Structural Analysis - II
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Lecture – 25

Good morning. We have been spending the last four lectures looking at moment distribution and I had promised that today, I would take up a problem that would include sway. Last time, we looked at a problem where the only load that was applied was the sway load and therefore, computation of the no-sway reaction was relatively simple. This time, we will take up a problem that will deal with the entire spectrum of issues that you have in using moment distribution for a frame problem that has sway degrees of freedom.

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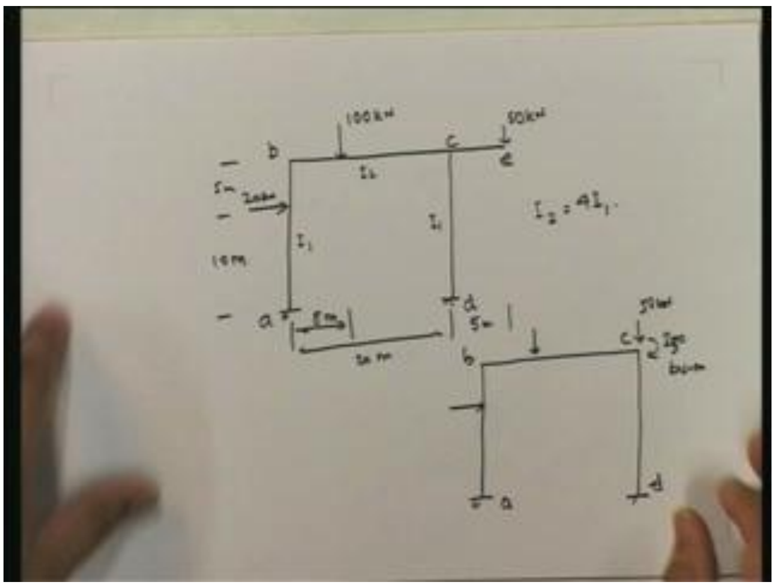

Let us take one of the old problems that we have all ready looked at using the displacement method. This was the problem that we had looked at long time ago and I am just going to redo this problem. The advantage of this is that if we use the moment distribution method…. We already know the answer to this problem when we have used the displacement method. Therefore, if the moment distribution method gives you the same result, it gives you confidence that the moment distribution method actually does solve the problem in a similar manner. This is your essential problem – e is the same, material is the same. This problem can actually be done in this fashion. If you remember, we had transferred that c load as a 50 Kilonewton load here (Refer Slide Time: 04:23). By the way, this was 5 meters. This is 250 Kilonewton meter and then you have all these other things. This is the problem that I am going to be actually looking at. In this case, what is the first step? Well, you know that there are three degrees of freedom, these are the two rotations and there is a sway degree of freedom.

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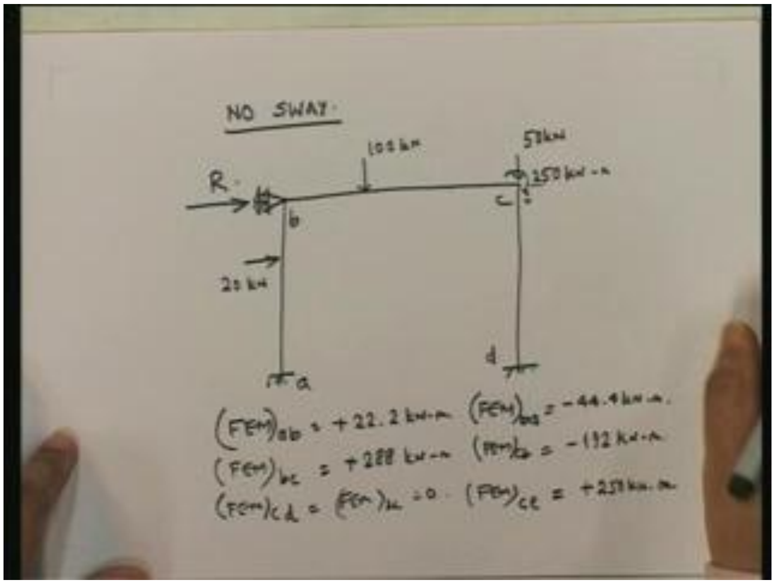

