**Structure Analysis - II**
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**Lecture – 32**

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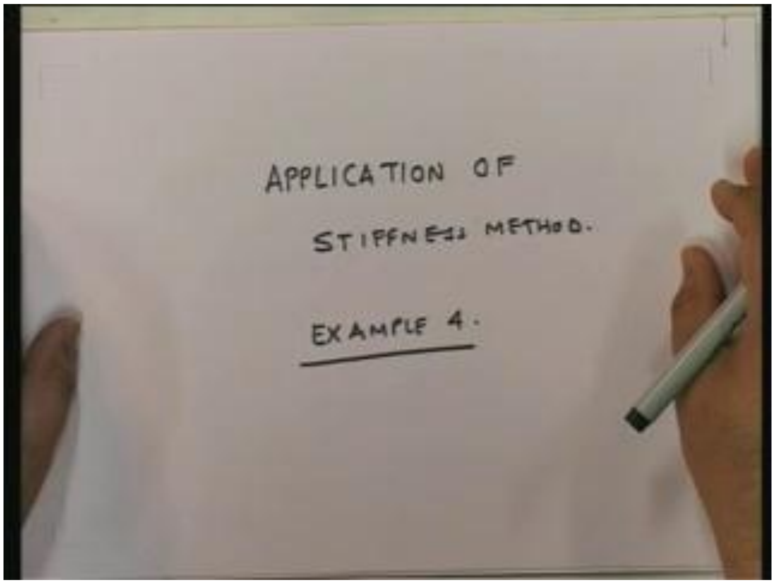


Two examples of how to apply the stiffness method in analyzing structures – today, we are going to continue with it, we are still going to look at only loads on the structure. From the next lecture onwards, we will start looking at other effects other than loads and what effect they can have on the structure, or in other words, how to consider those in the application of the stiffness method.

As I said, I am going steadily into more and more complicated problems but ultimately, as I told you, I am never going to exceed more than two degrees of freedom in all these examples for the simple reason that hand computation is not possible. However, the method that I develop is valid for all degrees of freedom – you can have 150 degrees of freedom if you so desire but the only thing is that at the end of it, you are going to get 150 by 150 matrix, which you will only be able to solve using a computer or maybe a program over calculator in today's world. Let us look at the example that I am going to present in today's lecture.

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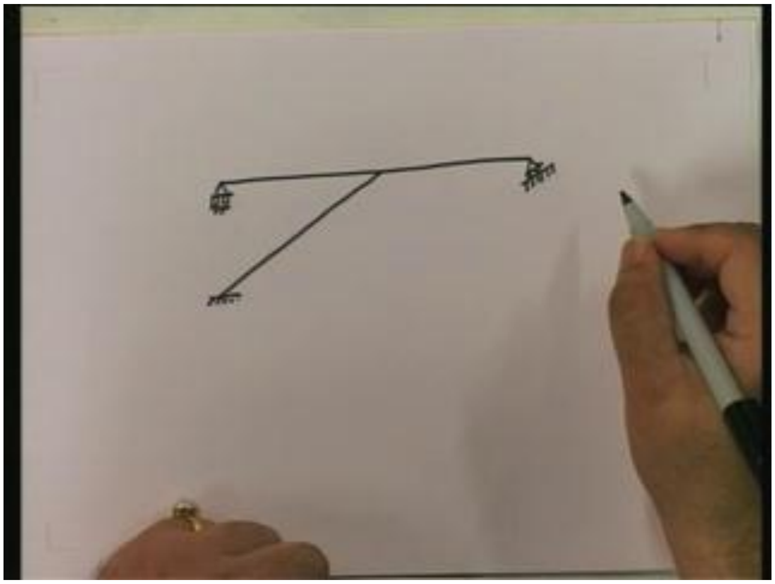

Do understand this: when I say I am steadily getting into more complicated problems, I am essentially really referring to complexity in these kind of problems, which can only come in the complexity of the kinematic relationships rather than any other complications because once you apply the matrix stiffness method, everything else is just one step after another.

The advantage of this method is that each member by member, you can look at first what are the fixed end moments… first, you decide what is the member type that you consider; next, you compute the fixed end moments; third, you compute the relationship with the member end loads and member end deformations.

Next, you compute the kinematic relationship between the member end deformations and the structure degrees of freedom. These are the steps that you follow – there cannot be any further complications in this. The only complication that I can bring in is how to draw kinematic relations and in fact, in the entire steps over here, the only major thing that really has a role to play is the kinematic relationship.

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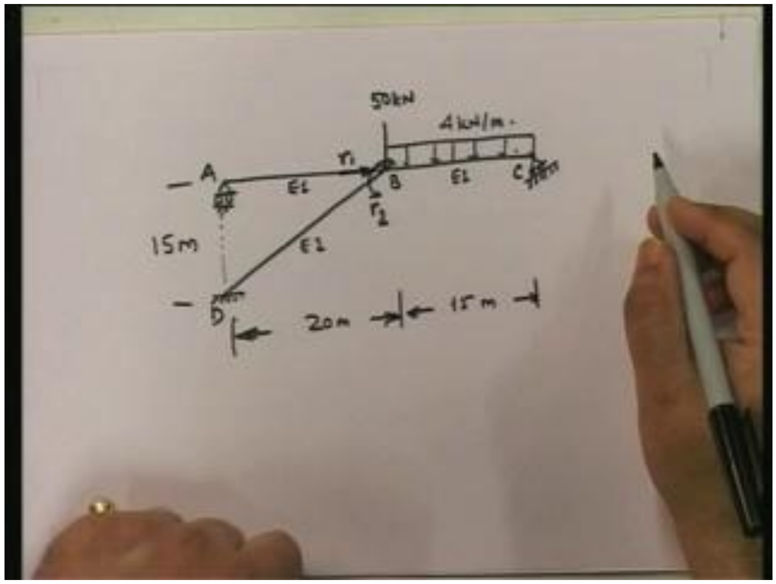


In this particular problem, I have A, B, C, and D and the loads are the following: I have 50 Kilonewton load here, $r_1$ here, $r_2$ here, this is EI, this is also EI and this is also EI (Refer Slide Time: 05:42); this length is 15 meters, this by the way is the same point, 20 meters, from B to C is 15 meters; furthermore, there is UDL on this and this intensity is 4 Kilonewton per meter. This is a two degree of freedom structure and these are the degrees of freedom – satisfy yourself that this is a two degree of freedom structure. I am not going to going into finding out the kinematic indeterminacy or the number of degrees of freedom – by now, I expect that you should be able to do that and I have chosen these two as my degrees of freedom in this particular problem.

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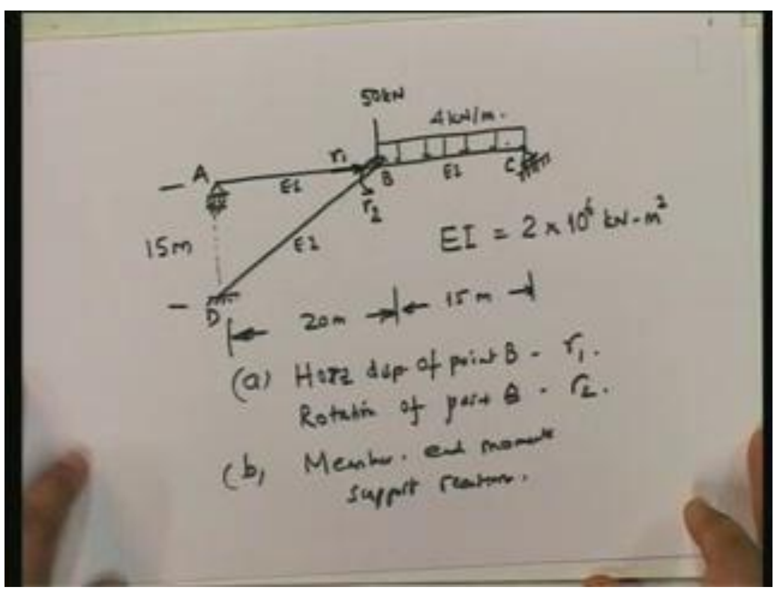


Furthermore, I am given here that EI is equal to 2 into 10 to the power 6 Kilonewton meter squared (Refer Slide Time: 06:54). The question here is to find a), the horizontal displacement of point B – that is why I have taken that as $r_1$ and the rotation of point B, which is why I have taken that as my $r_2$ and part b) is member end moments and support reactions. These are the things that you have been asked to find out in this particular problem.

