

**Geotechnical Earthquake Engineering**  
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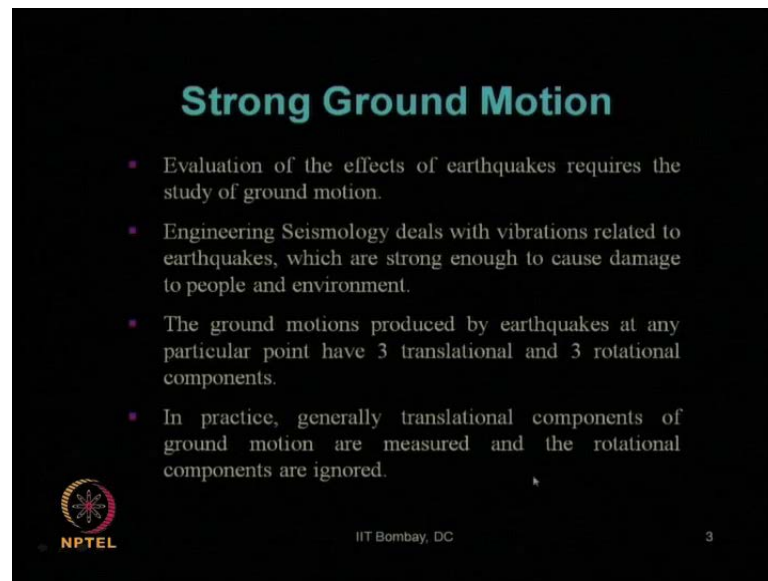
**Module - 4**

**Lecture - 13**

**Strong Ground Motion (Contd...)**


Let us start our today's lecture on the video course on geotechnical earthquake engineering. Let us look at the slide here; on this video course geotechnical earthquake engineering, we are currently going through module 4 on strong ground motion.

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**Strong Ground Motion**

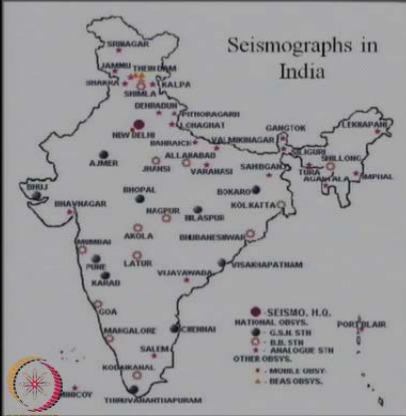
- Evaluation of the effects of earthquakes requires the study of ground motion.
- Engineering Seismology deals with vibrations related to earthquakes, which are strong enough to cause damage to people and environment.
- The ground motions produced by earthquakes at any particular point have 3 translational and 3 rotational components.
- In practice, generally translational components of ground motion are measured and the rotational components are ignored.

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So, a quick recap what we have learnt in the previous lecture, like strong ground motion, they evaluate the effect of earthquake, and for that study we need to estimate the strong ground motion properly. And strong ground motion typically produce during an earthquake in six directions that will be 3 translational and 3 rotational components, but for general practice, we mostly use the 3 translational ground motions for any engineering design. And most of the cases 3 rotational components are ignored.

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### Strong motion seismographs



- Designed to pick up strong, high-amplitude shaking close to quake source
- Most common type is the accelerometer
- Directly records ground acceleration
- Recording is triggered by first waves
- Difficult to differentiate S and surface waves

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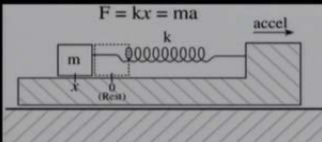
These are various locations of strong motion seismographs located all over India, which record P wave, S wave and surface wave, whenever they are not in the shadow zone of either of these waves.

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### Accelerometer

Types of Accelerometers:

- Electronic** : transducers produce voltage output
- Servo controlled**: use suspended mass with displacement transducer
- Piezoelectric**: Mass attached to a piezoelectric material, which develops electric charge on surface.



Principle: An acceleration  $a$  will cause the mass to be displaced by  $ma/k$  or alternatively if we observe a displacement of  $x$ , we know that the mass has undergone an acceleration of  $kx/m$ .

Generally accelerometers are placed in three orthogonal directions to measure accelerations in three directions at any time. Sometimes **geophones** (velocity transducers) are attached to accelerometers to measure the seismic wave velocities.

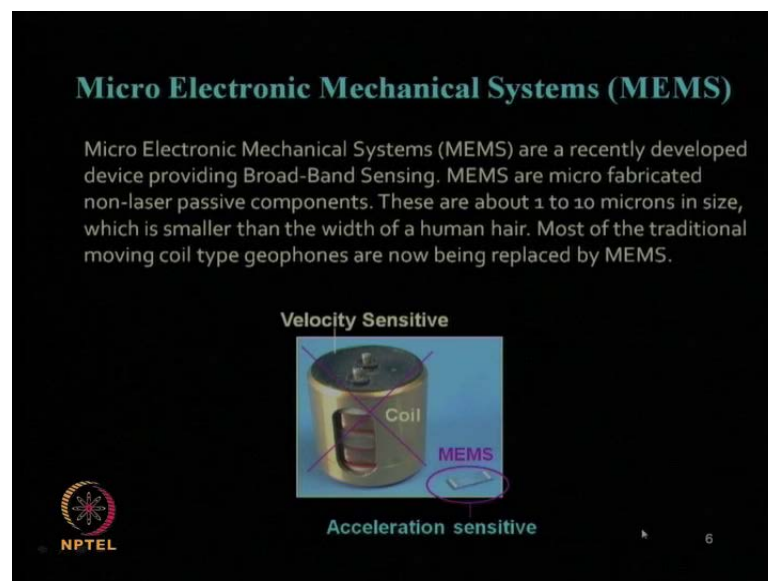
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This is the basic principle on which an accelerometer works. We have seen, there are typically three major types of accelerometers like electronic, servo controlled and piezoelectric. But the basic principle or the basic fundamental on which this accelerometer works is a single degree of mass spring system, that is if the block due to

earthquake shaking if it moves or accelerate in this direction, the mass which is hanging to measure that acceleration will be moved by  $x$  direction by  $x$  amount in this direction; and the spring force will be equal to that mass times acceleration to maintain the equilibrium of the system. So that is the principle that is either knowing this displacement  $x$  we can convert it to the acceleration that is  $k x$  by  $m$  or by knowing the acceleration you can convert it to the distance or displacement by which it has moved that is  $m a$  by  $k$ .

