Structural Analysis – II
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Lecture No. 39

Good morning. As you know already, we have been looking at influence lines for statically determinate structures. To begin with, we introduced the concept of the Müller–Breslau principle and then, we have spent the last two lectures looking at influence lines for statically indeterminate. Again, to reiterate, we said the last time we spoke of qualitative influence lines, Müller–Breslau principle, quantitative only the direct approach and in the direct approach, we saw that the moment distribution method gives us the quickest way of computing.

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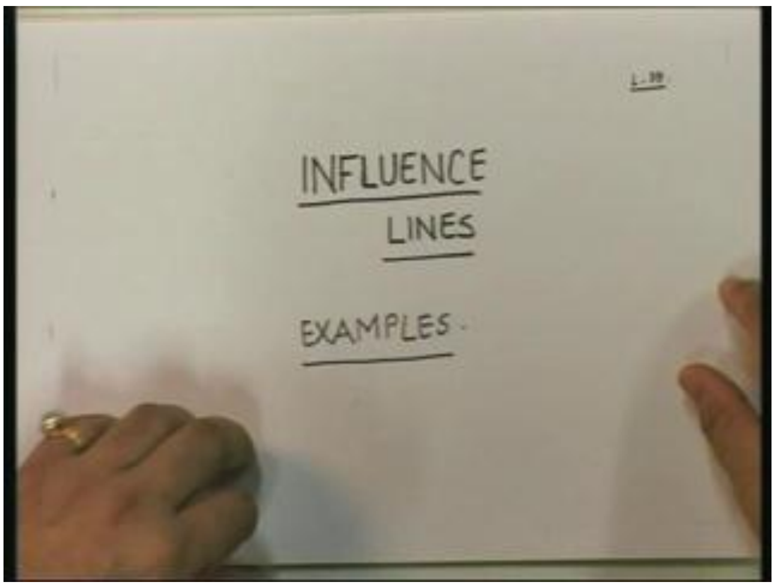

Let us now continue looking at influence lines. Hopefully, we will try to look at one or two examples in this particular lecture. I am going to be moving a little bit faster because by now, I do not have to reiterate every small step that I take – why am I releasing, why am I doing this; by now, you should be able to be fairly comfortable with that. Let us take an example.

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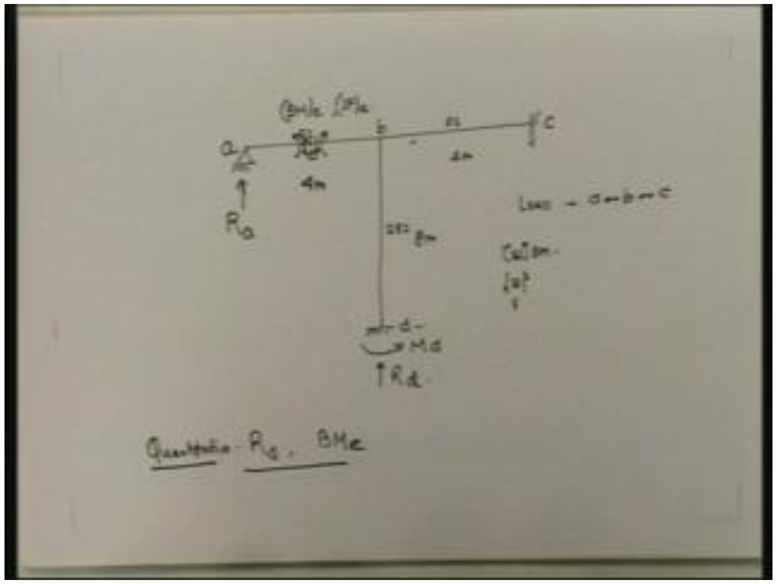

Now we are going into real examples. The load only moves in a, b and c – vertical loads only move between a, b and c. The flexural rigidity for this is EI, the flexural rigidity for this is EI and the flexural rigidity of this is 2 EI (Refer Slide Time: 03:42). We are given the fact that bending moment positive is this way and this is shear force positive – shear force and this is bending moment. This span is 4 meters (Refer Slide Time: 4:31), this is 4 meters and this is 8 meters height.

We have to find out influence line for R_a, for M_d, for R_d and for the bending moment and shear force – bending moment at e and shear force at e; this is the center point (Refer Slide Time: 05:32), it is 2 meters. This is the problem: both qualitatively find out the influence lines for this, so for these are qualitative and quantitative, find out R_a and bending, only quantitative is R_a and BM; qualitative – all of the others. Let us go through these step by step. We can do this reasonably quickly. Qualitatively, let us look at what happens.

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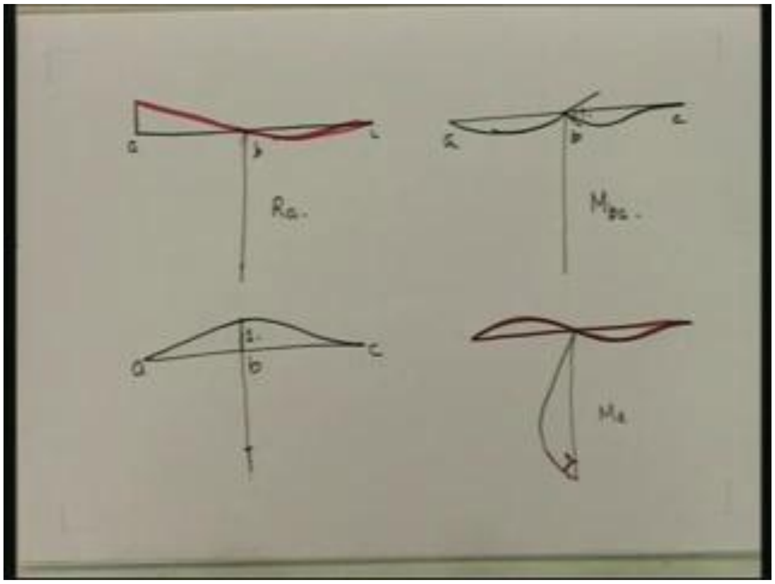

