**Structure Analysis - II**
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**Lecture – 33**

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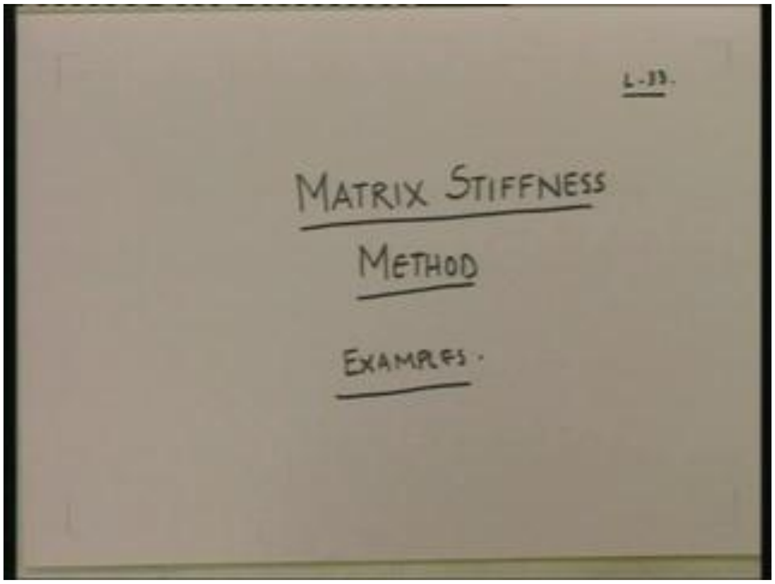


Good morning. Today, we are going to continue looking at the matrix stiffness method and the specific problem. Till now, we have been looking at a member that is essentially subjected to flexure because we have neglected both the shear and the axial deformations in the member. Today, I am going to take up an example in which you will have a member that is essentially subjected to only actual forces and we cannot neglect the effect of axial deformations. What do we do in such a situation? Today again, we are looking at matrix stiffness method application with respect by taking examples. Let me show you the problem statement and then we will see how to tackle it.

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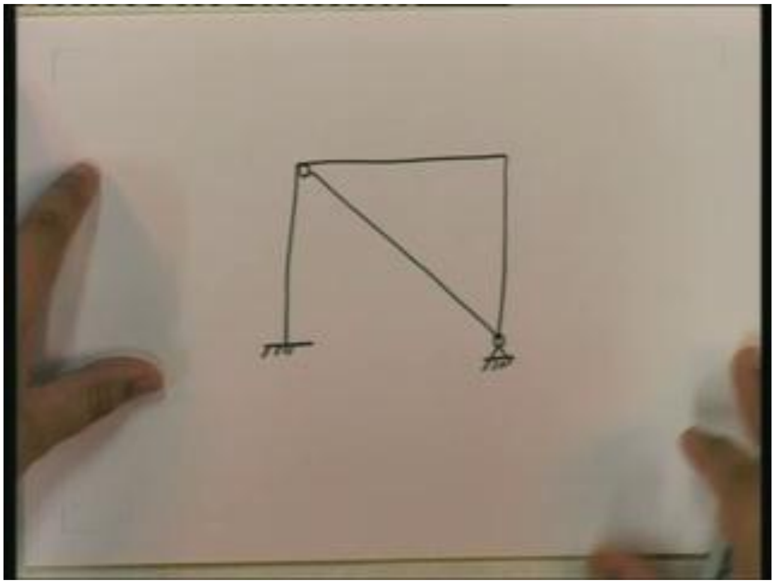

This is the problem that we are looking at: what you have is a frame and this is more in terms of a braced frame; this is a bracing (Refer Slide Time: 03:20). Bracings typically are pinned at two ends because essentially if you look at what happens to bracings, bracings in a frame are essentially to stiffen the frame and to take axial forces only. Bracing members are not normally designed for flexure. This is a braced frame a typical braced frame.

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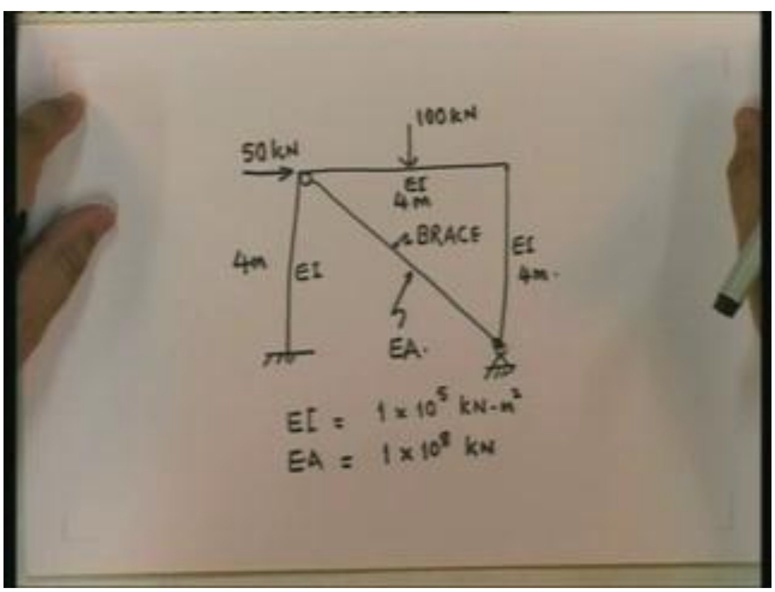


Let us look at the typical loading that you would have: you would have this kind of a loading and this kind of loading (Refer Slide Time: 03:59). Let me give some numbers: this would be EI, this would be EI, this would be EI; these are all frame members; note that this pin is only the connection of this brace – it is pinned at both ends, so essentially this is only subjected to axial and so, the only thing we have to do is to define its axial rigidity. Let me define some... this could be 4 meters, this could be 4 meters, this could be 4 meters this could be 4 meters, this could be 100 Kilonewtons and this could be 50 Kilonewtons. Let me say that EI is equal to 1 into 10 to the power of 5 Kilonewton meter squared and EA is equal to 1 into 10 to the power of 8 Kilonewton. These are just numbers that I am fitting in to ensure **to e**xplain what this is. Let me put A, B, C, D. Here if you look at it, we have to look at the members.

