Structural Analysis-I1
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Lecture – 28

Good morning. We were talking about the matrix analysis approach, especially the flexibility-based approach right now.

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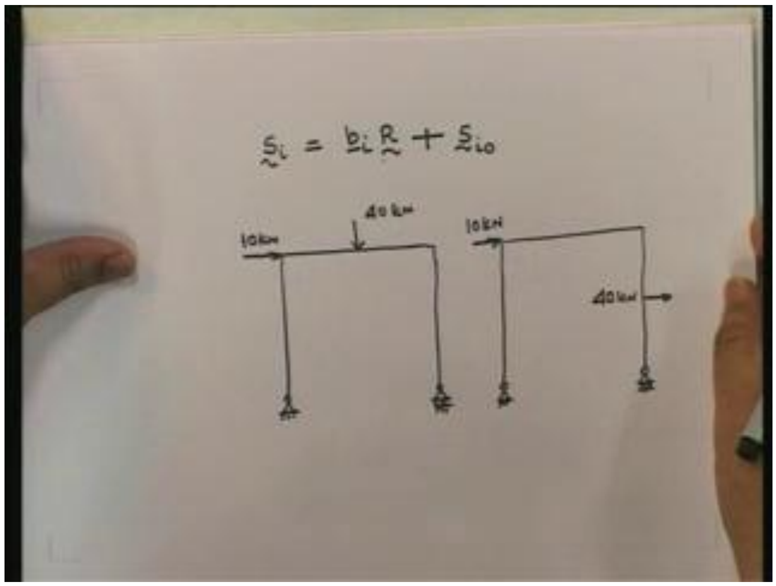

In the last lecture, if you remember, I talked about the fact that S_i was equal to b_i into R where R is the loading corresponding to the degrees of freedom, in which case it is essentially joints because degrees of freedom are defined as joints. In addition to that, you have, as I was saying, this (Refer Slide Time: 02:13) and we were discussing why we need this additional term which we did not need. We were looking at this, we had solved this problem and we did not have anything here but when I looked at this problem (Refer Slide Time: 03:20), I did not have an S_{io}; remember, we did not use any S_{io}, we just used S_i equal to b_i R and solved the problem. As soon as I came here, I said there is an S_{io}. What is the actual difference between the two? In this case, I do not have the S_{io} and in this case I have the S_{io}. Let us actually look at this. Let us forget this 10 Kilonewton because 10 Kilonewton is coming here (Refer Slide Time: 03:48) and anyway its superposition is valid and therefore you do not have a problem in putting it together.

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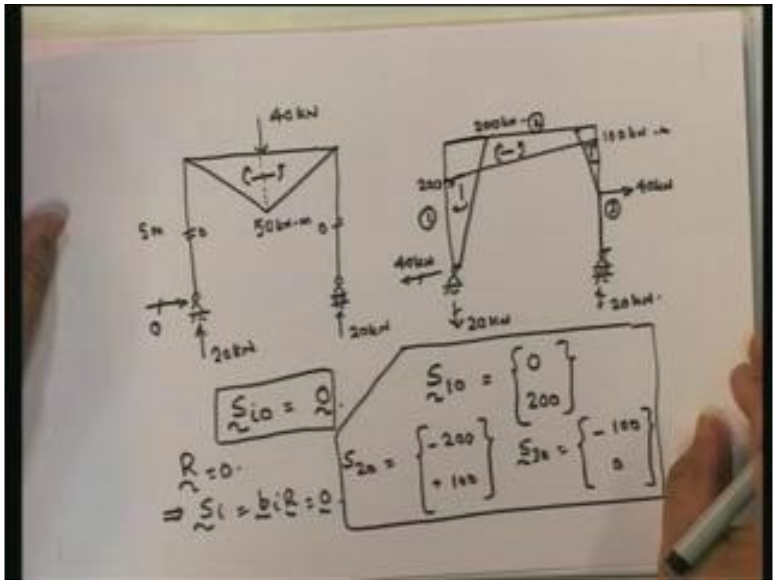

Let us just consider the 240 Kilonewton loads and see what happens. In this case, we do not have any S_{io}. Why? Let us solve it and you will see that you will get 20 Kilonewton, 20 Kilonewton, this is equal to 0; therefore if you look at the bending moment diagram, for these two, the bending moments are 0 and the bending moment over here turns out to be…(Refer Slide Time: 4:55). Now remember these were all 5 meters in length – every one of them was 5 meters. This is how the bending moment diagram looks.

If you look at it, what are the member end moments over here due to this load (Refer Slide Time: 05:17)? You will see that the member end moments at all points are 0. Therefore, actually the reason why we did not consider it was this was 0 for all the members. However, if you look at this, you have 40 Kilonewton load here (Refer Slide Time: 05:56); now, let us try to see what happens. You will see, first of all, that there is going to be 40 Kilonewton load here and due to this 100, you will have….

Let us draw the bending moment due to this – just this one alone and you will see that you get something like this: this value is 200 Kilonewton meter, the same value over here and then, this one reduces it; over here, it goes from 200 to 100 Kilonewton meter; this is in this fashion and ultimately, here also, this is going to be 100 and this is going to be in this fashion, so over here, 100; this is going to be 0, so it is going to be 0 from here to here (Refer Slide Time: 07:55) and from here to here, it is like this, this is going to be equal to 100.

