Structural Analysis – II

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Lecture – 40

Good Morning. Today we are going to be looking at influence lines and its applications.

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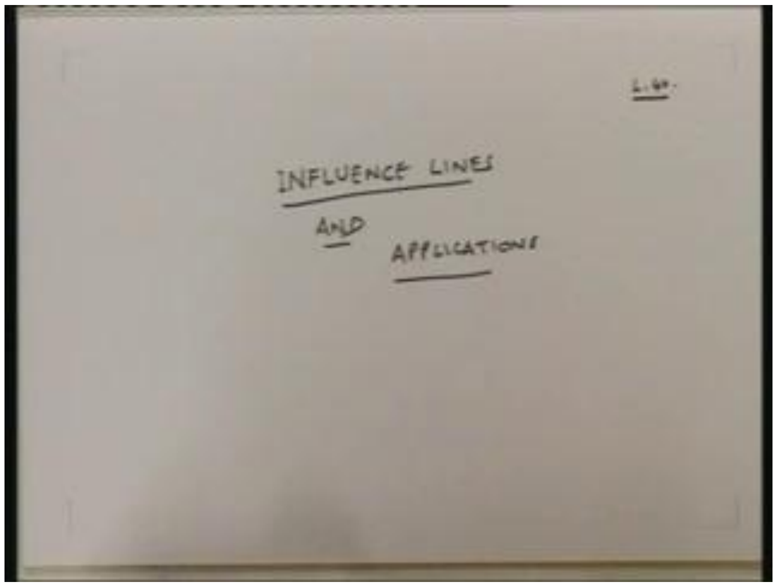

Last time I had given you a problem and I had given you the answers. Today we are going to take up that same problem and work through it. It will be the last problem I will look at and then I will look at how to apply it. We will see how to apply it. Hopefully at the end of by this lecture we will know what to do with influence lines. We are going to be looking specifically at the applications. Let us go back to the problem that we were looking at.

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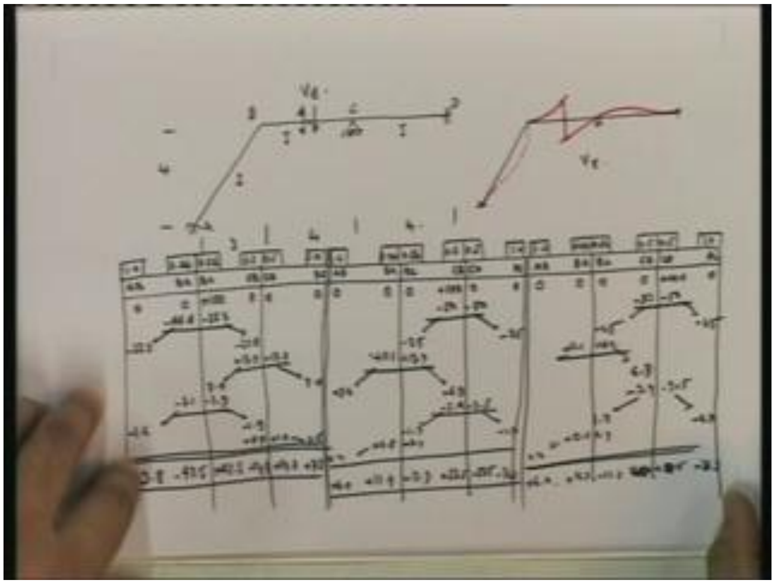

This is A, this is B, this is C and this is D. We know the qualitative. I am only looking at the quantitative. The quantitative is the center point of the V_E. For that this is I, this is I and this is I, this is 3 and 4 this is 4 and this is 4. The vertical load goes from B C D. If we look at it what is the first step? By the way we have drawn the influence line for this. I just put it together for you. If you remember this one - this theta and this theta are the same and this one of course will go like this. This is the influence line for E. As the load proceeds from B to C to D let us see. When the load is between B and C you set up a fixed end moment at BC and CB and then when you go from C to D you set up a bending moment at C and D.

We have to tackle 3 problems. We are not going to tackle DC because it is a fixed end. The fixed end moment BC is going to land up being there. It is not going to be a moment distribution but we need to do 3 sets of moment distribution and I am going to do all 3 sets together. The first set is here, the second set is here and the third set is here. This is going to be AB. There are 3 of them - BA, BC, CB, CD and then DC. This one again is going to be the same AB, BA, BC, CB, CD and DC and finally the last one AB, BA, BC, CB, CD and DC. In this particular case we are going to first tackle plus 100, 0, 0, 0.

In the second case we are going to handle 0 plus 100 0 0 and the last one we are going to handle 0 0 0 plus 100 and 0. These moment distributions we have to do for fixed end moment at BC CB and CD, the DC we do not need to do that because that is the fixed end that is going to show up directly at DC. The distribution factors both of them are continuous. We are going to have I upon 5 and I upon 4. I upon 5 and I upon 4 is going to give you this is I upon 5 this is I upon 4, this one is going to be I upon 4 I upon 4 so it is going to be point 5, point 5 and this is going to be 1, this is going to be 1. Here same thing 0.44 0.56 this is going to be 0.5, 1 here, 1 here 0.44, 0.56, 0.5, 0.5 and 1.0.

