**Structural Analysis II**
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**Lecture – 06**

Good morning. Today, we are going to continue with looking at the application of the force method for analyzing statically indeterminate structures. Last time, if you remember, I had said that today we are going to consider flexible supports and see how that can be included in the force method procedure to be able to analyze the structure. Let us look at what we have.

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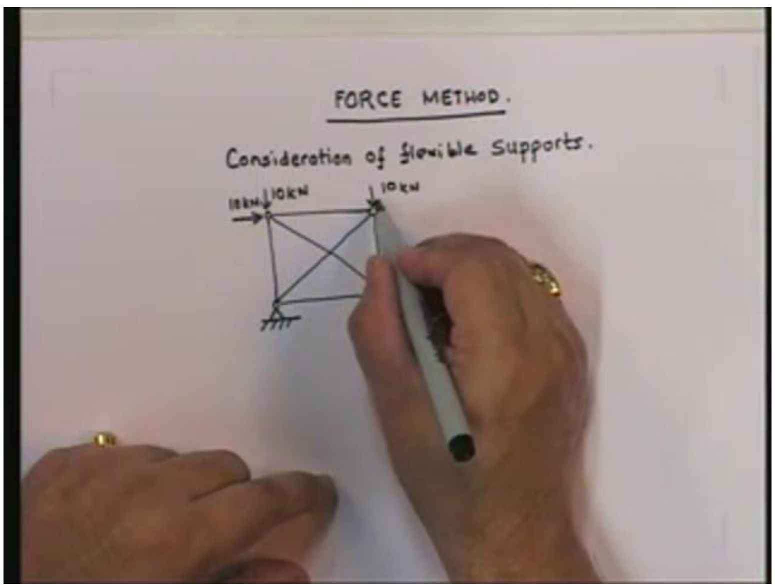


This is the same truss that we had considered last time, excepting that the roller here has been replaced by a flexible spring support where the flexibility is given as 1 over EA. EA is the value that we have for all the members, actual rigidity for all the members; all of them are EA. If you remember, that is what we had considered last time. I am just taking a flexibility value; I am taking it to be 1 upon EA. It is not necessary that this has to be 1 upon EA; it can be any flexibility that I define for this spring. In this particular case, I am defining it as 1 upon EA because I am going to solve it using 1 upon EA. However, you can use any value of f and you will see that whatever procedure I develop will be equally valid for that procedure.

How do you solve this problem? The last time we saw how to get the support reactions, what did we do? By the way, this is a single degree of static indeterminacy problem. This is the same problem as we dealt with last time; I am not going to go into all those details again. This time, I am just trying to see how we can consider the effect of additional flexibility in the supports. I have considered this support to be flexible. When would your support be flexible? In real life, when would this happen? This would happen in case you had a pier where you considered the axial deformation of the pier. If you consider the axial deformation of the pier, then you see that the pier becomes axially flexible since the pier is essentially subjected to vertical axial forces

because that is how the load is transferred through the pier; what happens is essentially that we are now considering the pier to be axially deformable. What we are saying in this particular case is that the pier flexibility is given as 1 upon EA; that is all we are saying in this particular case.
How do we solve this problem? The problem goes in the same procedure as last time; you have to develop your statically determinate base structure. I am going to have exactly the same thing as last time. (Refer Slide Time: 05:07) I am cutting this member so that I get a statically determinate structure. Then, I need to continue having this and these are the loads. This is my base structure.

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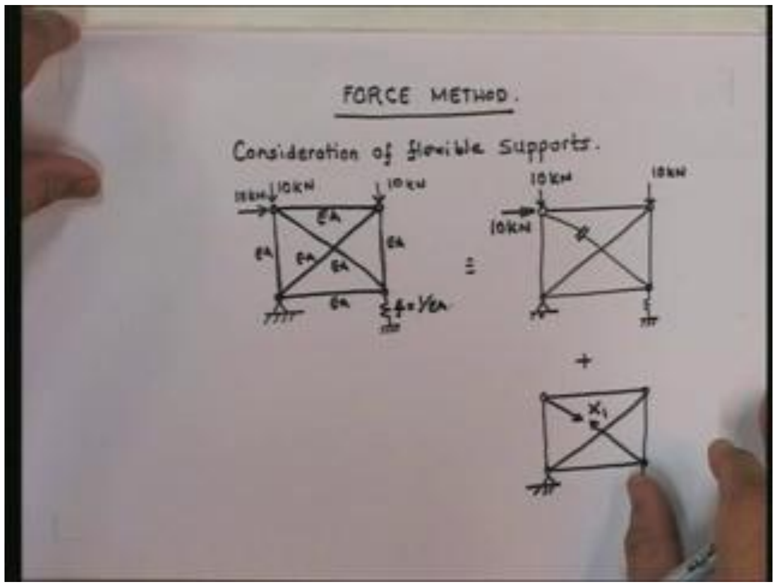


This plus this is equal to this. In essence, there is no difference in the solution procedure just because you have flexibility.

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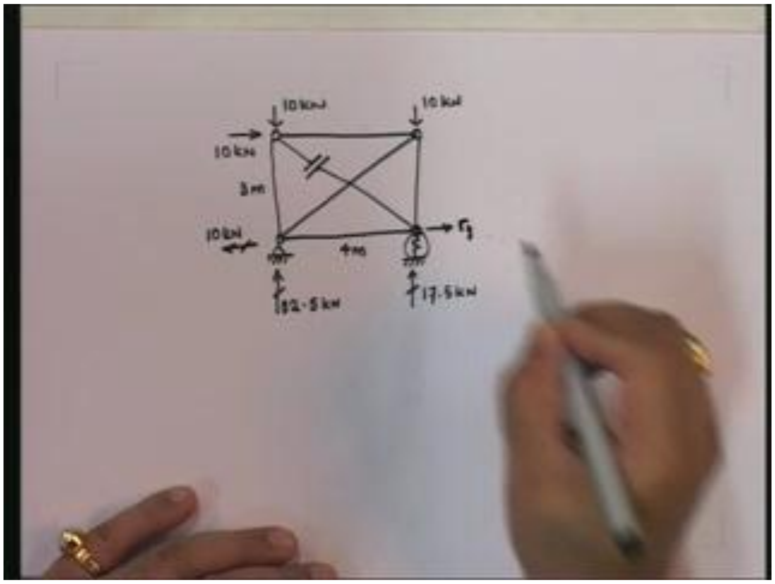


