Structural Analysis – II
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Lecture – 38

Good morning. We have been looking at influence lines for the last couple of lectures and today we are going to continue looking at influence lines. If you remember, in the first lecture I introduced you to the concept of influence lines, what an influence line was. Then we looked at how to get the influence line for structures and we saw that there were essentially two methods. One was the direct approach in which you use equilibrium to obtain the influence line. The disadvantage of this method is that you need perforce a statically determinate beam or frame, whatever you want to call it, before you can use equilibrium.

Secondly, it is a more time-consuming procedure obviously because you have to evaluate every response quantity and you have to get the expression for that response quantity as the load moves along the path that it is supposed to move. Therefore, we saw that the Müller–Breslau principle is based on the method of virtual displacement. The Müller–Breslau principle was an easy way of getting the influence lines.

We also saw that the Müller–Breslau principle can both give a qualitative and a quantitative assessment of the influence line of any response quantity for a statically determinate structure but when we came to a statically indeterminate structure, we saw that the Müller–Breslau principle can only be used to get the influence line in a qualitative fashion. The Müller–Breslau principle cannot give the influence line quantitatively. Therefore, we saw that to get a qualitative assessment, the Müller–Breslau principle is okay but to get a quantitative assessment, we have to go back to the direct method.

The direct method is based on equilibrium and we saw an example last time as to how for a specific structure, because we could evaluate, we knew how to evaluate the fixed end moment and the fixed end moment directly was the bending moment, we could get a statically determinate structure and we could solve it. Unfortunately, that was a very specific case because we looked at a specific problem where we had a single-span beam with one fixed and one pinned. Therefore, we knew the fixed end moment and the fixed end moment was directly the moment itself – bending moment at that particular point. Therefore, from that, we could directly get static equilibrium. However, it is not always true that this is valid.

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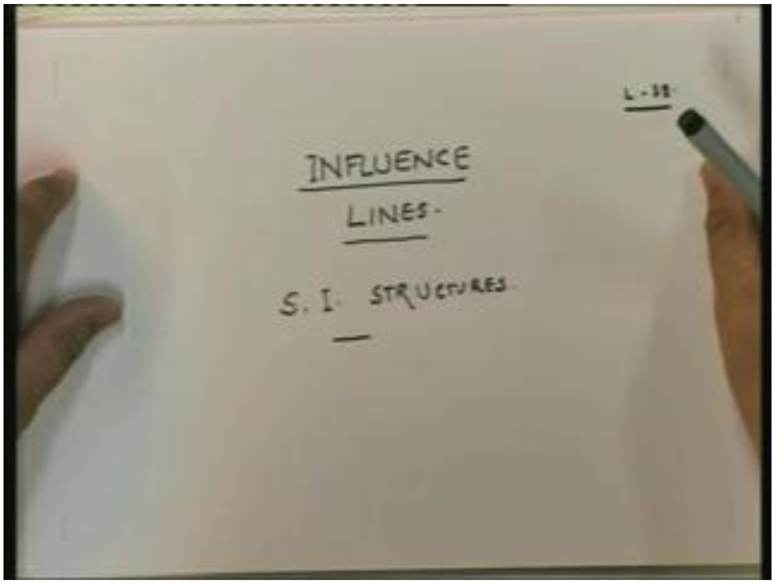

Today, we are going to look at influence lines and it is going to be statically indeterminate structures and somewhere where you could not directly get the statically determinate beam by finding out fixed end moment. In a single-span beam, it was not a problem; as soon as we go in to multi-span beams, you cannot do it, but then the direct approach is the only approach to get a quantitative assessment of the influence lines. Therefore, we have to figure out how to get a statically determinate structure where equilibrium can be used for multi-span beams or more complicated structures – let us look at this. By the way this is the thirty-eighth in the series of lectures for this particular course that I have done. Let us look at the problem.

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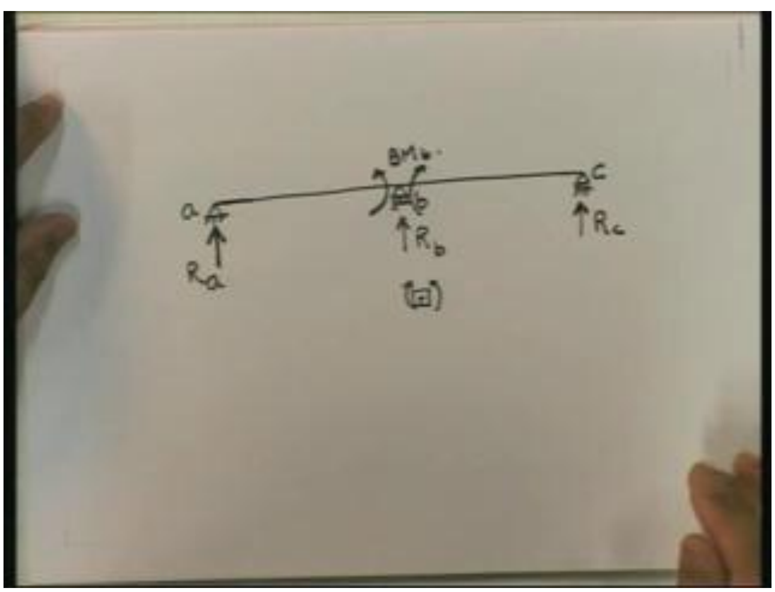