For R_a, what we have to do is…. This is the system and for R_a this goes up (Refer Slide Time: 06:35), this point goes nowhere and so, what we get is…. Note that this goes and this is fixed so this has to be like this, goes like this (Refer Slide Time: 07:01), fixed goes; **and** this is going to be less because this also has to go. However, the influence line is only over here. This is the influence line for R_a. If we want to do it for bending moment M_{ba}, this is for M_{ba}, let me draw it this way. For M_{ba}, what happens is that this goes this way and this goes this way where this and this has to be 1. That is the bending moment – this is for M_{ba}. Then, we need to find it out for R_d. For R_d, what do we do? Think about it.

We release this and push this up by 1 (Refer Slide Time: 08:16), push this up by 1, this goes up by 1. What is going to happen is that this is going to go and this is going to be 1 and this is a, b, c. Note that always… a, b, c, we do not care what happens to this part because the load does not travel on that. Is that clear? Therefore, even if you have a frame, the influence line is only to be drawn the shape that the part on which the load travels, that is the only part that comes into the influence line – remember that, never forget that.

Even if I have shown a frame here, I am only drawing the influence line for abc, because that is the only position, that is the only kind of extent the load travels – we have been given that. Then finally for M_d; for M_d, this has to go, this is going to be this way and so, our thing will be this way and the influence line, this has to be equal to 1 but note that the influence line is this; even though we have drawn this part, it has no role to play. This is my influence line – remember that. This is for M_d although I have given this (Refer Slide Time: 10:31). I am not going to draw anymore; okay, let me just draw the bending moment at e for your benefit.

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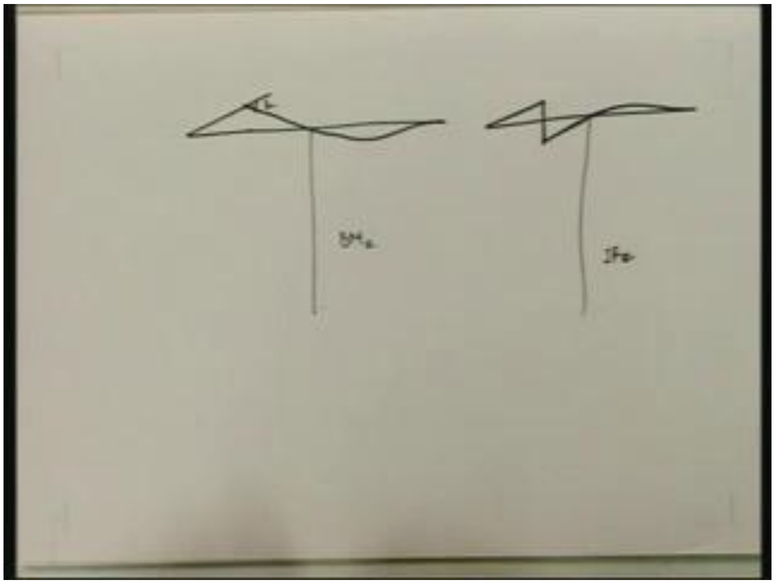

Bending moment and shear force – let me draw that. If I have bending moment at a, then this is going to go straight and then this is going to go in this fashion, where this is a straight line and this is going to be equal to 1. That is going to be bending moment at e. For the shear force at e, it is going to be this and the shear force, positive is… this being pulled down and that being pulled up, so it is going to be this way; of course, this is going to go this way. Again, this is the only part that matters; this is going to go this way, it is going to be less than… if this did not exist, it is because this stiffness reduces that, that is all. This is for the shear force at e. Qualitative influence lines is fine.

Now, we need to go into computations of this for this thing. The thing that we need to know is from which position does this go? Let us look at that. Quantitatively, we need to know this goes ab, bc and therefore, when it goes ab, $(FEM)_{ba}$, when it goes bc, it is going to be fixed bc and cb, but cb is anyway fixed, so it does not matter whether cb is fixed or not. Therefore, we do not apply a fixed end moment – that fixed end moment is going to go directly and at ab, we do not provide because M_{ab} is always going to be equal to 0. Therefore, the only ones we need to provide are bc and cb.

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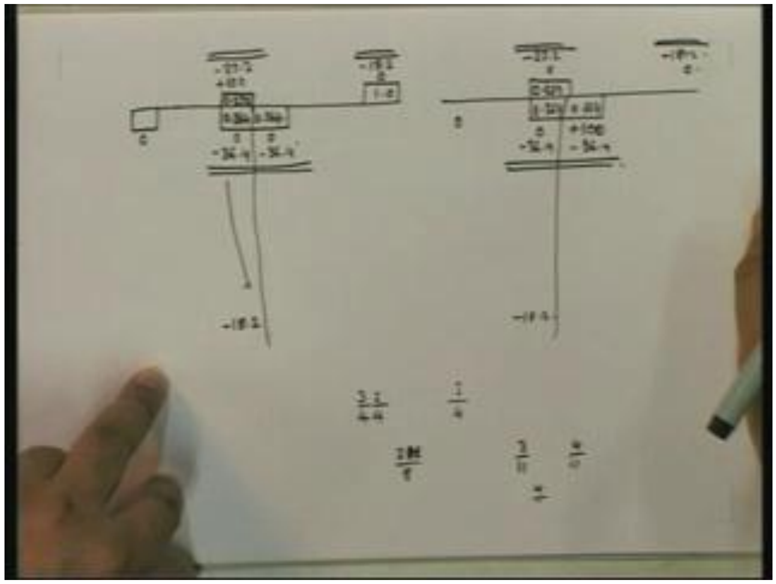

