Structure Analysis – II
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Lecture – 35

Good morning. Today, we are going to be continuing with our exposition on the stiffness method where we are going to be looking at a few more examples. I will look at one example and then I will essentially establish how to consider thermal stresses in the stiffness method.

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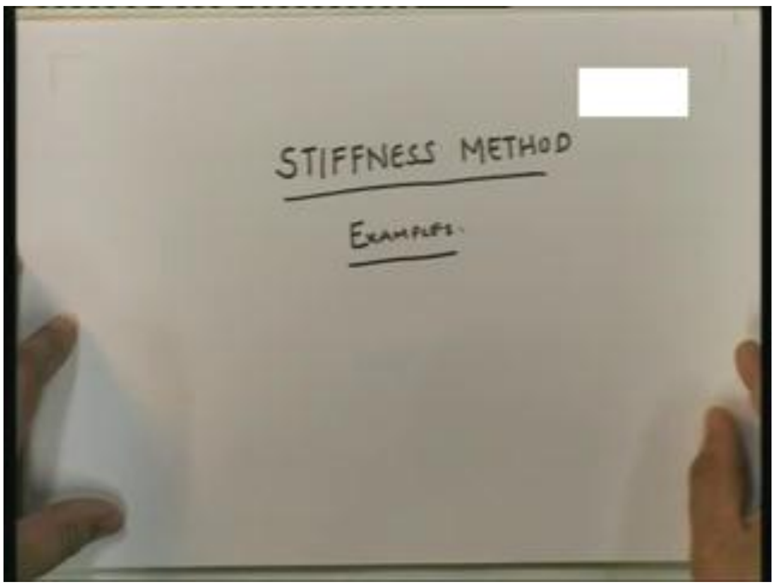

Today's lecture is on the stiffness method and we are going to be looking at examples. The first example was the one that I had presented in the last lecture at the end.

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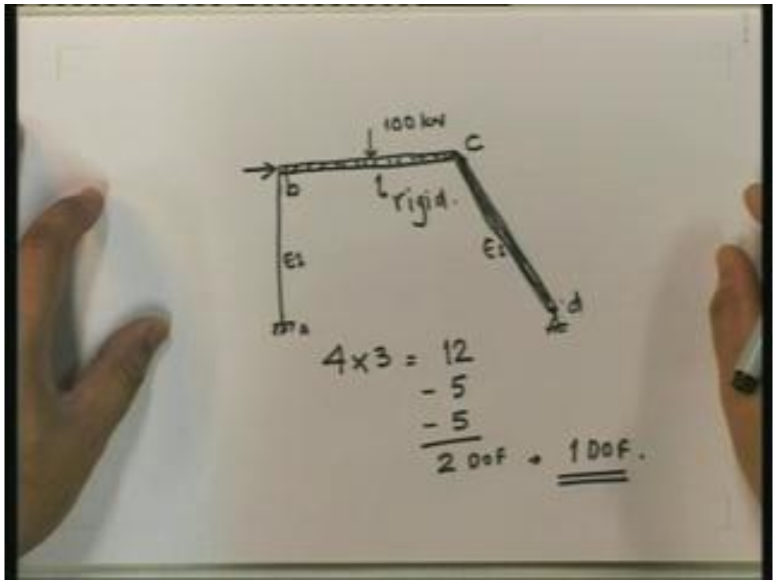

This was the problem that I had solved in the last lecture: a, b, c, d. this (Refer Slide Time: 02:38) member is now flexible, this member now becomes rigid. This is EI, this is EI and this is rigid. This is a more realistic problem where the columns are flexible and the beams are normally monolithic with the slabs and therefore, these are T-sections and the EI values for beams are typically much larger than those of columns. One of the simpler ways to deal with it when one member has a significantly larger EI than the others is to assume that it is flexurally rigid because after all, the rotations over there are going to be small and we might as well neglect it.

How many degrees of freedom do we have? If you look at it, let us go through it. This was the problem. How many degrees of freedom? It will still hopefully remain two; the only factor that becomes that it is two is that this is a degree of freedom. Think about it: can you have a rotation at this point at b without this? Think about it: if this rotates, then this has to go up, if this goes up, then the only way this can go up is by this moving and if this moves, it moves horizontally. Therefore, the two degrees of freedom actually are this and this but this is no longer a degree of freedom, so let us write down, prove to you that this is actually a single degree of freedom system and for that, let us go through the steps.

How many joints? 4 into 3 is 12. Then, how many restraints? 1, 2, 3, 4, 5 so five restraints. How many constraints? It is 5; 1, 2, 3, 4, 5. That means there are two degrees of freedom but one of them happens to be this end rotation, where anyway we know that the moment is there, so I have only one degree of freedom – essential degree of freedom; there is a second degree of freedom here but I do not want to find out the displacement right now. This goes as a single degree of freedom system and how do we solve this particular problem? Let us see. First and foremost, ab and cd are the members.

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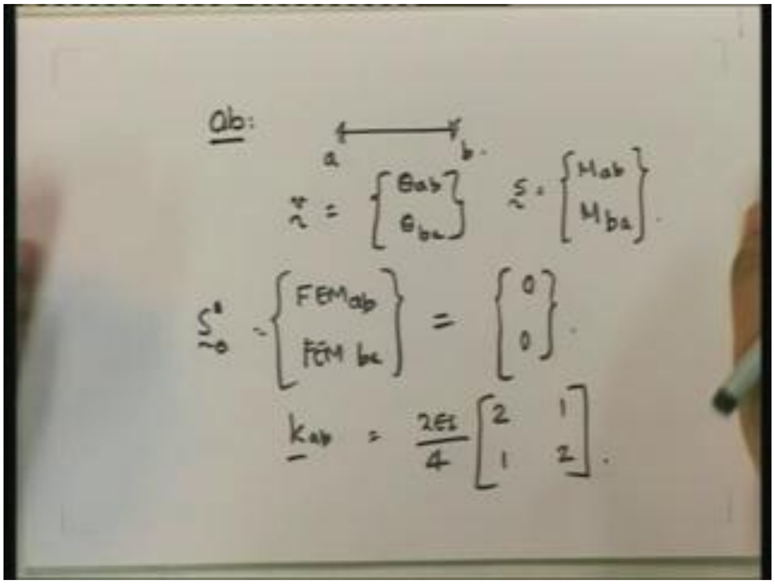