This is a two-span beam and we have to find out the influence line for various parameters. I am going to define some responses for which we will have to find out the influence line and today, we are going to see how this can be applied to get the influence line both qualitatively as well as quantitatively. Let me put few of the response quantities. Obviously, in this particular beam, the moving load is going to go from a to b to c. Let us look at R_a, let us look at R_b, let us look at R_c and let us look at $(BM)_b$. Let us just do these four.

This kind of bending moment actually implies that it is this way (Refer Slide Time: 07:59). This is the internal positive and that is why this is also positive because this is the right phase, this is this way, this is the left phase, this is this way. This is the internal force, this is the external force that balances the internal force. Note that whenever we do, we have to externalize all forces before we can apply the Müller–Breslau principle. Let us see how we solve this particular problem.

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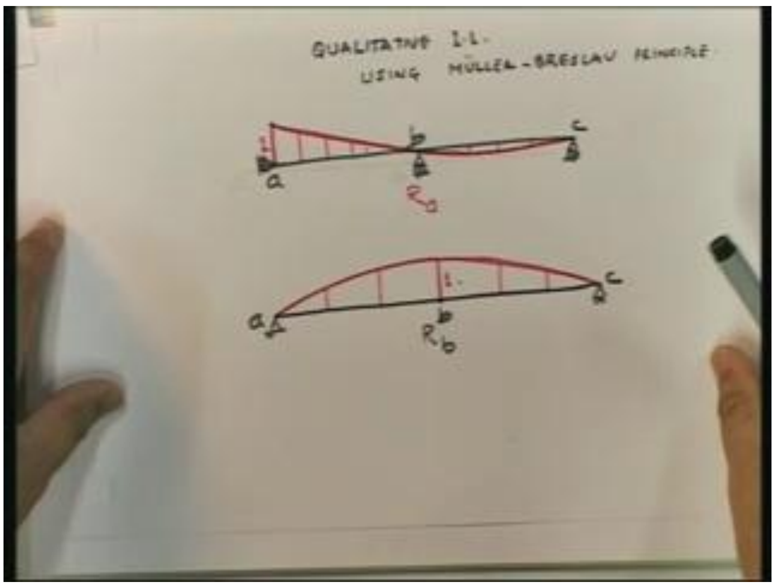

First, qualitative: We are going to do qualitative influence lines and we will be using the Müller–Breslau principle. We draw this. For R_a, this becomes this way pinned and then you have to give a unit displacement here and then… this is for R_a (Refer Slide Time: 9:53) Let me do it for R_b. Understand that all we are doing is giving unit displacement. It is the same principle whether it is a statically determinate structure or whether a statically indeterminate structure. As Müller–Breslau principle says release the restraint corresponding to the response you are trying to find out, give it a unit displacement and the deflected shape of the structure is going to represent the influence line.

Therefore, this represents influence line. Note something interesting: in this particular case, you will see that this is going to be symmetric – this side and this side are going to be identical. Here this is going to be 1, there are going to be lot more displacements here than are going to be here. Here, this displacement, this displacement, this displacement **are a**ll the same, this is equal to this, this equal to this, they are the same, it is symmetric. That comes from the boundary end conditions being symmetrical; you are giving a displacement at the point, obviously, it is going to be symmetrical. Now, let us look at R_c and bending moment.

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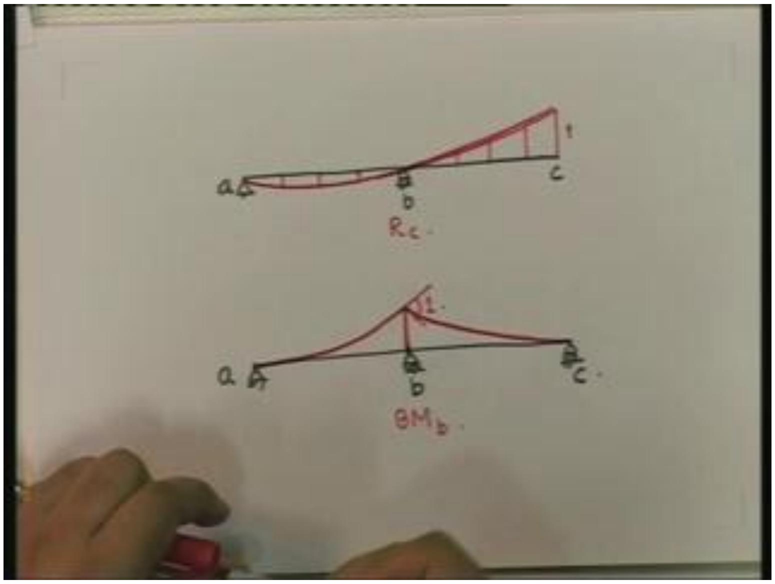

