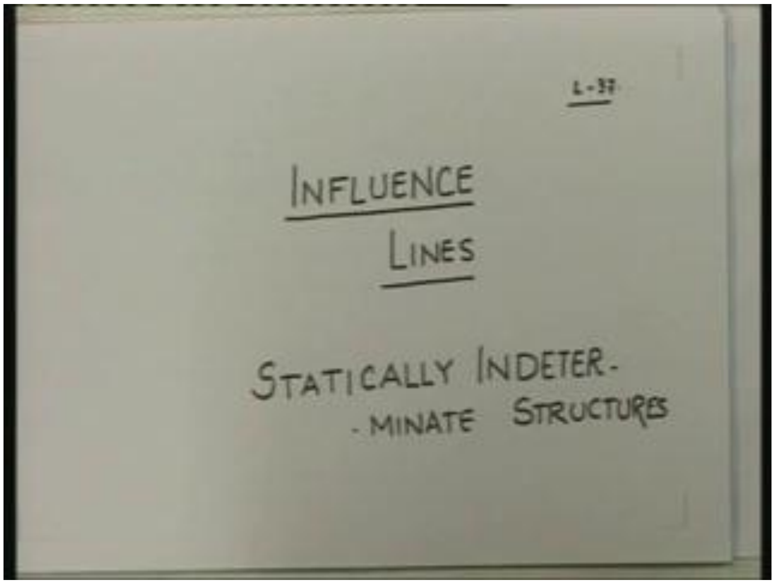
Structural Analysis - II
Prof. P. Banerjee
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 37

(Refer Slide Time: 01:31)



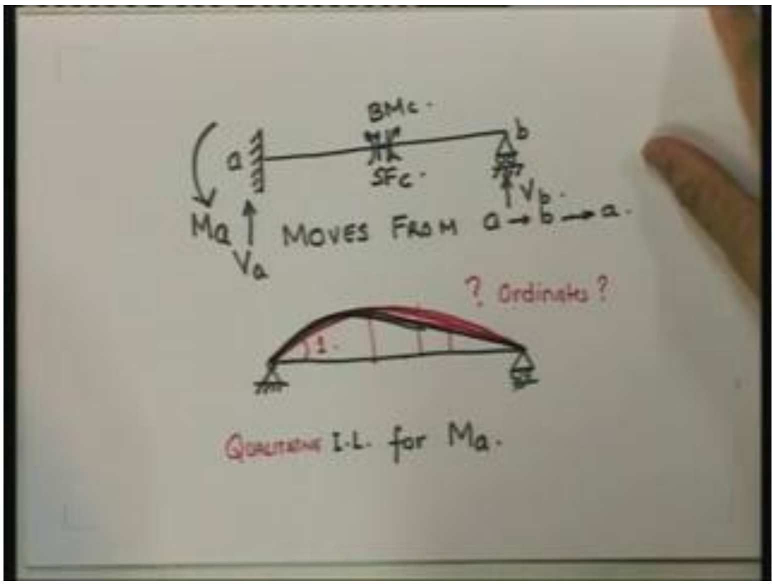
Good morning. We have been looking at influence lines and the concept behind influence lines. In the last lecture, I reviewed the concept for statically determinate structures, which I think you have already been exposed to – it was just a review – and also a review of the Müller–Breslau principle to show the application of the Müller–Breslau principle for statically determinate structures to lead into what we are going to be doing today, which is influence lines for statically indeterminate structures. Today, we are going to be looking at influence lines for statically indeterminate structures and how do we look at the influence lines for statically indeterminate.

For determinate structures, the Müller–Breslau principle itself application since for a statically determinate structure, when you remove the restraint corresponding to whatever it is that you are finding out, you make it into a first-order mechanism. Therefore, all influence line for a statically determinate structure are straight lines and since they are straight lines, you can find out the ordinates at every point and therefore, you know exactly what the values are for a statically determinate structure. You do not need to follow any other approach to determine what the influence line values are at different ordinates for different positions of the moving loads.

Today, we are going to be looking at statically indeterminate structure and we will see that for statically indeterminate structure, to draw the influence line there are two approaches. One: to draw the qualitative influence line, to get a feel for how the influence line looks, we use the Müller–Breslau principle. However, in statically indeterminate structures, even if you remove one restraint, it does not make it a mechanism.

Therefore, typically, influence lines for a statically indeterminate structure are not straight lines and therefore, the Müller–Breslau principle application will only give us a qualitative form of the influence lines. To actually get the ordinates, we need to go back to our direct approach, which is using equilibrium to obtain the coordinates. For the next three or four lectures, this is what I am going to be looking at. Today, I am just going to introduce the concept and see how to qualitatively evaluate or qualitatively determine the influence lines for a statically indeterminate structure.