For ab, what is it? The member is fixed at a and continuous at b and therefore, v is equal to $theta_{ab}$, $theta_{ba}$ – that is v. What is S? S is M_{ab}, M_{ba}. $(FEM)_{ab}$, $(FEM)_{ba}$ is equal to S_0 is equal to 0, 0 and K_{ab} is equal to 2EI and this is 4 meters, this is 2 1, 1 2 – this is your member ab. Now, bc is a rigid member and therefore you do not consider bc as part of the members.

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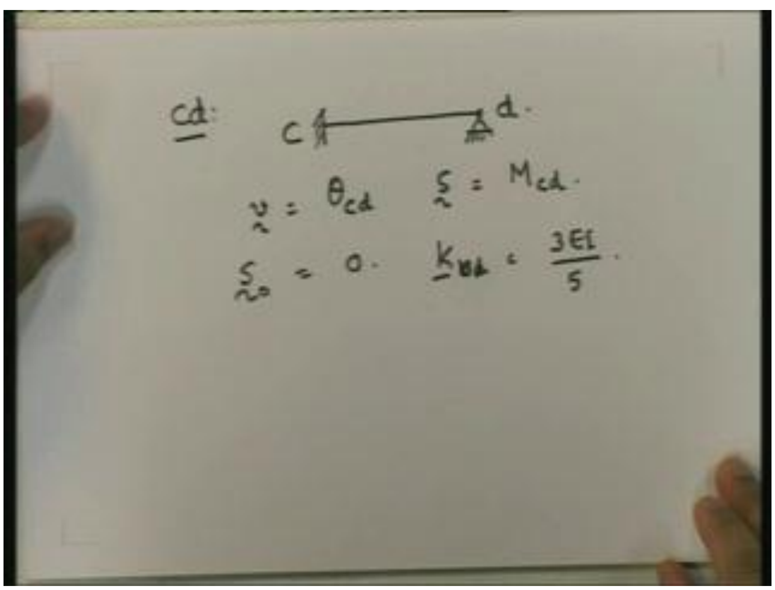

Therefore, the next consideration would be for cd; cd is continuous at c so it is fixed and at d it is hinged; this is c, d and therefore, v in this particular case is $theta_{cd}$, S is equal to M_{cd}, S_0 is equal to 0 (there is no load) and K_{cd} is equal to 3 EI by length which is 5. I have found out for each of the members. Now, I need to find out to how to get the member degrees of freedom related to the kinematic relationship between the member degrees of freedom and the structure degrees of freedom for which I have to do the kinematic relationship.

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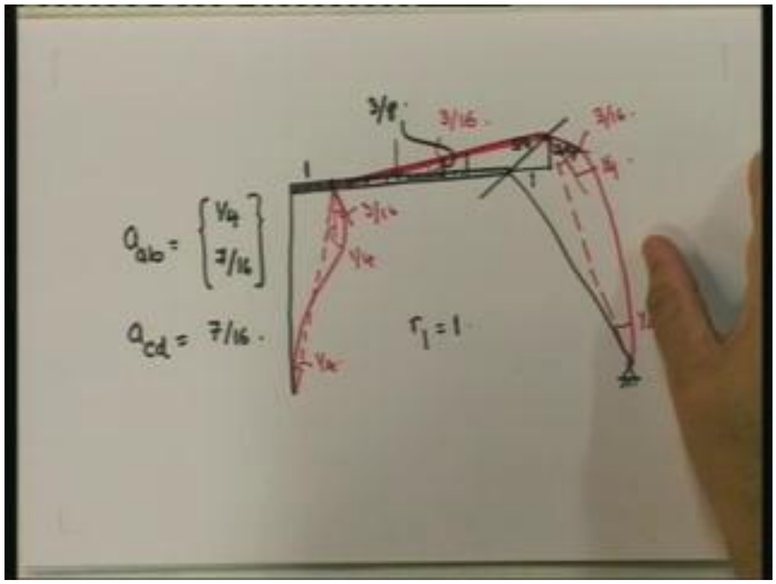

Therefore, if we look at this particular situation we have, now note that this is the rigid one (Refer Slide Time: 08:49) and then we have this which is continuous and here I have pinned. If I give it a unit displacements r_1 is equal to 1, this would imply that this would go here, this point would also move here but it cannot move only that way – it has to move perpendicular to this and this perpendicular is… if this is 1, this is 3 by 4 and this is 3 by 4 and the displacement pattern looks like this: if this has gone here and this point is here the rigid member is going to remain straight – that is the most important thing. What is this angle (Refer Slide Time: 09:59)? This angle is equal to 3 by 4 and upon 4 this becomes 3 by 16.

