Structural Analysis - II
Prof. P. Banerjee
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 34

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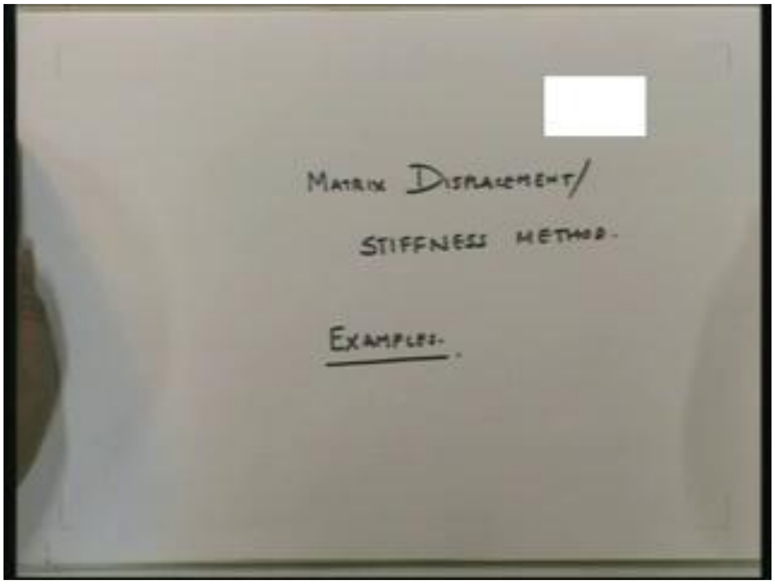

Good morning. We are continuing by looking at examples related to the application of the matrix displacement method or the stiffness method and that is our today's lecture: matrix displacement stiffness method with examples. Let us continue looking at more examples and today I am going to introduce to you a situation where maybe one of the members is rigid.

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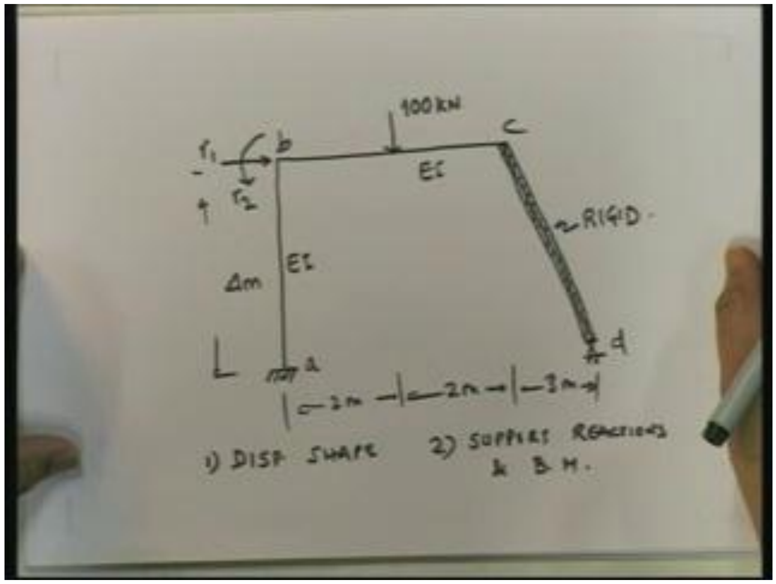

In other words, its flexural rigidity is significantly higher than the other two members. How do we deal with such a problem? Let us look at the problem that we are looking at. This is the problem: a, b, c, d where a is fixed, b is continuous, ab has EI flexibility, bc has EI flexibility and cd is the rigid member. This is actually a two degree of freedom structure and the r_1 and r_2 that I have defined over here (Refer Slide Time: 03:01) – the displacement and rotation at b are my degrees of freedom.

What has been asked of you is the typical structural analysis problem: Find out the displaced shape and the support reaction and the bending moment given this load (Refer Slide Time: 03:19) – this is the only load that is there on the structure. Let us now look at it.

What I will do is I shall look at the solution process for this problem to begin with and then, I will change this problem around a little bit to introduce to you another concept of the application of the stiffness method. What is the first step? Identifying the degrees of freedom and all the other forces etc. The next aspect is to find out the degrees of freedom for each member; first of all, find out the type of element; remember that there are only two types of elements that we have discussed: one is the standard fixed-fixed element and the second one is the modified element with fixity at one end and hinge at the other end, pinned at the other end.

We have to decide which one to take and then proceed. Note over here that any member that is rigid is not considered a member in the stiffness method itself. How does the effect of that member come into the picture? The effect of that member actually comes into the kinematic relationship. Although cd is a member and we have to find out the bending moment diagram for cd also, in the stiffness method since it only refers to flexible members cd will not be considered a regular member in the analysis. In effect, there are only two members in this particular problem: ab and bc.

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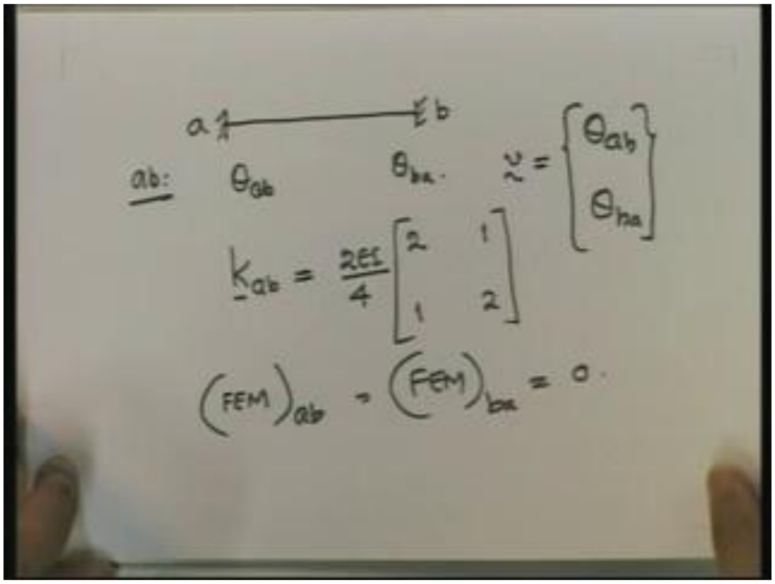

Let us look at ab. If you look at ab, a, b – this is the standard element (Refer Slide Time: 5:35) because a is fixed and b is continuous, therefore, you have this element. The degrees of freedom are theta$_{ab}$ and theta$_{ba}$ these are the two member end deformations that we are interested in. My v is going to have theta$_{ab}$ and theta$_{ba}$. What about K? Obviously then, K of member ab is the standard 2 EI upon L – in this particular case, L is 4 meters – into (2 1, 1 2). This is my K$_{ab}$ and finally I should find out the fixed end moments. Since there is no member load, fixed end moments are both 0. This is for member ab. Let us look at member bc.