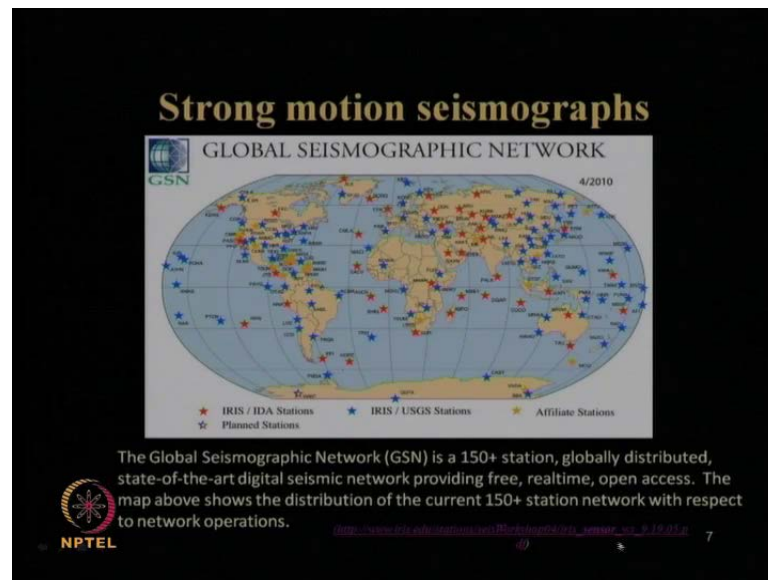
So, either way we can estimate this value through this accelerometers what it does actually. And geophones are commonly known as velocity transducers; that is they measure the velocity of various seismic waves, which we which arrive during the earthquake process.

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Then we have seen also the latest development in this area, the instrumental development is the use of microelectronic mechanical systems or MEMS in short it is called. These are as thin as human hair, you can see the size compare to the olden days the velocity transducers such big coils etcetera. So, now these velocity transducers are replaced by these acceleration based MEMS instrument for common measurement of this acceleration time history.

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These is the global seismic network - GSN available in this site; more than 150 stations globally which record and provide real time open access and free earthquake motions all over the world through this strong motion seismograph located at various parts of the world.

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**Ground Motion Parameters**

An earthquake history can be described using **amplitude, frequency content, and duration**.

**Amplitude:** The most common measures of amplitude are

- PGA: Peak ground acceleration (Horizontal- PHA & Vertical- PVA)
- EPA: Effective peak acceleration
- PGV: Peak ground velocity ( PHV & PVV)
- EPV: Effective peak velocity
- PGD: Peak ground displacement

**Frequency Content:** The frequency content of an earthquake history is often described using Fourier Spectra, Power spectra and response spectra.

**Duration:** The duration of an earthquake history is somewhat dependent on the magnitude of the earthquake.

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Then we have discussed the importance of various ground motion parameters. There are three major ground motion parameters; one is amplitude frequency content and duration, and we have seen that to define the damage of any civil engineering construction or

design or to define an earthquake itself, we need not be, it need not be sufficient to express it in terms of any one of this parameter, but it should be a combination of this three parameters, because importunacy of all the three parameters, we have discussed; that is amplitude, frequency content and duration.

Now within amplitude, the common measures are through peak ground acceleration or effective peak acceleration or peak ground velocity or effective peak velocity and peak ground displacement. So, these are the common forms of estimating the amplitude of an earthquake, ground motion; then frequency content that can be estimated using either Fourier spectra or power spectra or response spectra. And the duration of an earthquake the duration of earthquake history is also very important. So, these three are the major parameters, which defines a ground motion completely.

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**Measurement of ground acceleration**

A seismograph can be illustrated by a mass-spring-dashpot single degree of freedom system.

The response of such system for shaking is given by

$$m \frac{\partial^2 u}{\partial t^2} + c \frac{\partial u}{\partial t} + k u + m \ddot{u}_g = 0$$

where  $u$  is the trace displacement (relative displacement between seismograph and ground),  $u_g$  is the ground displacement,  $c$  is the damping coefficient,  $k$  is the stiffness coefficient.

Ref: Kramer (1996)

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Now, to estimate the ground acceleration, this is the basic principle on which a mass spring dashpot single degree of freedom system acts on a seismograph. So, we have derived this expression that is how this is a forced vibration similar to our previous course on soil dynamics for NPTEL, we have derived this expression here also that is the  $u$  is the relative displacement or trace displacement between the ground and the super structure;  $u_g$  is the ground displacement this is the function of time of course, during an earthquake how much ground displaces;  $c$  is the damping coefficient; and  $k$  is the stiffness coefficient.

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### Measurement of ground acceleration




If the ground displacement is simple harmonic at a circular frequency  $\omega_g$ , the ground acceleration amplitude is calculated from the trace displacement amplitude using the equation of acceleration response ratio:

$$\frac{|u|}{\frac{\partial^2 u_g}{\partial t^2}} = \frac{1}{\omega_0^2 \sqrt{(1 - \beta^2)^2 + 4\xi^2 \beta^2}}$$

where  $\omega_0$  is the undamped natural circular frequency

$\beta$  is tuning ratio, given by  $\omega_g / \omega_0$

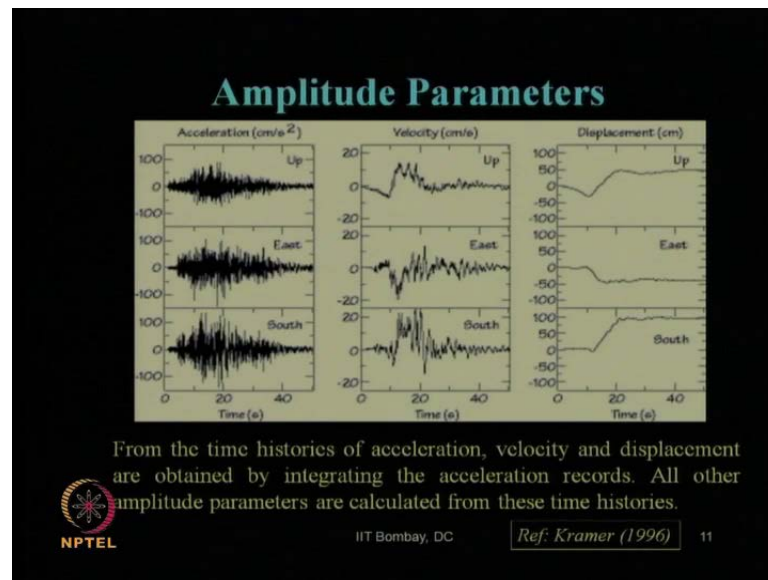
$\xi$  is damping ratio, given by  $\frac{c}{2\sqrt{km}}$



Then we have also seen, what is the acceleration response ratio, acceleration ratio with respect to this parameters, where  $\omega_0$  is the undamped natural circular frequency, and  $\omega_g$  is the circular frequency of the ground; if we replace or if we express the ground displacement that  $u_g(t)$  in the form of a simple harmonic equation through the use of this circular frequency  $\omega_g$ . And this  $\xi$  or  $\epsilon$  denotes the damping ratio, which can be estimated from this expression  $c$  by  $2\sqrt{km}$ , and  $\beta$  is called the tuning ratio or similar to frequency ratio, what we have learnt in the soil dynamics course; so  $\omega_g$  by  $\omega_0$ .

So, this way we can estimate the acceleration response ratio, which is recorded and also with respect to the ground acceleration which of course, is known to us, from the known ground displacement profile.

