Structural Analysis – II
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Lecture – 36

Good morning. Today, as promised last time, I am going to be starting off by looking at influence lines. Therefore, we are going to be discussing influence lines in today's lecture.

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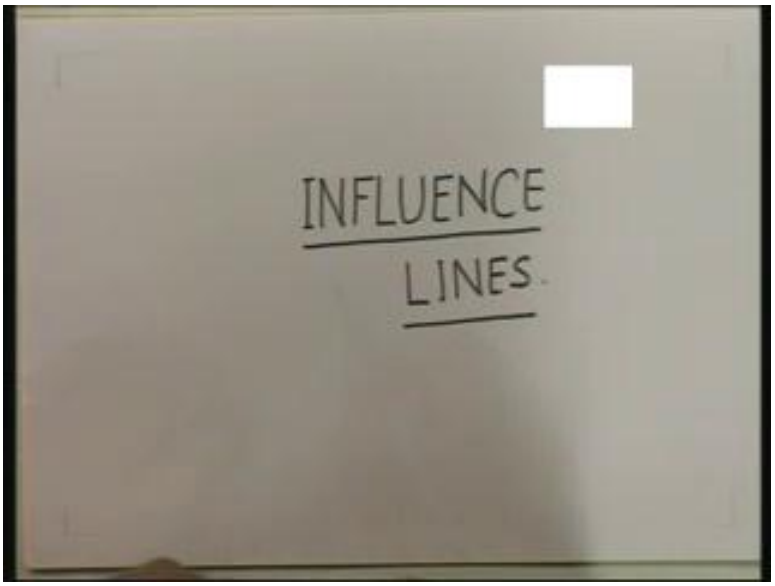

What are influence lines? ‘Influence lines’ is the name of a topic where we consider the effect of moving loads.

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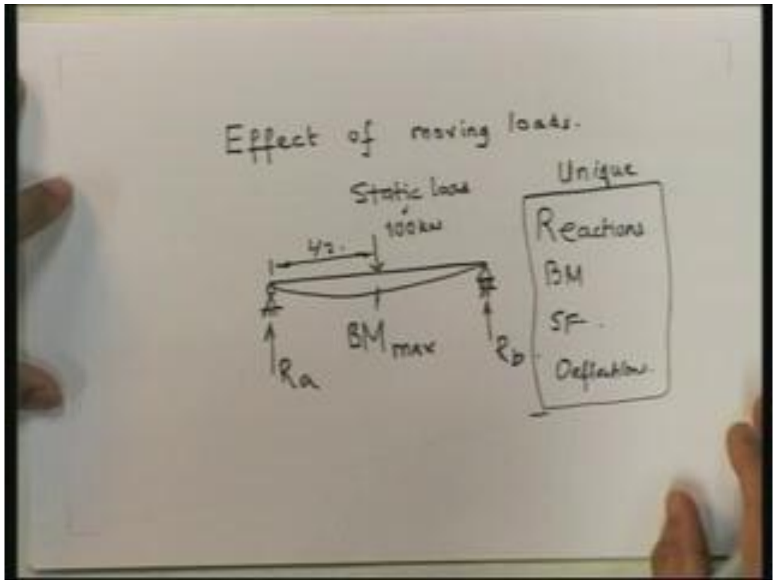

Essentially, influence lines are there to consider the effect of moving loads. What happens? Let us just take a simple beam. Let us say I have a fixed load of 100 Kilonewton acting at L by 2. I know everything: I know what is the reaction here (Refer Slide Time: 02:54), I know what is the reaction here, I can find out this is going to deflect like this, the maximum bending moment will be at the center and I can find out the maximum bending moment, $reaction_a$, $reaction_b$, this is my designed load and I can compute my reactions, bending moment, shear force, deflections – everything; unique; this is for a static load. Think about it. Here, what is going to happen? Suppose this load was not a static load.

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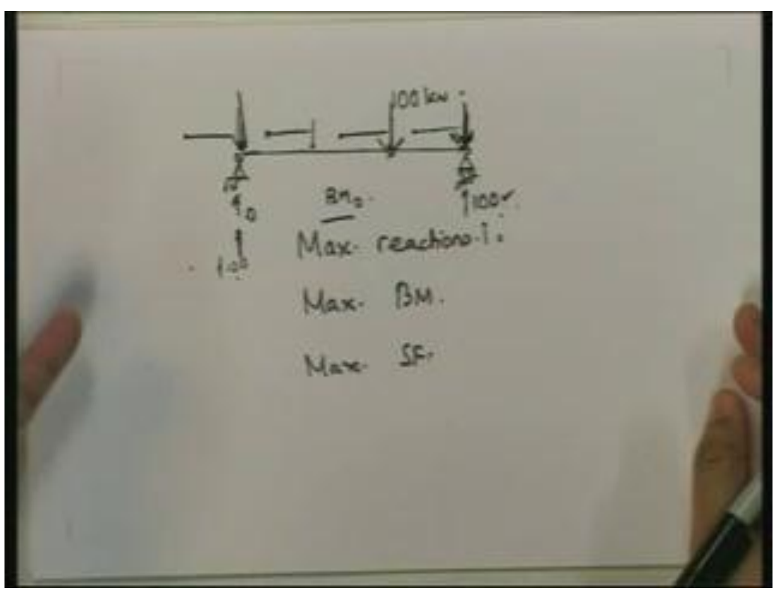

Suppose this was the load which on the same beam could be here (Refer Slide Time: 04:18), it could then move here, move here (Refer Slide Time: 04:23), move here and ultimately, go here and then move off. This load is a moving load, it is moving with a particular velocity. It may be moving very slowly or may be moving very fast. It is the same load by the way – 100 Kilonewton. Now, the question is: what is the maximum reactions? What is maximum bending moment? What is maximum shear force? Right now, I am just going to stick to the forces. How do I know?