What you have in this particular case is the following if I look at the base structure. This is my base statically determinate structure. If I have a statically determinate structure, remember that I am taking exactly the same thing as last time. The first step in analyzing this structure is to find out the reactions; and as we saw last time, the reactions are identical. This is going to be 10 and 10 and then due to this, 30 by 4; this is going to have 17.5 Kilonewtons, 2.5 Kilonewtons and this is going to be 10 Kilonewtons.

If you look at this, this is identical to…. Therefore, till now we have not considered any aspect of this flexibility; we have not included this flexibility into the entire system. However, the most important point here is that whereas in the previous case this deflection was 0 because it was a roller support, in this particular case there is going to be a deflection; but, the overall point is that this deflection does not affect the forces in the structure at all. What it may do is that it may affect the displacements in the structure. Therefore, the whole point is that even if I solve this structure and I were to find out what the value of $X_1$ would be, you would see that as far as the forces in the members are concerned, whether I have a flexible support here or not will not matter at all. In this particular problem, I am going to try to evaluate something else. Remember I said last time that we could find out this displacement in addition to finding out all the forces in all the members. Now if I do that, you see it is not just a question of finding out the forces in the members and the deformation in the members; I also need to find out what this displacement is going to be due to these forces (Refer Slide Time: 09:41).

How would I do that? Remember, to find out the displacement at this point due to all these forces, I need to apply a force there. Now under this system, I am trying to find out two things – I am trying to find out the separation at this point and I am trying to find out the displacement at this point. Both these are things that I need to find out together and I have the flexible support. How do I do this? In terms of this particular structure, all I need to do is to find out the forces.

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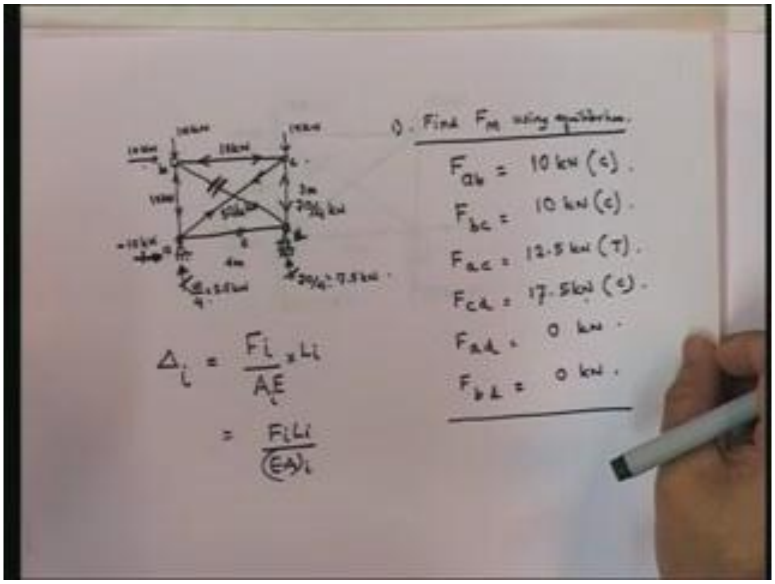


I do not want to go back and do the same thing all over again; I am going to just put the previous structure that we had done. If you look at this, these are the forces in all the members; what we need to do now is find out the forces due to these two.

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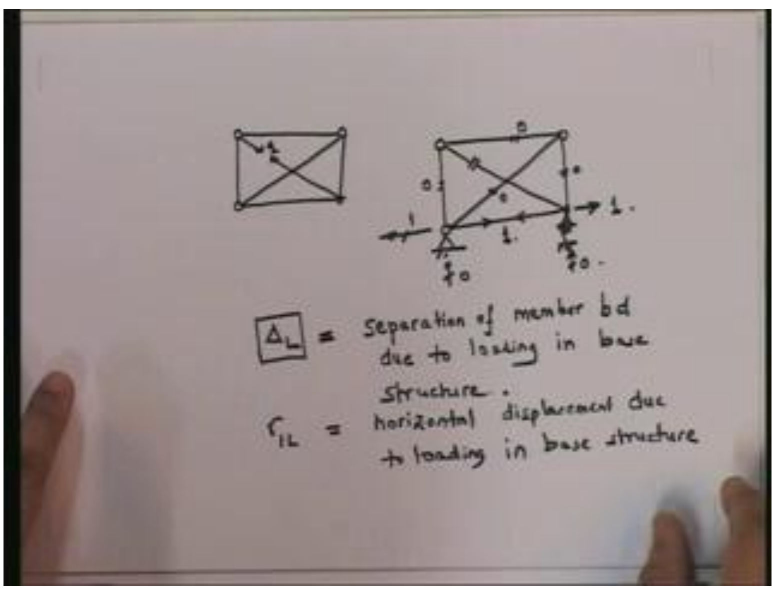


To do that, one is to apply a unit force here and find it out. That is the unit virtual force so that I can find out what the displacement at this point is in the statically determinate structure (Refer Slide Time: 11:56). I am applying a unit force and this is identical to what we had solved over here.

