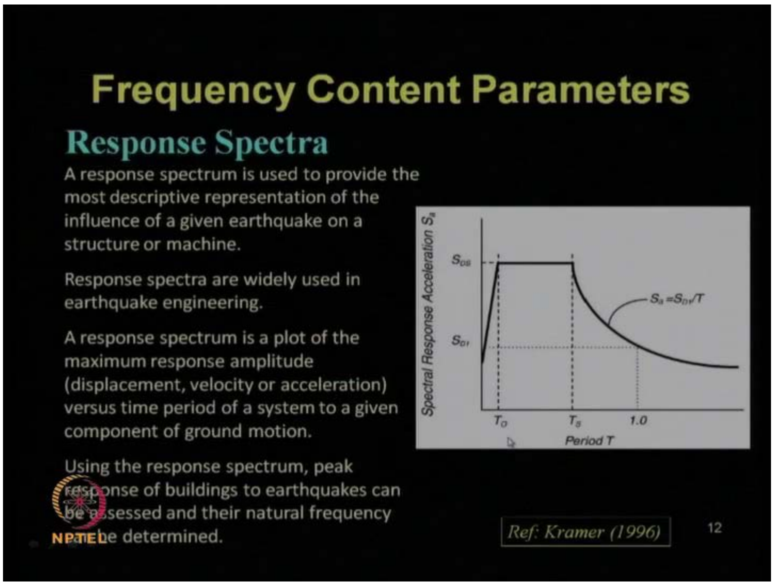

Now, we will continue our today's lecture with the derivation of the response spectrum. So, let us go back. This is the typical picture of the response spectrum – how to arrive at the response spectrum, because this is the maximum used frequency content parameter in the earthquake engineering.

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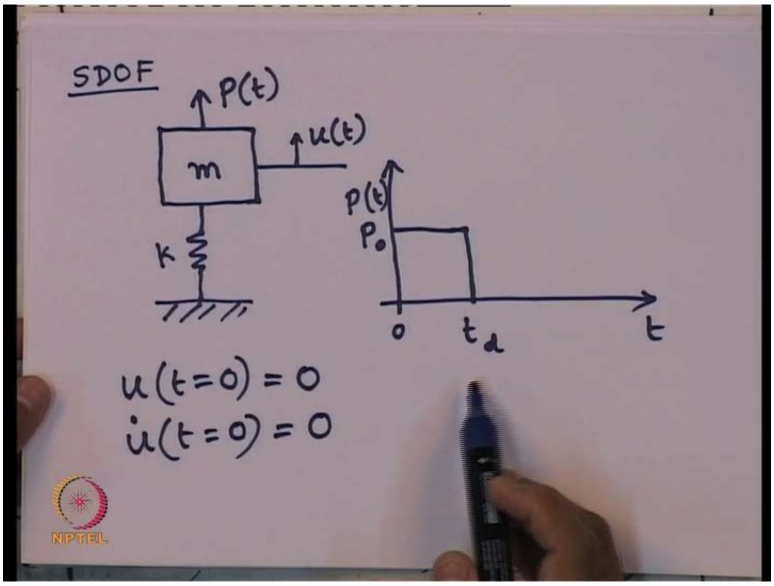

Let us take a simple mass-spring-dashpot single degree of freedom model, which we have already discussed in our previous video course on soil dynamics. But, for clarity, let us complete it once again over here. Let us say, this is a single degree of freedom system as we know, is SDOF system with mass m and spring constant k. And, u t is nothing but that single degree of freedom, which defines its position at any instant of time; and, p t is nothing but the externally applied dynamic load on the system. Suppose if we have the variation of this p t with respect to time; how this p t varies with respect to time? Let us say, it varies like this.

Let us take a very simple example first. Then, we can idealize it to a generalized case. Say this is the dynamic load, which is acting on the system; that is, from time 0 to time t d, it is a constant force, which is acting on the system is p naught; after t d, there is no force acting on the system. What does it mean? From time 0 to t d, it is forced vibration and beyond time t d, it is a free vibration case. So, that is the simple problem, which already we have discussed in soil dynamics. So, let us see the solution of it. For that, we should have the initial conditions be known, because that is the basic information should be known to us. Let us say u, that is, displacement at t equals to 0 say it is 0; and u dot at t equals to 0, that is, the velocity at time t equals to 0. Let us say that is also 0; that is, I am considering a system, which is starting from absolute stationary condition with 0 displacement and 0 velocity at the initial starting of the dynamic time t. Now, as we have already identified, it is having two phases of vibration: one is forced vibration case up to time t d and then the free vibration case beyond time t d.

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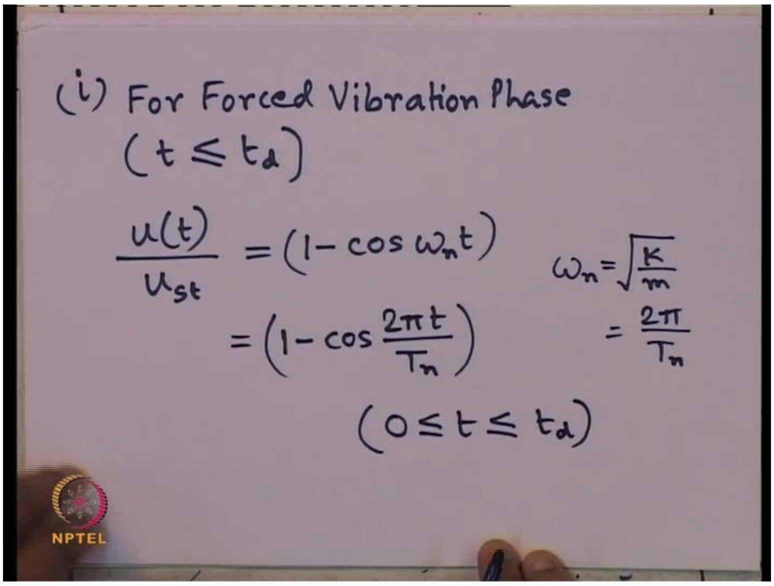

Let us see how the variation is occurring. So, let us consider for different portion, that is, for forced vibration phase; that means, t any time should be less than are equals to that t d, because force is acting till the time t d. So, any time if we consider between this range, that is nothing but a forced vibration phase. So, what is the solution of this one? If we refer to our former course on this soil dynamics, we will see that, u of t with u st – u st is nothing but static value of the displacement – can be expressed as 1 minus cosine omega n t for a constant force like p naught is acting on the system. So, that was the solution already we have derived in our solid dynamic video course. If you have gone through that, you can easily understand. I am not again repeating that portion; I am trying to derive beyond that, what is the response spectrum. So, if you are having any doubt, you can refer to that course and clarify by yourself.

