Structural Analysis - II

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Lecture – 05

Good morning. In the last lecture, we looked at the application of the force method to analyze statically indeterminate truss. It was a simple truss with a single degree of indeterminacy; as we go along, we will run into more and more complicated problems. Therefore, I am always going to start with a simple problem. Last time, we looked at a simple problem at how to use the force method and the basic concept of the force method is what I established. Let us write that down again. What was the basic idea behind the force method?

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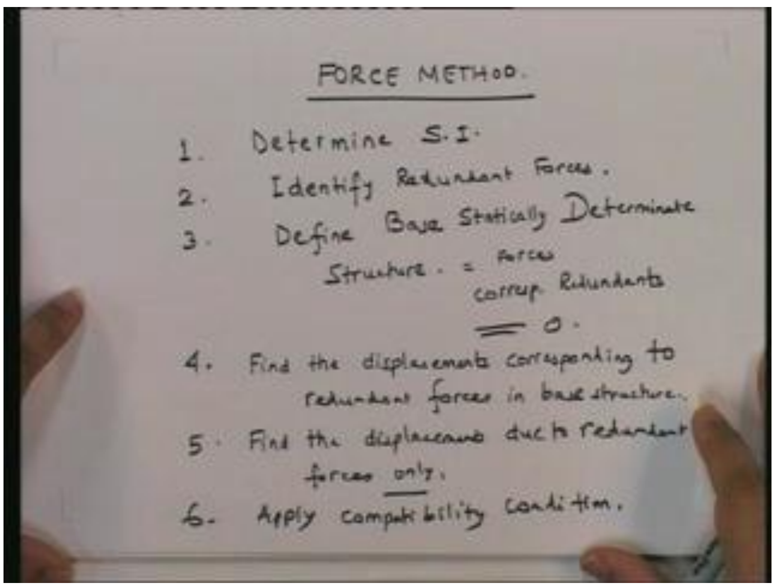

The basic idea behind the force method can be written down in the following manner: one, determine static indeterminacy; two, identify redundant forces. Now, how many redundant forces? Whatever is your static indeterminacy, you need to identify that many redundant forces. What do you mean by redundant forces? In other words, we are going to define a base statically determinate structure in which the forces corresponding to the redundants is equal to 0. That is the reason why we identify the redundant forces, so that we can define a base static determinate structure in which all the forces corresponding to the redundants is equal to 0. What is the next step? The next step is to find the displacements corresponding to redundant forces in the base structure. Define the base statically determinate structure; find the displacements corresponding to the redundant forces in the base structure. What you do is, you define the base statically determinate structure, find the displacements corresponding to the redundant forces in the base structure, find the displacements due to redundant forces only and step six – apply compatibility condition. These six steps totally define the force method and that is what we have done in the previous lecture. If you remember, let me quickly review the previous lecture.

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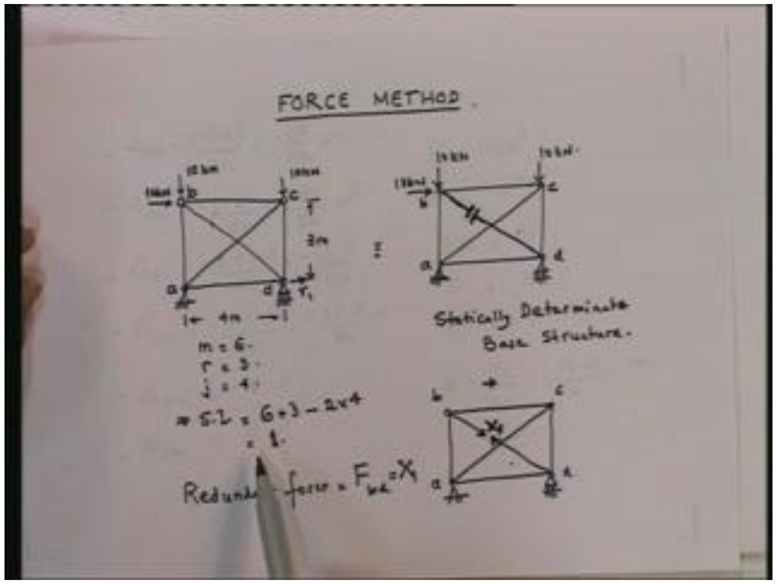

This was our base structure. We identified the static indeterminacy and we identified that the redundant force was the force in member bd, which we called as X_1. Therefore, this was the base structure where we had the loads and the force on bd was equal to 0. Then, we added the structure where the force due to X_1 was given and we found out the forces; we essentially found the displacements corresponding to the redundant forces in the base structure. We found out the displacement here due to these forces; find the displacements due to redundant forces only. We found out the displacements in this structure due to the redundant force and then we applied the compatibility condition (Refer Slide Time: 06:27). What was the compatibility condition?