If the load was here, this would be 100, this would be 0 and bending moment would be 0, would it not? If the load was here, this would be 100, this would be 0 and bending moment would be 0. If it were here, this would be something else, this would be something else and the question here is that… the question over here is that for this moving load, what is the maximum bending moment?

In other words, what is the maximum…? Before we answer that question, we need to answer this question: which is the position at which you are going to get maximum load? This is a moving load and you can obviously understand just by looking at this, if I were looking at this particular reaction, the position where this is maximum is when the load is here but the load at that point here is. However, the load is here, the maximum is 100; therefore, you can understand that for this 100 Kilonewton moving load, the maximum reaction here is 100, the maximum reaction here is 100 and bending moment etc. How do we do this? The only idealistic way of doing this is, let me do this.

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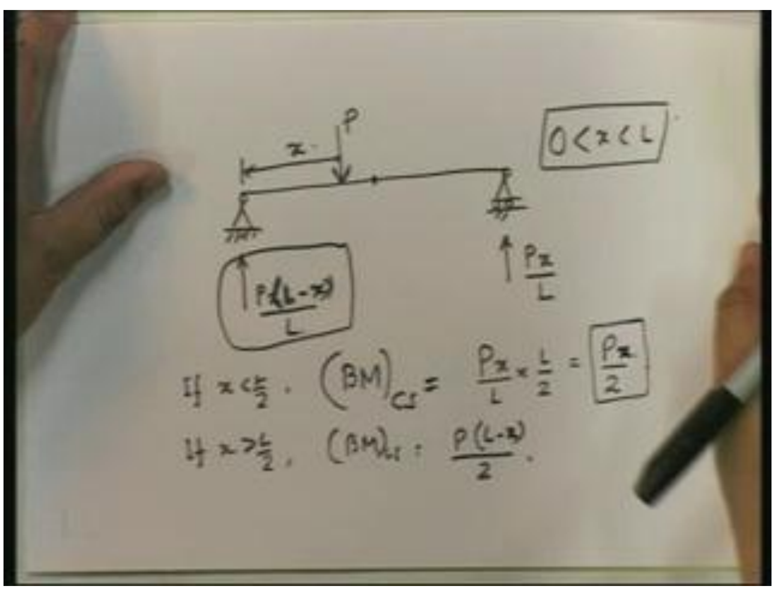

Let me put the load at a position x from the left end. Now this load is P (Refer Slide Time: 07:09) – it could be any load, put it equal to x and then find out the reaction here, the reaction here. What would be the reaction over here? The reaction over here would be Px upon L, the reaction over here would be P into (L minus x) upon L. What would be the bending moment at the center? The bending moment at the center span would be equal to Px assuming that the load is on this side it would be… if x is less than L by 2 then bending moment at this would be into L upon 2 which would be Px upon 2 and if x is greater than L by 2, bending moment at center span would be P into (L minus x) by 2.

The question here then becomes what would be the maximum? For that, I would have to put x is equal to 0 and from x is equal to 0 I would vary x is equal to L, so 0 is less than x is less than L – vary it and find out the maximum value. If you put it in here, put x is equal to 0, this is equal to P (Refer Slide Time: 09:16); put x is equal to L, this is P, put x equal to L by 2, this will be P by 4, put x equal to less than that and you will see that this will always be less.

Therefore, x is equal to 0 would give you the maximum (Refer Slide Time: 09:40), x is equal to L would give you the maximum here and if you are looking at bending moment at center span x equal to L by 2 would give you the maximum value of….. Now, you can do this easily by putting P is equal to x. Suppose my point of interest was this reaction then, my influence line actually becomes, what is this value as x moves from 0 to L?

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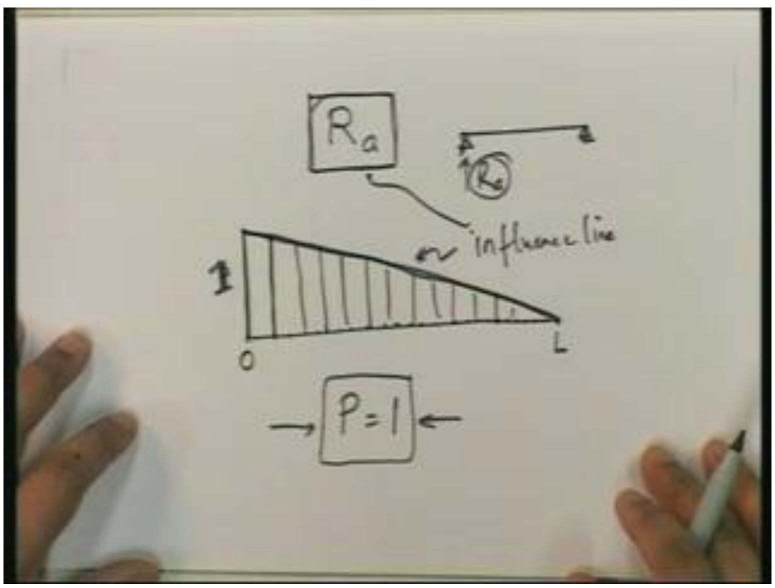

In other words, the way to influence line would be that, suppose I were to draw the influence line for reaction at a…. You always draw the influence line for a particular response parameter – influence line cannot be done for everything. The point is that you can only do things for a particular parameter in the influence line. What is the influence line? The influence line is this that, I draw my x-axis going from 0 to L. What does every point over here represent? Every point over here represents a position on the beam of the load, of the vertical load. What is the value at x is equal to L? You will see that this is equal to P. What is it? If you look, if you look at the expression, is this a linear in x? It is linear in x. What is it when x is equal to L? You will see that this is equal to 0.