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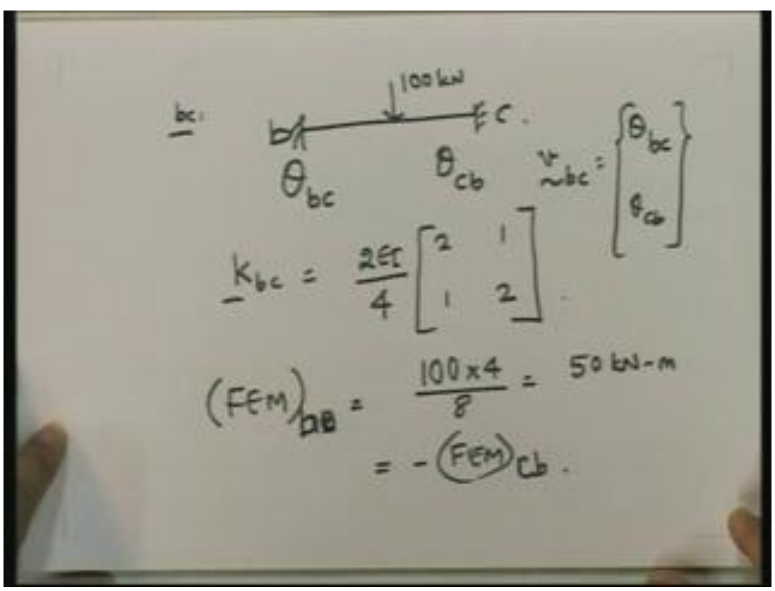

Member bc is rigid. At b it is continuous and it is continuous at c also so the element is of this form, this is b, this is c. The degrees of freedom are $theta_{bc}$, $theta_{cb}$ and therefore v of bc is equal to $theta_{bc}$, $theta_{cb}$. Note that this one has a load in the member. K_{bc} is the same standard one: 2EI upon L and L again is 4, so it is 2 L (2 1, 1 2). The fixed end moment is PL upon 8 $(FEM)_{bc}$ we know that so it is going to be 100 into 4 divided by 8, that is 50 Kilonewton meter which is $(FEM)_{cb}$ (Refer Slide Time: 08:31). Thus, we have evaluated what the degrees of freedom are, what the stiffness matrix is and what the fixed end moments are. The next step is the kinematics.

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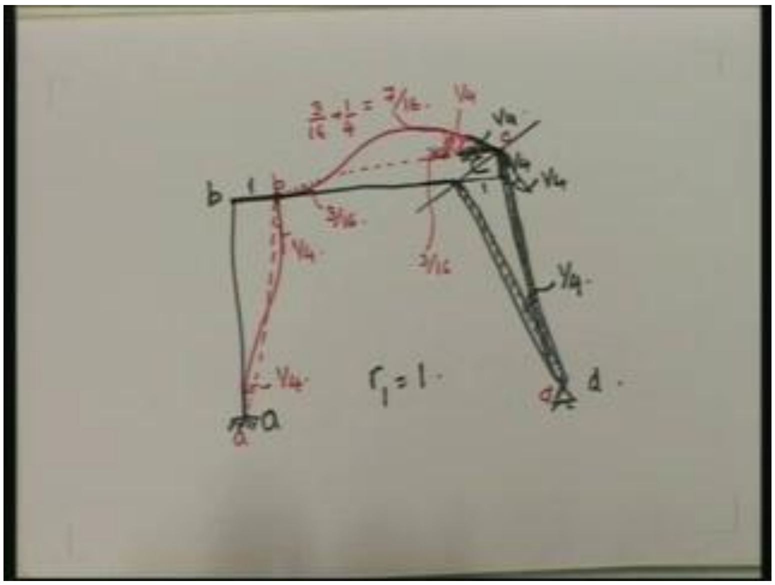

Here r_1 is equal to 1. This point – where will it go? Since this is fixed, it can only move in this direction (Refer Slide Time: 09:22), so r_1 is equal to 1 means b moves here. Now, this point also has to move horizontally by 1, but note that since this is an inclined member it cannot go this way so it has to move along this, so it comes over here whose horizontal component is 1 and then from geometry you will see that this is equal to 3 by 4 and this is equal to 5 by 4.

This is the point that I was trying to make: Look at it, what happens over here is that this member which is rigid is going to go in this way. What is this angle? This angle is equal to 5 by 4 and the length is 5 meters, so this angle is 1 by 4. Note that since this has rotated clockwise by 1 by 4, to keep continuity, this also, the tangent has to go up by 1 by 4 (this is 1 by 4 – the tangent) so that this continuity is maintained – this is an important point that I cannot highlight. The rigidity of this ensures that if this rotates by 1 by 4 then the tangent at this point… (Refer Slide Time: 11:01) remember that this is a continuous member, this is the original position, so if this goes clockwise by 1 by 4 so will this have to go to keep continuity because originally, it is this way – this and this; now since this has rotated by 1 by 4, this also has to rotate. If we look at the displaced shape over here this is how it looks. This one will go by 1 and if we look at it from the chord, these are the tangents and this tangent is equal to 1 divided by length and so is this: 1 by 4. Here, remember this tangent is this way, so this tangent is this way, so this one will go in this fashion but over here, the tangent will have to be off by 1 by 4. If we look at this and we join this, then this is the chord connecting c, b, a, d. If you look at this particular one, how much is this angle? This angle has to be equal to 3 by 4 divided by 4, so this is going to be equal to 3 by 16.

What about this (Refer Slide Time: 12:50)? From the tangent if you look at this particular situation, what is this angle equal to? Between the horizontal and this; between horizontal and this, this is going to be equal to the same because this is the same angle, so this is 3 by 16 but from here, the tangent is 1 by 4, so from to the tangent is equal to 3 by 16 plus 1 by 4, which is equal to 7 by 16. This one we need not consider. Therefore $theta_{ab}$ is equal to 1 by 4, $theta_{ba}$ is equal to 1 by 4. Are these anticlockwise? From the chord to the tangent both are anticlockwise, therefore you have plus 1 by 4 and plus 1 by 4. Over here from the chord to the tangent, this is 3 by 16, so $theta_{bc}$ is equal to minus 3 by 16 and $theta_{cb}$ is equal to… from the chord to the tangent clockwise so it is minus 7 by 16. That is all we need – cd does not come into the picture because cd is a rigid member.