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So, what are the amplitude parameters, which we are getting either it can be in the form of a acceleration, it can be in the form of a velocity or it can be in the form of a displacement. So, this is the acceleration record. So, from the time histories of acceleration, velocity and displacement, these are obtained by integrating the acceleration records. So, basic record is your acceleration record, which you can get. Then by integrating or using different centers like if you velocity centers, you will get velocity; record all other amplitude parameters can be calculated from these time history. You can see these three are showing typical acceleration time history record in a seismograph, in three orthogonal directions in the horizontal case or transactional case of acceleration.

If we look at the slide, the top picture this one is showing up means, it is the vertical acceleration time history. This y axis will be the acceleration scale, you can see over here unit is given in centimeter per second square unit, and x axis is in time scale in second. So, it started over here, reached a peak and then slowly it vanishes. So, it defines how long the acceleration were working or acting. So, this defines also the duration of your earthquake, which needs to be estimated properly we will see in the subsequent slides later on. And these gives you the maximum value of acceleration at ground level, it will be p g v sorry, it will be PGA - Peak Ground Acceleration, but in vertical direction.

So, it is basically PVA, what we have seen in the definitions, if we go back, it is PVA Peak Vertical Acceleration. So, this maximum value highest value in this plot, whatever highest value whether it you are getting here or here, that highest value you need to be, you need to take as a peak value of this vertical acceleration. Similarly in two horizontal direction like east west and north south direction, you can also have the acceleration time history record. From this time history record of acceleration, if you integrate it with respect to time, how you integrate? Curve under the profile, you can get the velocity or if you can express it in terms of the function  $t$ , you can integrate with respect to  $t$ ; you can automatically get the velocity profile with respect to time, in the vertical direction, in lateral direction in east west and north south direction.

Similarly further if you integrate it with respect to time, you will get the displacement profile. So, this gives the vertical displacement, this gives us the horizontal east west direction displacement, this gives us the horizontal north south directional displacement. As you can see these are cumulative that is once you integrate obviously, curve or area under the curve, it will come, so that is why you are getting it cumulative value. So, displacement once it occurs, it remains further stationary over here. So, this is the, in the vertical, this is the peak displacement in vertical direction; this is the, this is the peak displacement in east west direction; this is the peak displacement in north south direction. So, that automatically gives us only the peak value, not the effective value; here there is no meaning of the effective displacement, as I have already discussed in the previous lecture.



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**Peak Acceleration**

Most commonly used measure of amplitude of a ground motion is the Peak horizontal acceleration (PHA). It is the absolute maximum value obtained from accelerogram.

Maximum resultant PIA is the vector sum of two orthogonal components. Estimation of PHA is most important for any design. PHA and MMI relationship (Trifunac and Brady, 1975) are often used.

PVA is not that important and  $PVA = (2/3)PHA$  is commonly assumed for design (Newmark and Hall, 1982).

Peak acceleration data with frequency content/duration of earthquake is important. Because for e.g. 0.5g PHA may not cause significant damage to structures if earthquake duration is very small.

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So now, let us discuss in detail about various this amplitude parameters like peak acceleration. Most commonly used measure of amplitude of a ground motion is the peak horizontal acceleration that is vertical is also important, but horizontal is most important that is PHA. So, it is the absolute maximum value which is obtained from your directly that accelerogram, whatever you are recording the absolute maximum that itself is nothing but our PHA or PGA. So, maximum result in PHA is the vector sum of the two orthogonal components that is in east west direction and north south direction whatever maximum value you are getting from your accelerogram, if you take the vector sum that is root over of PHA in north south direction square plus root over of that under root PHA north south square plus PHA east west square, that will give you the vector sum of maximum resultant of this PHA. And that estimation of PHA is most important for any design; this is the value which is one of the most important parameter, which we use for any civil engineering design.

And PHA and MMI relationship are often used for estimation of PHA, that is at a site if you know what is modified Marcalli intensity scale of an earth quake, the estimation of which we have already learnt through the process of how it felt by the observer and also by the amount of damages. The Trifunac and Brady in 1975, they proposed the expression which can be used to estimate the PHA, if you do not have any other way to measure it. And PVA that is Peak Vertical Acceleration is not that important, but it is not so. Now-a-days researchers differ from this statement, this statement actually was given

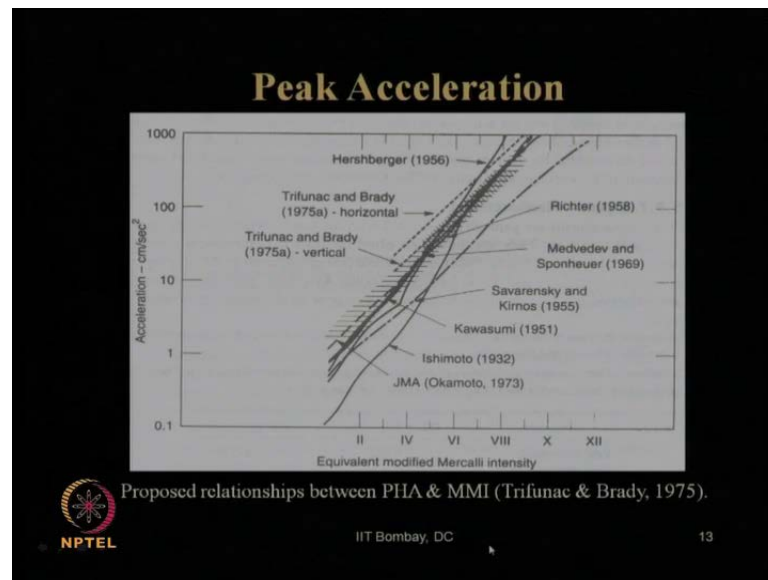
by Newmark and Hall way back in 1982, but after 82 several research work has been conducted, and it has been shown that yes PHA is the most important, but it does not mean that PVA is not important; it is also important, but may not be as important as PHA.

And PVA most of the time, it is considered as some factor of PHA that is it can maximum go up to PHA, it can never be more than PHA; that is from the observed results people have, researches have already mentioned. Typically for the design purpose, people use about two-third of PHA for the design for any structure or any civil engineering construction, they use the peak vertical acceleration, if they do not have the record of the vertical time history; in that case for the design, they will use two third of PHA as the design value. But if you have the recorded value obviously, it is better to go by that value for your design.

Peak acceleration data with frequency content and or duration of earthquake is important that is what we have mentioned just in the previous lecture, because the reason also given over here. Example like 0.5 g of PHA may not cause significant damage to structure if the earthquake duration is very small; that is such a high value of PHA, 0.5 g is pretty high. That value of PHA, if it occurs for a very small duration; that means, it may not be very much damaging to a structure say, it occurs for say for one second or two second; does not matter; because your structure will not get sufficient time to respond to that it needs certain time after the earthquake to respond to that system.

So, if suppose at a site, a PHA value of 0.35 g, which is much lower than this is continuing for a duration of say 20 seconds that will be more damaging to the structures than this 0.5 g PHA. So, simple example can be given like suppose if we slap the cheek of anybody's continuously very quickly, it will remain static, that is it will not get sufficient time to respond to this, if the duration is small, even though the magnitude is high. But even if the magnitude is small, but if we continue it for longer period then obviously, it will pain a lot. So, it depends on how much response time also you are giving along with the amplitude. So, that is why it is shown over here that it is not necessarily that 0.5 g PHA will cause a large damage to a structure, if the earthquake duration is very small.