Obviously, we are going to solve this using the matrix approach of the displacement method, which is essentially the stiffness method. Now, let us proceed on to the solution. What is the first step? You have already completed the first step – figured out how many degrees of freedom and identified the degrees of freedom. The next step is to find out the fixed end moments for each member.

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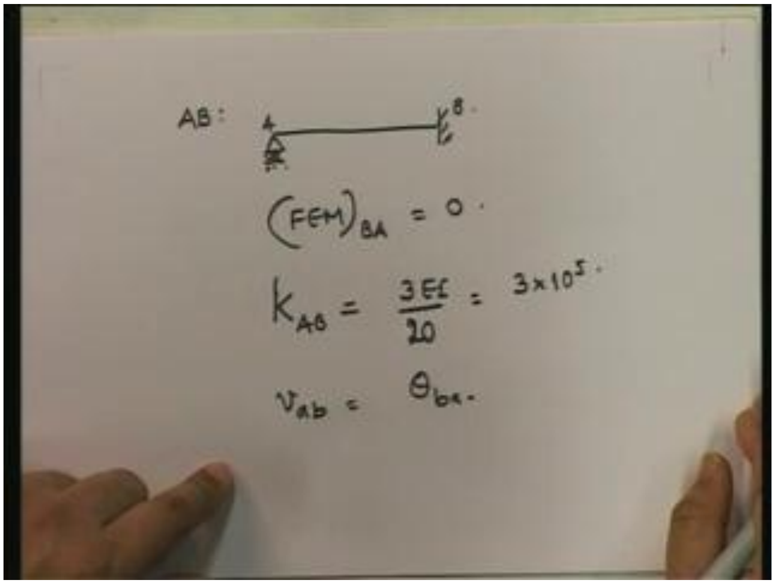


First, member AB: what is the type of member AB? The type of member AB is… we are going to take it in this form, this is A and this is B. Therefore, in this member, what are the fixed end moments? You know this is a modified member, so the only fixed end moment is going to be for BA and that is going to be equal to 0. Furthermore, what is $K_{AB}$? Member it is equal to 3EI by L, L is 20 in this particular case, which if I substitute becomes 3 into 10 to the power of 5 and the $v_{ab}$ is equal to $\text{theta}_{ba}$. These are my identifications for each member.

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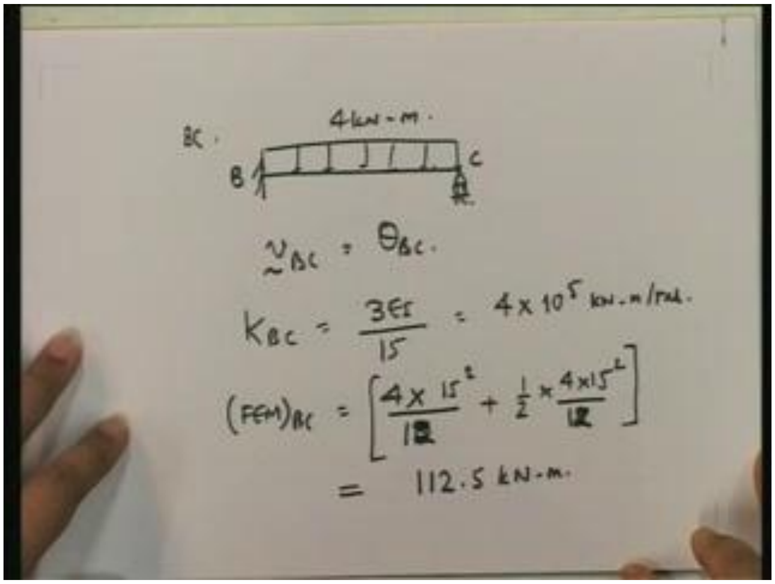


Then, I will go into computing for BC. What kind of a member for BC do I consider? For BC, I consider the member to be in this fashion, where this is equal to this, this is, B this is C and the loading that we have on the member is 4 Kilonewton meter (Refer Slide Time: 10:26). $v_{BC}$ is equal to theta$_{bc}$ only. $K_{BC}$ is obviously going to be 3EI upon L, L is 15 in this particular case so this is equal to 4 into 10 to the power of 5 Kilonewton meter by radian and $(FEM)_{BC}$ is going to be equal to (4EI into 15 squared by 12) plus (half into 4 into 15 squared by 12), so this is going to be equal to 112.5 Kilonewton meter. I have done this for BC now and now, finally CD.

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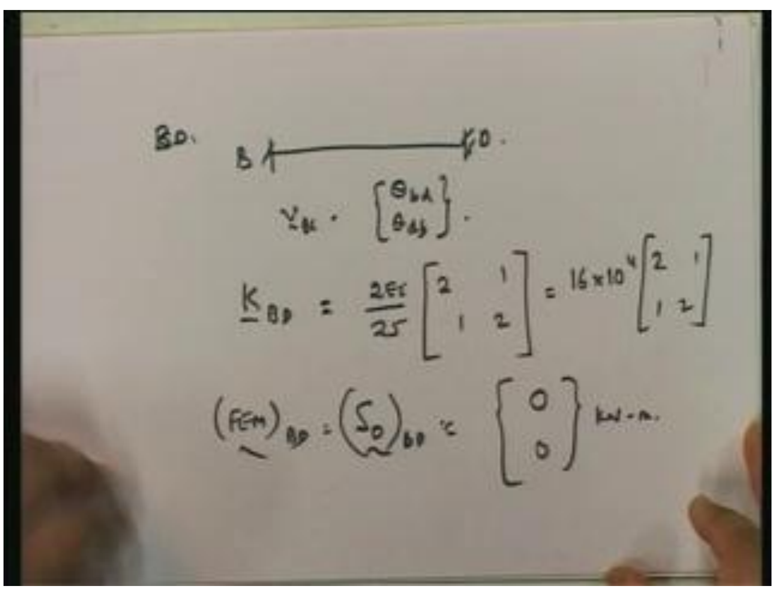


For BD, the form is: this is B, D and therefore, v of BD is going to be equal to (theta$_{bd}$, theta$_{db}$) and $K_{BD}$ is going to be equal to 2EI upon L so this is going to be equal to 2EI upon 25 into (2, 1, 1, 2) which is equal to 16 into 10 to the power of 4 into (2, 1, 1, 2) that is $K_{BD}$ and $(FEM)_{BD}$ which is essentially $S_0$ of BD, is equal to 0, 0; there is no load, so 0 Kilonewton meter. Therefore now I have written down degrees of freedom for each member, these are the member end deformations (that depends on what is the element that you are using), then you have to write down the stiffness matrix (that also depends on what are the degrees of freedom) and I have finally evaluated the fixed end moment for each member. This, in essence, is everything done for each member. The next step is to find out the kinematic relationships, which relate the member end deformations, in other words, member degrees of freedom to the structure degrees of freedom.