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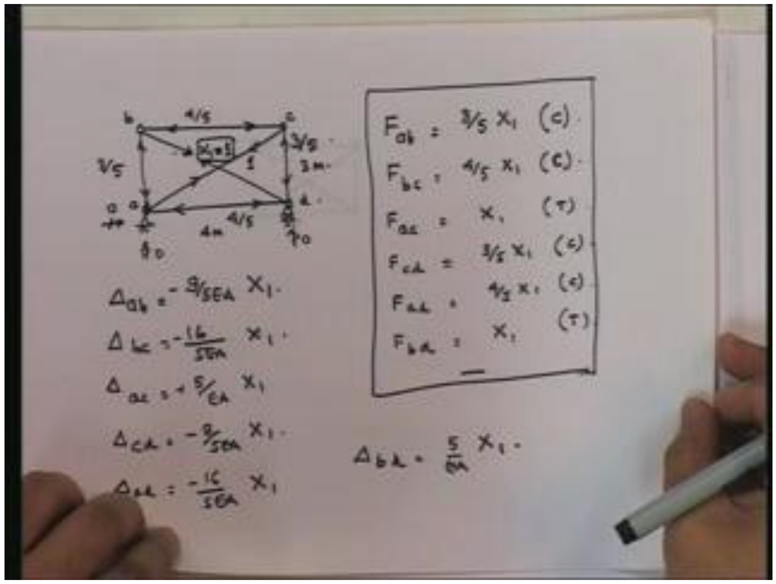


If I put $X_1$ is equal to 1, then $F_{ab}$ is equal to 3 by 5, 4 by 5, 1, 3 by 5, 4 by 5 and 1.We have already looked at all of these last time; therefore, there is no need to solve this particular problem; it is identical. To find out the other displacement, what do I need to do? Note that again in this particular case what I am doing is to find out this displacement, I need to apply a force at this point. Now let us look at what happens. This is my flexible support. As far as the forces are concerned, let us see what happens here. If I apply a unit force here, then the reactions, this is going to be 1; if you look at these reactions, these reactions are going to be equal to 0. What does that mean? That means that due to this displacement all that we have is a unit force in this and all others are 0.

This is the virtual force that I am applying here; I am finding out the virtual forces in all the members and the virtual support reactions that we have. The point that I am trying to make over here is that you just need to go through the same step procedures to get your displacement. What do you do? There are two steps; let me write down. This is $\text{delta}_L$. How do you find out $\text{delta}_L$? $\text{delta}_L$ is due to the load alone, which is due to this (Refer Slide Time: 14:15). How do you find out the $\text{delta}_L$? We have already gone through the steps last time; I am not to go back and do the same thing. You can find out this deflection and this deflection in this structure by just writing down the virtual work equations where the real deformations at this point are actually applied. The real deformations are multiplied by the virtual forces developed due to this and this (Refer Slide Time: 14:46). Understand that this $\text{delta}_L$ that I am finding out is the separation of member bd due to loading in the base structure; this is the separation of member bd due to loading in the base structure. How do I find this out? I find out the forces in all the members due to this, the virtual forces, and multiply them with the real deformations due to the loading. We have already got this earlier; there is no difference in this particular case.

The next thing that I am going to find out is $r_1$ due to L – that is the horizontal displacement due to loading in the base structure. How do I find this out? I find out the deformations in the member and then find out all the virtual forces in all the members. If you look at it, what really happens? You will have $r_1$ in the real structure, that is, $r_{1L}$ into 1; that is, the work done by the external forces is equal to 1; that is, member ad 1 into the deformation of this member $delta_{ad}$ in the original structure that is due to this loading. We have already found that those out earlier.

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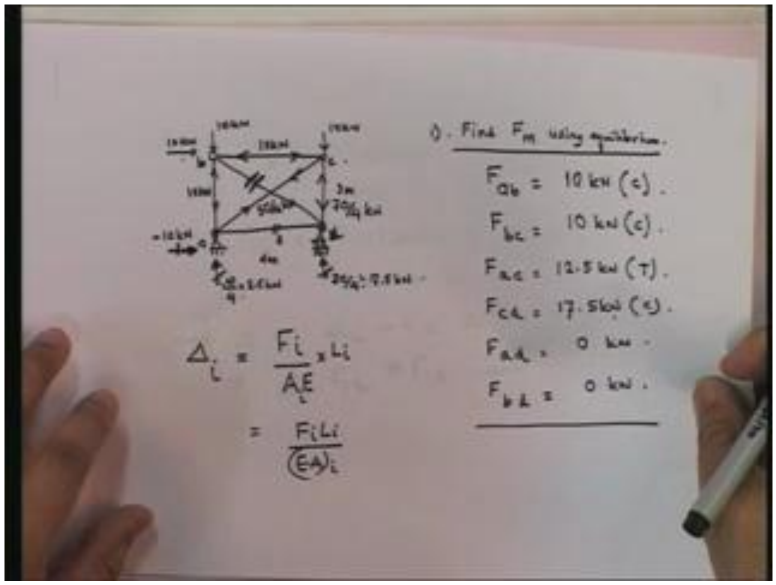


What is this delta? This delta is equal to 0. If it is equal to 0, then what is the delta in this? 0. What is the deflection at this point due to the loading that we have? 0. We can find that out; this is very easy to find out. This I have found out in the base structure due to the loading.

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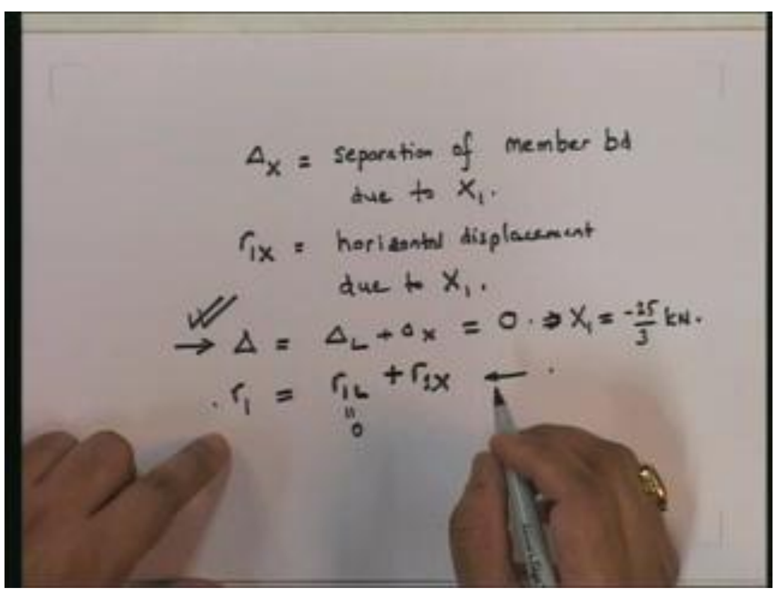