The first step is essentially to solve the problem – the no-sway problem. How does this problem look with no sway? You essentially have.… To stop the sway, I put a support here (Refer Slide Time: 05:40) so that this does not allow this and essentially, we solve the entire problem to find out this R. Now, first things first: we need to write down the fixed end moments – this we have already done earlier, so I am not going through the steps. We know that this is plus 22.2 Kilonewton meter, $(FEM)_{ba}$ is equal to minus 44.4 Kilonewton meter, then $(FEM)_{bc}$ is equal to plus 288 Kilonewton meter, $(FEM)_{cb}$ is equal to minus 192 Kilonewton meter, $(FEM)_{cd}$ is equal to $(FEM)_{dc}$ is equal to 0.

Another point here is you have to include the fact that $(FEM)_{ce}$ is equal to plus 250 – the reason for this is that although I have put the load here at this particular point, it is very important to understand that the total joint reaction includes… when you take the equilibrium of the joint, you will see that this 250 Kilonewton meter… note that I am putting this as plus 250, but this is actually a load applied here (Refer Slide Time: 08:00) – I can consider it as a moment applied here and so that would be the moment that I need to consider in the joint equilibrium; that is the reason why it is different from the other concept; the only thing is that ce will not take any this thing – it is not a member that takes any part in the stiffness.

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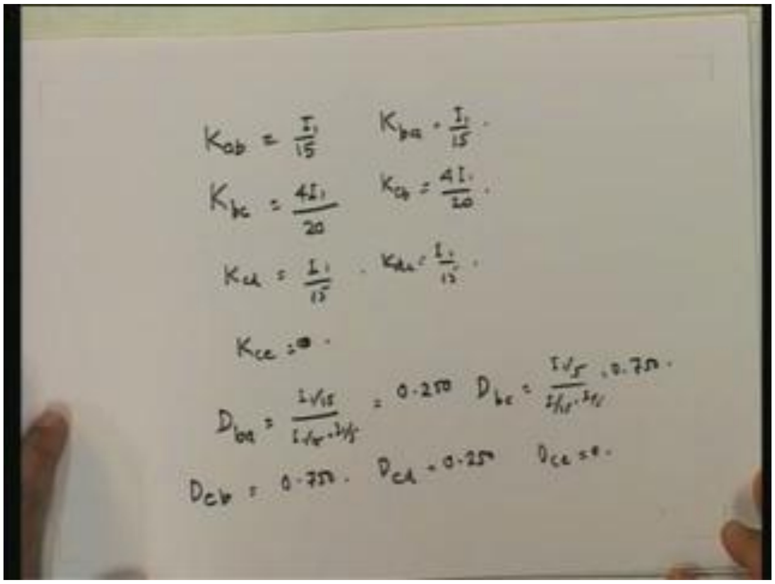

When we compute the relative stiffnesses, ab is a normal member, so K_{ab} is going to be I_1 upon 15; K_{ba} is equal to I_1 upon 15; if you look at bc, we saw that the moment of inertia of bc was 4 times I, so we have K_{bc} as 4 times I_1 upon the total length which is 20; K_{cb} is similarly 4 I_1 upon 20; K_{cd} is I_1 upon 15; K_{dc} is I_1 upon 15; note that K_c is equal to 0 because it is a cantilever. Therefore, this is included in the computation at c. There are two distributions that we have to do: distribution at b and distribution at c. Therefore, you can see D_{ba} is going to be 0.25, D_{ba} is I_1 upon 15 upon (I_1 upon 15 plus I_1 upon 5), so D_{ba} becomes 0.250 and D_{bc} becomes I_1 upon 5 upon (I_1 upon 15 plus I_1 upon 5), which is 0.750; at joint c, we have D_{cb} is equal to 0.750, D_{cd} is equal to 0.250 and D_{ce} is equal to 0 – these are the distribution factors.

Carryover factors… this is a normal problem, so carryover factor has half from each side excepting that when you have a carryover from a to b, it is 0 because a is the fixed end; carryover from d to c is 0 because d is a fixed end; from b to a, carryover is half; from b to c as well as from c to b, it is half, half and carryover from c to d is half – these are standard things, so once we have done that, we can now put together the no-sway problem. If you look at the no-sway problem…. Let me put it down in this fashion.