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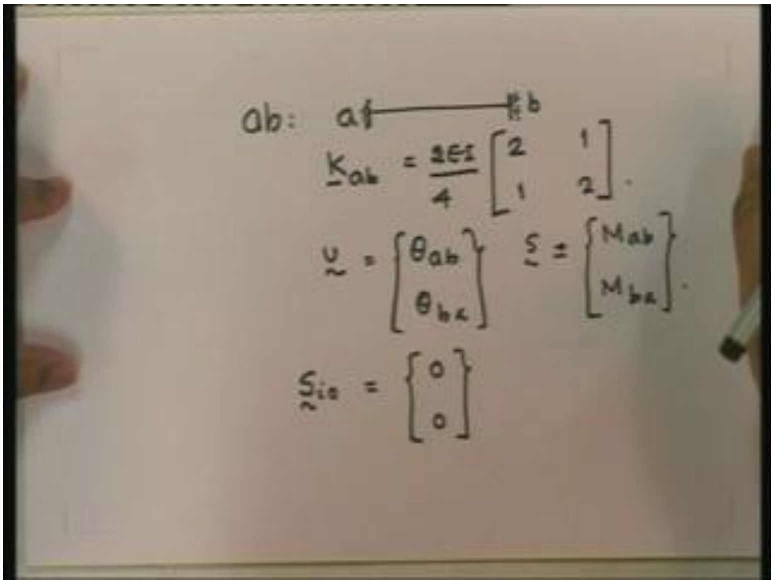


Now, ab is a member defined in this way and its $K_{ab}$ is defined by 2EI upon L into (2, 1, 1, 2). This is a, this is b, v is going to be theta$_{ab}$, theta$_{ba}$ and S are going to be $M_{ab}$, $M_{ba}$ and since there is no loading, $S_{i0}$ is going to be 0, 0. This is our definition of member ab. By the way, let us go back here.

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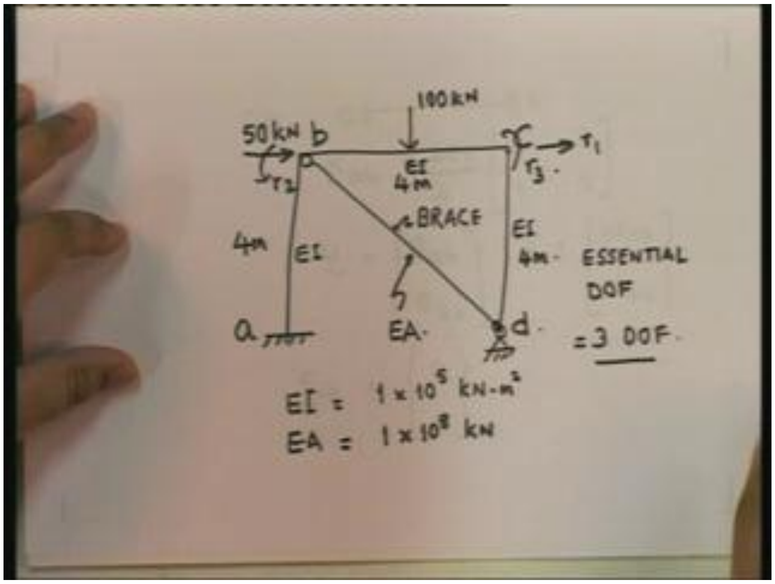


How many degrees of freedom does this structure actually have? Essential, in other words, there is a degree of freedom which is theta$_D$ but since moment at that point is equal to 0, this is essentially a three degree of freedom structure, which are one, $r_1$, two, $r_2$ and three, $r_3$ (Refer Slide Time: 08:13). This is member ab.

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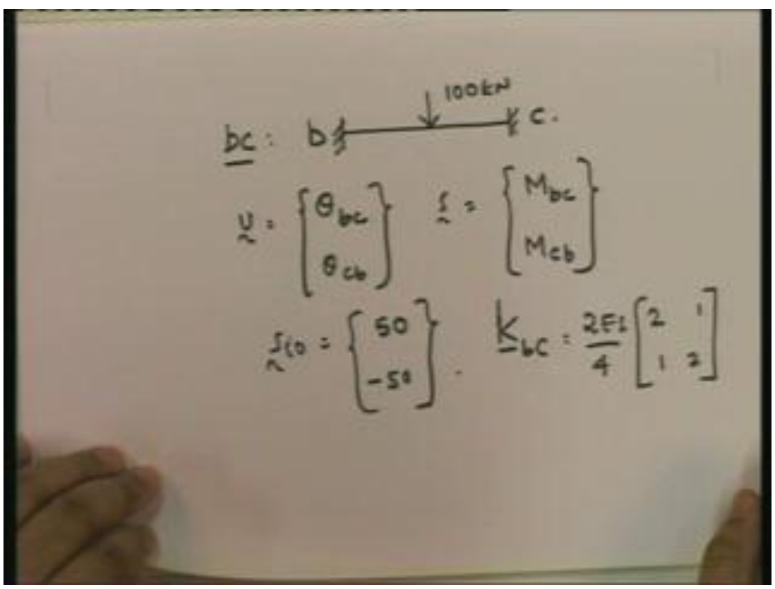


Similarly, member bc is continuous over both b and c and therefore, what we have over here is v is equal to theta$_{bc}$, theta$_{cb}$, S is equal to $M_{bc}$, $M_{cb}$ and $S_{i0}$ is equal to.... Here, there is a loading (Refer Slide Time: 09:11) and the loading is 100 Kilonewton per meter and this is going to be PL upon 8; PL upon 8 is going to be plus 50, minus 50 and $K_{bc}$ is going to be 2EI upon 4 into (2, 1, 1, 2). So much for member bc.

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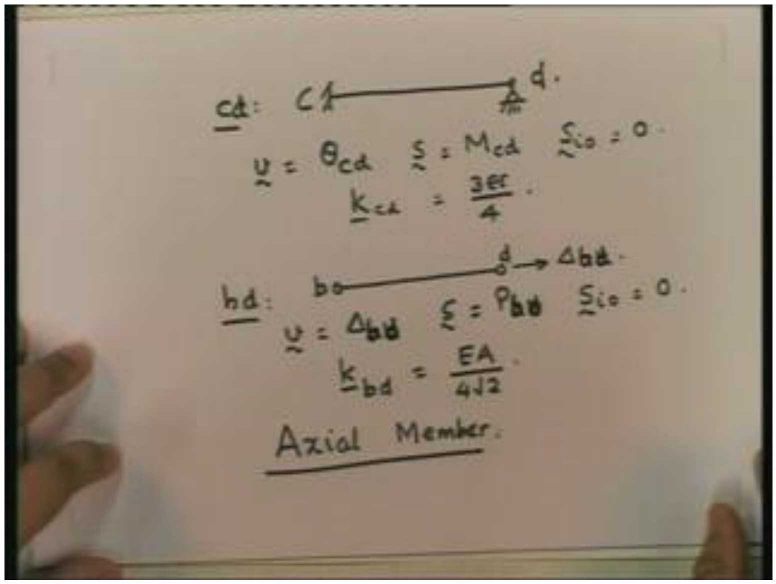