This is the bending moment diagram due to this. Therefore, you will see that even if we do not have any R_i, you are going to have S_i, so if this is 1 (Refer Slide Time: 08:21), this is member 2, this is member 3. You will see that for S_{10}, it will look like 0, 200; for S_{20}, it is going to be equal to minus 200, plus 100, and for S_{30}, it is going to be minus 100, 0 – these are…. This is the point that I am trying to make: typically, you have bending moments; if you look at it in this particular case, R is equal to 0 and therefore, S_i into b_i R is equal to 0. However, this is not 0 because you have this and therefore, there has to be an additional term that takes care of this aspect; in this particular case, we did not have it because this happened to be 0.

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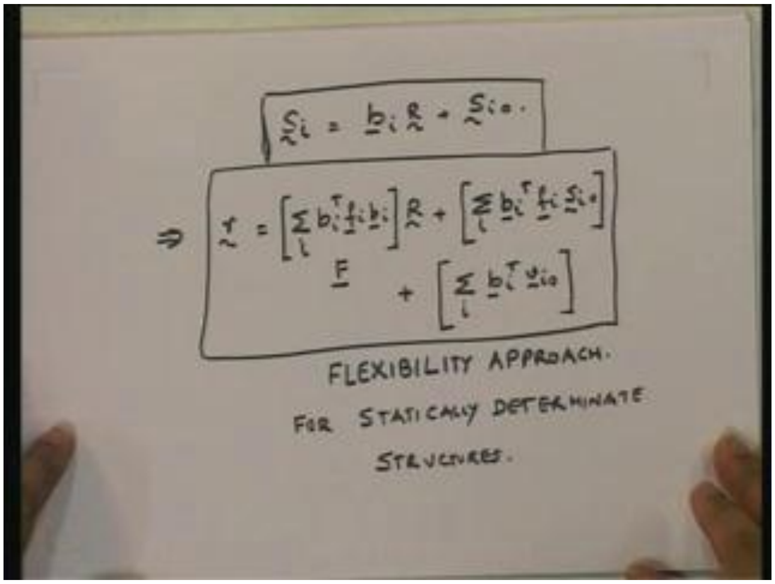

Once you put that in, we again come back to the fact that once S_i is equal to b_i R plus S_{i0}, this implies that r is equal to (summed up over all the members) b_i transpose f_i b_i into R – this part is the F, the structure flexibility matrix – plus (summed up over all the members) b_i f_i into S_{i0} plus (summed up over all the members) b_i transpose v_{i0}. Now, understand that this (Refer Slide Time: 11:02) is relating the joint loads to the joint displacements – this is directly given by FR. In addition to that, this aspect only happens if you have a member load that gives rise to bending moments at the member ends – so this is the effect of the member; and this (Refer Slide Time: 11:28) is the additional effect of the member rotations due to the member load. All these together account for and this is the total for the flexibility approach for statically determinate structures.

What you have in here is that we have set up how to get the member forces as well as the joint displacements using the flexibility approach, but this is true for a statically determinate structure because we said that you can actually solve for S_i, given the loads. Now, how can we extend this to include the effect of statically indeterminate?

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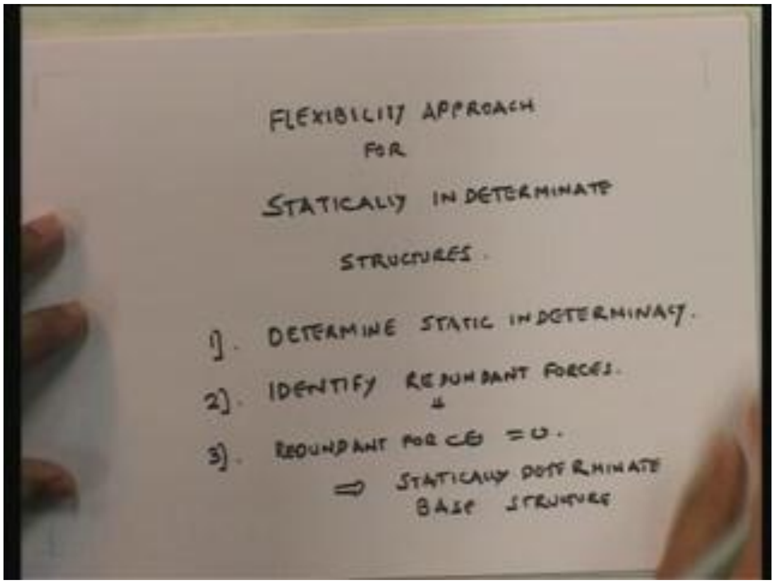

We are now using the flexibility approach for statically indeterminate structures. How do we apply this? In reality, it is actually very very simple; the reason behind it is that even in a statically indeterminate structure, first you determine static indeterminacy and then identify the redundant forces; then once you identify the redundant forces, redundant forces equal to 0 implies statically determinate base structure. Since you have that…. all that we have developed over here is the only additional thing. Let us see what happens.

