Structural Analysis- II
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Lecture – 29

Good morning. In the last lecture, I introduced you to how to extend the flexibility approach to include statically indeterminate frames and how to solve the entire problem – that is called as the matrix force method. Then, we started looking at a particular problem. Just to review the problem to you let us see what were the steps.

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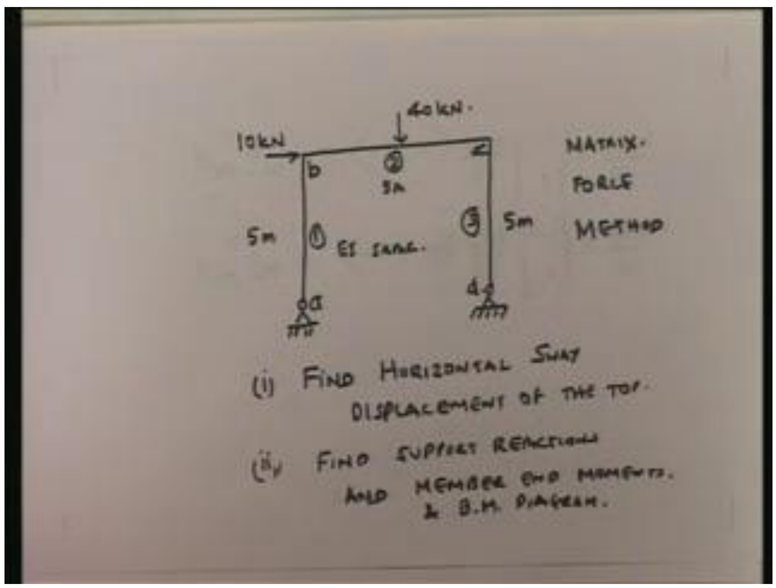

This was the problem; using the matrix force method you had to find out the horizontal sway of the top and the support reaction and member end forces and the bending moment diagram.

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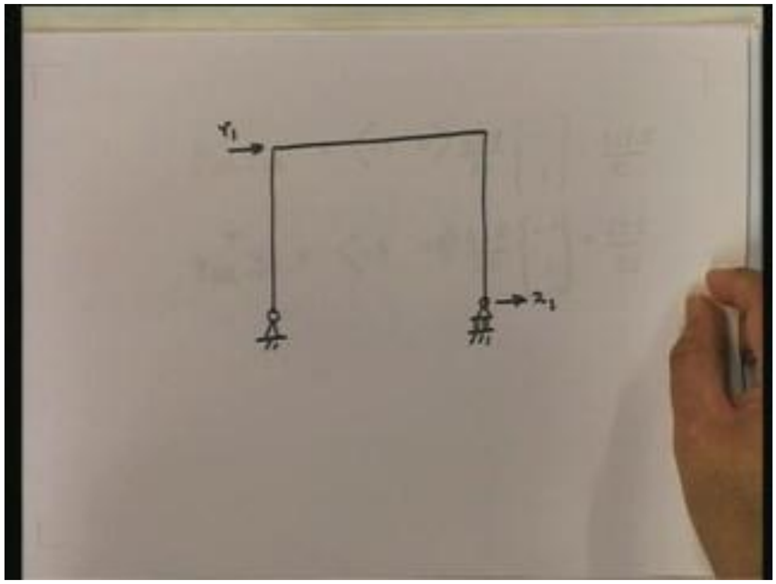

Then the way we went about it was that these were the two degrees of freedom that were defined and ultimately you went ahead and found out first and foremost the displacements the S_{i0} and the v_{i0} and once you found those out, then the next step was to find out the b_{i0} and b_{iR} and the b_{iX} also we found out by doing equilibrium.

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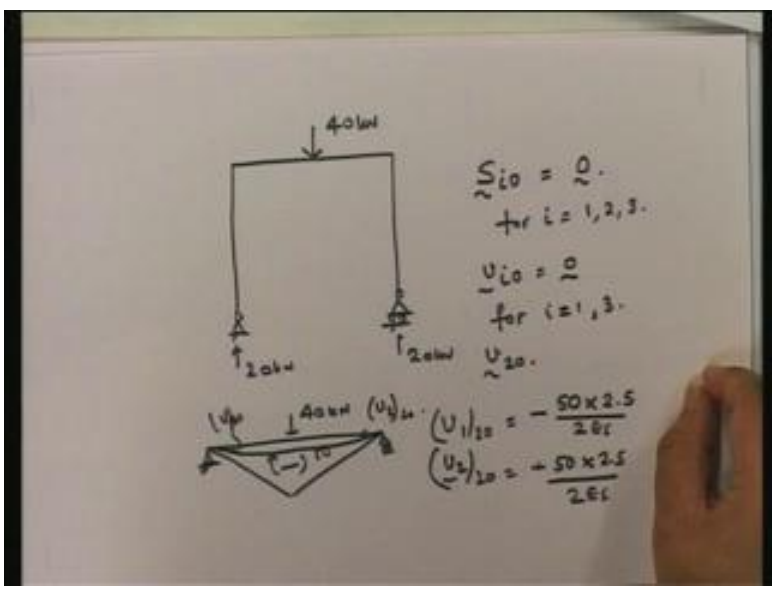

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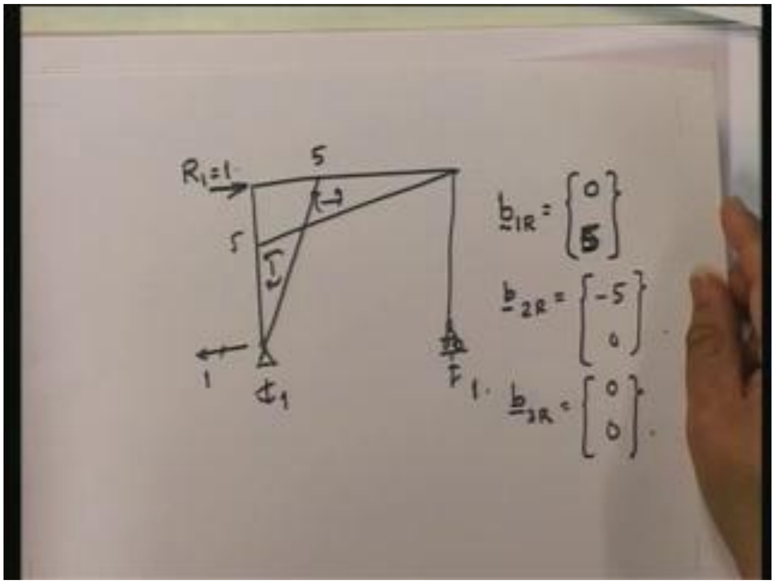