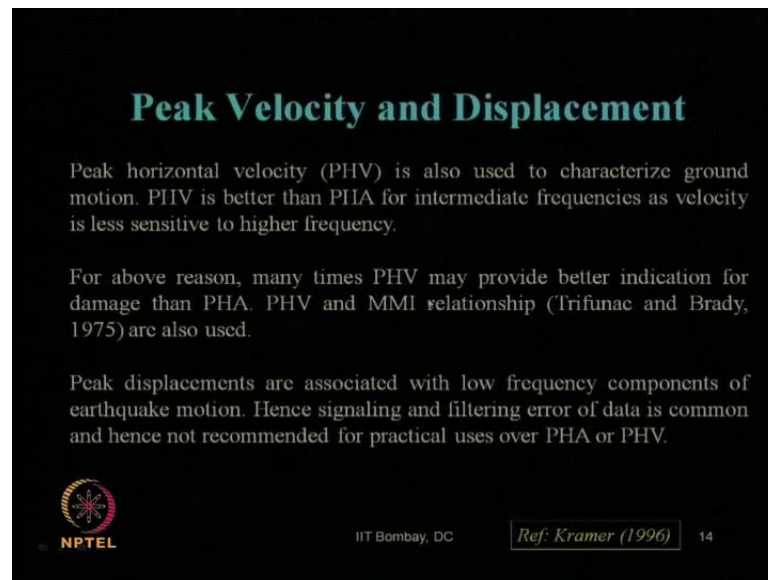
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Now, this picture, this result shows the proposed relationship between that peak horizontal acceleration and that MMI scale, what I was mentioning which is given by Trifunac and Brady in 1975. Professor Trifunac is a professor at university of southern California in USA. He worked on this area extensively, and he proposed that acceleration in the unit of centimeter per second square compared with equivalent MMI scale various intensity scale of MMI. So, for horizontal, he has proposed this line as the equation, one can use suppose it is a 6 MMI intensity, what can be the corresponding PHA for using this line.

Similarly for vertical also he has proposed another line you can see over here; vertical acceleration gives by this dotted line, which is little lower than the horizontal, you can see over here. And other researchers like Hershberger in 1956 proposed this line, Richter proposed this line, and various other researchers including the Japan metrological agency - JMA also have given different recommendations. So, these are various recommendations are equations proposed by various researchers, and the among this you can see over here, the latest one is proposed by Trifunac and Brady both for PHA as well as for PVA. So, once you know your MMI using this relationship or using this chart you can estimate your PHA or PVA, which you can use for further for your design.

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**Peak Velocity and Displacement**

Peak horizontal velocity (PHV) is also used to characterize ground motion. PHV is better than PHA for intermediate frequencies as velocity is less sensitive to higher frequency.

For above reason, many times PHV may provide better indication for damage than PHA. PHV and MMI relationship (Trifunac and Brady, 1975) are also used.

Peak displacements are associated with low frequency components of earthquake motion. Hence signaling and filtering error of data is common and hence not recommended for practical uses over PHA or PHV.

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Coming to next parameters that is peak velocity and peak displacement; peak horizontal velocity that is PHV is also used to characterize the ground motion. PHV is better than PHA, why? This peak horizontal velocity is sometimes better than peak horizontal acceleration, because for intermediate frequency, velocity is less sensitive to compare to the higher frequency; that means, if your earthquake is having an intermediate frequency or low frequency, what is that typical range I said? About 1 hertz to about 4 hertz.

So, if it is towards the intermediate frequency, then many a times, this PHA value will try to get recorded, not correctly or properly, in that case PHV is a better estimate, because when you are integrating the PHA over time, you get a entire frequency range to be observed. So, that is why intermediate frequency are also getting well recorded through this PHV. For this for this reason, many a times, this PHV that is peak horizontal velocity may provide a better indication for the damage than this PHA.

So, if we are using the damage estimation through this values of peak horizontal velocity and acceleration in many a times peak horizontal velocity will be better than the peak horizontal acceleration. And that peak horizontal velocity versus MMI relationship also given by Trifunac and Brady in 1975 - that can be used. Whereas, the peak displacements are associated with low frequency components, very low value of the frequency, they are associated with the peak displacement of earthquake motion. Hence signaling and filtering error of the data is very common, because at low frequency you

may get some error in your recording station itself. So, you may not record it properly, and hence it is not recommended for practical use over the peak horizontal acceleration and peak vertical peak horizontal velocity. So, which is the best one? PHV and PHA, they are the better estimate for the amplitude estimation of any earthquake than your PHD - that is peak horizontal displacement.

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**Other Amplitude Parameters**

**Sustained Maximum Acceleration:** The absolute values of highest accelerations that sustained for 3 and 5 cycles in acceleration time history are defined as 3-cycle sustained and 5-cycle sustained accelerations respectively.

**Effective Design Acceleration:** The<sup>h</sup> acceleration which is effective in causing structural damage. This depends on size of loaded area, weight, damping and stiffness properties of structure and its location with respect to epicenter.

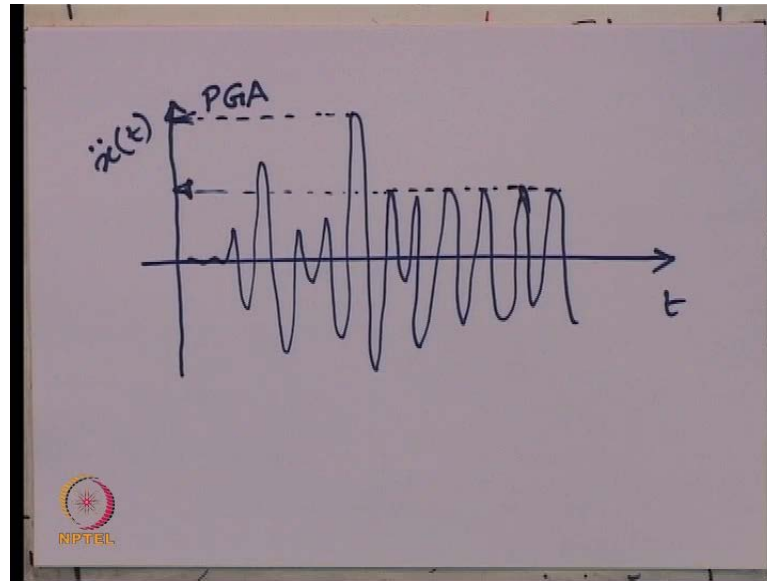
Kennedy (1980) proposed EDA as 25% higher than 3-cycle PHA recorded in filtered time history.

Benjamin and Associates (1988) proposed EDA as the PHA after filtering out accelerations above 8-9 Hz.

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Now let us see in this slide, what are the other amplitude parameters like sustained maximum acceleration. What is known as sustained maximum acceleration? The absolute values of highest accelerations that sustained for 3 and 5 cycles in acceleration time history, those are defined as either 3 cycle sustained or 5 cycle sustained acceleration respectively.