In this case, omega n is the natural frequency of the system, which is nothing but root over k by m; which is nothing but once again 2 pi by T n. T n is the natural time period.

So, if I express it in terms of t n, it will look like 1 minus cosine of 2 pi t by T n. And, this is valid as I said for a time, which is between 0 to t d. So, this is one set of solution as we already know about it. So, now, let us see what will be the response in the free vibration phase.

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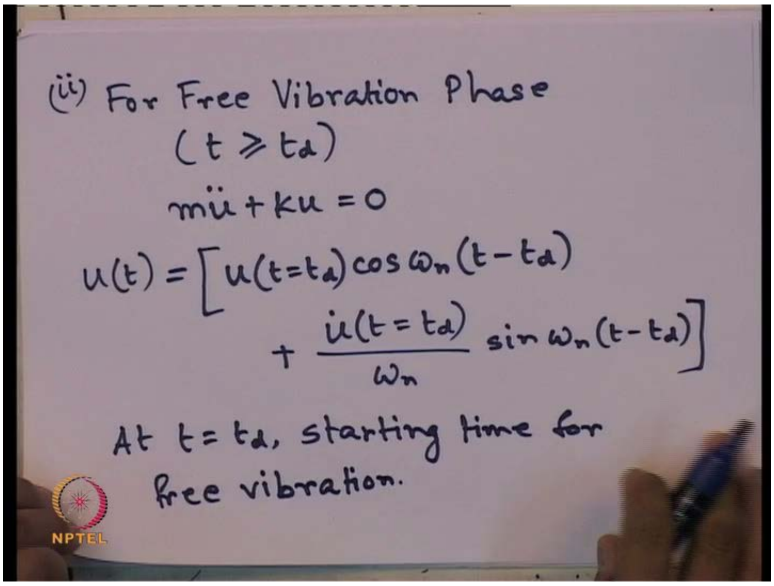

In the second phase, that is, for free vibration phase, that means, the time exceeds that value of that t d; that is nothing but free vibration, because you have taken out the load. What is the equation of motion? You have basic equation of motion m u double dot plus k u equals to 0 from our chosen model. Now, u of t – the solution of that t equals to – can be written as u at t equals to t d times cosine of omega n t minus t d; that is, the initial displacement for this case of free vibration. What is the initial displacement for this free vibration? That is the last point of the previous phase or forced vibration phase; at t equals t d is the nothing but the starting point of free vibration. Plus the velocity u dot at t equals to t d by omega n times sine of omega n t minus t d. There is solution as we know for any free vibration phase that, initial displacement times cosine of omega n t plus initial velocity by omega n sine of omega n t.

In this case, t is nothing but beyond that time t d. So, that is why, t has been replaced by t minus t d. And, the initial time has been taken as where that t equals to t d condition was existing. So, at t equals to t d, it is the starting time for the free vibration. That is the reason as I have already mentioned – starting time for free vibration phase. Now, with this, what we can rewrite this as solution, let us see over here.

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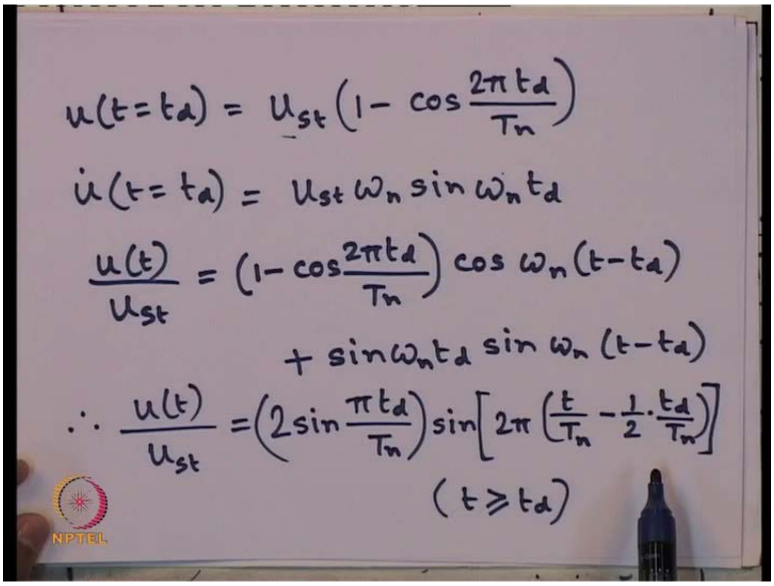

Next, we can write that, u at t equals to t d will be equals to u of s t 1 minus cosine 2 pi t d by T n. This is the solution we can write from the forced vibration phase, because that was the solution we have seen – u of t by u of s t equals to 1 minus cosine 2 pi t by T n. So, I have just placed t equals to t d over there. So, that is the initial condition. And, initial velocity u dot should be for this second phase of free vibration, can be u of s t. If I differentiate this, omega n sine of omega n t d. Therefore, what we can write, u of t by u of s t – static, can be written as the initial value is 1 minus cosine of 2 pi t d by T n times cosine of omega n t minus t d. I am writing now the complete solution that, u of t for second phase, that is, the free vibration phase. I have taken this u s t out in the denominator. That is why, we can write it like this – plus sine of omega n t d times sine of omega n t minus t d, because omega n was in the denominator. So, it gets cancelled. So, remaining portion is this one; I have already taken this out. So, this is the simplified form. Therefore, if I further simplify it, what I can write it?

u of t by this u of s t can be written as… This is a trigonometric function as you can see, the variation. We can simplify it and rewrite it as 2 of sine pi t d by T n times sine of 2 pi times t by T n minus half of t d by T n. So, this complete solution for the second phase is valid for which time? So, this is valid for t greater than equals to t d. So, what we can see in this case that… If you look at the value or the function of that u t by u s t, it is the function of this t d by t n ratio. That is why I have expressed it in this form – t by t n ratio and t d by t n ratio. Let me express it in a more simplistic way.