You look at the kinematics. The kinematics is very, very important when you have a rigid member. The thing is that if this had not been a rigid member, then this would have gone something like this (Refer Slide Time: 14:33) and this would have remained the same. But since this is a rigid member as soon as this has to rotate, from the tangent this one turns out to be 1 by 4 because it is rigid and as soon as that is 1 by 4, this also has to rotate – this is an important point that all of you should understand in the kinematic relationship wherever rigid members are considered.

Today, my focus is going to be on the effect of rigid members in the analysis of frames. For r_1 is equal to 1, this is the displacement pattern. Now, let us look at r_2 is equal to 1.

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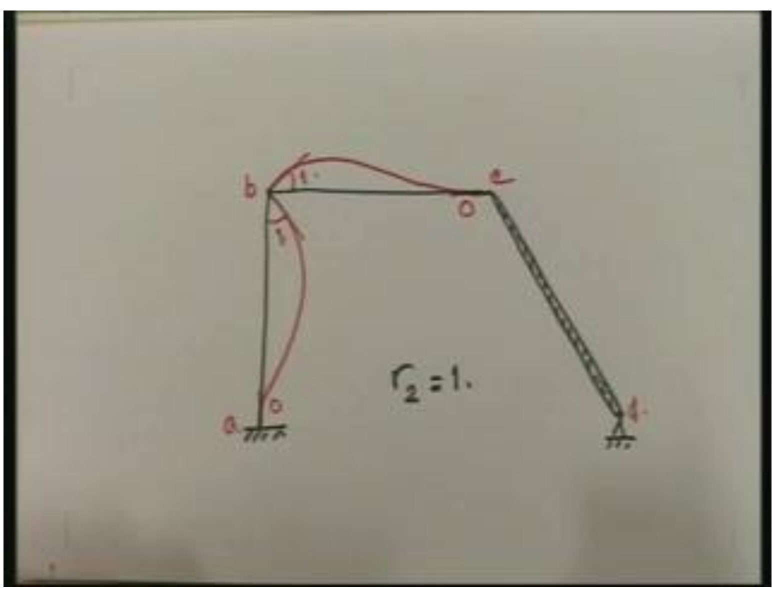

Here r_2 is equal to 1 is rotation. Always remember that rotation is always easier to do, this is the rotation – let us look at it. Can this joint (Refer Slide Time: 15:45) rotate without causing any displacement? Sure, it can – it can go in this fashion and I will show you. Please note one very important thing and that is that I always show displacements as exaggerated and it is obvious that this is not…. In other words, the tangential motion does not increase the length of this. Therefore, even though I write 3 by 16 etc… it is not really 3 by 16, it is 3 by 16 of r_1, r_1 itself is very small; this is 1 into r_2, r_2 itself is very small; it is just that I am taking r_1 and r_2 equal to 1 to essentially establish…. Here, what is the rotation? 0. Rotation here.… This is b, a, c and d, so ab is 0, ba is 1, bc is 1, cb is 0. Now, can we write down the 'a matrix' for this particular structure?

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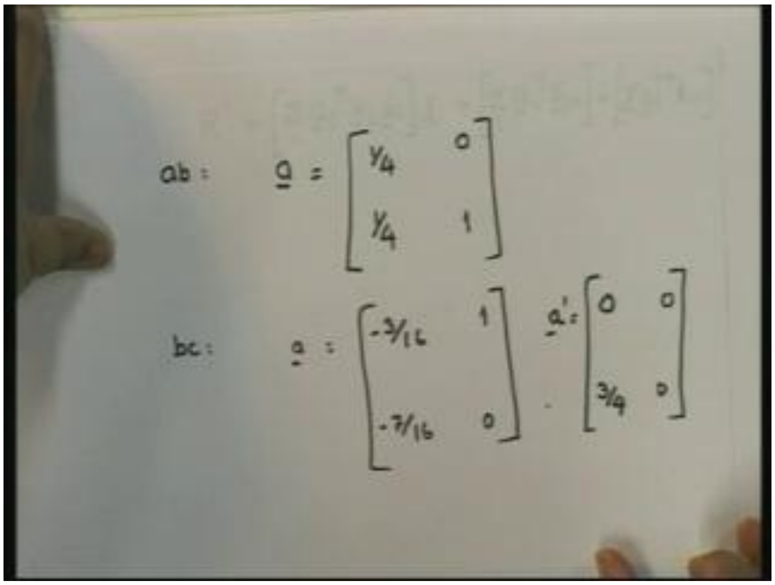

For ab, the 'a matrix' is equal to 1 by 4, 1 by 4 and for r_2, it is equal to 0, 1; for bc, a is equal to minus 3 by 16, minus 7 by 16 and 1, 0 for r_2 – this is my a matrix (Refer Slide Time: 17:54).

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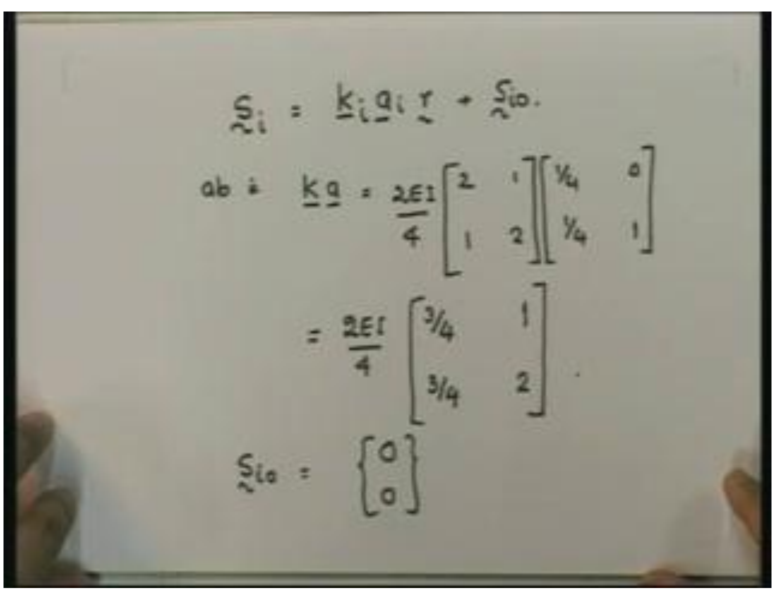

The next step is relating S_i; S_i is equal to K_i into a_i r plus S_{i0}. What we have to establish is K_i for a_i; for ab, Ka is equal to 2 EI upon 4 into (2, 1, 1, 2) into one-fourth, one-fourth, 0, 1; this is equal to 2 EI upon 4 into 2 by 4 plus 1 by 4, so this is 3 by 4, 2 into 1, so this is 1, this is 1 by 4 plus 2 by 4, 3 by 4, and this is 2 – this is for ab and S_{i0} is 0, 0 – this is for ab and let us find out that for bc too.