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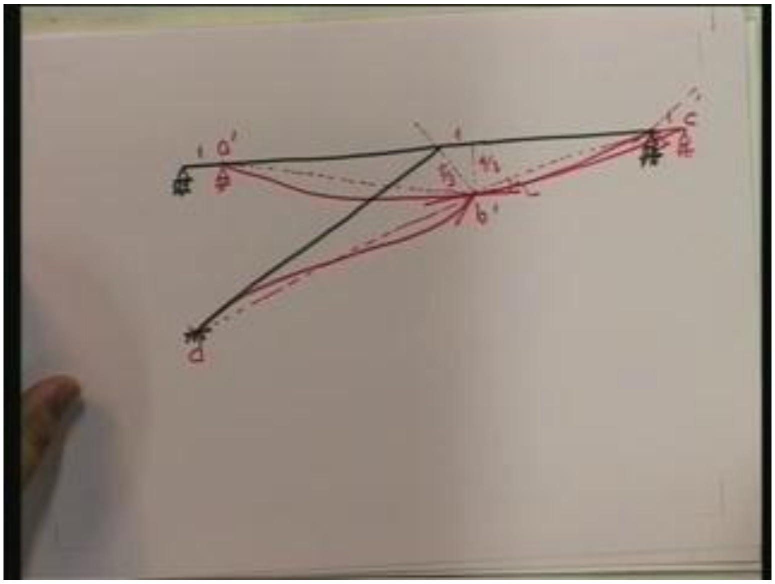

I am going to draw it slightly enlarged. How do we find that out?

I am going to be giving $r_1$ is equal to 1 which essentially means that this point (Refer Slide Time: 15:35) has to move horizontally by 1, but if it moves horizontally by 1, if you note, this particular member is fixed here, so the only way this can move is in this plane. These can move this way, so what we essentially have is that this point comes; if this is 1, this is 4 by 3 and this is 5 by 3; this point comes here, this point moves here by 1, this point moves here by 1 so what we have these come here.

note that I am always drawing this exaggerated to essentially established the point. By the way, these have to remain in this way because there can be no rotation at this point. So once we have this we can draw it in this way it does not matter; here, this one goes this way and here, this one goes this way. Let us join the chords: this is the chord for AB, this is a prime, this is my b prime, this goes actually here, this is my C prime and my D does not move anywhere; this is the chord of AB, this is the chord for BC and this is the chord for BD.

What are the rotations? This is this way, this is this way, this is this way. So the rotations here are going to be this, this is going to be equal to 4 by 3 into… By the way, this is not going to go here (Refer Slide Time: 18:07), this one is actually aligned this way, these cannot go this way, this has to move along this direction, so if we see this direction, this is given as 4 here and 3 here.

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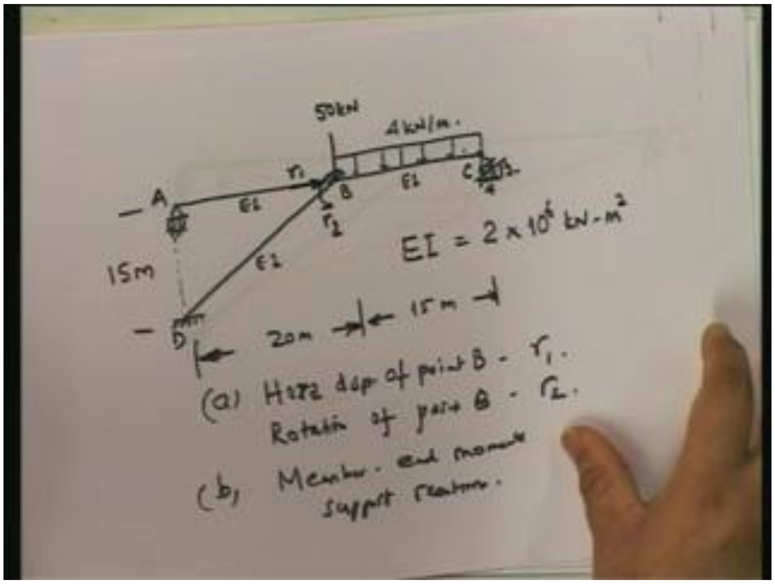


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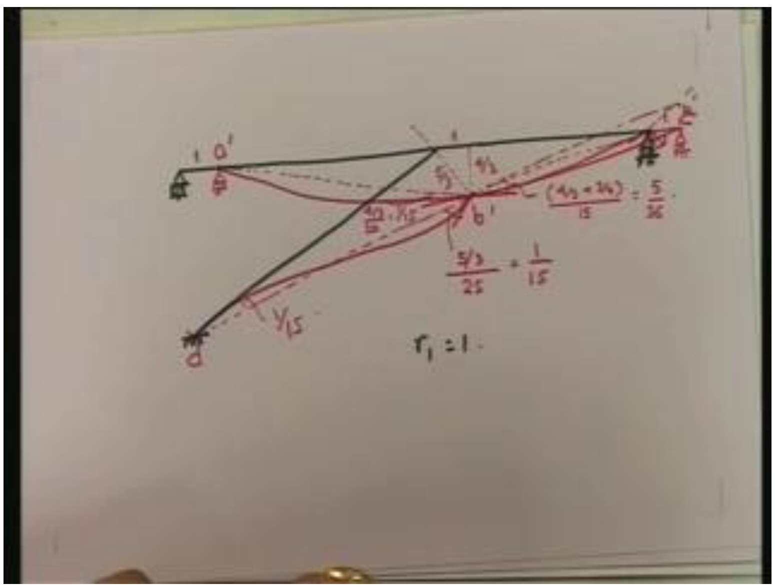


If this moves, this moves by 1 and this moves by 3 by 4, this actually is this way, so the total movement is going to be equal to 4 by 3 plus 3 by 4 – that is the total distance moved – divided by 15 (Refer Slide Time: 19:01). If you look at that, this is 4 by 3, this is going to be equal to 12, so 12, 16 plus 9, it is 25 upon 12 and 25 upon 12 is going to be equal to… you will have 5 upon 3, so this in essence becomes 5 upon 36 – this is this rotation. How much is this rotation? This is going to be equal to 5 by 3 divided by 25 which is equal to 1 by 15; this (Refer Slide Time: 19:55) is going to be equal to 4 by 3 divided by 20 which is equal to 1 upon 15 and this of course is the same, 1 upon 15. This gives you all the member end rotations, that: theta$_{ba}$ is equal to 5 by 3 of $r_1$, this is equal to 5 by 36, these are 1 by 15, 1 by 15.