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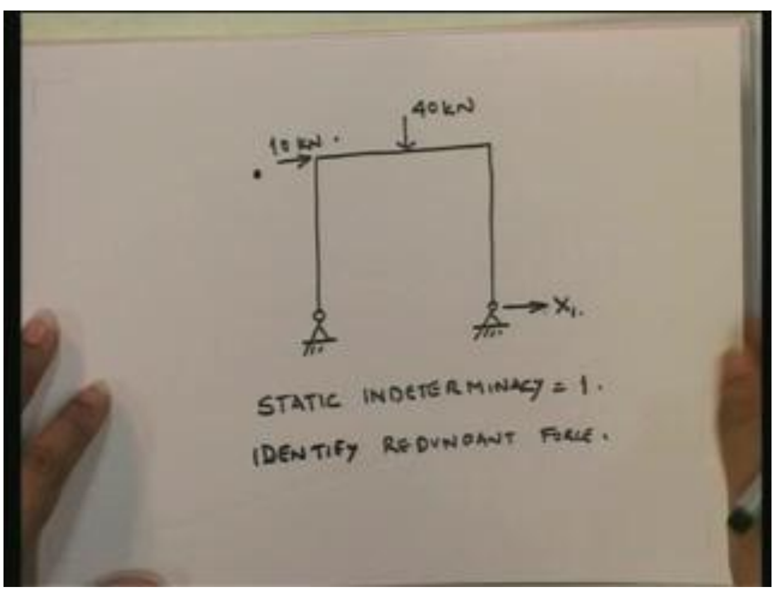

It is exactly the same problem excepting that it is statically indeterminate. What is the level of static indeterminacy? You will see that static indeterminacy is equal to 1, then identify redundant forces and once you identify the redundant force let me say that my redundant force is 1, I will identify this as my X_1. In other words, this is the force that I am making redundant, so as soon as I put that equal to 0, what is the structure that I get?

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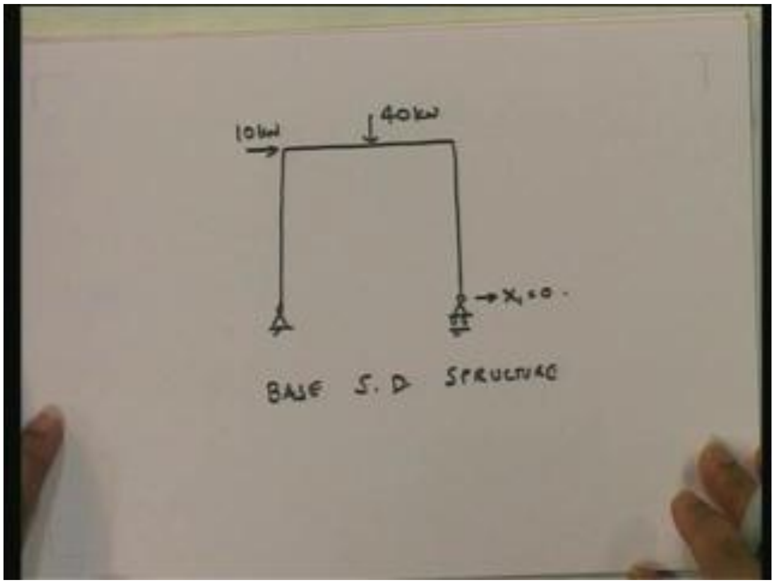

This is my base structure – this is my base statically determinate structure. If you look at this structure, this is identical to the structure that we have just solved in the last lecture, is it not? We have taken a statically indeterminate structure. What is the difference? In this particular case is, let us see what the difference is. In the previous case, what did we do? We took this load and we found out the displacement at this point (Refer Slide Time: 18:32) – we found out the horizontal displacement at this point and the horizontal displacement at this point. In this particular case, I am going to do exactly the same thing.

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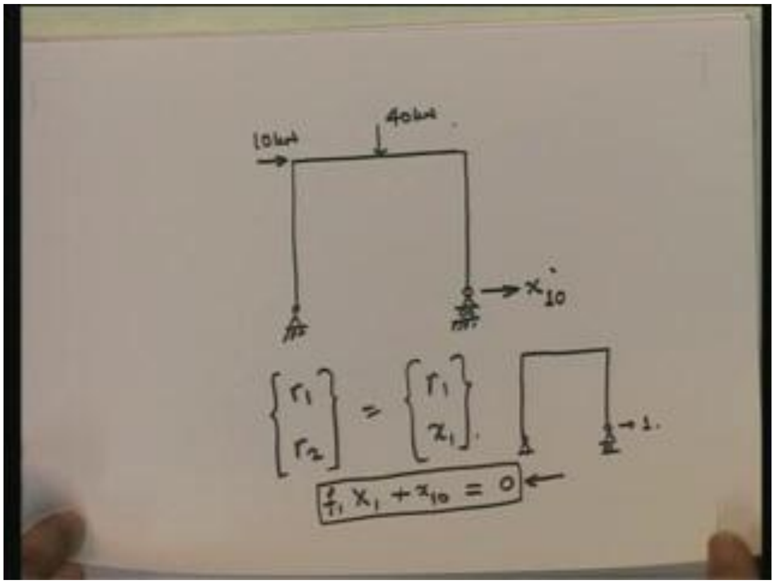

How would you solve this problem in the original situation? Due to these loads, you would find out this displacement (Refer Slide Time: 19:20). Remember I called it r_1 and r_2 this time; however, in this particular case, I will call this x_1. What is the reason behind it calling it x_1? Because in this particular case, remember that this is the degree of freedom associated with the redundant force and since it is the redundant force, I will give a different notation. I have just given it a different notation, but as far as I am concerned, it is still the same – r_1 and r_2 in the previous case is identical to what I am calling as r_1, x_1.