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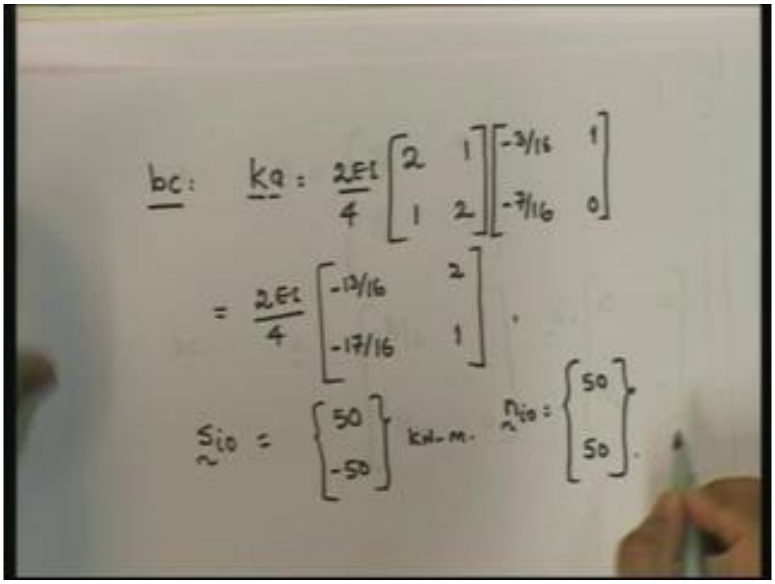

If we find out for bc, you will see that Ka is equal to 2 EI by 4 into (2 1, 1 2) multiplied by a, which is (minus 3 by 16, minus 7 by 16, 1, 0). If you put this together, what you get is 2 EI by 4 and minus 6 by 16, minus, so this is minus 13 by 16; similarly, this minus is going to be minus 17 by 16; this is going to be equal to 2 and this is going to be equal to 1. S_{i0} is going to be equal to 50, minus 50 Kilonewton meter. We have written these down.

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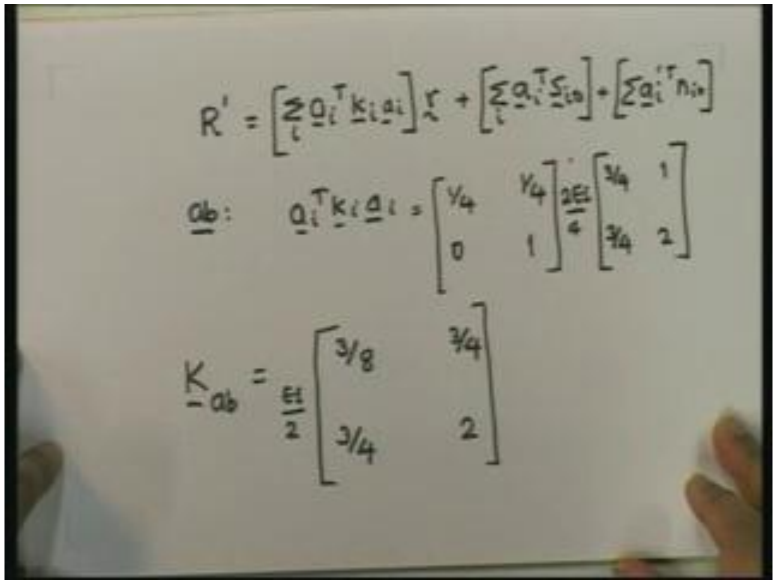

The next step now is to find out; R effective is equal to summation over all the members a_i transpose K_i a_i into r plus summed over all the members that have fixed end moments plus (Refer Slide Time: 22:09) and in this particular case, I just wanted to tell you that… How do we get a_i trans a bar? You will see that 'a bar' essentially is for member bc, which is the loaded member, we need to find out what is the vertical displacement of b – that is 0. **So** we can write down for bc because bc is the only member that 'a bar' is equal to bc – that is 0, then the vertical displacement of c which is 3 by 4, so this is 3 by 4 and the displacement corresponding to r_2 is vertical 0 0, so we can put down 0 0.

An important point to note over here is that.… What are n_{io}? Since it is 100 at the center, the reactions are going to be 50 50 upwards (Refer Slide Time: 23:22). This is what we have and therefore for ab, since there is no fixed end moment and there is no fixed end reactions obviously, therefore these two terms do not exist for ab and the only term that exists is a_i transpose K_i a_i. For ab, Ka is already given, so a transpose becomes 1 upon 4, 1 upon 4, 0 1 – that is a transpose multiplied by 2 EI upon 4 into what we have already computed; 3 by 4, 3 by 4, 1 2 and this is equal to 1 by 4, 1 by 4, I will put EI upon 2 outside, so you have 1 by 4 into 3 by 4 that is 3 by 16 plus 3 by 16 that is equal to 3 upon 8, here you have 3 upon 4, this way you have 1 by 4 plus 2 by 4, 3 by 4 and this way you have 2. That is a_i transpose K_i or we can call this as the contribution to the structure stiffness matrix of ab. Now, what we need to do is we need to find out the same thing for bc.