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So, if we see the plot of acceleration versus time, suppose the acceleration versus time plot is somehow going like this. Now in this case, earlier we have discussed that highest value say this is the maximum value that will be your peak ground acceleration. So, that is your PGA; it can be PHA or PVA that is either it can be a peak horizontal acceleration or vertical acceleration, depending on what acceleration you are recording, whether it is in the horizontal direction or in the vertical direction, that is the peak value. But what is this sustained maximum acceleration, it says that the acceleration, which sustains for minimum three cycles; suppose this value, this value, this value are same and they are remaining for at least three cycles. So, this value of acceleration will be 3 cycle sustained acceleration.

And suppose if it continues for 5 cycle say like this, then we can say this as 5 cycle sustained acceleration that is the highest or maximum value, which is continuing for at least three cycles or at least 5 cycles are known as sustained maximum acceleration corresponding to that number of cycles either 3 or 5.

Now let us look at this slide. Effective design acceleration, what is that? The acceleration which is effective in causing the structural damage that is called effective design acceleration or EDA. So, this is the minimum threshold acceleration, at which the structural damage starts occurring. This depends on size of loaded area weight damping and stiffness properties of structure, and its location with respect to the epicenter. And

what are the various suggestions? Like Kennedy in 1980 proposed that effective design acceleration EDA is as 25 percent higher than the 3 cycle PHA that is this 3 cycles sustained maximum acceleration recorded in a filtered time history; filtered means, in the acceleration time history record, you have filtered out or taken out the noises that is known as filtered time history. So, in that time history typically it has been found so far that whatever is the value of that 3 cycle PHA or 3 cycle sustained maximum acceleration, the 25 percent extra or if you multiply that value with 1.25, that will give you the design value of effective acceleration. So, that is the value of EDA, which needs to be used for design.

And Benjamin and associates in 1988 proposed that EDA as the PHA after filtering out accelerations above 8 to 9 hertz that is, you should not take any accelerations, which are having frequency value of more than 9 hertz. So, that is what they proposed that below 9 hertz frequency whatever acceleration time history record you have that is after filtering out 8 to 9 hertz, above that frequency range whatever remaining acceleration time history record you have, corresponding to that whatever peak horizontal acceleration you get, that can be taken as EDA. Because as I said earlier the typically about 3 and half to 4 hertz is a typical range of damaging earthquake frequencies, and with that range, you will find that the recommendation is up to maximum 9 hertz you considered the frequency range to take the value of PHA, which can be used as your design acceleration or effective design acceleration EDA.

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**Frequency Content Parameters**


The frequency content of an earthquake history is often described using Fourier Spectra, Power spectra and response spectra.

**Ground Motion Spectra - Fourier Spectra**

A periodic function (for which an earthquake history is an approximation) can be written as

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin(\omega_n t + \phi_n)$$

where  $c_n$  and  $\phi_n$  are the *amplitude* and *phase angle* respectively of the  $n^{\text{th}}$  harmonic in the Fourier series.

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Now let us come to the next important parameter that is after the amplitude, we said that frequency content is another important parameter to quantify any earthquake. So, what are the frequency content related parameters, let us see. The frequency content of an earthquake history is often described by using Fourier spectra or power spectra and response spectra. So, these are the three spectra by which the frequency content of any earthquake can be expressed.

So, ground motion spectra in terms of Fourier spectra, how it can be represented let us see. As we all know about the Fourier series in mathematics, how it is applied to find out the frequency content of an earthquake, we have to see over here. So, it is the periodic function, for which an earthquake history is an approximation, because we know that earthquake history is a random motion, it is not a periodic motion. But you can approximate that random earthquake history motion into a periodic function, which can be expressed in the form of a Fourier series. So, that is why the Fourier spectra is originated. So, what it says? Suppose the earthquake displacement  $x$  of  $t$ , which is the displacement, which is function of  $t$  can be expressed in this periodic format, which is nothing, but the equation of a well-known Fourier series. So,  $c_n \sin(\omega_n t + \phi_n)$ . So, this is expressed in this periodic function in terms of this sinusoidal function.

So, where this  $c_n$  and  $\phi_n$  these are nothing but the amplitude of your that Fourier function or the displacement function expressed in Fourier series term, and  $\phi_n$  is the phase angle of the  $n$ th harmonic that is the  $n$ th cycle of the motion in that Fourier series. Clear? And this is the initial displacement at the starting of your earthquake time history. Fine. So, this is the initial amplitude.



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## Frequency Content Parameters

The **Fourier amplitude spectrum** is a plot of  $c_n$  versus  $\omega_n$

- Shows how the amplitude of the motion varies with frequency.
- Expresses the frequency content of a motion

The **Fourier phase spectrum** is a plot of  $\phi_n$  versus  $\omega_n$

- Phase angles control the times at which the peaks of harmonic motion occur.
- Fourier phase spectrum is influenced by the variation of ground motion with time.



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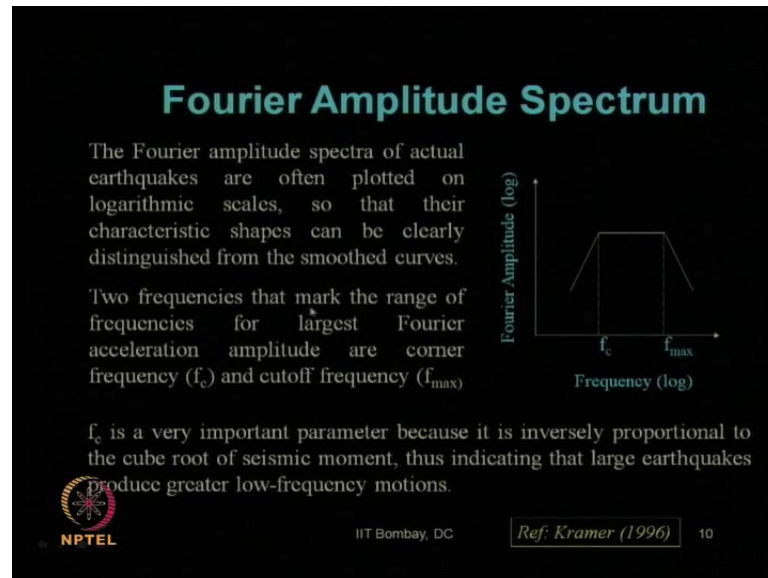
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So, after expressing the Fourier spectra, what we can say about this - that the Fourier amplitude spectrum, it is nothing but a plot of that amplitude  $c_n$ , which is nothing but the amplitude of  $n$ th cycle. So, that  $n$ th cycle amplitude versus  $\omega_n$ . So, if we plot that  $c_n$  versus  $\omega_n$  that will give us the Fourier amplitude spectra. So, it shows how the amplitude of the motion it varies with respect to the frequency. So, that is as the frequency changes, how your amplitude changes with frequency variation that gives us the frequency content; and it expresses the frequency content of a motion automatically. So, the Fourier's phase spectra, so this is the amplitude spectra, and another is Fourier phase spectra; Fourier phase spectrum is nothing but it is a plot of y axis will be your  $\phi_n$  versus  $\omega_n$  is your x axis.