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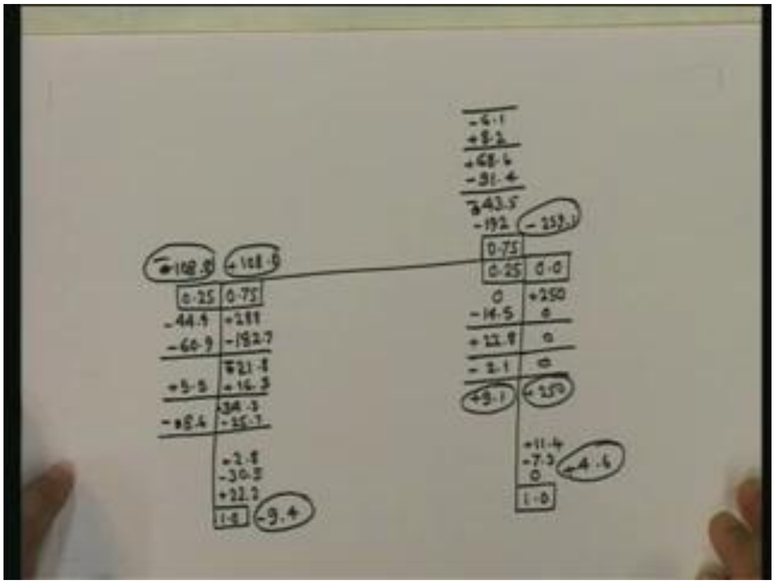

Again, being consistent over here, you will see that this is 1.0; this one over here is 1.0, then we have over here (this is for ba) – the distribution factor is 0.25; for bc, this is 0.75; again this one is for cb, so it is 0.75; this is for cd and ultimately, I am going to put this for ce, but this is going to be 0.0; the only thing is that here we have plus 250. What do we have here? We have plus 22.2, here we have minus 44.4, here we have plus 288, here we have minus 192, here we have 0 and here also, we have 0. I have written down all the fixed end moments and I have written down all the distributed factors. Now, what I am going to do is I am going to do this in this fashion: I am going to do the distribution at both joints b and c.

If you look at this, there is a plus 243.62, 243.6 of, there is a plus 243.6, so we have to do a negative 243.6 and distribute it – that is going to become equal to 182.7 and this is going to be minus 60.9; this distribution is over, so done here (Refer Slide Time: 14:25). Do the distribution over here: we get 250 minus 192 – that is going to be 58 and 58 has to be distributed. If we distribute 58 in 3 by 4, we get minus 43.5 (the this thing is plus 58, so minus 58 has to be distributed) and this is going to be minus 14.5 and this is 0 – distribution done, put it here. Now, the carryover from here to here is going to be minus 91.4, carryover from here to here is going to be minus 30.5, carryover from here to here is going to be minus 21.8, minus minus 21.8, carryover from here to here is going to be minus 7.3 – you have done all the carryovers.

Now again, we do the distribution and so I am not going to spend too much time on this – I am just going to put down the values and you know what the values are going to be. This is done, so I can put a…; then I do this over here; this is going to be minus 91 distributed, it is going to be plus 68.6 and plus 22.8; here, we will have plus 11.4; this is distributed (Refer Slide Time: 16:26); here, we will have plus 34.3; there is a carryover here, which is going to be plus 8.2; we have all ready done the carryover here; the final carryover of this to here is going to be plus 2.8. Next, we again do this, it is going to be minus 8.6, then minus 25.7 close this; this one needs to be done, so this is going to be minus 6.1 – closed; this is also closed; this is 0; this is going to be minus 2.1, close, 0.

You can keep doing this; I am not going to go any further – you know the steps. I am going to write down the ultimate value that you are going to have at this point. At this point, you are going to have minus 108.8 at this end (Refer Slide Time: 18:04), here you are going to have plus 108.8, here you are going to have minus 9.4, here you are going to have minus 259.1, here you are going to have plus 9.1, here you are going to have plus 250 and here, you are going to have plus 4.6 – these are the final moments that you get. When do you stop? You stop when your distribution gets to be less than 1 percent of the smallest value. Is the smallest value over here? The highest value is 288, so 1 percent would be about 2.8 and as soon as your carryover becomes less than 2.8, you stop the entire process; the last process in a moment distribution should always be the distribution – it is just that you do not do carry over, which is less than 2.8 here in this particular case. Once we have these moments, what is the next step? The next step is you put it down on a figure.

