Structural Analysis - II
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Lecture - 04

Good morning everybody. In the last three lectures we have looked at the tools that we would require to do structural analysis and that is, namely, finding out the static indeterminacy, the kinematic indeterminacy or degrees of freedom and then we looked at the virtual work method and the two principles that I used to solve various analysis problems, for example, finding out displacements and forces in a structure. Today, we are going to be starting off with what is known as the forced method for analyzing structures. I am going to start off by looking at a truss structure to begin with and then we will move on to beams and later frames. So let us look at a simple truss.

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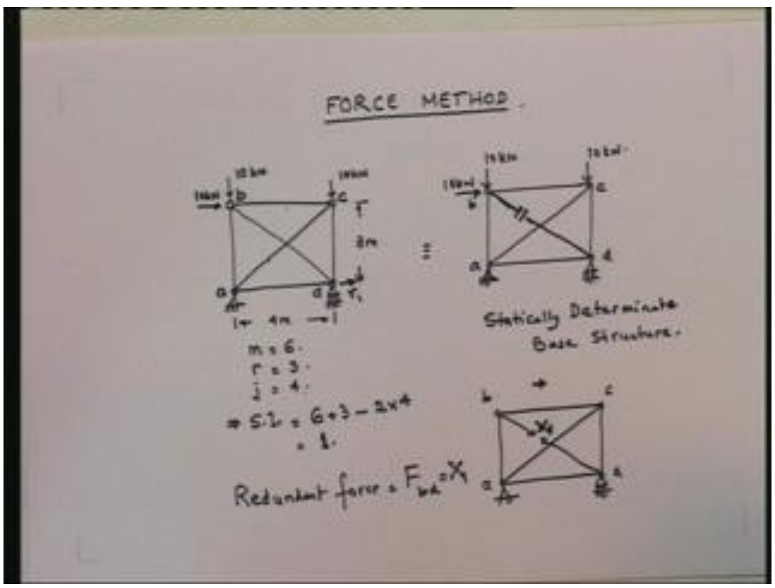

Let us say that the structure is subjected to this kind of loading. This is the loading on the structure (Refer Slide Time: 2:52) and this is the truss structure we have. Understand that in the force method, the starting point is to find out the static indeterminacy of the structure. Let us go through the steps; number of members 1, 2, 3, 4, 5, 6 so 6 members, number of reactions 1, 2, 3; number of joints 1, 2, 3, 4 . Static indeterminacy it is equal to number of unknowns; one for each member, plus the number of reactions minus the number of equations of equilibrium that you have which is 2 per joint and this is equal to 1. So this is a single indeterminate structure. It is a truss which has a single degree of indeterminacy. Now we go into the steps of how to use the force method to analyze. Let us state the problem. The problem is, I want to find out the forces in all the members plus I also want to find out the horizontal displacement of this point under this loading (Refer Slide Time: 4:48). So that's my problem statement: Evaluate the forces in all the members and find out this displacement. We will find out the displacement later, first let us find out all the forces in all the members. Obviously, since it is a statically indeterminate structure you cannot find out all the forces in all the members using just the pure equations of equilibrium since it is statically indeterminate to the first degree what you have is, the number of equations of equilibrium is one less than the number of unknowns that you have. So essentially if we can establish one other independent equation then we can find out all the unknown forces. That is essentially the crux of the force method.

How do we go about finding it out? The first thing that you do in the force method is, you find out the statically determinate base structure. In other words, you take the structure, eliminate one of the unknown forces because it is a single degree of indeterminacy, so you have to eliminate one of the unknown forces then what happens is you have a Statically determinate structure. If you look at it in this particular case we have cut this member. Once we cut this member, we know that the force in that member is zero. How many equations of equilibrium do we have? We still have the same number of joints, so the number of equilibrium equations is still going to be 2 into 4.

Only thing is that the number of members has been reduced by one because one of the members we know is going to have zero force so it becomes a statically determinate structure. Let us take this problem, let us solve this problem and to that we take the same statically determinate structure. In reality this problem is really just a super position of these two problems, let me just number them here; a b c d, a b c d, a b c d. What we are saying is; that from the original structure we get the statically determinate base structure where the member bd has been cut so that there is no force in the member. And we say that this original structure actually can be solved. You can find out the forces in these structures, by finding out the force in these structures plus the force in all the members and reactions due to the unknown force in member bd.

What is the unknown force? We don't know what the unknown force is. So we assume that it is an unknown X_1. If you look at this, there is no force here, we have the force that bd actually has so if you add all of them up you will see that this is exactly this. In other words the redundant force, for this structure is force in member bd which is an unknown value X_1. But suppose we knew X_1, then we could solve this problem. This becomes a statically determinate structure because we know the force in one member. This is the basic idea behind the force method. You take the actual statically indeterminate structure; make it into a statically determinate base structure by, identifying certain redundant forces and putting them equal to zero in the base structure. Once you have the base structure, then you add to the base structure the structure being subjected to the redundant force. If you add the two of them together you got the original structure. That is the basis of the force method. However this is still an unknown, how do you find out this? Let us see how we solve that. Let us look at the base structure firstly and analyze it. Assume that all six members have the same EA values.