Remove the restraint at c, this is for R_c, give it a unit displacement and find out the deflected shape. You will notice that R_a and R_c have just the opposite, they are just the opposite; we will see that later when we do a quantitative analysis. I am just showing it to you because again this and this are just the opposite of each other. Therefore, what we got for R_a which was this and this way. Between ab, whatever the deflections are and between bc, whatever the deflections are for R_a, those are going to be between ab which is what in between b and c and it is just the opposite. Finally for the bending moment: for the bending moment a, b, c and what we have to do is give it a pinned and make it go this way.

Again, this is going to be symmetrical about point b. I do not know what this value is. However, I know that this has to be equal to 1 because again M_b right, so $(BM)_b$ and therefore, I have to give a unit displacement corresponding to the response quantity and so therefore, it goes this way. Why does it go this way? Because if you look at it, the moments do this, so it is going to go this way and this relative between the two tangents have to be equal to 1 at this point. The Müller–Breslau principle gives you the qualitative assessment. Now, the question comes: how do I quantitatively assess it?

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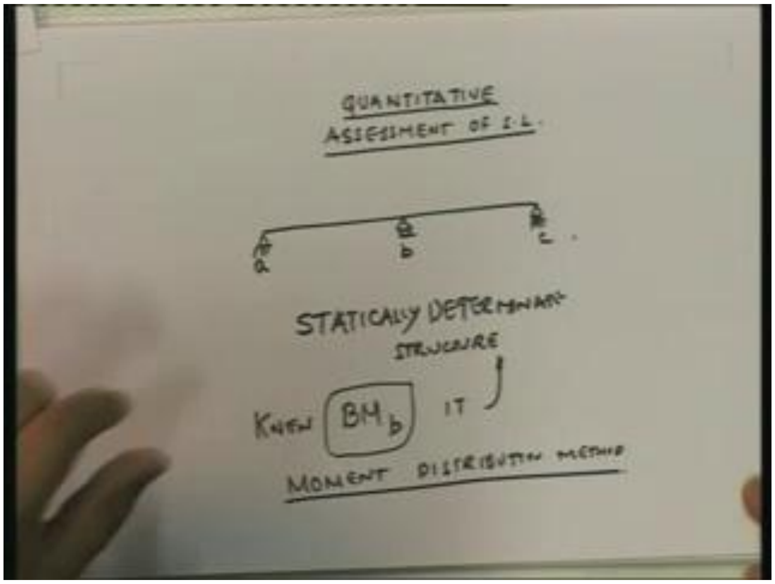

Quantitative assessment: What do I need to do for the quantitative assessment of influence lines? Remember that the only way that you can do quantitative assessment is by using the direct equilibrium approach. For direct equilibrium approach, note that if I know in this particular case a, b, c… for both member ab and bc, for example, if I knew the $(BM)_b$, would both ab and bc be statically determinate? Note that if I knew the $(BM)_b$, ab and bc would become statically determinate. Therefore, entire thing hinges around finding out the $(BM)_b$. But how can I find out the $(BM)_b$? The only way I can find out the $(BM)_b$ is to analyze the statically indeterminate structure and find out the bending moment at b. Which are the approaches we know? The flexibility approach, the stiffness approach or call them the force method and the displacement method or the moment distribution method.

Note that in both the force method and the displacement method what do you find out first? Do you find out the bending moment directly? No. You first find out actually the displacements corresponding to the degrees of freedom in both the force method and the displacement method; in the force method, you find out redundant force and from that solve the statically determinate and in the displacement method, you find out the displacement.

In other words, you do not directly find out any quantity without first finding out something else and then deriving it, but in this particular case, we do not need to know that under the load what is the displacement, we do not need to know under the load what would be the support reaction at c; I may not want to find out the influence line for support reaction at c – you understand the point. You have to understand another point that you actually have to solve this using the force method and displacement method you would have to actually solve it for different positions of x and find out the redundant or the displacements so it is actually solving many force methods and displacement methods. You can do it, I am not saying you cannot do it but it is not an efficient way of finding out an influence line because in an influence line.
All I am interested in is to find the ordinates of the influence line so that I can find out the particular response quantity and the best method which directly finds out the $(BM)_b$ without

finding out something else is the moment distribution method. Therefore, the whole thing over here is to make it a statically determinate structure. We know here that if we knew the $(BM)_b$ it would be a statically determinate structure, so we use the moment distribution method. How do we use the moment distribution method? Let us see.