So, that is nothing but here phase angle in the  $n$ th cycle of your earthquake cycle, whatever phase you are getting that you have to plot with respect to that frequency of an earthquake  $\omega_n$ . So, phase angles control the times, at which the peaks of the harmonic motion occur, because you know from this plot of phase angle that is at what interval of phases, this peak of earthquake motion is going to occur at the particular location. So, that is how it is not only giving this Fourier amplitude spectra the value of the amplitude, but it also gives from this plot of phase spectra that is at what interval that peak or the maximum will occur in this harmonic motion.

So, the Fourier phase spectrum is influenced by the variation of the ground motion with respect to time, which is quite obvious because that is the input parameter for you to derive at Fourier amplitude or response spectrum.

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So, let us look at the picture, how it looks like, a Fourier amplitude spectra. Let us look at this slide, the Fourier amplitude spectra of actual earthquake are often plotted on a logarithmic scale, because why to plot in logarithmic scale? You can identify the intermediate small variation also if you stretch it in the logarithmic scale that is the advantage of using the logarithmic scale for any variation.

So, these are often plotted on the logarithmic scale, so that their characteristic shapes can be clearly distinguished from the smooth curve. So, as I was mentioning, this y axis will be your  $c_n$ ,  $c_n$  is nothing but Fourier amplitude in the  $n$ th cycle of earthquake,  $n$  is particular cycle if you peak up. And x axis is your frequency or  $\omega_n$  that is also in the log scale. So, this is in log scale this is as well in log scale.

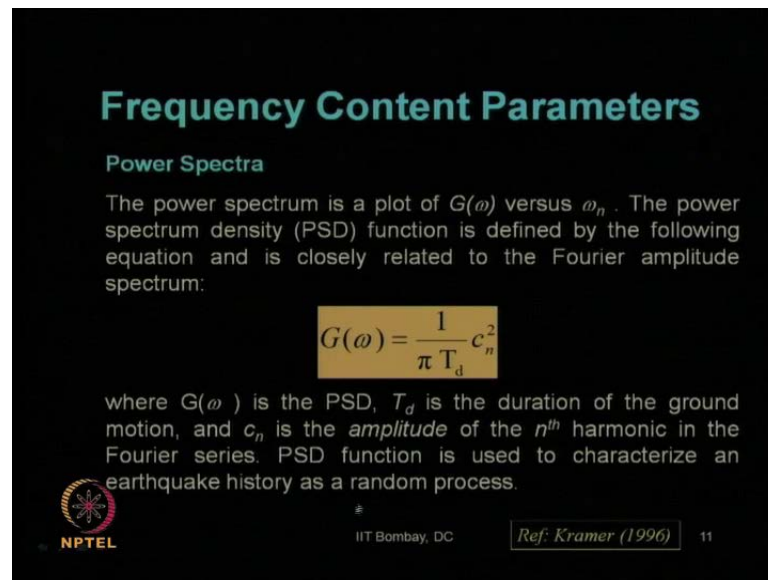
So then typical Fourier amplitude spectra will look like something like this. So, you will have a constant Fourier maximum amplitude, which will range between two values of the frequency, which are denoted here as  $f_c$  and  $f_{max}$ . Let us see what are those. So, these two frequencies that mark the range of the frequencies for largest Fourier acceleration amplitude, these are called corner frequency  $f_c$ , and the cutoff frequency  $f_{max}$ .

So, these are the definitions of corner frequency that is the minimum value of the frequency, at which the earthquake attains or reaches its maximum amplitude. And what is the  $f_{max}$  or cutoff frequency? It is nothing but the maximum value of the frequency at which that maximum value of earthquake amplitude or Fourier amplitude sustains. So, this is the range with which your maximum value of your Fourier amplitude spectra will continue. So, in this case,  $f_c$  is a very important parameter; why? Because it is inversely proportional to the cube root of seismic moment; thus it indicates that large earthquake produce greater low frequency motions.

So, that is the reason why we were mentioning earlier that what is considered in literature, the range of frequency we can take as per Benjamin and associates that up to 9 hertz of frequency, we should take. Above 9 hertz frequency we generally discard; below 8 to 9 hertz frequency whatever earthquake occurs? The peak value of the horizontal acceleration we considered as the effective design acceleration, for any structural design and more so because the low frequency motions, they cause more damages to the, because they are the large earthquake. That I will tell you when we will compare this low frequency that means, high time period why it is occurring, and why it is a more dangerous to tall buildings etcetera. So, it is a way related to the cube root of the inversely proportional to the cube root of seismic moment

So, automatically it gives you a kind of correlation between the seismic moment magnitude and the frequency content or the frequency value, through these Fourier amplitude spectra. So, that is the use of Fourier amplitude spectra, but as I have already indicated, it is approximated that random motion of earthquake has been approximated through this periodic function, through use of this Fourier series, Fourier series function.

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## Frequency Content Parameters

### Power Spectra

The power spectrum is a plot of  $G(\omega)$  versus  $\omega_n$ . The power spectrum density (PSD) function is defined by the following equation and is closely related to the Fourier amplitude spectrum:

$$G(\omega) = \frac{1}{\pi T_d} c_n^2$$

where  $G(\omega)$  is the PSD,  $T_d$  is the duration of the ground motion, and  $c_n$  is the *amplitude* of the  $n^{\text{th}}$  harmonic in the Fourier series. PSD function is used to characterize an earthquake history as a random process.

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Coming to next type of spectra, which is also used some times to express the or to obtain the frequency content parameter, like it is called power spectra. So, what is power spectra? Power spectrum is a plot of  $G(\omega)$ , this is again another function which is expressed in the power spectra form versus  $\omega_n$  that frequency once again, frequency of the earthquake. Remember this is not the natural frequency I am talking about this is the frequency of earthquake motion.

So, the power spectrum density, it is called PSD - it is a function which is defined by the following equation, this equation, and it is closely related to Fourier amplitude spectra. So, similar to the Fourier series, the power series is another form of the mathematical expression, which can be used to express the earthquake response to obtain the frequency content. So, this is the function, power function  $G(\omega)$ ;  $G(\omega)$  is can be written in the form of  $\frac{1}{\pi T_d} c_n^2$ . So, where this  $G(\omega)$  is nothing but the PSD that is power spectrum density, and  $T_d$  is the duration of the ground motion that is nothing but time period, time period or time duration of the ground motion, which is related to your  $\omega_n$  of course, as we know  $\omega_n$  and  $T_d$  is correlated like the time period and frequency is related.

And  $c_n$  this  $c_n$  is the amplitude that displacement amplitude of the  $n^{\text{th}}$  harmonic in the Fourier series, which we have already discussed while discussing the Fourier series spectra. So, this power spectrum density function is used to characterize an earthquake

history as a random process; so unlike your Fourier spectra, where we have expressed that random process into a sinusoidal or a periodic function, if we go back into that equation. So, that random behavior of an earthquake displacement is assumed to be as a periodic function through this expression of Fourier series in the Fourier spectra.

But in the case of power spectra, it only takes the maximum values of those or the amplitudes of those Fourier spectra, and then express it is as it goes in the random process related to your variation of this earthquake frequency  $\omega$ , which is expressed through this  $T$ , clear? So, here this is the only advantage is it need not to consider the periodic function, it can be applied for a random process.