I am just going to show you notionally, I am not going to do it precisely. I am applying and remember: this one, there is nothing (Refer Slide Time: 13:48), this one this and here, I have this, for this, we have this side and this, this is 1.0. Let me go back and have a look. This is EI and 4 meters. This is going to be I upon l, three-fourths of I upon l, so this side, we have three-fourths of I by 4; this side, we have I upon l, so we have I upon 4; this side, we have 2 I upon 8; so this is going to be 3 by 16 I plus I upon 4 and I upon 4. What we have then is 4, 4, 11 by 16, 11 by 16, 3 upon 16 divided by 11 upon 16 is 3 by 11 and this is 4 by 11, 4 by 11. What we have is 0.272, this is going to be 0.364 and 0.364, this is going to be 0.728, yes 0.364 – these are the distribution factors that you have over here.

The first one is going to be plus 100 here, 0 here, 0 here and of course, this is 0 and so is this. We need to distribute it and we distribute it. This becomes minus 27.2, minus 36.4, minus 36.4, this one goes here, minus 18.2, this one goes here, so this becomes this, this becomes minus 18.2 and that is it. Now, we go on to the next one. It is the same thing that we have. It is going to be 0.364, 0.364, 0.272 and this time, this is plus 100, this is 0, this is 0, this is 0, and so is this. When we do this, this becomes minus 27.2, minus 36.4, minus 36.4 and this becomes minus 18.2, close, close, close. Nothing goes here and here, we get minus 18.2 (Refer Slide Time: 17:40).

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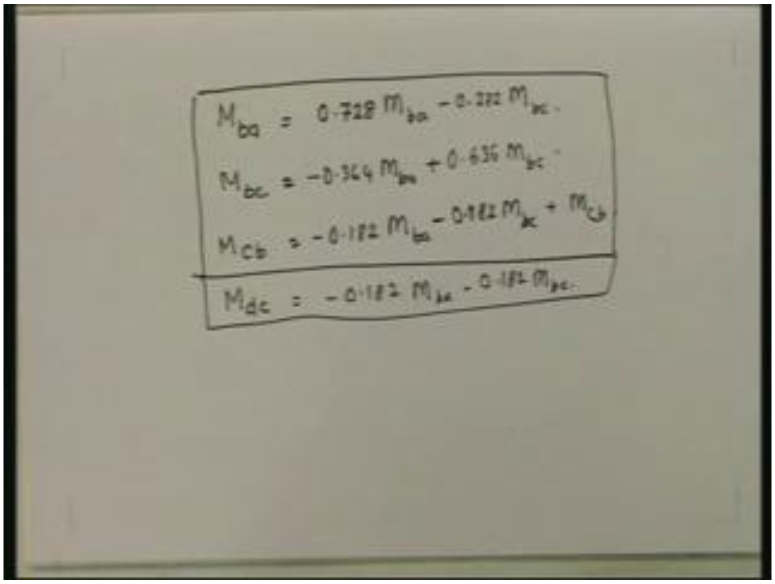

If we look at this, we see that M_{ba} is equal to 0.728 of m_{ba} and minus 0.272 of m_{bc} – this is the fixed end moments. Then, M_{bc} is equal to minus 0.364 m_{ba} plus 0.636 m_{bc}. M_{cb} – this is the moment at that end is going to be equal to minus 0.182 m_{ba} minus 0.18 m_{bc} plus m_{cb}. Note that I have not done this moment distribution. Why? Because at a fixed end moment if I apply a fixed end moment, what is going to be the moment? The fixed end moments, so that is going to be directly this, it is not going to get distributed anywhere else. These are the expressions that I get from my moment distribution because if you look at this, if this is 100 (Refer Slide Time: 19:43), then this is 72.8, this is minus 36.4.

By the way I have not done the others because those do not… In this particular case, I have only asked you to find out R_a and bending moment at e. If I had asked you to find out M_d, then you would of course have got it in terms of minus... let me do that, let me do that, let me find out M_d so that I would need to find out M_{dc} would be equal to minus 0.182 m_{ba} minus 0.182 m_{bc} – that is what M_d would be, so we will find that out. I have got these. Once I have got these, then the next step is of course the same procedure that we have already gone through and without much ado. We have already done this in the previous case.