You would find out x_1 here, you would find out the displacement here – that is the step; I am just going through the steps; you would find out the displacement here. What would the next step be? The next step would be to apply a unit force over here (Refer Slide Time: 20:27) and find out the displacement at this point. Therefore, we can say that this would be x_{10} and I would say that f_1 X_1…. What is this (Refer Slide Time: 20:46)? This is the displacement at this point due to X_1 is equal to 1. Then, you would say f_1 into X_1 is the displacement at this particular point plus x_{10} is equal to 0, right? This is the compatibility condition that you would satisfy in the normal force method. In this particular case, understand that this problem once this is the case.

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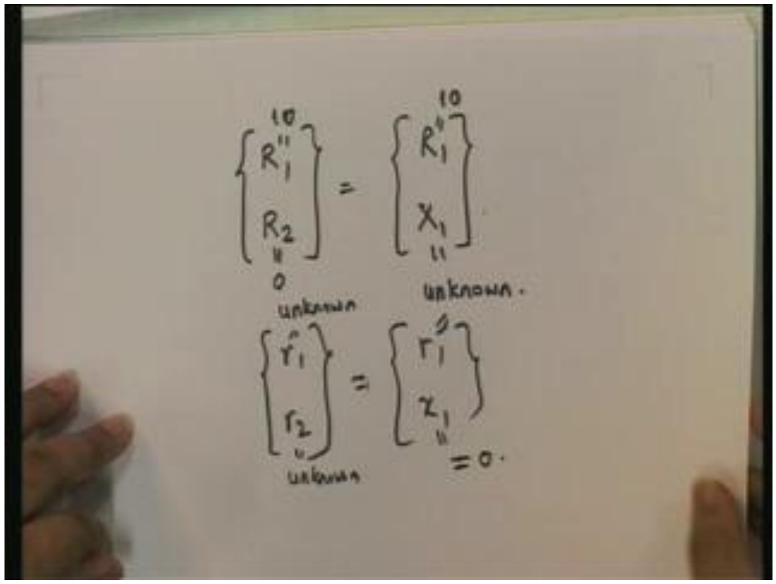

Then, note that in the original case we had R_1, R_2; in this case this becomes R_1 and R_2 is my redundant force. In the previous case, this was equal to 10 and this was equal to 0 – this is what we knew because it was a statically determinate structure. In this particular case, this would still be 10 but this is unknown but look at this: r_1, r_2 is equal to r_1, x_1; this is the statically determinate case in the previous case, this is the current case (Refer Slide Time: 22:15)**.** Here, this and this both were unknown; in this case, this is unknown and this is equal to 0 because I am considering the effect of x_1 and r_1, so this is 0. Now, I am not solving this particular problem because we have all ready found out this thing for this.

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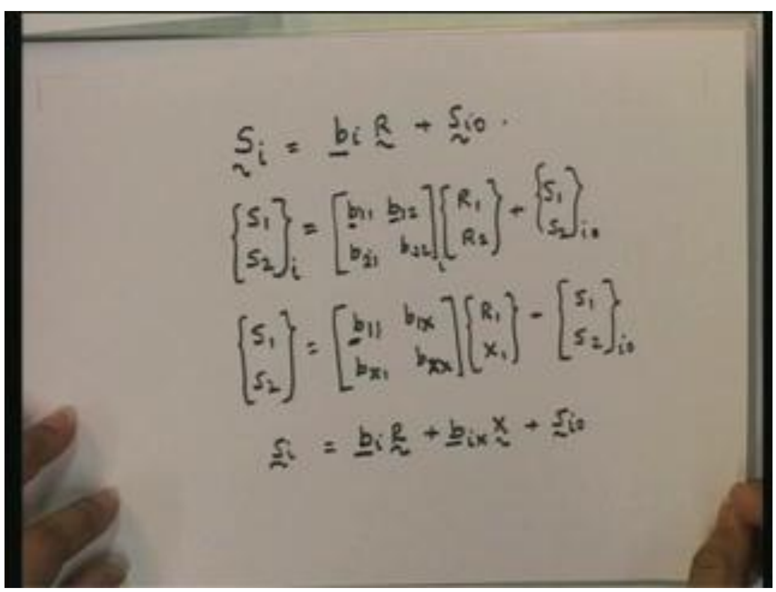

In the earlier case, for each member what would you do? Find out S_i is equal to b_i into R plus S_{i0}. Understand that once I accept that x_1 is a redundant load, it behaves exactly like an external load but the only problem with x_1 is that whereas for most of the situations we know the loads, here, x_1 is the load that we do not know. I can write this one in this fashion: I can write it as $(S_1, S_2)_i$ is equal to… in the previous case., this would turn out to be equal to b_{11} , b_{11}, no (Refer Slide Time: 24:00), b_{21} b_{22} into R_1, R_2 plus S_1, S_2, this is i0.

In this case, all that happened.… Remember we computed these (Refer Slide Time: 24:26). How did we compute this? We computed this by putting R_1 is equal to 1 and finding out what the member end moments were. We found this by putting R_2 is equal to 1 and finding out what the member end moments are. Therefore, in this particular case… here, understand that R_2 is x_1, so here all that would happen is (S_1, S_2) into b_{11}, now b_{1x}, b_{x1}, b_{xx} into (R_1, X_1) plus this still remains the same because this is nothing but the effect of the loads. No difference absolutely – these two are identical to these, this is equal to this, this is equal to this; the only thing is that whereas in the previous case since R_2 was an external load for statically determinate structure, R_2 was a determinate load, now we have exactly the same statically determinate structure except that x_1 is a redundant force and therefore unknown. If I go through the steps, I can say that this is equal to b_i R plus b_{ix} X plus S_{i0}. Do you see this? Since this is nothing but R and X together, I can actually write it in this fashion.