Now what is the next step? I also need to find out the $\text{delta}_X$. What is $\text{delta}_X$? It is the separation of member bd due to $X_1$; I can find out this also. What is the real structure? (Refer Slide Time: 17:58) The real structure is the one with $X_1$ applied here. Find out all the displacements and deformation forces in the members and the deformation in all the members; then take this as the virtual force and I can find out this $\text{delta}_X$. I have already found this out the last time also. If you remember, the $\text{delta}_L$ that I got last time was this (Refer Slide Time: 18:21), the $\text{delta}_X$ that I got last time was this (Refer Slide Time: 18:27).

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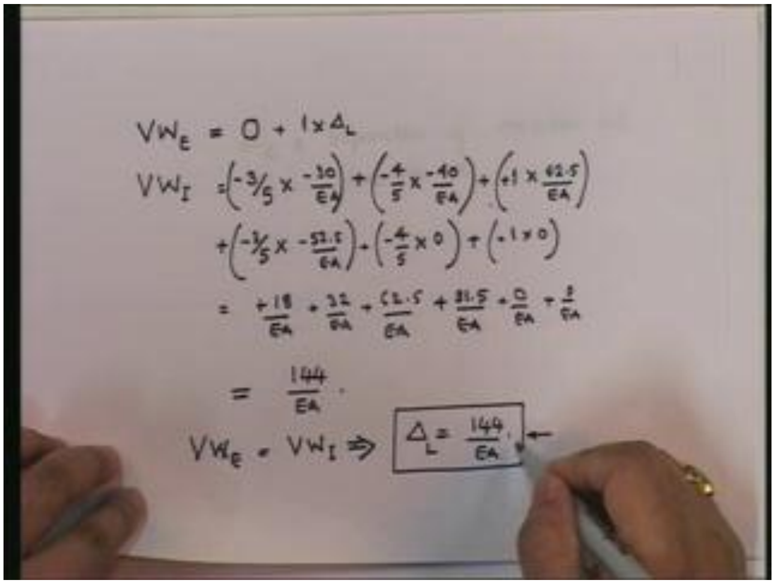


Therefore there is another aspect I need to consider, that is $r_1$ due to X which is the horizontal displacement due to $X_1$. How do I find that out? In that case, my real structure is this structure; I can find out the forces and the real deformation in all the members and my virtual structure is this structure (Refer Slide Time: 19:04). I can find out the virtual forces which I have found out and then I can do the virtual work to get $r_{1X}$. Therefore, what is the final solution? The final solution you will see is delta – that is the deformation – the separation in the original structure is going to be equal to $\text{delta}_L$ plus $\text{delta}_X$ which by the way is equal to 0 because there is no separation. $r_1$ is equal to $r_{1L}$ plus $r_{1X}$. Note that this gives me… like last time, I found out that $X_1$ is equal to minus 25 by 3 Kilonewtons; there is no difference in the flexibility at all.

Once I find out $X_1$, I can find out $r_{1X}$ because $r_{1X}$ will always be in terms of $X_1$ and since I know $X_1$, I can find this out, add it to this one, which is a number, and I have got my displacement at this point. Now you may ask in this particular case, how does having the flexible support affect this entire process? It actually does not affect this at all; it actually does not affect this at all; it does not affect this at all (Refer Slide Time: 20:27). Does this affect this one? I am not going into the details; you can actually solve it yourself.

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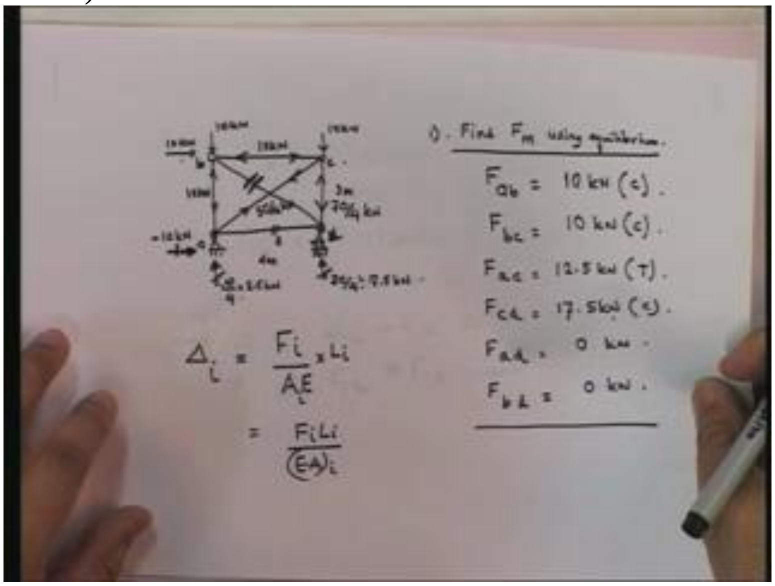


You will see that this one is not going to give anything primarily because if you look at this problem which we solved, the primary problem is that since this is equal to 0, this is equal to 0; you see that $r_{1L}$ turns out to be 0 (Refer Slide Time: 21:03). Since $r_{1L}$ turns out to be 0, $r_1$ is only due to $r_{1X}$ and that is not affected by the flexible support. You may well ask what does the flexible support do for this particular problem. Actually, the flexible support does not affect all the things that we have found out. What would the flexible support affect? It would affect, for example, if you were to find out this displacement.