Now, what is the value of the reaction when x is equal to 0? This is the value. This is the reaction. What I am actually doing is in this structure, this is my R_a, I am just drawing this. If the load was at this point what would be the reaction R_a? It would be this. If the load was here, what would be the reaction R_a? It will be this. Therefore just by looking at the ordinate corresponding to any value of x, I can find out the value of the reaction at a. Now, what is done is the influence line is for P is equal to1. In fact, if P is equal to a unit load, then this ordinate is known as the influence line, so this is 1, this is my influence line – the influence line for R_a. Note that the influence line is for particular response parameters – please understand that. Now, how would I find out the influence line?

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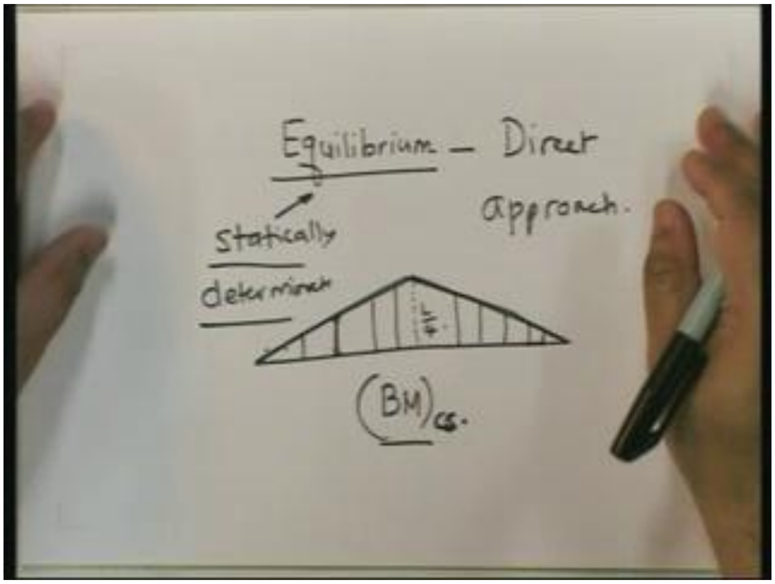

The best way and the easiest way is equilibrium – direct approach. Suppose this is my reaction (Refer Slide Time: 13:46), find it out and then plug in the values of x and find out this value. Of course, here P is equal to 1 – you have to put unit load vertical. If my bending moment in the center span was my this thing, I would put if x was less than equal to 0, this is less than L by 2 this would be the value, this would be the value if it was greater than L by 2.

If I were to plot it, it would look like this, unit load and this would be the influence line for the bending moment at the center span. If the load was here, I could find it out for any value of x. Therefore, what this ordinate means is that if the unit load was here, what would be the bending moment at the center span would be given by this ordinate – that is the definition of the influence line for a particular response parameter, for a particular structure. Now, if you had a statically determinate structure, then equilibrium is one way – it is the direct approach and you can obtain it as absolutely directly. However, what you tend to do is you tend to not use the equilibrium approach for statically determinate structures.

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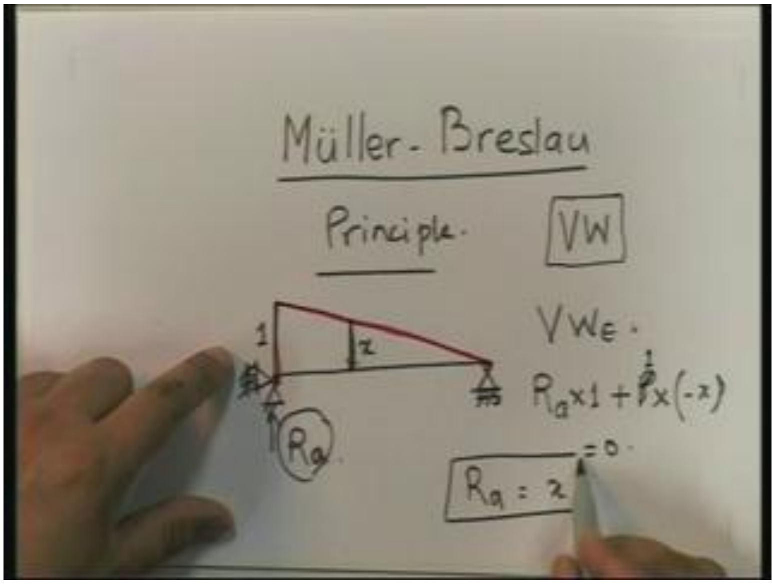

Why? Because there is another very, very useful approach known as the Müller–Breslau principle. What does the Müller–Breslau principle say? I am trying to find out the influence line for R_a. Now, the way the Müller–Breslau principle works is it says the influence line for R_a is obtained by releasing, so instead of the hinge, you release this (Refer Slide Time: 17:23) and give a unit displacement corresponding to this and then what will the displaced shape look like? This is a hinge, this is a hinge, so if it was two hinges, how would this member look? This member would look like this – the deflected shape of the member would look like this and the Müller–Breslau principle says that this deflected pattern is directly equal to the influence line. Note: you can see obviously that this does look like the influence line for R_a, which we have already done.

Let me just first try to understand where the Müller–Breslau principle actually comes from. The Müller–Breslau principle actually works on the virtual work principle. What it says is this: suppose I had load at this point (Refer Slide Time: 18:35), the load is at this point, then I could find out the reaction. How? Well, put unit displacement. This is the method of virtual displacement – put displacement corresponding to the support reaction that you are trying to find out and then the work done, the external work done is equal to reaction$_a$ into 1 plus P, now P is downwards, P is not this, it is 1 into what is this value? Let us call it x. 1 into minus x because the load is downwards, this is going upwards is equal to 0. R_a is then equal to x and that is the Müller–Breslau principle. It is based on the virtual work principle.

Remember I had said originally that the method of virtual displacement can be used to replace the equations of equilibrium for a structure and this is what the Müller–Breslau principle actually does. What is says is whatever … you want to find out the influence line for whichever response quantity, release the restraint corresponding to that response quantity, give a unit displacement corresponding to that response quantity and then the deflected shape would represent the influence line because remember that the load that you have is only a unit vertical downward load that you are finding it out for – that is the basis for the Müller–Breslau principle. Let me look at this particular case.