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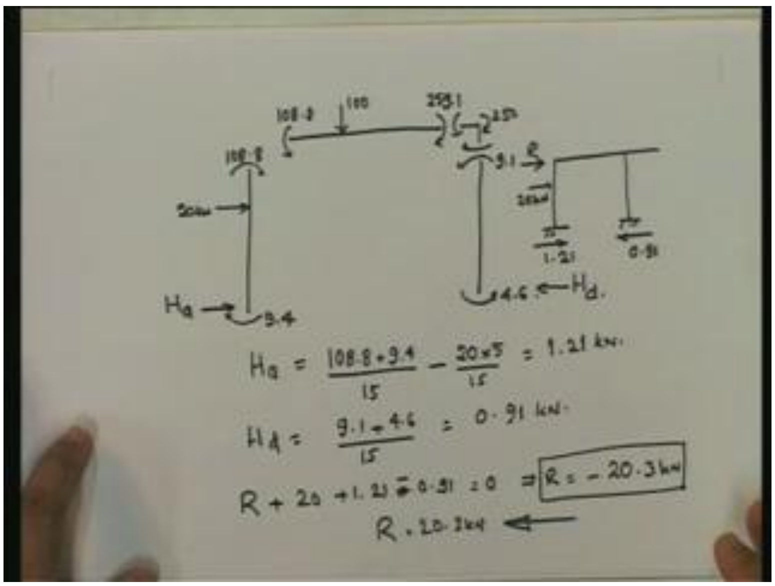

What you have is…. For each one, you know the member end moment; the member end moment – minus means it is going to be this way, so it is going to be 9.4; on this side it is going to be clockwise so it is going to be 108.8 – these are Kilonewton meter, I am not writing them down – and here, you have 20 Kilonewton meter. Understand that for this, I can find out the force over here. If I look at this particular force, what do I get over here? If you take this moment and you subtract this aspect of it, you are going to get that this force... this is this way, so this will be this way, so it is going to be in this direction and this is my horizontal at a.

Similarly, I need to find out at d. You put it all together – this is going to be 100, this is going to have 108.8 here, here this is going to be 259.1, here this is going to be plus 9.1, this is going to be plus 4.6 and understand that here, you have this and this and here you have a net moment of 250 so that they balance each other out – here, these balance each other out automatically. All we are interested in is actually finding out how much…. This is going to be H at d. Let us calculate what H upon a H_a is going to be.

H_a is going to have two components: 1 component coming from the moment, that is going to be equal to… that is this way, it will be this way, so it is positive, so it will be (108.8 plus 9.4) divided by 15 and then you have the other load in the other direction and that is equal to 20 into 5 by 15. If you look at this, this turns out to be equal to 1.21 Kilonewton. Let us look at H_d: this and we are going to have this, so H_d is going to be equal to (9.1 plus 4.6) upon 15, so this is going to be equal to 0.91. Understand that if I look at this entire structure, H_a, H_d; H_a is equal to 1.21 and H_d is equal to 0.91, we have a reaction R here (Refer Slide Time: 24:00) and you have a load of 20 Kilonewton here. If you look at that R, this is going to be R plus 20 plus 1.21 minus 0.91 is equal to 0. This implies R is equal to minus 20.3 Kilonewton. In other words, the loading, the reaction is in this way.

If you look at that, what does that essentially mean? That means that actually the structure wants to sway in this direction but you are restraining it and therefore you generate a 20.3 Kilonewton force so R is equal to 20.3 Kilonewton force in this direction – that is something that we have done. Remember I had said that if R was 0, you would know that there is no sway, but here you have R in this direction, which essentially means that the structure is trying to move in this direction – in the sway case, you have to consider the structure to be moving in this direction, so let us see what happens.

