Structural Analysis II

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Lecture - 07

Good morning. In the last lecture, we looked at how to tackle various kinds of conditions other than loading in the analysis of statically indeterminate trusses. Today, we are going to concentrate on the same thing for beams and one of the additional points which I had forgotten to mention in my last lecture. I looked at support flexibility, lack of fit, thermal stresses; there is also the consideration of support settlement.

In this particular lecture when I am looking at beams, I will also look at what affects support flexibility as well as support settlement. These are two different things. A member can be axially rigid and can displace rigidly; that is settlement. When does that happen? Support flexibility is where we are considering the fact that a pier can deform axially. However, let us consider the situation of support settlement. When do we get the situation of support settlement? The pier is axially rigid; in other words, there is no deformation; it is axially rigid. The only problem is that the pier support, which is the foundation, is not rigid; the foundation deforms under the load that the pier puts on the foundation and it settles. If the foundation settles, then the top of the pier also settles. If the top of the pier settles, then the support of the beam on top of the pier will also move and that is support settlement. Today, we are going to be looking at beams.

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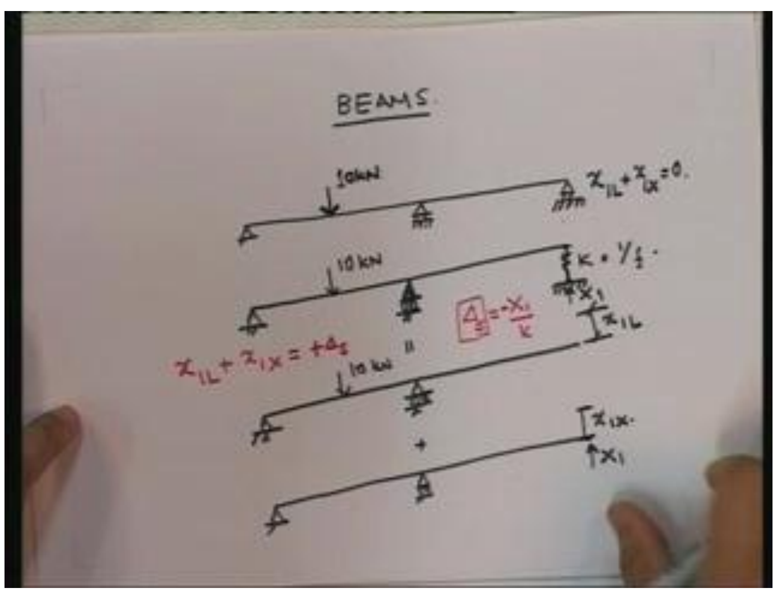

This is the problem that we had looked at last time which was a two-span beam with a load in the center span of one of them; and we solved how to get the forces in all the members as well as all the supports. This time, I am going to look at support flexibility. What happens when I have a flexible support? Let me not consider both of them flexible; let me just consider one of them flexible. Let me take that this continues to be a roller support, but this is flexible. I have this load and this is flexible with a spring constant K, which is equal to 1 upon f, the flexibility constant. This is the problem.

Now, how would the problem change because of this flexibility? Let us see. The problem still remains the same; in other words, I can solve this problem by considering this as my base structure; so, I am removing this… plus this. Now, I have solved this problem. How is the entire thing going to be different? The difference to be noted is that here we computed X_{1L}, if you remember; this is the X_1 due to load. Then we computed X_{1X}; and in the last case where this was this way, we took the compatibility condition is equal to X_{1L} plus X_{1X} is equal to 0, because in the real original structure, what was the displacement? 0.

However, when you have a flexible support, what happens? This is no longer true (Refer Slide Time: 06:38); so, what do you have? Let us see what happens. This is under this load. This one and this one are identical to what we had computed for this case. There, we had put it equal to 0 and computed the value of X_1. Here, in the actual structure if I have a load X_1 in the support, what is going to happen to the force in this member? Think about it. You will see that if there is a force X_1 here, this spring is going to be subjected to compression; if it is subjected to compression, then what happens? This member is going to compress; this point cannot go anywhere; so, if the member compresses, what is going to happen? This point is going to come down.

How much is it going to come down by? That depends on what is the deformation in the member. How do we compute the deformation in the member? If you look at it, the spring is subjected to a compressive force of X_1; and if it is subjected to a compressive force of X_1, then what is the deformation of this? The deformation of the spring is going to be equal to X_1 by K and minus. Why minus? Because if X_1 is this way, the deformation is compression; and if deformation is compression, then what happens to this point is it actually goes down by the value of $delta_s$.

Therefore, for this particular situation when we have the flexible support, the equation becomes x_{1L} plus x_{1X} is equal to minus $delta_s$. Why minus? Because x_{1L} and x_{1X} are taken as positive in this direction; and if you look at $delta_s$, it is actually downwards. Now, I am going to put $delta_s$ to be positive when the spring expands. When the spring expands, it is positive; if the spring compresses, this is negative. If $delta_s$ was positive, this would also be in this direction; so, it is going to happen that x_{1L} plus x_{1X} will be equal to plus $delta_s$.