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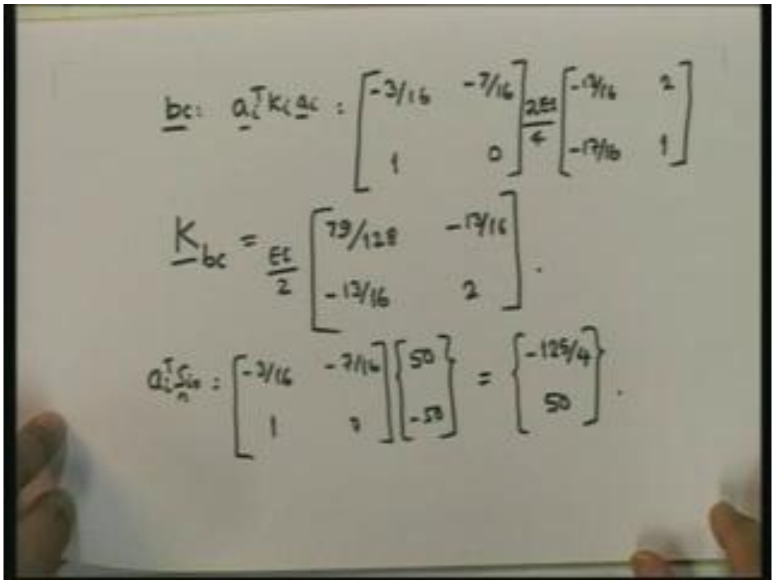

Of course, for bc we also have to find out the a_i transpose S_{i0} as well as a_i prime transpose into n_{i0} because bc has member loads. What do we have here? First is a_i transpose K_i a_i. Now, a transpose for this will be minus 3 by 16, minus 7 by 16, 1, 0 and the K_i a_i has already been obtained, that is equal to minus 13 by 16, minus 17 by 16, 2 1. If we look at that, this becomes EI over 2 and what do we have? 39 upon 256 and 119, 119 and 39 is 158 upon 256, so 1, sorry 39 and 119 is 158, so it will become 79 upon 128, this one is going to be equal to minus 13 by 16, this is going to be minus 6 plus minus 7, so that is going to be minus 13 by 16 and this one is going to be 2. That is the contribution to the structure matrix of bc. I also need to find out a_i transpose n_{io}, so if I do that, a_i transpose S_i is going to be equal to minus 3 by 16, minus 7 by 16, 1, 0 multiplied by S_i, which is 50, minus 50 and if we look at it, it is going to be equal to minus 50, so I can take it as 1, so it will be minus 10 upon 16 into… so it will be minus 500 upon 16, 500 upon 16 will become equal to minus 125 upon 4 and on this side, I will have 150, so this is going to be 50. This is going to be a_i transpose S_i.

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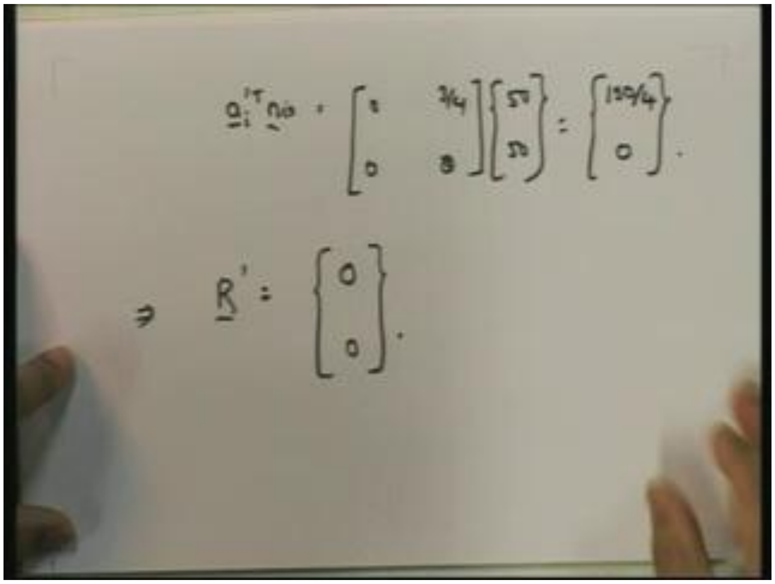

Finally, I need to do a_i prime transpose into n_{i0}. a_i prime transpose is going to be equal to 0, 3 by 4, 0, 0 into 50, 50 is going to be equal to 150 by 4 and 0. By the way, what is R prime equal to? You will see that there are no nodal loads, so R prime has to be equal to 0, 0. Let me put it all together; I am doing the summations.

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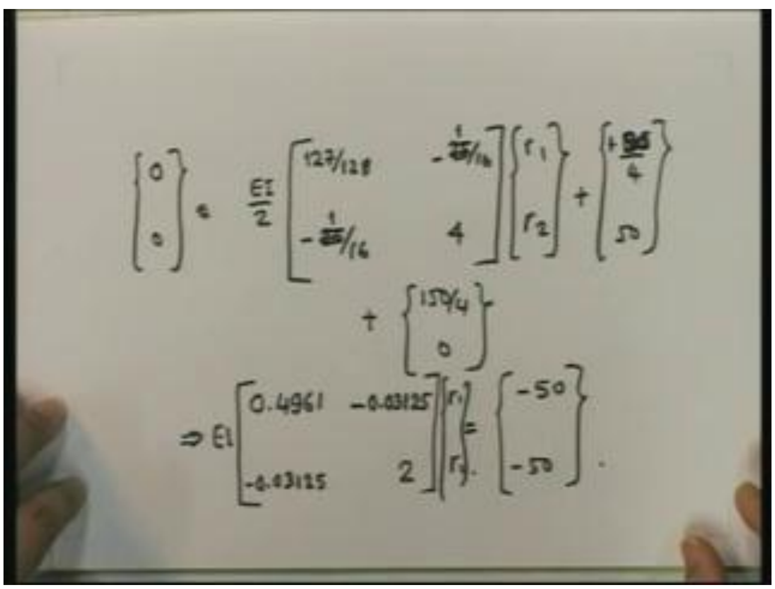

I have 0 0 equal to… now I need to do K_{ab} – it is given by this (Refer Slide Time: 30:08) and K_{bc} is given by this, so this is equal to…. I can put EI by 2 outside and inside, I will have 79 plus 48, 79 by 48 is 127 upon 128, here I have 13 by 16 and 12 by 16, so I will get minus 25 upon 16 (13 by 16 plus 12 by 16, minus 25 by 16) and here also, we will get minus 25 upon 16 and over here, you have 2 plus 2, which is equal to 4 and this times r_1, r_2 plus of course that is minus 125 by 4, plus 50 plus my a_i transpose, which is 150 by 4, 0. This is the relationship that I have and if I plug in this, what will we get?

I will put this thing down, put this on this side and put the EI outside and then what we get over here is… 127, so this becomes 0.4961, this one is 2 and this one is 25 upon 32; let me go back and check whatever we have written down; this is equal to 3 by 8EI r_1 plus EI upon 2 – that is M_{ab} and M_{ba} is equal to 3 upon 8EI r_1 plus EI r_2 – correct; right now, I am just going back and checking the statements that I have made, so Ka I have 13 upon 32EI into r_1 plus EI r_2 plus 50 and in the bottom, we have minus 17 upon 32 plus EI.