Member cd: This member is continuous over c but pinned at d and therefore its v is equal to $theta_{cd}$, its S is equal to $M_{cd}$, its $S_{i0}$ is 0 and its $K_{cd}$ is 3EI upon L that is for member cd. Now we have another additional member bd. What kind of a member is it? It is actually a member of this type; pinned at both ends and only subjected to $delta_{ab}$ (Refer Slide Time: 11:03). Its v is equal to the axial deformation, its S is equal to $P_{ab}$; since there is no axial force in the member, its $S_{i0}$ is going to be 0 this is b, this is d, so this is $delta_{bd}$, $delta_{bd}$, $delta_{bd}$ and K of bd essentially relates to this and this and you already know that since $delta_{bd}$ is equal to $P_{bd}$ into L upon EA, **so** $K_{bd}$, which is $P_{bd}$ in terms of $delta_{bd}$ is going to be equal to EA upon L, and L here in this particular case is 4 root 2. (Refer Slide Time: 12:24) This is an axial member.

In fact, let me just point out that this is something that we had not discussed and I am introducing today, but the concept is the same – whether it is a flexural member or it is an axial member; the only thing that happens is the member degree of freedom for flexure is the rotations from the chord at the two ends whereas for an axially loaded member, it is actually equal to delta, which is the axial deformation and the load is the axial force in the member. In fact, I will just expand this; this, of course, is only a member that is subjected to axial; you might have a situation where you have a member that is subjected to both axial and flexure. How do we consider that?

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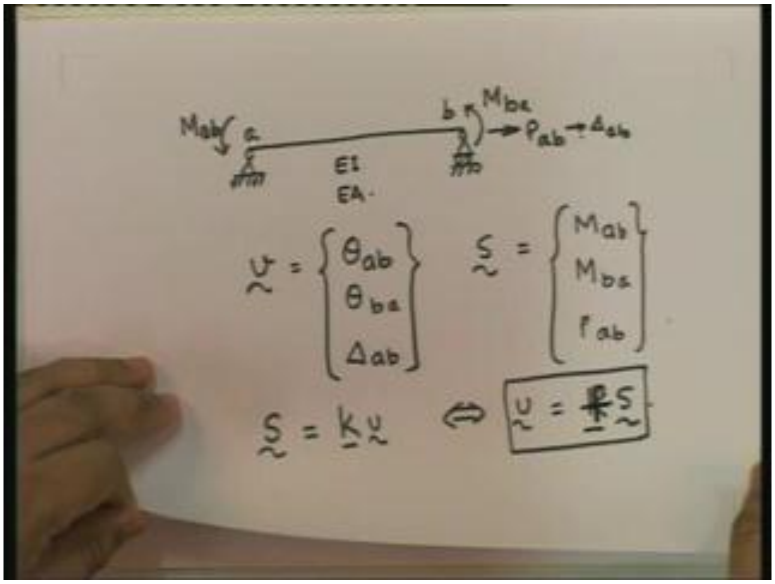


I am going to divert a little bit, come back to this problem a little bit later and treat how we may have a problem where we have a member. I am going back to the basics: Starting from flexibility and then going off from there, because you can always invert the flexibility matrix and get the stiffness matrix for a member – I have already shown that many times, so let us go back to this. Here, what we have is $M_{ab}$, $M_{ba}$ and $P_{ab}$. This is the situation and therefore here the flexural rigidity is EI and the axial rigidity is EA. This is a situation where we are considering both axial deformations as well as flexural deformations in a member. This typically has to be done although we tend to neglect it in a general situation, but suppose we want to consider it, how do we write down the v?

If you look at it, the v in this particular case are equal to $theta_{ab}$ (I am just saying a, b), $theta_{ba}$ and $delta_{ab}$; $delta_{ab}$ is here (Refer Slide Time: 15:47); this is the deformation – the change of length of member ab axially; this is different from the earlier situation. S is equal to $M_{ab}$, $M_{ba}$, $P_{ab}$. If S is equal to Kv... sorry, flexibility – this is what we find out by actually going through the process. If you look at this, you will see that if I give a force P, you are not going to generate any bending moment and therefore, you will see that $theta_{ab}$ and $theta_{ba}$ are equal to 0 if $M_{ba}$ and $M_{ab}$ are equal to 0.

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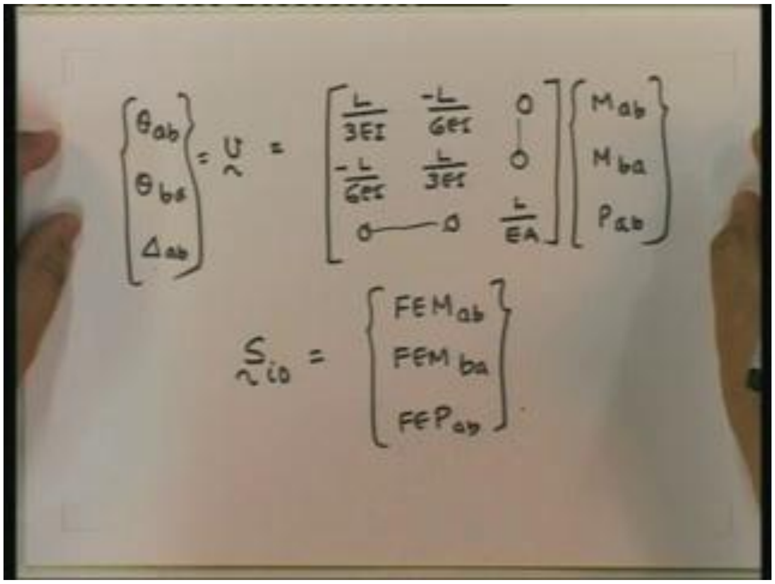


Putting that in, we can see actually that v is equal to (L upon 3EI, minus L upon 6 EI, 0, minus L upon 6 EI, L upon 3EI, 0, 0, 0, L upon EA) into $M_{ab}$, $M_{ba}$ and $P_{ab}$. This is relatively easy to prove because v is nothing but equal to **...**(Refer Slide Time: 18:36) and for a straight member, $P_{ab}$ does not give rise to theta$_{ab}$ and theta$_{ba}$ and $M_{ab}$ and $M_{ba}$ do not give rise to any delta – that is the reason why these are 0s. Therefore, delta$_{ab}$ is equal to PL upon EA we already evaluated earlier. This is all that happens. If you have a member where both flexural and axial deformations have to be included then all you have is this term but you will see that in a straight member these two terms are 0, so actually the axial deformations are uncoupled from the flexural deformations. But this in essence is what happens. Then, of course, you have to find out $S_{i0}$, which are essentially $(FEM)_{ab}$, $(FEM)_{ba}$ and fixed end axial force in ab $(FEP)_{ab}$.