There is going to be 'carry over' in all the directions. We know how this entire thing is going to happen. I am just going to put down some of the numbers. So you know how to look at it minus

44 minus 55, this is cancelled, (Refer Slide Time: 9:20) here you get minus 22.2, here you get minus 27.8, this 27.8 gets distributed 13.9 13.9, this carries over here to 7.0, this is carried over here to 7.0, this gets minus 3.1 minus 3.9, this gets carried over to minus 1.6, this gets carried over to minus 1.9 and then this gets done as plus 0.9 plus 1. We will stop there because we have reached less than 1 percent and close it at this point. This becomes minus 23.8, this becomes minus 47.5, this becomes plus 47.5, this becomes minus 14.9 plus 14.9 and plus 7.5.

Similarly, we do it for this (Refer Slide Time: 10:49). This is going to be minus 50 minus 50 break carry over minus 25 minus 25 gets done with plus 11.1 and plus 13.9. This one comes as plus 6.9, this goes as plus 5.6, this comes out as minus 3.4, minus 3.5, minus 3.5 goes as minus 5.17, we then come over here this is plug this is minus 1.7, this becomes plus 8 plus 0.9, this carries over to plus 4. We close it here and whether this I have just done the so this becomes plus 6 this becomes plus 11.9. This becomes minus 11.9. This becomes plus 53.5. This becomes minus 53.5 and this becomes 26.7. This one goes exactly the same way; minus 50 minus 50 - this goes minus 25 minus 25 plus 11.1 plus 13.9, this carries over to 6.9, this becomes minus 3.4 minus 3.5 carry over 1.37 here 1.7, this carry over this becomes 0.8 and 0.9, this carries over 0.4 close and this becomes plus 6.0 plus 11.9 minus 11.9 minus 53.5 plus 53.5 and this 46.5 (Refer Slide Time: 13:03) and this becomes minus 26.7.

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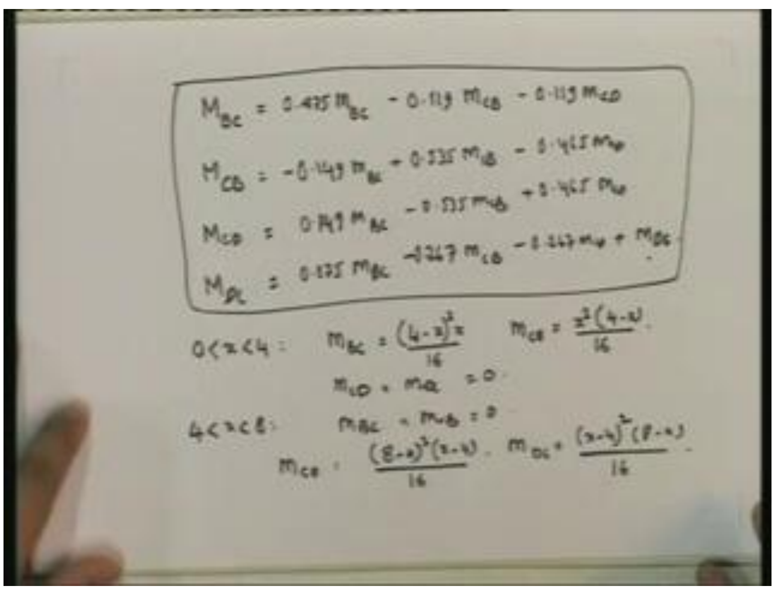

Essentially we are writing down that M_{BC} is equal to… we are not interested in AB and BA because a load does not travel on AB and BA. Those are always going to be equal to 0. We are only interested in the once that whether fixed end moments are non zero. We have 0.475 M_{BC} minus 0.119 M_{CB} and then minus 0.119 $M_{CD.}$ Then we have M_{CB} is equal to minus 0.149 M_{BC} plus 0.535 M_{CB} minus 0.465 M_{CD} and M_{CD} is equal to 0.149 M_{BC} minus 0.535 M_{CB} plus 0.465 M_{CD} and finally M_{DC} is equal to 0.075 M_{BC} minus 0.267 M_{CB} minus 0.267 M_{CD} plus M_{DC} this is the fixed end moment at DC. These are our expressions. When 0 is less than x is less than 4 M_{BC} is equal to 4 minus x squared into x all upon 4 squared. M_{CD} and M_{DC} are equal to 0. When you have between 4 x and 8 M_{BC} and M_{CB} are equal to 0 and M_{CD} is equal to 8 minus x the whole squared into x minus 4 upon 16 and M_{DC} is equal to x minus 4 squared into 8 minus x upon 16. These are the fixed end moments.