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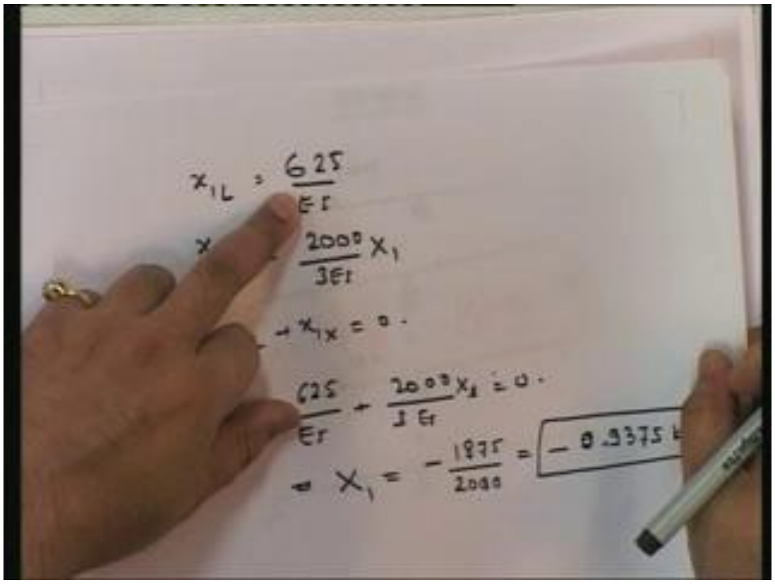

If I plug that in and if I take the values that I got from the previous case, what was my x_{1L} equal to? 625 upon EI. So, this was x_{1L} is equal to …. Let me put it on another sheet.

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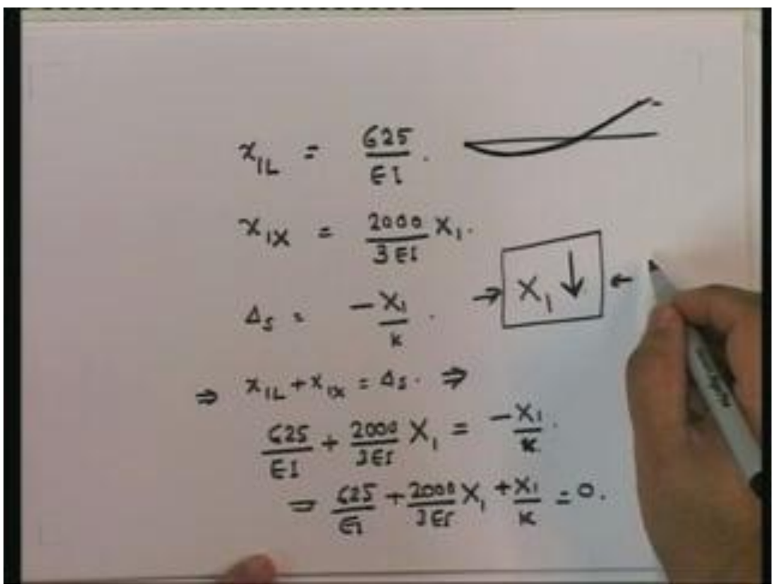

x_{1L} was equal to 625 by EI, x_{1X} was equal to 2000 upon 3 EI into X_1 and what is $delta_s$ equal to? minus X_1 upon K. Therefore, by putting that into the compatibility condition, which is x_{1L} plus x_{1X} equal to $delta_s$, what we get over here is 625 upon EI plus 2000 upon 3 EI into X_1 is equal to minus X_1 upon K; K is a known factor because it is a spring constant. We know the flexibility; therefore, we can find out X_1 directly because this will become nothing but 625 upon EI plus 2000 upon 3 EI into X_1 plus X_1 by K is equal to 0 and you can solve this for X_1.

Note that because of the foundation flexibility, the value of X_1 changes. If you look at this, the value of X_1 actually goes down due to the foundation flexibility. That makes sense because if you really look at it, what is happening here? If you really look at this, this one, if you look at the deformation pattern, what is going to happen? This is going to go like this and this. In reality, because of the…. Due to the loading, you are going to have a compression in the spring, but due to the deflection you are going to have an extension in the spring. Therefore, the compression in the spring is negated a little by the extension and therefore the value of X_1 goes down due to the foundation flexibility. This is how you consider the effect of foundation flexibility. Note that in this particular case, as opposed to the truss, the foundation flexibility directly has an effect on the internal forces in the member. The bending moment diagram will change because the support reaction has changed. Now, let us look at the situation of foundation settlement, how foundation settlement has an effect.

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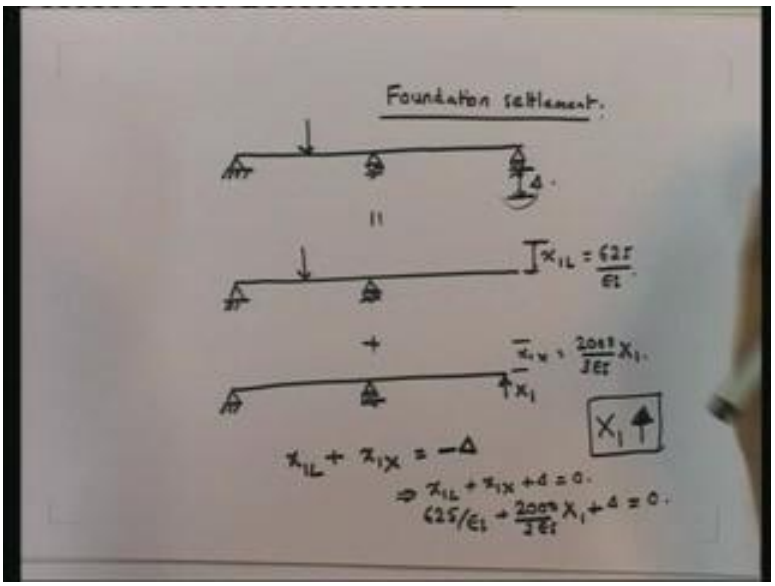