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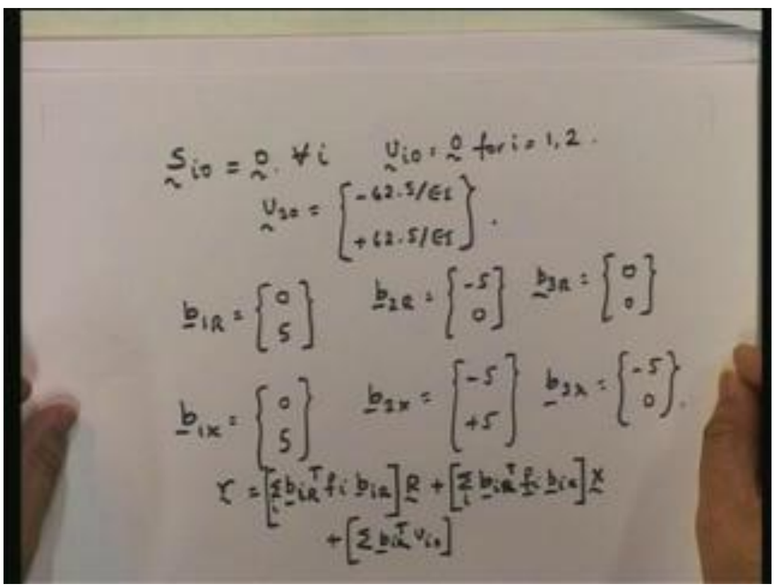

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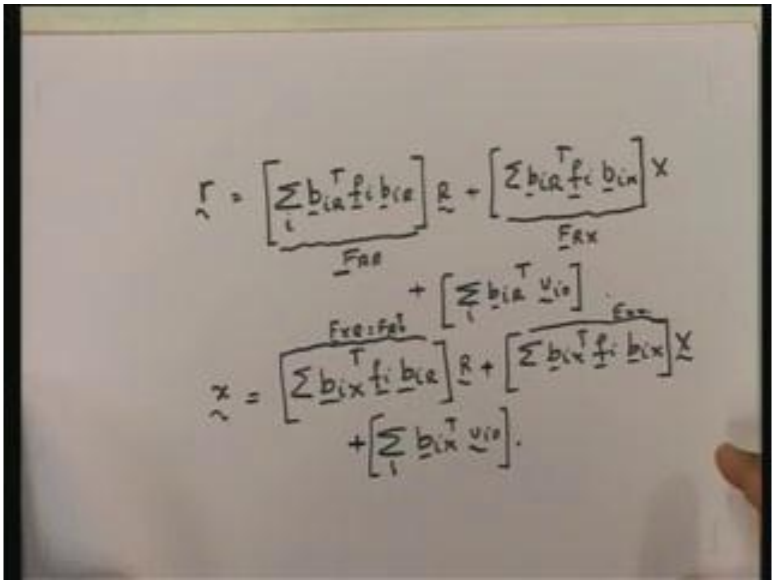

Then, I listed out all of these things and ultimately, this was the equation that you had to solve. Here, r was equal to this (Refer Slide Time: 03:30) – you can do this if you just rewrite it; this into R plus b_{iR} into X, so this is F_{RR}, this is F_{RX}, this is F_{XR}, you will find out that if you take the transpose of this, you will get b_{iR} f_i transpose b_{iX}; f_i transpose is itself and therefore you see that F_{XR} is equal to F_{RX} transpose and then you can find out F_{iX}; these are the additional terms due to the v_i; I do not have the terms for S_{i0} over here because in this particular problem, we have already seen that S_{i0} is 0 and therefore, I am just dropping those terms out. Now, all I need to do is evaluate all these terms and then put them into the equation.

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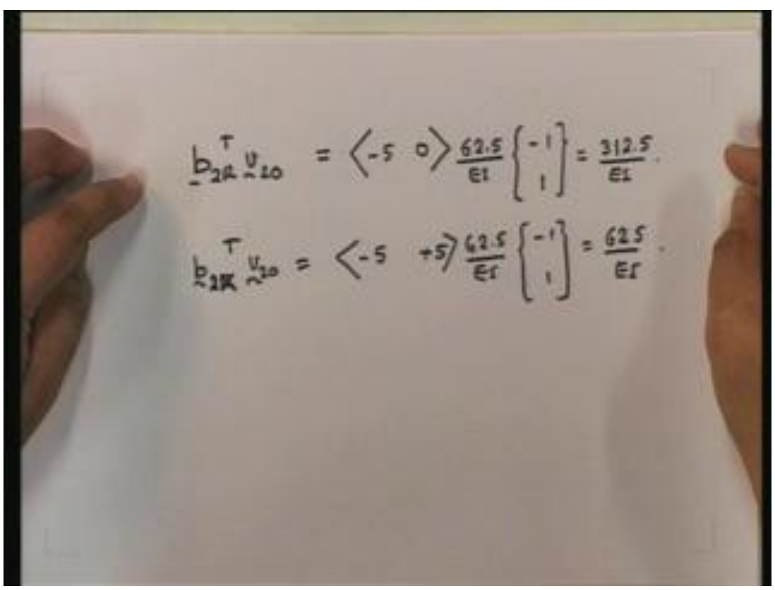

If you look at this, the first step is to find out the summations. Note that since v_{i0} is equal to 0 excepting for i is equal to 2, the summation term only has these two terms and I have found them out**;** b_{2R} transpose v_{20} is 312.5 upon EI and b_{2X} v_{20} is 625 by EI. What is the next step? The next step is to find out all those b_{iR} and b_{iX}. We are going to go through the steps and we will do it member by member.

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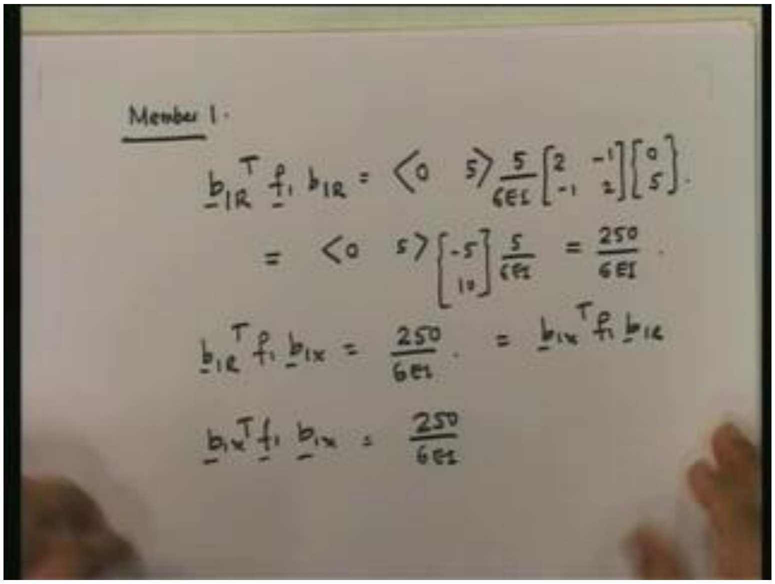

First and foremost, member **1:** Let me find out b_{iR} transpose. What am I doing? I am actually evaluating. I have evaluated this term because in this term, excepting for b_i is equal to 2 everything else is 0; this term is equal to 312.5 by EI and this term is equal to 625 by EI – we have already done this.

The next step is to find out this (Refer Slide Time: 05:47), this and I am going to find out this. If I put that in, I get for member 1 b_{1R} transpose f_1 into b_{1R}. What is b_{1R}? b_{1R} transpose is going to be (0, 5). What is f_1? l upon 6EI, l is 5 upon 6EI into (2, minus 1, minus 1, 2) and here, we have (0, 5). We are going to go through all the steps so that you know exactly how we are solving this. If I look at this particular thing, you will see that you will get 0, 5 and inside minus 5, here 10 and this is equal to 5 upon 6EI and this is equal to 250 upon 6EI.