Remember I talked about temperature? If you have temperature, if you have the neutral axis expanding, then you actually have developed a fixed end force that you can compute easily. This in essence is the overall but by and large, since these are uncoupled, we do not consider flexure and axial together and that is the reason why if I go back to the problem that I was looking at, I have one member that is an axial member and therefore, the axial member only has axial force and this is EA upon L. We have written down, we have four members, three degrees of freedom, we have written down all the relationships for all the members. What is the next step? The next step is kinematic relationships – take every degree of freedom, put displacement equal to one and then find out what the member deformations are under that particular loading.

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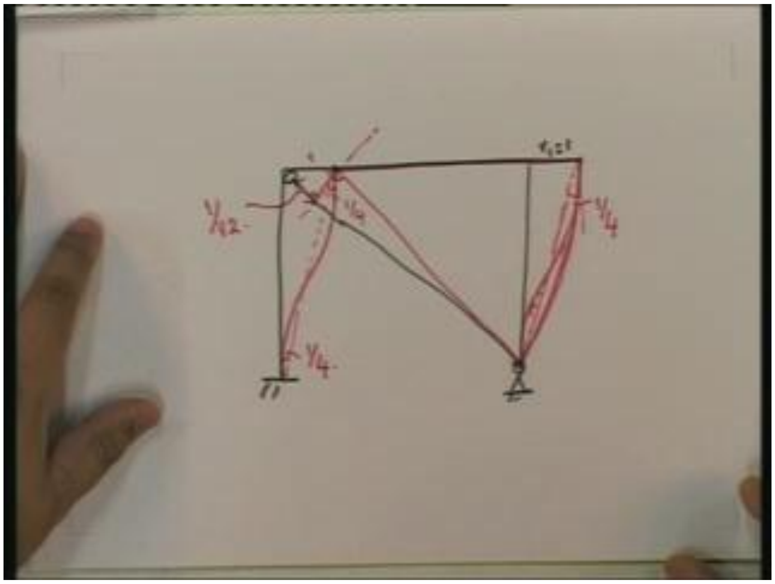

I am going to give $r_1$ is equal to 1. What is going to happen? $r_2$ and $r_3$ have to be equal to 0 so this will also come here. Note that these members are axially rigid so they cannot deform, but this member can deform and it does deform, in fact. The only way it can go is this way and so, if I look at it, this is how it goes. We have here this. By the way, this one does not go this way, this one goes this way because here you have 0, so we have this; this is my theta, this is going to be 1 by 4, again this is going to be 1 over 4, this is going to be 1 over 4.

Now, the other thing that we have to find out is how much has this member shortened by. To find out how much it has shortened by, we need to draw the perpendicular because note that whatever it has moved perpendicular by, that does not change length and so when we drop a perpendicular here, this is the amount of change of length, this is 45 degrees, this is 1, so this is going to be 1 over root 2, this is 90 degrees, so this is sine of 45 degrees and sine of 45 degrees is 1 upon root 2. That is the most important thing and then of course, we have $\text{theta}_2$ and $\text{theta}_3$ equal to 0, so you can plug that in. Those are relatively easier.

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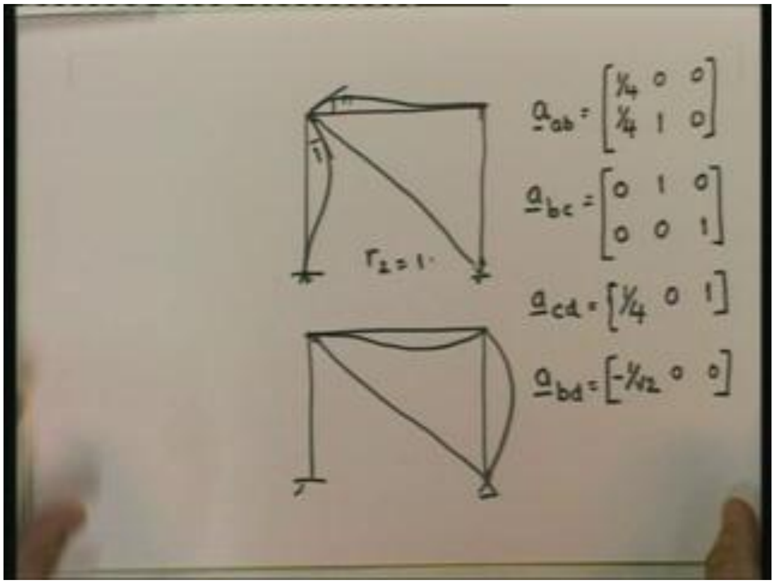


You get theta, you get this, you get this (Refer Slide Time: 24:51) – that is for $r_2$ is equal to 1. Note that this point goes nowhere and therefore this does not change length. The only thing that you have is this is equal to 1 and this is equal to 1. When you put $r_3$ is equal to 1, all you get is this, this member remains the same. Therefore, if we were to write down our as, $a_{ab}$ is equal to... corresponding to the first degree of freedom, this is 1 by 4, 1 by 4, corresponding to the second degree of freedom, this is 0 1, corresponding to the third degree of freedom this is 0 0.

If we look at $a_{bc}$, corresponding to the first degree of freedom it is 0, 0, corresponding to the second degree of freedom it is 1 0 and corresponding to the third degree of freedom, it is 0 1. Then, we have $a_{cd}$: corresponding to the first degree of freedom it is from the chord to the tangent is one-fourth, second one, 0, third one, 1, and $a_{bd}$: what is $a_{bd}$? This is the axial shortening or lengthening due to the unit displacement and in this particular case what you have is (Refer Slide Time: 27:01) a shortening due to $r_1$ and shortening is defined as minus, so you have minus 1 over root 2. Now what is the shortening in these two? It is 0 0.

You will see essentially the overall concept, whether you have an axial member or whether you have a flexural member or whether you have a member that has both flexural and axial, all that happens is that you have to consider both the effects together. That is why in this particular case since bd is only an axial member, this (Refer Slide Time: 27:47) corresponds to the axial deformation in the member bd due to $r_1$ is equal to 1, due to $r_2$ is equal to 1 and due to $r_3$ is equal to 1. In essence, there is no difference in the entire scheme of things whether it is an axial member or whether it is a flexural member. Let us go through some of the numbers. I am not going to be solving this entire problem because all I wanted to introduce was the concept of this.