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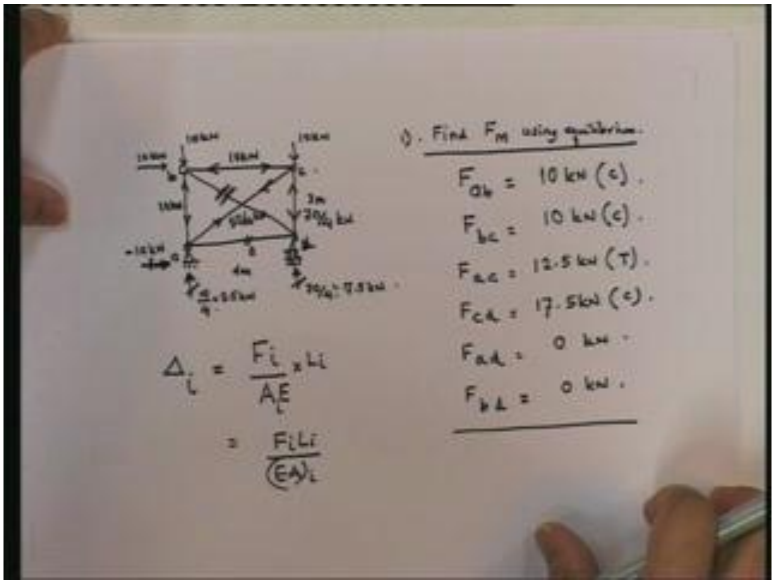

I am looking at only the base structure. The statically determinate, where the redundant force is zero; we will analyze that structure. Note that I am going to keep this member there, because the members still exists. It is very important to understand that the member still exists. It is just that in a statically determinate structure, we have assumed that the member force is zero. Let us now see the steps. This is a, b, c, d. I have 10, 10, 10, and today I am going to actually analyze all the structures. We are going to actually find out all the member forces by actually solving it.

The first step is to find forces in members using equilibrium. I am going to using the method of joints. Let us first find out the unknown reactions. I am going to just find out the reactions without actually writing down all the equations; I leave it up to you to write down the equations because by now you will already have taken a course in basic structural analysis and finding out forces in trusses, beams and frames are things that you are expected to know. So, I shall not actually go around specifically solving statically determinate structure and how to find out the forces in them; I will make an assumption that you know all of those. However in this particular case I will actually go through the steps without actually writing the steps down. The first and foremost, these are the unknown reactions. Sigma F_x is equal to 0 of the entire structure; that gives me that this is equal to -10. The direction that I am showing, is actually in the opposite direction 10; I am putting down -10 because I have shown it in this direction. That gives me Sigma F_x equal to 0.

Sigma Fy is equal to 0 gives me that this plus this is equal to 20. However, that still does not help us. I am going to take moments about point A of all the forces and from that I can find out this value. What are the forces? This does not have any moment, this has 10 into (by the way remember that this is four and this is three meters) what you have over here, this is going to be 10x4 clockwise plus 10 into 3 anti-clockwise divided by 4, because this clockwise, this clockwise, this nothing, this nothing, this one anti-clockwise. This is going to be equal to 70 by 4 value because 10 into 3 plus10 into 4 is equal to 70, 70 divided by 4 is 70 by 4. Since this plus this is equal to 20, this is going to be equal to 10 by 4 which is equal to 2.5 Kilonewton and this is equal to 17.5 Kilonewton. Now we have got the reactions for the support. Now I can use the method of joints to find out all the forces in all the members.

Let me start of with the simple one; member B. Note that this member although it exists I know this force is zero in this. If I take Sigma F_x is equal to 0 of member B you will see that this is going to be is equal to 10 Kilonewtons and similarly if I take F_y I will get this is equal to 10 Kilonewtons. I have found out the forces in these two members and member bc I have a compressive force of 10 Kilonewtons, in member ab I have a compressive force of 10 Kilonewtons. Now, I go to member joint A. What is the force in this member? The force in this member is going to be equal to… if I take Sigma F_x is equal to 0, I am going to get force in ad plus the horizontal component of the force in ac plus the member in ad is going to be is equal to 10. Similarly, you will see that the vertical component of this member ac plus this 10 (Refer Slide Time: 17:58) which is downwards, minus this 2.5 which is this way.

Once you put it all together, here you will see that Sigma F_x is equal to 0 gives me this as more force and if I find out Sigma F_y is equal to 0, you will see that this is equal to 70 by 4 Kilonewtons. This is zero and from that we can find out that this becomes 30 by 4 and 40 by 4 and so this is equal to a tension force of 50 by 4 Kilonewtons. If you look at this particular situation, you will see that the equilibrium of this joint is satisfied automatically and you will see also the vertical of this is satisfied. The forces F_{ab} is equal to10 Kilonewtons; this is compressive. F_{bc} is equal to10 Kilonewtons compressive. F_{ac} is equal to12.5 Kilonewtons tensile. F_{cd} is equal to17.5 Kilonewtons compressive and member ad is 0 Kilonewtons, no force. Note that I also have member F_{bd} since I have cut it I know that is going to be 0 Kilonewtons. This is not the true forces in the Original structure, it is in this structure (Refer Slide Time: 20:36) .