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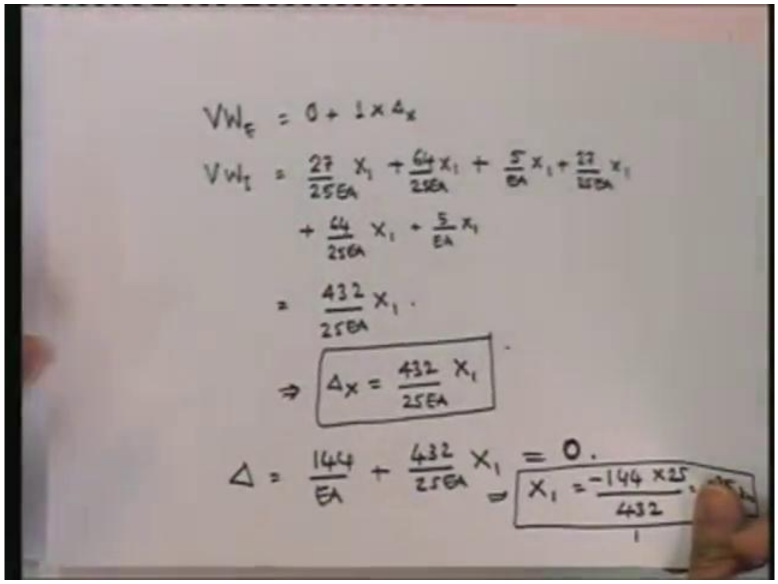

The compatibility condition was that the actual delta due to the loads alone, due to the redundant, was equal to 0. From that, we solved and we got the redundant forces. That is the force method.
In this lecture, I am going to be applying the force method to a beam. Understand that you have to be able to find out the displacements; to find out the displacements in this structural analysis that I am going to be doing, remember I said to find displacements I will always use the virtual force method. Therefore, the whole process over here is based on the virtual work principle. Remember I said that I was consistently going to use only virtual work. Where do we use virtual work? We use virtual work to find out the displacements. That is the overall concept that I am going to be using over and over again.

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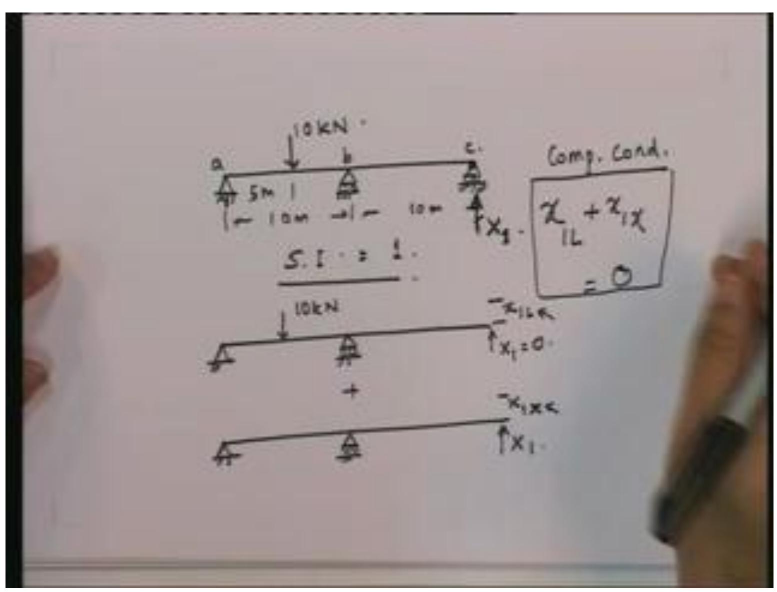

Let us look at a statically indeterminate beam. Let me put a loading here; let us say that these are 10 meters and this load is 10 Kilonewtons. This is a two-span beam; each span is 10 meters and the load is applied at the center of the span; this is 5 meters; this is a, b, c. Think of it as a two-span bridge supported on two abutments and a central pier. We have a static load of 10 Kilonewtons applied at the center of span ab and we have to find out the bending moment diagram for this structure. That is what we have to do in this particular case. We have to find out the bending moment diagram for this.

What is the static indeterminacy of this? We have done this over and over again. The static indeterminacy, if you look at it, there are four reactions, three equations; so static indeterminacy is 1. What is the base structure? I will make this the base structure. I will identify this force as my redundant. Since it is only single indeterminacy, I need to identify one and I will take this one. You could have taken any one of them; it is up to you. The only thing is that you cannot take a redundant that will make the structure unstable; that is all. You have to maintain the structural stability; as long as your structural stability is maintained, it does not matter how you define it. You could have taken this (Refer Slide Time: 10:24), you could have taken this, you could have taken this, any one of the verticals, reactions could have been taken as redundant. You can do that; it is not a problem.