Exercise: Find out the displacement in this structure due to these loads considering two situations, one where you have the roller support.

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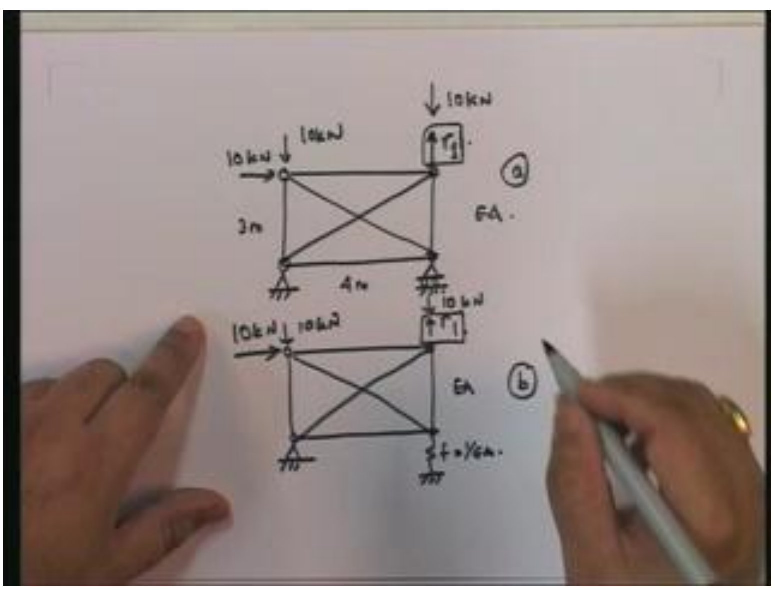

Do you understand the problem? I want you to find out the deflection at this point in this statically indeterminate structure due to this loading, where this is 4 meters, 3 meters; all of these members have a flexural rigidity of EA. I also want you to find out this displacement due to the same set of loads, excepting that the roller support is replaced by a spring whose flexibility constant is 1 upon EA where EA is the flexibility of all the members; find these displacements out. I am giving this as an exercise to you so that you can actually apply the principles that we have been talking about. Find out the displacement at this point due to these loads. This is your first problem, problem a; then find out this displacement, which is problem b, due to this.

I am going to tell you right upfront that in this particular case, you will see that this displacement and this displacement are two different values. That means that the flexibility of this support has an effect on it. Therefore, there are certain responses that are affected by the flexible support; there are other responses that are not affected by the flexible support. I just gave you two examples where the flexibility of the support does not affect the response. However, I have also given you one exercise where you will see that the flexibility of the support does have an effect on the displacement quantity that I am trying to find out.

Where do we go from here? What we have to do is we have to find out what other effects you can have in a structure which may affect the static analysis of the structure. Let me look at another classical situation; I am going to remain with trusses for this particular lecture because I want to look at all the effects that you may have to consider in addition to the effect of the static indeterminacy.

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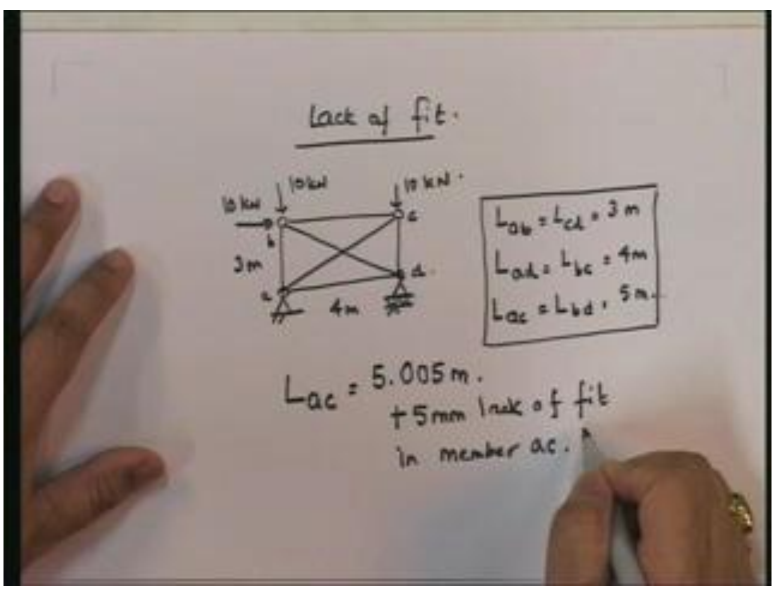


Let us look at what I call as lack of fit. What is lack of fit? Let us say that this is the same structure. I am going to go back to the roller support and I have these loads a, b, c, d; this is exactly the same structure. This is 4 meters, this is 3 meters; so, member ab is 3 meters, member cd is 3 meters, member ad is 4 meters, member bc is 4 meters and member ac and bd are 5 meters each; 3, 4, 5 triangle. These are the actual lengths; let me write it down. Length of ab is equal to length of cd is equal to 3 meters; length of ad is equal to length of bc is equal to 4 meters; length of ac is equal to length of bd is equal to 5 meters.

This is the geometry that I have (Refer Slide Time: 27:18). When I give this geometry to someone, what are we saying? Please make the length of ab 3 meters, length of cd 3 meters, length of ad 4 meters, bc 4 meters, ac and bd 5 meters; this is what we have given. However, whenever you are putting together members, how do you do it? You never have a member of size 3 meters exactly. What you have is probably two members of 10 meters each. What you have to do is you have to cut them up so that you get exactly the sizes that you have. There is also the factor that what we call as 10 meter length may not be exactly 10 meter length. Therefore, there is this aspect that even though we want ab and cd to be 3 meters, ad and bc to be 4 meters and ac and bd to be exactly 5 meters, in reality, when someone actually cuts the members and puts them, you might have a situation where some members are not exactly the size that they are supposed to be.