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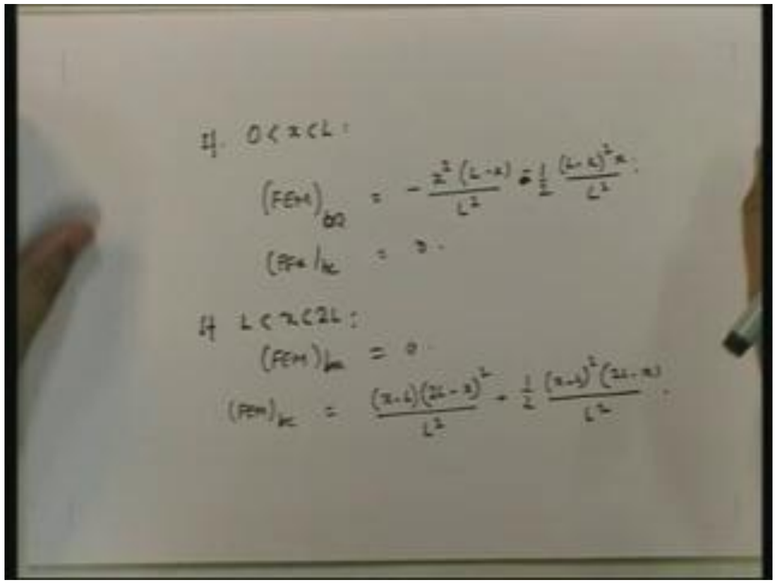

We know that if 0 is less than x is less than L, then $(FEM)_{ba}$ is going to be equal to minus x squared into (L minus x) upon l squared minus half (L minus x) square into x upon L squared and $(FEM)_{bc}$ is equal to 0. If L is less than x is less than 2 L, $(FEM)_{ba}$ is equal to 0, $(FEM)_{bc}$ is equal to (x minus L) into (2 L minus x) squared upon L squared plus half into (x minus L) the whole squared into (2 L minus x) upon L squared. Note that the only thing that we have been asked to find out are three quantities.

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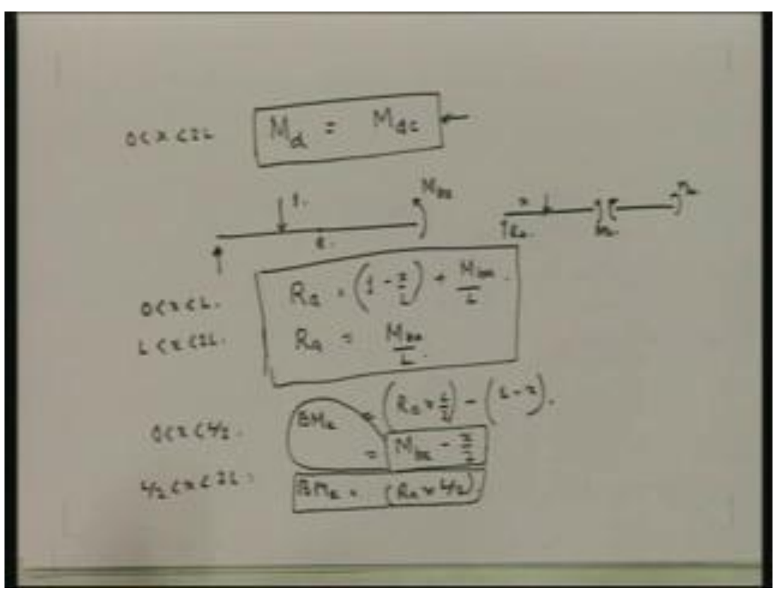

Those three quantities if you look at it is M_d. M_d at all times for 0 less than x is less than L by 2 is equal to M_{dc}. What are the other ones? The other two things that I have been asked to find out are R_a – I need to find out R_a, so let us see what R_a is going to be. I have over here M_{ba} and I am only interested in finding out R_a and the bending moment at e. When a load is between 0 and x by L, R_a is equal to (1 minus x by L) plus M_{ba} by L – we have already done this, I am not going to go into that.

When x is between L and 2 L, R_a is equal to M_{ba} by L. That gives me the expressions for R_a. Now, for the bending moment at a, when load is between 0 x and L by 2, bending moment at E, let us look at what bending moment at E would be like. You would have a bending moment in this fashion. If we look at it, M_{ba} and the load is over here and so x is between and you have R_a here (Refer Slide Time: 24:49) and this is the bending moment. If it is between this and this and I take moments about this point, what do we get? We get R_a into L by 2 – that is being this thing, so it is R_a into L by 2 and here, we have this one going the opposite way, so this is going to be minus 1 into L upon L minus x. This is the bending moment and you can see that this is also equal to M_{ba} minus x by L into L by 2, x by L into L by 2 is minus x by 2, so we can actually put it in this fashion. If you substitute into this, you will see that this is exactly the same as this, so there is nothing new in it. We will probably use this – this is the easier one to use. What happens when x is between L by 2 to 2 L? Bending moment is just equal to R_a into l upon 2. Once I have this, then I can write it down.

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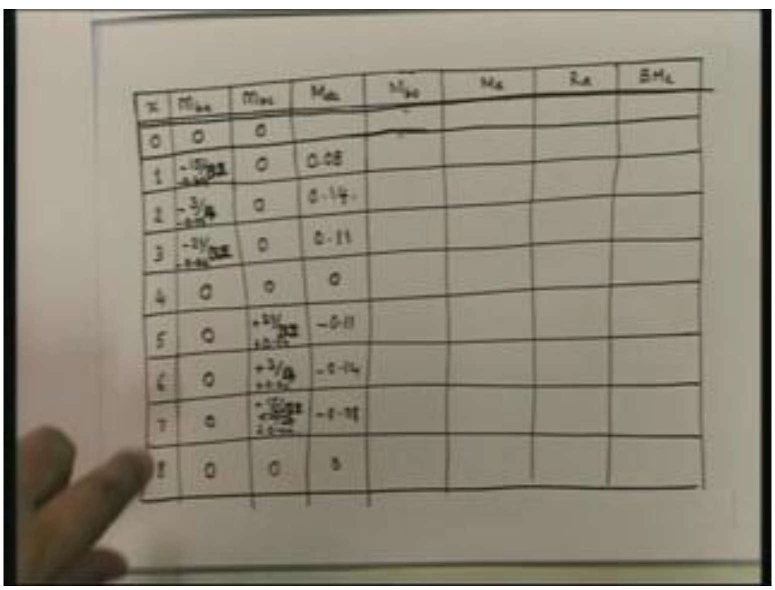