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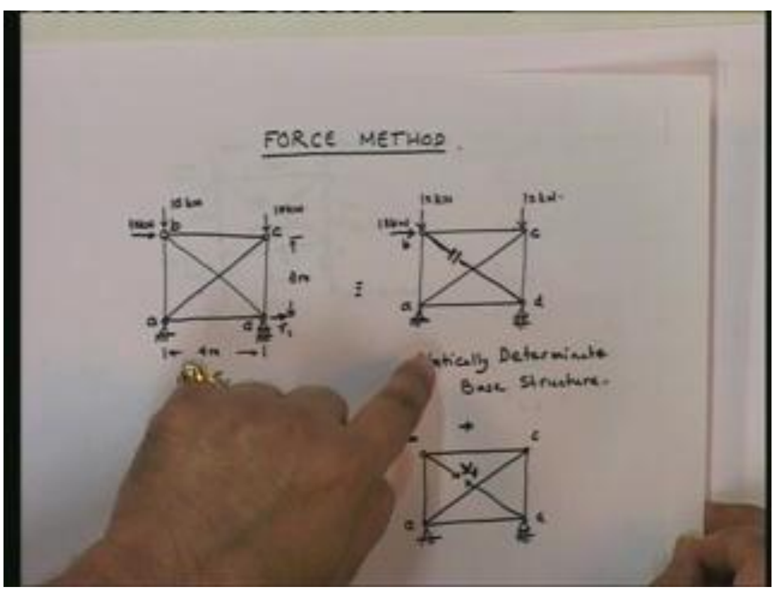

These are the forces that I found out for this particular structure which is the statically determinate base structure. I have found out the forces in all the members. What is the next step? The next step is actually to find out the deformations in all the members. What are the deformations in each member? Please look at it; these are all axial forces. So the deformations in the member i you can find out, is going to be equal to F_i - Force divided by A; that is Stress divided by E which gives me the strain. The strain is uniform, therefore integrated over the whole length gives me the axial deformation. If I rewrite this, this comes out as F_iL_i $(EA)_i$. Force in the member, axial force in the member, length of the member, the axial rigidity of the member. This way I can find out for each one what the corresponding delta is going to be equal to.

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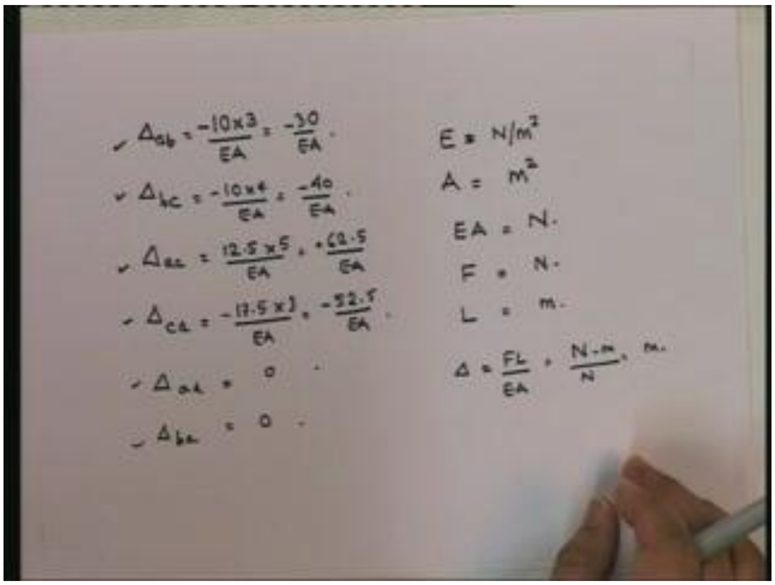

Let me write down those values. Given these forces I can write down that: $delta_{ab}$ is going ot equal to 10 (Note I am going to put minus because tension is taken to be positive and since its compressive is going to be 10) multiplied by Li what is Li; L_{ab}? Lets, look at the length of L_{ab}, L_{ab} is equal to 3 meters divided by (Note I have said all EA are the same) So $delta_{ab}$ is equal to minus 30 upon EA. Let us look at $delta_{bc}$ this is also is 10 Ton bc, length is 4 meters, so this is going to be equal to minus 10 into 4 minus 40 upon EA.

Let us look at $delta_{ac}$. $Delta_{ac}$ Force is 12.5 tension, so its 12.5 tension, what is the length? Length is ac. So this 4, and this 3, and this is 90 degree, and this is a hypotenuse 5. Total length of ac is 5. So it is going to be 12.5 EA this is plus, so it is going to be plus 62.5 upon EA that is $delta_{ac.}$ $Delta_{cd}$, the force is 17.5 compression and the length is 3, so we can put it down as minus 17.5 multiplied by 3 divided by EA, so this is minus 52.5 EA. Finally $delta_{ad}$ and $delta_{bd}$ what are they equal to? Since the Forces are 0, 0 so the deformations are going to be zero.

Let me ask you this question, this is something that we always tend to forget about when doing structural analysis - Note that you have to have dimensional consistency. Here, I have said that all the EA values are the same; I have not explicitly stated the EA value. What are the units of E? Units of E are Newton per meter squared. They are normally given in Newton per meter squared; force per area; that is the value of E because that is stress by strain. Stress is Newton per meter square, strain does not have any units, then the units of area is in units of meter squared. What is

EA in terms of? It is in terms of Newton. What is force in terms of? Well I have given in Kilonewtons over here but kilonewtons is nothing but 10 to the power of 3 Newtons. So the basic units is Newton and the units of length is meter. When I write down delta is equal to FL upon EA let us see what I am doing actually Newton meter upon Newton meter units consistent. Delta which is the displacement or deformation has to be in meters.