What is the next step? Once we have figured out from the moment distribution the relationship between the member end moments and the fixed end moments we also have the expressions for the fixed end moments while the load is traveling from B to C to D. We have to write down all the expressions for it. If we look at it now the only thing that we are doing is V_E. Understand that we are only interested in V_E.

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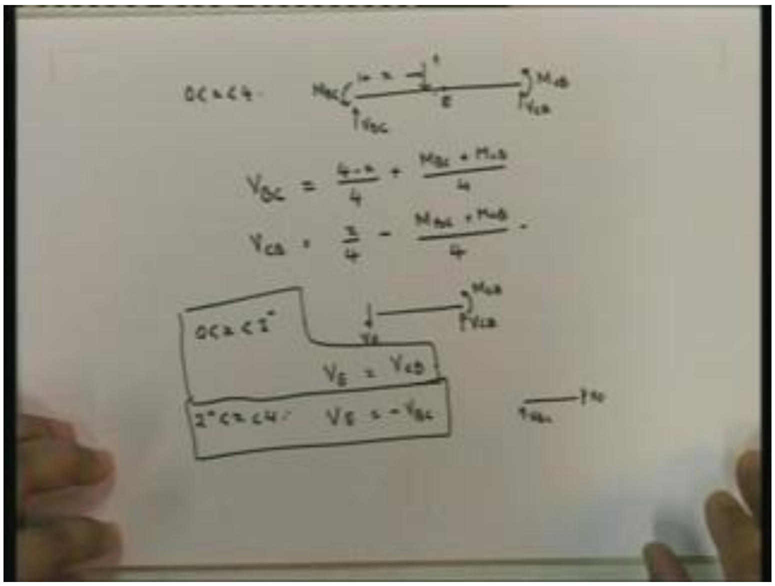

Let us say load is between 0x and 4. One of the things that we do know - let us look at this. This is M_{BC} (Refer Slide Time: 18:24) this is $M_{CB.}$ In addition we have the load 1 at a distance x. Therefore if you look at this one I will call it as V_{BC} and I will call this as $V_{CB.}$ If you look at this in this way V_{BC} is going to be equal to 4 minus x upon 4. From this part and the other part is plus M_{BC} plus M_{CB} upon 4 and V_{CB} is equal to x upon 4 minus M_{BC} plus M_{CB} upon 4. Once we know these now our E point is here, when 0 is between 2 minus that means the load is between B and E. That means the shear force definition will have this and will have this this is $V_{E.}$ V_E is going to be equal to V_{CB} and when the load is between this V_E is equal to we will see that this is V_{BC} and this is the positive V_E then this is going to be minus V so for this V is equal to this and for this - these are the expressions for V_E when the load is between B and C. We now need to just find out what is going to be the $V_{E.}$

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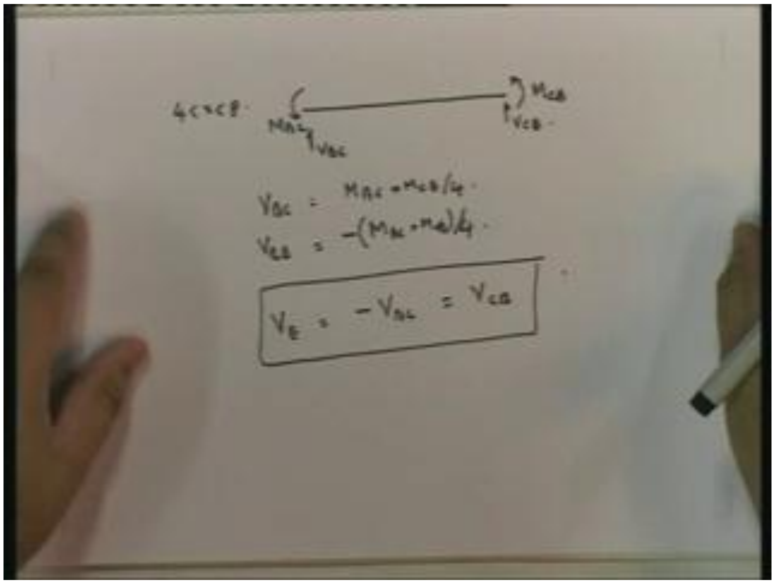

If the load is between 4 and 8 then BC becomes M_{BC} M_{CB} the load is no longer there. We have V_{BC} and V_{CB} and we see that V_{BC} is equal to M_{BC} plus M_{CB} by 4 and V_{CB} is equal to minus of M_{BC} plus M_{CB} by 4. If we look at it then V_E is equal to minus BC or which is also equal to $V_{CB.}$ This is minus and this is minus (Refer Slide Time: 22:23). Now let us look at how to compute. What we do is we compute.