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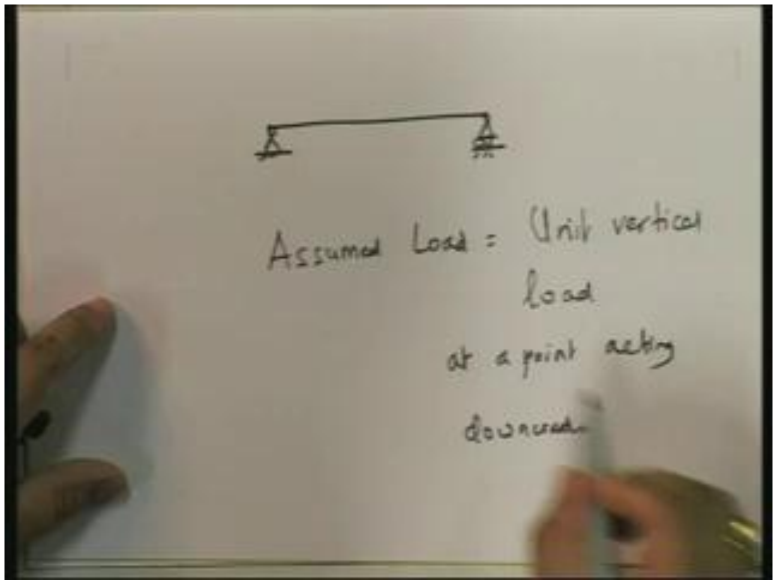

Note that whenever we are drawing influence lines, we do not have any load because what is the assumed load? The assumed load is unit vertical load at a point acting downwards. When we are looking at influence lines, we do not have a load at all because it is always unit vertical load at a point acting downwards.

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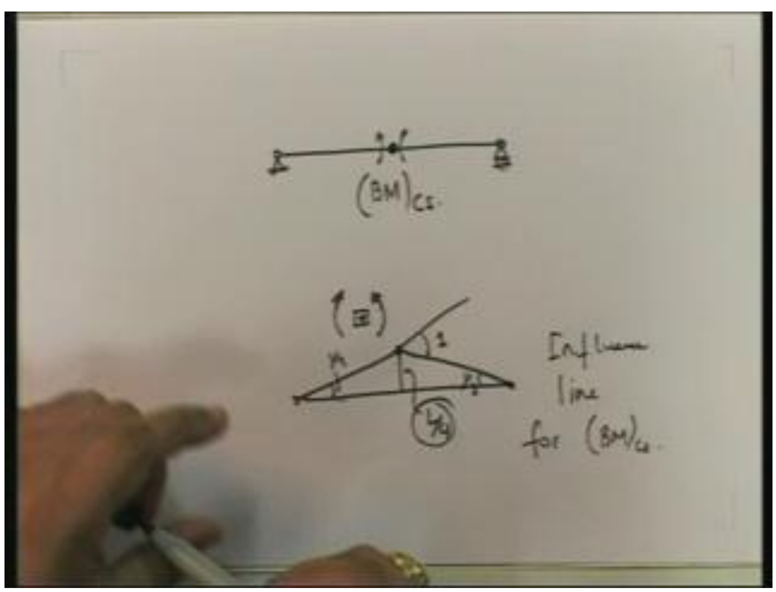

Now, how do we find out? Let us say I have this member and I want to find out the bending moment, the influence line for the bending moment at the center span. Müller–Breslau principle: what is the restraint corresponding to the bending moment? Remember: the corresponding bending moment is that the rotation has to be continuous – that is how you generate a bending moment; if rotation was not continuous, then you will not generate a bending moment. Therefore, how do I remove the restraint? I put a pin here (Refer Slide Time: 22:44). As soon as I put a pin here, what happens is that the bending moment gets released.

Always you have to define positive. So I am going to define this as my positive bending moment. If you look at it, then this is for this side, so it will be like this (Refer Slide Time: 23:15) and this will be for this side. This is the action that you do. If you do that action, what happens? Center, this and this and what is the unit? The unit is the relative rotation, so this has to be unity. Note that if this has to be unity and this is L by 2, you will see that this is equal to half and this is equal to half and if you look at this, half into L by 2 is equal to L by 4. I have my influence line for the bending moment at center span using the Müller–Breslau principle. Today, what I am going to do is I am going to solve several problems on this aspect. I have looked at reaction, I have looked at bending moment; in bending moment, whatever is the positive, you give a reaction in that way and get unit relative displacement.

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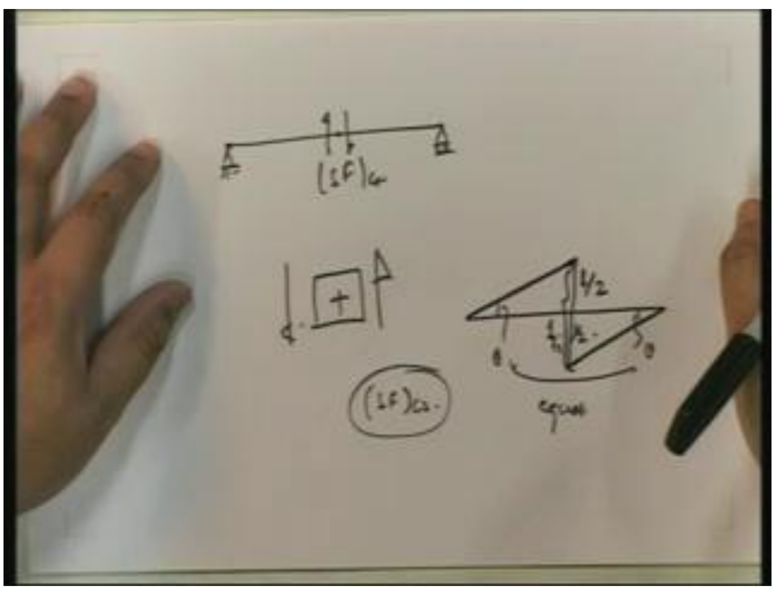