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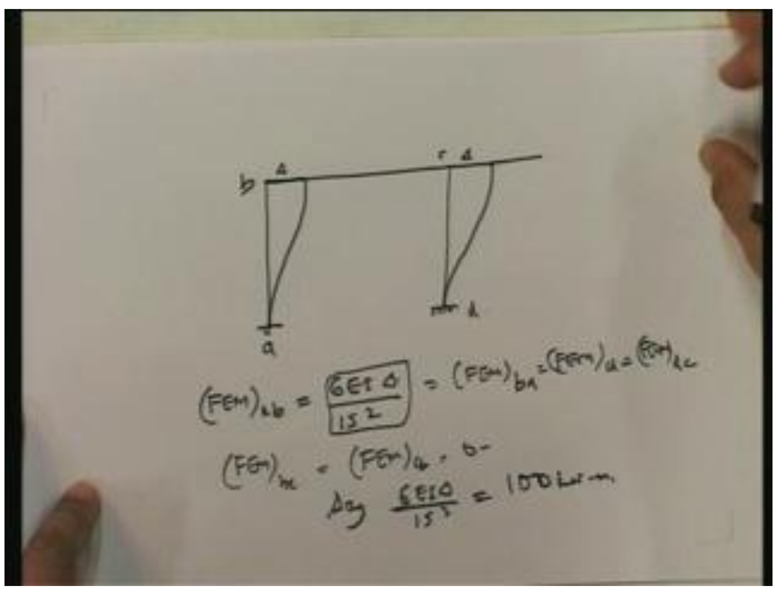

This is swaying in this direction, so let us make it sway in that direction. You will see that this joint will go here (Refer Slide Time: 25:49), this joint will go here and all you have is this; the other part remains straight because it is a cantilever. If you look at this, if we say this is delta and this is delta, what do we get? We get that the fixed end moment…. This is a, b, c, d. $(FEM)_{ab}$ is equal to plus 6 EI delta upon 15 squared, which is equal to $(FEM)_{ba}$ and you will see that is equal to $(FEM)_{cd}$ also, which is equal to $(FEM)_{dc}$. All of them have this. $(FEM)_{bc}$ is equal to $(FEM)_{cd}$ is equal to 0.

Having done that, we can now say that that the fixed end moment… this (Refer Slide Time: 27:06) – let us say that this is equal to 100; if we say this is equal to 100, then we can say that 6 EI delta upon 15 squared is equal to 100 Kilonewton meter – we are assuming it equal to 100. We will find out what it actually is a little bit later, because we do not know what the delta is; therefore we will find out the actual value of this a little bit later.

That is why I am saying that when you look at the sway problem, what do you actually do? You put it equal to 100 so any one of them to be 100 and you can compute all other fixed end moments automatically. In this particular case, you will see that since all the moments are equal to 100, we will get that all fixed end moments are equal to 100 – that is incidental in this particular case. The next step is to find out your…. The distribution factors remain the same –the distribution factors are what they are originally, so I am going to just put them down.

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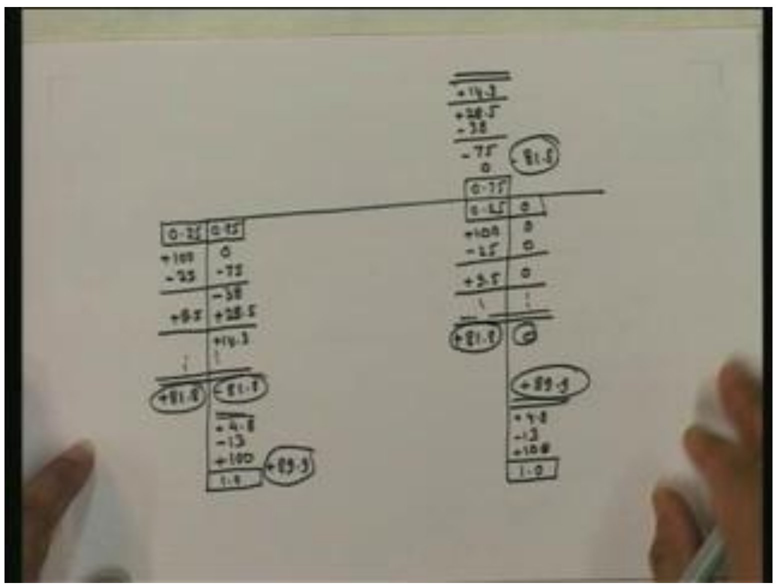