Once that happens, note that if this is the thing, this also has to move by 3 by 16, so the tangent becomes like this and this member comes like this. Note also that this tangent also has to rotate because this has rotated and therefore this goes this way and these rotations are 3 by 16, this rotation over here is also 3 by 16, this is hinged. We are always taking theta from the chord, so this angle – since this is 1 and this is 4, this is going to be 1 over 4, this is going to be 1 over 4 and this is going to be 1 over 4 and 5 by 4 upon 5 because that is the length of the member and this here is going to be 1 over 4. Since this is my only this thing, a_{ab} is going to be equal to ab is 1 by 4, ba is 1 by 4 plus 3 by 16, that is from the chord to the tangent, so if you have that, that is 7 by 16 and note that these are anticlockwise so they are positive so you have 7 by 16 and a_{cd} is equal to 3 by 16 plus 1 by 4 from the chord and this is also counterclockwise so this is 7 by 16. That is it. Note that this load over here – how much is that displaced by? That point, the central point – how much has it displaced by? The central point, by the way, has moved by 1. How much is this? This is going to be half of three-fourth, so this is going to be 3 by 8, so the load point has moved up by 3 by 8 – this is going to play an important role in our calculations a little bit later. I have computed the a_{ab}.

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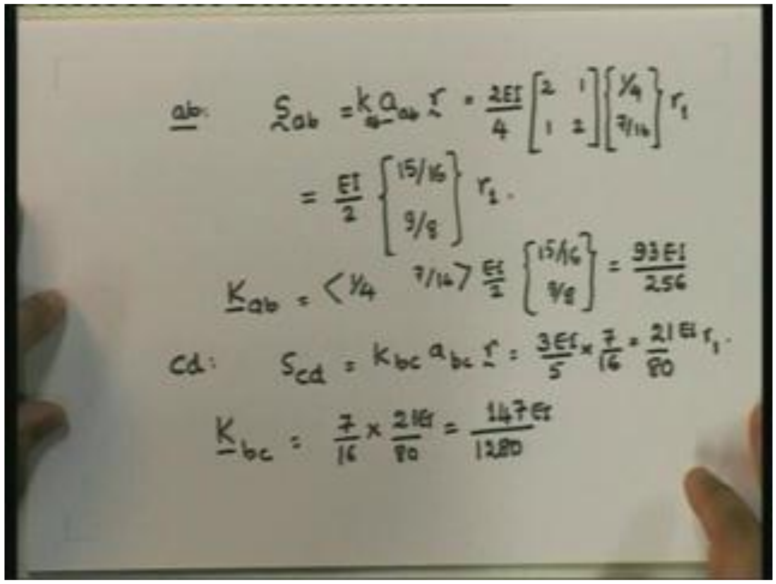

Note that in both the cases, the fixed end moments are 0 and therefore, all I have is directly S_{ab} is equal to a_{ab} into K_{ab} into r, which happens to be a 1 by 1. If I look at this, this becomes 2EI upon 4 into (2, 1, 1, 2) into (1 by 4 into 7 by 16) into r_1. This is equal to… I will make it 2 by 4 plus 7 by 16, 8 by 16, so this becomes 15 by 16, so EI upon 2, this is going to be 15 by 16 and this is going to be 14 by 16 will become 7 by 8, 7 by 8 plus 1 by 4 is going to be 9 by 8, this is into r_1 – that is S_{ab}. The contribution to K_{ab} is going to be equal to one-fourth, 7 by 16 into EI upon 2 into (15 by 16, into 9 by 8). If you look at this, this becomes 63 upon 128 and this becomes 30 upon 128, so 63 plus 30 is 93 upon 128, so 93 upon… so this is going to become 93 EI upon 256, because 128 into 256 – that is K_{ab}. Now, let us look at cd. For cd, S_{cd} is going to be equal to K_{bc} into a_{bc} into r, which is going to be equal to 3 EI upon 5, so it is going to be equal to 3 EI upon 5 into a_{bc} is 7 by 16, so this is going to be equal to 21 EI upon 80 into r_1. K_{bc} is going to be a transpose – a transpose is again the same so it is going to be 7 by 16 into 21 EI upon 80, which is equal to 147 upon 1280 EI (Refer Slide Time: 17:24). When we add both of them up, what do we get? We get this, the following.

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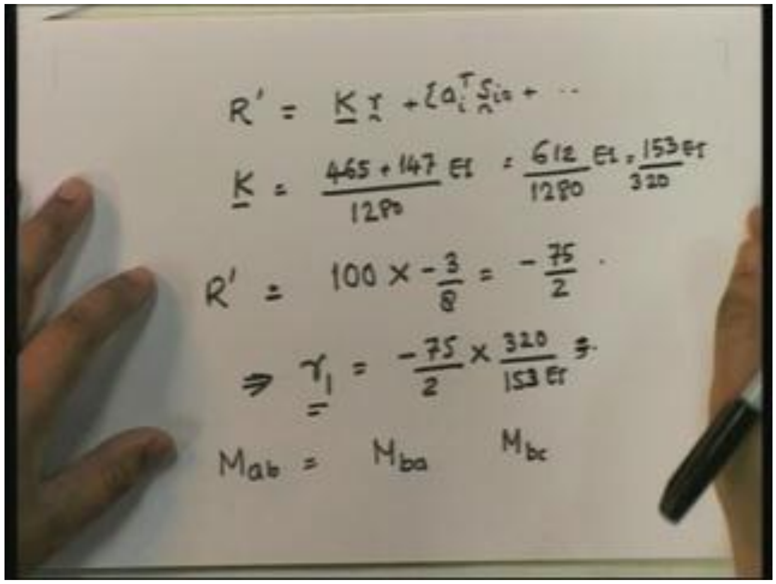

R is equal to K into r plus summation a_i transpose S_{i0} plus…. The major point to be noted here is that there are no S_{i0} and n_{i0}, so we do not need to worry too much about that. Therefore, the K is going to be equal to 5 as it is going to be 465 plus 147, so it is going to be 465 plus 147 upon 1280 EI, which is equal to… if you look at it, this becomes 612 upon 1280 EI and if we look at this, this can go through by 4, so this becomes equal to 153 upon 320 EI – that is K. What is R? How do I…? Note this is R prime.