Suppose I were asked to find out shear force, how would I do shear force? Note that shear force actually tries to get relative displacement of one edge corresponding to other and the only way this is stopped because the continuity and that is what develops a shear force. If I make it relative to each other but I have to ensure that the bending moment is not released – I am only finding out the… for the shear force, so what has to happen is that, Müller–Breslau principle says unit relative displacement, so the displacement between the two edges has to be equal to 1 but the slopes have to remain the same so that there is no relative rotation between the two ends. Therefore what I am doing is I am introducing a fictitious cut to release the restraint corresponding to the shear force; however, I am not making a hinge at that point, so it looks weird but this is the whole effect. How I ensure that there is no hinge is by ensuring that there is continuity of slope. You are maintaining continuity of slope without maintaining continuity of the beam – this is a very funny situation.

Let me say that I want to find out shear force at the center span. My definition is plus, minus this way (Refer Slide Time: 26:35). If you look at this way, this implies that this goes up and this comes down because if you look at this side, this is the one that is going to be pulling it down, so do this, then you get something like this and here you get something like this. The only thing that you have to ensure is that since this becomes… any restraint removal that you do in a statically indeterminate structure makes it a mechanism and in a mechanism you only get straight lines, so all you have to ensure is that this theta and this theta are equal. That will ensure that the tangent here and the tangent here are equal to are equal to each other and this is equal to 1.

If you look at this, this is equal to let us say that this is L_1, this is equal to L_2 (Refer Slide Time: 27:43). Now, these have to be theta, so by definition by equal triangles, you will see that these ordinates are half and half, these are 0, 0, so that becomes the influence line for the shear force at the center span. Suppose I were to ask you to find out the influence line for the bending moment at quarter span again, for the simply supported beam then how would I do it.

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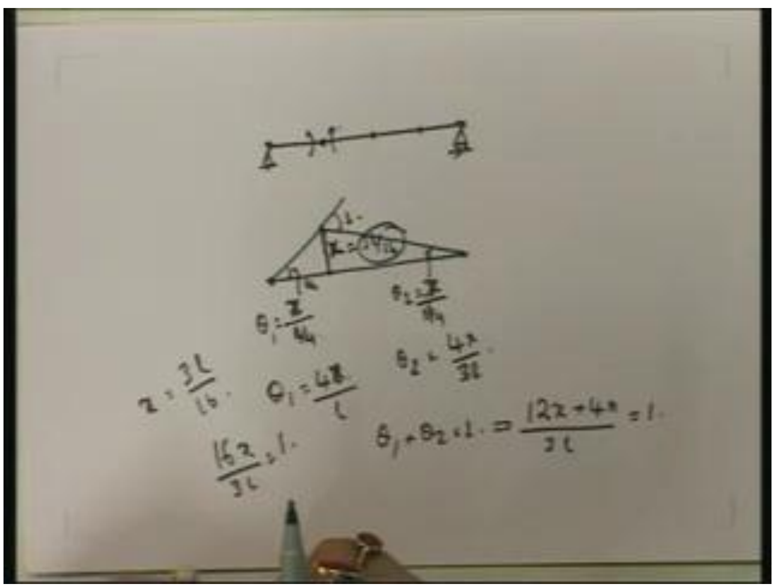

Quarter span; I have half, quarter, three-fourth, quarter span. Again, I need to do this. When I need to do this, this one will go this way and this way, where this angle has to be equal to 1 – the relative. How do we obtain the ordinates? Let us look at it. If this is 1, this is a constant length L, this ordinate is L, so this has to be L divided by 1 by 4 and this has to be equal to L divided by 3 by 4. This theta_1 is equal to this, theta_2 is equal to this. What does that mean? This means theta_1 is equal to 4 l where l is this; this is the unit – so I will call it x; this is going to be l upon 4, this is l upon 4, this is x upon 3 l upon 4, this basically becomes 4x upon l and theta_2 becomes 4x by 3 l but theta_1 plus theta_2 is equal to 1 and this implies that if 12x plus 4x upon 3 l is equal to 1, so

16x upon 3l is equal to 1, x equal to 3 l upon 16, so this becomes 3 l upon 16 and that becomes your ordinate for one-fourth span.

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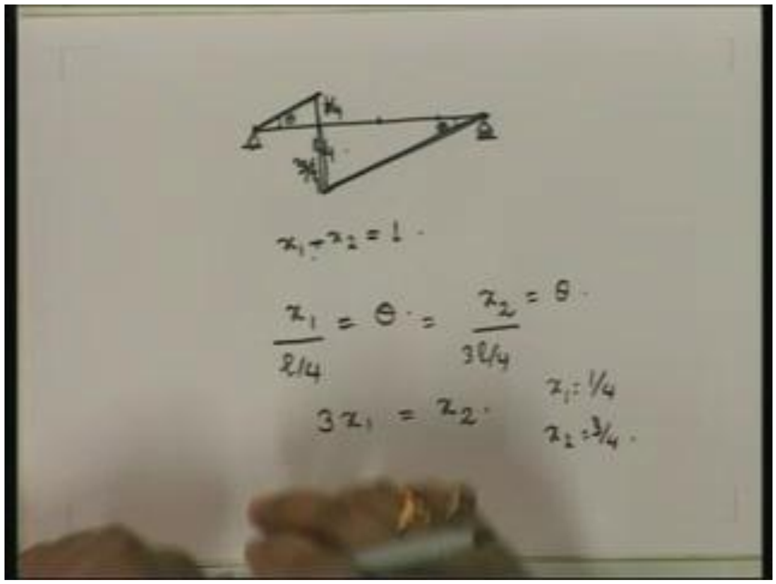