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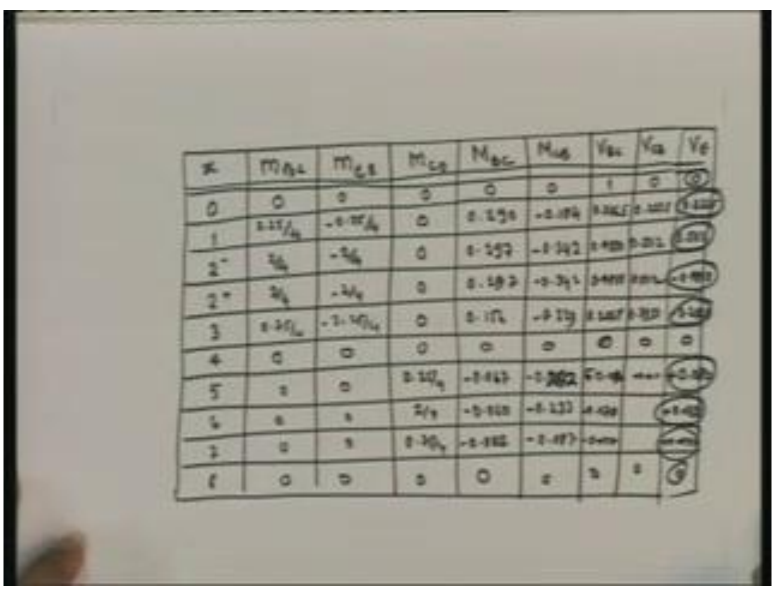

For computation again we draw this. We are going to have x and we are going to have M_{BC} M_{CB} M_{CD} we get small m and capital M, then we get V_C and finally we get $V_{E.}$ This is x, this is the fixed end moment, this is (FEM)$_{cb}$, this is (FEM)$_{cd}$ we do not need anything else for M_{BC} and

M_{CD} because V_E only depends on those V_{BC} V_{CB} and $V_{E.}$ then we put 0 1 2 minus 2 plus 3 4 5 6 7 8. Let us draw these lines. Why 2 minus and 2 plus? When the load is just to the left and just to the right of the E the sheer is different which we already computed.

Having done this just substitute x we get this 0, this 0, this 0, this is 0, this is 0, this is 0, this is 1, this is 0 and this is 0, this is the exponent (Refer Slide Time: 24:39)**.** Then for 1 we compute and this becomes 2.25 by 4, this becomes 0.75 by 4, this is 0 because M_{CD} is 0 when it is 0 up to here up to 4 and M_{BC} and M_{CB} are 0 when between 4 and 8 because, this is also 0, this is 8. When we compute this becomes point 290**,** substituting these M_{CB} becomes minus 0.184 so V_{BC} turns out to be 0.7765, this turns out to be 0.2235 and therefore V_E is 0.2235 because up to 2 minus it is equal to V_{CB}.

Plug this in (Refer Slide Time: 25:49). This becomes 2 by 4, this becomes minus 2 by 4 so when you compute this becomes 0.297, this becomes minus 0.342, this becomes 0.488, this becomes 0.5112, this is now 0.5112. At this point 2 plus all of these are identical because the moments cannot change, V_{BC} also is the same but now at 2 plus V_E is equal to minus of, so this is going to become minus 0.4888 and then this becomes 0.75 by 4. Now you keep putting in these values and you will get this is equal to 0.156, this is equal to minus 0.329, this becomes 0.2068, this becomes 0.7933, this becomes minus 2 0 6 8. At 4 all of these are 0 now here this becomes 2.25. This is 2 by 4 and this is 0.75 by 4.

Substituting those in we get minus 0.067 minus 0.060 minus 0.022. This is of course 0, 0, 0 and 0. Once we substitute these we get minus 0.262. This becomes minus point 0823, this becomes .08 plus 0.823 and this one is minus 0.233. This is minus 0.087. This becomes minus 0.733. This becomes plus 0.733. This one is minus 0.0273. This becomes plus 0.273. These are the numbers that I had done. I just evaluated the procedure for getting these.

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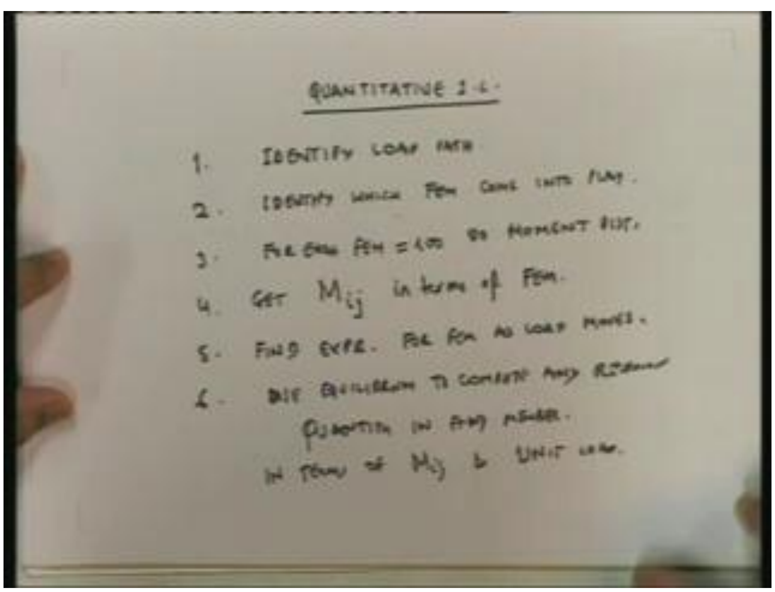