Let us take the same case. I have looked at foundation flexibility; now we are looking at foundation settlement. How will foundation settlement come in? Let us look at this. Let us say that this foundation settles by a delta value and we want to know what that effect is going to be equal to. How would I solve this problem? This would be equal to…. Again, you have to understand that the entire procedure is based on the same premise plus…. Here, what are we getting? We are getting a fact that…. we find out x_{1L}. Note that this x_{1L} is going to be the same as before; this is going to be 625 upon EI. We are going to find out x_{1X}. This again is going to be the same; it is going to be 2000 by 3 EI because there is no effect over here. However, when you write the compatibility condition, what will it be? x_{1L} plus x_{1X} is equal to…. Now, instead of 0 what is….? The displacement is this and if you look at this, what is this? (Refer Slide Time: 15:24) This is going to be minus delta because remember I am taking upwards as positive, so this delta is actually a negative value. It becomes a negative delta and if you really look at it, all that does is x_{1L} plus x_{1X} plus delta is equal to 0. If you really look at it, this is going to be equal to …. This is going to be 625 by EI plus 2000 by 3 EI into X_1 plus delta is equal to 0.

The value of X_1 goes up because of the support settlement and that makes sense because due to the support settlement, this is going to take up a much larger value of reaction because support settlement causes a larger reaction to come up. So, you see, foundation flexibility and foundation settlement actually does not affect the solutions of the statically determinate problem themselves. All that foundation flexibility and foundation settlement both do is that they affect directly the compatibility conditions that you have.

In the original case, you would have X_{1L} – the displacement due to load – plus the displacement due to the redundant force X_1 is equal to 0. When you have foundation flexibility, it is not equal to 0; it is equal to the displacement due to the flexibility of the foundation and that is related to the force in the foundation itself. When you have foundation settlement, all you have is that the value of the foundation settlement comes in directly and you can compute it from there obviously. Therefore, the only aspect of foundation flexibility and foundation settlement or in support flexibility and support settlement is that your compatibility conditions are affected; the analysis of the statically indeterminate are not affected. Hence there is so much for foundation-related matters.

Now, let us look at the other kind of situation where…. Today, if you remember, I am always looking at other kinds of loads, in other words, the effect of what is known as non-load effects. What are non-load effects? Foundation flexibility or support flexibility, foundation settlement or support settlement, thermal stresses, lack of fit. I have already talked about lack of fit in trusses and I am not going to talk about lack of fit in beams because it involves a little bit of a difference.

For example, a beam being longer does not really affect the stresses in the beam. Think about it. In this particular case, if this beam instead of being 10 meters was 10.1 meters, what would happen? The only thing that would happen is that these rollers would just move to the right and give no forces in the member. In other words, lack of fit does not normally have any effect on beams because lack of fit has an effect on axial forces; and remember that in beams, axial forces are not of major consideration normally. That is why lack of fit is not a major issue as far as beams are concerned; that is why I am not considering that right now.

However, thermal stresses can have a large effect. The only difference is that…. Again, uniform rise in temperature of the member gives rise to axial deformations; the uniform rise in the member does not really have a tremendous effect on beams, it does have in certain situations; we will look at such problems. We have got the entire course where we are going to be solving a lot of problems. What I am trying to do in the beginning is try to lay the theory so that later on we are going to be solving a lot of problems; and when we solve the problems, that is when you actually know that when even a lack of fit or uniform rise in temperature could have an effect in beams and frame structures.

However, there are certain situations where you cannot really say beam, frame, truss; it may be a combination. So, there are going to be various… but those I leave for solution of example problems directly. Right now, we are looking at, in essence, the theory or how to consider the various effects. Later on, we will see by solving actual problems how these have an effect in reality. Let us now look at a situation where you have thermal stresses.

I just said that uniform temperature rise has no effect on beams because they essentially lead to axial deformations and axial deformations directly do not have a large role to play in beams, because beams essentially are subjected to flexure. So, what kind of thermal stresses would have an effect on beams? Let us look at that.

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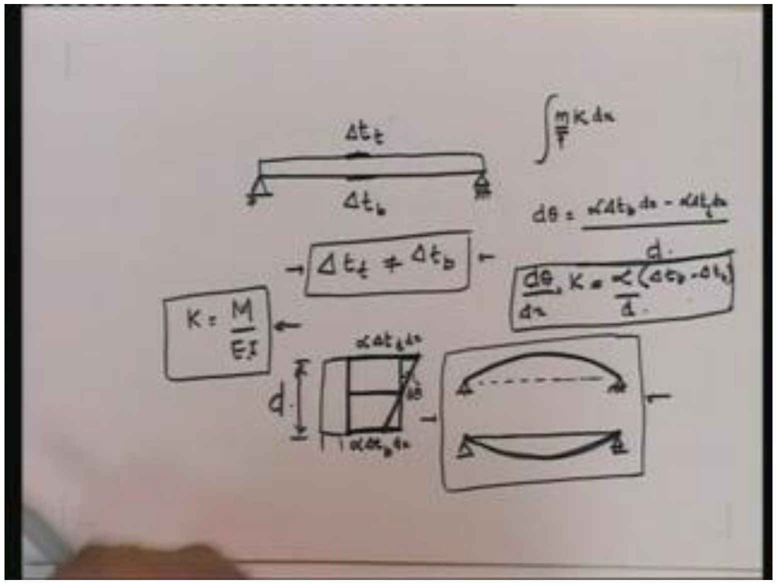