Let us look at b_{1R} transpose f_1 b_{1X} you will see that since b_{1R} and b_{1X} are identical both will give you identical results so you are going to have 250 on 6EI and b_{1X} …; this (Refer Slide Time: 8:02) is equal to b_{iX} transpose f_1 b_{iR} – satisfy yourself that this indeed is the case; b_{1X} is also 250 upon 6EI. I have found out for member 1. Similarly, I am going to find these for member 2 and 3.

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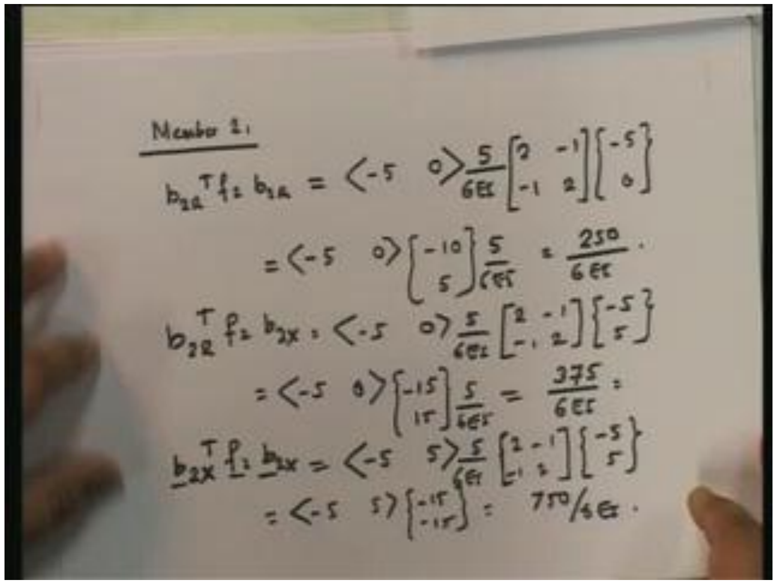

For member 2, I have b_{2R} f_2 b_{2R}; b_{2R} is (minus 5, 0) and into the same – all the f_is are the same because the lengths are the same, the EI's are the same and so here, this is equal to (minus 5, 0), which is equal to (minus 5, 0) multiplied by (minus 10, 5); if you look at this, you will have (minus 5, 0) into (minus 10, 5) into 5 upon 6EI and so this is equal to 250 upon 6EI. Now, b_{2R} transpose f_2 b_{2X} is minus , this is going to be different because if you look at this, b_{2X} is equal to (minus 5, 5) so this is going to be equal to (minus 5, 0) into minus 15 and here also this is plus 15 (plus 5 plus 10) into 5 upon 6EI (Refer Slide Time: 10:38) – this is going to be equal to 375 upon 6EI; this by the way is going to be the same as b_{2X} transpose f_2 b_{2R}, so if we do b_{2x} transpose f_2 into b_{2X}, you will get it equal to (minus 5, 5) into 5 upon 6EI into (2, minus 1, minus 1, 2) into (minus 5, 5), so this is equal to (minus 5, 5) into (minus 15, minus 15), which is equal to 750 upon 6EI so we have done it for member 2.

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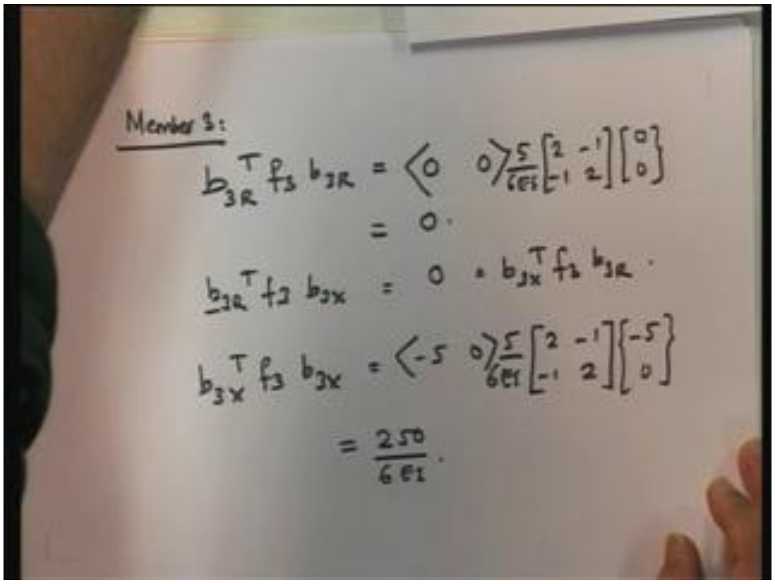

Now, finally, we have to do it for member 3. Let us see what we get for member 3. Once we have done it for all three members you will see that we can add up and actually solve the problem. Let us look at b_{3R} f_3 b_{3R}. b_{3R} is equal to (0, 0) into 5 upon 6EI into (2, minus 1, minus 1, 0) into (0, 0) and without much ado, we can see that this is going to be this (Refer Slide Time: 12:33). Similarly, you will see that b_{3R} f_3 into b_{3X} … even though b_{3X} is not equal to 0, you will see since this is 0 that this is equal to this and…. Finally, b_{3X} transpose f_3 b_{3X} is going to be equal to (minus 5, 0) into 5 upon 6EI into (2, minus 1, minus 1, 2) into (minus 5, 0) and this you will see is going to be equal to 250 by 6EI. Now, having done all of this, we need to add for each one and when we add them, this is what we get.

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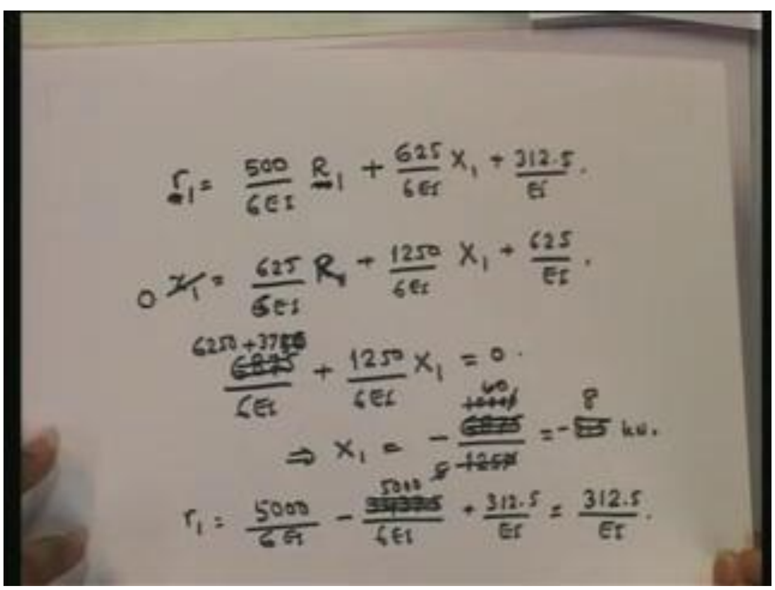

Now we are going to add up all of them. Thus you will have 250 plus 250 plus 0, so b_{1R} is equal to 500 upon 6EI R; note that this vector R is just a vector of 1 and this is what you will see – that this is nothing but r_1, this is nothing but R_1 (Refer Slide Time: 14:20) plus b_{iR} transpose f_i X. If I add all of them up, you will see 250 plus 375 plus 0, so this is going to be equal to 250 plus 375 is equal to 625, 625 by 6EI X_1 plus b_{iR} 0, which if you remember was equal to 312.5 – that was positive, so plus 312.5 upon EI.