How do I get R prime? For R prime, note that in the previous case, even though in this particular case, I have a load over here (Refer Slide Time: 19:04) which looks like a load that is on a member but note that bc is not a flexible member and since bc is not a flexible member, any load on a rigid member is considered equivalent to a nodal load and therefore, since the effect is not on a member, here, the members are ab and cb, since those are not there, you have to consider this not as a member load because bc is not a member – note that bc is a rigid member and therefore it is not one of the considered members. What we have to do is Find out the work done by the 100 Newton and this 100 is going to do negative 3 by 8, so the work done would be 100 into negative 3 by 8, which basically becomes minus 75 by 2. This then says that r_1 is equal to minus 75 by 2 multiplied by 320 upon 153 EI. This is equal to whatever this value comes out to be (I am not going to do it) that is my r_1. As soon as I know my r_1, I know my deflection pattern and since I know r_1, I can also find out what M_{ab}, M_{ba} and M_{bc} are and these can be immediately evaluated. Once I evaluate those, for example, what is M_{ab} equal to?

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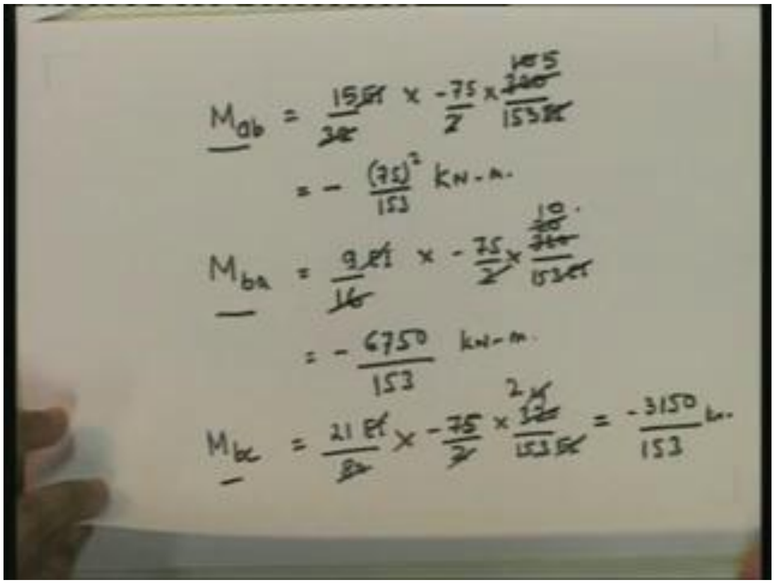

Let me just put it down. M_{ab} is equal to 15 by 32EI into minus 75 by 2 multiplied by 320 upon 153 EI; EI, EI cancels, 320 cancels 10, 10 cancels 5, so it is going to be equal to minus 75 squared upon 153 Kilonewton meter. M_{ba} is equal to 9 by 16 EI multiplied by minus 75 by 2 into 320 upon 153 EI; EI, EI cancels out, this goes 20, this goes 10 and so, what we have is 775 into 9 becomes 675, 6750 upon 153 Kilonewton meter. M_{bc} is equal to 21 EI upon 80 into minus 75 by 2 multiplied by 320 by 153 EI; EI, EI cancels, this goes 4, this goes 2, so what we have is 1575 into 2, 1575 into 2 is equal to minus 3150 upon 153 Kilonewton meter. Therefore, these are my M_{ab}, M_{ba} and M_{bc}. Once I have found out my r_1 and M_{ab}, M_{ba} and M_{bc}, now how do I solve this problem? Displaced shape – no problem, all you have to do is.… This is my displaced shape, only thing is that it will be in the opposite direction, so this will come down, this will go in this direction, this value I know and therefore, I can find out all the other values automatically and that is what you would expect: since the load at this point would come down and therefore, this would have to move in this direction and in the opposite direction. Now, onto the support reactions and the bending moment diagram.

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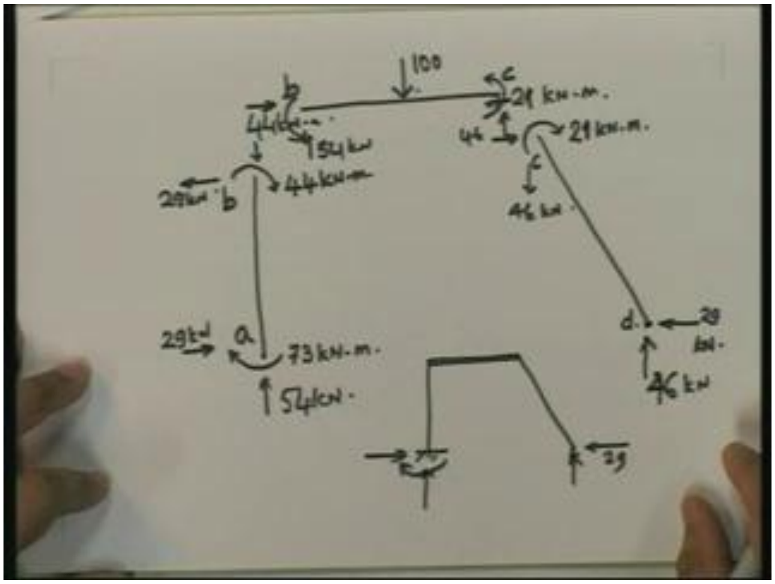

I am just going to again put it. Here, what do I know? I know what M_{ab} is. This is going to be minus, so this is going to be clockwise (Refer Slide Time: 24:43) and clockwise approximately about 73 Kilonewton meter – this is what it comes out to be approximately; this is also going to be clockwise and this is going to be equal to about 4 into 15, 60, so it will be about 44 Kilonewton meter. I am giving you approximate numbers, I do not know anything about this one but I know this one (Refer Slide Time: 25:23).