(Refer Slide Time: 05:03)

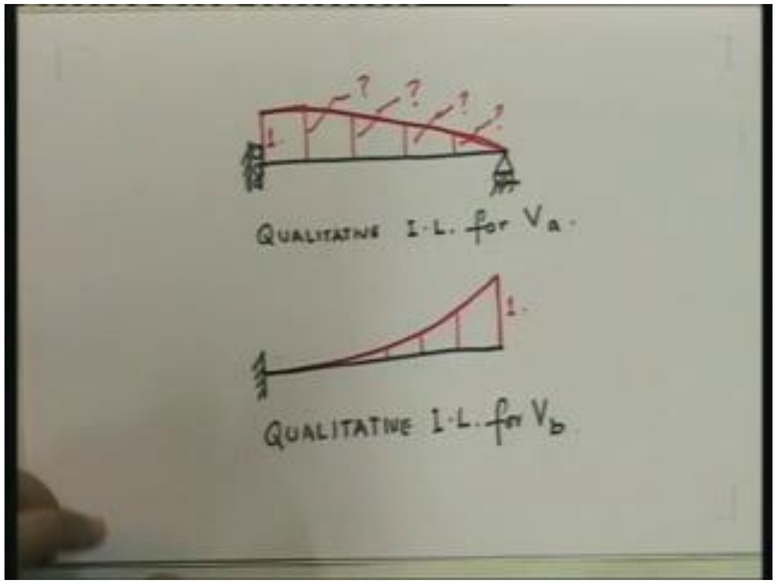


Let us first look at this. This is the structure and the moving load moves from a to b or b to a – it does not matter, it moves between a and b. How do we draw the influence line for a particular response quantity? Let us look at some of the response quantities. Let us say that I want to find out the influence line for M_a, the vertical reaction at a, the vertical reaction at b and at the center I want to find out bending moment at c as well as shear force at c. We want to find out the influence lines for all of these.

Let us start one by one. I want to find out the influence lines for this. M_a – what do I do to find out for M_a? I release influence line M_a – I release and give a unit rotation. If I give a unit rotation in this direction (Refer Slide Time: 07:49), this is how the influence line looks, where this is 1. What have I done? Müller–Breslau principle: release and then given a unit rotation corresponding to the influence line and this is the shape. What are the ordinates? I do not know.

However, qualitatively, this is how it will look. In other words, when the load is here, it is going to be 0, when the load is here, it is going to be 0 and in between, it is going to reach a maximum somewhere between the half-way point and this and it is always going to be positive. In other words, the M_a is always in this direction and the value itself I can always find out. I do not know the ordinates, but.… Qualitatively, qualitative influence line for M_a. Next, I want to draw the influence line for V_a. For V_a, what do I do?

(Refer Slide Time: 09:44)



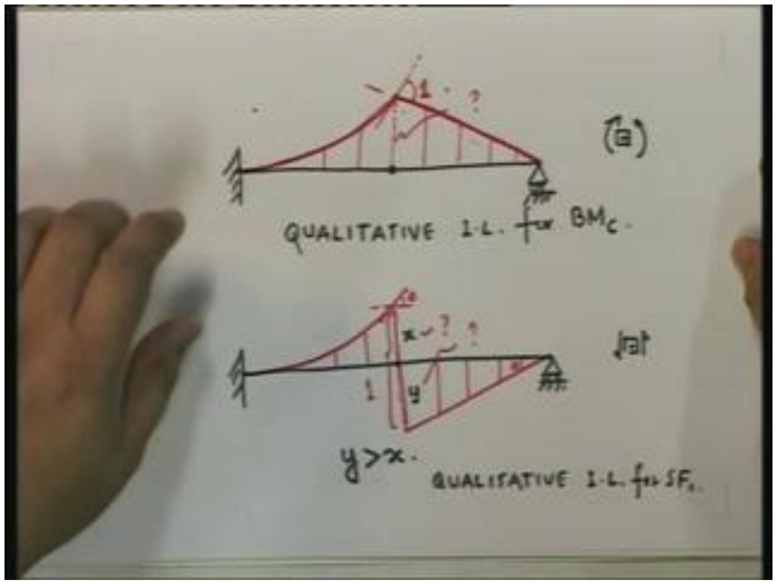
I have released that and then I give a unit displacement in the direction of the force and the influence line looks like this. The slope here is 0, this is 1, and here it is 0 – that is it. Again, the values – I do not know; I do not know the values, so what do I do? I release this but note that it is still fixed, so it is going to go up 1, so this is going to remain horizontal – the tangent, then it is going to go in this way. I am going to look at the others by and by, but what have we learnt here in these two influence lines?