Suppose I wanted to find out the shear force at one-fourth span, how would I do it? Well, shear force at one-fourth span, l by 4, how would I do it? This is x_1, this is x_2, this is theta and this is theta by definition because the slope here and the slope here have to be the same. So what we have is x_1 plus x_2 is equal to 1 because relative displacement has to be equal to 1 so x_1 plus x_2 is equal to 1 as well as x_1 upon l by 4 is equal to theta, x_2 upon 3 l by 4 is equal to theta. So what essentially it means is that this one is equal to this, so what we get here is that 3 x_1 is equal to x_2. However, x_2 is equal to $3x_1$ so if we substitute in you will get x_1 equal to 1 by 4 and x_2 is equal to 3 by 4; this is one-fourth and three-fourth and that is the influence line for the shear force at one-fourth span. Do this and satisfy yourself that you can actually get both of these by the direct approach. Satisfy yourself that the influence lines that I have drawn for all the specific characteristics actually come out to be them. The main advantage is that the whole idea of finding the influence line…, actually it is an equilibrium problem but by using the Müller–Breslau principle we change it into a geometry problem and geometry is always much easier to do than equilibrium. Today, I am only going to be looking at statically determinate structures just to introduce you to the concept of how to apply the Müller–Breslau principle. Let us look at various structures for which the Müller–Breslau principle can be used.

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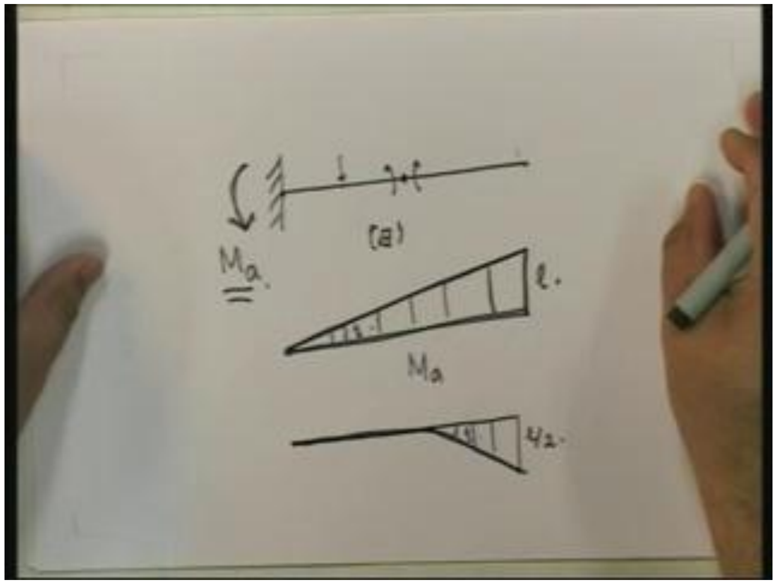

Let us look at this. This is my structure – this is a statically determinate structure and I want to find out the influence line. Note that the vertical load can only go from here to here – this is a factor that is very important; it is a unit vertical load, so it can only go from here to here. Now, I want to find out the influence line for this – the moment M_a and I am going to assume that this is positive. What do I do then? Take this structure and I release this and I give it a unit…. Note that I have only released it – I have not let it move up and down. All I have done is released it and I have given it a unit rotation here – I give it a unit rotation here. What does the beam become? Unit rotation here, this is unity, so what does this value become? It becomes l. If you look at this, the influence line for M_a is this, where each ordinate over here gives me the value of the moment due to the load being at that point. Think of the load at this point (Refer Slide Time: 36:10). What would M_a be equal to? Unit load at this point of length l, so M_a would be equal to l.

Suppose in this particular problem I wanted to find out the bending moment at this point (Refer Slide Time: 36:25), what would I do? This is positive. What would I do? I would put a restraint here and find out how this would go, so think about it; introduce a hinge and do this. Note this part is fixed over here, so if you look at this when I introduce a hinge over here, the simplest thing that would happen would be this and then unit, this would be unit because that is the relative rotation between the two sides, I have introduced a moment here and this would be l upon 2.

Think about it and you will see that it is indeed minus l upon 2 – if you have the load here and this you have taken as positive, this is sagging and you would see that if you put load here, you would get hogging, which is basically minus l upon 2. What does this mean? This means that when the influence line for this portion is 0 and only kicks in here. Why? Put the load here. What would be the bending moment here? You will see that the bending moment everywhere beyond this point is 0 and therefore you would not have any bending moment and that is reflected by the Müller–Breslau principle.

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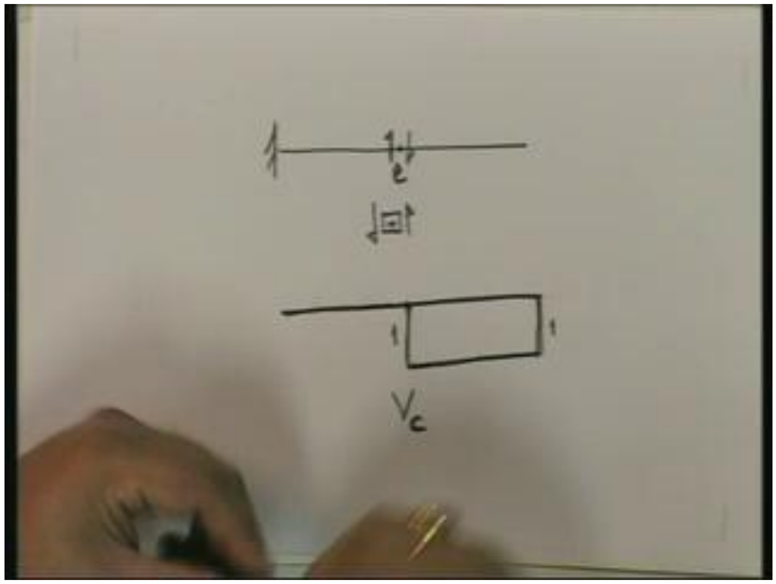

Suppose for this same structure I wanted to find the shear force at this point (Refer Slide Time: 38:22) and my positive is this way, essentially what I am doing is I am going to force them to do this. Now, how am I going to get the shear force at this point? Think about it. The only way I would get shear force is… in other words, I am just pulling it this way. Now this point cannot do anything.