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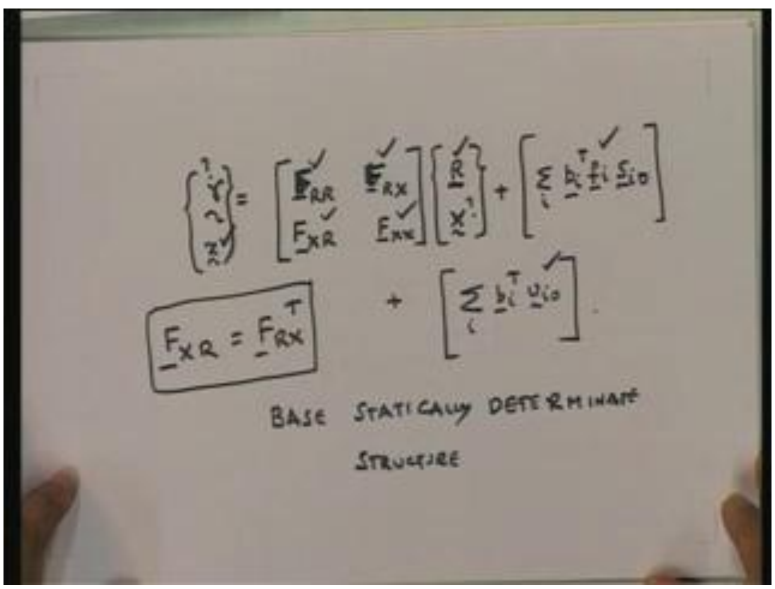

I am going to say that r is equal to F_{RR}, F_{RX}, F_{XR}, F_{XX} into (R vector, X vector) and then here this is x vector plus… I am going to put inside all the standard items, which is (b_i transpose f_i S_{i0}) plus sigma over i b_i transpose v_{i0} – note this equation that I am writing is for the base statically determinate structure. This is identical to what we had for the statically determinate structure excepting for a small problem and that is that in this particular case, we know these but we do not know these. These of course are known – all; we can find these out; since the flexibility matrix is symmetric, you will see that this is true. I do not know this (Refer Slide Time: 29:11), however, compatibility conditions give me that I know this, so here you will see this problem; I know this, I do not know this; so if I rewrite this you will see that this b_i will include both the effect of R as well as X.

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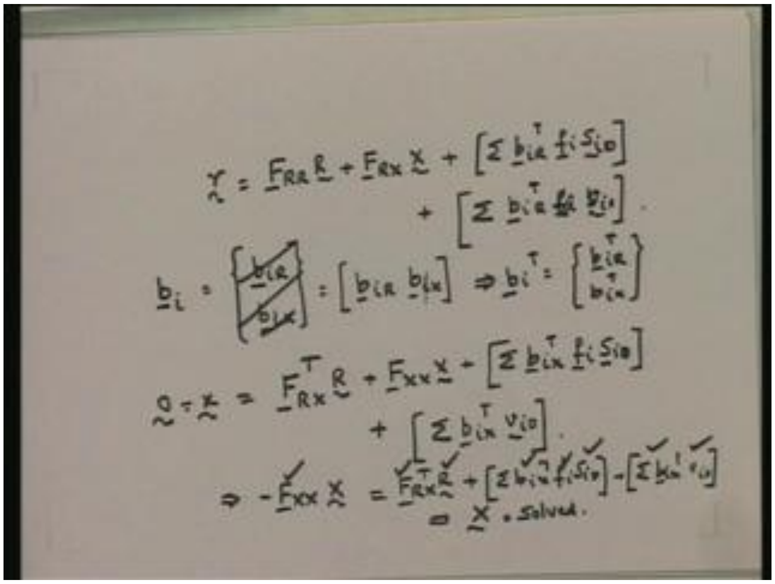

If you look at this particular thing, you will see that this can be written in this form. The top set of equations will become r is equal to F_{RR} R plus F_{RX} X plus summation… (now, these are just corresponding to the R) f_i – this is nothing but the structure levels, so this does not have anything to do… plus summed over all the length only due to the R, so this is the part of the matrix that belongs to only the Rs. In other words, b_i can actually be segregated into b_{iR} and b_{iX}. This should actually be [b_{iR} and b_{iX}]. This corresponds to the internal forces due to the external loads and (Refer Slide Time: 32:04) these are the internal forces due to a unit redundant. This is equal to b_i transpose f_1 v_{i0} and you will see that x is equal to F_{RX} transpose into R plus F_{XX} into X transpose plus sigma b_{iX} transpose.

When you do transpose you will see that this implies that b_i transpose is going to be equal to b_{iR} transpose, b_{ix} transpose and that is how it happens – b_{iX} transpose f_i S_{i0} plus b_{ix} transpose into v_{i0}. Note that typically, this is a 0, typically – this is the zero vector. Therefore, I can write this as minus F_{XX} X is equal to F_{RX} transpose R plus… plus….Note that I know this, I know this, I know this, I know this, I know this, I know this, I know this, I know this and I know this (Refer Slide Time: 34:20), so I can solve this for $X_{i.}$ Once I solve for X_i, I can substitute that in here and I can get the displacements. What about my S_i?