What is it? This value is going to be equal to again clockwise, so this is going to be clockwise and the value is going to be approximately 20, so it is going to be 20, so this is going to be 306, 90, so it is going to be about 21 Kilonewton meter – 20 point something. Here, this is 0, I know that this is 0, the displacement here is going to be 0. Now, let us go through the steps. I know nothing about this because this is not a member. This is a, this is b, this is b, this is c, this is c, this is d. There is no moment at this particular point. Since there is no moment at that particular point, for bc we can find out the member end moments purely from equilibrium. Since I have 44 over here, from equilibrium this will have to be 44; here again from equilibrium this has to be 21 Kilonewton meter and this has to be 44 Kilonewton meter. I know that these are going to be the moments.

What else do I know? Since I know this, I can find this, I can find this (Refer Slide Time: 27:03) and this and since this is 117, 117 divided by 4 –, I am again doing approximate numbers – is going to be equal to 117 this way divided by 4 is going to be equal to 29 – so 29 Kilonewton meter, 29 Kilonewton meter. Once I know this, again this has to be 29. Now note that there is this 100 force, so this has to be this way 29, so therefore, here I need to have a force this way. Since I have force this way, this is going to be 29 and therefore I am going to have a reaction here which is equal to 29 – I know this reaction.

How much is this going to be? This is going to be in this fashion (Refer Slide Time: 28:05), so this 29 into 4 is 117 plus 21 is going to be 138, 138 divided by 3 because this one is this way, so this one has to generate this way (Refer Slide Time: 28:33), it is going to generate this way and this is going to be equal to 117 plus 21 which is 138 by 3 and 138 by 3 is going to be equal to 46, this is 46 Kilonewton. Since this going to be 46, by definition this has to be equal to 46 and this is 46 this has to be equal to 54; if this is 54, then this has to be 54 and therefore, this has to be 54. Now the question is: does this check? Let us look at this.

What we generate over here from the 100 is 50, 50 but for this, it is going to generate an additional one, so this is going to be equal to 44 and 21 is 44 and 21 is going to be equal to 65, 65 divided by 4 is 15. Let us look at this again because there has to be equilibrium – without equilibrium, you cannot satisfy this particular problem. There is no moment here, so if I take moments about this point, this 46 into 7 has to be equal to… Let us check – let us go back and check what we are doing here. This is 117 this way. Essentially, what happens now is therefore if you look at the reactions, the reactions are this way: the reaction over here is 29, 54, 73 the reaction here is 46 and 29.

I have found out all the reactions and once I know these, I can draw my bending moment diagram very easily. I think this particular example was being presented to you essentially for the last point that I wanted to make as far as low defects were concerned that if a load is on a member that is rigid, then it is not a loaded member because bc, which is a rigid member, will not have any effect and we have to consider the equivalent load on the left-hand side.
I hope that after this, you should not have any problem by applying the stiffness method for the solution of any structural problem as far as we have only considered effects of loads on the structure.

Let us look at certain other effects now and the only other effects that I am going to spend any time.… Note that support settlement is not a major problem in this particular case because all that means is if there is support settlement, that means there is a degree of freedom corresponding to that support settlement direction and the only thing that happens is that we do not know the reaction at that point but we know what is the left-hand side, I know the displacement. I can always solve for it because after all, I just get one additional degree of freedom and that degree of freedom – I just need to solve it to be able to get any this thing. Now, what I want to consider is the last thing, which is how we consider temperature in structures and that is going to be in the stiffness method because we already know how to consider it when you have, when you use the force method, the flexibility approach. Let us now move on to that quickly.

Once we know temperature, how to consider temperature, we will have understood the complete concept behind the stiffness method. Let us now look at how to consider the stiffness method. Again, since it is a member load, I am going to first consider, how to consider it… I am going to first talk about how to consider it at the member level and once we consider at the member level, then considering at the global level becomes a trivial issue because after all, again, all it does is that S becomes K_{ab} a a r plus S_{i0} where 0 is the fixed end moment, so all I need to find out is for temperature, how do I compute the fixed end moment? Let us go back.

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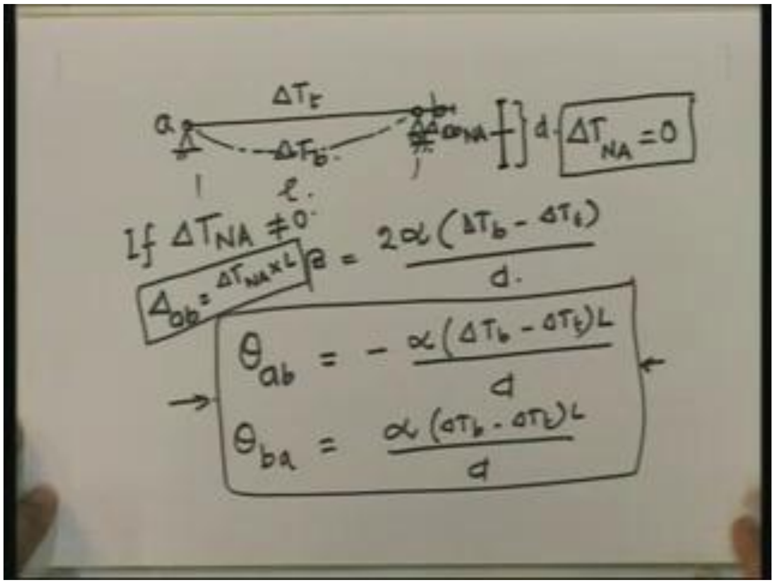

Let us review the situation as we had already dealt with it earlier, let me see what happens. Let me take a situation with both the top and the bottom. Note that here since we are only considering flexure, we are going to assume that the neutral axis does not suffer a temperature differential; if it does, all it means is that there is an axial deformation in the member and that is all there is to it. Let us consider the situation where you have delta_t; this is the top fiber – it has delta T_t and the bottom fiber has delta T_b. We have already seen that this gives rise to a curvature beta which we defined as 2 alpha into (delta T_{bottom} minus delta T_{top}) upon d where d is the depth of the beam, the difference between the top fiber and the bottom fiber – this is d (Refer Slide Time: 35:43) – to the depth of the beam.