Similarly x_1 is equal to 625 by 6EI X_1 plus 250, 250, 500 plus 7 times 50 is 1250, so it is 1250 upon 6; this is R_1 (Refer Slide Time: 15:40), this is X_1 plus we have got it as 625 upon EI and therefore, we have it as 625. Now, we know this is equal to 0, we know that this is equal to 0 and we know R_1 is equal to 10 so we have 6250 plus 6250 so this is going to be equal to 6875 upon 6EI plus 1250 upon 6EI X_1 is equal to 0 and we solve for X_1. X_1 is equal to minus 6875 by 1250, which is equal to minus 5.5 Kilonewtons.

We have X_1 is equal to minus 5.5 and therefore, R_1 is equal to 5000 upon 6EI minus 3437.5 upon 6EI plus 312.5 upon EI. Here there is a change**;** this is wrong here, this is 6250 plus 3725 (Refer Slide Time: 18:28); 625 into 6 is equal to 6750, 6750 is going to be 10000 so this is going to be 10000; this goes, so this goes 5, 25, 40, this is minus 8 Kilonewtons and so when you do 8 over here you get 200, 5000 and what you are left with is 312 upon EI – that is your R_1; you know your R_1, you know the sway – we have found out your sway; ultimately, once you have found out your sway, the next step is finding out the rotations.

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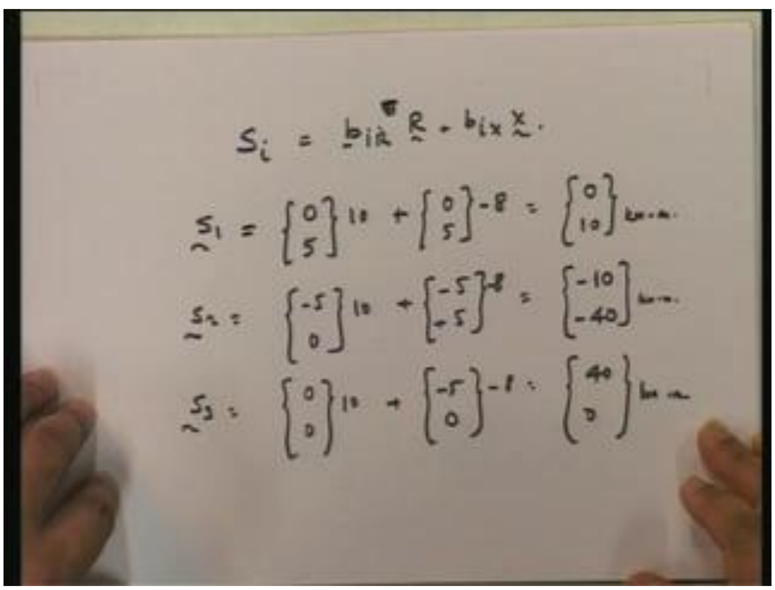

If you look at the member end forces, the member end forces will be given by S_i is equal to b_{iR} R plus b_{ix} X. Let us look at S_1: S_1 is equal to plus S_{i0} (S_{i0} is 0 for all of them), so b_{iR} b_{1R} is equal to (0, 5) into 10 plus b_{1X} into minus 8; this you see will turn out to be (0, 10) Kilonewton meter; S_2 is equal to (minus 5, 0) into 10 plus (minus 5, plus 5) into minus 8 and what we finally get is (minus 10, minus 40); and S_3 is equal to (0, 0) into 10 plus (minus 5, 0) into minus 8, so I get (40, 0) – these are my member end moments.

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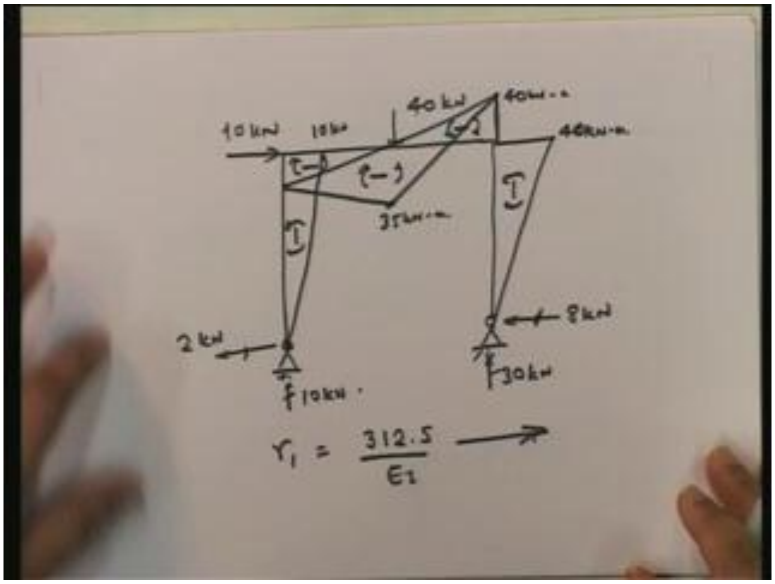

If I have put these in, let us see what we finally get. This is my x_1, minus means in this direction. Then, if I take moments about this point, you will see that I will get 10 into 5, 50 plus 100, 150 divided by 5, this is your member end forces and if I were to plot the bending moment diagram, it would look like this. Here this is in this fashion, here, it is in this fashion and here it is in this fashion (Refer Slide Time: 24:21) and these are the member end moments that you already have. This is your bending moment diagram. This one is this way – that is only due to the load but there is a superposition over here and that is 50 so this is going to be and 40, so at the center it is going to be equal to 40 minus 10 upon 2, it is going to be 15 in the middle, so this is going to be 35, so it will go like this and like this (Refer Slide Time: 24:57) and here, I have this and over here, this is going to be 35 Kilonewton meter.

This is the total bending moment diagram and also you know what R_1 is, it is equal to 312.5 upon EI in this direction. That is solving this problem using the matrix force method. I suggest that you solve this problem using your standard force method and satisfy yourself that whatever answers we have gotten are the answers that you would get using the standard. I can assure you that you would get exactly the same if you went ahead and did it that way. So much for the matrix force method as used for frames. Now, these are using flexural members. Now, I want to spend the next bit of time looking at a problem where you have truss, in other words, the axial deformation is the key. Let us take a problem for that.