Steps for quantitative influence lines: first identify load path, identify which fixed end moments come into play. When you know the load path you know exactly from which fixed end moments are going to be non zero as the load moves from which ever point to which ever point we wanted to move. Identify which come into play. For each FEM equal to 100 do moment distribution. As many number of points at which you develop fixed end moments you are going to get that many moments to distribute.

For example, in this particular case we had when it was between B and C we had BC and CB operating. When it is between C and D we had CD and DC, DC was the fixed end so we did not do a moment distribution for that but for the other 3 BC CB and CD we have to do the moment distribution. Once you do all of those then you get member end moments M_{ij} in terms of fixed end moments. Find expressions for fixed end moments as load moves.

In other words, for each member that you have, you put it at a point x and you find out the expression for the fixed end moments. Once you get the find out the expression for fixed end moments you get the expressions for M_{ij} as the load moves. Then do equilibrium or should I say use equilibrium to compute any response quantity in any member in terms of M_{ij} and unit load. Once you do that then you have got it and then you can put in different values of x and evaluate whichever response you need at that point and that gives you the influence line for the for the particular response quantity. For example, in that problem that I have just done the response quantity of interest was V_e and so therefore we only obtain the expressions for V_E in terms of M_{BC} and M_{CB} and when the load was in bc also in terms of the load.

When the load moved away from BC and the load did not come in, it was only in terms of M_{BC} and $M_{CB.}$ So much for how to get influenced lines both qualitatively and quantitatively for any structure, be it statically determinate. In the statically determinate structure the Muller Breslau principle gives us both the qualitative as well as quantitative because whenever we remove the restraint corresponding to the response quantity of interest a statically determinate structure becomes a first auto mechanism and all displacements are straight lines. Therefore, we do not need any method we also get the quantitative. When you have a straight line and you know the end you always can find out anything as we have already seen. Therefore, the Muller Breslau principle gives us both the qualitative as well as the quantitative for a statically determinate structure.

For a statically indeterminate structure we can still continue using the Muller Breslau principle but it only gives us the qualitative sense of the influence line. To get a quantitative sense you have to go through the direct approach in which you have to make each member into statically determinate and then solve for it. So much or influence line. Now the question arises - see till now whatever we have done there has been a reason for it. This is the first time I looked at a problem when I say look let us first look at the influence line and then we will see what the applications are.

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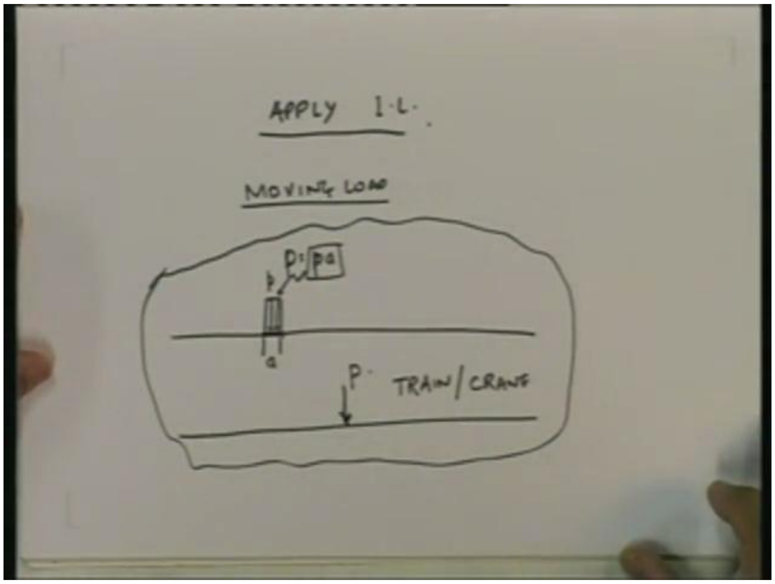

Where do we apply influence line? It is obviously moving load. What kind of moving load can we have? We can have something like this that over some a we have some load intensity which is given uniformed intensity P_a so that total load is P into a. When does this kind of a thing happen? This happens when you have a truck or a car traveling over a bridge. There is an area over which the load is distributed.

On the other hand, you may have a situation like this; effectively point load where you have a load which is P this is capital P this is small p (Refer Slide Time: 35:50). Load is P and it is acting at a particular point when you can have this kind of a thing. This can happen when you have a train on a railway track or a crane. Essentially the load is transferred through a steel wheel which has in a sense a point contact. These are the **two** kinds of moving loads that we have. How do we tackle this? You see the overall point here is this. Let me just take this particular example that we have just done to look at.