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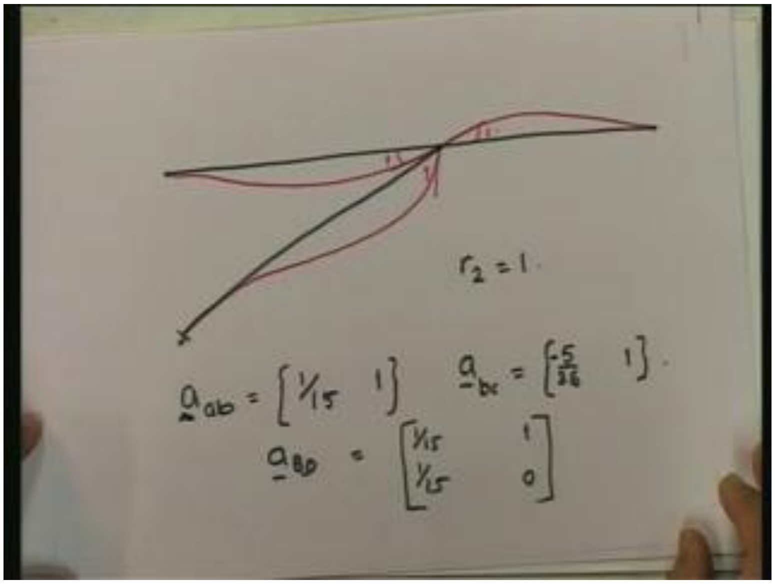


Now we are also going to look at $r_2$ is equal to 1 and I am going to do that. $r_2$ is equal to 1 is going to give me that this point cannot move anywhere because that would imply that these points are here too and $r_2$ is equal to 1 would imply this (Refer Slide Time: 21:04), this, this, where this is 1, this is 1 and this is 1, $r_2$ is equal to 1.

We can essentially now put together the $a_{ab}$ is equal to $r_1$ and what is $r_1$? 1 by 15, in this it is 1, because remember this is theta$_{ba}$ in terms of $r_1$ and $r_2$. Then, $a_{bc}$ is going to be equal to… again this is theta$_{bc}$ in terms of $r_1$ and $r_2$, so theta$_{bc}$ for $r_1$ is equal to 1 is 5 by 36 and for $r_2$ is equal to 1 is this. Finally, $a_{BD}$ is both theta$_{bc}$ and theta$_{cb}$ and this, if we look at it from the chord to this, this is negative, from the chord to the tangent, this is negative and here this one turns out to be from the chord, positive, 1 upon 15 and this one is going to be equal to 1, 0, 1, 0, B, D. These are my a's.

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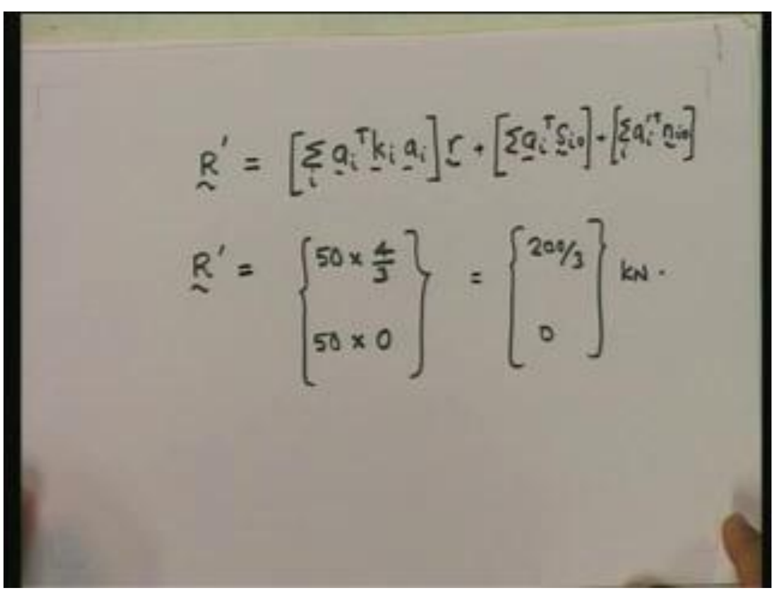

I have already got my $K_a$, everything etc so now, I can put that my R vector and this is the modified R vector – you will see what the modification is. R is equal to in brackets over all i $a_i$ $K_i$ $a_i$ into r plus summation $a_i$ transpose $S_{i0}$ (what is $S_{i0}$? these are fixed end moments) plus summed up over all of them the $a_i$ transpose $n_{i0}$ which are the reactions that come from…

Now let us see what R is. These are the nodal loads that are not included in the member loads; the only nodal load is the 50 Kilonewtons, so let us find out what that is. So what we have to find out is how much that displaces by for $r_1$ is equal to 1 and how much does it displace by for $r_2$ is equal to 1. Therefore, what we have is 50 multiplied by whatever, (Refer Slide Time: 25:00).

Let us look first at $r_2$. When $r_2$ is equal to 1, how much does it move vertically by? 0. That means 0. Now, for $r_1$ is equal to 1, how much does this move vertically by? 4 by 3. This is going to be 50 multiplied by 4 by 3 and it does not move down, so it does do positive work. This is going to be equal to 200 by 3 and 0 Kilonewton – this is R prime. Now, let us find out what the $a_i$ transpose $S_{i0}$ are. For AB, what is $S_{i0}$? Let us take for each one.

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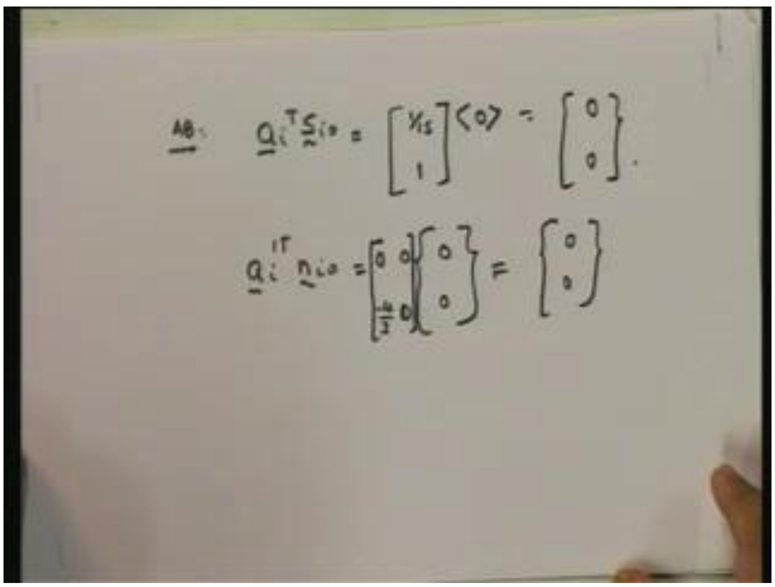

For AB, let me find out what $a_i$ transpose $S_{i0}$ is. We have evaluated; for AB, the fixed end moment is 0. What is $a_i$ transpose? $a_i$ transpose is equal to 1 by 15, 1 into 0, so this is going to be equal to 0, 0. Note that since there is no member load, the $n_{i0}$ are also going to be 0 and therefore it does not matter what you do – you are going to be having it equal to 0, but what should be the $a_i$ transpose? The $n_{i0}$ are 0, 0 but $a_i$ transpose if you look at it is going to be equal to…. how much does this move up and down by? 0. This is going to be equal to 0 and how much does this move down by is 4 by 3**,** it was upward so this would be minus 4 by 3 and for this one how much does this move by? 0 0. Therefore this is going to be equal to 0, 0. I am going to actually put those in so that you know exactly how to compute it.