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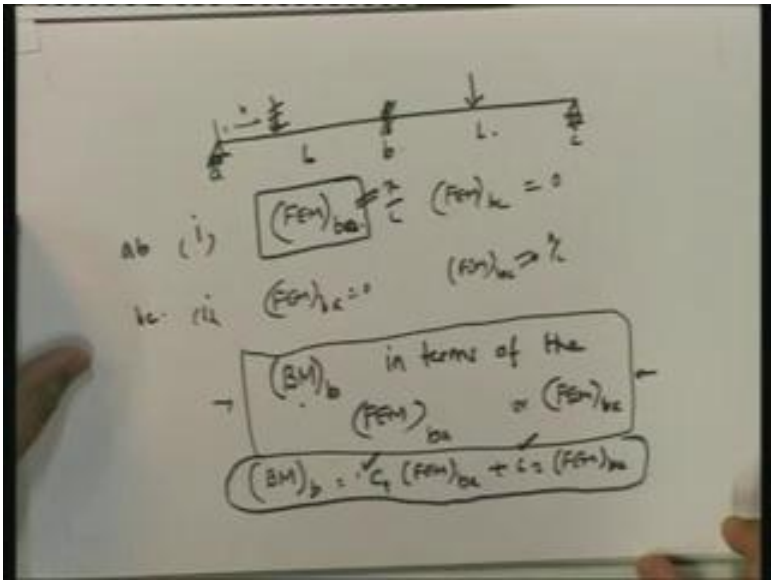

In the moment distribution you will actually fix this, do you not (Refer Slide Time: 20:59)? These you do not fix because these are ends, remember? We can do the modified 3 by 4 I upon L and use the modified member, if you remember. In this particular case the load could be between a and b. Suppose the load were between a and b, what would happen? You would get $(FEM)_{ba}$ and what would be the $(FEM)_{bc}$? It will be equal to 0. You get a fixed end moment and then what would you do? You would release it, you put this fixed end moment, find out etc. While the member is between a and b, we could actually find out the $(BM)_b$ by moment distribution by first finding out $(FEM)_{ba}$ and then doing it.

Now, suppose it is between b and c now, it is not here (Refer Slide Time: 22:13), it is here. Now, what happens? In that particular case, the second case, between ab and now the member is between bc, then $(FEM)_{ba}$ is equal to 0 and $(FEM)_{bc}$ is what you calculate. The point here is, if you could somehow find out the $(BM)_b$ in terms of the fixed end moment either ba or bc depending on where the load is, then you see, can I find out the fixed end moment given a load? Fixed end moment – I can find it out. If this is x, then this will be in terms of x and L etc; this is L, this is L, this also would be in terms of x and L etc.

In other words, given a position of the load, I could find out the fixed end moment and if I could find out bending moment in terms of this and this, then I could find out this in terms of x and L and then, that is it, we have solved the problem because once I know bending moment for a given position, I can find out everything else.

The whole point really boils down to this. All we need to find out is for example, possibly if we could get something like this is equal to c_1 $(FEM)_{ba}$ plus c_2 $(FEM)_{bc}$, if we could get an expression like this and we knew c_1 and c_2. Note that when the member is in ab, I know that $(FEM)_{bc}$ is 0, this would drop out and since I know this and I can find this in terms of x by L, I can find out bending moment and make both ab and bc as statically determinate structures. Suppose it is between b and c, then this would be 0, this would kick in, so all I need to do is really find out c_1 and c_2. If I can find out c_1 and c_2, I have solved my problem and therefore, we do now is the following.

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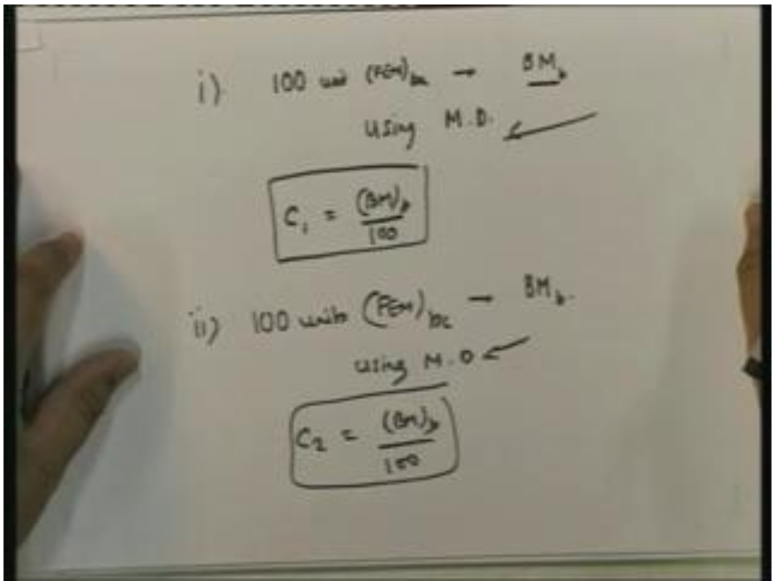

What we do is we find out c_1 and c_2 separately. How? All I need to do is apply 100 units fixed end and find out what the $(BM)_b$ is using moment distribution and whatever I find this (Refer Slide Time: 25:43), then c_1 is equal to $(BM)_b$ upon 100. Next, this is separate, I give 100 units $(FEM)_{bc}$ and find out the bending moment at b using moment distribution and then, c_2 is going to give me bending moment upon 100. This is the first case, this is the second case. Two different loads and two different moment distributions so let us do that.