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## Frequency Content Parameters

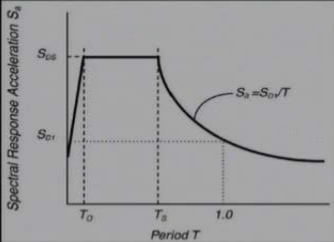
### Response Spectra

A response spectrum is used to provide the most descriptive representation of the influence of a given earthquake on a structure or machine.

Response spectra are widely used in earthquake engineering.

A response spectrum is a plot of the maximum response amplitude (displacement, velocity or acceleration) versus time period of a system to a given component of ground motion.

Using the response spectrum, peak response of buildings to earthquakes can be assessed and their natural frequency determined.



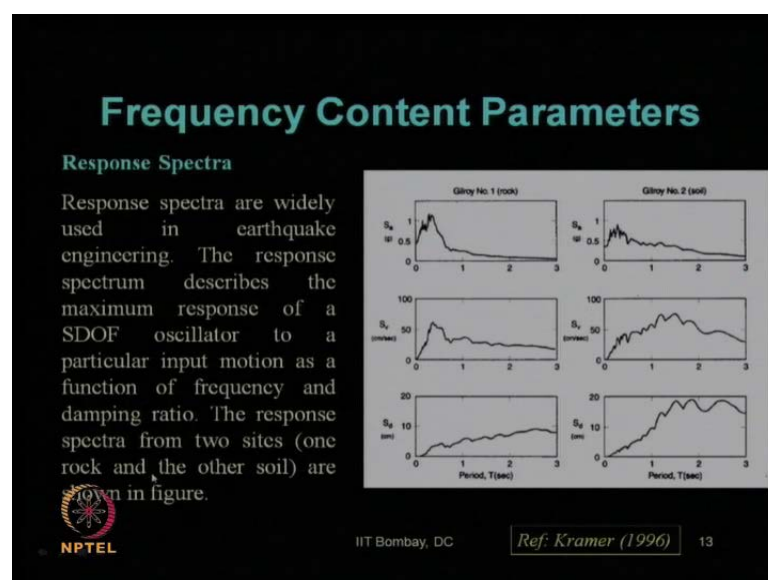
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Coming to the next or I should say, most practically used concept of frequency content parameter is the response spectra. So, a typical response spectra is looking like this, the y axis is the spectral response, it can be a displacement it can be acceleration, it can be velocity accordingly the name will change to a as shown in this picture, it is the acceleration response spectra. Because y axis shows the acceleration  $S_a$ ; if it is the  $S_v$  that is if it is the spectral response velocity, then we call it as velocity response spectra; if it expresses the spectral response displacement, then we call it as a displacement response spectra. And what is plotted in the x axis? x axis is nothing but the period, period of ground motion that capital T.

So, this variation typically gives the result like this that can be estimated for a particular value of your assumed damping constant or damping ratio. We will come to that derivation very soon, let us see with the basic definition a response spectra is used to provide the most descriptive representation of the influence of a given earthquake on a structure or machine. And response spectra are widely used in the earthquake engineering, because every where we will see later on that we are using for any design, the response spectra of an earthquake motion. So, this is the way to represent the variation of earthquake random motion in more logical or understandable manner, which can be further used for our engineering design for earthquake resistant, design etcetera

So, a response spectra is a plot of the maximum response amplitude. So, that maximum response amplitude as I said, it can be displacement or it can be a velocity or it can be an acceleration as you decide to express your y axis accordingly. So, that verses the time period of the system to a given component of a ground motion, and using that response spectrum, peak response of buildings to earthquakes can be assessed, and their natural frequency also can be determined using this response spectra. We will see its application extensively later on, also we will talk about the various seismic design code including our Indian seismic design code, where this response spectra is thoroughly used.

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Coming to this slide where again we are talking about the response spectra, if you see here in this picture response spectra at two different site; one is rocky site and another is

soil site are shown here for the same particular earthquake. And various response spectra are plotted; the top one is for the acceleration response spectra  $S_a$  in terms of  $g$ ; so these values are 0.5  $g$ , 1  $g$  like that, and this is also  $S_a$  that is the acceleration response spectra in rock and in soil site.

The second one the central one shows  $S_v$  that is in the unit of centimeter per second. So, this the velocity response spectra for rock site and this is the velocity response spectra for the soil site. The last one at the bottom one is the  $S_d$  in centimeter unit, so this is the displacement response spectra for the rock site, and this is the displacement response spectra for the soil site. So, if you look here, the typical values of the response spectra in the rock site, this is pretty high compared to the soil site. Whereas the displacement if you look at here, finally we are talking about the displacement here for rock site it is pretty low compare to the soil site.

So, response spectra are nothing but these are widely used in earthquake engineering as I already said, the response spectrum describes the maximum response, it can be maximum displacement or maximum velocity or maximum acceleration. So, that is maximum response of a single degree of freedom oscillator. So, we have to use the single degree of freedom system to a particular input motion that is, whatever earthquake input motion you have, you apply that input motion to an idealized single degree of freedom mass spring dashpot system or MSD module, which we have learnt in our another video course on soil dynamics.

So, we will see here also, how we can reach to a response spectra or the maximum response of a single degree of freedom system of a mass spring dashpot system or MSD model or oscillator under a particular input motion. So, this will be considered as a random motion if you apply to a SDOF system, how to solve that? We will come to the solution very soon. So, as a function of the frequency and it is a function of damping ratio, because we were knowing whenever we are finding out the maximum response or the suppose it is maximum displacement, we always expressed that maximum displacement as a function of the initial displacement or in addition to that, we use the value of the damping ratio; the damping ratio was a very important parameter.

If you remember, it was a function of nothing but damping ratio, and what was the function, another function - frequency ratio, if you remember that in our other video

course on soil dynamics, if you have seen that video course or have gone through that video course you will know that single degree of freedom system, its response or if I want to find out its displacement at any point of time, we it has to be expressed in terms of its static displacement or initial displacement, and in terms of its damping ratio, and in terms of its frequency ratio that is the applied frequency to the natural frequency of the entire system. So, the same thing, we will see here also in subsequent lecture.


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**Duration**

Duration of an earthquake is very important parameter that influences the amount of damage due to earthquake. A strong motion of very high amplitude of short duration may not cause as much damage to a structure as a motion with moderate amplitude with long duration can cause. This is because the ground motion with long duration causes more load reversals, which is important in the degradation of stiffness of the structures and in building up pore pressures in loose saturated soils.

Duration represents the time required for the release of accumulated strain energy along a fault, thus increases with increase in magnitude of earthquake.

**Relative duration** does not depend on the peak values. It is the time interval between the points at which 5% and 95% of the total energy has been recorded.