The influence lines are that the Müller–Breslau principle can be applied to get a sense of the ordinates – we can get a sense of the ordinates. However, we do not know what the ordinates are because the lines are no longer straight. For a statically determinate structure, when you release one restraint, it becomes a mechanism, we get straight lines and the Müller–Breslau principle directly gives me the influence line, the type of the influence line as well as the ordinates of the influence line, but in statically indeterminate structures, we only get a qualitative sense of the influence line; however, we know nothing about the ordinates because we do not know what the ordinates are going to be using the Müller–Breslau principle alone.

But let us continue looking at it because even when we compute (we will see how we compute later on), we need the qualitative assessment to be able to check that our computations that we have carried out are correct. It is very important to use the Müller–Breslau principle even for statically indeterminate structures even though you do not directly get the ordinates but you get a feeling of how the influence line is going to look; you also get a feeling of where it is going to be positive, where it is going to be negative etc.

Getting back to this particular… we have done it for M_a and V_a and now we want to do it for V_b. How would we do it for V_b? For V_b, we release that; when we release that, this becomes a cantilever and we give it a unit displacement. This is how it looks; the influence line actually resembles the cantilever shape function. One of the approaches is not a very popular approach, but you can see that I can put a unit load here, find out this shape and I could find out this shape for a cantilever and that shape would directly give me the ordinates then, but I am not going to follow that method for this approach at all.

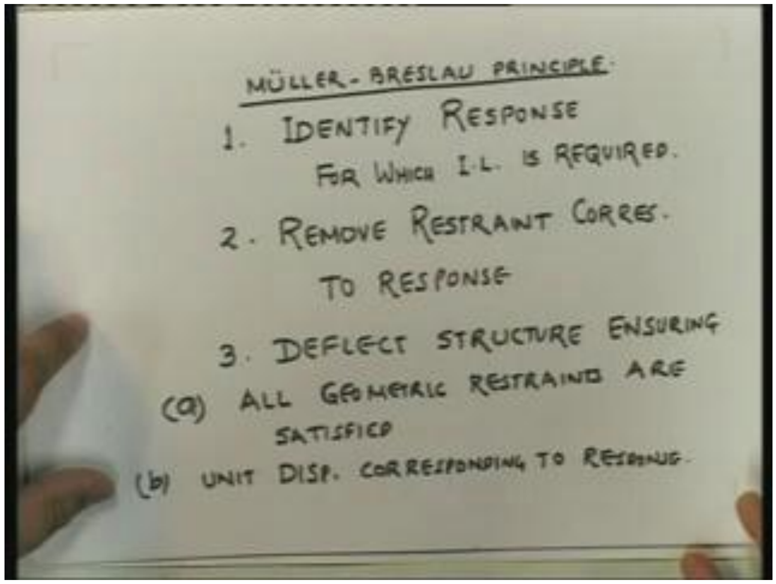
(Refer Slide Time: 15:30)



Let us look at it. For the bending moment at c, how will the structure look? For bending moment, I release the relative rotation and then I do this. If I do this, what happens? This part is straight – this part is a straight line, this is a curved line and the tangent, this is equal to 1, this is the tangent at this point and this straight line – the angle is 1. How much is this value? I do not know; also, I do not know about others. All I know is that this relative rotation is 1. Again, you will see that if we define bending moment as positive this way, then this is the sense of the bending moment diagram at c.

Finally, let us look at the shear force at c. How do I do it? If this is to be the positive sense, then this one on the right will have to move up and this will have to move down, so it is going to look something like this. This angle theta (Refer Slide Time: 18:20) and this angle theta have to be identical and this one will be 1. All I know is that if this is x and this is y, because this is a cantilever and this theta has to be equal to this theta, obviously y will be greater than x – that much I know, but what will be the values? I have no idea. This is the qualitative influence line for shear force at c. This is for bending moment at c, shear force at c, this is for the reaction at a, reaction at b and the moment at a. We can find these out. This is the pure application of the Müller–Breslau principle.