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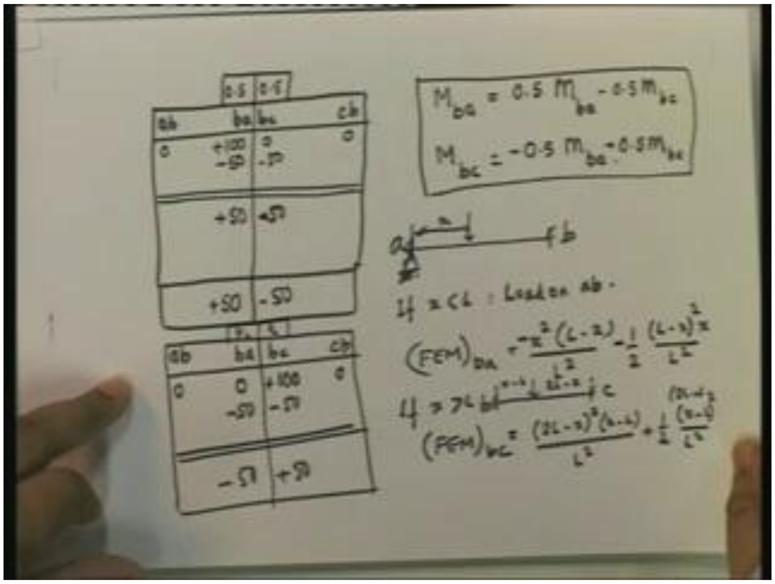

I am going to do the moment distribution moment; distribution is simple, go back and look at your thing. You will have ab, ba, bc, cb. For this, it will be 3 by 4 I by L, 3 by 4 I by L, 0.5, 0.5. What is the fixed end? First case, fixed end moment, I am applying plus 100, 0, 0, 0 (Refer Slide Time: 27:26). I am only applying a $(FEM)_{ba}$ because my whole goal here is to actually find out c_1, which relates $(BM)_b$ to the $(FEM)_{ba}$. How would I…? First is going to be distribution: minus 50, minus 50 and look at this, nothing is going to happen here because no carryover, no carryover, they are… it is already 3 by 4 modified, so what we get is plus 50, minus 50.

In other words, the $(BM)_{ba}$ is equal to 0.5 of the fixed end moment; instead of writing that down, I am going to write it in this fashion – m_{ba} and M_{bc} is equal to 0.5 $(FEM)_{ba}$. Let us now do the same thing, another moment distribution where ab, ba, bc, cb and now, I am doing the $(FEM)_{bc}$. I will do plus 100, this is 0, this is 0, this is 0, again the half, half, this is going to be minus 50, minus 50 and that has done it, minus 50, plus 50 and therefore, this is going to be minus 0.5 fixed end moment and this is going to be plus 0.5.

Note these are the $(BM)_{ba}$ (Refer Slide Time: 30:09) and bc, these are the fixed end moments, in other words, we found out the moments in terms of the fixed end moment at ba and bc, so now we have this expression. We have this situation. Wait a minute, this is this way, a, b; if the load is at x, if x is less than L, that means load on ab, then the $(FEM)_{ba}$ is equal to… think about it, it is going to be x squared into (L minus x) upon L squared, this is minus because it is anticlockwise, this is also minus half (L minus x) squared x upon L squared. You do it; I am not… I have done enough of this to let you know how you can find this out directly, so I am not going to ….

This is the fixed end moment if x is less than L. If x is greater than L, then $(FEM)_{bc}$…. Note that here I am going to be taking x, this is b, this is c and x is actually from this end, so this is going to be equal to x minus L – x is from a – and this is going to be 2 L minus x because x, remember, is going from 0 to 2 L. For this, fixed end moment is going to be equal to again, b is going to be positive, so it is going to be (2 L minus x) the whole squared into (x minus L) all over L squared plus half (x minus L) the whole squared into (2 L minus x) all over L squared. We have got the fixed end moments etc. I am going to write them down again here.