We have dimensional consistency and that is very important to know when you actually solve a problem. Suppose you are given E in the standard form in SI units which is GPA or giga Pascals which is 10 to the power of 9 Pascals, and Pascals is Newton per meter squared. When you say that some value is given in giga Pascals, you have to multiply by 10 to the power of 9 to bring it into Newton per meter squared. If you are given a force which is in kilonewtons you have to multiply by 10 to the power of 3 to bring it into Newtons. You need to bring all the parameters to the same basic units otherwise you are going to always have a problem.

However, here I am just trying to state that you need to bring everything down if you want to make F in Kilonewtons. Then EA also should be in kilo Newtons. If you make this area into millimeter squared then E also needs to be in Newton per millimeter squared and your delta will be in millimeter. As long as you are consistent you should not have a problem with units. I have found out the deformations in all the members. What are these deformations due to? These deformations are due to the loads that are applied on the structure. What is the next step? Let us first go through all the steps and then I will put it all together. The next step is actually finding out the forces in all the members due to the redundant force.

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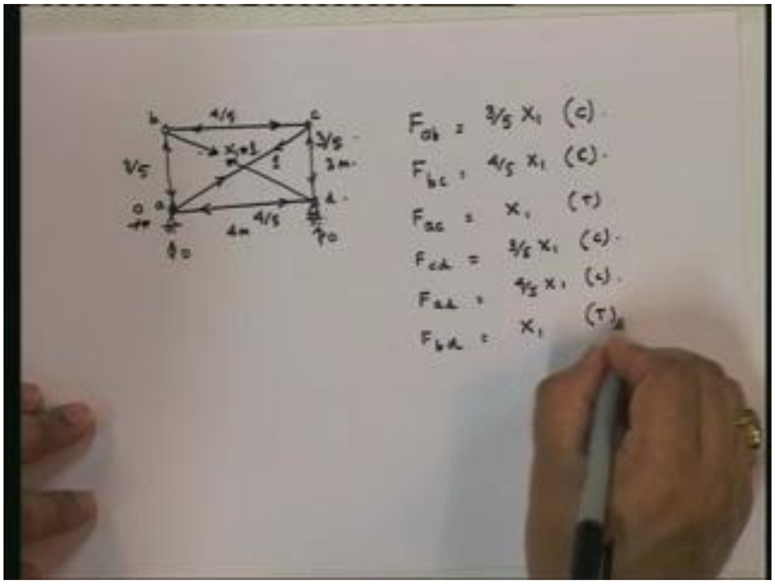

I know that it will always be in terms of X_1, so I can find out the coefficients by actually putting $X_1$is equal to1 and then solving the problem. I am going to solve this problem now. This is (Refer Slide Time: 29:10) actually this plus this so for this I have found out the forces and deformations, for this the second one I need to now find out the forces. X_1 is equal to 1 and due to this I need to find out what the forces are. Remember that this is still 4 meters, it is still the original structure. What about the reactions? This is a force that I am applying in a member. It is an internal force so it is equal and opposite. Therefore there is no net force acting on the structure and therefore all the support reactions are actually 0. Sigma F_x gives me this is equal to zero; sigma F_y gives me this plus is this equal to zero. Then, I take moments about this point. Since these two are acting equal and opposite at the same point this is going to be 0 and this is going to be 0.

Now I need to find out, what the forces in all the members are? Let me first take joint b. In joint b what is the force in this member? The horizontal component of this plus the force in this member is going to be equal to 0. Then the vertical component of this plus the force in ab is going to be equal to 0. From this I can find out that these are going to be equal to 4 upon 5, this is going to be equal to 3 upon 5. Let us look at the other force; if you look at this particular situation, what is the force? I know this force. What are the two unknowns? I do not know the force in this member and this member.

Sigma F_x is equal to 0, Sigma F_y is equal to 0. Sigma F_x is equal to 0 will give me that the force in this member, the horizontal component of this plus this is going to be equal to zero. If you do that you will see that this works out to 1. Once you do that then you can find out what is the force in this member because the vertical component of this is going to be equal to this. I am just writing down the values, you should be able to find them out. What about the force in this member? You will see that this will turn out to be 4 by 5. I found out the forces in all the members so let me write them down.

Force in $_{ab}$ is equal 3 by 5 X_1(X_1 is equal to one and this is compressive; assuming X_1 to be tensile)
F_{bc} is equal to 4 by 5 X_1 (compression)
F_{ac} equal to X_1 (tension)
memebr$_{cd}$ equal to 3 by 5 X_1(compression)
mamber$_{ad}$ equal to 4 by 5 X_1 (compression)
member$_{bd}$ equal to X_1 (the one we will apply)
We have found out the forces in all the members due to an unknown force X_1 . What would be the force in the original structure?