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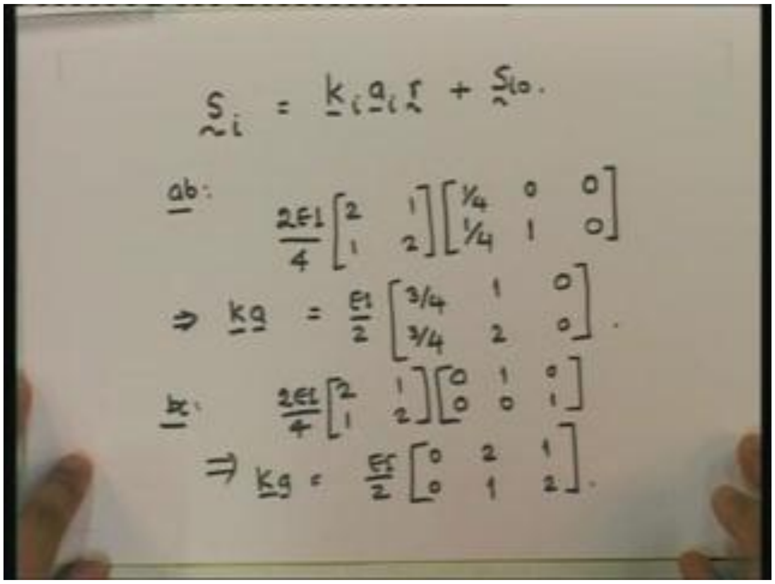


Ultimately, what we have is; $S_i$ is equal to $K_i$ $a_i$ into r plus $S_{i0}$. In that way we can find out for each member and for the flexural members ab, bc and cd**;** let me do it; for ab what we have is $K_i$ is equal to 2EI upon 4 into (2 1, 1 2) and my $a_i$ is equal to 1 by 4, 1 by 4, 0 1, 0 0 (Refer Slide Time: 29:33). This implies that K into a is equal to… I am going to keep EI upon 2 outside and the inside then becomes 2 by 4 plus 1 by 4 is 3 by 4, this way 3 by 4, this way 1, this way 2 and this way 0 and 0. Similarly, we can do it for bc. For bc we have 2EI by 4 into (2 1, 1 2) into $a_{bc}$, which is 0 0, 1 0, 0 1 so this implies that $K_a$ is equal to EI upon 2, the first one is 0 0, the second one is 2 1, the third one is 1 2. This is $K_a$ for bc and this is $K_a$.

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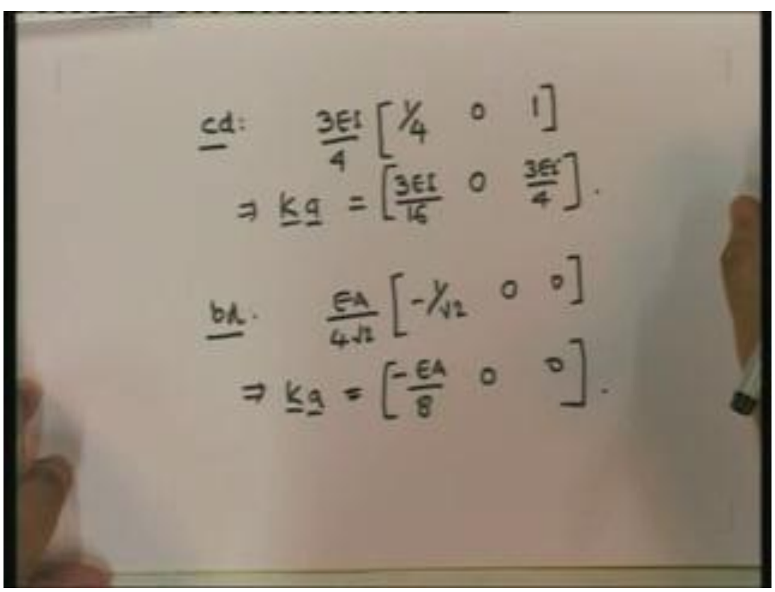

Let us write it down for cd. For cd, $K_i$ is equal to 3EI by 4 and cd is 1 by 4, 0 1. This basically becomes 3EI by 16, 0 3EI upon 4 that is for cd, this is $K_a$ for cd. For bd, this is equal to EA upon 4 root 2 multiplied by minus 1 over root 2, 0 and 0 so this implies $K_a$. Note that whether it is a flexural member or whether it is an axial member, the concept is still the same, only thing is here, we have EA and so if I look at it, this becomes minus EA upon 8, 0 0. We know the values of EI and EA, so we can plug those in.

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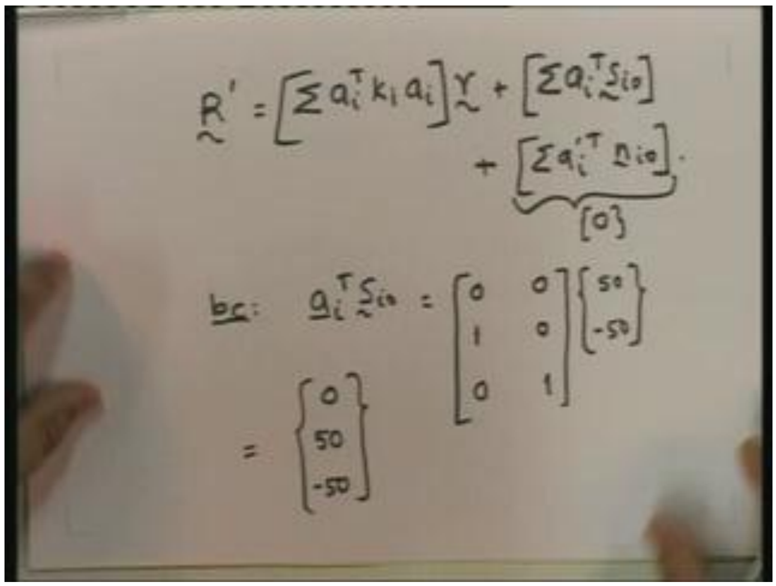


Now, once we have that, ultimately, we have this: R prime is equal to summation $a_i$ transpose $K_i$ $a_i$ summed over all the members into r plus summed over all the members of $a_i$ transpose $S_{i0}$ plus summed up over all the members $n_{i0}$. Let us first find out this (Refer Slide Time: 33:22). We know this one only exists for bc – this only exists for bc it is 0 for all the others and the major thing is that the $n_{i0}$ are the reactions.