However, in this particular case, I opt to take the reaction at c as my redundant. Therefore, what is my base structure? My base structure becomes this because this $X_1$is equal to 0. Remember that in the base structure, this reaction X_1 is equal to 0; of course in this I have the load. This is equal to this is equal to this plus this (Refer Slide Time: 11:24). Do you agree that this plus this is equal to this? It is obvious. Therefore, what are the steps here? Understand that this is the redundant; so, what do I have to do? In this case, I have to find out the displacement X_1 due to the loading; then, I need to find out X_1 due to the redundant. What is the actual X_1 in this particular case?

Since the displacement of this point is vertically restrained and is going to be equal to 0, all we are saying is that our compatibility condition is X_{1L} plus X_{1X} is equal to 0; this is our compatibility condition. The whole point over here is to find out these two displacements. Once we find out these two displacements then we add them equal to 0 and we get our equation. That single equation we can then solve for X_1; so, let us go back. We are going to now start solving the problem. Let us now solve this problem.

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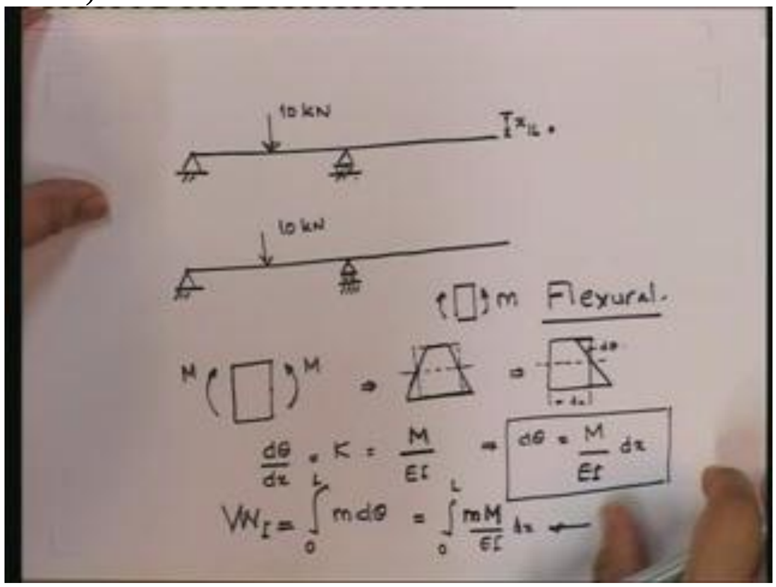

Let us look at the base structure. The objective of this problem is to find out the vertical displacement of this point – that is the whole idea behind this. This is a statically determinate structure and you have to find out the displacement. Simple; I am going to be using the principle of virtual force. Since I am finding out displacement, I have to use the principle of virtual force. What are the steps that I go through? Note that the basic assumption over here is that only flexural deformations; in other words, only flexure is considered, shear deformations are neglected. Now you know how to compute flexural deformations using the virtual work principle. We will go through it step by step.

When I have a member which is subjected to a moment, what happens under this effect? Under this effect, this structure, plane sections remaining plane, it goes something like this; at the neutral access, it remains the same dimension as it was here (Refer Slide Time: 14:53). The top part compresses and the bottom part expands; the plane section, this section, remains plane; original is this. After deformation, it becomes something like this; so, plane section is remaining plane, the simple beam hypothesis.

What we are interested in essentially is the movement of this plane relative to this plane; we can actually draw it in this fashion. This plane has not moved; only this plane has moved (Refer Slide Time: 15:42). Deformation is all relative; this is what we are interested in. If you look at this, what is the curvature? By definition, curvature is the rate of change of theta per unit length. Curvature in a lot of places is given as kappa. In simple beam theory, this is given by M upon EI; curvature is given as M upon EI; that is your curvature. Therefore, if my curvature is this, what is this? (Refer Slide Time: 16:38) This is length dx. What is d theta? d theta is given by M upon EI into dx. Given an M and given an EI value, the d theta is equal to M upon EI into dx. That is the real deformation due to an applied moment M.

If you want to find out the virtual work done by this deformation when undergoing a virtual moment small m, then the total virtual work done, internal virtual work done over this infinitesimal area is going to be equal to small m into d theta which is for the infinitesimal length dx; that integrated over the entire length is going to be my work done (Refer Slide Time: 18:01). Note that since m is in this direction, theta is in this direction, they are positive and d theta is given this way so this becomes 0 by L mM upon EI into dx. The internal virtual work done due to flexure is given by the moment at a particular point multiplied by the real curvature, integrated over the whole length.

If we do that, what do we have to do then? If we are using the virtual force principle, we find out the bending moment due to the actual load, we find out the bending moment due to the virtual force applied corresponding to the displacement that you are interested in finding out. Then, you are going to integrate it over the whole length and that will give you the work done; that you can equate to whatever external work that you need to do. This is due to flexural deformations only and in a beam, we only consider flexural deformations; it is fundamental. What we need to do here is find out the displacement in this particular case. To find out this displacement, what we do is find out the bending moment due to this force (Refer Slide Time: 19:43). Then, we find out the bending moment diagram due to…

We are trying to find out this displacement and because we are trying to find out this displacement (Refer Slide Time: 20:00), you have to apply a virtual force corresponding to this displacement. Once you find out the force due to this displacement, you need to apply a virtual force, find out the bending moment due to that; then, you need to go through the virtual force method.