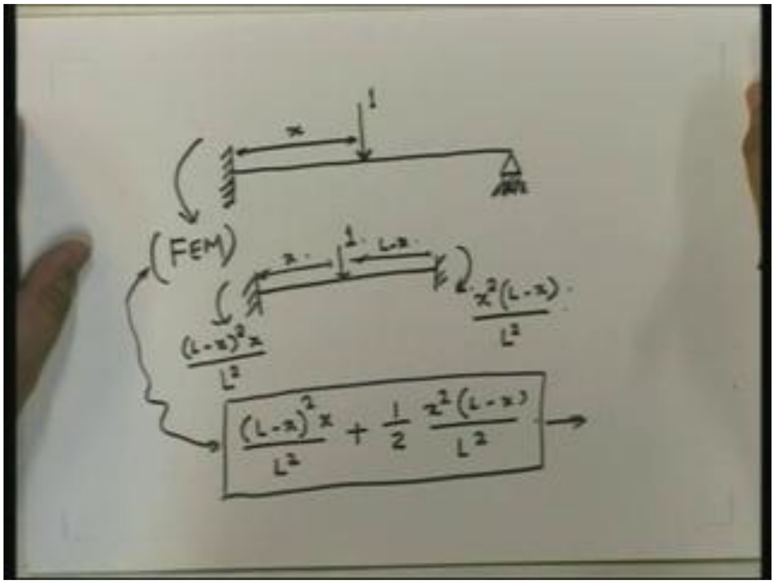
(Refer Slide Time: 19:52)



Essentially, just to write down what the Müller–Breslau principle does. One: identify a response for which IL is required. Two: remove the restraint corresponding to response. Three: deflect the structure, ensuring a) all geometric restraints are satisfied and b) unit displacement corresponding to unit displacement corresponding to response. Once you do that, you have got the influence line for a statically determinate structure, the Müller–Breslau principle gives you not only the qualitative shape, it also gives you the quantitative values at every point of this thing; if you have a statically indeterminate structure, the Müller–Breslau principle only gives you the qualitative values of the influence lines.

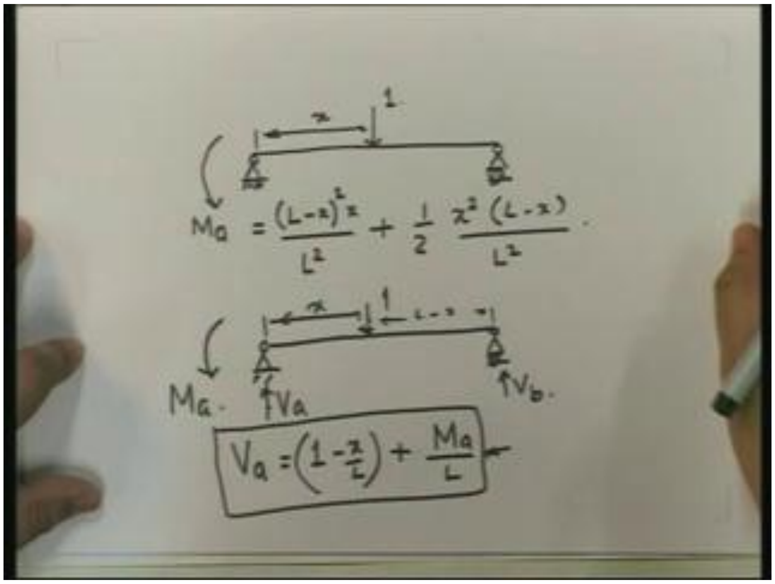
We see that the Müller–Breslau principle is a very useful tool for influence lines, but for statically indeterminate structures, it essentially gives you only a qualitative and just getting qualitative influence lines is not good enough because you need to know what the values are. Why do you do influence lines? Because ultimately, we will show that a little bit later that the reason why you draw the influence lines is actually to find out the values of a particular response quantity given a particular load. Therefore, to find out the value of a response quantity, you need to know the quantitative values also. For a statically determinate structure, the Müller–Breslau principle gives it. How do we do it for the statically indeterminate structure?

(Refer Slide Time: 24:34)



For that, we actually need to go back to our first principles. That is, put the load here at a distance x – I am still dealing with the same problem. Can I use equilibrium for this? This is a statically indeterminate structure. How do I do it? Can I find out something for this? Does this propped cantilever remind you of something? (Refer Slide Time: 25:38) Look back to the displacement method, the member. Does this remind you of a particular kind of member? Does this not remind you of the modified member with a fixed end at one end and roller at the other end? Now, do we know what the fixed end moment is? We know the fixed end moment for this, do we not? We have already done this. This is unit load (Refer Slide Time: 26:28) what is this (Refer Slide Time: 26:33)? This becomes L minus x. If you look at this, this one is given by (L minus x) squared into x upon L squared. What is this? This is this way. This is equal to x squared into (L minus x) upon L squared. We have already done this, do you not remember? Now for this one, to get the modified fixed end moment, how does it go? Well, release this, put an opposite one, carry over half. This fixed end moment is equal to (L minus x) squared into x upon L squared plus half x squared (L minus x) upon L squared. I can find out the fixed end moment directly using this.