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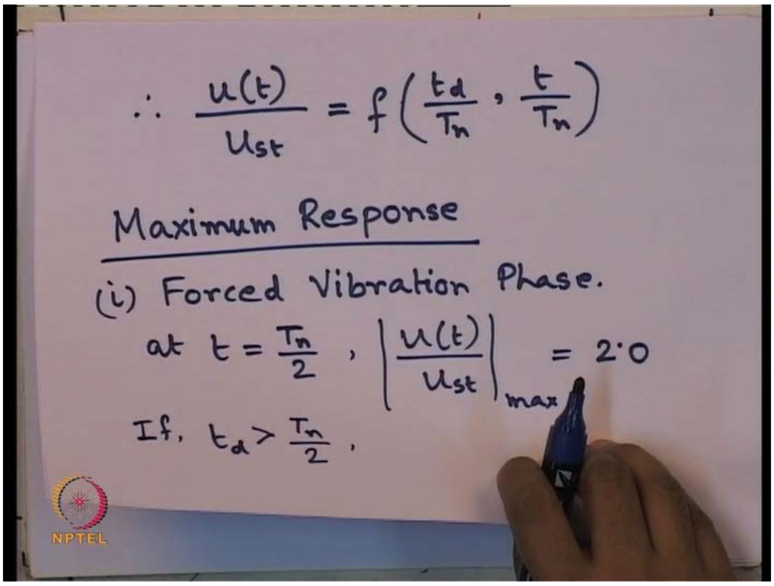

Therefore, my final output say is that, u of t by u of s t, that is, the dynamic displacement with respect to the static displacement is the function of what parameters? That t d by T n ratio and that t by T n ratio. So, that means, any dynamic response or dynamic displacement is dependent on not the individual values of this duration up to which the earthquake is acting or the dynamic load is acting, nor it is dependent on the individual values of total duration or the individual value of the natural time period of the system; but, it is dependent on the ratio of this, that is, how long the dynamic load was acting over the natural period and how long that total time you are considering. What does it mean? Suppose earthquake duration was for say 20 seconds and I am interested that, after one hour of earthquake, what is the damage on my structure; then, this function tells us clearly that, in that case, in the solution, you have to put t d as 20 seconds, but t has to be 1 hour, that is, 3600 seconds. So, it is clear that, the damage we are considering for any structure whether substructure or superstructure anything, it is not only a function of the earthquake duration; it is a function of the – at time, where you are interested to, but not that individual value. It is a related to a ratio of that natural period of the system. That is the reason.

If you have understood this solution clearly, this automatically tells us that, it is not necessarily that all the earthquake damages are going to occur during the earthquake or soon after the earthquake; it may happen that, the structural damage or any damage as such due to the earthquake in the soil also, may occur after a large time has spend after the stopping of the earthquake also. So, that is what I wanted to show through this solution. What it says? It says, the dynamic displacement or the response is not only a function of that ratio of duration of earthquake to the natural period, but it is also dependent parameter on at what time you are interested to know the behavior of your structure. So, we have to be careful about both these two parameters – this as well as this ratio.

Once knowing that, let us find out what is the maximum response. Then, we can probably be able to appreciate it. Maximum response of the structure subjected to this case. In the forced vibration phase, that is, the first phase, if we look at, at t equals to t n by 2; that means, if our time is equals to t n by 2, then what will be the value of the u t by u st? Maximum value, maximum amplitude of that will be equals to 2; that is, if you put it in this solution, I will show you once again, that is, what was my solution for this case. If I bring it here, yes, this was the solution, if you look at here within the forced vibration phase. So, I am talking about within forced vibration phase. So, obvious reason is t has to be within t d. Now, what I have chosen, this t value I have chosen as t n by 2; that is, the ratio I have taken as… t by t n ratio I have taken as half – 0.5. So, if it is so, if you put that value, what it is becoming? It is becoming minus of cos pi. So, plus 1. So, 1 plus 1 – 2. That is the possible maximum value of this solution. So, that is why I have said, it gives us the maximum value of this u t by u st max is 2 at this point. Now, if the value of this t d is greater than this t n by 2, then what happens? Then, we have to see two solutions. What are those? We will look at here. So, let us mention these as a response; let us identify this as a response – u t by u st; that is, with respect to static.

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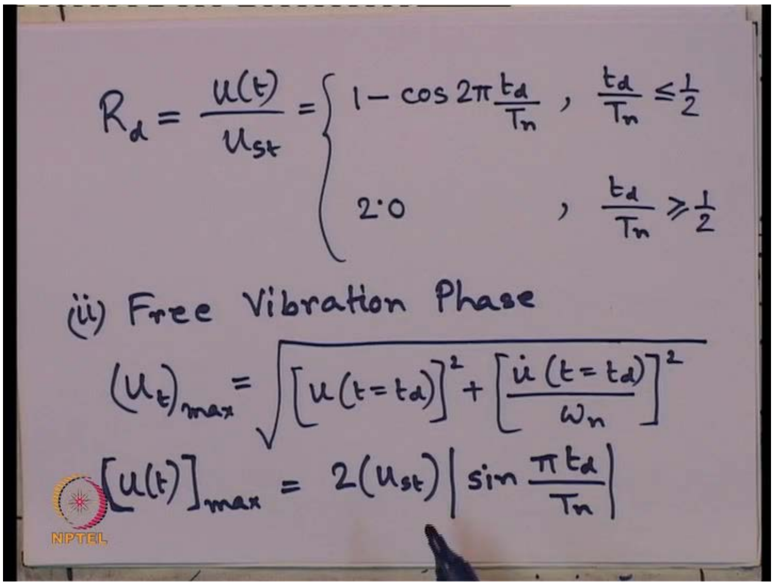

If I want to show the response R d let us say, response; which is nothing but u t by u of st I am defining. It can be expressed by two solutions: 1 minus cosine of 2 pi t d by T n and 2. These are the maximum values. This is applicable when the t d by T n ratio is less than equals to half. And, this is applicable when t d by T n ratio is greater than equals to half. So, that is what we said just now that, maximum response is very much dependent on this ratio as you can see. If this ratio is less than equals to half, then the maximum response will be computed like this; based on whatever value of cos you are getting that will be coming. But, if this ratio is more than or equals to 0.5; in that case, always the maximum response will be 2 times; that is, the dynamic response will be 2 times of the static.

Now, let us see in another phase, that is, the free vibration phase. For second phase, that is, for free vibration phase, what is the maximum value of this u t? If I write down that, u t of max, that will be nothing but the amplitude, whatever you are getting at t equals to t d – that square – plus that u dot t equals to t d by omega n – that one square. Are you satisfied with this expression? Let us look at here. As I said, the solution of this will be in the form of… For free vibration phase, I brought this here; u of t is expressed as initial displacement times cosine of omega n t plus initial velocity by omega n times sine of this. So, what can be the maximum value of this u t? Obviously, it will be root over this amplitude square plus this amplitude square. So, that is why, u initial square plus this velocity by omega n this square. That will give us the maximum value of this entire solution. So, u of t maximum. That is why I said, can be written as this.