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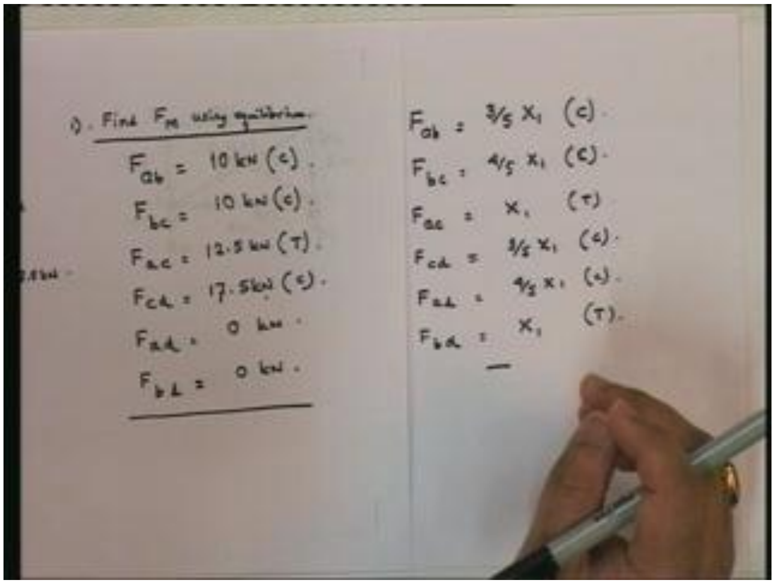

In the original structure
F_{ab} would be equal to 10 plus 3 by 5 X_1
F_{bc} equal to 10 plus 4 by 5 X_1
F_{ac} would be 12.5 plus X_1
F_{cd} would be 7.5 plus 3 by 5 X_1
F_{ad} would be 4 by 5 X_1
F_{bd} would be X_1 with the corresponding signs.

Essentially you see, I can find out the forces provided I know the value of $X_{1.}$ How do I find out the value of X_1? I have found out the forces. In the force method there is the additional equation to establish the value of $X_{1.}$ Let us look at the original problem.

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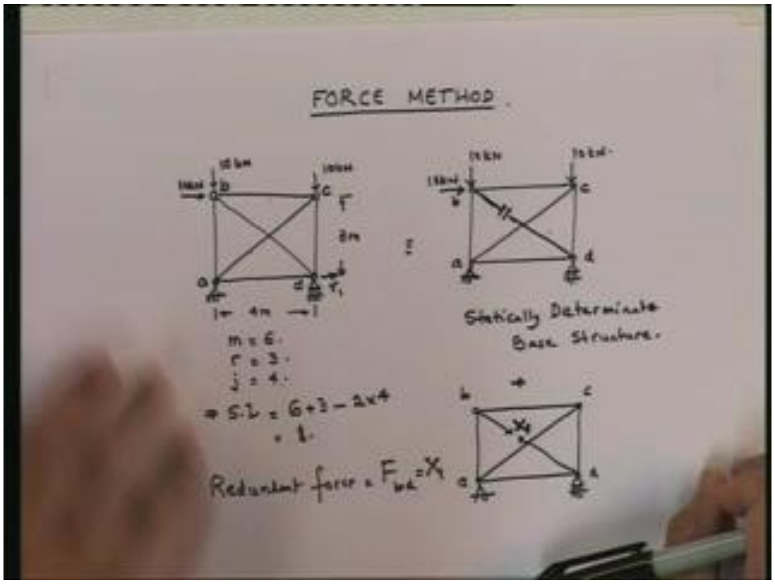

If you look at this, the original problem, what we have done is that we say that this is equal to this plus this. The only problem here is that everything is fine except that we do not know the value of X_1. We need to an equation find out the value of X_1. I made a cut here; under the action of these forces what is going to happen to the cut? You see, all the joints will displace, the members will deform and because of all of that what is going to happen? If this member moves along this direction relative to this, which is bound to happen because they all move independent of each other, there is going to become a separation. There is going to be an opening or an overlap depending on how b and d move. If d moves closer to b there would be an overlap if d moves away from b there is going to be an opening. But the major point is that I cannot stop once I made a cut from these two points which actually in the real structure are the same point. But once I made a cut and I let the structure deform, these two points are going to get separated.

Does that happen in the real structure? No, in the real structure any point over here is not going to separate they are going to move whichever way, they are not going to separate. What is going to happen? Let us have a look at the second point. When I apply X_1 what do I do? Note that these two joints, I have applied a force X_1 across the joint. Since these two are independent points when I apply X_1, they are likely to overlap (assuming X_1 is positive). When X_1 is negative they will pull apart. In both these structures what is happening? The points which I have over here, are overlapping and it is opening up, depending on which is positive and which is negative. But in the real structure does it happen? Do the points overlap? Does any point overlap with itself? It cannot since it is a continuous member. In reality, in this structure, there is no overlap or shortening. Do you understand what we get? We get what is known as a compatibility condition. What we do is, we find out how much this has opened by and then add the two and the two together have to add up to zero because the point cannot go anywhere. The point is stuck to each other. So that is the independent equation.

We need to find out how much displacement I get at this point and we need to find out how much displacement I get this point; add the two things up and put it equal to zero because this point cannot go anywhere; they cannot separate or overlap in the real structure and therefore that gives us the additional equation that we are interested in. Essentially, we have to find out how much this has opened by under this situation and how much this closes by under this situation. How do I do this? I need to find out a displacement. How do I find out a displacement using the method of virtual work, use the method of virtual forces. Actually, I am trying to find out the displacement at this point relative to each other. I actually apply a virtual force at this point and compute the work done and the total work done has to be equal to zero; that gives me the displacement. What is that force? Note that the force is exactly the same as a redundant force.