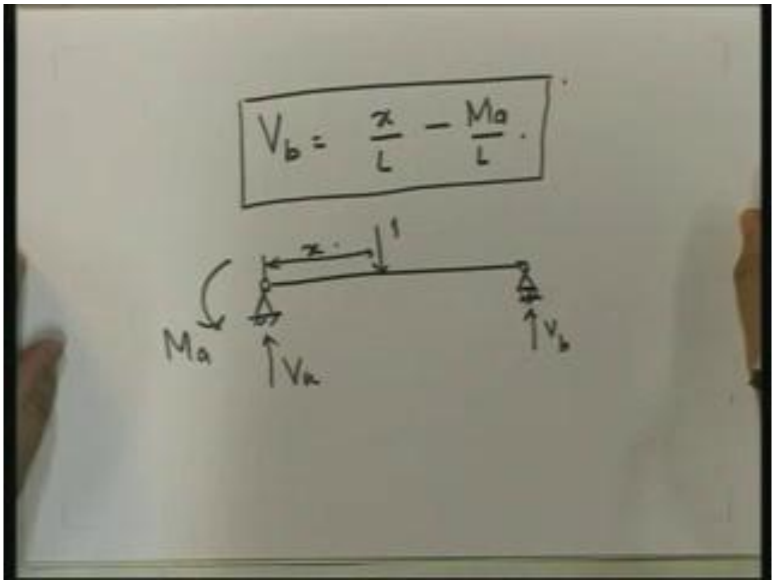
(Refer Slide Time: 28:18)



Once I find out the fixed end moment, think about it, this problem then becomes this problem with unit load, where I know this (Refer Slide Time: 28:34). This, which I can call as M_a, is going to be equal to… because this is a fixed end, so that is M_a, which is equal to (L minus x) squared x upon L squared plus half x squared into (L minus x) upon L squared. If you look at this, is this not a statically determinate structure, where I know this moment? Actually, I know both the member end moments – this moment is equal to 0 and I know the value of this moment. Given length L and given the position of the load, I know this value; if I know this value and since this is a simply supported structure, I can find out all the others using equilibrium. Let us go through the process. What is that?

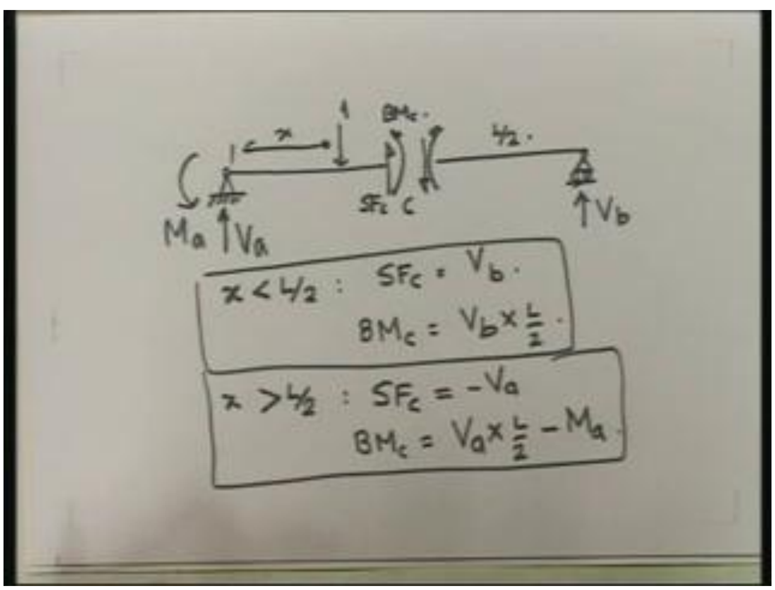
One of the things I wanted to find out was M_a – I have already found that out directly. The next was finding out V_a. What is V_a going to be equal to? Let us look at it. This is x, this is V_a, this is V_b (Refer Slide Time: 30:14), then, of course, there is M_a. If you look at it, V_a is equal to 1 minus x by L – that is the part of this load that comes here. How did I find that out? Take moments about this particular point and you will see that this will be V_a into L is equal to 1 into L minus x; this is L minus x remember that; so when you did that, this becomes this plus now if you look at this, this is doing this, so this is going to be this, it is going to be plus M_a by L. Now, I know M_a, I know V_a. What is V_b going to be equal to?