Let me start by looking at a simply supported beam. This is a statically determinate beam. In a statically determinate beam, what kind of thermal stresses would give rise to problems? Let us see. Let us have a situation where the top of the beam is heated. When can this happen? Let us think of this as a room (Refer Slide Time: 23:04). A room is typically colder than the outside, assuming Indian condition summer. If it was winter, then probably the outside temperature would be less than the inside temperature. However, the point that I am trying to make is that the delta t at the top and the delta t at the bottom are not equal to each other. In other words, the temperature differential exists; there is a thermal gradient across the beam. Thermal gradient – what does that mean? The temperature here and the temperature here are not the same. When there is a thermal gradient setup, what actually happens? Let us look at a particular phase (Refer Slide Time: 24:11). What happens? If you look at it, before the temperature went up, this was the phase. Now, the temperature at the top goes up by delta t; so, what will happen? Note that I am looking at a cross section, so this is the beam (Refer Slide Time: 24:27).

What is going to happen to that beam? This point, because it has been subjected to delta t, it is going to go alpha delta t top. This is going to be the strain at this point; and what is going to be the strain at this point? (Refer Slide Time: 24:53) It is going to be alpha delta t bottom. Since these two are not the same, these two are not going to displace the same; as long as the temperature gradient is uniform, in other words it is linear from the top to the bottom, you will see that each one will deform separately and if you look at the neutral axis, the neutral axis has gone in this fashion, but if you look at the section, it has actually rotated; 'rotated' means there is a flexure.

If there is a flexure, what is going to happen? If you look at this, delta t was higher than delta b; this is how the member would go in flexure. The bottom would go in compression, the top in tension. If the inside temperature was higher than the outside temperature, then the member would deform in this fashion. So, a differential temperature actually leads to flexure in a simply supported beam. Not only does loading lead to flexure, differential temperatures also lead to flexure; and how much is that flexure? Let us look at that. Remember that this is equal to d theta (Refer Slide Time: 26:54). If this is delta$_x$, then this goes dx, dx; and d theta is going to be equal to alpha delta t_b dx minus… because remember that d theta is positive in this direction, that is why it becomes negative; alpha into delta t top into dx all upon the depth of the beam.

Therefore, d theta by dx the curvature is given by alpha into delta t bottom minus delta t top upon d. This is the real curvature due to a differential temperature and once you have the real curvature then how do you find out anything? This is the real curvature. What we did was that we also evolved that the real curvature was M upon EI when you have loading. When you have loading, this is M where this is the moment due to the loading; and this is kappa due to that. Therefore, what does virtual work give us? Virtual work gives us m Kappa dx. Whether that K is due to M upon EI or whether it is due to differential temperature, it makes not one bit of difference. This is the virtual moment established due to the virtual force system that we develop in the structure. Let me now go back to the problem that we were looking at originally.

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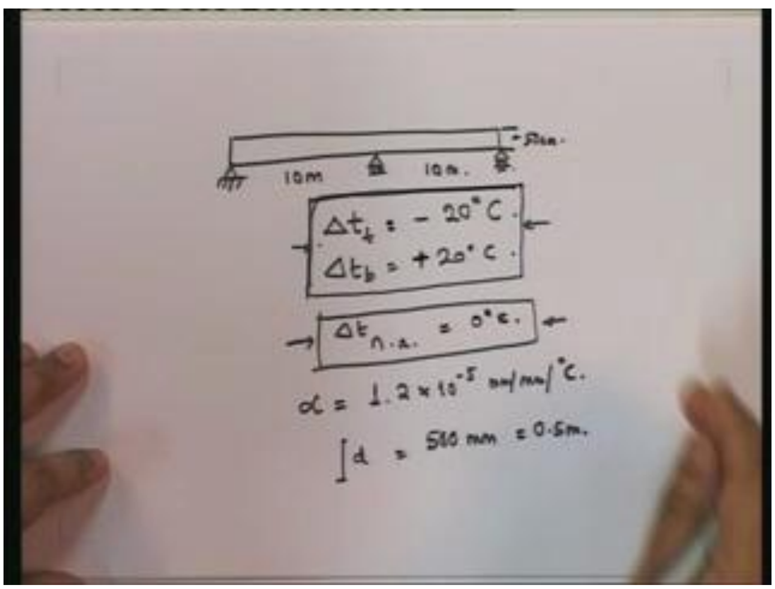

Let us take this situation. This is the structure and I have a situation where the delta t top is equal to minus 20 degrees and delta t bottom is plus 20 degrees. Let me put this kind of a situation. Why have I put this situation? Because if we look at this, what is the delta t at the neutral axis? You will see it is 0 degrees. In other words, this temperature differential leads to pure flexure because there is no strain in the neutral axis; that is why I am setting up a pure flexure kind of a situation. There is no other load and I have to find out the bending moment in this indeterminate truss due to this temperature differential. Let us see what happens. Did this temperature differential have any effect as far as forces were concerned? You will see that in a statically determinate, temperature only gives rise to flexure without giving rise to any forces; there are no forces or bending moments in the structure. However, you will see that in a statically determinate situation you do have this. Let us try to solve this problem.