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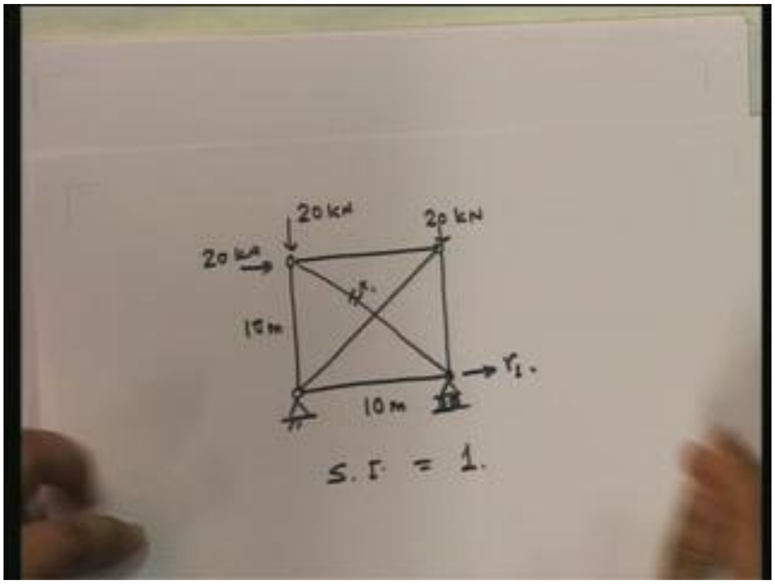

This is the problem and here, the question asked of you is the following: I need to know how much this has gone by. So I want to find out the horizontal displacement at this roller and I also want to find out what are the member forces. This is again static indeterminate – it is the problem that we have looked at earlier: the static indeterminacy is 1 and I am going to assume that this is my x_1 (Refer Slide Time: 27:41) – the internal force is x_1. How I am going to solve this problem? All I am going to tell you is how I am going to set up the problem and that will give you the details of how to solve the problem. The numbers I am not going to go into. What we are going to do here is assume that this is 10, we will take this also as 10.

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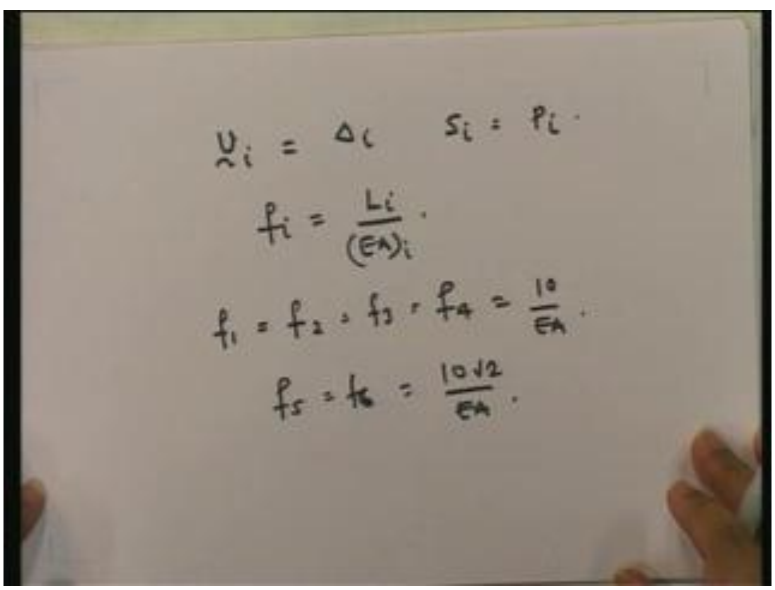

Therefore, let us take down the steps over here. First and foremost, what are the f, the flexibility matrix? You will see that v_i is nothing but delta – it is just a single degree of freedom. S_i is nothing but P_i and f_i is nothing but L_i upon $(EA)_i$. In this particular problem, we will assume that the EAs are the same for all the members.

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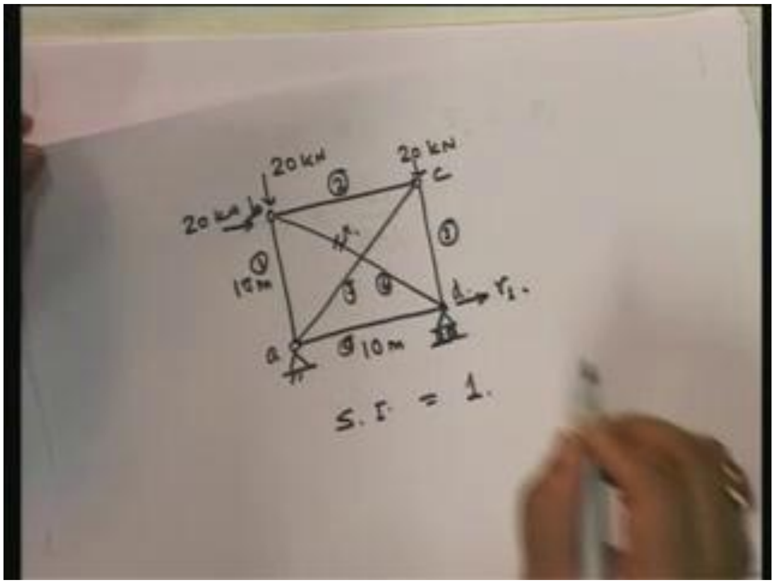

Then, if we put it in this fashion, a, b, c, d, let me put this member 1, member 2, member 3, member 4, member 5, member 6. f_1 is equal to f_2 is equal to f_3 is equal to f_4 and all of them are equal to 10 upon EA; f_5 and f_6 are equal to 10 root 2 upon EA – these are the flexibility matrices.

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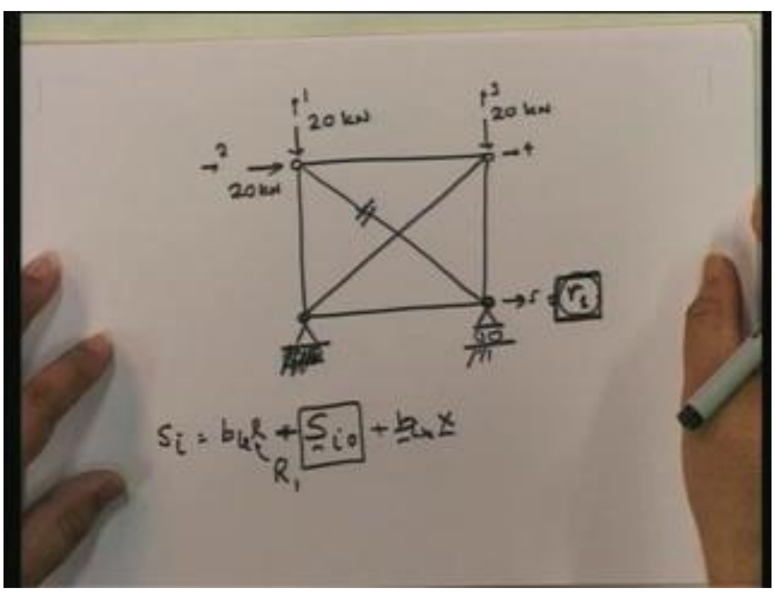

Now, what is the next step? The next step is actually to find out the equivalent statically determinate. This is the hinge, this is the roller. I have this member and this member exists but it is cut, so the force in this member is equal to 0 (Refer Slide Time: 31:00). This is the point I was trying to make: in the force method, typically in this particular case, it is very easy to solve because you would have all the degrees of freedom. How many degrees of freedom does the structure have? You will see that this is 0 (Refer Slide Time: 31:18), here, you will have 1, 2, 3, 4 and 5. 5 is the only one that you have been asked to find out; you have not been asked to find out 1, 2, 3, 4.