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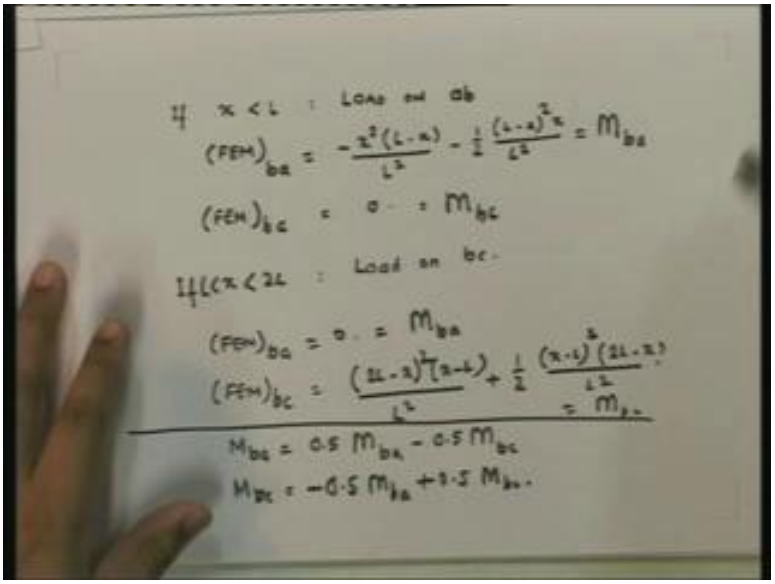

If x is less than L, that is load on ab, then $(FEM)_{ba}$ is going to be equal to minus x squared (L minus x) over L squared minus half (L minus x) the whole squared x upon L squared and $(FEM)_{bc}$ is equal to 0. If x is between L and 2 L, that is load on bc, then $(FEM)_{ba}$ is equal to 0 and the $(FEM)_{bc}$ is equal to (2 L minus x) the whole squared into (x minus L) upon L squared plus half into (x minus L) the whole squared into (2 L minus x) upon L squared. Finally, we also know that the $(BM)_{ba}$ is equal to 0.5.

This I am denoting as m_{ba} (Refer Slide Time: 35:27), this I am denoting as m_{bc}, this I am denoting as m_{ba} and this I am denoting as m_{bc}. I am writing down… the reason I am doing this… it is not for anything else, it is still the fixed end moment; it is just that writing down long expressions does not serve a purpose; I am using a smaller notation, that is all, there is nothing, there is no other reason for this. Bending moment at bc is equal to minus 0.5 m_{ba} plus 0.5 m_{bc}. Now, we have these and finally, if I know the moments, then let us see what we can do, let us look at it.

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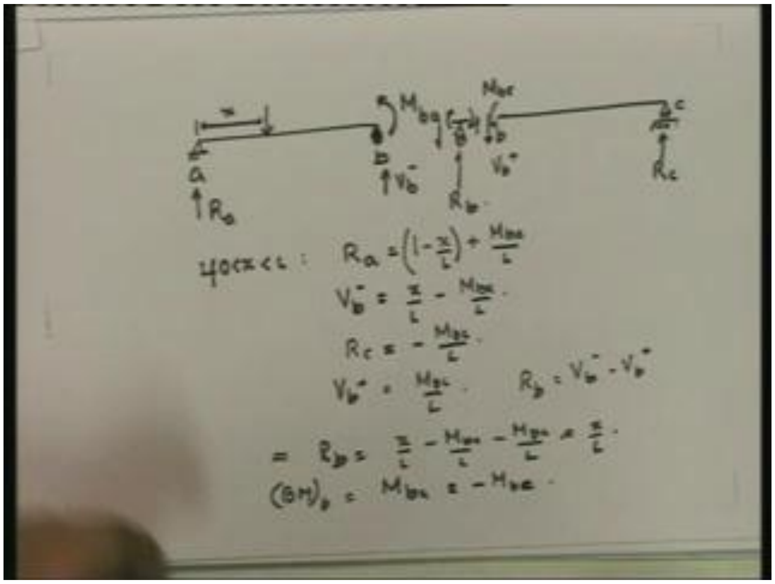

Essentially, this is member ab where we know M_{ba}. Now if I know M_{ba}, let us look at what I can find out. By the way, let us assume that the load is in. If we look at this, then you can see that R_a and I will call this V_b (Refer Slide Time: 37:24) because this is an open section where…. This is the support b, so you have made a cut and then we have bc; here, if I have like this, then this goes this way and this is M_{bc}, this is R_c, this is V_b minus – this is just to the other side, this is V_b plus and if you look at it, this one is going to be positive this way, this one is positive this way, therefore this one comes down here and this one goes up here. Let us see that R_a directly is equal to R_a.

We can find out the expression for R_a if x is between 0 and L, then R_a is equal to… one aspect of it is 1 minus x over L that is directly from the load, the other one is from the bending moment and this is going to be this upon L, so that is R_a. Let us look at what happens to V_b minus. V_b minus is equal to x by L minus M_{ba} upon L. Let us look at what happens here. What is R_c equal to? R_c is equal to minus just directly… so R_c is going to be minus M_{bc} by L. V_b plus is equal to M_{bc} by L and since R_b is equal to V_b minus minus V_b plus, R_b is equal to V_b minus, minus V_b plus, it will become x by L minus M_{ba} by L minus M_{bc} by L. We will come to it. Also, if you look at $(BM)_b$, the $(BM)_b$ is equal to M_{ba} because this is positive (Refer Slide Time: 41:28).