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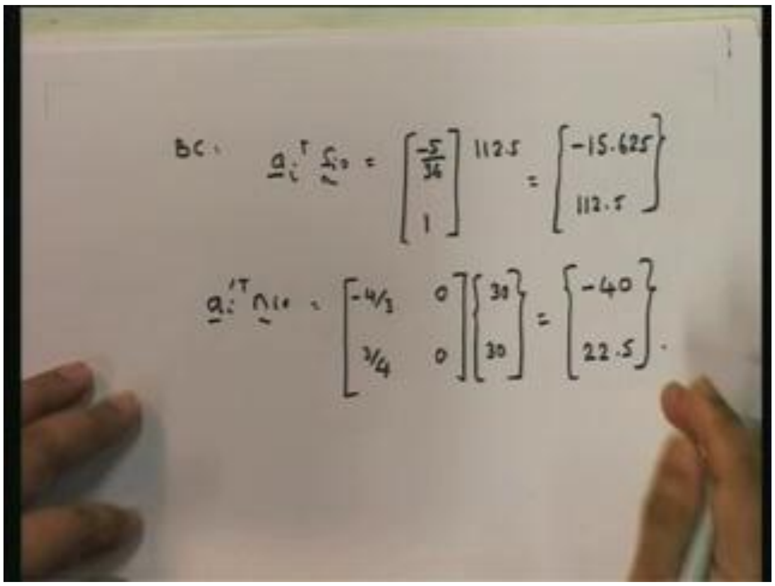


For bc, let us look at $a_i$ transpose $S_{i0}$. What is $a_i$ transpose for this? Again, $a_i$ transpose is going to be equal to minus 5 upon 36 and 1. What is $S_{i0}$? We have computed fixed end moment for bc. Where did we compute it? 112.5, so 112.5 and if we compute that, what do we get? We get it equal to minus 15.625 and 112.5. What about $a_i$ transpose $n_{i0}$? $a_i$ transpose are going to be equal to…. Let us look at how much these move up and down by due to this thing. The left-hand side moves down by minus 4 by 3. This is going to be minus 4 by 3. How much does the other side move up by? It moves up by 3 by 4. This is going to be up 3 by 4. Then for $r_2$ equal to this, we are going to see that both are going to be 0 and then what is my a? You will see that for this particular case, what is my? Let me put those down. My n, which are the reactions at these points – they are going to be equal to… due to this 4, it is going to be equal to 4 into 15 is 60, so this is going to be 30 30. What I have ultimately is 30 30 and if you look at the work done, this is going to be equal to minus 40 and this is going to be equal to 22.5 – I have found these two out.

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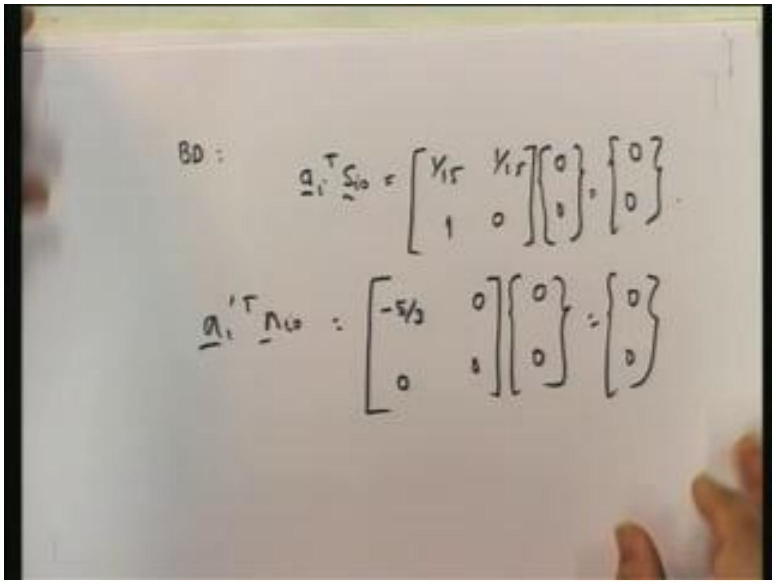

Finally for BD, there is no load, so I am not going to belabor the point. Well, anyway, let us belabor the point, there is no problem. What is $a_i$ transpose? $a_i$ transpose is going to be equal to 1 by 15 1 by 15 1 0. What are the member end moments? 0 0. What is this? 0 0. Let us do $a_i$ transpose $n_{i0}$. What is it? Let us look at that. Again for $r_1$ is equal to 1, how much does B move by? It moves down by… so it is 5 by 3, this is 0 and for $r_2$, these are going to be 0 0. What are the reactions? 0 0. What do we get? 0 0. We have got it – we have got all the members' $a_i$, etc.

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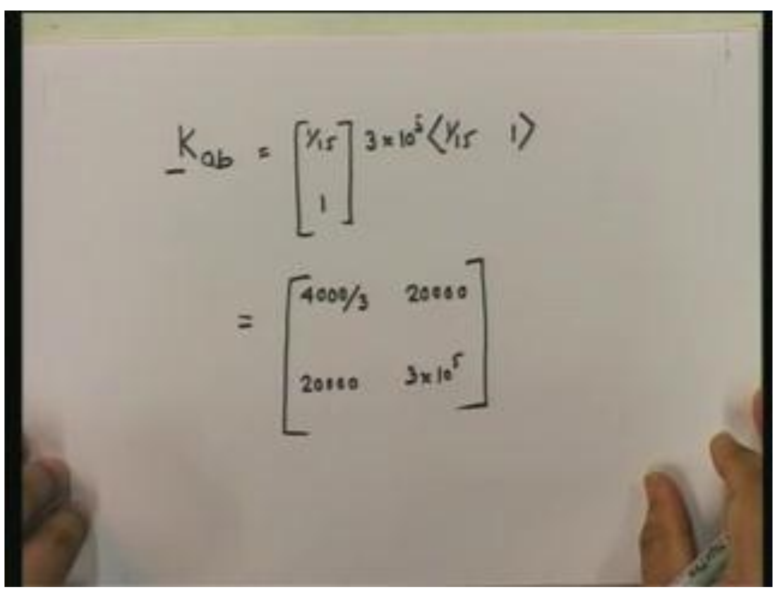

We can find out the contribution of ab to K and you will see that this is going to be equal to $a_i$ transpose and what is $a_i$ transpose? It is going to be equal to 1 by 15, 1 and what is the EI? $K_b$ is equal to 3 and 10 to the power of 5 into a; a is 1 upon 15, this and so this is going to be equal to 4000 by 3, this is going to be equal to 20000, this is going to be equal to 20000 and this is going to be equal to 3 into 10 to the power of 5 – that is my $K_{ab}$.

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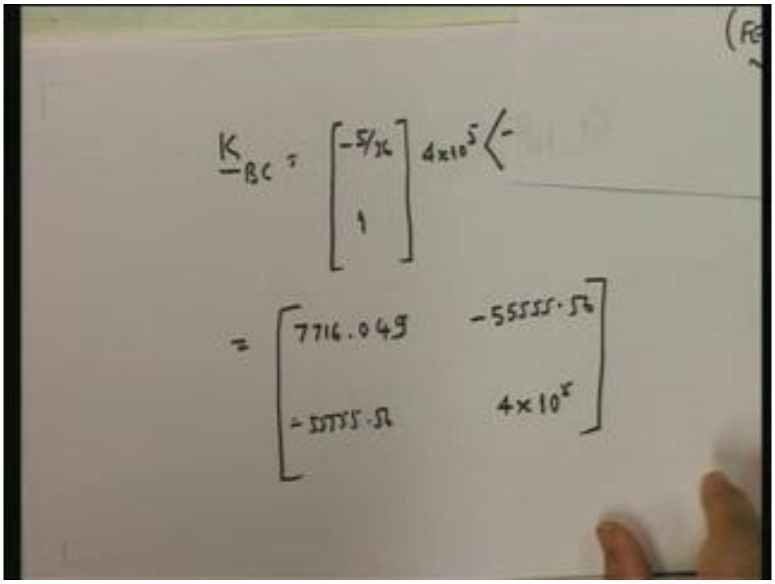

My $K_{BC}$ is going to be equal to minus 5 by 36, 1, this is 4 into 10 to the power of 5 into minus 5 by 36, 1, so this is going to be equal to 7716.049, minus 55555.56, minus 55555.5 6 and this is equal to 4 into 10 to the power of 5 so this gives me $K_{BC}$. Then, what else do we have?