Let me put a few things in here. I am going to say that alpha is equal to 1.2 into 10 to the power of minus 5 millimeter per millimeter per degree centigrade. Note that this is the strain per unit temperature, that is alpha; this is all I need right now. Let us say that the beam depth is equal to 50 centimeter; that is going to be 500 millimeter, which is equal to 0.5 meter. As long as you use units consistently, you will never have a problem in getting any solution. I am still keeping the same thing, so this is 10 meters, 10 meters. Therefore, the problem statement here is due to this differential temperature where the top is minus 20 degrees and the bottom rises by 20 degrees; in this case, there is a temperature differential of 40 degrees between the top and the bottom. What would be the forces in the supports and the bending moment diagram for this statically indeterminate truss? I am given that the depth of the beam is uniform. In other words, this is the beam where the beam is 50 centimeter deep. This is my problem statement. How do I solve this problem?

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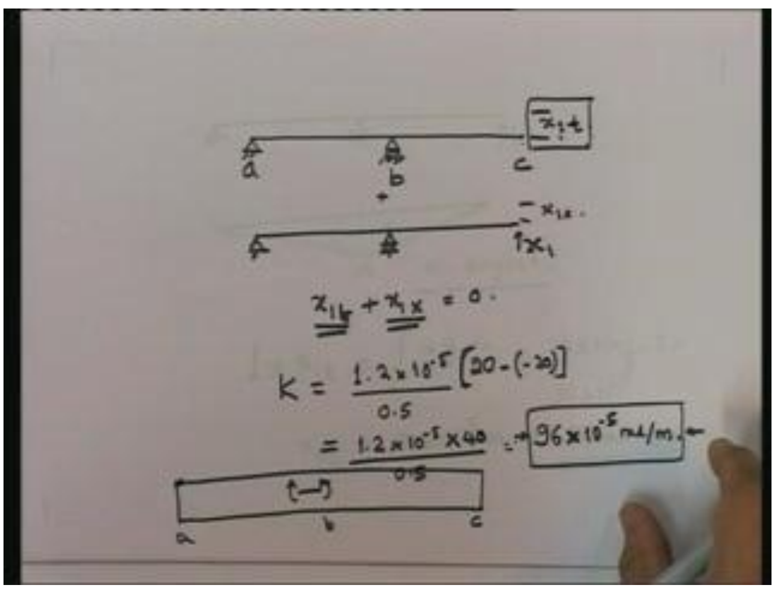

Again, we go back to the basics. This is my structure. What I need to do is I need to find out how much this has gone up due to the differential temperature. Note that if this does not go up or down due to the differential temperature, you will not have any support reaction here, you will not have any x_1; if you do not have any x_1, the temperature will not have any effect on the structure. Therefore, we have to first see whether there is a displacement here due to the temperature. Then of course, we add up this and put it equal to 0 and you know, I mean all those. Find out this displacement x_1 and the compatibility condition is x_{1t} plus x_{1x} is equal to 0. Note that if this is equal to 0, this is equal to 0 and this is equal to 0 (Refer Slide Time: 34:10), the next one is equal to 0; if X_1 is equal to 0, there are no forces in the members and there is no effect of the differential temperature. So, the whole thing hinges on whether there is an x_{1t} due to the differential temperature; that is our whole focus over here. Let us see what happens. Now, I have already done this here; what is the effect of the differential temperature?

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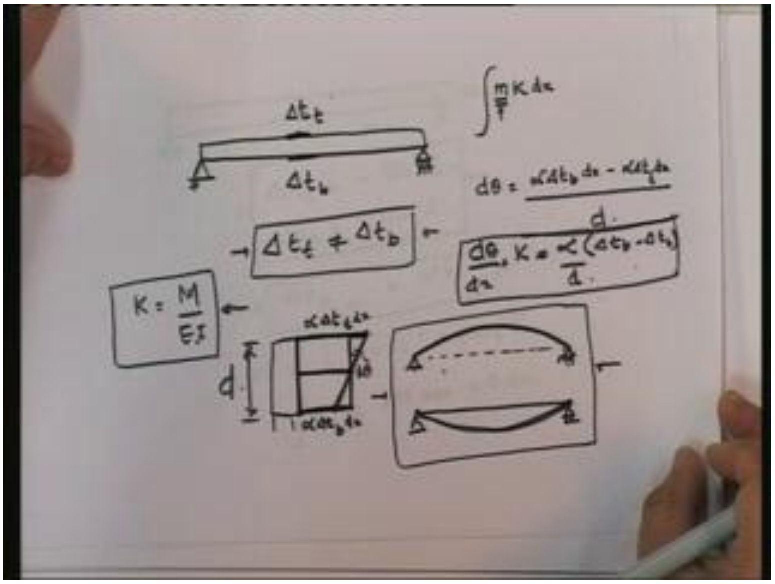

This is the curvature that happens due to differential temperature. So, if I put that, what is …? Note that the differential temperature is the same; in other words, all along the bottom you have plus20 degrees, all along the top you have minus 20 degrees. So, what is the curvature? The curvature, if you look at this, is uniform from here to here; and what is that curvature value equal to in this particular case? It is equal to alpha, which is 1.2 into 10 to the power of minus 5 divided by 0.5 meters; that is alpha into delta t. If you look at this, what is this? Delta t bottom is plus 20 degrees, minus delta t at top, so minus 20 degrees. If you look at this, this is equal to 1.2 into 10 to the power of minus 5 into 40 upon 0.5. If you put that together, you get 96 into 10 to the power of 5 radians per meter. You can see that the units of alpha are millimeter per millimeter per degree Centigrade; since millimeter per millimeter does not really have any units, we can say per degree Centigrade.