Here, 1.0, 1.0, 0.25, 0.75, 0.75, 0.25, 0. Here, what is it? (Refer Slide Time: 29:24) $(FEM)_{ab}$ is plus 100 plus 100, 0, 0, plus 100, 0, plus 100 – this our starting. Ultimately, if you look at it, the first step, this is minus, so you will have minus 25, minus 75; here you will have minus 75, minus 25 that is distributed. Next is 0 of course; next, minus 38, minus 38 – this is the carryover from here to here; from here to here, minus 13; from here to here, minus 13; so we are done with the carryover.

Again, we just have this, so this is going to be one-fourth, so this is going to be plus 9.5 and this is going to be plus 28.5; this is going to be again plus 28.5 and plus 9.5, 0; carryover is plus 4.8, plus 4.8; and here carryover is going to be plus 14.3 plus 14.3. Keep doing this – what is going to happen? Ultimately, we are going to be doing the same process. This one needs to be distributed. I am just assuming that you will be doing all the steps, so you continue with the steps – it has not ended here; you are going to have continuous steps here and these are going to be 0.

Ultimately, I am putting down the final value. The final value over here is going to be plus 81.8, this is going to be minus 81.8; this is going to be equal to plus 89.9, this is going to be minus 81.8. Note that it is anti-symmetric, so you are going to have exactly the same values coming through. This is going to be plus 89.9 and this is going to be 0. We have done the moment distribution – I have not gone through all the steps. Again, where do you stop? You stop when the carryover is less than 1 percent of the largest original fixed end moment. The largest original fixed end moment was 100 and therefore, as soon as the carryover becomes less than 1, you stop the process. These are the final, I am just putting down the final ones that we have. If you look at it, what does this mean? Having gotten these, we need to find out what is the moment that we get over here.

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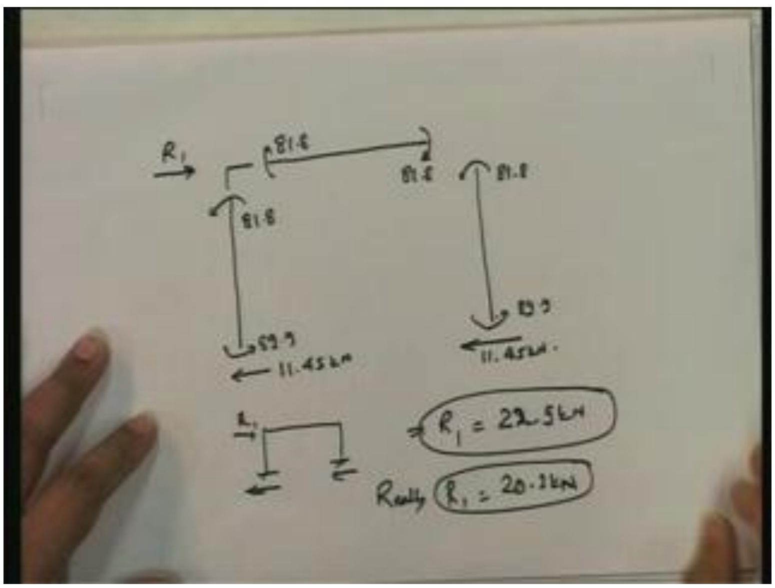

We are essentially saying what kind of R_1; here, this is like a load R_1 which would do give us this. This R_1 we need to find out. How do we find out that R_1? We actually put down the values that we get here. This is going to be equal to plus 89.9, 81.8, here this is going to be 81.8, this is 81.8, 81.8, 89.9. If we put this in, this one turns out to be equal to 89.9 plus 81.8 divided by 15, which turns out be 11.45.

Similarly, here we get 11.45. If you look at the entire structure which is this, we know the reaction here and here and this is R_1, you will see that R_1 is equal to 22.9. The main thing is if R_1 was equal to 22.9, then the moments over here that you would have was equal to 0, but understand that we got our reaction at that particular point to be equal to 20.3; so essentially, really, R_1 is equal to 20.3, because the sum of the two together is the actual load that you have at that particular point, which is equal to 0.