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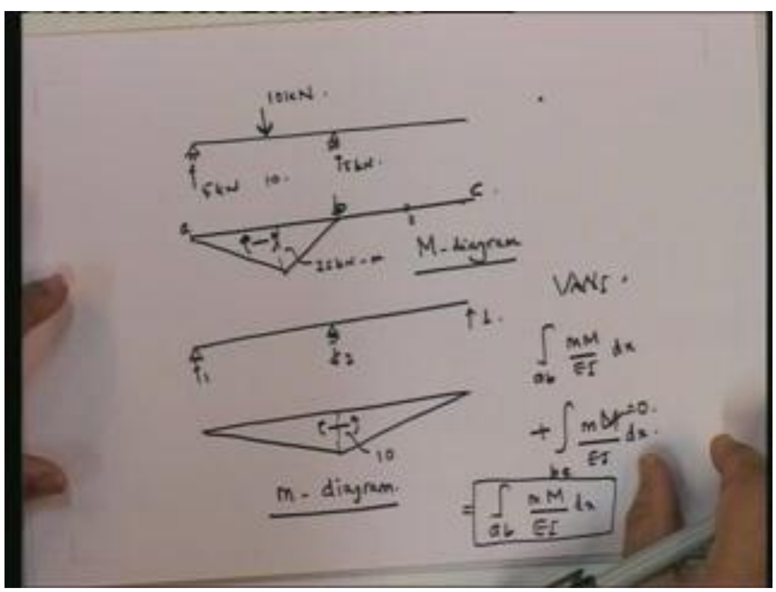

Let us do this process – find out the bending moment due to the applied load. The applied load is 10 Kilonewtons. How do you find out the bending moment? In this particular course, I am not going to actually go through all the steps of finding out bending moment, shear force, axial force diagrams; never. I am going to assume that you know all of this. If you find it difficult understanding how I am doing all these – finding out the bending moments and shear force diagrams – I recommend that you go back to your first course in structural analysis and review how to get bending moment diagrams. During this lecture, I am going to be just going through the steps as if you know how to find out the bending moment diagram; I am not going to go through the steps of finding out the bending moment diagram.

Here, 10; this is going to be 5, 5 (Refer Slide Time: 21:46); and the bending moment diagram is going to look like this. Note that this is 10 meters; this is going to be equal to 25 Kilonewton meter and the sense of curvature is going to be in this way. It is my point never to say 'positive moment' or 'negative moment'; I always draw the bending moment diagram and then show the sense of the curvature, so that you know what kind of moments are being applied at that particular point. This is the bending moment diagram for your real load.

What is the next one? We need to find out the bending moment diagram for a virtual load because we need to find out the displacement at this point. When you find this out, you will see that this is equal to 1, this is equal to 2 and the bending moment diagram is going to look like this. This is my M diagram; this is my small m diagram. Now, the internal work done is going to be equal to integral over ab small mM dx plus integral over small m capital M upon EI into dx. We will assume that the EI for both spans are the same, but if you look at bc, what is M in bc? M is equal to 0. This essentially becomes integral; since this M is equal to 0, this becomes only over ab – this integral (Refer Slide Time: 24:31). You already know how to do these integrals when you have straight lines; you have already done this in your solid mechanics course.

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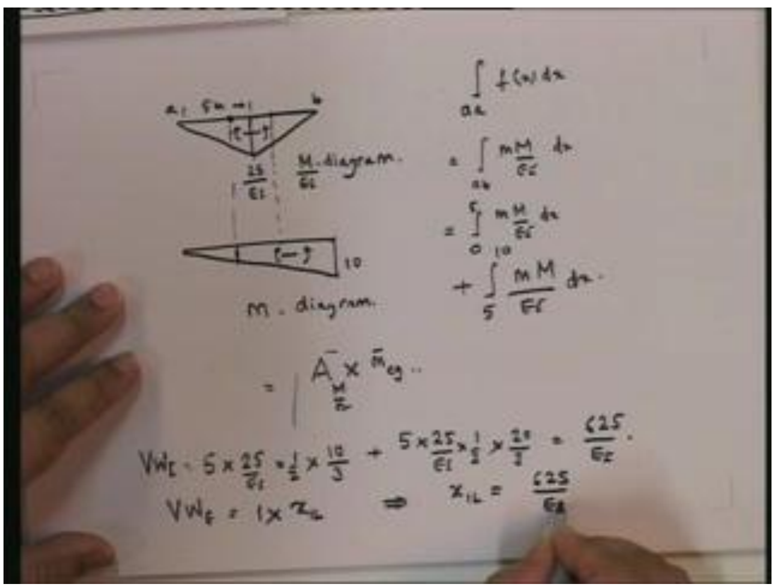

In this particular case, this is going to be 25; this is over ab. Since we are only integrating over ab, this is my capital M diagram; this is at 5 meters; then, I have my small m diagram. One way of doing this is finding out the expression from a to midpoint, then another expression for M from midpoint to b and then taking this also as a function of X, integrating from 0 to 10 and you can actually get it analytically.