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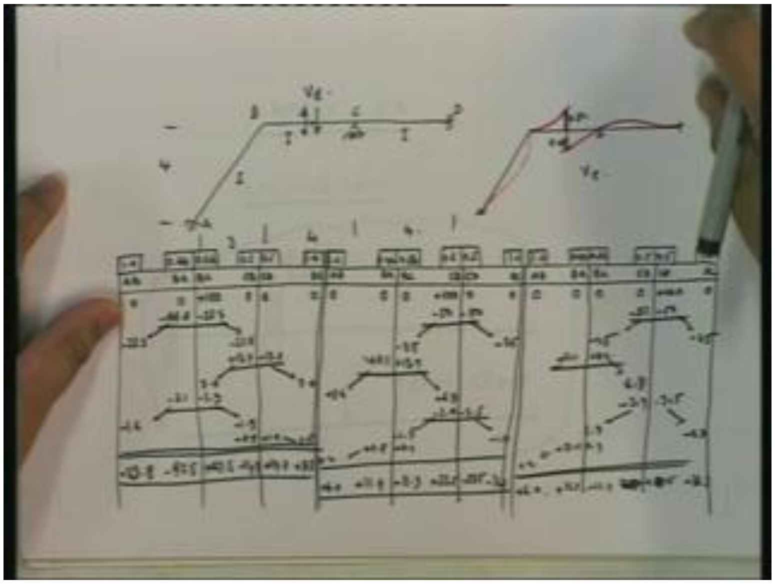

We have this influence line and now we know the values also. This is (Refer Slide Time: 36:45) point 511 and this is point 489 what is this do actually. This tells us to get the maximum value of V_E which is the position of the load we have. This is the position we have.

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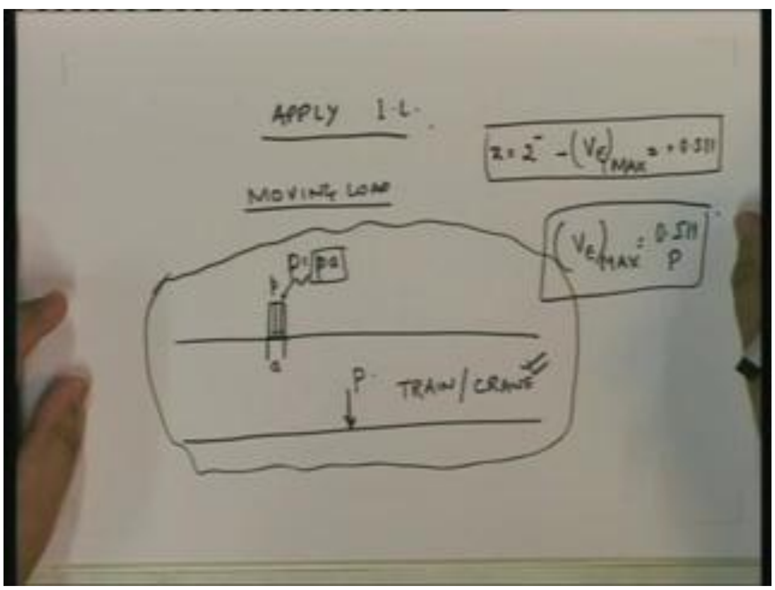

For maximum value of V_E we need to put it just to the left. So, x equal to 2 minus is V_E max and the value for a unit load is equal to plus point 511. When we have this, for this load, this moving we know what the maximum V_E is going to be, we know where the load has to be applied, it has to be applied at this point and once we apply the load at this point you can do analysis and you can get V_E max is equal to 0.51 P.

I am just giving you the example V_E but this you can do for anything, any response quantity. For example, what is the maximum support reaction? We know what the maximum support reaction is. We can also find out the position where it is, we also know what the maximum value is for the unit load. So when you have a train or a crane all you need to do is whatever value you get for the response quantity you multiply it by P. When you are moving a load you can never find out and that is the reason why we have influence line, but now we know for a point load how to apply the influence line to get the maximum response for a moving point load.

What happens when you have something like this? Let me look at V_E.