Therefore, if you look at it, the diagram would look like this: unity, unity at the point at which you have this; this slope and this slope are the same. Is that clear? Because this part as soon as you make a cut over here, this bit goes by this and this does not have to rotate. Remember you always go for the simplest displacement pattern that can satisfy your kinematic conditions, so this is what happens. When you make a cut here and you have to keep this slope and this slope same, the easiest way to do is keep this and move this down; then the slopes remain the same, this becomes 1.

Again, if I put a load here (Refer Slide Time: 39:52), what is the shear force here? You will see that the load shear force will be negative 1 and it will remain negative 1 till it goes beyond this, which I meant becomes 0. This is my V at point C – influence line for the displacement at point C. Now, let us get slightly more complicated. We have done fairly simple things and I hope you have figured out how to get it for support reactions that are linear, support reactions that have moments and also draw the influence line for the bending moment and shear force. Now, let me take an example that would be slightly more complicated.

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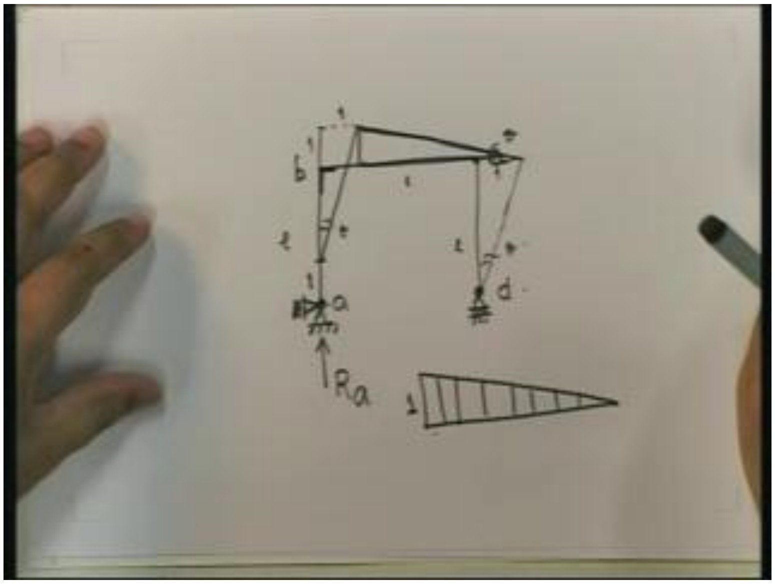

Let me consider a situation like this: this is the frame – this is a statically determinate structure. Where does this thing move? Think about it. I am going to draw the influence line. Actually, this is a crane moving over here – a crane goes from here to here and for that, I need to draw the influence line because ultimately for the crane, I need to know what would be the maximum support reaction, I would need to know what would be the maximum bending moment – all of those kinds of things I need to find out. Therefore, the vertical load is moving from here to here only.

Let me say that in this particular case, I would like to find out the influence line for reaction at a, so let me say a, b, c, d, reaction at a. Now, note that the hinges are here and here, so I release this and I give a unit displacement corresponding to this. If I give it a unit displacement corresponding to this, this will go here, this will go 1 here but it is very important to understand that I cannot draw it this way. Why can I not draw? What is the problem of drawing it like this? Think about it. In other words, I am saying why cannot the reaction at this point be the same as what we have done earlier, which is this? There would be a problem. Why? Because if this member went up like this and this thing went, this would imply that this, which is a continuous joint, would be hinged and that cannot be true because the only thing that we are doing is releasing this and making it become something like this.

For this, how will it be satisfied? The only way that it would be satisfied would be if this went this way (Refer Slide Time: 43:51) so that here some l, so that this would be l, so that you would have a continuity over here and then, this would also have to remain and then you have something like this. This would be the displacement pattern. How much would this move by? Let us see. Let me say that this is l, this is l and this is l.

Now, we have to ensure that this theta (Refer Slide Time: 44:38), this theta and this theta are the same. You will see that this one, this displacement is l theta, so that means if this is 1, theta has to be 1 upon l and if this one upon l, this is 1, this is 1 and this is the displacement pattern exactly; this one remains here and this one goes up, but what is the influence line here?

Is this displaced shape the influence line? No. Understand that the only thing that we are interested in is the influence line is the displaced shape of the part of the member on which the load acts. On which member does the load act? The load acts only on the beam, not on the columns. Therefore, if you look at that, what does that look like – the influence line? The influence line looks like this (Refer Slide Time: 45:51). Look at that influence line – that influence line looks like this; 1, 0. Is there any difference between the influence line for this and the influence line for the simply supported beam? No. Why? That is because this one (Refer Slide Time: 46:34) and one shift has absolutely nothing to do with that because it is only the vertical ordinate that actually tells us what the reaction is when the load is placed in various positions. Therefore, the point here is that you will see that this R_a – whether it is for a simply supported or whether it is for this frame, it makes no difference. That is for R_a.