I am going to use the simpler method which says that the area under this curve…. Look at this – this is essentially an area under the curve. When you are doing integration of any function over dx, what are you doing? You are actually finding out the area under this fx curve (Refer Slide Time: 26:22).

Now, this I am going to divide by EI; so, I will call it the M upon EI diagram. If you look at this M upon EI diagram, this is the small m diagram and what you are doing is, over m, M upon EI which is.... This multiplied by this into dx is equal to the area under any one of these curves.

Note, of course, that this has to be separated from 0 to 5 because the expression for m is different. It is going to be this plus 5 to 10 mM upon EI into dx. This part and this part have to be done separately; this integral I am going to use is given by the area under any one of these curves multiplied by the …. Let us say for example the area under the M upon EI curve, multiplied by the m bar value, the m value at the cg of this area. This is how integration is done and if we use that, you will see that the m upon EI is given in this particular case. This is 0 to upon 25; the area is 5 into 25 upon EI into half length into base, multiplied by cg; cg is two-thirds of the distance from here (Refer Slide Time: 28:27); that is at 10 by 3. At 10 by 3, what is the value? Over here, the value is going to be 10 upon 3; that is for this one.

Next, let us take this area. Note that both are of the same sense; that is why it is positive. Similarly, if you take this one (Refer Slide Time: 29:00), that is also going to be 5 into 25 upon EI into half multiplied by cg. The cg is at this point and that is 10 by 3 from here; this value will be 20 by 3. If you add all of them up, you will see that this is equal to… 10 by 3, 10, 5; so, you are going to get 625 upon EI. That is the work done by the internal flexural deformations. What is the work done by the external? External is equal to 1 into x_{1L}; that implies that x_{1L} is equal to 625 upon EI. What does that mean? This means that due to this load (Refer Slide Time: 30:14), this displacement is equal to 625 upon EI.

If you look back at the original problem, we have found this out (Refer Slide Time: 30:28). The next step is to find this out. How do you find that out? Simple. Due to this load, we find out the bending moment diagram; then we apply a unit load here and find out the virtual bending moment diagram. Note that all of it involves this load itself for which we have already found out the bending moment diagram. I am going to now utilize that.

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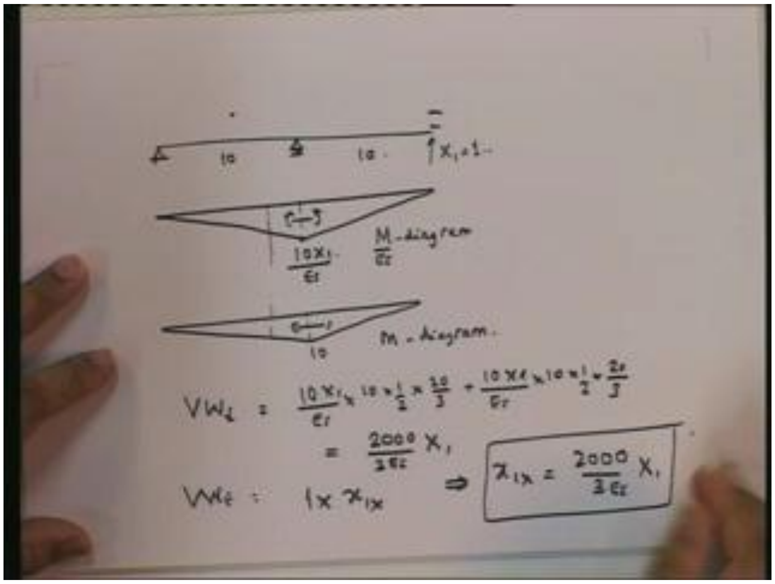

I am going to apply X_1 is equal to 1, find out the work done and then the total displacement is going to be multiplied by X_1, the unknown. For this, we have already got our bending moment diagram; we have already drawn it. It is like this and this is equal to 10. Again, this is the real capital M diagram; the M diagram will have 10 into X_1 upon EI. That is the bending moment M upon EI diagram for the actual load X_1. Now, for the unit load X_1, which is my small m diagram, this is going to be equal to 10. This is my small m diagram because I am trying to find out the displacement at the same point.