Again, if we look back at the shape corresponding to this, there is no vertical displacements of these points and in $r_2$ and $r_3$, there are no vertical displacements. In this particular case, we can see that this is going to show up as a zero vector – this does not exist and this will only show up for bc. Let us do this for first bc; for bc, this is the only one: $a_i$ transpose $Si_0$ is going to be equal to $a_i$ transpose would be 0 0, 1 0, 0 1 and $S_{i0}$ we have already seen was equal to 50 and minus 50 and if we look at that this becomes equal to 0 then 50 and minus 50.

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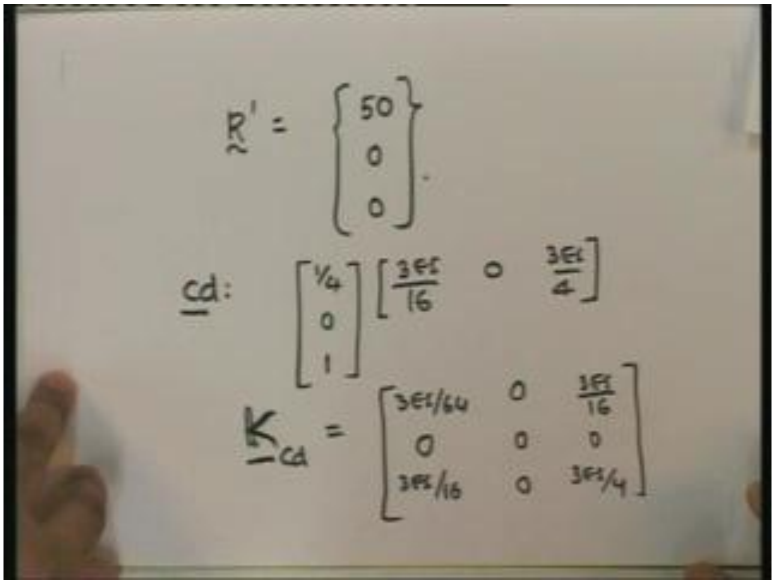

Now, if we look at the R and we find out the work done by the nodal forces, we will see that in this particular case, this becomes 50 and these do not do any work. Next, we have to find out the $a_i$ transpose into $K_i$ $a_i$ for each member and I will first do it for the members which are the smaller ones and for those, just trying to find where I have written down the... here, Ka.... If I pull these together, I have for cd, $a_i$ transpose is going to be equal to 1 by 4 and 0 1 and here we have 3EI upon 16 0 3EI upon 4 and if I do that you will see that this becomes essentially 3EI by 64 0 3EI by 16, then we have 0 0 0 and then we have 3EI upon 16 0 3EI upon 4 – this is the contribution of cd to the structure stiffness matrix (Refer Slide Time: 36:59).

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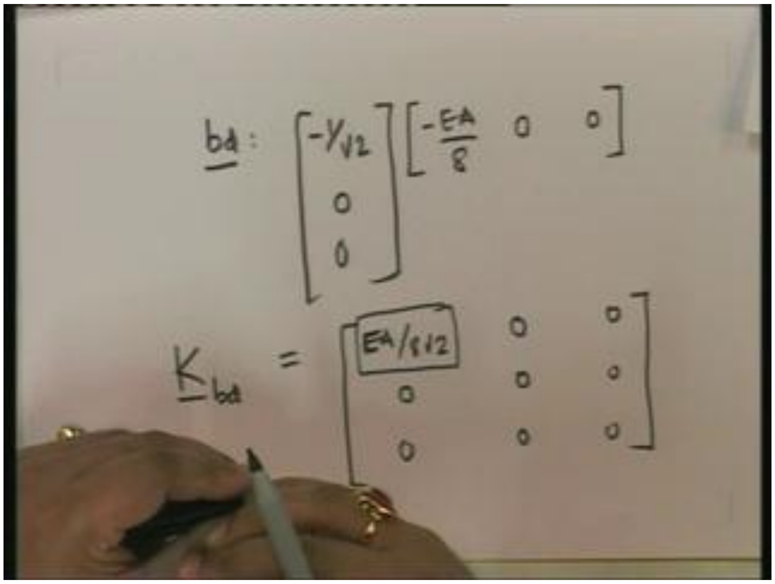

Similarly, we can find out for bd. Then, $a_i$ transpose becomes minus 1 over root 2 0 0 and here we have minus EA upon 8 0 0 and if you look at this, this essentially becomes EA over 8 root 2 0 0 and everything else is 0 (Refer Slide Time: 37:43). Why do you think that is? That is the contribution of bd to the stiffness matrix. If you look at it, what does this mean? It means that it only provides rigidity towards $r_1$. That is true because when you have $r_2$ and $r_3$, the bracing, the member does not deform.

In fact, the bracing member is only provided to provide an additional rigidity to the lateral displacement – that is the whole purpose of the bracing member; it is not there to take vertical loads; when there is lateral displacement, it is to provide and therefore, it is not surprising that it adds to this. We have done for cd and bd; can we do it for ab and bc?

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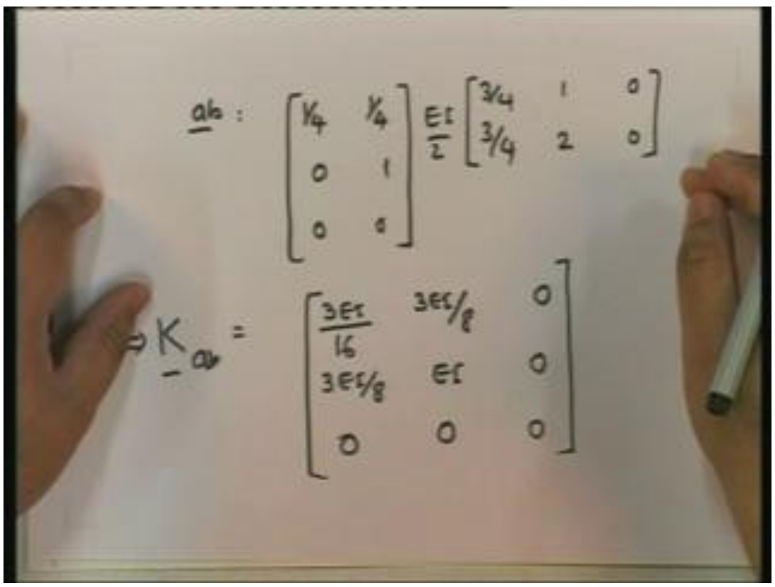


For ab it is going to be equal to $a_i$ transpose into $K_i$ $a_i$; $a_i$ transpose for ab is going to be equal to one-fourth one-fourth, 0 1, 0 0 that is just $a_i$ transpose and then, $K_i$ was equal to EI upon 2 into (3 by 4 3 by 4 1 2, 0 0). If we multiply those we get the following: this implies the contribution of ab to the stiffness matrix (Refer Slide Time: 39:39) is equal to… if you look at the first one, it is going to be 3 by 16 plus 3 by 16 that is going to be 3 by 8 so this is going to be 3EI by 16 the first one; the second one: this way is going to be 0 into 1 by 3 so this is going to be 3EI by 8; this one 0 so this going to be 0; now we do the second one: this is going to be 1 by 4 plus 2 by 4 so 3 by 4 so 3 by 8, so it is going to be 3EI by 8; next one, 0 into 1 and 1 into 2 so this is going to be EI; the next one is going to be 0 0 so this going to be 0; and the third one is going to be 0 0 and 0. In other words, ab does not contribute to the stiffness corresponding to $r_3$ – that is not surprising; there is no displacement in ab due to theta $r_3$ and therefore it will not contribute. Whenever there is no deformation in a member due to a particular degree of freedom it does not contribute to that degree of freedom. We are now done with ab.