The $(BM)_b$ is equal to M_{ba} or minus of M_{bc}; in fact, M_{ba} is equal to minus of M_{bc} and you will see that since M_{ba} and M_{bc} are minus, this actually lands up being x over L (Refer Slide Time: 41:54). Let us now proceed. If the load is between a and b, these are the expressions. We have found out everything – we have found out R_a, R_b, R_c and the $(BM)_b$ – the expressions.

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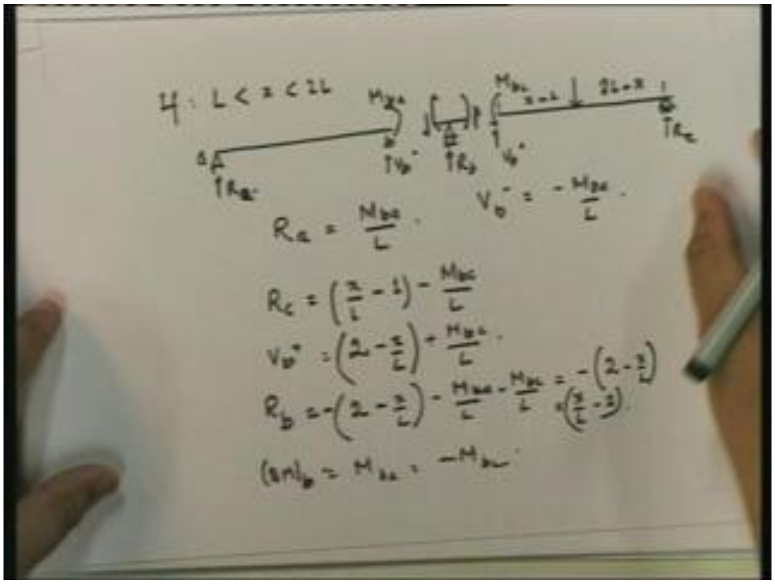

If L is between x, what happens here is this**; as** far as a and b are concerned then we have this, so this remains as M_{ba}, this is V_b minus, this is R_a (Refer Slide Time: 43:00), this is R_b, this is going to be this way, this is going to be this way, this is going this way, this is going this way and on this side, I have M_{bc}, this is M_{ba} (Refer Slide Time: 43:20), M_{bc}, R_c (Refer Slide Time: 43:24) and now this position **is** such that this is 2 L minus x. This is when x is between L and 2 L**.** Then, what would R_a be equal to? You will see R_a is directly equal to M_{ba} by L, V_b minus would be minus M_{ba} by L and then, R_c would be equal to… this would be x L (Refer Slide Time: 44:25), so R_c would be x by L minus 1 and this (Refer Slide Time: 44:44) is from the loading itself and then, the other part is minus M_{bc} by L – that is R_c and then, V_b plus, that is this one – V_b plus, V_b plus would be equal to 1 minus this, so it would be essentially 1 minus this will be 1 minus this will be 2 minus x by L – that is from this part – plus M_{bc} upon L and this implies that R_b is equal to, here it is V_b minus this so this becomes minus (2 minus x upon L) and then minus M_{ba} by L minus M_{bc} upon L which basically means minus (2 minus x over L) which is essentially x over L minus 2 and $(BM)_b$ still continues to be M_{ba} or minus M_{bc}. Now, we have all the expressions and I will just illustrate what these would be for a particular value.

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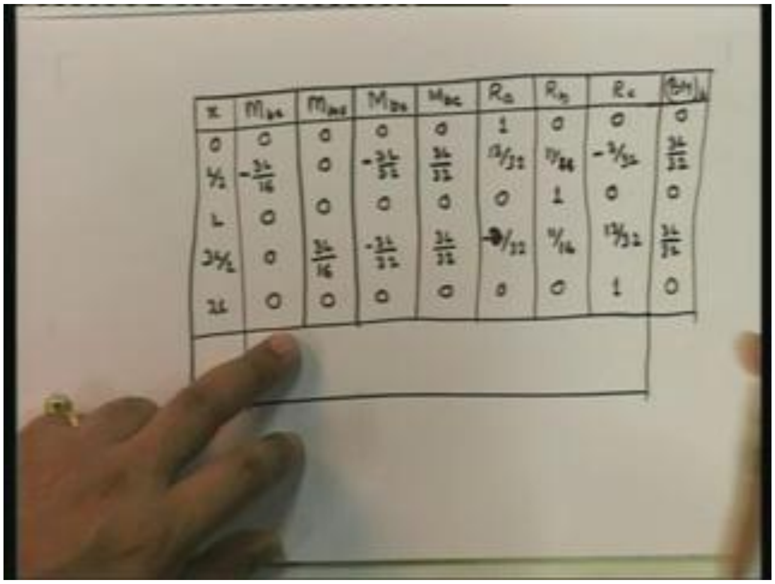

You can obviously find out more details as we go along, but I am just going to do this for specific quantities.