Once I have that, I am just going to go ahead and calculate the internal virtual work. Internal virtual work is nothing but … This is 10 meters, this is 10 meters; so the internal virtual work is $10X_1$ upon EI into 10 into half multiplied by the value at this cross section, which is equal to 20 by 3. Again, we do the same thing on this side. You will see that you will get the same $10X_1$ upon EI into 10; that is the area and this is the moment at cg. What you get here is 2000 by 3EI into X_1 – that is the internal virtual work. What is the external virtual work? You have already got it – 1 into X_{1X}. This implies that X_{1X} is equal to 2000 upon 3EI into X_1. Therefore, we have computed this and now we apply the compatibility condition.

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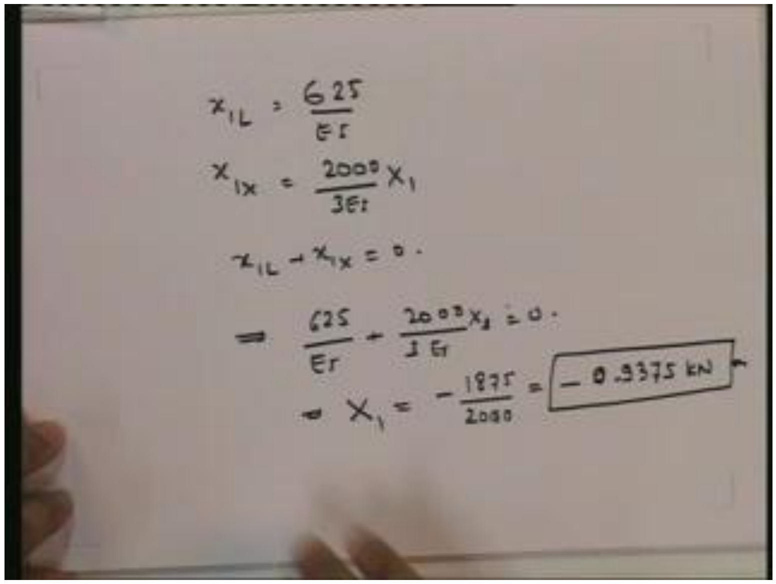

What is the compatibility condition going to give us? X_{1L} is equal to 625 upon EI, X_{1X} is equal to 2000 upon 3EI into X_1, compatibility condition x_{1L} plus x_{1X} is equal to 0. This implies that 625 by EI plus 2000 upon 3EI into X_1 is equal to 0. This implies that X_1 is equal to minus 1875 upon 2000, which is equal to minus 0.9375 Kilonewtons. Once I have this, note something very interesting. How do I get the bending moment diagram for the entire thing?

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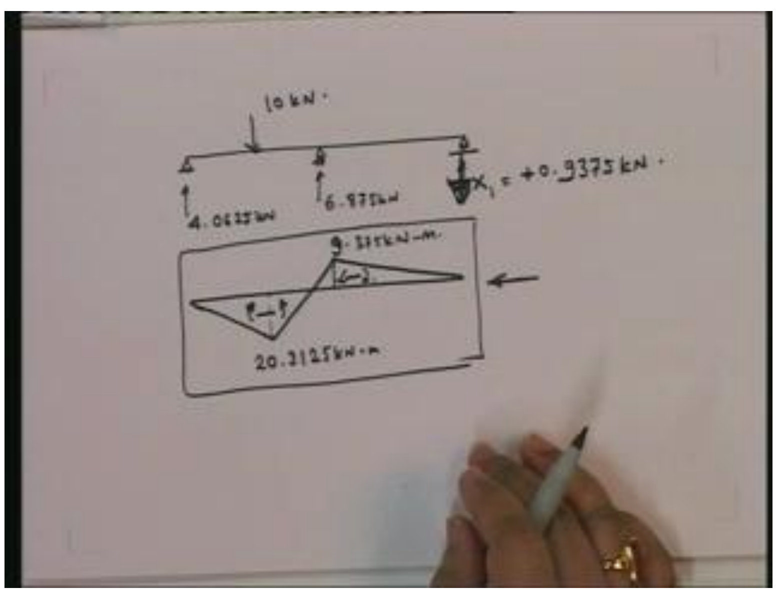

Note that once I have X_1, now I have this structure. I now know X_1 is equal to minus 0.9375 Kilonewtons. I have this load. Now, can I find out my reactions? Of course, I can find out; I can find out everything; I can draw the bending moment diagram. Actually, let me draw this. Due to this load, it is going to be 5, 5 here. When this is minus, it essentially means that the load is in this direction plus (Refer Slide Time: 36:16).

The load at this point is acting downwards; hat makes sense because when you do this, this tries to pull it out and this load acts downwards to pull it back there; that is what this means. Therefore, this is going to be equal to 1.875; this is going to be 6.875 Kilonewtons; this is going to be 4.0625 Kilonewtons. This going to be the total loading that you have, reactions; once you have the reactions, you can of course draw the bending moment diagram.