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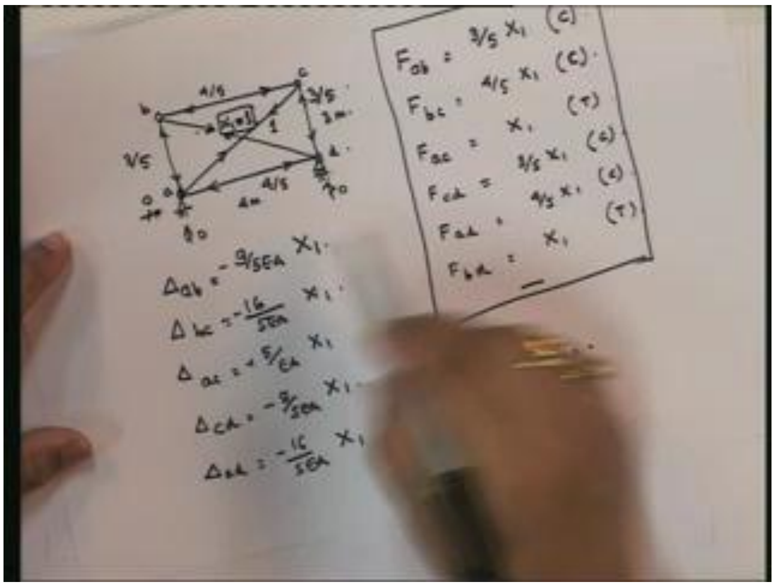

When I actually solved this, I can assume, if I take this to be the virtual force, what will be the forces in all the members? I know the forces I am going to have, so if you look at it using the method of virtual force, what is the displacement at this point? What is the displacement at this point? The displacement at this point can be given by the virtual work equation.

External virtual work done by all the external forces:

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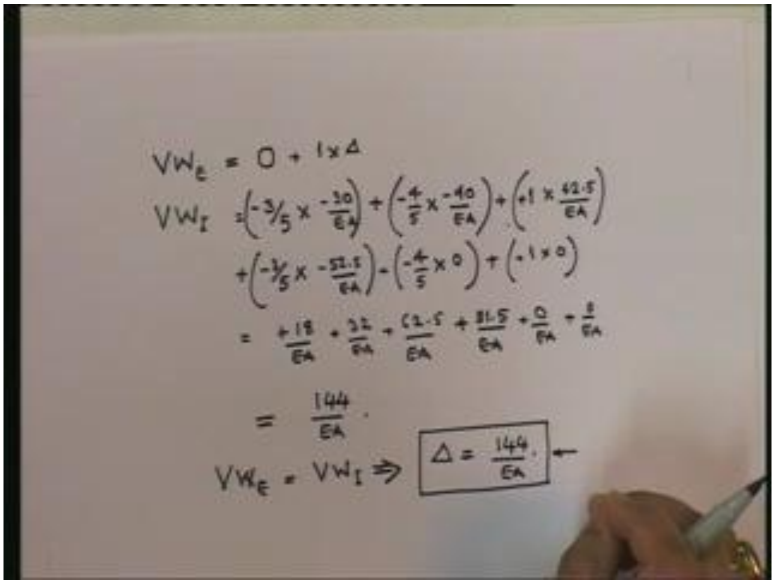

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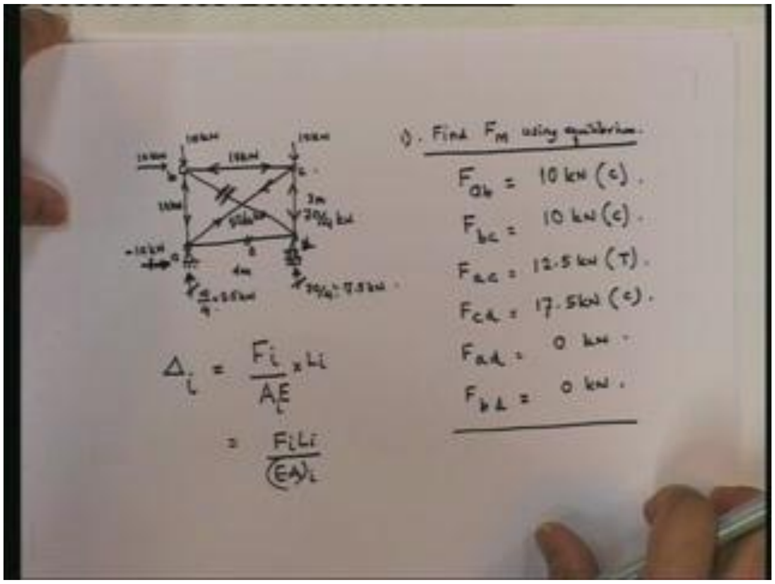

This is the real structure, I am trying to find out the displacement at this point because of all these forces. I need to find out the real displacements and real deformations. What are the displacements? Real displacement corresponding to any force here. Real displacement is going to be equal to all these forces. What is the work done by these forces? The work done by these forces are equal to (Let’s go back).