(Refer Slide Time: 31:27)



V_b is going to be equal to x upon L minus M_a upon L. In this particular case, I know M_a, I know V_a, I know V_b where the load is x. Therefore, the main thing over here is that I I have found these three. The next thing that I wanted to find out was the moment and shear force at the center. Let us see how we can solve for that.

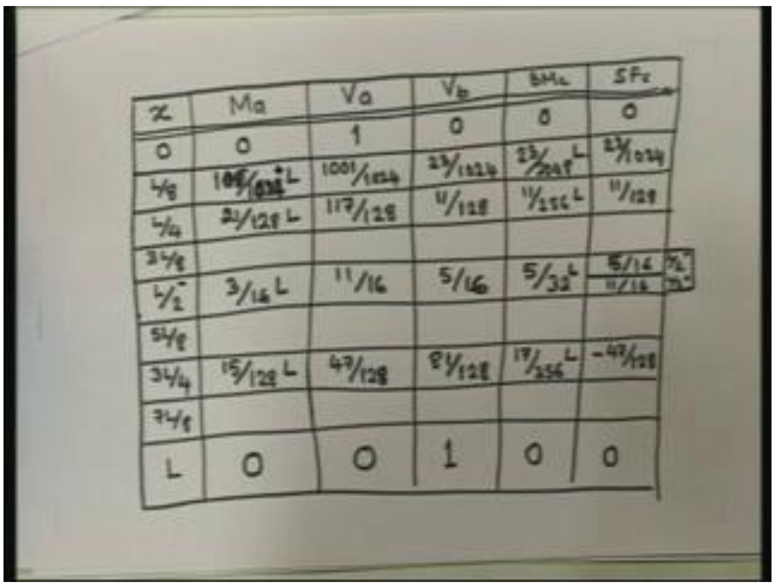
(Refer Slide Time: 32:31)



Let us look at this situation, let us assume that x… so, the shear force and bending moment are this way, this way, this is V_a (Refer Slide Time: 33:11), this is M_a, this is V_b and this is L over 2 – this length, because this is point c, this is shear force at c and this is bending moment at c. If x is less than L by 2 then shear force at c is equal to look at it V_b directly, bending moment at c is equal to… if you look at this, this does this, this is equal to V_b into L over 2.

If x is greater than L by 2, then look at what happens. If x is greater than L by 2, then this comes on this side (Refer Slide Time: 34:35) and if you look at the shear force, the shear force is equal to minus V_a because shear force plus V_a because this is now over here – this unit load is now over here, so it is going to be shear force plus V_a is equal to 0 because the 1 is here right now, x is greater than L by 2, so this going to become V_a plus SF_c is equal to 0, and bending moment is going to be equal to… let us look at it: this is opposing it, it is going to be (V_a into L over 2) minus M_a. The values of shear force and bending moment are different for x is less than L by 2; the position of the load is different. Now, the point is that I have found out the expressions for all the quantities that I wanted and let me now draw.

(Refer Slide Time: 35:59)



x	Ma	Va	Vb	BMc	SFc
0	0	1	0	0	0
L/8	105/1024 L	1001/1024	23/1024	23/2048 L	23/1024
L/4	21/128 L	117/128	11/128	11/256 L	11/128
3L/8					
L/2	3/16 L	11/16	5/16	5/32 L	5/16 (L/2⁻) / −11/16 (L/2⁺)
5L/8					
3L/4	15/128 L	47/128	81/128	17/256 L	−47/128
7L/8					
L	0	0	1	0	0

Let me just say 1, 2, 3, 4, 5, 6, 7, 8 and 9. First is x value and next M_a. Then, once we get M_a, the next is V_a and V_b and once we get V_a and V_b, the next row are bending moment at c and shear force at c. What is my x by L? This will be 0, L over 8, L over 4, 3 L over 8, L over 2, 5 L over 8, 3 L upon 4, 7 L upon 8 and L. With these expressions that I have got, can I plug in the values of M? I am now going to put in the various values of x into the expression for M_a that is here, M_a and I am then going to go through the steps, so let us look at it. x is equal to 0 – plug that in and you will see that M_a is equal to 0.

If you look at L over 8, you are going to see that this is going to become 7, let me do the computations. It is going to become 7 by 8 the whole squared into 1 by 8, so it is going to be 49 by 64 into 8 is going to be 512, so this is going to be 49 upon 512 plus if you look at it, it is going to be 1 upon 8 squared into 7 upon 8, that going to be 7 upon 15, so that is going to be 7 upon 512, that will be 7 upon 2, so this is going to be 98 plus 7, 98 plus 7 is going to be equal to 105, so 105 upon 1024, this is 105 upon 1024 (Refer Slide Time: 40:21). Can I find out my V_a for that? Let me see what the V_a is going to be.