Here, we have 40 degree Centigrade; that multiplies, the top becomes nothing, no units, the bottom is in meters, so what we have is per meter. What is kappa? Kappa is the rate of change of theta with length; so, kappa is d theta by dx. What is theta given in terms of? Radiance. What is length given in terms of in this particular case? Meters. So, it is radiance per meters, 96 into 10 to the power of minus 5.

This is uniform. In other words, if I were to draw the real kappa, it would be uniform across the board and it would be equal to 96 into 10 to the power of minus 5 radians per meter. If I put a, b, c, a, b, c, due to the temperature, I have a constant flexure across the entire beam of 96 into 10 to the power of minus 5 radians per meter. So, does that give an effect? I need to find out what that leads to in terms of this. What do I do?

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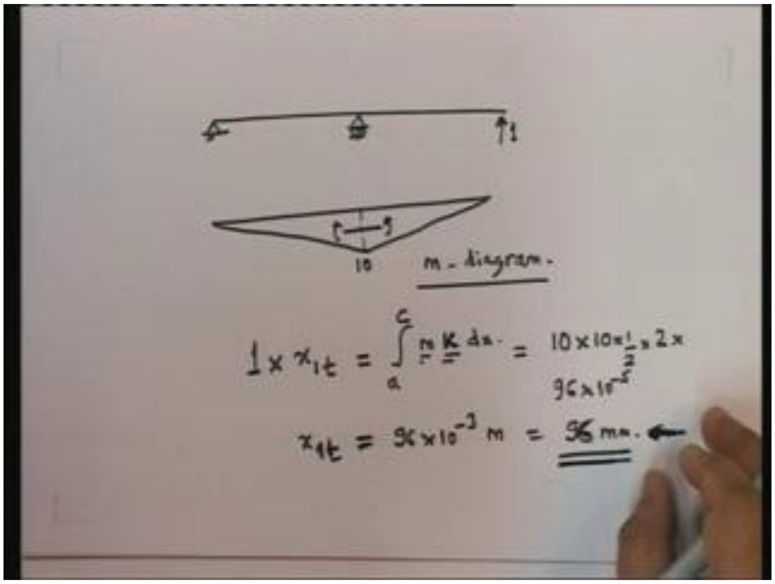

Again, using the principle of virtual work, which is really to find out a displacement, I give a unit; corresponding to the displacement, I give a unit virtual force and I find out the internal forces. If I look at the internal forces, what do I get? If you look at the curvature, the curvature is also this way; because the bottom actually increases and therefore, you have a situation where the top compresses making the flexure positive in this way (Refer Slide Time: 39:02). So, this one is going to be 10 units; this is my m diagram.

What I need to do is I need to find out x_{1t}. It is going to be 1 into x_{1t} – that is the work done. External virtual work is equal to this and the internal virtual work is going to be over from a to c m into kappa dx. Remember this m M upon EI; M upon EI is nothing but the real…. So, this is the virtual moment, this is the real flexural deformation and this is integrated over the entire length. If you put that, this is going to be equal to …. What is the moment? Again, integral area of the moment…. if you look at this particular one, the area would be equal to 10 into 10 into half into 2 multiplied by kappa, which is this value; so, 96 into 10 to the power of minus 5. What happens here? x_{1t} turns out to be equal to…. This 10 plus 10 is 100, you get 96 into 10 to the power of minus 3 meters, which is equal to 96 millimeters. Note that x_{1t} turns out to be 96 millimeters, which is not a very small displacement. You see, because of the temperature differential of 40 millimeters, x_{1t} is equal to 96 millimeter. Now how do I find out the forces? I have to find out x_1.

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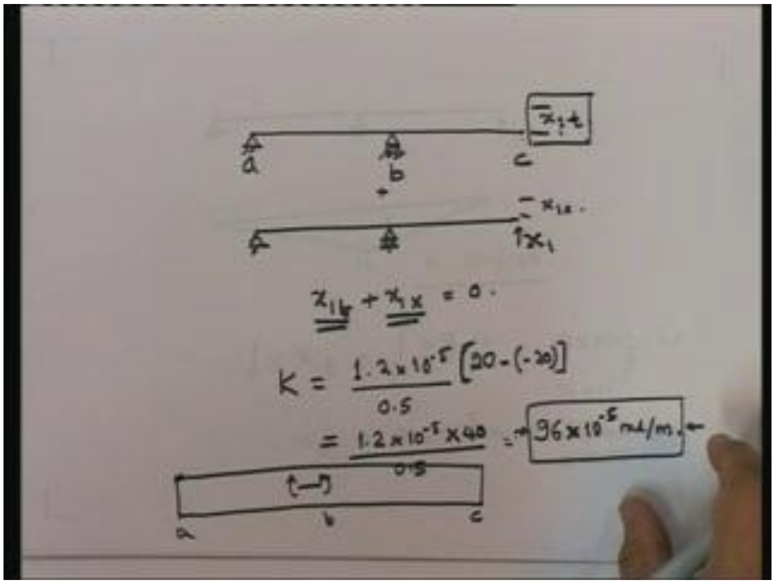

Note that I have found out x_{1t}, I need to find out x_{1x}. I have already found out X_{1x} earlier. Remember? What was the x_{1x}?