You will see that in this particular case, the bending moment diagram will look like this, with this equal to 9.375 Kilonewton meters at this point (Refer Slide Time: 37:21). The value over here is going to be equal to this into 5. That is going to be equal to 20.3125 Kilonewton meters; this is going to be in this way; this is going to be sagging and this part is going to be hogging. You have got the bending moment diagram using the force method.

I know that you probably have studied the force method to begin with. However, I have spent two lectures looking at the basic concepts of the force method just to kind of review if you have studied; if you have not studied, then there is sufficient detail in the last two lectures that I have talked about, to give you an overview of how to use the force method to analyze a structure. When you have flexural deformations, the basic concept remains this – if you have the statically indeterminate structure, define redundant forces. Once you have redundant forces, the base structure has redundant forces equal to 0. Find out the displacement corresponding to the redundant force in the base structure, apply this redundant force, find out the displacement of that particular point and then superpose the two displacements, because after all, the two structures together form the actual original structure.

Superpose the two displacements and find out what the displacement in the real structure is and apply the compatibility condition to find out the value of the redundant force. Once you have found out the value of the redundant force, you have essentially a statically determinate structure for which you can find out the axial force in the members in a truss or the bending moment diagram and shear force diagram for a beam structure. Now, I would like to spend a little bit of time on how to take this forward for a frame. I will just spend 10 minutes talking to you about how to use this for a frame. Let us take a simple frame.

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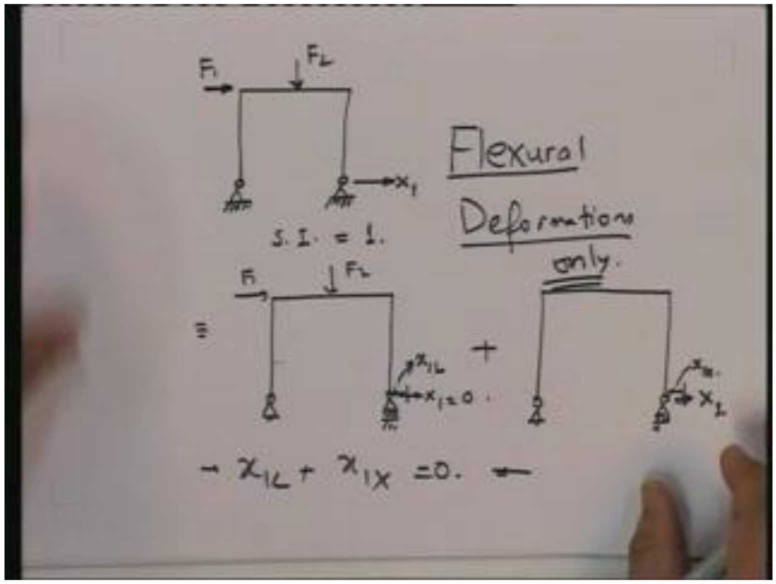

I am not going to solve this problem today because during the next few weeks, we are going to solve enough number of problems; I want to just introduce the concept to you. Let us say that I have gravity loading and some wind loading on this structure. What is the redundancy of this structure? If you look at it, you will get static indeterminacy is 1.

Which one would I take in this particular case? You can take any one of them; it does not matter; but, I will take this as my redundant, in which case my base structure automatically becomes this, because this is equal to 0 in the base structure. Let me say F_1, F_2; so, I have F_2, F_1. This is equal to this plus this (Refer Slide Time: 41:59). Therefore, how do we solve the problem? Again in this case, due to these loads, we find out what is the displacement corresponding to the redundant. Since this is free to move, it will move. Then, we find out due to this how much this moves by; that is x_{1X}. In the real structure, how much does this point move by? Nothing; so, x_{1L} plus x_{1X} is equal to 0.

The point I would like to make here is how do I find out this x_{1L} and x_{1X} in a frame? We have already done that for a beam. In a frame, how do we do it? Understand that the only difference between a beam and a frame is that it has a different way of computing bending moment diagrams; that is all. Otherwise, even in the frame we only consider flexural deformations. Why? We neglect axial deformations, they are very small; we also neglect shear deformations because they are small.

After a few lectures, I am going to actually look at the relative value of the axial deformations and shear deformations in normal situations and show to you that in normal situations, you can neglect axial and shear deformations; but in some specific kinds of situations, you may not be able to not consider axial and shear deformations. In those cases, you might have to consider all three; but then, this is the beauty of the virtual work method because all you need to do is find out the work done due to the flexural deformations, find out the work done due to the shear deformations and find out the work done due to axial deformations. Remember work is a scalar;

you can add up all the work done by all the deformations and get the total work done. Let us look at the flexural deformations in the frame; I am just going to go one step. Let us look at what it is that we are interested in; we are interested in finding out this displacement due to these loads (Refer Slide Time: 44:51) and this displacement due to this load. Let us see what happens.