We have already done this and we saw that this gives rise (Refer Slide Time: 35:48), so due to this curvature, you are going to have theta and theta_{ab} – this is a, this is b – is equal to minus alpha… now this is length l, so it is going to be alpha L upon d. theta_{ba} is equal to alpha…. I have already established these in the force method and the flexibility approach that we looked at earlier. This is my theta_{ab} and theta_{ba} due to the curvature. This is assuming that the neutral axis delta T at neutral axis is equal to 0. If it is not, then all it does is that in addition to this, if delta T_{NA} is not equal to 0, all that means is that in addition to theta_{ab} we also have a delta_{ab}, where delta_{ab} is equal to delta T_{NA} into L – that is all. What does that mean?

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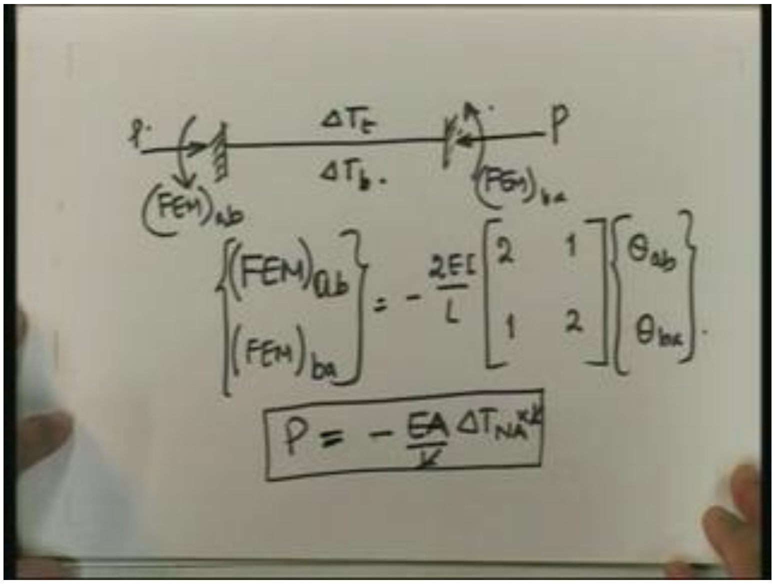

But then, you see, our structures, our basic member that we have is this. Then, I will talk about the modified member a little bit later and here, we have delta T_t and delta T_b. Now, how do I compute the fixed end moments as well as in this particular case, if there is an extension, you also have these forces that come into picture. Therefore, what is that equal to?

We find out the fixed end moment exactly the way we find out that that remember all that we knew is that this one may be due to a load or may be due to a temperature. How did we compute the fixed end moment? Look back, think back at how we computed fixed end moments for members and you will see that all we need to do is find out the fixed end moment such that the other direction is going to give you that theta minus of theta_{ab} and theta_{ba}.

The $(FEM)_{ab}$ and $(FEM)_{ba}$ are going to be equal to minus into K_a, K_a is 2EI upon L into (2 1, 1 2) into theta_{ab} and theta_{ba}. These are my FEM_{ab} (Refer Slide Time: 39:55) and this is my $(FEM)_{ba}$, this is my P, this is my P. We can show that P is equal to nothing but… this is minus because it is opposing but then you know I am just proving it, this is equal to EA delta T_{NA}; I am sorry, EA by L into delta T_{NA} L so L, L cancels, this becomes EA delta T_{NA}. I am not going to go into this but let us see what this comes out to be. This comes out to be equal to… I am going to plug in the values of this thing.

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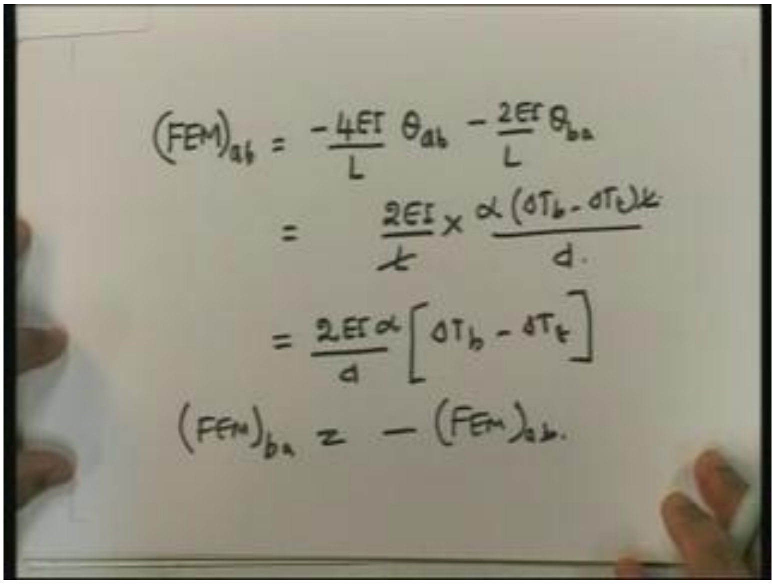

Therefore, $(FEM)_{ab}$ is equal to minus 4 EI by L into $theta_{ab}$ minus 2EI by L $theta_{ba}$ but then, $theta_{ab}$ is minus of $theta_{ba}$ so this becomes equal to 2EI by L into $theta_{ba}$ which is equal to alpha delta into (T_b minus delta T_t) L upon d; L, L cancels and so, this becomes 2EI alpha upon d into (delta T_b minus delta T_t). If you look at the other one, this is going to be $(FEM)_{ba}$ is going to be equal to minus $(FEM)_{ab}$ – these are the fixed end moments due to the rotation. If we have this kind of a situation, how how do we solve this problem? Let me now just take you through the problem that we solved last time and instead of the load of 100 Kilonewtons I am going to put it equal to temperature differential of 100 Kilonewton. Let me take the problem that we were talking about.