Now, if I simplify it, u of t – this max – u of t max can be written as – on simplification, by putting the known expression – 2 of u s t times mod of sine pi t d by T n. Where from I got it? Let us look at the solution of this. Then, we will be able to understand. Yes, here was the solution. If you remember this one, if you see, this was the solution for free vibration phase at the end. So, what is the maximum value of this solution – this portion? This amplitude; remaining is sine. So, if sine is 1, then it will be maximum. So, if it is maximum, 2 times u st sine of pi t d by T n; that is what I have written over here. And, it can be amplitude – either minus or plus. So, with this, absolute response will be – let me write it down once again.

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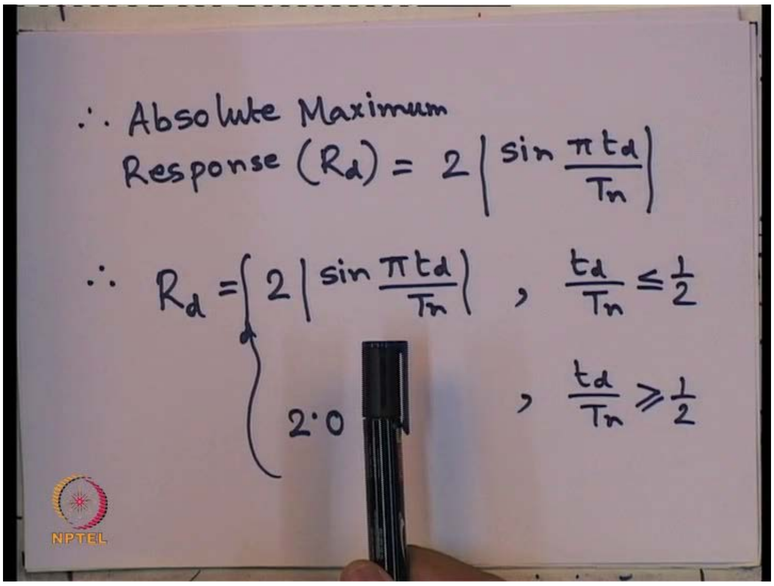

Therefore, the absolute maximum response, that is, R d, what we are expressing; that can be written as 2 times of that sine pi t d by T n. This is for the free vibration phase. Therefore, if I write it in a combined form, I can write it as 2 times mod sine pi t d by T n; two solutions actually: this is one case; another case is 2. When this case is valid; when this t d by T n ratio is less than equals to half; and, this condition when t d by T n ratio is greater than equals to half. See this is the complete solution. What is maximum value of this again? When this sine parameter is 1; that will give us that value of 2 – maximum value. When it will occur? That will occur when this ratio t d by T n is more than equals to half; otherwise, if it is less than equals to half, it has to be two times of whatever the sine value gives us. So, this is the response in free vibration zone. And, in the forced vibration zone, if you look at, this was the maximum response. Say everywhere, it is the function of that t d by T n; and, that t by n concept will come to you only when you are interested about a particular time when you want to study the effect on the structure; as I said, after 1 hour; after 30 minutes; after 10 minutes; like that. So, that is the reason; if you see over here, here I have not considered the damping portion.

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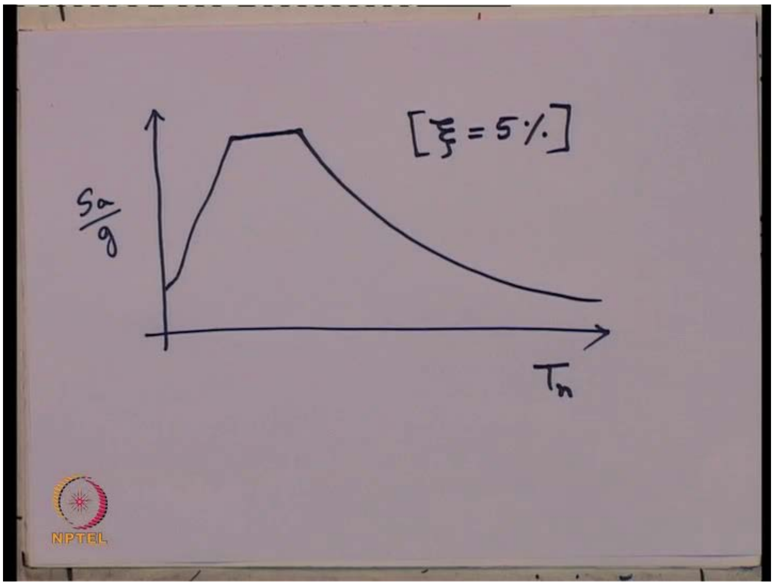

But, if you consider the damping portion; if you remember my soil dynamics lectures, the typical value of this plot – the response spectrum, which we show that, T n and say S a by g; it can be acceleration response spectrum; it can be displacement response spectrum; it can be velocity response spectrum. This looks like this for a particular value of the damping ratio say 5 percent damping or something like that. For a particular damping ratio, this behavior we can obtain. And, if I want to plot this u t max by s for our case, how it should look like? Let us see.

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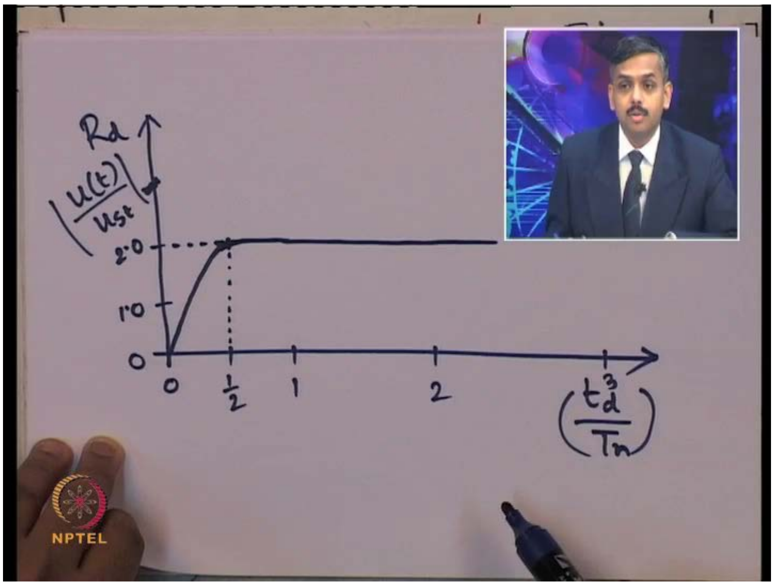

I am now plotting this value of R d versus t d by T n. This is the variation of t d by T n with respect to R d. R d as I have defined; so, u t by u s t – maximum of that I am plotting. So, as I said over there, it is 0; say this is 1; say this is 2; this is 0; let us say this is half; this is 1; this is 2; this is 3; like that. So, at half, if I draw over here; variation will be something like this. So, maximum response – it occurs at the t d by T n ratio of this one when it is subjected to that constant force type vibration for a finite time period. But, if it is a random, similar process has to be used to get the solution and get this response like this. So, that similar procedure we adopt for the earthquake random motions also; which probably you have to solve using the (( )) integral process, which I have discussed thoroughly in the soil dynamics course for any kind of motion. But, everything finally, will be a function of this – how long it is occurring – that earthquake motion with respect to the natural time period; and, at what point of time you are interested to find out the value or response.