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So, the response spectra from let us have a look at this slide. So, the response spectra from two sites one rock and the other soil are shown in this figure as we can see over here. Now, let us see another parameter which is known as duration, this is another important parameter out of those three to quantify any earthquake. As I said three important parameters are amplitude of earthquake, then frequency content of earthquake, and then duration of earthquake. So, if we talk about the duration of an earthquake, it is very important parameter that influences the amount of damage due to earthquake

So, a strong motion of very high amplitude of short duration may not cause as much damage to a structure as a motion with moderate amplitude with long duration can cause that I have already mentioned in my previous lecture - that it is not necessary that a very large magnitude or amplitude earthquake will always cause a major damage; if it occurs for a very short duration, it may not make large damage, because the structure did not get enough time to respond to that large magnitude. Instead of that, if we have a moderate



magnitude of earthquake, but which is lasting for a longer duration that may be more damaging to the structure, because the structure gets enough time to respond to that, to behave to that.

So, let us look at here. So, this is because the ground motion with long duration that is the longer time, for which it is lasting causes more load reversals that is once load is applied, another case load is taken out. So, that is the load reversal. So, if you make your structure to go through more number of load reversal that is, once you apply load, then taken out, then apply, then taken out. So, one cycle of load reversal as many as load reversal you will apply to a structure that will cause more damage rather than single peak value which probably will not cause any load reversal to the structure. So, that is what it is showing over here, if you look at this slide.

So, which is more important in the degradation of the stiffness; so, more number of load reversal will cause more degradation of the stiffness of the structure, and it builds up pore pressure in loose saturated soil. So, as well as what it does in case of our soil or geo technical engineering? In case of soil, it builds up the pore pressure if that favorable condition exists over there with number of more longer duration or more load reversal, it generates more and more pore pressure, which does not get time to dissipate.

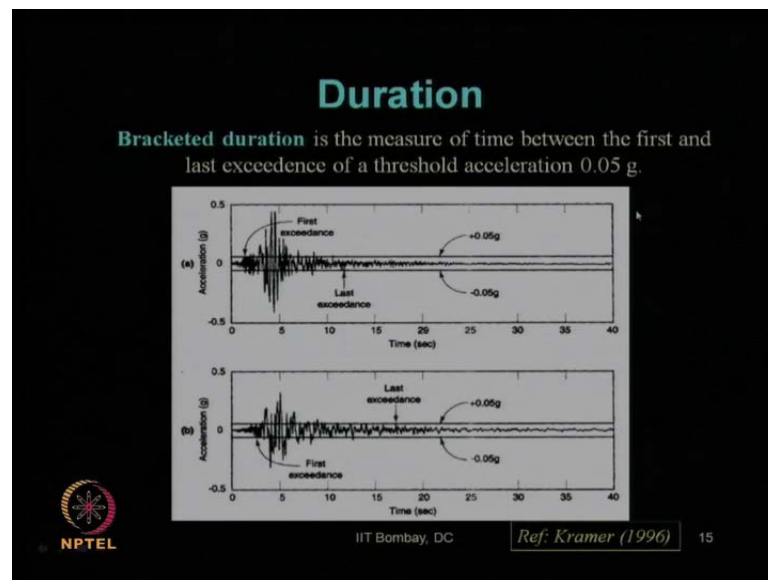
So, automatically that also creates a problems liquefaction or a loss of effective shear strength, which ultimately reduce the strength of your foundation soil etcetera and of course, the structure reduces its stiffness, due to this more number of load reversal. So, if you get a longer duration always that is a more damaging than a shorter duration earthquake. So, duration represents the time required for the release of that accumulated strain energy along a fault; that describes the duration of the earthquake motion; that is how much time it requires to release the entire accumulated energy during this earthquake process.

So obviously, it increases with increase in the magnitude of earthquake, because in high magnitude earthquake typically, it will release more amount of energy. So, more amount of time typically will be required. And what is relative duration? This is another definition relative duration does not depend on this peak value. The duration depends on this peak value, but the relative duration does not depend on the peak values; it is the time interval between the points at which 5 percent and 95 percent of total energy has

been recorded. So that means, it is the duration or the time, which is recorded between this 5 percent of energy release to the 95 percent of the energy release.

Let us look at the slide over here. So, this is the 5 percent energy release to 95 that is the time, at which earthquake energy say total 100 percent, 5 percent of that has been released up to the time up to which the 95 percent of the total energy has been released that this time to this time is called as the relative duration of an earthquake motion. It is not dependent on the peak value, say peak value may come within the say, it may happen within one first 1 percent or may be later on 99 percent, but that need not be considered as a duration though in duration it is mentioned as like that, but relative duration corresponds to this two amount of release of the total energy.

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Now, let us see another definition of another type of duration, let us look at the slide. It is known as bracketed duration, what is bracketed duration? It is the measure of the time between the first and last occurrence of a threshold acceleration; what is threshold acceleration? This threshold acceleration is nothing but a minimum acceleration which will cause damage to structure that is the minimum value of acceleration which will start causing damage to our structure, which is typically taken by engineers as 0.05 g.

So, that is the value of acceleration, which is taken as minimum or threshold acceleration, above which it is considered as a damaging acceleration, which may cause damage to the structure. So, what the bracketed duration says, if suppose your

acceleration verses time profile looks like this. So, this is a random motion like this, you have to look for in that acceleration time response or time history data - basic data, after filtering out of course, the noises etcetera; wherever your value of that 0.05 g, if you see the scale this is the 0 scale, this is 0.5 g. So, you have the demarcation over here, 0.05 g 0.1 g, 0.15 g like that. So, you draw a line of 0.05 g in both the direction that is plus or minus, because we are considering only the amplitude of it.

Now look at the acceleration time history profile, where that first time it has touched that line, you got it? So, suppose this is the value, where it first touched that threshold acceleration of 0.05 g. So, this will be taken as first exceedance of the value; that is beyond this, it exceeded that value of 0.05 g; clear? So, that will be starting time. Again all these are seen above 0.05 g; and it need not be always that everything has to be 0.05 g in all cycle. If you look here very carefully, see in this zone, where I am showing here through this cursor, here it is still below 0.05 g, but you have to still consider and look for the value where beyond that there is no 0.05 g is occurring; you got it? That is the last exceedance of that value of 0.05 g.

So, that time is the last point of time, which has to be considered. So, this time to that time, this duration is called as bracketed duration, which is important for our civil engineers to design any structure, whether sub structure or super structure under earthquake condition. So, this bracketed duration is having a huge amount of applications, because of this consideration above this threshold acceleration limit. So, that is the time period we are more concerned about. Suppose it may longer for more time, here if you see it is probably between 2 seconds to about 12 seconds; so bracketed duration for this earthquake motion which is shown in this picture is about 10 seconds. Whereas the total earthquake duration may be considered as say if you consider this values also may be up to 29, this is 20 about 20 seconds or so. So though the total earthquake duration is 20 seconds, but bracketed duration which is more important for us as a civil engineers are 10 seconds; that is what I wanted to point it out here over here.

So, this is another example where it is shown the acceleration time profile and this is the first exceedance, this is the last exceedance of that 0.05 g value, whether it is plus or minus, and that difference of time gives you the bracketed duration of this earthquake motion; whereas the actual duration may be pretty long like say 25 seconds or something

like that. So, with this, we have come to the end of this lecture; we will continue further in the next lecture.