In this particular case, let me take the situation where let us say when we cut $L_{ac}$, it actually turns out to be 5.005 meters instead of 5 meters. What does that mean? That essentially means that there is a plus 5 millimeters lack of fit in member ac. In other words, the member ac is actually 5 millimeters longer than it is supposed to be. What effect does that have on the structure? If it was a statically determinate structure, it would have no effect on the forces in the members; all that would happen is that the structure would no longer be exactly a 3, 4, 5 kind of triangle; there would be some small displacements and the nodes would be shifted slightly. However, there would be no additional forces in the members due to the lack of fit alone.

However, in a statically indeterminate truss, that is not true. I am now going to show you that due to this lack of fit there are going to be additional forces in the members. To do that, what I am going to do is I am going to forget about the fact that you have a loading on the structure. What I am doing is I am taking the problem as two parts. One part is that I am finding out the forces in all the members due to the loading alone; I have already done that last time; I am not going to repeat that here. What I am going to do is…. What is the additional effect of this lack of fit that I have because of the fact that member ac is 5 millimeters longer? That is the only problem that I am going to be solving here right now.

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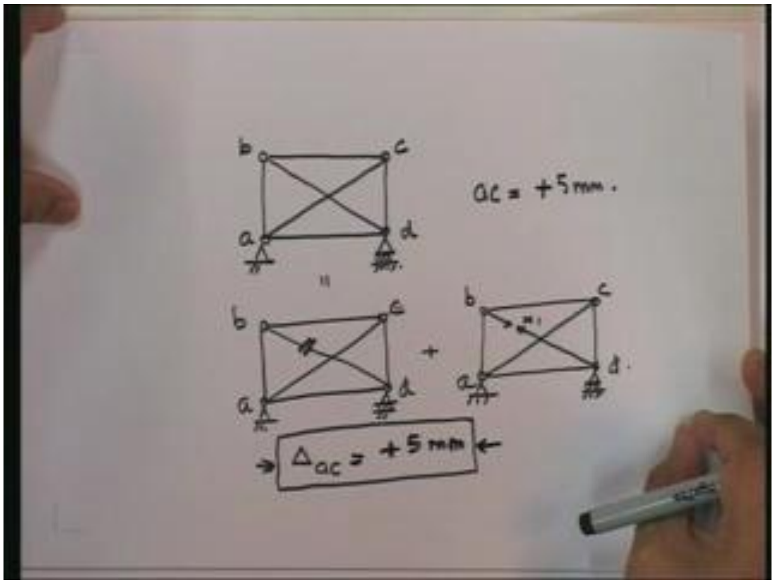


Therefore, we have this situation that member ac is longer. What is going to happen? You have this structure; no loading on it at all, this is a, b, c, d. The only thing that we are trying to find out over here is that ac actually has a positive lack of fit. In other words, what we have is, member ac is 5 millimeters longer than it should be. I can look at this problem in this manner that in the base structure….

I am going to still solve this problem as two problems; it is still two problems plus…. Only thing to note is that there is no loading; there is no loading. The second problem is …. This is the same problem excepting that in this structure, there are no forces in any member; and this is how you solve a lack of fit problem. I can say that in this particular structure, $delta_{ac}$, that is, the lack of fit turns out to be a deformation in member ac. Do you understand what I am saying? I am saying that in this structure, which is the base structure, none of the members have any forces in them, none. However, the lack of fit essentially means that I am treating this problem…. Due to some reason, $delta_{ac}$, the ac which is supposed to be 5 meters, is actually 5 millimeters longer. This is the original structure and I am saying that this member ac is longer by 5 millimeters; I can take that longer as a deformation in the real structure. This is the deformation (Refer Slide Time: 34:24).

Now, I need to get the effect of this deformation. Note that I still have this 3, 4, 5 and this cut that I have made was in the 3, 4, 5 triangle. Certainly, ac is longer; so, what is going to happen? Actually, this point cannot go anywhere; to ensure that this is longer, this will have to move somewhere along this direction. Then it will move 5 millimeters in this direction; because of that, there is going to be a separation at this point induced due to this. I need to find out what the separation over here is. How do I find that out? Again, use virtual displacement. If I use virtual displacement, what do I have? Let us go back.

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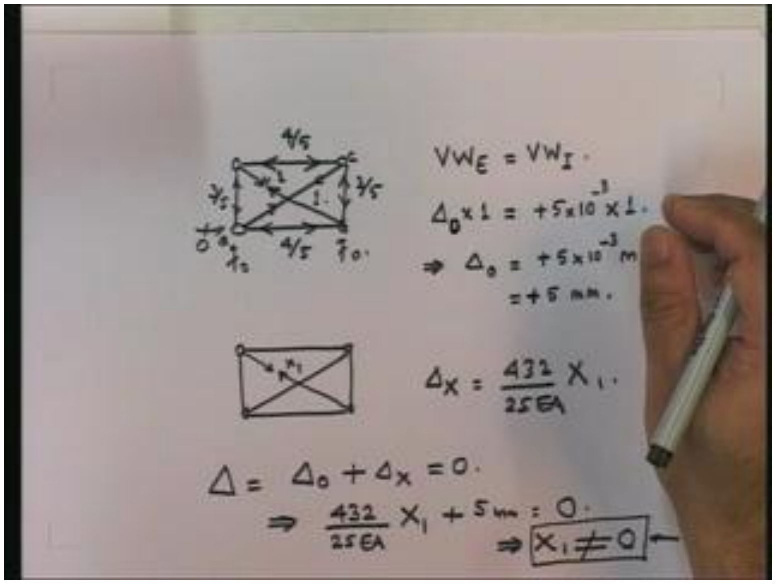


Apply a unit virtual force here. I have already solved this particular problem. What did we get? We got this.