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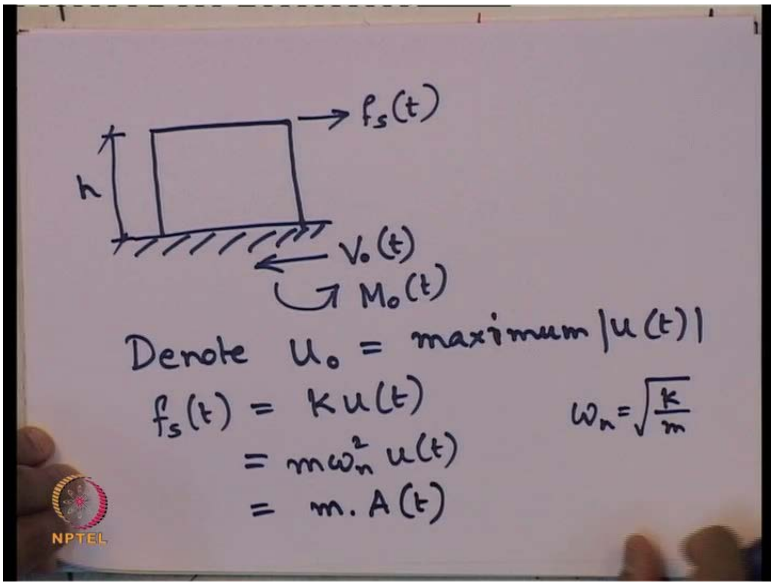

Another parameter I want to highlight over here. Let us look at here; like it is called the pseudo value of acceleration response spectrum and pseudo velocity response spectrum and pseudo displacement; how it is defined? Let us say, this is a typical structure of height – h, which is subjected to this force f s t over here. I have a (( )) shear here – V naught t let us say; and, a moment – resisting moment – let us say M naught t; but, the structure… And, let us denote that, u naught as the maximum value of that u of t – the function, whatever it is subjected to any kind of dynamic loading; that f s t is nothing but a spring force acting on the structure with k as the spring constant or the stiffness times u of t; which if we simplify it further, it can be expressed as m omega n square times u of t. Why? Because as we know, omega n nothing but root over k by m. So, automatically, k is nothing but m omega n square. So, in this case, we call it as m times A of t. What is then A of t? If you look at here, A of t is nothing but…

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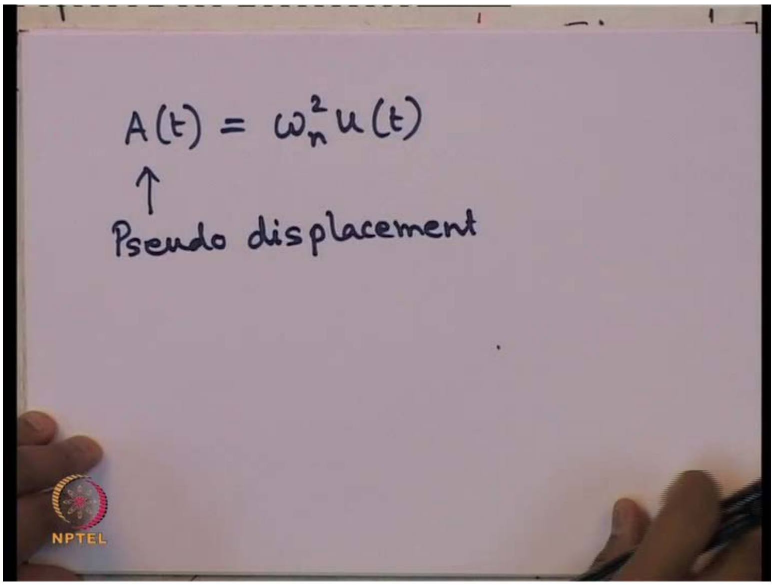

As per our written expression, A of t is omega n square times u of t. So, this A of t is known as pseudo displacement. So, this is pseudo displacement. This A of t...

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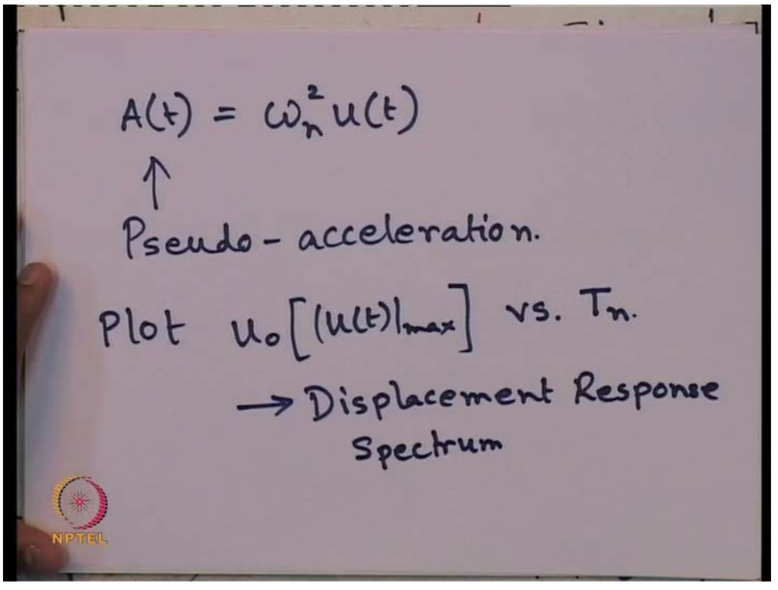

A of t can be written as omega n square u of t. This A of t is called pseudo-acceleration response spectrum. So, this is a… If we plot this value of u naught… What is u naught? As I said, it is u t max. If you plot this u naught versus this T n, that is known as displacement response spectrum. And, if you differentiate it further – this u naught with respect to time, what you get over here?