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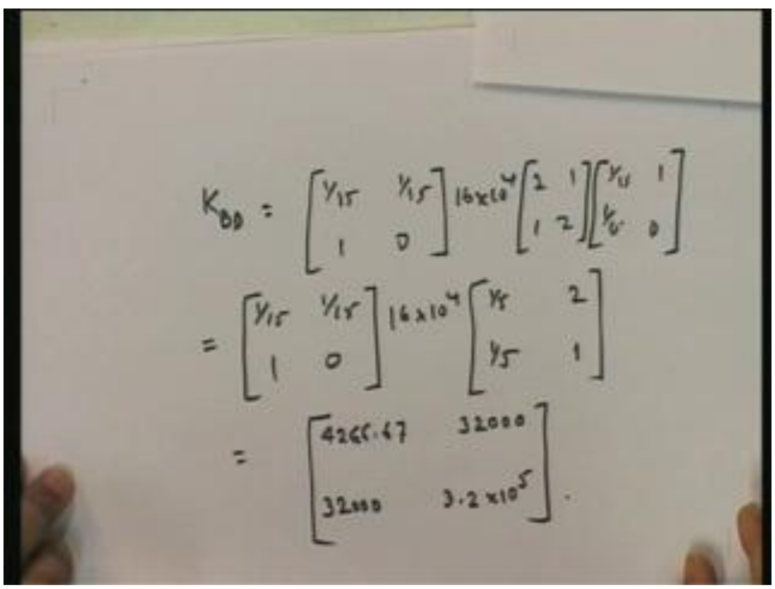

Finally, $K_{BD}$, which is equal to (1 upon 15, 1 upon 15, 1, 0) into K and K for this thing is going to be equal to 16 into 10 to the power of 4 into (2, 1, 1, 2) into (1 upon 15, 1 upon 15, 1, 0), this is going to be equal to (1 upon 15, 1 upon 15, 1, 0) into 16 into 10 to the power of 4, this is going to be 3 by 15, which is 1 upon 5, this is going to be 1 upon 5, this is going to be 2, this is going to be 1. Ultimately, this is going to be equal to 4266.67, 32000, 32000 and 3.2 into 10 to the power of 5.

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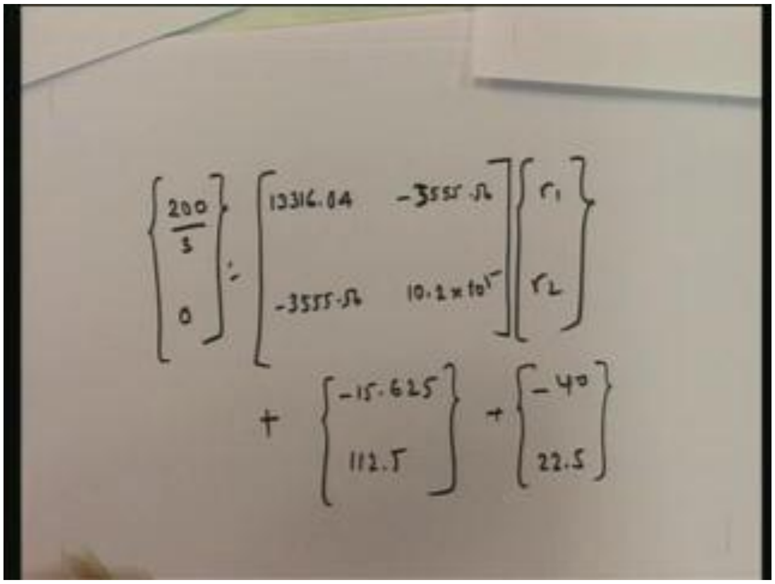


When we put all of these together, what you ultimately get is that r is going to be equal to 200 by 3 and this was 0 multiplied by…. when we add all of them up, we get 13316.04, minus 3555.56, minus 3555.56 and this is going to be equal to 10.2 into 10 to the power of 5 into ($r_1$, $r_2$) plus the only one that comes in; both BD and AB are going to be 0, 0, so what we have is minus 15.625, 112.5 plus (minus 40, 22.5) I have made a mistake here.

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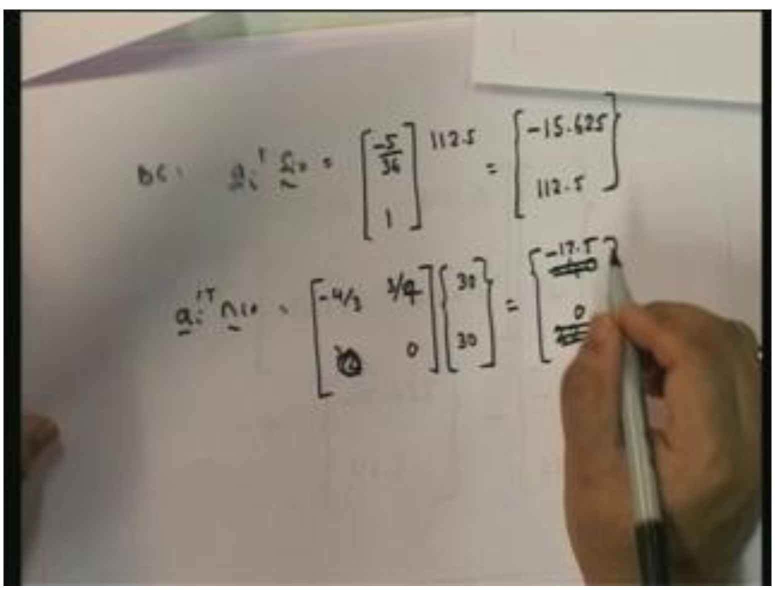


This is $a_i$ transpose and the transpose would be 3 by 4 and this would be 0, 0, here, I would get 0 and this one would turn out to be equal to minus 17.5.

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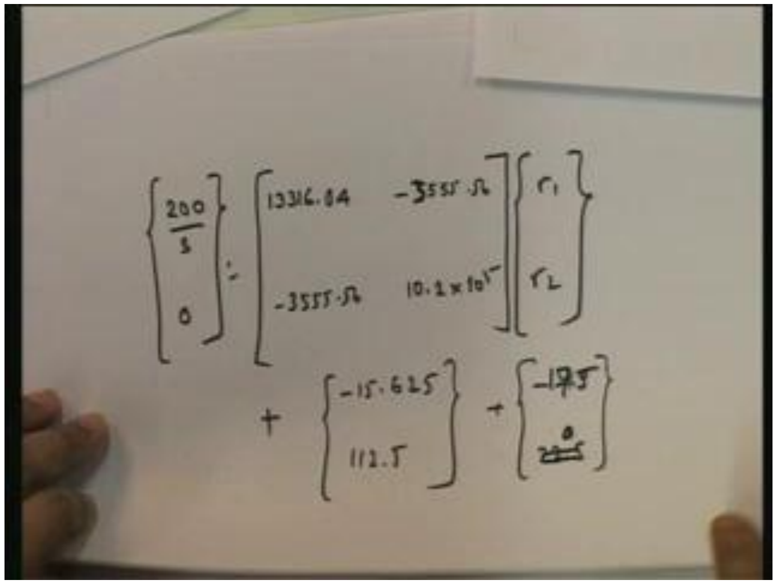


This is actually minus 17.5 and 0 (Refer Slide Time: 38:50).