In fact, when you actually solved it on the computer, you would consider all these five degrees of freedom and therefore, these displacements would become joint loads. However, since I have asked you to only find out this (Refer Slide Time: 32:03) and this hand calculation… I cannot deal with five degrees of freedom plus one which is the sixth redundant degrees of freedom and I would have a 6 by 6 matrix that I would have to solve. I am not quite interested in that particular case.

If this (Refer Slide Time: 32:24) is my only degree of freedom that I am interested in, understand that these then become actually not joint loads – technically they are joint loads, but in this particular case, since I am only interested in finding this out and this out, so these do not come as members and so I am going to solve these separately and I am going to say that these are going to give me my S_{i0}.

Remember I have said that you could solve it, that S_i to be equal to b_i into R plus S_{i0}. In this particular case, R is the only one corresponding to… this is R_1 here, of course here in this particular case, there will also be b_{iX}. I am now going to club all these loads because they do not correspond to this one as these loads – member loads. I can find out the member loads to begin with, so let me find out the member loads.

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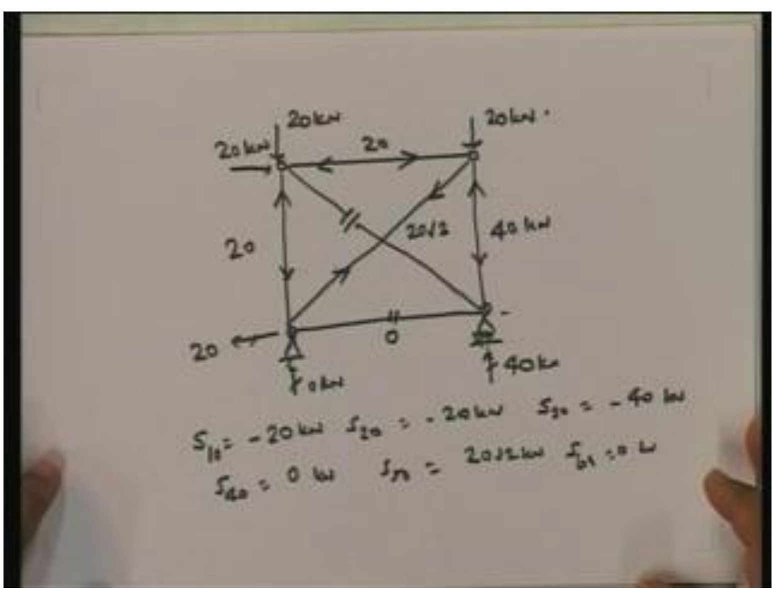

Until and unless our joint loads happen to be corresponding to a degree of freedom that you are interested in finding out, you will never consider those loads as joint loads – you will club them together and forget about it. I am going to just solve this problem to begin with and get my S_{i0} for each one – I am just going to forget it at that point of time. Therefore, let me solve for this. What am I going to get? Here, this is going to be equal to 20. Then, if I take moments about this point, I will get 20 into 10, 200 plus 200, 400 divided by 10 – you get 40 and this is equal to 0. If I have this particular situation, let us go to a specific point: since this is 0, I can find out that this is going to be equal to 20, similarly, this is going to be equal to 20.

Then, what else do we have? Let us look at this particular joint. In this particular joint, the horizontal of this is going to tie up with this, so this is going to be equal to 20 root 2 and therefore, this is going to be equal to 40 and this is going to be equal to 0. You will see that at this particular joint, 20 the vertical matches with this and the horizontal matches with this, so this is 0, this is 0; here (Refer Slide Time: 36:20), there is only vertical, so this horizontal is 0 and this is anyway 0. Now, here I get that S_{10} is equal to minus 20, S_{20} is equal to minus 20, S_{30} is equal to minus 40, S_{40} is equal to 0, S_{50} is equal to 20 root 2 Kilonewton and S_{60} is 0 because that happens to be the redundant force.

In other words, what I have done actually is to include all these loads as this thing, because the only one that I am interested is here. Next, what do I need to find out? I need to find out b_{iA}. By the way, will there be any v_{i0}? There will not be any v_{i0} because there are no member effects; the only time that you will get v_{i0} is if you have a temperature stress or you have a lack of fit; in this particular case, there is none, so v_{i0} is equal to 0 and I need to find out b_{iR} and b_{iX}.

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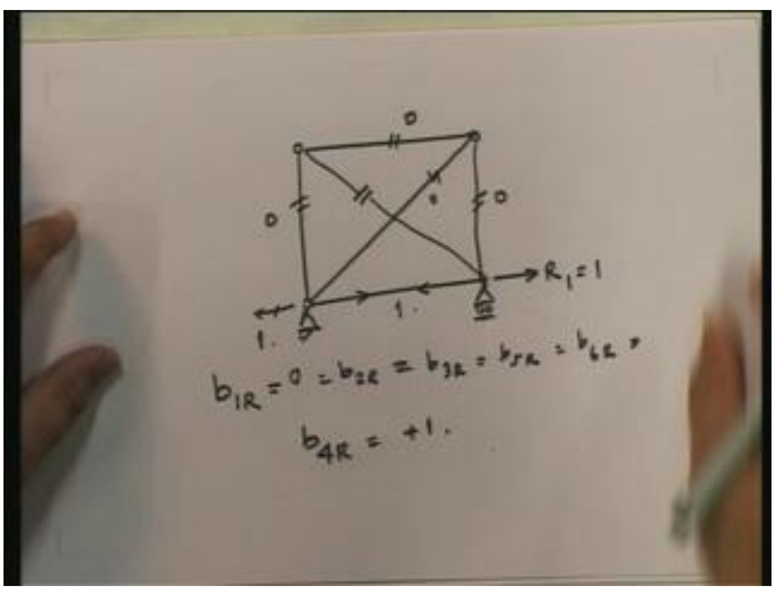

How do I find out b_{iR}? This is the displacement, I need to apply R_1 is equal to1 (Refer Slide Time: 38:08) and find out all the member forces just for R_1 is equal to 1. Obviously, this is going to give me this, you will see that this has to be equal to 0, this has to be equal to 0, this has to be equal to 0, this has to be equal to 0 and so the only thing that you have is 1. b_{1R} is equal to 0, b_{2R}

is equal to 0, b_{3R} is equal to 0, b_{5R} is equal to 0 and b_{6R} is equal to 0 – all of them are equal to 0; the only thing that you have is b_{4R} which is equal to plus 1.