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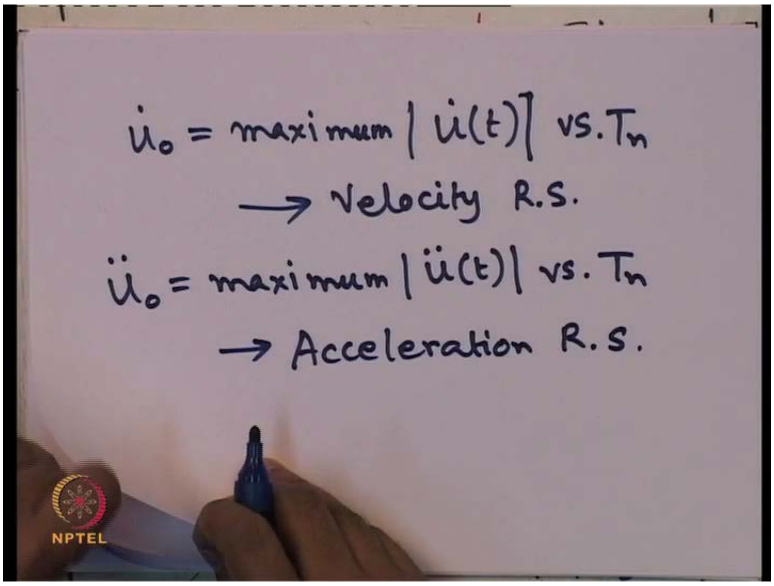

That u naught dot, which is nothing but maximum of this u dot t versus that T n. That plot will give us velocity response spectrum. If you further differentiate it with respect to time, u naught double dot is nothing but maximum of this u double dot t versus the plot of T n. That gives us the acceleration response spectrum.

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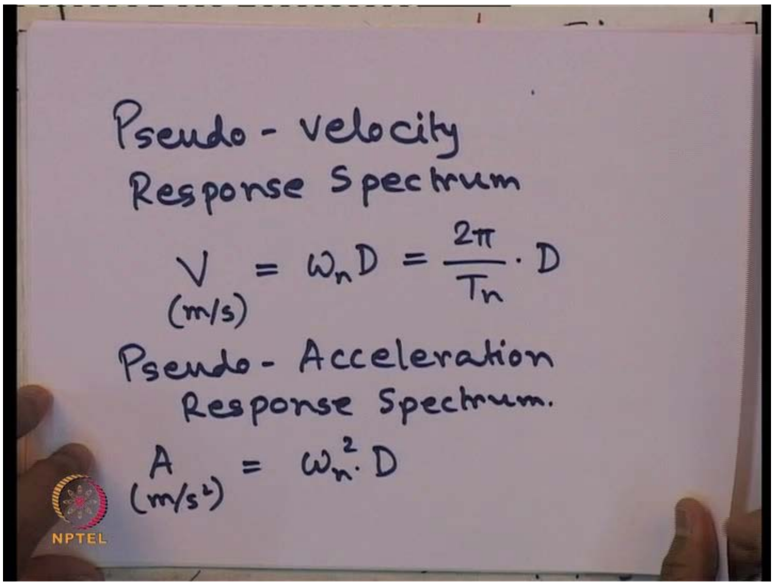

So, for the chosen solution of A of t what, we have seen, pseudo-velocity response spectrum can be obtained as let us say V and the unit will be velocity unit say meter per second, will be omega n square times D or omega n times displacement – 2 pi by T n times D. And, pseudo-acceleration response spectrum will be A in terms of meter per second square unit; that will be omega n square times this D; D is pseudo displacement. So, what I have written over here earlier, if I correlate; you can see over here. This is known as pseudo-acceleration; which is nothing but omega n square – natural frequency square times the displacement profile with respect to time – omega n square times u of t. So, that will give you the pseudo-acceleration response spectrum. So, like that, we can draw any response spectrum – whether it is a pseudo-acceleration response spectrum or pseudo-velocity response spectrum or pseudo-displacement response spectrum.

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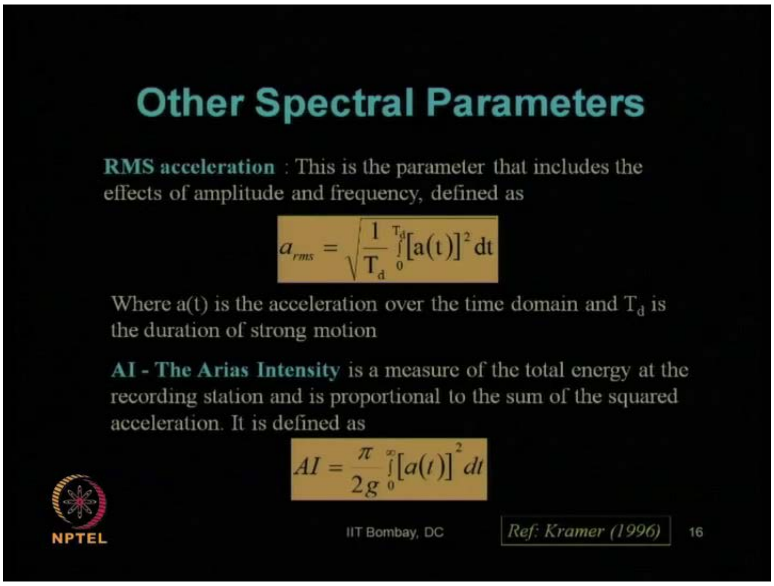

Now, let us come back to our topic. Now, let us start some more parameters like other spectral parameters. What are the other various spectral parameters, which generally are used? One is called RMS acceleration – nothing but root mean square acceleration. So, this is the parameter that includes the effects of amplitude as well as frequency. So, this acceleration parameter – root mean square acceleration – it considers both effects of the amplitude as well as the frequency. It is expressed in this form – a of RMS equals to root over 1 by T d integrate 0 to T d a of t whole square times dt; a of t is nothing but the actual variation of acceleration with respect to time. If you integrate over the time period that T d and then find out the root mean square, this is nothing but the… You are taking the mean by dividing it and root your taking. So, root mean square acceleration you will get by this over the time domain and t d is the duration of the strong motion.

And, AI is another important spectral parameter, which are commonly used for design. AI is nothing but – full form is arias intensity. This arias intensity is the measure of the total energy at the recording station and is proportional to the sum of the squared acceleration. So, it is defined as arias intensity, is pi by 2 g integrate 0 to infinity a of t whole square dt. So, that acceleration response with respect to time, whatever you have during an earthquake; if you integrate it over 0 to infinity by taking a square of that and then multiply it by pi by 2 g, that estimates the total energy, which is measured during a recording station, which is known as arias intensity.