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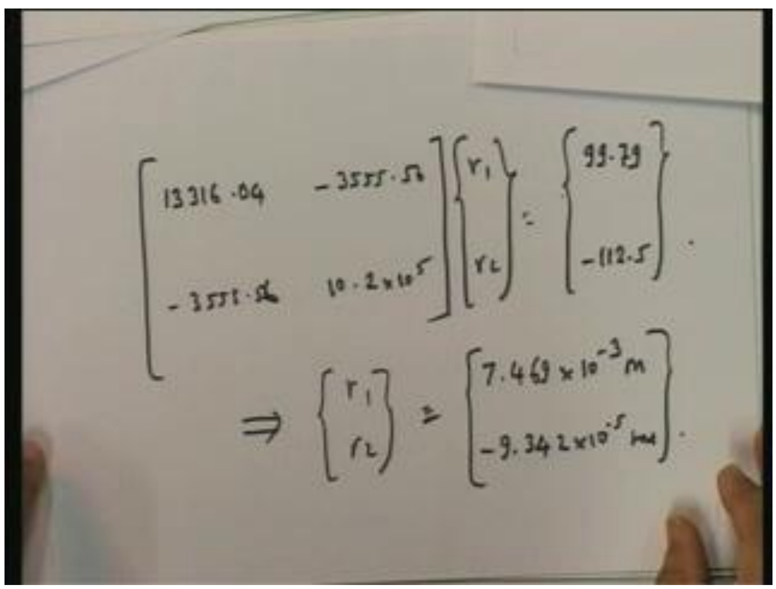


Ultimately, what we are left with is this: (13316.04, minus 3555.56, minus 3555.56, 10.2 into 10 to the power of 5) into ($r_1$, $r_2$) is equal to 32 and 66, so 99.79 and over here, it is minus 112.5 and you can solve this (Refer Slide Time: 39:57) for $r_1$ and $r_2$; $r_1$ and $r_2$ in this particular case is equal to 7.469 into 10 to the power of 3 and minus 9.342 into 10 to the power of 5 radians. This is the procedure that we use to solve for $r_1$ and $r_2$. Once we have solved for $r_1$ and $r_2$, the next step is to find out the member end moments.

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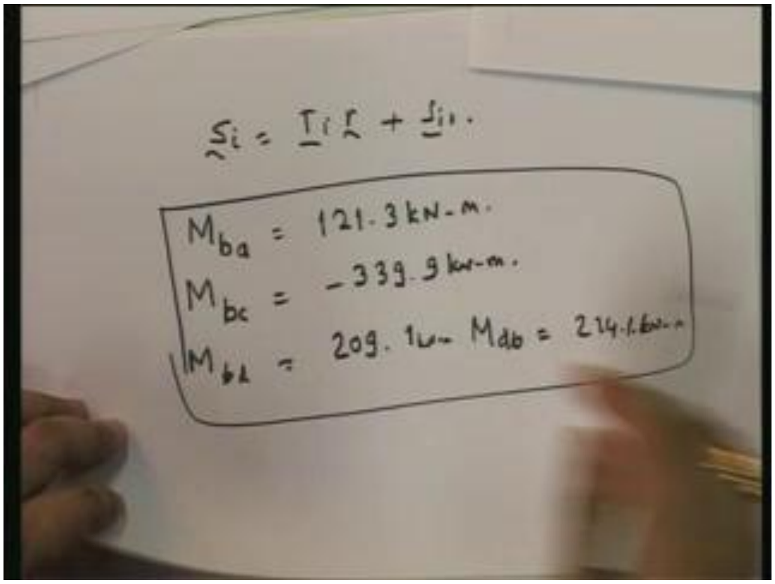


There you put in; $S_i$ is equal to $T_i$ into r plus $S_{i0}$. Once we do that we are going to get that $M_{ba}$ is equal to 121.3 Kilonewton meter, $M_{bc}$ is going to be equal to minus 339.9 Kilonewton meter, $M_{bd}$ is going to be equal to 209.1 Kilonewton meter and $M_{db}$ is 214.1 Kilonewton meter. I will leave it to you. Once you find out the member end moments, then you can find out the reactions at every end and from there, you can find out the support reactions. We have found out the member end moments.

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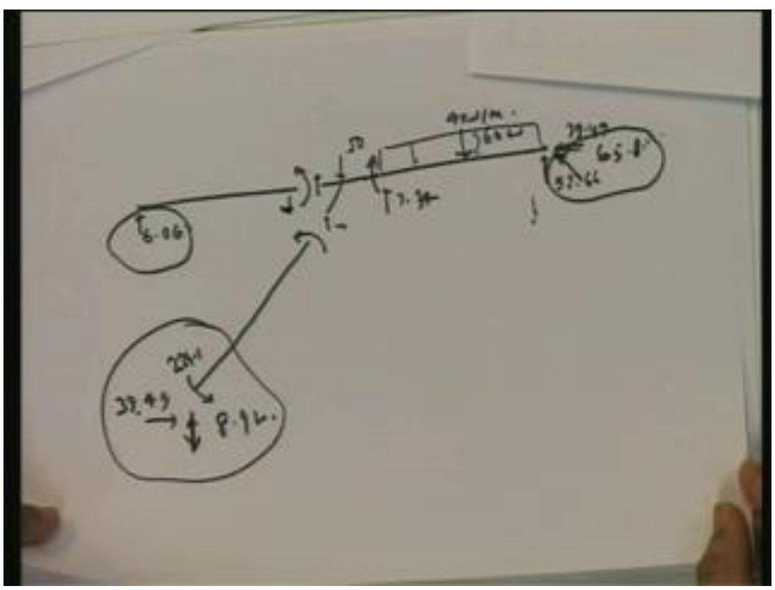

Here, you have it as this way, 0 (Refer Slide Time: 43:13) and then here, you have it going anticlockwise and here also, you have it going anticlockwise. From this, you can find out the reactions: they will be in this manner and they are going to be equal to 6.06, this is also going to be 6.06, here, this is going to act in this way. Once you put this, you are going to get this is equal to 4 into 15, 60 Kilonewton, so you will have two parts to it: one part will be this aspect, so here, you get 52.66, 39.49 you can compute the forces there and this will be 7. 34, there is going to be this way, this way.

Ultimately, from this, we can find out what this and this are and then we can find out what reaction comes over here and what reaction comes over here. The reaction that comes over here is directly 39.49 which is this one and the reaction over here which is a vertical reaction is going to be 65.8; this is a roller, roller on an end gives you both vertical and horizontal; then you can find this out, this is going to be equal to 52 and 62, it is going to give you this way, so essentially, what we have over here is going to be this way and this way the load is going to be equal to 8.92 and the moment over here is going to be 224.1.

For this support, there is a fixed support, this is for this roller support and this is for this roller support and that gives you all the support reactions that you have. The overall point is, in this particular problem, if you look at it, was probably that you had to consider different kinds of issues in this and if you do it step by step as I said, what you need to do essentially in this particular kind of situation is this: start the problem by considering member by member.