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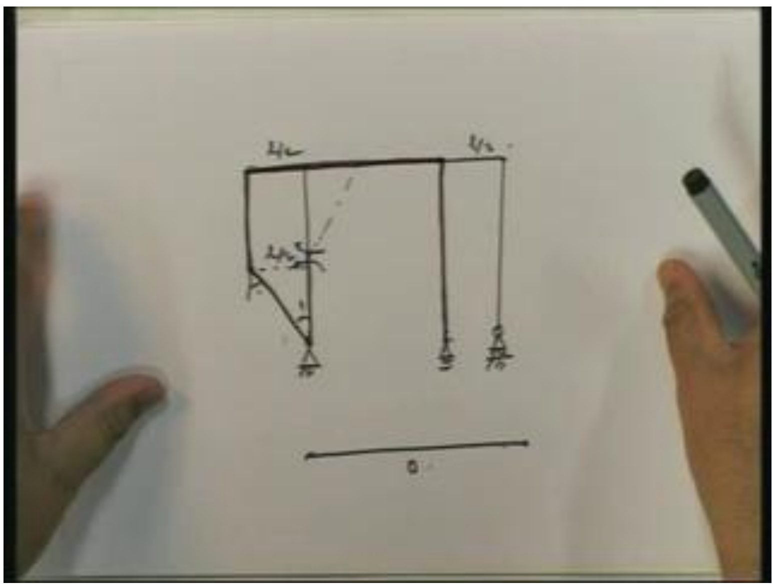

Let me ask you the same problem. Let me say I would like to find out the influence line for the bending moment at this point. What do I need to do? All I need to do is I need to put a hinge here and rotate it so that I get unity. Let us assume that this remains this way and I have this but now, can this go like this? It cannot for the simple reason that if this has to go in this fashion, what is going to happen? This will have to move down and there is no way that this can move down then this becomes an issue. Therefore, obviously, this (Refer Slide Time: 48:51) cannot remain straight and this cannot go this way because this would ensure that you have something like this.
What would you have to do? Think about it. The only way is this has to be moved here (Refer Slide Time: 49:06). Therefore, take this and move this such that you get a situation that this does not move; this cannot rotate because if this point rotates, this point cannot go anywhere so this point cannot go anywhere vertically; if this point cannot go anywhere vertically (Refer Slide Time: 49:32), then the only way is that this can move is only in this direction – it cannot move in

any other direction because this cannot move up and down, this cannot move up and down, the only way is this can go up and down.

If you really look at it you will see that ultimately the bending moment looks like this: this member is going to look like this, this has to be equal to 1, so this is 1; if this is 1, this becomes l over 2, this becomes l over 2, this becomes l over 2, this has to remain straight so that this remains straight and if this remains straight, this also remains straight.

My displaced shape is this but again understand one critical parameter and that is that this is the displaced shape but what is the influence line for this bending moment? The influence line for this bending moment is this (Refer Slide Time: 51:13) and what is that influence line? If you look at it the influence line is 0. What does that mean? That means if the load moves here there is never any bending moment at this point and you can satisfy yourself that these two members actually do not deform. This is the bending moment. Suppose I wanted to find out the shear force at this particular point. Let me do that. This is the final thing that I am going to be doing in today's thing – that same frame.

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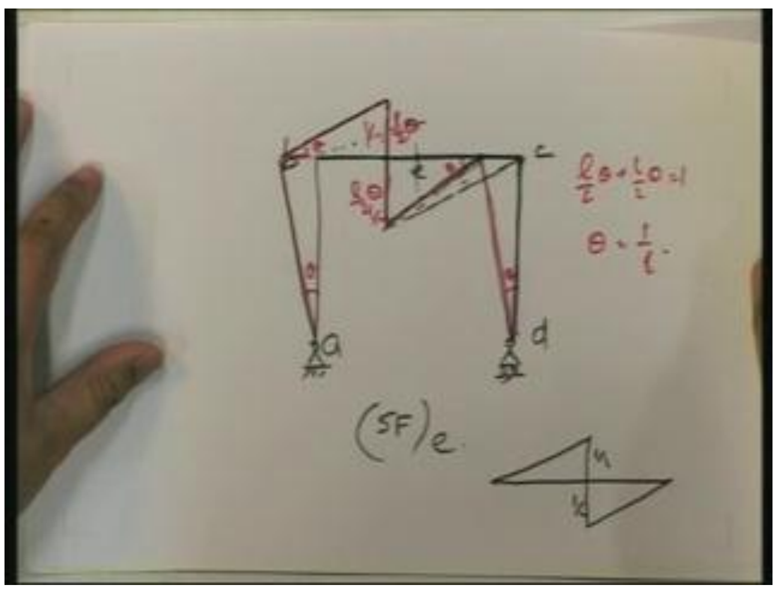

It is continuous here (Refer Slide Time: 52:07) it cannot have a hinge there, continuous over here, this is a, b, c, d, e and I want to find out the influence line for shear force at e. How do I find that out? I have to give a relative displacement. The relative displacement has to be such that this goes like this and this goes like this, so that is going to be the thing. However, this is not possible because this again would entail. How would I have to do it? I would have to move this this way, move this this way and then, and this point also would move this way.

If I were to draw the deflected shape, it would look like this (Refer Slide Time: 53:33) where this angle and this angle are the same and then, this angle and this angle have to be the same to ensure… and so, this angle and that angle has to be the same. All the thetas have to be the same and the only way that you can get those thetas is that this theta is equal to…. What is this equal to? This (Refer Slide Time: 54:17) is equal to l by 2 theta, this is equal to l by 2 theta. Now, l by 2 theta plus l by 2 theta is equal to 1, so theta is equal to 1 upon l.

If you have 1 upon l, then what do you get? You will be getting… half, half and if you look at this, it has no role to play, if I look at the… it would go like this, half, half and as far as the shear force at this point is considered, it does not make a difference what I have in the frame or not. Again, the entire thing boils down to the fact that influenced lines are obtained through kinematic relationship. The Müller–Breslau principle lays stress on getting kinematic relationships that would be compatible. Therefore, you always have to give a unit displacement corresponding to the degree of freedom or to the release that you are going to do and if you have a statically determinate structure and if you release, it becomes a first-order mechanism, so you are bound to get only straight lines. You have to get only straight lines as influence lines for a statically determinate structure.

I hope I have been able to establish the concept of influence lines as well as the concept of the Müller–Breslau principle. Next few times, I am going to be spending more time on the application of the Müller–Breslau principle – first for statically determinate structures and then, we will be moving on to statically indeterminate structures.
Thank you very much.