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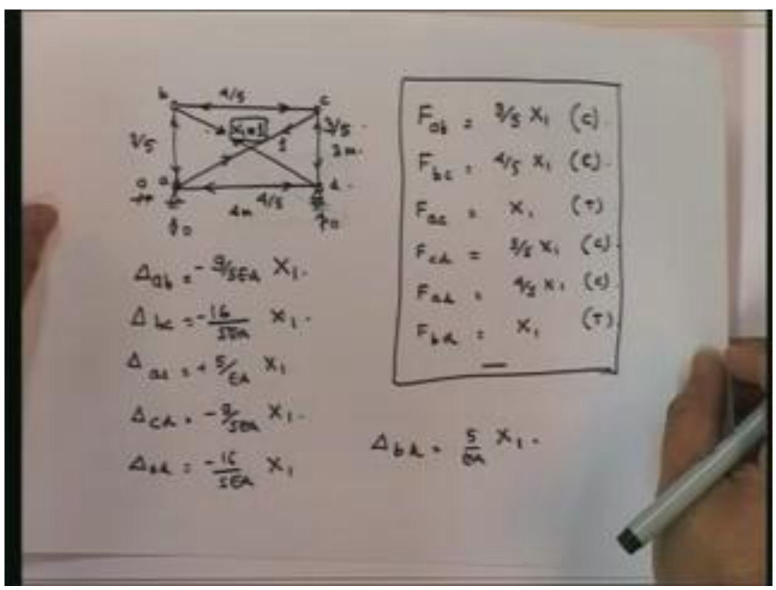


I am going to just copy that down without actually doing the analysis. This is 4 by 5, this is 3 by 5, this is 4 by 5; all the support reactions are 0, 3 by 5 and 1. These are the virtual forces set up due to applying this. Now, what do I have to do? I am going to use the virtual work principle; I am going to say that external virtual work is equal to internal virtual work. What is the external virtual work? $Delta_L$. Note that this $delta_L$ that I am writing down is not due to loading; so, I will call it $delta_0$; it is the separation at this point due to the base structure. The real displacement

$delta_0$ multiplied by the virtual force gives me the external virtual work is equal to the work done by all the real deformations. What are the real deformations? (Refer Slide Time: 37:12) ab is 0, bc is 0, cd is 0, ad is 0, bd is obviously 0 and ac is plus 5 millimeters.

If you see, what I have to do is …. All these virtual forces are multiplied by their corresponding real deformations; all deformations are 0 excepting in member ac. What is the member ac? plus 5 millimeters. If I want to put it into units of meters, it is 5 into 10 to the power of 3 multiplied by…. What is the force virtual force in the member? 1. This gives me the separation in the real structure due to…. It is plus 5 into 10 to the power of 3 meters or plus 5 millimeters.

What does that mean? (Refer Slide Time: 38:20) Since this is longer, what has happened is that this has overlapped. We have computed that; I have computed the $delta_{0.}$ Now, what do I need to compute? I need to compute the delta due to $x_{1.}$ I have already have done that last time; I am going to just take that down. Due to $x_1$, I need to find out the displacement at this point; this I have already done last time. If you remember, I had already done it and that $delta_x$ turned out to be 5 upon EA into $X_1$ (Refer Slide Time: 39:13). These were all due to the internal forces.

What did I have? I had the situation where…. Just one second, I will compute that for you. The $delta_X$ is equal to 432 upon 25; this still remains the same (Refer Slide Time: 39:42). Therefore, the $delta_X$ is equal to 432 upon 25 EA into $X_1$. Therefore, what is the total delta? Total delta is equal to $delta_0$ plus $delta_X$ equal to 0, which implies that 432 upon 25 EA into $X_1$ plus 5 millimeters is equal to 0. From that, I can find out the $X_1$ value depending on what the value of $E_1$ is. The main point to note is that $X_1$ is not equal to 0. Do you understand? Once $X_1$ is not equal to 0, all the member forces will depend on $X_1$ because given $X_1$, all the forces in all the members can be found out. Understand this – I had this situation where I did not have any load; the only thing that happened was that ac was 5 millimeters longer than it should be; because of that, you have forces in all the members. Therefore, a lack of fit actually gives rise to additional forces in the members in a statically indeterminate structure.

What have we considered till now? We have considered how to find out forces in a truss due to loading; we have done that. We found out the effect of the flexibility of a support on the truss. We saw that the flexibility of a support does not directly have any effect on the forces in the members; however, it can affect some of the displacement quantities. The third thing that we looked at was lack of fit – what effect does 'lack of fit' has on a member? We saw by an illustrative example that a lack of fit in a statically indeterminate truss which is not subjected to any load whatsoever gives rise to member forces. We have found out using the virtual work principle, the method of virtual force, how to get the particular values of all the forces due to a given lack of fit.

Remember, since we are dealing… I want to establish this once and for all. Since we are dealing with linear structures, any effect on the structure can be considered separately. Since the method of superposition is valid for linear structures, you can just find out everything due to one effect, then everything due to another effect and just keep adding it up. The sum total of all the effects is just going to be solving the problem for each individual effect and then adding up to get the final effect.

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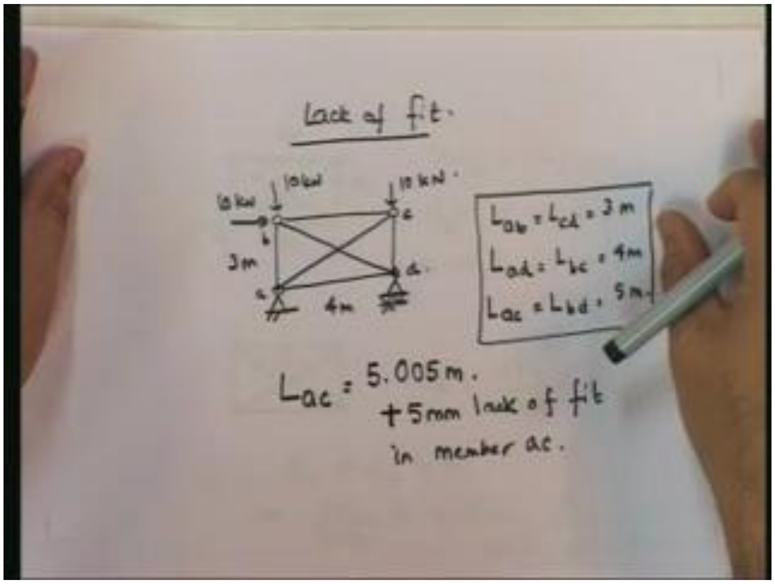


Therefore, even if I had this situation where I had this case… You see this? In this case where I have a loading in addition to which I have a lack of fit, I can solve this problem in two ways. I can say forget the lack of fit; I am going to first consider the effect of the loads, find out all the member forces due to the loads alone. Then, I am going to forget about the loads completely and I am going to take only the lack of fit. I am going to find out all the member forces due to the lack of fit alone.