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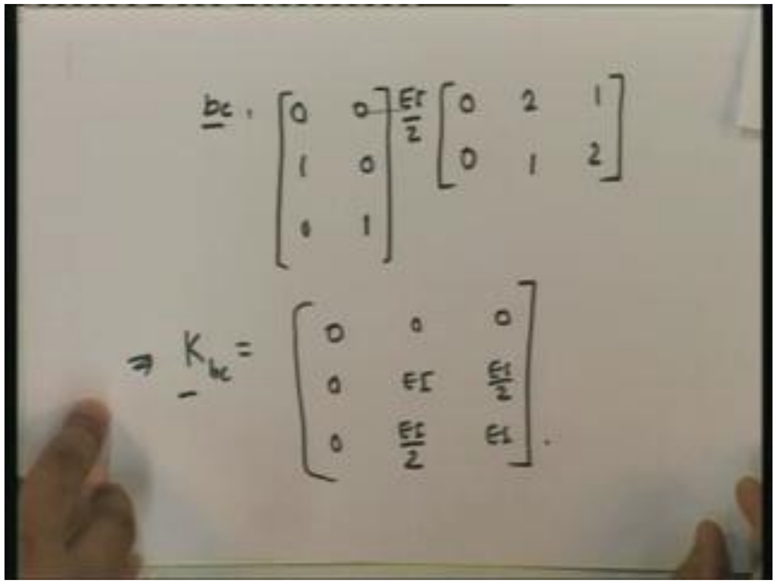

We move on to bc. For bc, $a_i$ transpose is nothing but 0 0, 1 0, 0 1 and over here, we have EI by 2 into (0 0, 2 1, 1 2) and let us see what happens. This first one is 0 into 0, so it is 0; 0 into 0 is 0, 0, 0 into 0 is 0 – the first one is here; the second one: this is 0, this one is 2, so this is EI, third: if you look at it, it is 1, so this is EI over 2; third one: this is 0, this is EI by 2 and the fourth one is 2, so this is EI – we have got $K_{bc}$. Let me put up all the four here and add it up.

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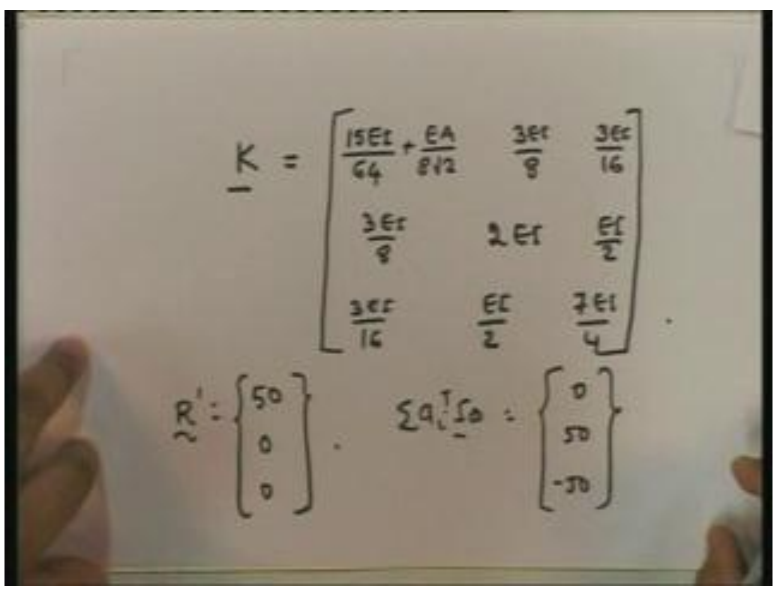

Since all the EIs are the same, my stiffness matrix, my K is going to be equal to 12 over 16, so this is going to be 15EI upon 64 plus EA by 8 root 2, the second one is going to be 3EI upon 8, the third one is going to be 3EI upon 16; then we again look through this, we get 3EI upon 8, we look through this we get 3EI upon 16 and if we look at the third one we get EI, EI and 0 so we have 2EI, we have EI by 2 0 0, so this is EI by 2, here EI by 2 0 0, so it is going to be EI by 2 and the last one is going to be EI plus 3EI by 4, so that is going to be 7 EI by 4 – this is my K; my R – I am just repeating it – is going to be 50 0 and 0 and my summation $a_i$ transpose $S_{i0}$ is going to be 0 then 50 then minus 50.

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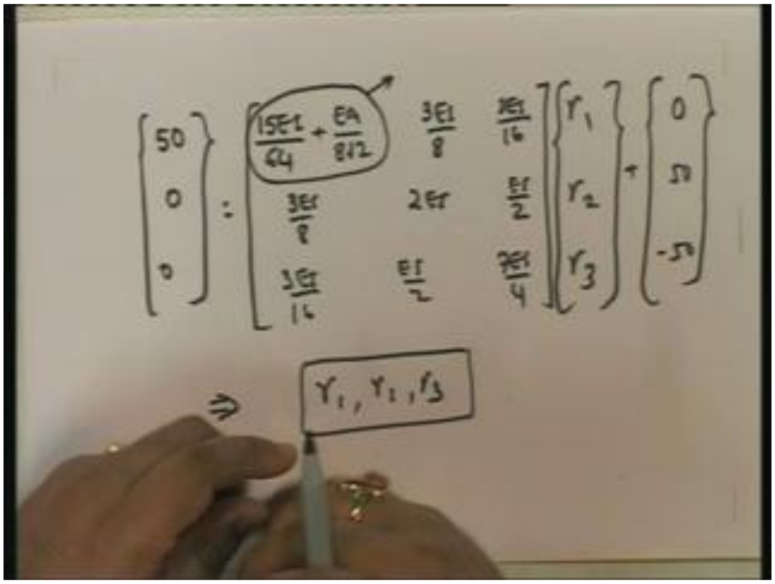

If we look at the final solution, what we get over here is going to be equal to 50 0 0 is equal to (15 EI by 64 plus EA by 8 root 2 then 3EI upon 8 3EI upon 16 3EI upon 8 2EI, EI upon 2, 3EI upon 16, EI upon 2, 7 EI upon 4) into ($r_1$, $r_2$, $r_3$) plus 0 50 and minus 50. Ultimately from this we can solve for $r_1$, $r_2$, $r_3$. I am not going to solve this because if you look at it, you will see that in this particular case it is a 3 by 3, you can plug in the values of EI and EA.