If we look at this, this is minus 13 upon 16 (Refer Slide Time: 33:41) and K_{ab} is 3 upon 4, so actually if you look at this, when you add the two, you do not get minus 25, you get 12 and you get minus 13 plus 12, so you get minus 1 upon 16 and this is also minus 1 upon 16 – that is what I was checking and then this equation becomes minus 0.03125, minus 0.03125 is equal to… and on this side, we have it equal to minus 125 and plus 150; just let me check this over again – let us do all the checks properly.

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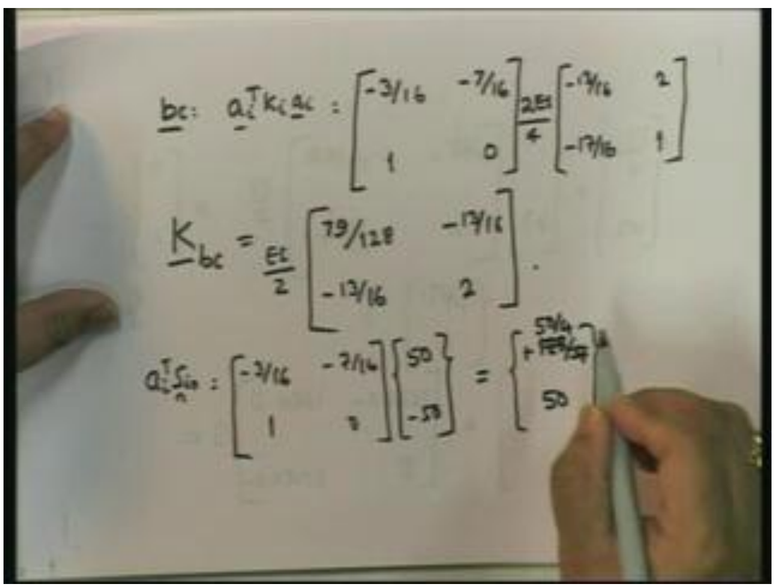

What we get over here is minus 3 upon 16 into 50 plus…. Again, here is a mistake: minus 3 into 50 (Refer Slide Time: 35:17) and here we get plus, so basically you will get 4 upon 16 positive, so this is going to be plus 4 upon 16, which is… so this is going to be plus 50 upon 4. Here, we have plus 50 upon 4, so plus 50 upon 4 plus 150 by 4 is equal to 200 by 4, which is 50, so 50 and 50, so what we get on this side is minus 50 and minus 50, of course into sorry r_1, r_2. This is the final equation that we get.

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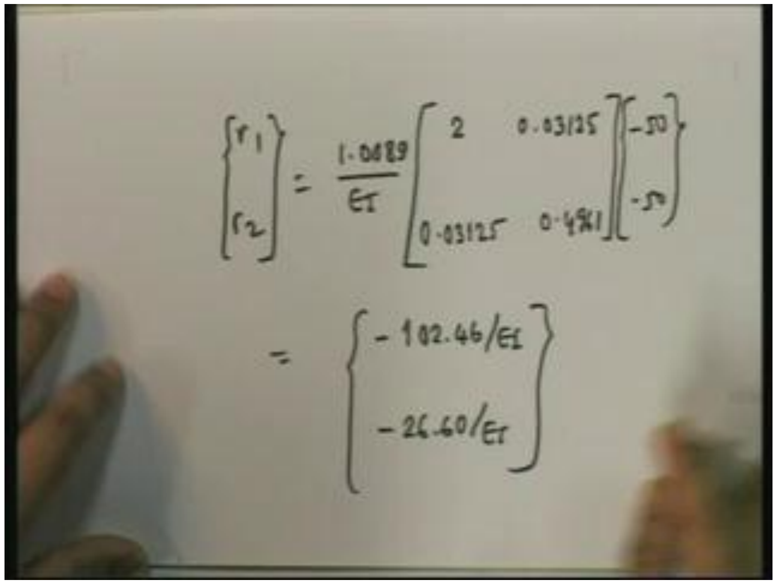

If we look at this equation, the solution turns out to be equal to… you can solve for this: r_1, r_2 is equal to 1.0089 upon EI and here, I have (2, 0.03125, 0.03125, 0.4961) into (minus 50 and minus 50) is equal to minus 102.46 upon EI, minus 26.60 upon EI – this is my r_1 and r_2 (Refer Slide Time: 37:14). I know what my r_1 and r_2 are and once I have got my r_1 and r_2, can I draw my displaced shape? Let us try to do that; we will draw the displaced shape. How do we draw the displaced shape? Well, let us look at it.

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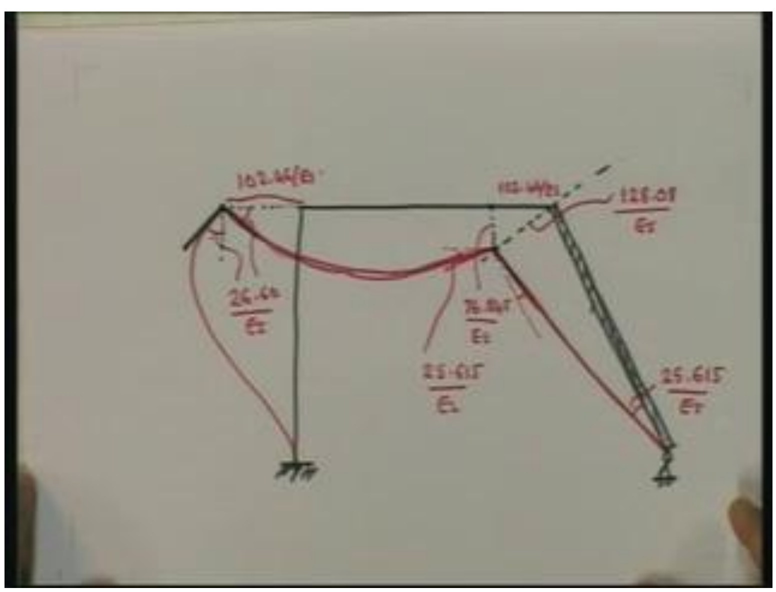

This is this way, this is this way and this is the undisplaced shape where this is rigid. We know that the r_1 which is this is minus, which basically means r_1 was positive in this direction (Refer

Slide Time: 37:59), so this will have moved by 102.46 in this direction. Furthermore, the anticlockwise is considered as positive, so minus 26.6 is that if this was straight, then it has moved clockwise, so this is your tangent at this point for this and therefore, this also will have moved 90 degrees.