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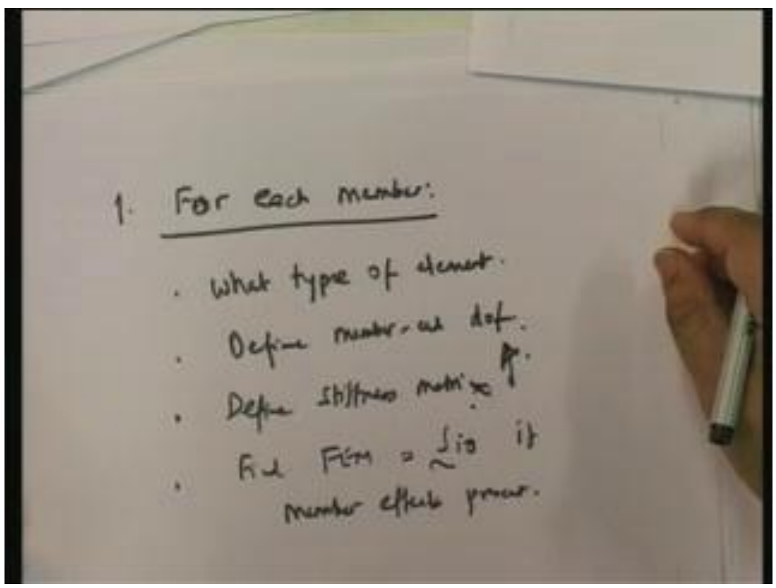


First is of course the degrees of freedom, then…. The first step is for each member, what do you find out? First, what type of element do I use? Do I use the standard fixed-fixed element or do I use the modified element with a hinge at one end and fixed at the other end? Once you do that, the next step is automatically; define the member-end degrees of freedom.

In other words, if you have both the members fixed, then the rotation at both the ends would be the degrees of freedom; if you had pinned at one end and fixed at the other end, then only the fixed end rotation becomes the member-end degree of freedom. Three: define the stiffness matrix corresponding to this degree of freedom – in other words, if you have both ends, then the member stiffness matrix will become 4EI upon L, 2EI upon L, 2EI upon L, 4EI upon L; if you only have a pin at one end and fixed at the other end, then there is only that this thing, then you have $M_{ba}$ is equal to 3EI upon L into $theta_{ba}$ – that defines the matrix. Finally, find the fixed end moments, which are essentially $S_{i0}$ if member effects are present. That is the first step. For each member, you have to figure out what type of element to use, define the member-end degree, define the stiffness corresponding to the degrees of freedom and then find out the fixed end moments.

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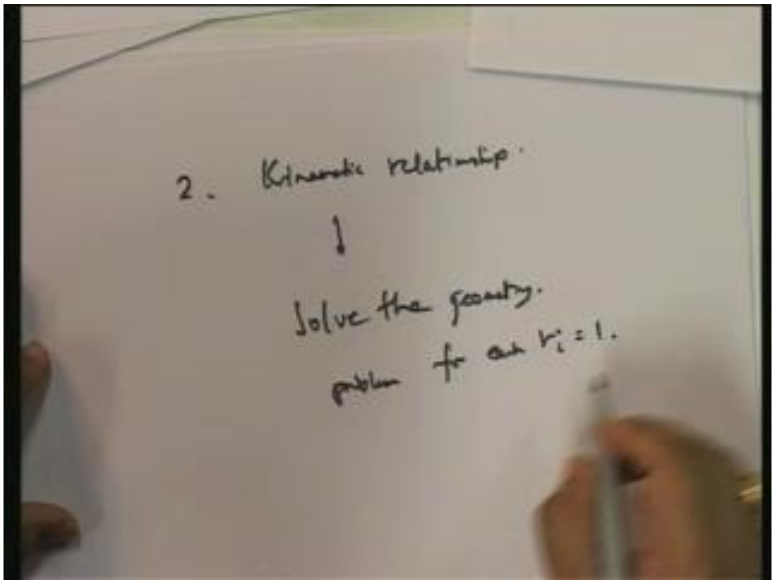


Once you have done that, the next step is kinematic relationships: solve the geometry problem for each $r_i$ is equal to 1 and all others 0 – solve the geometry problem; provide $r_1$ is equal to 1 and all others equal to 0 and find out what the displaced shape looks like, put $r_2$ is equal to 1 and all the others 0 and find out what it looks like; this way, you would solve the kinematic relationship.

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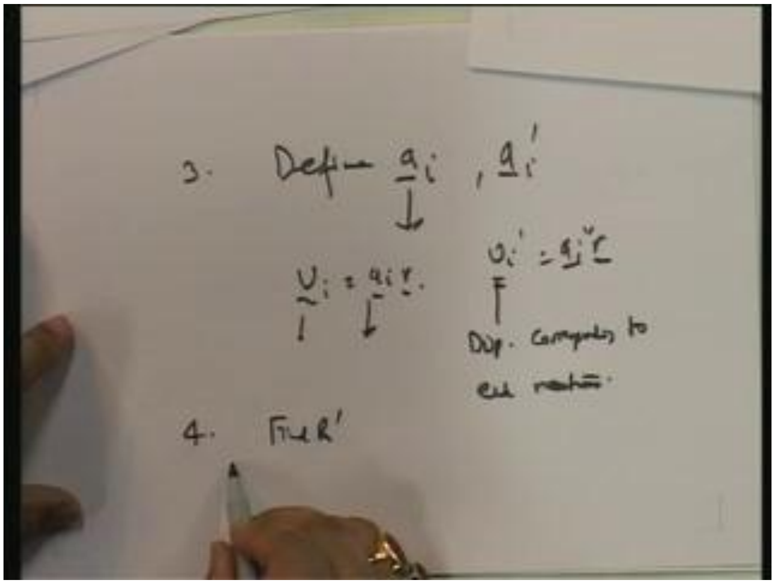


Once you have that, then define $a_i$ and a prime i; $a_i$ is essentially $v_i$ in terms of $a_i$ r and these are (Refer Slide Time: 50:12) the displacements corresponding to the end reactions; these are the standard ones, defining the member end deformation in terms of this thing. Once you have this, the fourth is find R prime, the effective load vector; the effective load vector becomes the loads at the nodes multiplied by the corresponding displacements for each degree of freedom – that is what you would find out.

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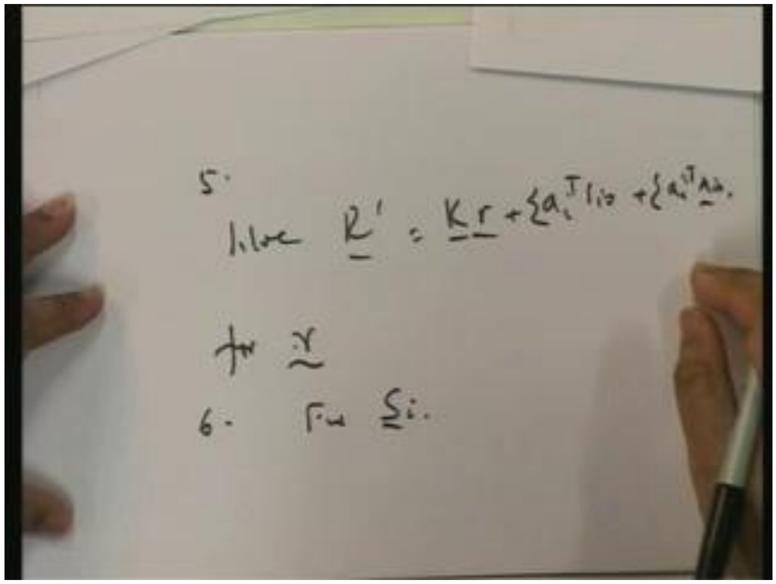

Once you do that, the final thing is solve R is equal to K r plus summation ($a_i$ transpose $S_{i0}$) plus summation ($a_i$ transpose $n_{i0}$) for r. Once you find r, six, find $S_i$ and once you find out $S_i$, the problem becomes a statically determinate problem for which you can solve for anything. This, in essence, is the entire problem.

In the next lecture, I am going to solve one more problem where I am going to introduce the factor that one of the members may be flexurally rigid – how does the kinematic relation look in that particular case? Then, I am going to introduce all the member load effects like temperature etc. Thank you very much.