This time, I will do it with L by 4 so that we can actually find out more details. This is going to be x (Refer Slide Time: 27:26) and the overall distance that it travels is from 0 to 8. I will have 0, 1, 2, 3, 4, 5, 6, 7, 8. The first ones we have to find out are m_{ba} and m_{bc} – these are the fixed end moments and those are the first things that you have to evaluate. Once you evaluate that, if we look at it, we need look at this so all we need to do is to evaluate M_{dc} and M_{ba} because these are the only ones that we need. We are going to do that. I am going to evaluate M_{dc} and M_{ba}. I do not need to evaluate any of the others. That will give me directly M_d, R_a and the bending moment at e, the bending moment at e. Let me now do this.

We have done this now. We know that m_{bc} is going to be 0 while the member goes between 0 and 4. We also know that m_{ba} is going to be 0 when it travels between 4 and 8 and of course, when it is at the end, these are going to be 0. All we have to do really is evaluate these and this one we know, it is 3 by 16 (Refer Slide Time: 30:03) and this is negative and this is going to be positive 3 by 16, here plus 6, this is not correct, this is plus 3 by 16 at the center span. Let us find out at three-quarter span.

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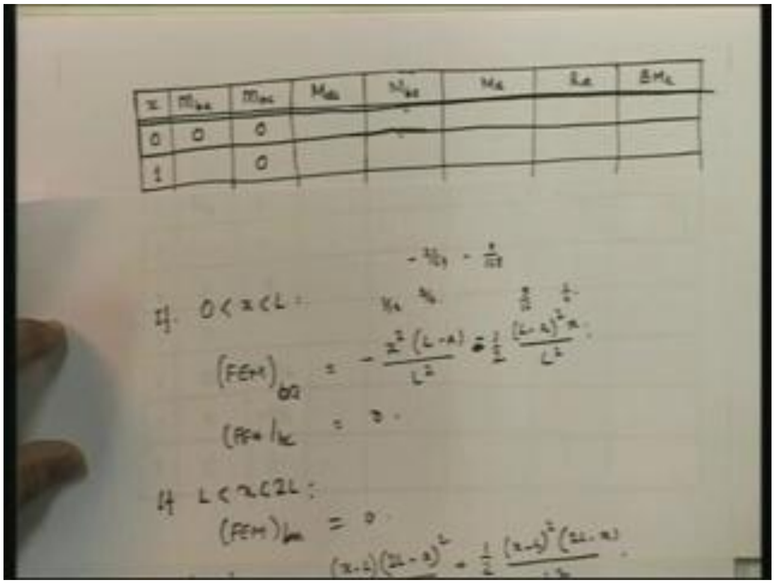

I am going to plug in the value of x equal to one-fourth in this (Refer Slide Time: 30:39). When I plug in one-fourth, I get 1 upon 16, this is going to become 1 minus, so this is going to be three-fourths and this is going to be 9 upon 16 and this is going to be one-fourth. What we have over here is minus 3 upon 64 and then we have minus half of 9 upon 128, so this is going to be equal to 6, so minus 15 upon 128. The other one with three-fourths is going to be just the opposite, so this is going to become minus 9 upon 64 and the other will become 3 upon 128 and so when we do this, 18, we get minus 21.

What you have here is that since we are doing this thing, this is going to be 18 and 3, plus 21, this is going to be plus 21 upon 128 and this is going to be plus 15 upon 128. If you look at this, this turns out to be… as the load comes in, the bending moment increases and this is more and this is less. Now, let us look at directly M_{dc}. This one if you really look at it, this into L, so this is going to be 60, so this is going to be 4. I will have to do this again, this is going to be 32 – this is into L, L is 4 here, this is 3 upon 4, this is going to be 21 upon 32, 21 upon 32, 3 upon 4, 21 upon 32.

If we write those down in numbers, you will see that you get 0.4 on 28, 12, 12, so that is going to be 4.44, this is going to be 0.75, this is going to be equal to 0.6, 6 will make it 192, that is going to give me 12, so it is going to be 0.64; this is going to be minus, minus, minus, this is going to be minus 0.64, this is going to be minus 0.75, this is going to be minus 0.44. When we plug those values in here in M_{dc}, you get minus 0.182 into 0.44 and if you look at that, that basically becomes minus, so it is going to be plus, so plus 0.072, so this is going to be 0.08.

Then if you do this into 3 by 4, it is going to be 0.75, this is going to be 3 by 4, 0.55, 0.55, 3, so this going to be 0.55 divided by 4, that is going to be 0.14 and in 0.64, so this is going to be 0.11, this is going to be equal to 0 and this is going to be minus, note that these are plus, plus, plus, so this M_{dc} is going to be minus, it is going to be minus of 0.11, this way, it is going to be minus 0.14 and minus 0.08 and 0. Let us look at for M_d. This is exactly how it looks. This is M_d and the point that I am trying to make is this way you can find out m_{ba}, plug it in and you will get m_{ba} and then, you can plug in these. Once you get M_{ba}, you can find out R_a values and you can find out moments.