Note another thing that, this point would have moved by 102.6; therefore, this will have moved at 90 degrees to this; this is my point and if I look at this particular point and I were to draw the displaced shape, the displaced shape would look like this: this one will have moved and note that this is rigid – this is rigid and so it has to move in a straight line.

How much is this? If we look at this, this is equal to 102.46 divided by EI and this one will turn out to be 3 by 4 of that, so 3 by 4 of that is equal to 76.845 upon EI and this is equal to then 128.08 upon EI. If this is 128, then this angle turns out to be equal to divided by 5, which is equal to 25.615 by EI; if this go this way, note that this is the straight line – that means this is also equal to… and therefore this one will also have moved, this has moved anticlockwise, so this will also have moved anticlockwise and this should be it, where this is equal to 25.615 by EI to the horizontal.

If we look at this, this is going to look like this, this and this, where this angle and this angle – both of them are equal to 26.60 upon EI and this displacement is equal to 102.46 by EI. This is the displaced shape. This is straight, the angle here is 26.6, this one is 78 point and note that since this is this way, this will actually go this way and then go in this manner and this. This is the displaced shape that we have for the structure. Once we have the displaced shape and we also have our r_1, now I have got my S_i for both ab and bc.

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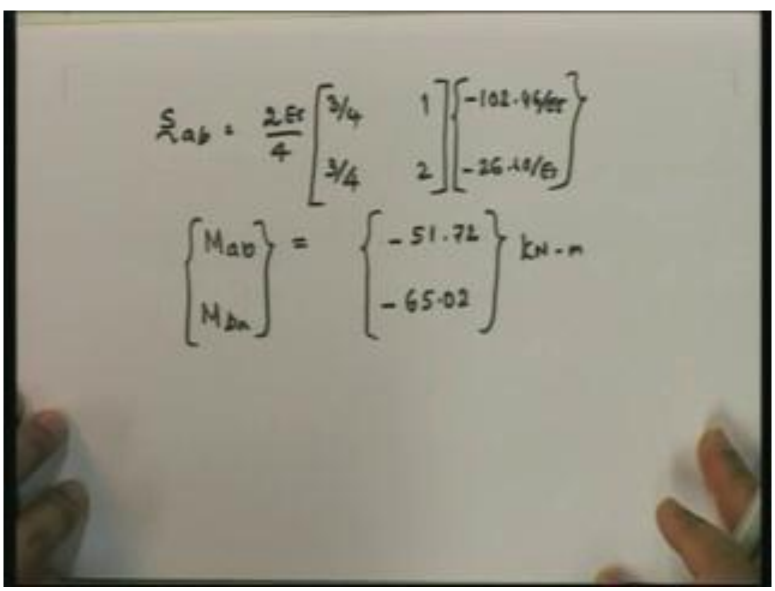

Therefore, S_{ab} is going to be equal to 2EI upon 4 into… we have calculated this earlier – this is K_i a_i, this is equal to this into r_1 and r_2, r_1 and r_2 are 102.46 upon EI, minus 26.60 upon EI, there is no S_{i0}, so that is 0, so S_{ab} is equal to minus 51.72 and minus 65.02 Kilonewton meter – this is M_{ab}, this is M_{ba}. Having done that for ab we can find **for** S_{bc}.

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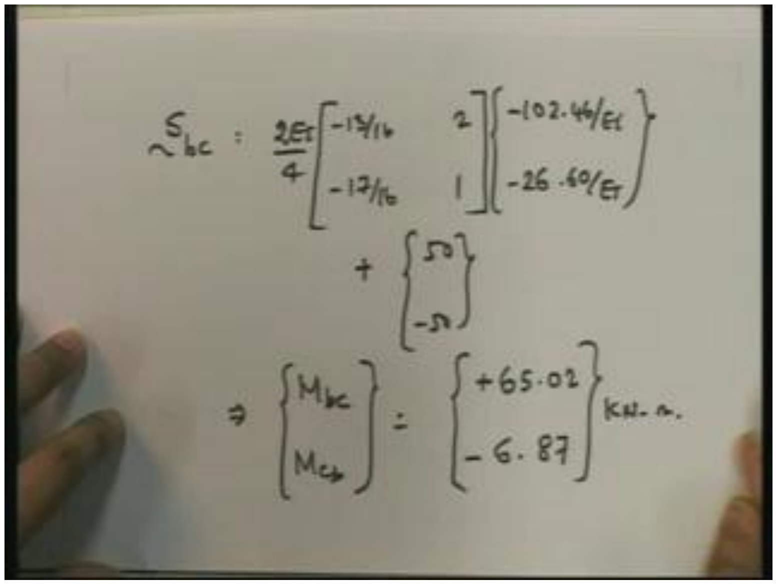

S_{bc} is equal to 2 EI upon 4 into (minus 13 by 16, minus 17 by 16, 2 1) into minus 102.46 upon EI, minus 26.60 upon EI plus S_{i0} which is 50 and minus 50. These imply that in this particular case, M_{bc}, M_{cb} are equal to plus 65.02 and minus 6.87 Kilonewton meter. Once you have got the member end moments we can now take the free body to be able to get the support reactions.

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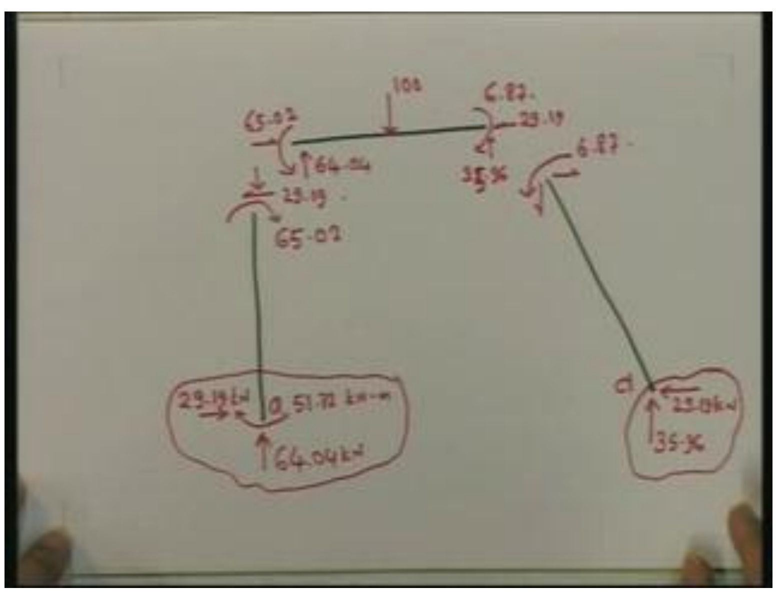

Now note one thing. What are the things that we know? We know that this is equal to 51.72 (Refer Slide Time: 45:01), we know that this is equal to 65.02, so what we know is we know this and therefore, we will know the shears here because there is no other load and the shears are going to be in this direction and they are going to be equal to… this plus this divided by 4, which works out to be 29.19, 29.19 – these we know. Furthermore, what else do we know?