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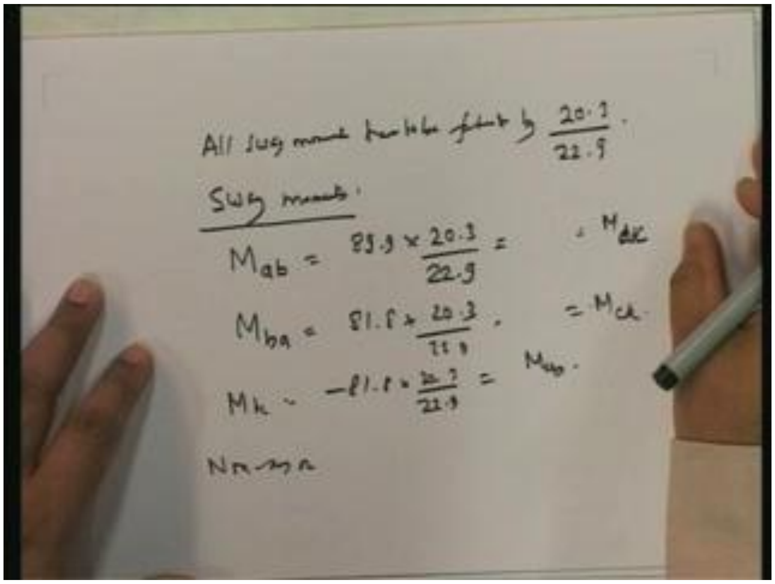

R_1 should really be 22 – that means all the sway moments have to be factored by 20 by 3 upon 22.9. Therefore, if you look at the sway moments, they turn out to be equal to…. M_{ab} is actually given as 89.9, but in reality, it is this (Refer Slide Time: 36:43); M_{ba} is equal to 81.8 into 20.3 upon 22.9; M_{bc} is equal to minus 81.8 into 20.3 upon 22.9, which is also equal to M_{cb}; you will see M_{ab} is equal to M_{dc} and M_{ba} is equal to M_{cd}. When you add these with the non-sway moments, what do the final moments turn out to be equal to?

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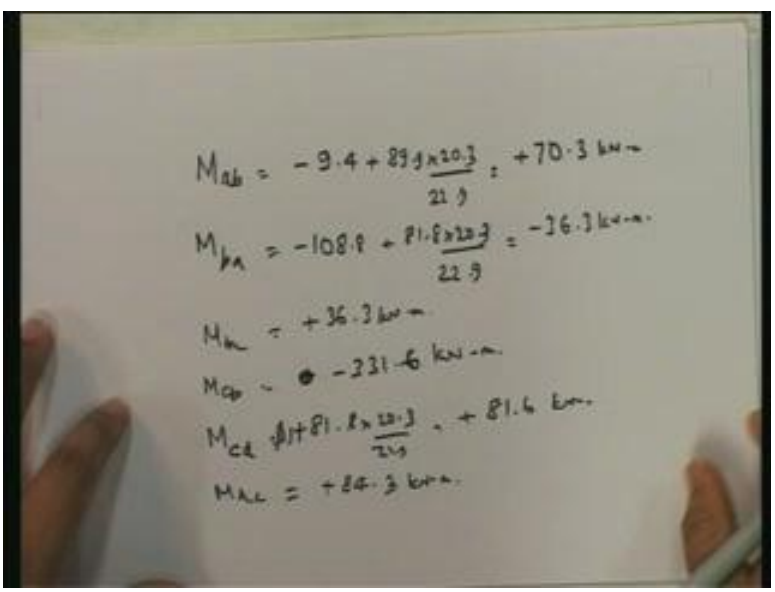

You will see that you will get M_{ab}…. When you include both the sway and non-sway, the non-sway was equal to… minus 9.4 plus 89.9 into 20.3 upon 22.9, which is equal to plus 70.3

Kilonewton meter (Refer Slide Time: 38:04). M_{ba} is going to be equal to minus 108.8 plus 81.8 into 20.3 upon 22.9 – this is equal to minus 36.3 Kilonewton meter. Similarly, M_{bc} is equal to plus 36.3 Kilonewton meter. If you add them up, you get M_{cb} equal to minus 331.6 Kilonewton meter (259 and plus). M_{cd} is equal to 9.1 plus 81.8 into 20.3 upon 22.9 – that is equal to plus 81.6 Kilonewton meter. M_{dc} is equal to plus 84.3. Once we have these, then my final member end moments turn out to be….