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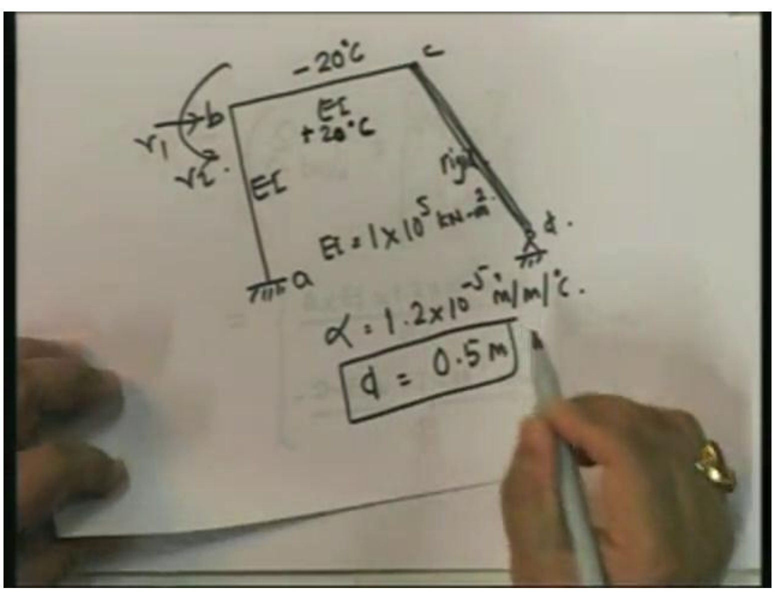

I am looking at the last week's lecture's problem. This was the problem: EI, EI, rigid, a, b, c, d. Now, the only thing that I have over here is that this is minus 20 degrees and this side is plus (Refer Slide Time: 44:04). The reason why I am doing this plus and minus is to ensure that delta NA at 1 is equal to 0. How do we solve this particular problem? Two degrees of freedom; without going into the detail, we have done it already r_1, r_2.

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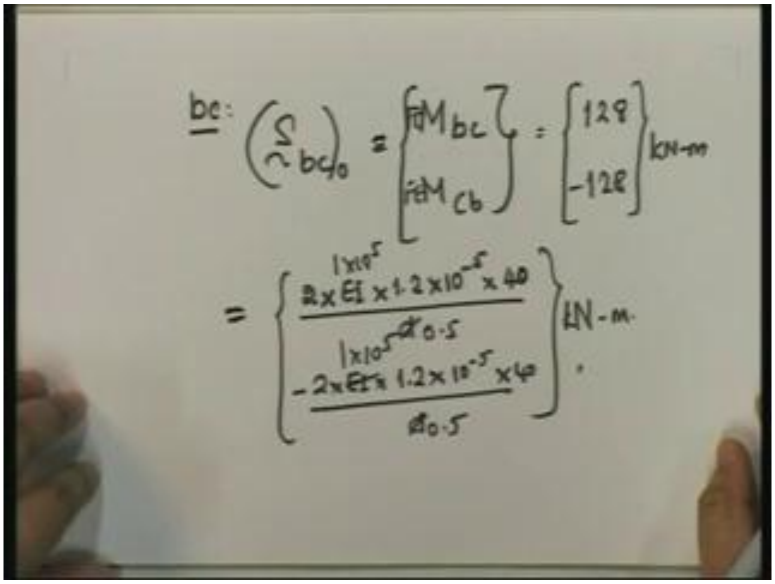

All that happens is you see, ab is still a member without any loads, so I am not going to go into the computation of ab. The only thing that happens in bc is that $(S_{bc})_0$. In the previous case, we computed it for 100 Kilonewton load at the center and we computed PL upon 8, PL upon 8 and went through and went through with that. In this particular case, all that happens is that S_{bc} is equal to M_{bc}, M_{cb} is equal to… Now, note that in this particular case, we can take alpha equal to 1.2 into 10 to the power of minus 5 meter per meter per degree Centigrade (Refer Slide Time: 45:42).

If we plug that in we get that the fixed end moments, these are the fixed end moments that are equal to 2 into EI into 1.2 into 10 to the power of minus 5 into delta T_{bottom} minus delta T_{top} is 40 divided by d – this is my fixed end moment. Note: what are the units of this? Per degree centigrade. This becomes a dimensionless value. What is this (Refer Slide Time: 46:32)? Newton meter square. Newton meter squared divided by meter is Newton meter and that is the units of the moment. This consistent unit and this one becomes 2 into EI into 1.210 to the power of 540 upon d, so there is consistent units and this one becomes 2 into EI into 1.2 into 10 to the power of minus 5 into 40 upon d. The only thing that happens is that in this particular case instead of the 100 Kilonewton force, you essentially have this moment. Therefore, once the moment, you know I can plug in a value of EI and d – I have not put that in, you will actually get it in… let us say if units of EI are in Kilonewton meter square, let me say EI is equal to 1 into 10 to the power of 5 Kilonewton meter squared (Refer Slide Time: 47:38) and let me say that the depth of the beam is 50 centimeters, so it is 0.5 meters. If we plug that in, we will put those values in, what do you get? You will see that this is 1 into 10 to the power of 5 and this is 0.5, 0.5, 1 into 10 to the power of 5 (Refer Slide Time: 48:06). What you get is 48 into 2, 96, so you get 128, so these become 128, minus 128 Kilonewton meter. Once you have 128 Kilonewton meter then all that you have to do is that this is my S_{bc} and everything else continues in exactly the same way. What are the steps?