This is going to be x and I am going to say 0, there is no L by 4 so I will just do it L by 2, then L, 3 L by 2 and 2 L. The reason why I am only doing these… of course, you just have to evaluate it. You should be able to evaluate that. You can find it as one-eighth of L, you can just try to find it at one-eighth of L that can be an exercise for you to do so that you can get a finer thing on the influence line – I am just doing it grossly.

The first thing we need to find out is the fixed end moment and I am going to put that as m_{ba}, then I am going to make fixed end moment at m_{bc}. From that, we can find out the moments at the ends ba and member ab, we can find out M_{bc} and once we find out these, then we can find out…. What can we find out? The quantities that we are interested in R_a, then R_b, R_c and then I will just draw another line, $(BM)_b$.

The first step is to find out these values. I am going to spread out all these in front of me so that I can find out each one of them directly. When this is between 0 and L, this, this and this are 0 (Refer Slide Time: 49:19) and then, when it is between L and 3 L by 2, this is 0 this we have already done. For this, I know that this is also equal to 0 and here, this is also equal to 0. Let me find out for L by 2. If I find it out for L by 2, what would be the value? Just plug in x equal to L upon 2, substitute it in and you will see that this becomes 3 L upon 16, then plug in over here too minus 3 L upon 16 because it is anticlockwise and this is going to be plus 3 L upon 16. Let us now do this. M_{ba} we know in terms of the fixed end moments at ba and bc, so this is going to be 0 0 (Refer Slide Time: 50:27).

What is the expression for R_a when x is between L and L? Just plug it in M_{ba} is 0, so you get x is equal to L, so R_a is equal to 1 and R_b is x by L, so that is 0, R_c is minus M_{bc} by L, that is 0 and the bending moment is M_{ba} which is 0. Let us now find out for this (Refer Slide Time: 50:57).

Obviously, M_{ba} and M_{bc} are equal to 0. R_a is equal to 1 minus x by L, x by L is 1 so this is 0, R_b is equal to x by L, so it is equal to 1 and R_c is equal to minus M_{bc} by L so this is 0 and bending moment is also 0. Let me find it out for here. This is going to be 0 and 0. Let me see what R_a is, it is M_{ba} upon L, so this is 0. What about R_b? R_b is equal to x upon L minus 2 x upon L is going to give me 2 minus 2 so this is going to be 0 and R_c is going to be equal to x upon L minus 1 so this is going to be 1, this is going to be 0.
Let me evaluate it here. If we look at this, M_{ba} is half of this (Refer Slide Time: 52:02), so this becomes minus 3 L upon 32. What is M_{bc}? It is minus M_{ba}, so this is going to be 3 L by 32 and R_a is going to be 1 minus x by L, 1 minus x by L is 0.5, 0.5 plus M_{ba} by L, M_{ba} is minus and so, this is going to be equal to 0.5 minus 3 by 32, that is going to be 16, so this is going to be 13 by 32 positive (Refer Slide Time: 53:02), R_b is going to be x by L, so x by L is going to be equal to half and R….

Now, V_b plus is actually this way (Refer Slide Time: 53:35) and this is this way, so V_b plus and this is going to be this plus this, so you are going to have plus this and so, this is not x upon L (Refer Slide Time: 53:50)**,** this is plus and plus. The other point that I would like to make is that M_{bc} is this way, R_b is equal to this and then, this is proper. Actually, this is equal to not just half, it is going to be this plus 6 upon 32, that is 3 upon 16, so it is going to be 11 upon 16 and R_c is going to be minus M_{bc} upon L, so that is going to be equal to 3 by 32 negative because here (Refer Slide Time: 54:52), you have this so it is minus M_{ba} upon L and the bending moment is going to be equal to 3 L upon 32. Now let us go over here. This is going to be 3 by 16, so M_{ba} is going to be minus 3 L by 32, this is going to be 3 L by 32 and therefore, you get exactly the same values, 13 by 32, 11 by 16, this becomes minus 3 by 32, this becomes 13 by 32 and this is 3 L upon 32 – this is the end.

If you look at this, you will see R_a is 1. At the center point, it is 13 by 32 which is less than half – as you expected it; over here, It is minus 3 by 32, much less than this – this is what you expected; it is 0 here, 0 here and 1 here – that is there. If you look at R_b, it is symmetrical 0, 11 by 16, you would expect that this is more than half and 11 by 16, symmetric and this one is just the opposite and this is how it is. If you look at the bending moment, you will see that it is indeed correct. In other words, we have illustrated how to use moment distribution to get the influence lines qualitatively for a particular structure.

I will stop here. I will continue over the next few lectures looking at other examples of this. Thank you.