V_a is going to be M_a upon L. By the way, this is into L here (Refer Slide Time: 40:39). Now, V_a is going to be 1 minus x upon L, x upon L is 1 upon 8, so this is going to be 7 upon 8 plus M_a upon L, so 7 upon 8 plus this. If you do this, this is going to be 128, so it is going to be 7 into 128, 7 into 128 is equal to… into 8, so this is going to be 7, this is going to be 56, 14, 19, 19 is going to be 896, so 896 plus 105, 896 plus 105 is 1001, so it is going to be 1001 upon 1024 – that is V_a. If you look at V_b, it is going to be equal to 1 minus that, so this is going to be 23 upon 1024. Then, if you look at shear force, V_b is going to be 23 upon 1024 and if you look at the bending moment, that is going to be multiplied by 2, half, so it is going to be equal to 23 upon 2048 L. Since this L by 8 is less than L by 2, these two are kicked in.

Now, let us look at the qualitative values that we have drawn. Let us have a look at that. Let us see if these seem to make sense to you. It is positive value at L by 8, positive. What about for V_a? It is almost 1. That makes sense. For V_b, it is almost 0, makes sense. For V_a, very small value and here also, a very small value, makes sense. This is beginning to make sense.

Now, I will just put down for the L by 2 value. This way, you can continue on, so let us just do a couple of more values. If you do L by 4, what will happen is this becomes 3, let us see what happens here. L by 4, this becomes 3 L by 4, so 3 by 4 squared becomes 9 by 16, 9 by 16 multiplied by 1 by 4 is 9 by 64, so this becomes then 9 by 64, so this one is 9 by 64 and this one will become 1 by 16 and this one will be 3 by 4, so this will be 3 by 64, so this becomes 9 plus half of 3 by 64, so this becomes essentially 18 by 128 plus 3 is 18 plus 3 is 21, this is going to be equal to 21 upon 128 L.

If that is the case, then let us look at what V_a is going to be. Again, plug in 1 by 4, this becomes 3 by 4, this one is 21 upon 128, so 3 by 4 becomes 32, 32 into 3 is 96, 96 plus 21 is 117 upon 128. This becomes 11 upon 128, this becomes then 11 by 256 L, this becomes 11 by 128. Let us look at L by 2; at L by 2, what does this become? At L by 2, this one will become 1 by 4 into 1 by 2, that is 1 upon 8, 1 upon 8 plus 1 upon 16 basically becomes 3 upon 16, so this is 3 upon 16 L. Then if you look at this, 1 minus half becomes half, 50 by 16 becomes 11 by 16. V_b becomes then obviously 5 by 16. What will happen is that I have to actually do…. For bending moment, there is no problem – you will see that you will get the same, but for shear force at L by 2, there will be an L by 2 minus and there will be an L by 2, there will be two values: one will be for L by 2 minus, one will be for L by 2 plus. The reason behind it is that the shear force just when the load is to the left of c and when it is just to the right of c are going to be two different values.

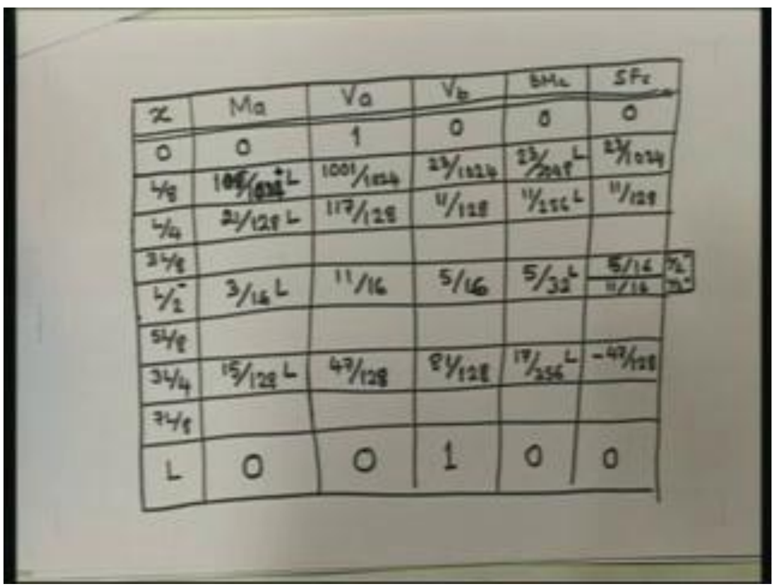
Let us look at that. We have already got these values. When x is equal to exactly L by 2, we cannot define this. Let us look at the bending moment. The bending moment is going to be V_b into L by 2 so this is going to be 5 by 32 L. Let us find out the bending moment using this approach: V_a into L by 2 where V_a is 11 by 16 into L by 2 is going to be 11 by 32, 11 by 32 minus 6 by 32 is going to be 5 by 32. Whether you use this (Refer Slide Time: 48:13) or this, the bending moment turns out to be the same. However, the shear force is different. If it is just to the left, it is equal to V_b and the value is 5 by 16; if it is just to the right, it is going to be 11 by 16 – this is L by 2.