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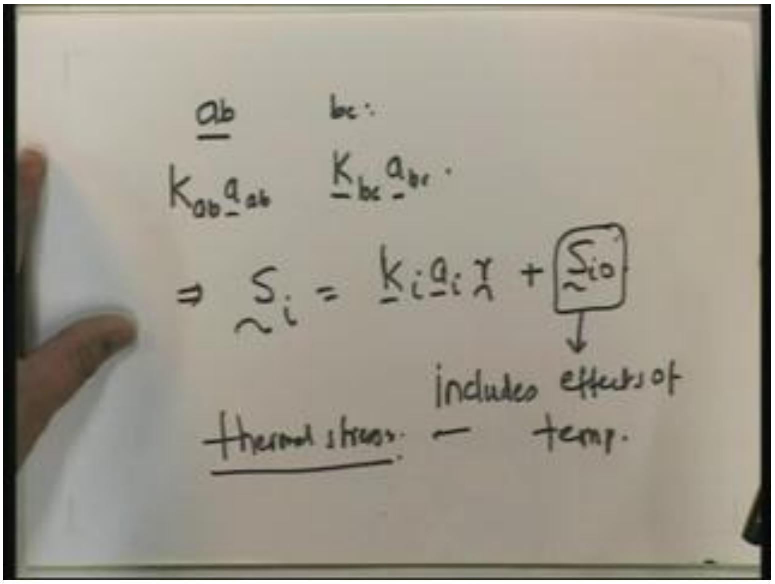

Once you have done this, you have both for ab and bc, you have K_{ab}, a_{ab}, K_{bc}, a_{bc} and so the S_i is going to be equal to K_i into a_i. Where do I get a_i from? I get a_i from the kinematic relationship. I am not going into the details of the kinematic relationship because I have already solved this problem. The only difference between this problem and the previous problem that I have already solved in the last lecture is that instead of the 100 Kilonewton force I have just considered it to be a temperature; therefore, there is no difference in (a, K_a) and all of those kinds of things: K_{ab}, a_{ab}, K_{bc}, a_{bc} all these are identical to what we have already seen. I am only trying to show you what effect the thermal has. I have already shown that the only way to consider thermal is to compute the fixed end moments and I have already computed the fixed end moments. All that happens is once you have this, this becomes this (Refer Slide Time: 50:15) where this now includes effects of temperature, which means thermal stress, this includes the effects of temperature and I can always get this.

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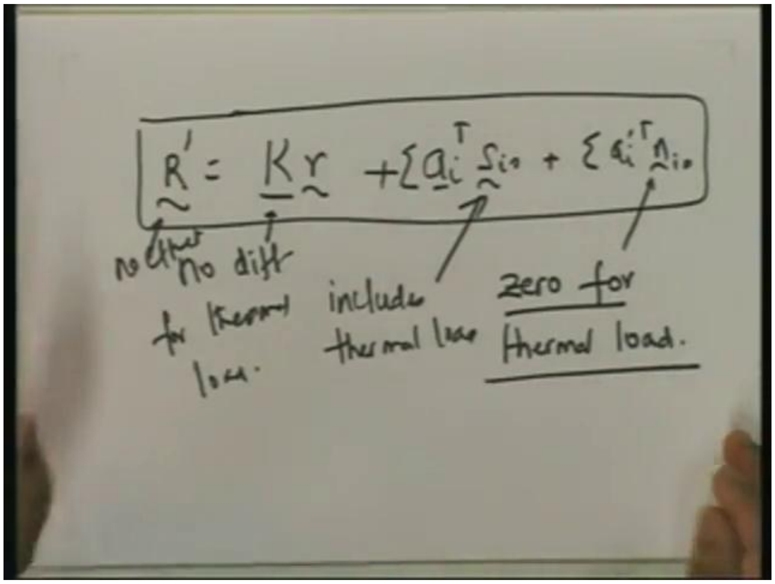

Then, I can also get ultimately this which is K into r – note that in these (Refer Slide Time: 50:55) there is no change – and summation a_i transpose S_{i0}. Note now that a_i bar transpose n_{i0} is 0 for thermal load because when you have a thermal load, you do not have any additional reaction; there is no reaction – there is only an equal and opposite moment that is developed at the fixed ends and those are essentially to account for the curvature that comes into picture because of thermal. This of course includes thermal load, no difference for thermal load. This is of course no effect of thermal.

If you look at this particular problem, the only part where the thermal effects at the member comes into picture is only in this, which is the fixed end moment, and that we have already included in our formulation. Therefore, if you really look at it, all we have to do right now is if you were to review it that wherever you have a temperature load effect and you are looking only at the flexural part of it, all it does is that you have to account for it and the fixed end moments; for fixed end moment, I have already given you the effects; this is the fixed end moment (Refer Slide Time: 53:10) where this is the temperature differential at the bottom, temperature differential at the top fiber and this is the depth of the beam, this is the coefficient of thermal expansion, this is EI. That is all there is to it. Of course, there is an additional effect here but if EA is equal to infinity, then all you have is that this effect has no effect whatsoever – all that happens is that you have a expansion and therefore, you have an additional displacement because of the expansion of the particular member; there is no effect on the structural composition as such.

So much for the displacement method and the stiffness method, which is the matrix displacement method and I hope that over the last many lectures, I have solved quite a few problems in the stiffness method and at the end of this, I hope you shall be able to apply the slop deflection method, the displacement method and the stiffness method, which is the matrix approach to the stiffness method.

From the next lecture, I am going to look at a completely different topic and that is influence lines, which is equally important. By the end of this lecture, I hope you will be able to solve any planar problem, planar frame problem using either the force method or the displacement method as well as both their matrix methods associated with them; the matrix method associated with force method is called the flexibility method and the matrix method associated with the displacement method is known as the stiffness method.

Thank you.