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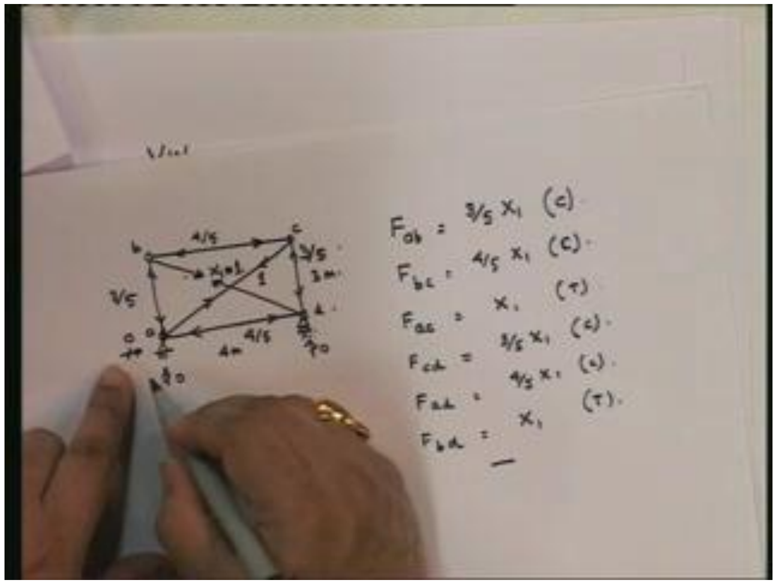

The virtual forces undergoing the real displacements; what are the virtual forces here? The only virtual external forces are these reactions which are all zero. There are no external virtual forces, so the work done by the external virtual forces is equal to zero because there are no external virtual forces. Let us look at the internal virtual work done. The internal virtual work done is how much? The virtual force here is this reaction, ab so I need to find out what is that ab. What is the force in ab? 3 by 5. So 3 by 5 multiplied by the real deformation in member ab. What is the real deformation in member ab? Remember we computed the real deformation. So I am going to put 3 by 5 multiplied by minus 30 by EA that gives me the work done by the virtual force in ab which is 3 by 5. Note that this is compressive, so the compressive force is going to be given by minus. Undergoing the real deformation in member ab which is minus 30 by 9.

The internal virtual work contribution by ab is going to be minus 3 by 5 into minus 30 by 90. That is the virtual work done by the real deformation in ab due to the loads undergoing the virtual force in the system. This negative implies that there is a shortening, this negative implies there is a compressive force. Let us go from ab to the next member which is bc. What is the virtual force in bc? The virtual force in bc is 4 by 5 X_1 compressive. If we put a unit force it is going to be 4 by 5. That is going to be compressive minus 4 by 5 multiplied by the real displacement which is minus 40 upon EA. Then I am going to add the next Force ac tension 1. It is going to be plus, multiplied by the real deformation in ac which is 62 point 5 plus so this is 62 point 5 upon EA. That is member ac. Next is cd; what is the force? 3 by 5 into compression, so the virtual force is minus 3 by 5. What is the real deformation in cd? The real deformation in cd is 52 point 5 so that is minus 52 point 5 by EA.

Then we go to next, I have done ab, bc, cd, next is ad. What is it? The virtual force is minus 4 by 5. What is the real deformation? 0; so multiplied by 0. Then I need to look at bd which is 1 tension, so it is going to be plus1. What is the real deformation in bd? 0. What is the total internal virtual work? You will see that is going to be equal to plus 18 upon EA plus 32 upon EA plus 62 point 5 upon EA plus 10 point 5 into 3 is 31 point 5EA plus 0 upon EA plus 0 upon EA so tha total virtual internal work done is equal to 50 plus 94 144 upon EA; That is the internal virtual work done by all the forces in all the members.

Note that the external virtual work done by all the forces is zero but we have neglected one, what is that? This is also an external work and what is the external work done by this force? This force is equal to 1 into…. Note, what we are trying to find out here, we are trying to find this overlap the delta. That is the reason why we applied the force X_1. Therefore 1 into this delta is the external virtual work. Once we put external virtual work is equal to internal virtual work; that is our principle of virtual work because the total work done is zero we get delta equal to 144 upon EA.

Note that since we have applied this force what we are actually computing is the opening up of the system and therefore the opening up is equal to 144 upon EA. The next step now is to find out what. We found out how much this has opened up by. We need to find out what is the displacement at this point. What is the real load? The real load is this. To find out the overlap what do we need? We need to apply the same virtual force. Once we have found out the delta here due to force $X_{1.}$ Note that these are now the real forces, so for these forces we need to find out what is the displacement. Note I have already done this; the real deformation is equal to delta_{ab} is equal to 3 into 3 9 by 5EA X_1,delta_{bc} is equal to 16 by 5EA X_1,delta_{ac} is equal to 5 upon EA X_1. Note this is negative, and negative because this is compression and this is tension, so this is positive. delta_{cd} is equal to 3 by 5 X_1; into the length of cd which is 3 meters so this is going to be equal to minus 9 by 5EA X1 delta_{ad} is equal to 4 by 5 compression, so this is going to be equal to minus 16 by 5EA X_1.

And finally delta_{bd} is equal to 5 upon EA X_1. These are the deformations due to these. Now to find out the displacement using the same virtual work principle you will get displacement is equal to external virtual work all outside excepting for this, is going to be equal to one into delta due to x. This original one (Refer Slide Time: 50:45) is delta due to load. This is delta equal to x. The internal virtual work is going to be equal to; if you look at this the displacement multiplied by this, we are going to get