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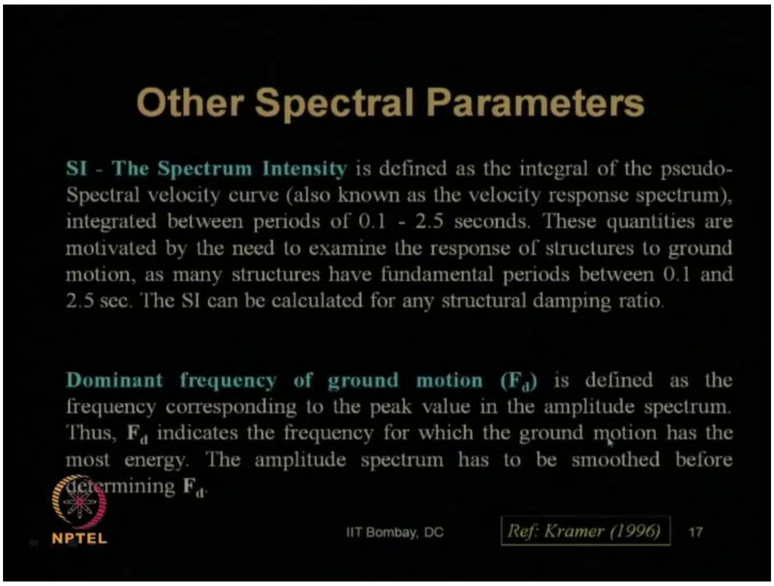

Another spectral parameter is spectrum intensity or SI. SI is defined as the integral of the pseudo-spectral velocity curve. Just now, I have discussed what is pseudo-spectral velocity curve, which is known as the velocity response spectrum. So, if you integrate that between the time period; that is, if you have that plot of pseudo-velocity spectrum versus T n between 0.1 to 2.5 second s; between that, whatever you get, if you integrate that curve, these quantities are motivated by the need to examine the response of structure to ground motion as many structures have fundamental periods between 0.1 to 2.5 seconds. That is the reason why the spectrum intensity – it is calculated for any structural damping ratio to determine their response or determine their damages. So, this is the typical range of the fundamental period of any structure – of our civil engineering structure. It will be within 0.1 second to 2.5 seconds.

And, in the reverse way, we can see, considering the structural period or fundamental period of a structure, if you take in hertz, how much it is coming? About 1 by 0.1. So, 10 hertz. And, this is 1 by 2.5. So, it will be inverse of 2.5. That much hertz will be the typical range of the natural or fundamental frequency of the system. So, if the earthquake frequency also comes within that, then here you have to take care of the structural damages, etcetera. That is the reason you have to integrate that velocity – pseudo-velocity response spectrum curve between this time period of 0.1 to 2.5.

Another spectral parameter is dominant frequency of ground motion – F d. What is f d? It is defined as the frequency corresponding to the peak value in the amplitude spectrum; that is, whatever amplitude spectrum you have, in that, whatever is the frequency which corresponding to its highest value or peak value, that will be the dominant frequency. Thus, F d indicates the frequency for which the ground motion has the most energy, because it is giving you the maximum amplitude. The amplitude spectrum has to be smoothened before determining the F d; which is quite obvious; as we have already understood, we have to smoothen it, filtering it, etcetera.

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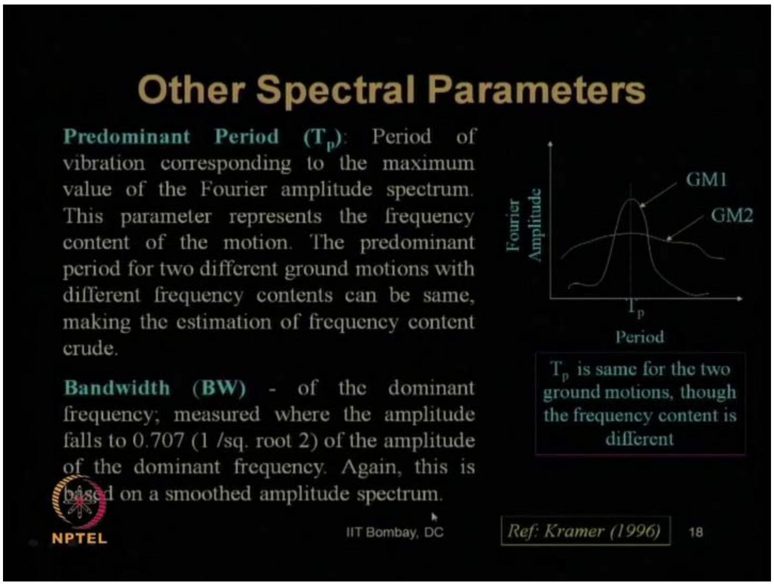

Other parameters like predominant period. What is predominant period or T p? Period of vibration corresponding to the maximum value of the Fourier amplitude spectrum; that is, you have to express the amplitude response in terms of Fourier series. So, as we have already seen Fourier amplitude versus the period, if you plot it like this, something it will like look like this. This is for two different ground motions: this is for ground motion 1; this is for ground motion 2. Can you see? For both of them, the predominant period is same; T p is same; that is, where it is occurring; the maximum value of the Fourier amplitude. So, this parameter represents the frequency content of the motion.

The predominant period for two different ground motions with different frequency content can be same making the estimation of frequency content very crude; that means, suppose for this one, if you plot it in the logarithmic scale, whatever is your frequency content and if you plot this in logarithmic scale, whatever is the frequency content, they are quite different. But, their predominant period, where the maximum value occurs is same. That is what you have to use the combinations of these various spectral parameters to identify the nature of any earthquake motion; not only in terms maximum amplitude, but also in terms frequency content. What is band width? Band width or BW is of the dominant frequency measured, where the amplitude falls to 0.707; which is nothing but 1 by root 2 of the amplitude of the dominant frequency. Again, this is based on the smoothed amplitude spectrum.

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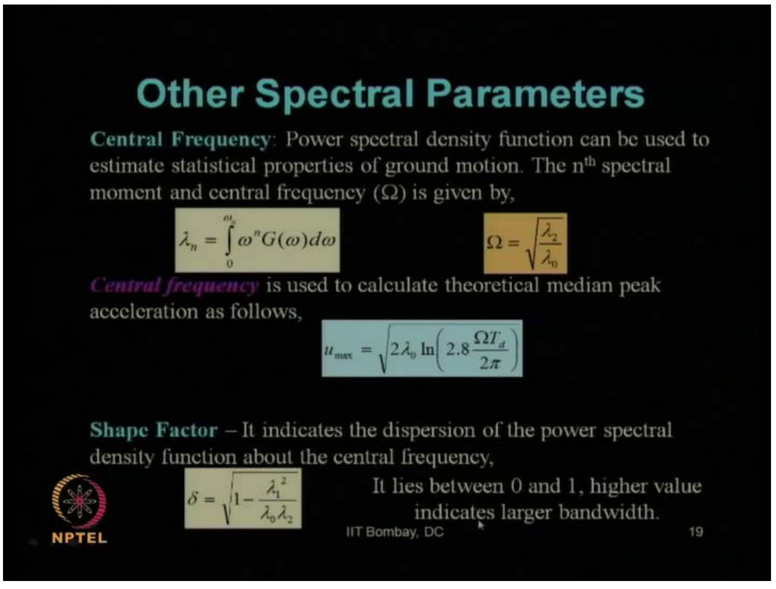