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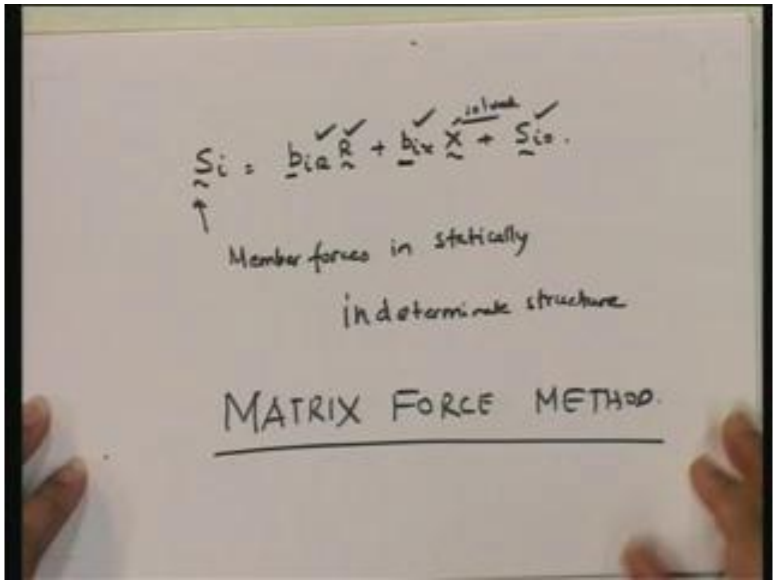

I have found out the displacements and my S_i is given by b_{iR} into R plus b_{ix} into X plus S_{i0}. Note this is known, this is known, this is known, this is known and this is solved (Refer Slide Time: 35:19). Therefore, I can find out my member forces in a statically indeterminate structure. This is known as the matrix force method. The beauty of the matrix approach is that the force method becomes a part of the overall flexibility approach – we are not deriving it separately. Therefore, the redundant force is just treated as another additional load, of course, which we do not know.

However, we know the displacement corresponding to that load so we can always solve for it. This is the major advantage that the compatibility equation that we get in the force method really is part of the entire flexibility method in itself. Once you have understood this, it is very easy to understand that there is absolutely no problem in using the matrix approach in the force method. What I am going to now do is essentially start off a problem; I am not going to be able to solve the entire problem – we will continue into the next lecture, but I want to tell you to look at a problem in its entirety and solve it.

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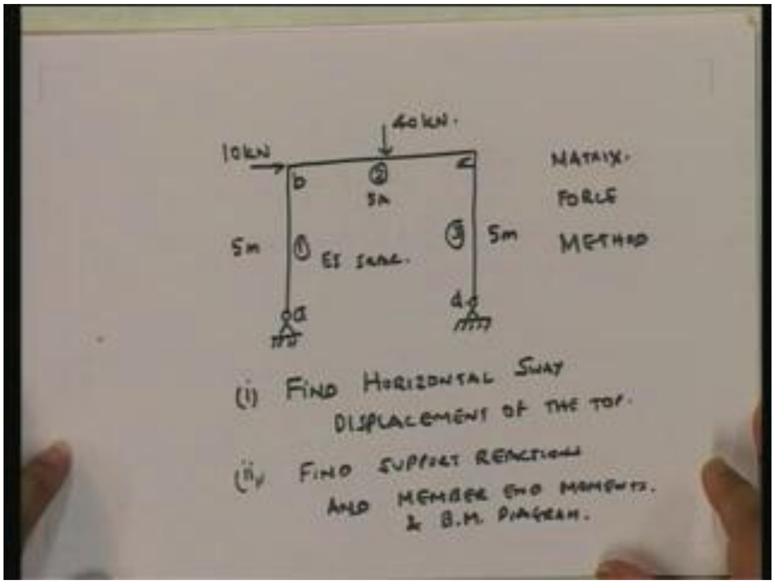

Up till now, I have just only briefly discussed this problem but now I am going to actually solve it; so let us take this problem and solve it in its entirety. This is my member 1, member 2, member 3 (Refer Slide Time: 38:11) and this is a, b, c and d; all the members have the same EI and all of them are 5 meters in length – this is at the center – all of them are the same EI. The problem is one: find horizontal sway displacement of the top and two: find support reactions and member end moments and bending moment diagram – this is the problem statement and we are going to be using the matrix force method, so we are going to be using the matrix force method.