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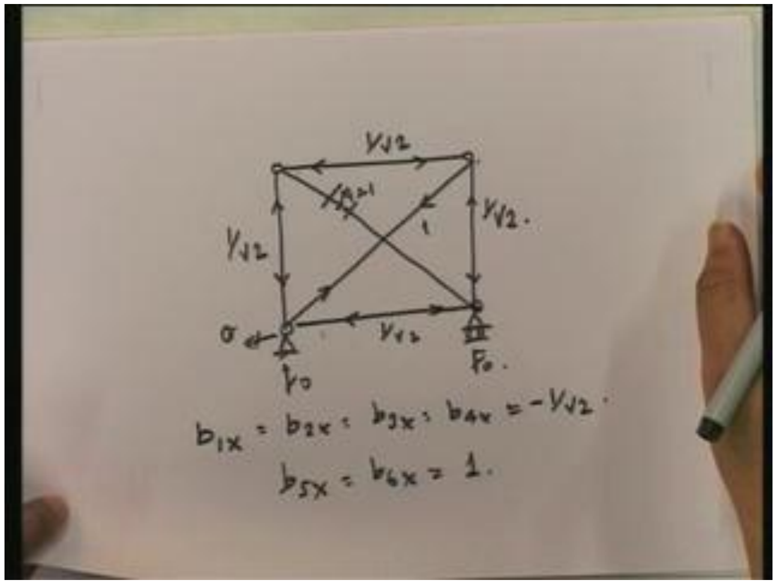

Now, put X_1 is equal to 1 so you essentially have X_1 is equal to 1 and find out all the net forces. You will see that the external reactions are all 0 because this is an equal and opposite force – it is an internal force and therefore, what are you going to have over here when you put X_1 is equal to 1? This is equal over here. This is going to be 1 over root 2; this one is going to be 1 over root 2; similarly over here, you will see that this will be 1 over root 2 and you will get this equal to 1; you will get that this is equal to 1 over root 2.

Therefore, b_{1X} is equal to b_2X, b_{3X}, b_{4X} is equal to minus 1 over root 2 because they are all negative and b_{5X} is equal to b_{6X} is equal to 1 – both of them are plus 1. Now, I have found out my S_{i0}, I have found out my b_{1R} and b_{2R}.

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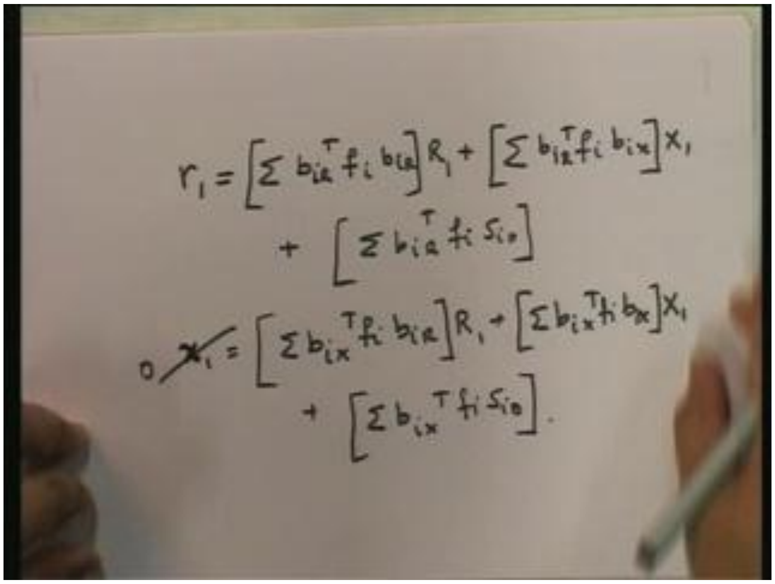

Next, what is the solution? If you look at the solution, you will see that r_1 is equal to (summation (b_{iR} transpose f_i b_{iR})) into R_1 plus (summed up (b_{iR} transpose f_i b_{iX})) into X_1 plus … now you will see S_{i0} come in so this is going to be equal to summation over the entire b_{iR} transpose f_i S_{i0} and x_1 (which by the way is the separation, so it is equal to 0) is equal to summation over b_{iX} transpose f_i b_{iR} into R_1 plus summation b_{iX} transpose f_i b_{iX} into X_1 plus summation b_{iX} transpose f_i S_{i0}. Now, let me first find out these two quantities (Refer Slide Time: 43:37) and then I will find out these two quantities.

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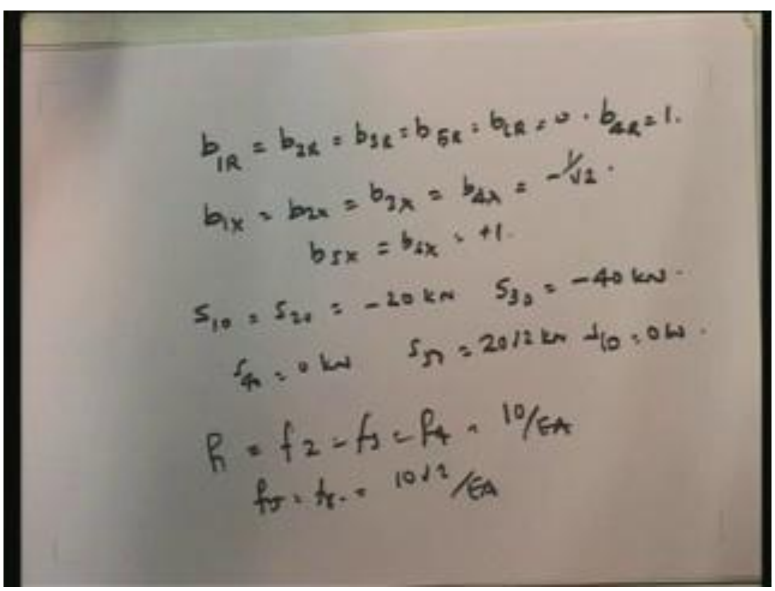

My b_{1R} is equal to 0 and I am only left with b_{4R} is equal to 1 and then b_{iX}, b_{3X}, b_{4X} are all 1 over root 2 and b_{5X} and b_{6X} are plus 1 – that gives me my b's. Then, what do I have? My S_i – S_{10} is equal to S_{20} which is equal to minus 20; then I have S_{30} equal to minus 40, S_{40} is equal to 0, S_{50} is equal to 20 root 2 and S_{60} is equal to 0 Kilonewton. Remember that this is identical to what I am doing… this is the problem that I had solved earlier too. The f_1 equal to f_2 equal to f_3 equal to f_4 equal to 10 upon EA, f_5 and f_6 are equal to 10 root 2 upon EA. Once I have all of these things together, now all I need to do is put together the terms that I have to get. If I put together those terms, you will see that they turn out to be equal to…. These are all single, single terms.