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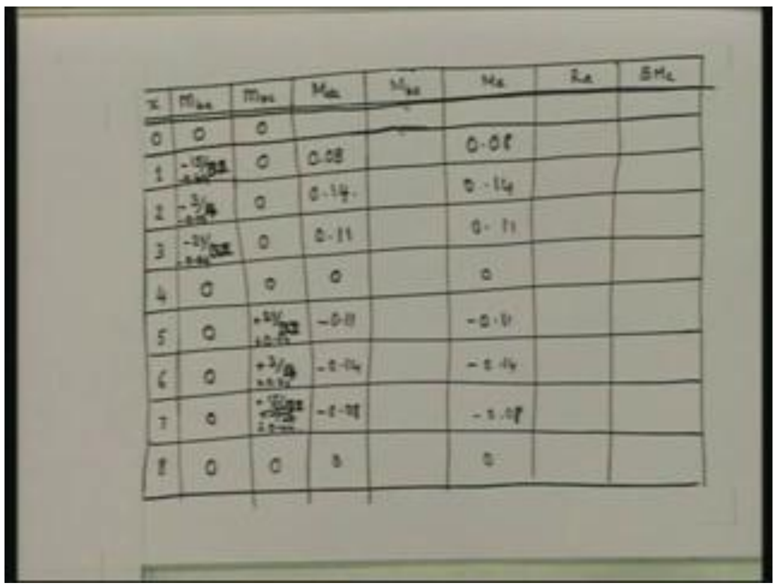

Once you know this, M_{dc} is directly this (Refer Slide Time: 36:28). Now the point that we are trying to make over here is that once you put this factor in, you can get R_a and bending moment, you can get M_{ba}. I am not going to do all of this. The overall procedure boils down to three steps. The first step is to determine where exactly your loads are. In this particular case, for example, load is going from a to c.

Now, a is hinged, it cannot have a fixed end moment. Therefore, when the load is between a and b, you have a fixed end moment at ba. Therefore, you need to do one moment distribution with ba. Then when it is between b and c, you need to do the moment distribution. There is going to be a $(FEM)_{bc}$ as well as cb, so you need to do a moment distribution with bc and cb. But note that the there is no need to do M_{bc} because that is a fixed end and you do not release that point, so there is no fixed end moment, so it only comes into M into M, the moment at cb – these are important points. Therefore, once you know the levels of moment distribution that you have to do, the next step is once you have done the moment distribution and you have to now write down what are the member end moments in terms of the fixed end moments – those are the coefficients you have found out from the moment distribution.

Once you have written those down, then you need to do the equilibrium of each member and get it from there. Is that clear? This is the step and now I am going to quickly look at another problem so that I can illustrate… one other type of problem to you. Right now, we have set up the entire step and now all we need to do is essentially put down all the values that you are going to get so that we do not need to do anything anymore. We were done with this, this is it. Just let me put my papers together and then we will quickly move on to the next one. The next problem is, let me take a particular example, let us take this example.

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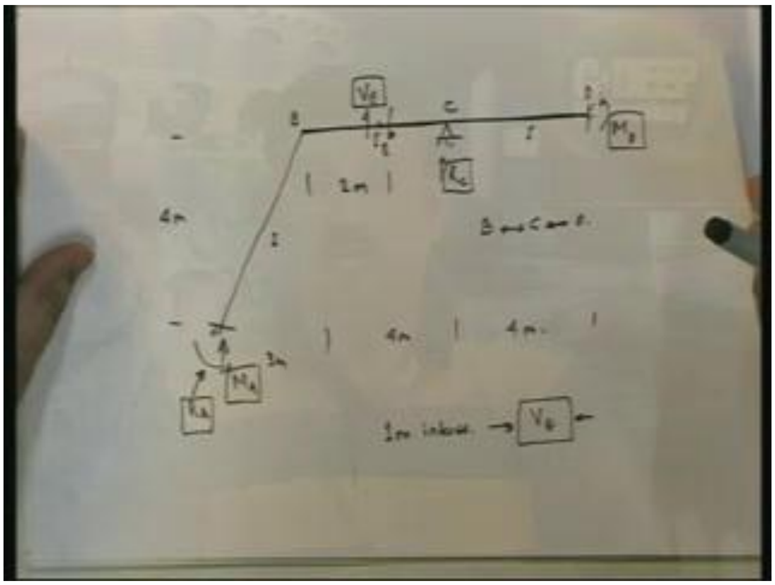

This is A, this is B, this is C and this is D; for this, I have I, for this, I have I, for this, I have I; this is 4 meters, this is 3 meters, from here to here is 4 meters and from here to here, it is 4 meters. The vertical load only goes between B and C and D – this is important; it does not go on AB. Is that clear? The vertical load is only in this zone. What does that mean? When we draw the influence line, we draw the deflected, use the Müller–Breslau principle and get the deflected shape. We only need to draw the deflected shape of BCD for the influence line.

Let me now say that the influence lines for M_a, for R_a, then at E, which is the center point, I need for the shear force, shear force, the reaction at C and… R_a, M_a, V_E, R_c and M_D. We have five for which we need to find out…… I leave this as an exercise for you – you are going to do it yourself, qualitatively over this entire scope at 1 meter intervals, you are going to find out V_E. I leave this to you as an exercise – I will give you the answer but I will not go and solve the details. However, what I will do is I shall use the Müller–Breslau principle and show you how this thing can be done.