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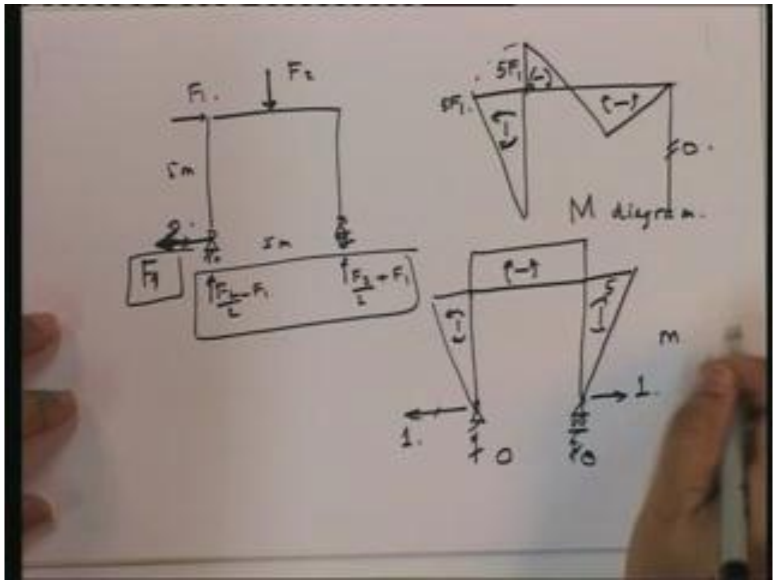

This is a statically determinate frame; you should be able to find out the bending moment diagram for this. How would you find out the bending moment? Find out these reactions. This reaction sigma F_x is going to be equal to F_1 and this, due to this load, is going to be F_2 by 2, F_2 by 2. Due to F_1, let us say that this is 5 meters and so is this (Refer Slide Time: 45:40). Then, this will become plusF_1 and this will become minus $F_{1.}$ These are the reactions; this is this (Refer Slide Time: 45:54). How will the bending moment diagram look? It is going to look in this fashion. I am going to draw it any which way I want because I am always going to show the sense; the sense is going to be in this particular case in this fashion.

What is the value over here? It is going to be equal to $5F_{1.}$ What is the value over here by the way of continuity? You have to have $5F_1$. What is going to happen over here? That depends on the relative values. What is the bending moment in this section? Note that there is no shear; if there is no shear, it is here. Therefore, in this particular case, it will probably go something like this and this. Here, it is going to be in this fashion; here, it is going to be in this fashion (Refer Slide Time: 46:47). The particular values will depend but you have got your M diagram for the frame, for the given loading.

Now what is your small m? Put unit load here and find out the bending moment diagram. Note that in this particular case, this is going to be 0, this is going to be 0 and this is going to be equal to 1. If you look at the bending moment diagram for the small m diagram, it is going to be 5, 5, 5 with this in this fashion; this is going to be in this fashion and this is going to be in this fashion. That is your small m diagram. Then, you follow the same procedure to compute mM upon EI dx for each member, add them up and you have your total work done and the external work is going to be 1 into x_{1L}; from that, you can find out your x_{1L}.

The basic point is that there is no difference between a beam and a frame as long as you consider only flexural deformations. The only thing that happens is that in a beam, everything is along one line. In a frame, it is spaced across the plane; so, you have a difference. The only thing is that the equilibrium conditions look different. Otherwise, once you have the bending moment diagrams, whether it is a beam or whether it is a frame, it does not really matter. Therefore, beams and frames are really going to be the same problem and I am not going to treat them in any different way. It is only that the equilibrium conditions give rise to different sets of equations in a frame as opposed to a beam; there is nothing else.

For example, in a beam you may not have horizontal forces at a hinge because you normally only have vertical forces on a beam, whereas in a frame you have both gravity loads as well as horizontal environmental loads due to wind or earthquake. Therefore, you would have horizontal forces, reactions at hinges; it actually does not matter one way or the other. After all, if you know how to compute the bending moment diagram for a beam or a frame, that is all that is required for you to be able to successfully apply the force method. We shall continue solving problems etc; but, I want to now introduce the fact that if you look at it, you see this (Refer Slide Time: 50:28). I have a situation where I have a reaction and the displacement corresponding to the reaction is always 0 because it is fully restrained.

Next lecture onwards, I am going to look at a situation where you have a partial restraint. In other words, you can develop a reaction, but the displacement in the structure need not necessarily be equal to 0. How would you treat those kinds of problems? Those are essentially flexible support conditions that you are aware of. Next time, we are going to look at something new – flexible support. We are going to look at other kinds of situations – what happens if a member is not exactly the size that in its geometry it is supposed to be; that is called a 'lack of fit'. Then, you may have temperature stresses, erection stresses. How do you analyze the structure for all those kinds of situations? We shall see how we can apply the force method for all those kinds of situations over the next few lectures.

Thank you very much and look forward to seeing you in the next lecture.