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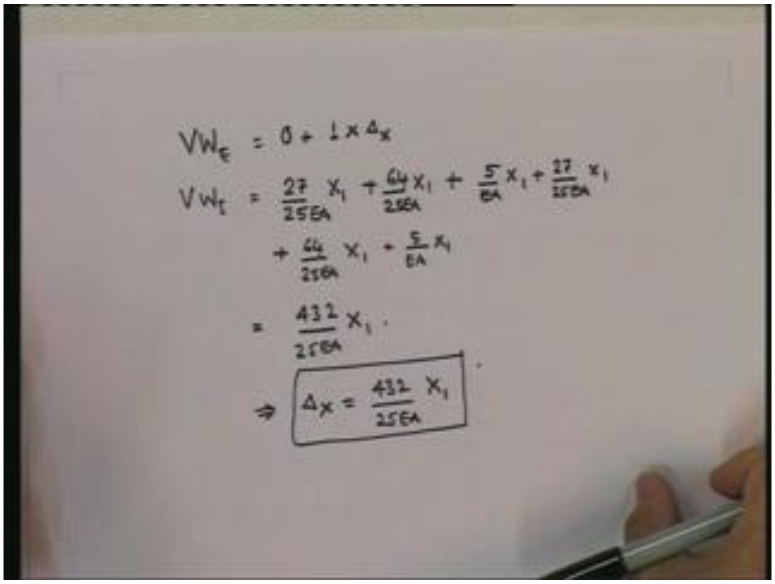

27 upon 25EA X_1, plus delta bc is going to be 64 upon 25EA X_1, plus 5 upon EA X_1, into 1 that is going to be 5 into X1, then CD is going to be equal to 27 upon 25EA X_1, AD is going to be 16 by 6 into 4 by 5 64 upon 25 X_1, plus 5 upon EA X_1 into 1; this is by internal virtual work.

If I add all of those up I get 27 plus 64 that is 91, 91 and there are 2, 182, 182 upon 25 182 plus 250 is 432 upon 25EA X_1. So, if you equate the two you get delta_x is equal to 432 upon 25EA X_1. Now I have computed delta_L, delta_x. What is the actual delta?

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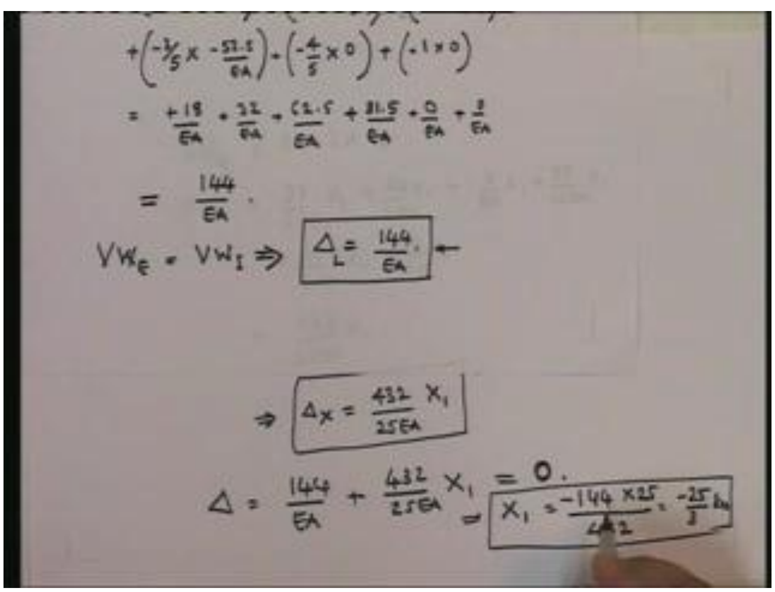

This going to be super position of the two because if you look at this, it is going to be the displacement in this plus the displacement in this and delta is going to be equal to 144 upon EA plus 432 upon 25EA $X_{1.}$ What is the actual delta? In the real thing delta is equal to 0. From this you can find out $X_{1\ is}$ equal to minus144 into 25 upon 432 equal to minus 25 by 3 Kilonewtons. I have found out the value of X_1, the force and once I find that out I can find out the force in all the members because all I need to do is this plus this now I know X1 so I just need to find out add of all of these up and I have got my X_1.

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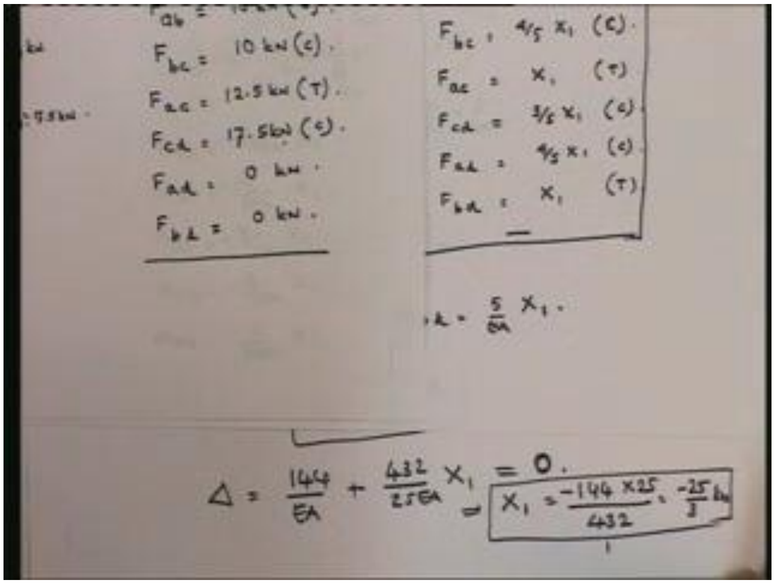

This in essence is the force method and I hope that with this simple illustration I have been able to illustrate the basic concept of the force method as applied to a single static indeterminacy truss. Thank you.