For this one, we do not know what the vertical reaction here is; there will be a vertical reaction – we do not know what that is. Therefore, what we have to now do is we have to look at this particular member – it has a 100 Kilonewton force acting here, we have a 65.02 moment acting here (Refer Slide Time: 46:04) and we have a clockwise moment of 6.87 acting here. Therefore, you have one aspect, which is this minus this divided by 4 – that is one and the other is 50 50 both sides, so when you combine the two you get that this is equal to 64.04 and this is equal to 35.96.

Once we know that, from this we know that this is going to be 64 and so over here we have 64.04 Kilonewton, this is Kilonewton meter, this is Kilonewton. I know the reactions at this end. Now let us look at this (Refer Slide Time: 47:03). If this acts here, this has to act here, this has to act here – that is going to be 29.19. Therefore, if we look at this by this side, we will have to have the situation that this will be 6.87 and over here, we have this and this; this is going to be equal to this and this is going to be equal to this. Therefore**,** over here, what kind of reactions do we have?

If we go through with this, you will get 35.96 and over here, this is going to be acting in this direction 29.19 and moment over here, if you check these out, you will see that the moment will turn out to be 0, so this is the reaction at this support and this represents the reaction at the fixed support at a, this is d; these are the reactions at the hinge and these are the…. This is the way we can find out the reactions and once we have these member end moments and these we can now draw the bending moment diagram.

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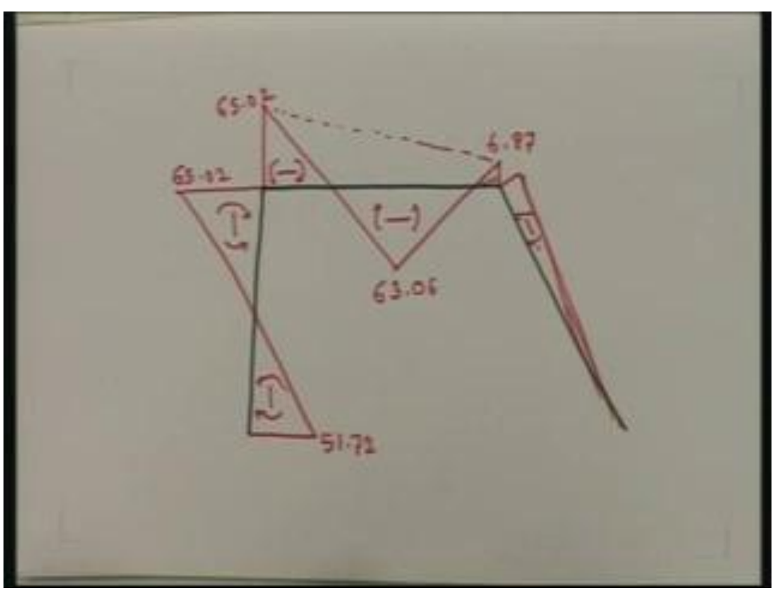
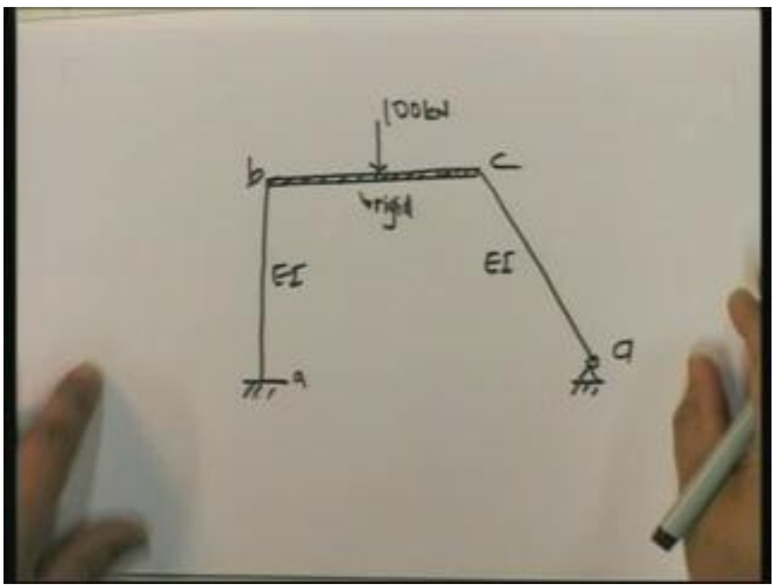


Once you know that, this is the final one: drawing the bending moment diagram. I will draw the final bending moment diagram without going into how I have done it – I will leave it up to you to check it out. This is 51.72 (Refer Slide Time: 49:02), this is 65.02, so what we have over here is this, this is equal to this kind of so that tension at this side, compression at this side and over here, you will have just the opposite way – this way (Refer Slide Time: 49:32), here you will have the same and here, you will have 6, so if we do this, this is going to be 100, so if we draw this, it will look like this, this is going to be the same 65.02, this you can compute is 63.06, this is 6.87 and over here, we have the same.

Of course, here, this is the bending moment diagram but that does not give rise to any flexure for the simple reason that this is a rigid member, so you do have a situation where you have this, but although this looks like this, the bending moment looks like this, there is no flexure; on these, you have flexure. I have got the bending moment diagram, I have got the reactions and I have got the displaced shape – complete problem solved. I am going to leave you with this problem and I want you to think about it.

(Refer Slide Time: 51:06)



Same structure: this is EI, this is EI, this one is now rigid, this is a, b, c, d; all I have done here is that in the previous case, this was rigid (Refer Slide Time: 51:44) and this was flexible. Now, this is a more realistic kind of situation where the two columns are flexible and the beam connecting the two columns is rigid – this is a more realistic kind of structure. Why is it considered rigid? Because typically, you might have a situation where the columns may be EI but the beam because it is a T-beam, it may be something like 10 or 15EI – for all practical purposes, you can consider it to be rigid. This is the same 100-ton load, ab, bc – think about this problem.

I am going to leave you today with this – think about this and we will take up this problem very quickly next time before we move on to the other kind of load effects or other kind of member local effects, which are important. Thank you very much. See you next time.