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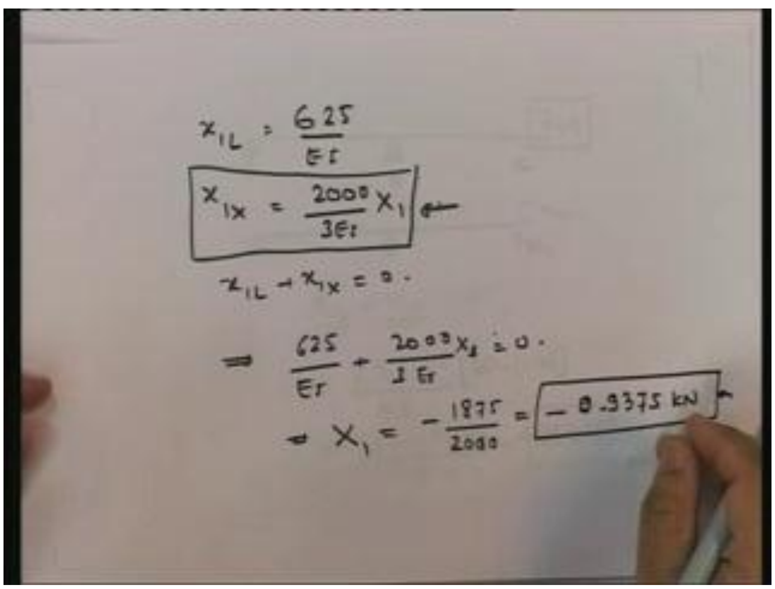

x_{1x} is this; this does not change. If I put that in, if I plug that in here, what do I get?

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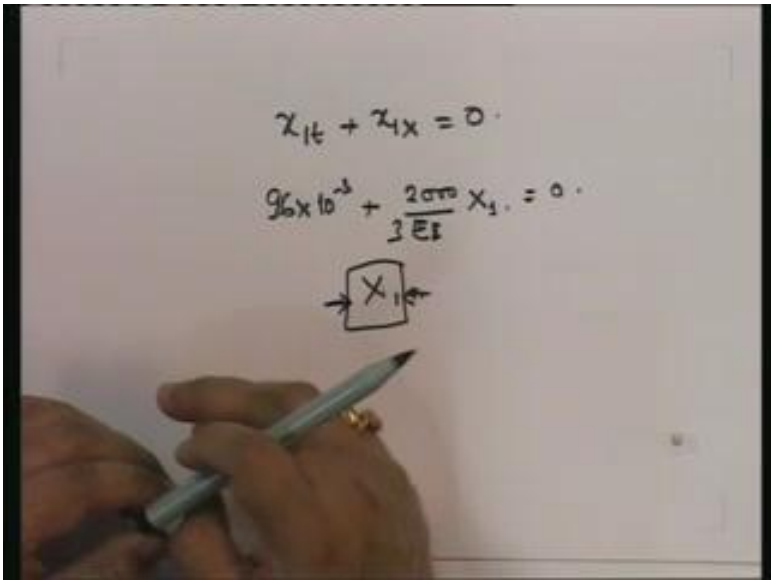

I get x_{1t} plus x_{1x} is equal to 0. x_{1t} is 96 into 10 to the power of 5 meters and this is 2000 upon 3 EI into X_1 is equal to 0. From that, you can find out the value of x_1. Once you find out the value of X_1, it is a statically determinate structure and if it is a statically determinate structure you can find out the bending moment diagram. I am not going into that detail because you know you need to put in the value of EI, etc…. to actually solve for it. In essence, I am just coming down to the fact of how to compute the effects of thermal stresses on beams.

You may say that you have not considered any loading, but I go back again to the same point that I made earlier: If I have a temperature as well as the loading, how would I do that with both of them together? I would consider only the loading, then I would only consider only the temperature, find out all the effects, add the two up, totally add it up – algebraic addition – and then I have the effect of all the things together. So, it boils down to the entire fact that you have a situation where you can find out temperature thermal stresses in beams.

Note that exactly in the same way you can consider frames too. Remember last time, two lectures ago, I talked to you about beams and frames in terms of new thought processes; there are no new thought processes because the only difference between a beam and a frame is geometry. A beam lies along one direction; a frame lies in a plane. That is the only difference between all these. What I would like to do today is I have developed the theory for the following things.

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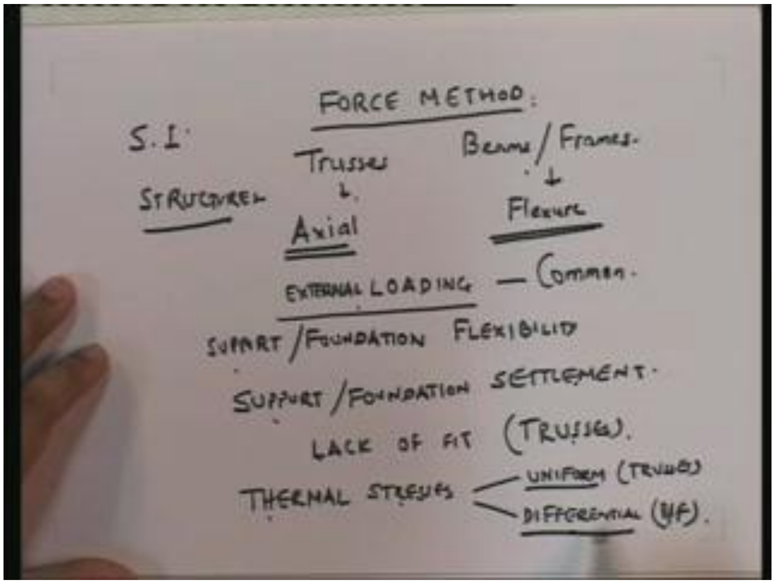

Force method: I have looked at the force method, I have applied the force method to both trusses and beams; the thought processes are the same. It is: Trusses/Axial - main load transfer - beams and frames/flexure - main load transfer. This is the essential difference between the two. We have looked at how to consider the force method for trusses as well as beams and frames. What are the kinds of things we have looked at? We have looked at external loading which is the most common situation. Our analysis is normally always for external loading. However, over the last two lectures, we have looked at a variety of other effects, we have looked at foundation flexibility.