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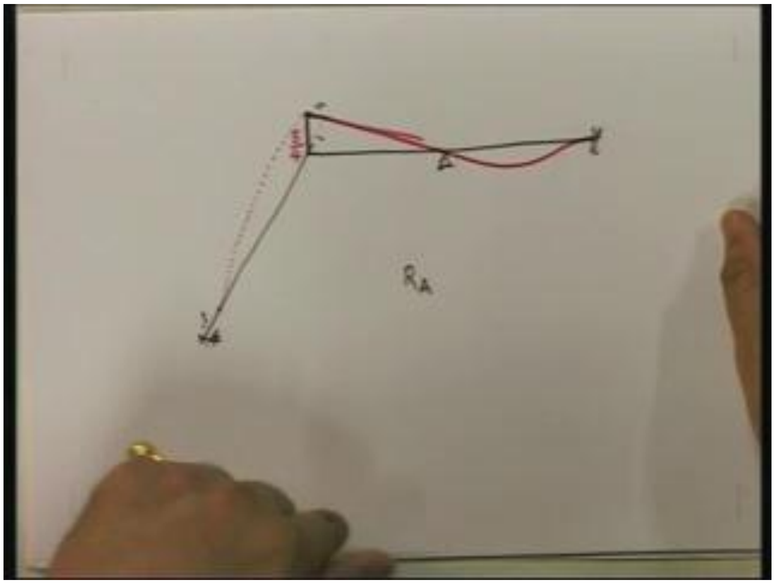

First, R_a**:** I am going to make it a question that is going to become very very important and that is the reason why I have given you this particular problem, nothing else.

Here, note that now going to become a fixed roller (Refer Slide Time: 43:25) which is going to move in this direction, so this way it has to be equal to 1. What does that mean? Ideally, this would go (Refer Slide Time: 43:46), 1. However, this cannot move, so therefore this has to now move perpendicular and how much will the perpendicular be in turn, think about it**.** If this is 1 (Refer Slide Time: 44:01) then the perpendicular is going to be 3 by 4 so that this then becomes 5 by 4. this is going to go like this and I am not going to draw this, this is going to go like this. Therefore, this is going to rotate in this direction if it has to move up; the tangent is not going to be this way, the tangent is going to be this way, so it is going to go like this, go here and then go this way and this is going to be 5 by 4. Note that this is for R_a.

Note this very importantly and that is the whole reason why I have given this particular problem and that is why I have given R_a in this direction instead of vertical. Vertical would have been very easy. But I gave it in this direction for you to understand that when this has to go this way, you have to satisfy the boundary conditions. Remember that the Müller–Breslau principle has to satisfy all the boundary conditions that you have in your specific problem – this is the point, this was the whole point, everything else is now trivial. However, I am going to go through with it. Let us look at the next one which is M_a. I am going to M_a.

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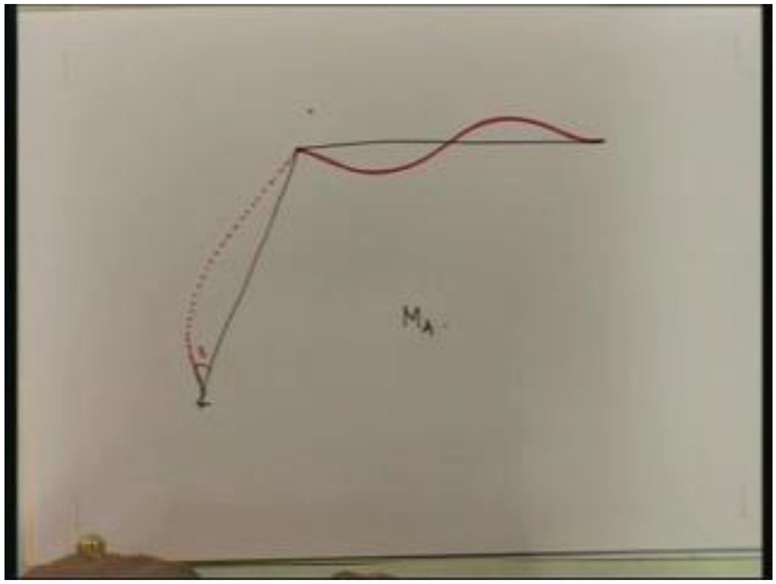

M_a means I rotate it so this becomes now a hinge and rotate it here. How will it look? It will look like this. When this goes this way, this would tend to do a little bit and so this is where it becomes interesting. We knew that this is 1 (Refer Slide Time: 46:44) and this is the influence line. However, qualitatively we know nothing over here. This is for M_a. Now, let me do it for R_c.

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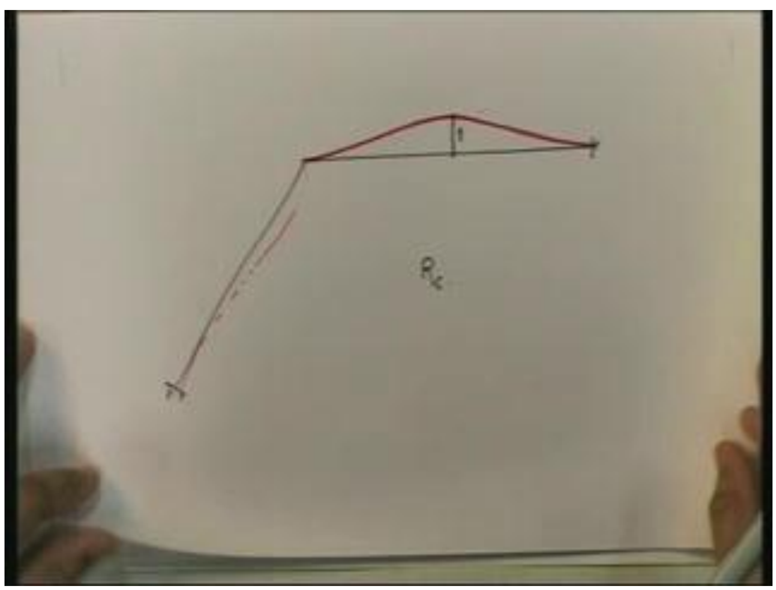

For R_c this is going to remain this, this is going to remain this, we are going to remove this and move it up and up by 1. How will this thing look? You will see that when this goes up, this is going to go like this, so this is going to go a little bit like this and so this is going to get a little bit like this (Refer Slide Time: 47:45), probably this will have an angle this way but this in essence gives you for R_c.