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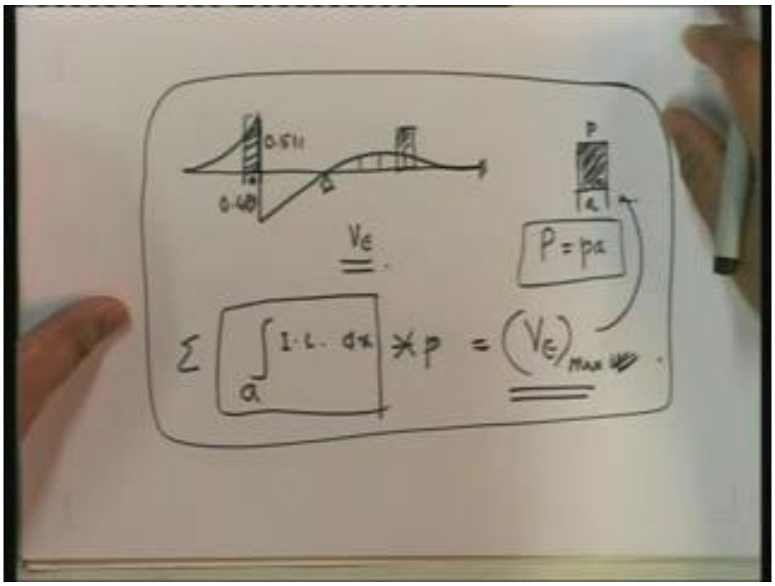

I have V_E, I am illustrating with V_E mind you, it is not if I am choosing, and so this is 0.511 and this is 0.489. This is what we have evaluated then we have evaluated for every point. Now the point here is, if we have a load which has a particular pressure over a particular area. This is a typical class AA loading……… if you look at the Indian Roads Congress code IRC: 6 then you will see that you have given this kind of a load. Of course what you have given is many loads together. The point that I am trying to make is that, once you have for one you can always place it any which way you want because once you have the load for one position you have all the other load positions automatically.

Now the total load P is equal to P_a. How do we solve this problem? Note that all we need to do is find out for this load what the maximum is, we need to find out the position. Which position do you think is going to be the position of the load? Automatically if you look at this you will see that this is the maximum. So automatically one edge of the load should be here and if the load sits like this you will get the maximum influence of the load on V_E. This is a and what we do is we find out the area under this curve. So the area under the influence lines over the distance a when the load is placed at the maximum over the x multiplied by P gives me, this is the pressure so this pressure multiplied by the influence line gives me the pressure at this point and then when we integrate it we get V_E max for this load.

How do we integrate it? We actually do not integrate it. What we do is we discretize it and then numerically integrate it. You got your V_E max. What might happen is that once you place one load the other load may come here, all you need to do is sum up the influence lines. Once you place one load at the maximum the others will come automatically. The distances are fixed, the wheels, axels, loads are fixed. This way we find out V_E max. I am illustrating it with V_E max. Therefore you see now, how for a given moving load how we get the maximum response in this particular $V_{E,}$ we could also do it for R_a R_D M_D M_a anything for this particular problem.

For your own particular problem you can always solve the specific problem and find out its effect. This is the reason why it is actually an analysis problem. We are actually trying to find out the maximum value of a particular response by sheer, the sheer force at a particular point, bending moment at a particular point, the reaction at a particular support, the moment at a fixed support so these are the things that we are interested in. And for whatever we are interested in, we draw the influence line for that. What we always do is that whatever load you have you always place the load where you get the maximum influence. Once you place the load at the maximum influence if it is a point load the value of the influence multiplied by the load directly gives you the maximum value of the response under the moving load and then if you are doing for distributed load like a tyre load then this is the procedure that you use (Refer Slide Time: 44:30).

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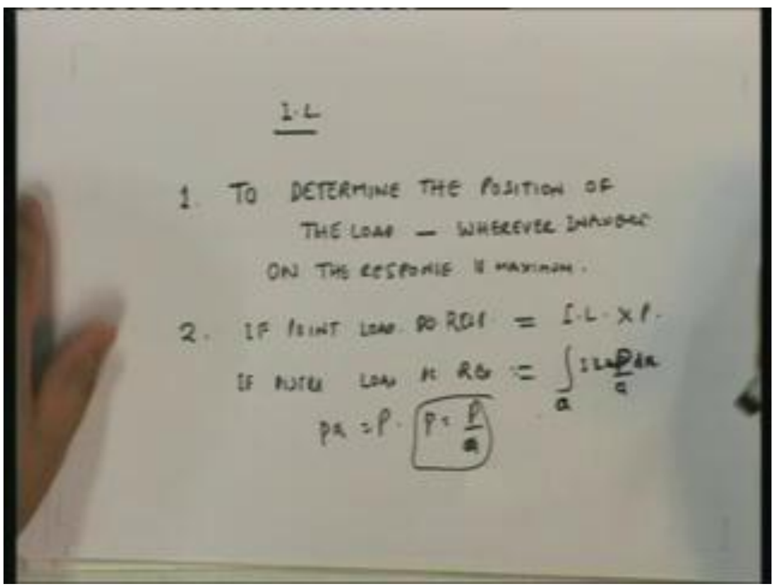

In essence the overall use of influence line is; 1) to determine the position of the load - wherever influence on the response - that is the reason why we call it influence line, the Influence on the response is maximum. Once we determine the position; if point load then design response is equal to influence line into load; if distributed load then design response is equal to integral over the distribution area or length into influence line into pressure into dx where p into a is equal. Basically P is equal to P upon a that is the pressure, therefore here what we do is load upon a multiplied by influence line over the length a you integrate it and that gives you the design response. This in essence is the overall thing.