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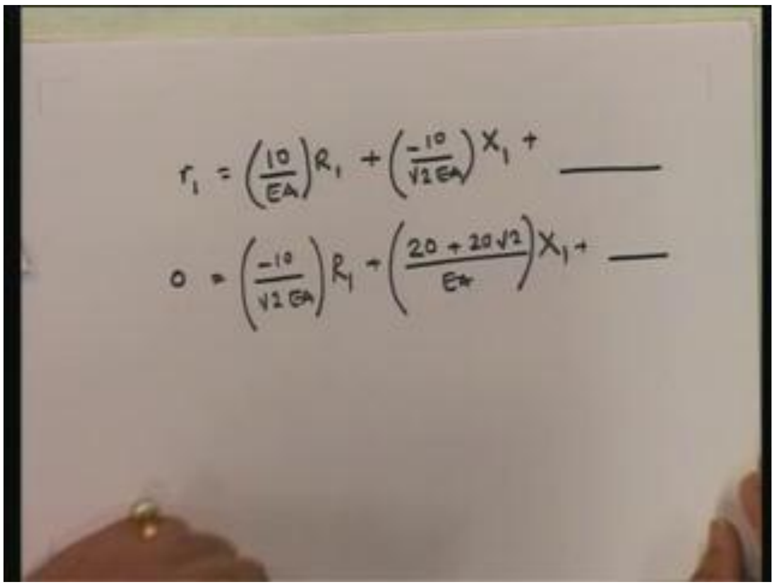

If you look at it, in b_{iR}, since all the ones are 0 excepting for b_{4R} (Refer Slide Time: 46:23), you will see that only b_{4R} will contribute and so this r_1 will become equal to 10 upon EA R_1 plus now if you look at it, b_{iR}, and because all the b_{iR} transpose are equal to 0 the only one that will come in will be b_4 so we need to get b_{4X} and that is minus 1 over root 2 so this is going to be 10 into R_1; and this is going to be equal to minus (10 over root 2 EA) into X_1 plus now here this is the term that we need to evaluate – this is the term (Refer Slide Time: 47:18) and we are going to evaluate this term.

Now, let me just put this together: you will see this will become minus 10 over root 2EA into R_1 and ultimately, we will see that in this particular case, you will see that this term… all the b_{4X}s are root 2, root 2, so 1 over root 2, so you will get one half and one half into 10 EA is going to be 5, 5 by 4 times is going to be 20, 20 and then you are going to have 20, and then you are going to have one 1 into 10, so it is going to be 20 root 2 upon EA into X_1 plus this term. Now, this term is the one that we have to sit and evaluate for each member and let us do that for each member.

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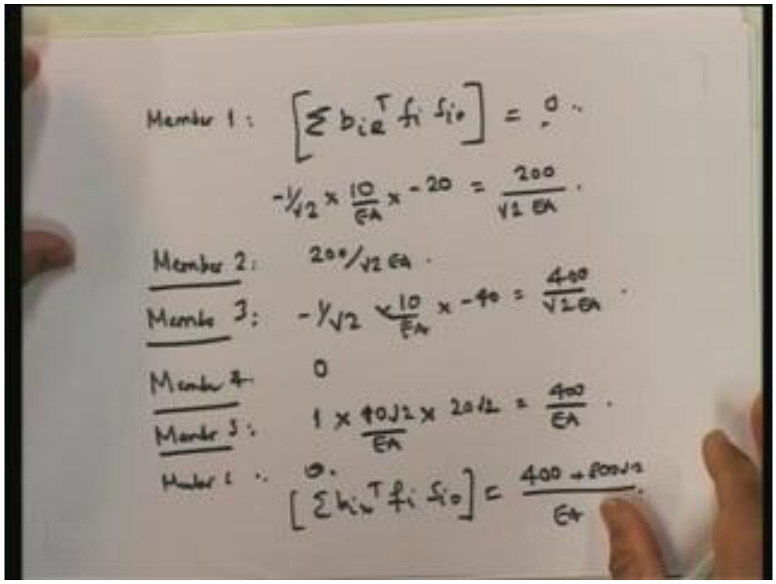

Member 1: note that in all these cases, b_{iR} transpose is equal to 0 excepting for b_{4R}, so the only one that I have to actually do is to find out b_4 S_4 but you see S_{40} is equal to 0, so you will see that summation b_{iR} f_i S_{i0} for this particular case is equal to 0; now for the b_{iX} transpose, now this is going to get this thing, so here I am going to do for member 1; member 1 b_{iX} is minus 1 over root 2 into 10 upon EA multiplied by S_{10} is minus 20; for this one, this is going to be equal to 200 upon root 2EA.

Member 2: for member 2, you will find that it is minus 1 over root 2, S_2 is also the same, so for member 2 also, this is 200 over root 2EA. For member 3, you will see that b_1, so it is 1 over root 2 into 10 upon EA multiplied by minus 40, so you have plus 400 over root 2EA. Then, you have member 4 and member 4 is going to have 0; S_4 is 0, so this is 0. Then, you have member 5 and member 5 is going to be equal to… b_5 is 1 and this is multiplied by 10 root 2 upon EA and this is going to get multiplied by 20 root 2, so this is going to be equal to 400 upon EA. For member 6, it is going to be 0. We add it up and you will get summation b_{iX} transpose f_i S_i is equal to (400 plus 800 root 2) all over EA.

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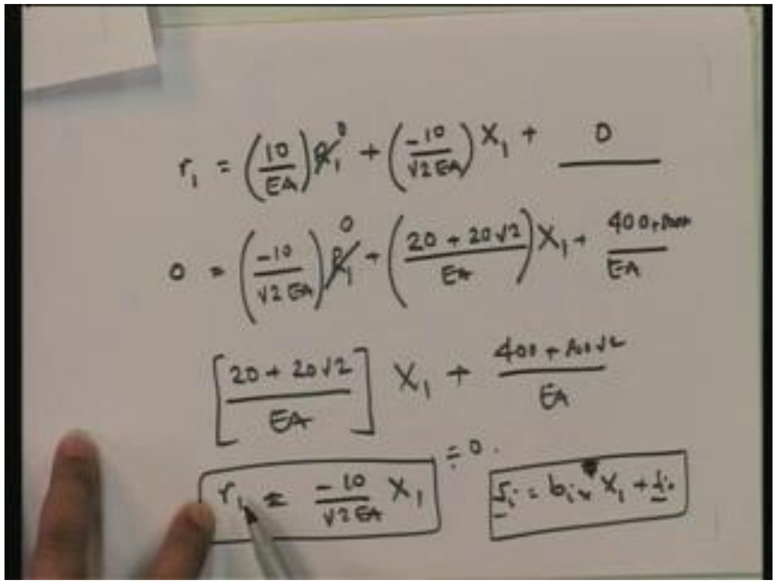

If we come back to this particular problem, you will see that this is 0 and this is (400 plus 800 root 2) upon EA (Refer Slide Time: 52:04). Now, what is R_1? R_1 in this particular case is 0, so all we get is that (20 plus 20 root 2) upon EA into X_1 plus (400 plus 800 root 2) upon EA is equal to 0 and from this, we can solve for X_1 and we get r_1 is equal to minus 10 upon root 2EA into X_1. Finally, my S_i: S_i are going to be equal to b_{iX} X_1 plus S_{i0}. You can solve for X_1 in here: plug this in, you have got this displacement, you have known this, you substitute these and you get your member end forces solved.

What I have tried to explain to you today is how to use the matrix force method to solve the statically indeterminate approach. The entire concept is built on the flexibility approach and whether it is statically determinate or indeterminate, the approach is identical excepting that in a statically determinate problem, you know all the capital Rs and you do not know the displacements, you know all the loads, you do not know the displacements.

For the statically indeterminate problem, you know some of the loads for which you do not know the displacements and you do not know some of the loads, which are the redundant forces, for which you know the displacements because those are the 0 compatibility displacements that you have – so that is the only difference.
Up till now, I have been looking at the flexibility approach. Next, I am going to be moving on to the stiffness approach of the method which is also the matrix displacement method although we never say matrix displacement method we call it the stiffness method. Thank you very much.