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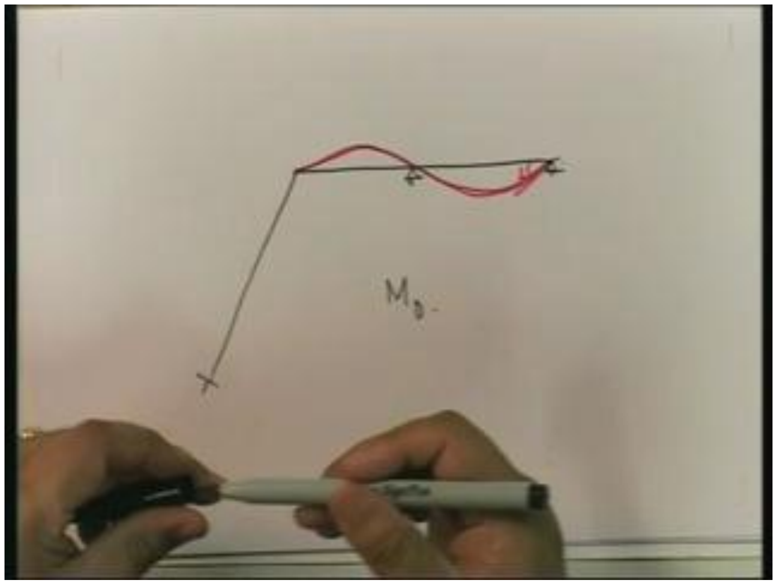

Now, M_d, let us make it for M_d. For M_d, now what do I do? I release this, make it into a hinge, this is anyway a hinge (Refer Slide Time: 48:36), this is fixed, make it into a hinge and then rotate it. What is going to happen here? You are going to see that this is going to go in this fashion and this is going to go this way and this way, so this is going to be where I know that this slope is equal to 1. This is for M_d. Finally, for the shear force also I am going to draw this because this is important. This is quantitative**.**

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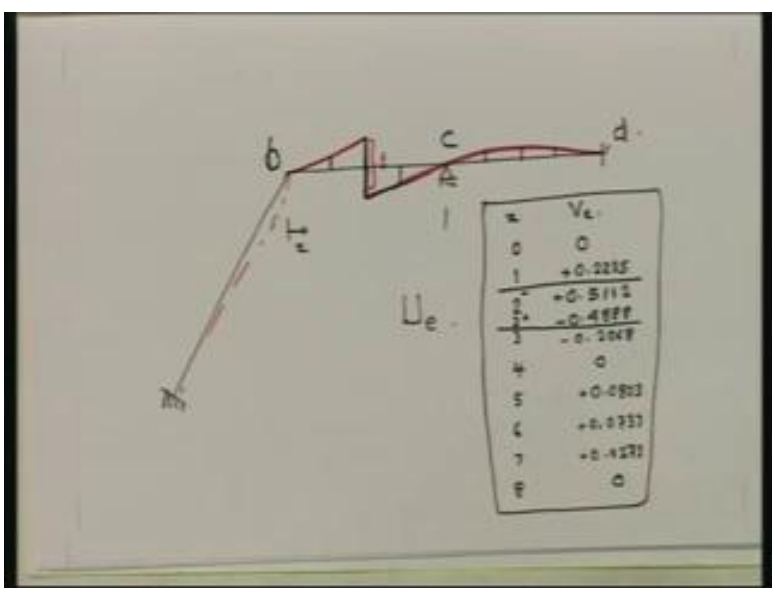

Then we have to actually get the quantitative also. This is fixed, this is hinged, this is fixed. Therefore, what we need to do is… here, get it up and here, get this to go down; when you make it go down, this this will go in this fashion and go this way, this is going to go in this fashion – this (Refer Slide Time: 50:10) and this have to be the same, so that this is going to be something like this; this does not matter. It is this, this and this and this total is going to be equal to 1. This is for V_e. Finally, as I said, I am not going to show you how I have done it but I am going to put down for x, so x is moving from b to c to d at 1 meter intervals. It is 1 meter **(**Refer Slide Time: 51:03) here, here, here, here, then here, here and here. At 1 meter intervals, I am plotting it starting from here – my x starts from here.

For 0, 1, 2, 3, 4, 5, 6, 7 and 8 I am putting down this as x, I am putting down the derived values of V_e for you – you are going to actually calculate it. For 0 this is 0, for 1 it is plus 0.2235, for 2 there is going to be a 2 minus and then there is going to be a 2 plus; for 2 minus this is plus 0.5112 and for 2 minus it is minus 0.4888 – this is for 2; for 3, it is minus 0.2068, for 4, it is 0, for 5, it is plus 0.0803, for this, it is plus 0.0733, this is 0.0273 and for 8 it is 0.

I am going to leave this with you; you have to solve this problem and get these V_e – use this. Next time, I am going to do it in detail and show it to you. Do not wait for that – try to get these values using the procedure that I have developed for you. Thank you. See you next time.