Let me just do it at 3 L by 4 and I am going to stop it at that and of course L, so let us look at what happens. Let me do first for L, this is going to be equal to 0, so this is 0. What about this particular one? V_a is going to be equal to 1 minus x upon L, so 0 plus 0, this is going to be 0. By the way, this is 1, this is 0, bending moment is 0, this is 0. Now, V_b is going to be equal to 1, now this here we have to put this in, shear force is minus VF_a, so this is 0 and V is V_a into L by 2 minus M_a, V_a into L by 2 minus 0 is 0.

Let us just find out one particular value in the middle – that is at L by 4. If we look at 3 L by 4, what do we get? Plug in the value of 3 L by 4 (Refer Slide Time: 50:01), you get one-fourth, so one-sixteenth, one-sixteenth plus 3 by 4 is going to be equal to 3 by 64, 3 by 64 and this becomes 9 by 64, this is 3 by 64 plus half, so this becomes 6 plus 15, so this becomes 15 upon 128 L.

If we look at this, this is going to be 1 by 4 plus 15 by 128, so this is going to be 1 by 4 is going to be 32, 32 plus 15, this is going to be 47 upon 128; if this is 47 upon 128, this is going to be equal to 81 upon 128. If we look at this, this becomes minus 47 and this one, if we look at it, V_a into L by 2, it is going to be 47 upon 256 and 47 by 256 minus 15, so this is going to be 30, so 47 minus 30 is going to be equal to 17, so this is going to be 17 upon 256 L.

(Refer Slide Time: 51:58)



x	Ma	Va	Vb	BMc	SFc
0	0	1	0	0	0
L/8	[illegible] L	1001/1024	23/1024	23/2048 L	23/1024
L/4	21/128 L	117/128	11/128	11/256 L	11/128
3L/8					
L/2	3/16 L	11/16	5/16	5/32 L	5/16 / -11/16
5L/8					
3L/4	15/128 L	47/128	81/128	17/256 L	-47/128
7L/8					
L	0	0	1	0	0

I am not going to do the other ones, but let us look at the qualitative values. We are going to end here today with this, let us look at this. This started off large – this is about 0.1, this is about 0.18, this one is less, so if you look at this, this is half (Refer Slide Time: 52:29), half is actually 3 by 16 and this is 21 by 8, this one is only 15 by 128, 0.1, this 0.1 at this point is almost the same as L upon 8, so you can see the shape indeed is the way we have shown it. Then, if you look at the influence lines for these, V_a starts off very slow and then starts dropping really, really, fast and if you look at this, it is almost 0.7 and this is less than… it is about 0.3. Therefore, it is really dropping fast, which is how this is.

On the other hand, if you look at this (Refer Slide Time: 53:20), this starts off really slow, here, this is about 0.3, whereas if you look at this, this is 0.7. In fact, this is going this way and if you look at it, it is 1 here, it is 0 here. The qualitative and the quantitative show similar tendencies and then if you look at this, we will see….

At this, what is this value? Now we know what this value is – 5 by 32 L. What is this value? We know this value is now 5 upon 16. What is this value? 11 by 16. Add the two of them up, you get 1. You look at this theta, you will see that this is the same theta and over here, 0.7, you will see that this is exactly half of that, you will see this is exactly half, this is linear. All these things that we saw qualitatively, we see quantitatively and just to ensure, how did we do the quantitative influence line? The only way we have done qualitatively is that we knew how to find out the fixed end moment. Once we know the moment at the end, then it becomes a statically determinate structure for which we used equilibrium and solved it. The only way to get quantitative is by the direct approach.

Therefore, you see that for influence lines for static indeterminate, you have a combination of the Müller–Breslau principle and the direct approach that we have discussed already for statically determinate structures. Thank you very much. I am going to continue with this and show you how the direct approach can be used. Here, we could get M_a directly because it is a fixed end moment. Later on, you will see that this is not obvious – we will have to use some other method. Thank you. See you next time.