What is the value of EI? The value of EI it is going to be equal to 1 into 10 to the power of 5 and the value of EA is equal to 1 into 10 to the power of 8.

If we look at this particular value (Refer Slide Time: 46:36), do you notice something? You will see that this stiffness value that you get is essentially overpowered by the value of the bracing. In fact, that is the whole point. The point why the bracing is provided is to provide lateral stiffness and that is where it has provided stiffness because $r_1$ corresponds to the lateral motion of the structure.

Once you have found out $r_1$ and $r_2$, then you can always go back and find out and show what those would be. How would I find out my $S_{i0}$? My $S_{i0}$ would be this. Since I know r and I have $K_a$ for all the members, I can find out $S_i$. And for the bracing member, the $S_i$ that will come in will only be the axial force. In other words, the point I tried to make today was that it does not matter whether a member has flexural deformation, whether it has axial deformations or a combination of flexural and axial because all that happens is the S, the v and the K for the member is different for each case, but it does not matter.

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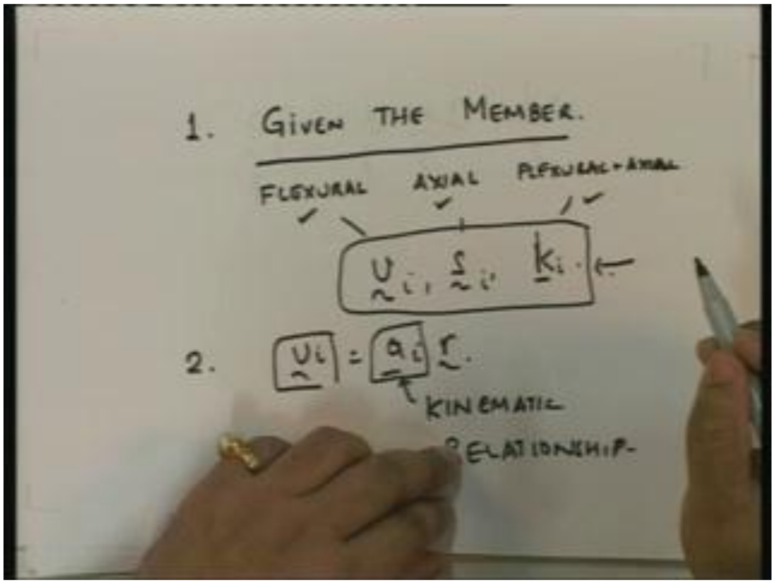


I am just putting down the steps. The first step is given the member, a priori we have to know whether the member is flexural, whether it is axial or whether it is flexural plus axial. if I have a frame member where you neglect axial deformation, then it is a flexural member; if it is a truss member, in other words pinned at both ends, then it is only axial member; if we have a frame member where we do not neglect axial deformations, then it is a flexural plus axial. Once you have that, then all you know is once you have this, you define your v, you define your S and you define your $K_i$ (Refer Slide Time: 49:33). Once you have this, the next step is $v_i$ is equal to $a_i$ r; in this, the kinematic relationship... this is the kinematic relationship; once you know what this, you can always find out what this is by giving each displacement corresponding to each degree of freedom.

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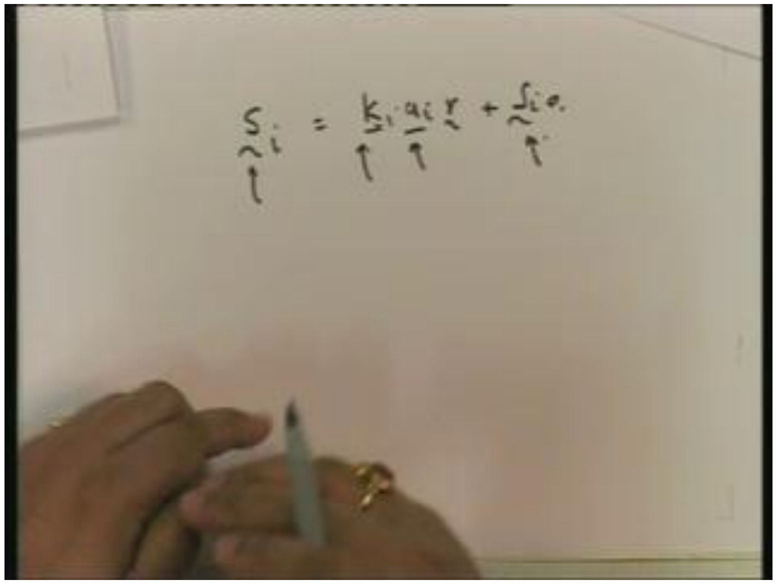

Then you see the third step becomes $S_i$ is equal to $K_i$ $a_i$ r plus $S_{i0}$. This I know, this I know, this I know, this I can find out. In essence, the point I am trying to make here is that in the stiffness method, the only difference it makes, whether you have a truss member, whether you have a frame member where you neglect axial deformation or you have a frame member where you do not neglect axial deformation, all that it does is it only affects this – nothing else; everything else comes out automatically.

Therefore, the beauty of the stiffness method lies in the fact that it does not really matter what kind of member you are dealing with – all you need to know is when you give each individual r equal to 1, you have to find out the $a_i$ that gives the member degrees of freedom in terms of the displacement degrees of freedom. I am going to stop here today; I have introduced to you the concept that axial deformations are included, not included does not matter. We are going to continue looking at more problems and each time, each example that I look at, I will highlight one important aspect of the application of the stiffness method. In general, the stiffness method is easy; what I am going to do is specific cases.

Next time, I am going to be introducing you to rigid members. Suppose I have a member that is both flexurally and axially rigid, what happens then? How do we consider that effect? Then, we are going to look at.... We have only looked at member effects where $S_{i0}$ depend only on loads. What happens when temperature becomes an issue? You have temperature effects. How do we include those? Those are all the things that I am going to look at in the next few lectures and hopefully, these will illustrate how the stiffness method, which is essentially the matrix displacement method, how that is utilized to analyze structures.

Thank you very much.