What if I have a situation where all the loads and the lack of fit are there together? If I have a structure where I have all the loads as well as a lack of fit, I can treat this problem as a superposition of two problems. One problem is where there is no lack of fit; the member ac is 5 meters and we consider the effect of the loads and analyze the structure for that. Then, I can solve another problem where there are no loads; there is only the lack of fit; solve that problem. Then, superpose the effect of the loads and the effect of the lack of fit onto each other. In other words, I add up all the analysis results that I get for the load alone to all the analysis results that I get for the lack of fit alone, add the two up and I have got the effect due to both the loading and the lack of fit together.

This principle of superposition is valid; therefore, you will see that I will put situations where I consider only one effect. The reason behind that is that if I consider each effect separately and add up all of them, I am going to get the combined effect anyway. This is of course true only for linear structures. In this course, we are only looking at linear structures; therefore, we can use the principle of superposition.

Coming back to where I was, I have considered the case of loading alone. How to find out the response of a statically indeterminate truss for loading alone? We have done that for flexible support, we have done that for lack of fit; we have done that. Now, we have to consider another aspect; if we consider that aspect, we will have considered all the cases that could possibly arise in a truss.

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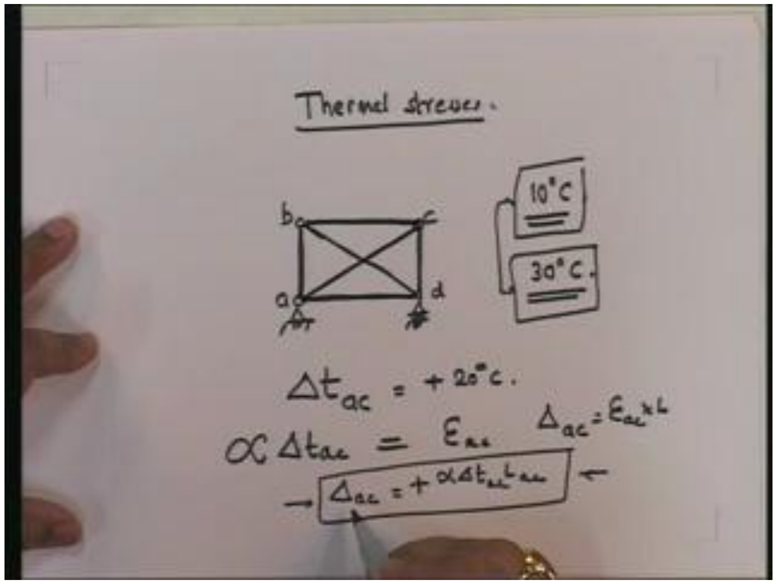


That is called as thermal stress. What are thermal stresses? Thermal stresses are forces developed in the structure due to a temperature variation in the structure. Let us take this situation that I put together. In this structure, there is no loading; all the members are perfect. In other words in this particular case, member ab and cd are all 3 meters, ac and bd are 5 meters, bc and ad are 4 meters; perfect. We have put it all together; we have constructed this truss; we have prepared the supports and put the truss up. Now, when we built this truss during the night, the temperature was 10 degree C; we put the truss up; and in the middle of the day, the temperature is 30 degrees. What effect is this going to have?

First of all, there is this aspect that these are ambient temperatures. There is this entire concept on how ambient temperatures actually give rise to temperature distributions in the structure. I am not going to go into how to treat ambient temperatures and go to obtaining what are the actual temperature variations that the members are subjected to. In this case, all I am going to do is I am going to consider that due to whatever ambient temperature, member bd is thermally insulated. What does 'thermally insulated' mean? It means that no temperature change has any effect on member ab. Member bc is also thermally insulated, member ab is thermally insulated, member cd is thermally insulated, member ad is thermally insulated. Unfortunately, member ac is not thermally insulated and it is seeing this additional 20 degrees change. What happens? That means that the $delta_t$ for member ac is plus 20 degrees.

What happens in this case? Think of what happens when a member is subjected to an increased temperature. What happens? The member elongates. How much does it elongate by? That is given by the coefficient of thermal expansion. What is the coefficient of thermal expansion? The coefficient of thermal expansion is alpha. Alpha is the strain that the member is subjected to due to a 1 degree rise in the temperature. Alpha is the strain in the member due to 1 degree rise in the temperature, the additional strain, axial strain. (Refer Slide Text: 50:44) This into the total temperature rise will give me the strain in member ac. What is the deformation on member ac? It is the strain into L. In other words, $delta_{ac}$ is alpha $delta_{ac}$ $L_{ac}$.

What does thermal stress do? Thermal stress leads to an elongation, a real elongation of the structure. How will I solve this problem? Do you see any difference between this and the lack of fit? None. Again, the real deformation in the member due to thermal stresses gives rise to $delta_{ac}$, which I can compute given alpha, $delta_{ac}$ and $L_{ac}$. Once I have this problem, you can treat this problem exactly as we have done the lack of fit problem. That in essence is your analysis of a truss for flexible support, lack of fit as well as thermal stresses.

Thank you.