Central frequency – what is central frequency? See it is a power spectral density function; that PSD what we have already discussed. Within that can be used to estimate the central frequency of the ground motion. The n-th central moment and the central frequency can be estimated like this. This lambda n is the n-th frequency; that can be estimated as 0 to omega n omega to the power n G of omega, that is, the power spectral density function times dw. And, that central frequency is defined as this second one – spectral moment divided by the 0-th one; that is, lambda 2 by lambda naught; root of that will give you the central frequency. This is another just estimate of a spectral parameter. So, central frequency is used to calculate the median peak acceleration using this expression. So, this value of this omega – capital omega you can use for that to calculate the u max value in this fashion. And, the shape factor – it is indicating the dispersion of the power spectral density function – that PSD about that central frequency. How it varies with respect to that central frequency, that delta can be obtained using this expression, that is, the lambda 1 – the first spectral moment square divided by 0-th one and the second one. Then, it lies between the value of 0 and 1; higher value indicates the larger amount of band width – the higher value of the spectrum.

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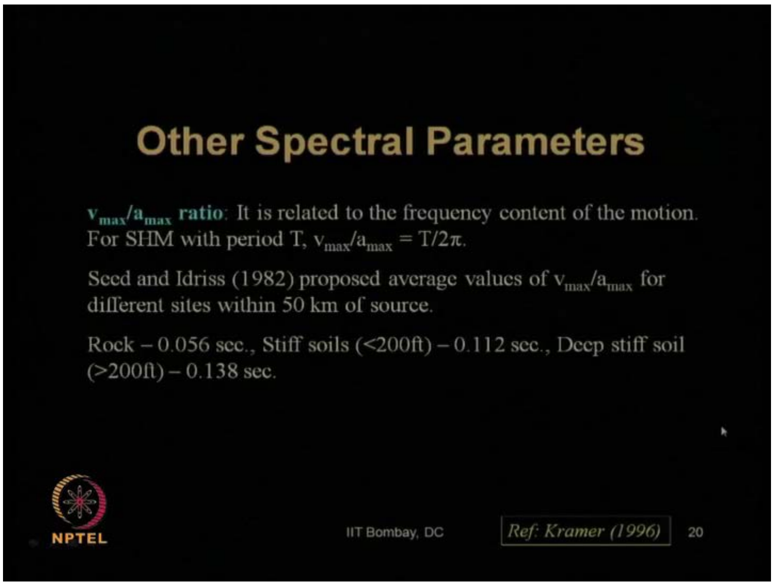

Another very important spectral parameter is nothing but v max by a max ratio. This is important. What is v max? v max is the maximum velocity, if you consider the simple harmonic motion is applied to a single degree of freedom system like structure; what I have already discussed about the derivation, etcetera. And, a max is the corresponding maximum amplitude; that is, from u t max if you find out the velocity max for a simple harmonic motion, it will be how much? Omega times u t.

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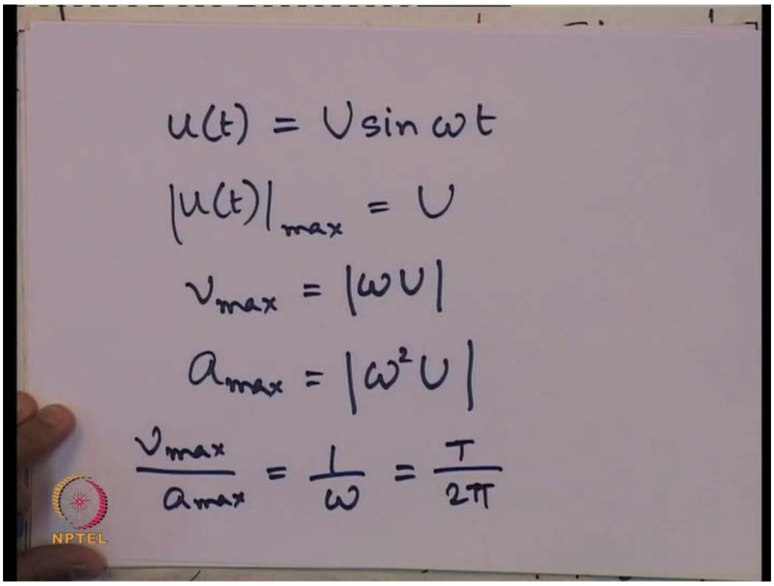

That is, if I want to express; suppose u of t is having some expression say u sine of say omega times t; then, u t – maximum of that is nothing but u. Then, what is the v max? v max is nothing but omega times u – mod of that, because velocity – you can find out by differentiating it with respect to time; this displacement. And, maximum value of that will be… This will become cosine; omega will come over here. So, omega times that u. And, what will be a max? Omega square times u – mod of that. So, that says in this slide whatever I have discussed just now – look at here – v max by a max ratio is related to the frequency content of the motion. For simple harmonic motion with a period T, this v max by a max can be represented as T by 2 pi. If I look at here; let us see here. v max by a max for this simple harmonic motion will be nothing but how much? v max by a max will be 1 by omega; which is nothing but T by 2 pi.

That is what it is shown over here. You can see here in the slide, v max by a max is nothing but T by 2 pi. Now, Seed and Idriss in 1982 proposed average values of that v max by a max for different sites within 50 kilometer of the source; that is, they have analyzed the values of v max by a max between the earthquake epicenter to a point of site of response for different types of sites, that is, rocky site, soil site, etcetera. And, what are the typical values they have mentioned, the average values? For rock, they have mentioned this value of v max by a max ratio. And, remember, what will be the unit of this? It will be the unit of this T; that is, in second. So, that is why, for rocky site, they have mentioned, the typical value is 0.056 second; for stiff soil, within the 200 feet; that is, the stiff soil available at relatively shallower depth. The value is 0.112 second. And, deep stiff soil – that is, the stiff soil is available beyond 200 feet; in that case, the value of this T by 2 pi or v max by a max ratio is 0.138 seconds. So, what we can see? This value of v max by a max ratio keep on increasing if we go from a stiffer to a softer soil. That is what it means. So, this gives us a very good estimation. But, remember, these are the typical average value. The actual value can differ. And, how this value can be used to estimate for estimation of the magnitude of another location or computation of epicentral distance? That we will see through an example problem in the next class. With this, we have come to the end of today's lecture. We will continue further in the next lecture.