Again, I use the word 'support and foundation' together. Then, we have looked at support foundation settlement. We have looked at lack of fit, which is essentially for trusses. We have looked at thermal stresses and this was separate, uniform for trusses. Why? Because you have to understand that uniform gives rise to axial. Differential for beams and frames. Why differential? Because differential is what gives rise to flexure. We have looked at all of these things for the analysis of statically indeterminate structures.

After these lectures, we should be able to solve using the force method. We should be able to analyze the forces and displacements in trusses, beams and frames for external loading, support flexibility, support settlement, lack of fit in a truss and thermal stresses that are uniform in trusses and differential in beams and frames. This is what we should be able to do. Of course, you may ask how I solve a particular problem. I am going to spend the next few lectures looking at a variety of problems so that you will be able to confidently use the force method to solve all these problems. Over the last few lectures what I have tried to do, in essence, is to lay out the theoretical basis. From here and out, I am going to be purely applying for example problems. This is the basic force method. Now I am going to do a couple of problems over the next couple of lectures.

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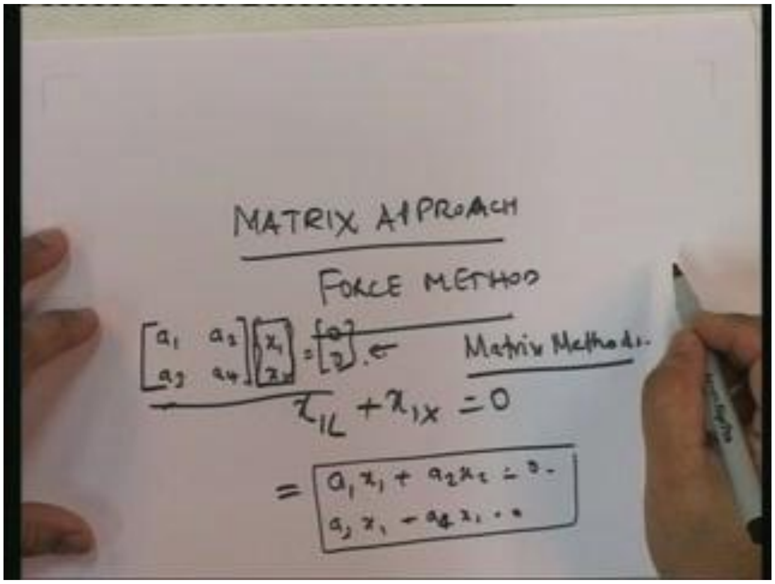

Then, we are going to move into the matrix approach for force method. Why the matrix approach? Essentially what you are trying to do is, if you are writing down the compatibility equation, it becomes something like x_{1L} plus x_{1X} is equal to 0. If you really look at this, this is more like a_1 x_1 plus a_2 x_2 is equal to 0; and then you can put a_3 x_1 plus a_4 x_2 is equal to 0. If you really look at this, this can be written in terms of a matrix as a_1, a_2, a_3, a_4 into x_1, x_2 is equal to 0, 0. This is nothing but a matrix that we can solve. What do we need to do? We need to solve for x_1 and x_2 and we can solve that using the matrix methods.

That is not the only aspect of the matrix approach. The matrix approach not only uses matrix methods but it also actually develops the entire procedure using a completely different theoretical premise. Of course, the basic concepts that we have looked at are on how to compute deformation, how to use the virtual force system; they all remain the same; it is just that they are set up in a different way. For trusses, you will see that they are really not very different, but for beams and frames, you will see the entire basis become quite different. I just wanted to introduce you today to this particular concept just to lay out the next few lectures that I am going to be talking about; I am going to spend the next couple of lectures actually looking at some example problems. We will look at one truss, a more rational truss; up till now what we have been doing is, we were looking at simple structures to essentially illustrate the method. You may say all this is fine but when I look at the actual truss, how do I solve that? We will look at that problem. We will take up a truss problem and incorporate as many things as possible into it so that you can gain confidence in solving the problems.

Then, I am going to take up a beam problem for an example beam problem and solve it all the way through. Then, I am going to take a frame problem and solve it all the way through so that at the end of the next couple of lectures, you are ready to use the force method to solve any truss or beam frame problems totally by yourself. Then, I am going to move into the matrix method to introduce you to a different way of solving and we will see quickly that the matrix method actually is a very very good way of using the force method and very quick way of solving, but the only thing is that it gets algorithmic and you lose sight of the basics. That is the reason why I have used the force method, to start off with, it in its basic form so that you can look at the physics of the problem. Later on, in the matrix method we will make it an algorithmic way of analyzing structures. Thank you very much. I hope you have enjoyed listening to my lecture as much I have enjoyed delivering it. Look forward to seeing you next time.

Thank you.