Just to ensure that I leave you with some problems to do I am going to give you two or three problems to do for your own self. I am going to define one problem, this is the same problem as what I have just done.

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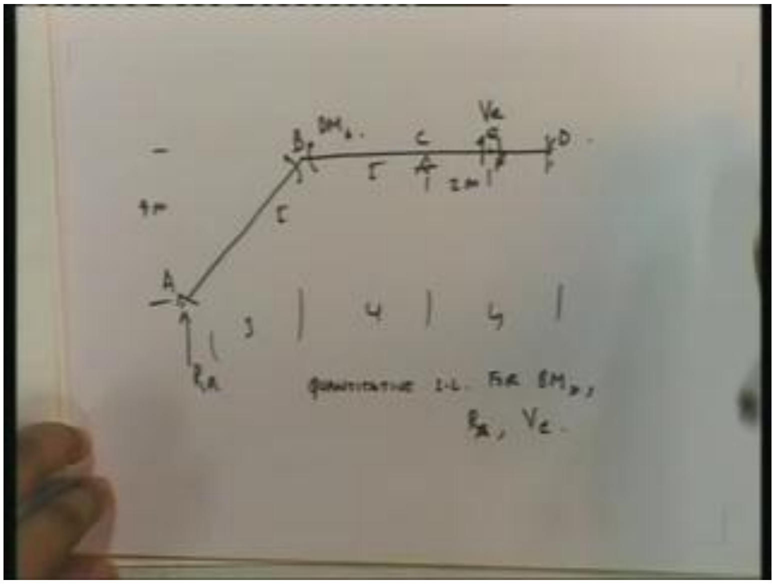

In this problem this is my e point which is 2 meters identical. This is the same problem in fact for M I I I this is 3, this is 4, this is 4 A B C D and I ask you to find out R_A, the sheer V_E and the bending moment at B, quantitative influence line for bending moment at B R at a and V at E. You find these out I have already done this I will give you the answers for your computation.

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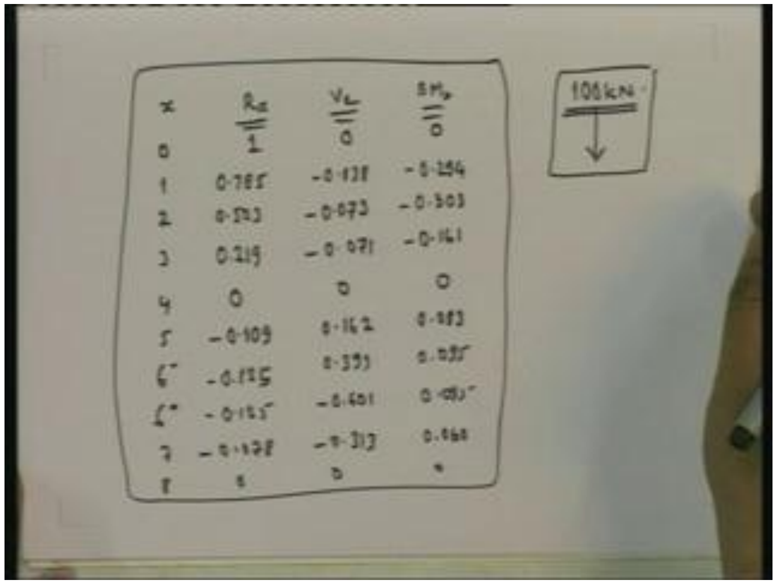

If you look at it this is x 0 1 2 3 4 5 6 minus 6 plus 7 8 and R_a V_E bending moment at b this is going to be 1 0 0. This is going to be 0.7855 minus 0.038 minus 0.294 0.503 minus 0703. This is going to be minus 0 3 0; for 3 it is 0.219 minus 0.71 minus 0.161, for 4 all these are 0 (Refer Slide Time: 49:55)**,** for 5 this is minus 1, this is 0.62 point 0.383. At 6 we have 0.125, this is 0.399 0.95 and for 6 plus these two remain the same, this one is minus 0.061; for 7 this is 0.078 minus 0.313 0.060 ultimately 0 0 0.

I have given you the answers you have to solve this problem yourself. Then I would like you to find out the maximum values for a point load of 100 kilo Newtons.

So find out the maximum value or the designed responses for all these 3 for 100 kilo Newtons.
I am going to stop here I hope that the lectures on the influence line has given you a little bit of an insight on how to obtain influence lines for structures and then how to apply them for actual problems to be solved.

Thank you very much. It has been a pleasure interacting with you. I hope that my lectures on the force method, the displacement method and then the matrix flexibility approach, the matrix displacement method, the moment distribution method and finally the influence line has given you some idea of structural analysis for statically indeterminate structures. Thank you very much. Bye.