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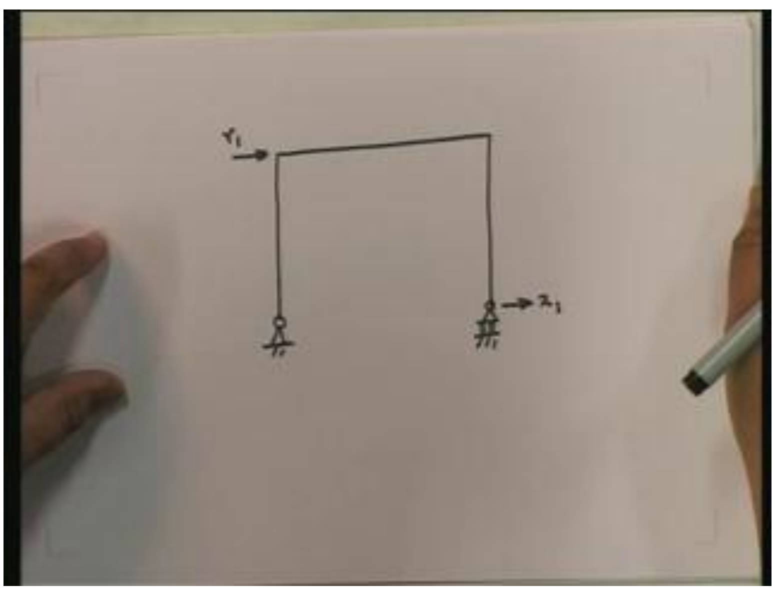

Already, we know that this is a single degree of freedom and we have identified the redundant force and therefore, I need to find out this displacement (Refer Slide Time: 40:45) and this displacement, these are the two displacements. Why do I need to find out this displacement? It is because this displacement is what I have been asked to find out. Why do I need to find out this displacement? It is because this is the displacement that is going to be equal to 0 in the original structure, and so now, we are finding this out. How are we going to find this out? We go through the steps. The steps are first find out v_{i0} and S_{i0} for the loading; remember v_{i0} and S_{i0} are only found out for member loads. Which is the only member load here?

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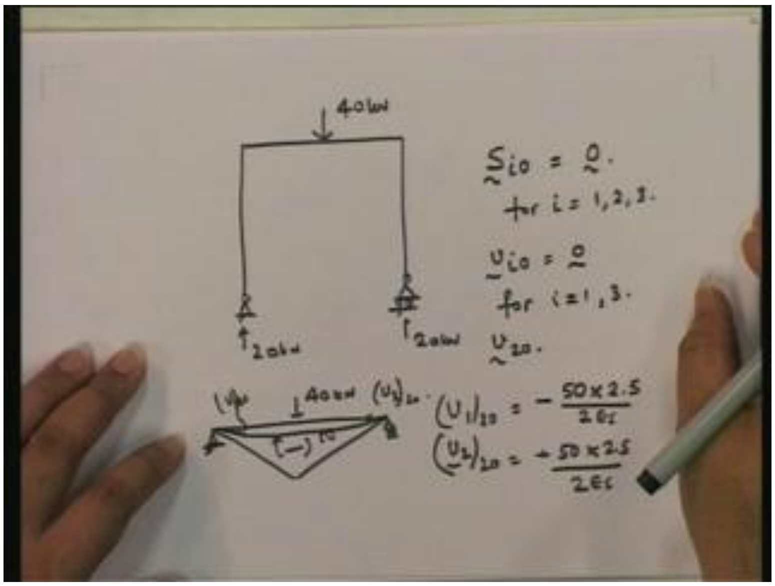

The only member load is this (Refer Slide Time: 41:51); so this gives rise to this and this so we know that S_{i0} is going to be equal to 0; we have already discussed this so all of them for i equal to 1, 2, 3. Now we have to find out v_{i0}; now, v_{i0} will only happen in the members that… so v_{i0} is equal to 0 for i equal to 1 and 3 and for v_{20}, we find out; this is the member given this load. Here, you have 50 and we have v; this displacement, this is going to be my $(v_1)_{20}$ and this is going to be $(v_2)_{20}$; so, $(v_1)_{20}$ and $(v_2)_{20}$ is going to be equal to minus 50 into 2.5 divided by 2 EI and this is going to be equal to plus 50 into 2.5 upon 2 EI (we have already found this out, I am just putting them down). We have found out $(v_1)_{20}$ and $(v_2)_{20}$. What are the next steps?

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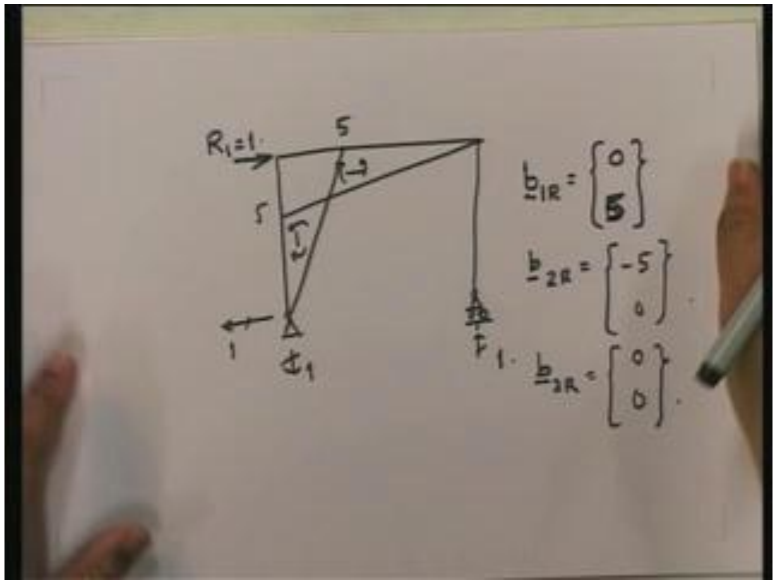

The next steps are to find out the b_is. If I put this, I get this equal to 1 and this is going to be this way, so 1, 1. Therefore, if you will look at it, you will get that this is equal to 5 and this is equal to 5, this one goes this way and this one goes this way (Refer Slide Time: 45:43) – we have already done this, so I am just putting down everything properly. Here, you will see that b_{1R} is equal to 0, 5 and b_{2R} is equal to minus 5, 0 and b_{3R} is 0, 0. There is nothing new, but I am just going through the steps again because I want to solve the entire problem.

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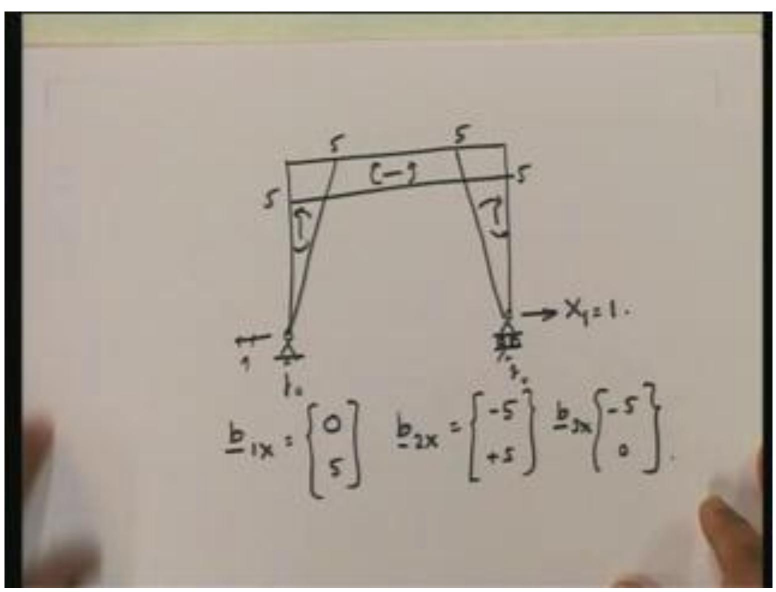

Remember that in the force method, we are doing everything to the base statically determinate structure; therefore, always I am actually solving for this. What is the next step? Put X_1 is equal to 1 and find out how the thing will look like. If you look at this, this is going to be like this, these are going to be 0, 0. What you have is… all these – I am just putting them down as if I have found it out, but if you cannot and if you do not know what I am talking about, I recommend that you go through and compute the bending moment yourselves. This is the bending moment for X_1 is equal to 1 and therefore b_{1X} is equal to 0, 5; b_{2X} is equal to minus 5, plus 5; and b_{3X} is equal to minus 5, 0. Till now, you have not seen anything that is different from what we have already done, so the only thing is that I have identified this as X and the other one as R. Let us go through the steps.

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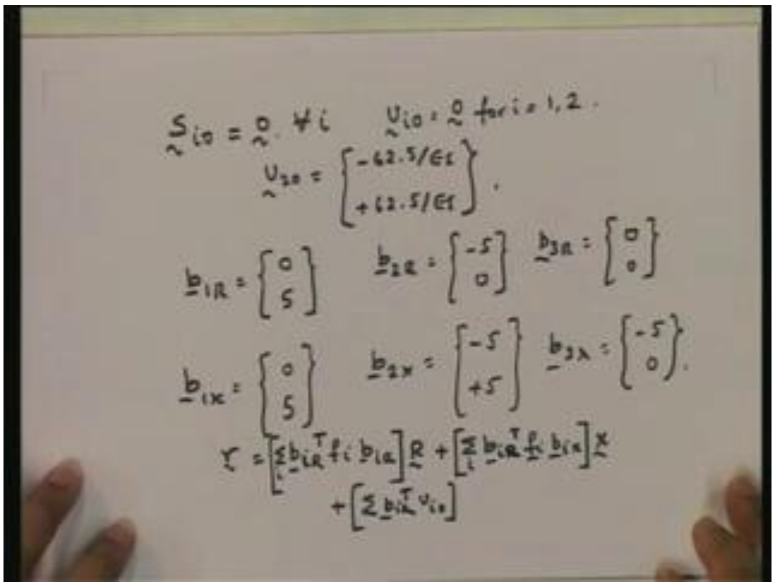

Therefore just putting down everything, I have got S_{i0} are all of them for all i; v_{i0} is equal to 0 for i equal to 1, 2 and v_{20} are equal to (minus 62.5 by EI, plus 62.5 by EI). I have got my S_{io}, I have got my v_{io}, now what I need is my b. b_{1R} is equal to 0, 5; b_{2R} is equal to minus 5, 0; b_{3R} is equal to 0, 0; b_{1X} is 0,5; b_{2X} is equal to minus 5, 5; and b_{3X} is equal to minus 5, 0. Now what? Now, you will see that r is equal to b_{1R}; I am going back to the equation that I had written down: remember that the equation that I had written down was that r was equal to (b_{iR} transpose f_i summed up over all the i b_{iR}) into R plus (b_{iR} transpose f_i b_{iX} into X) plus b_i transpose – in this case, S_{i0}, so all I am left with is b_i transpose, so this is going to be equal to b_{iR} v_{i0} and similarly, if I put X here, I will get b_{iX} f_i b_{iR} R plus b_{iX} f_i b_{iX} X plus b_{iX} v_{i0}; then, we put X is equal to 0 and we can solve for the problem. I am going to actually solve this problem in the next lecture, so I just wanted to introduce the entire concept and I hope you are beginning to appreciate all the things that I have been talking about for now.
Thank you very much.