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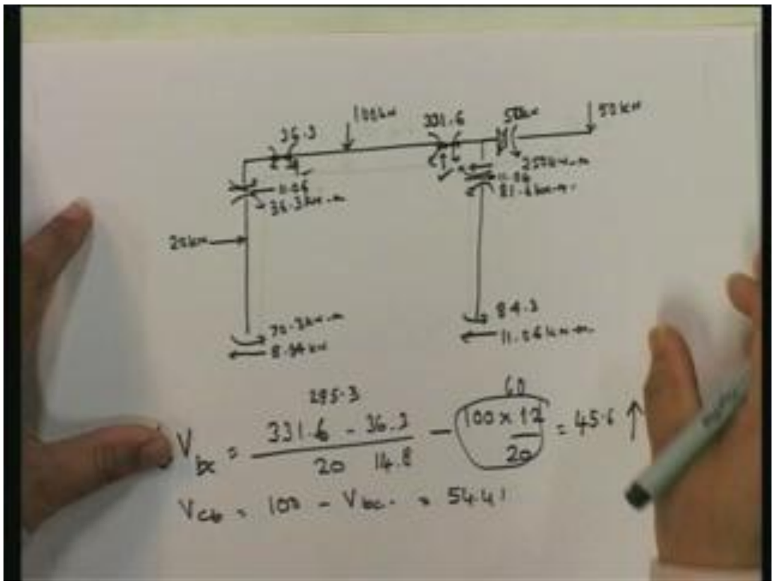

This is the final actual structure that I am going to put down. This is 50 Kilonewton, here we have 250 Kilonewton meters and 50 Kilonewton shear, so you will get just the opposite on that side. This one is plus 70.3 Kilonewton meter; this is going to be minus so it is 36.3 Kilonewton meter; I have this load here which is 20 Kilonewton meter; if I look at this and add the two up I get 8.94 Kilonewton; on this one, I get plus 84.3 and I also get this to be 81.6 Kilonewton meter; if you really look at it 160 divided by 15, you get 165.9 divided by 16 you will get it equal to 11.06 Kilonewton meter and over here, you will get this equal to 11.06 here, you will get 11.06. Let us just go through the steps. This will be like this (Refer Slide Time: 42:26) and the moment will be like this. What do we have on this one? You will see that I have a 100 Kilonewton force acting over here. What are the moments? M_{bc} is plus 36.3. Here, this is going to be this way minus 331.6. I have got the moments down and let us go through the procedure. Here, this one is going to be acting in this direction, this one is going to be acting in this direction; over here, you have this and you will have this; over here, you will have this; over here, you have this and you will have this. Over here, you will have this, so over here… (Refer Slide Time: 43:42); this is the opposite. If you look at it, this plus this 11.6 will give you net F_x equal to 0 which stands to reason.

If you look at this one, this is going to be this way, this is going to this way – just check it out; this is 331, this is 250 and 81.6; 250 plus 81.6 is 331.6, which is balanced – so there is moment balance, there is F_x balance, now all we have to do is do F_y balance. In this particular one, you have a net moment in this direction, so this is going to give rise to a force in this direction, so that is going to be equal to (331.6 minus 36.3) divided by 20 – this is going to be a force in this direction.

If I am taking force in this direction, this is this way plus I have minus (100 into 12) upon 20 – this is my net moment. If you look at this, the net force over here is going to be in this direction and it is going to be equal to… compute it. This is going to be this (Refer Slide Time: 45:27), now you can get this one, this one is going to be equal to your… this is your V_{bc} and your V_{cb} is going to be equal to 100 minus V_{bc}; you can compute this, you can compute this and you will see that they all balance each other out.

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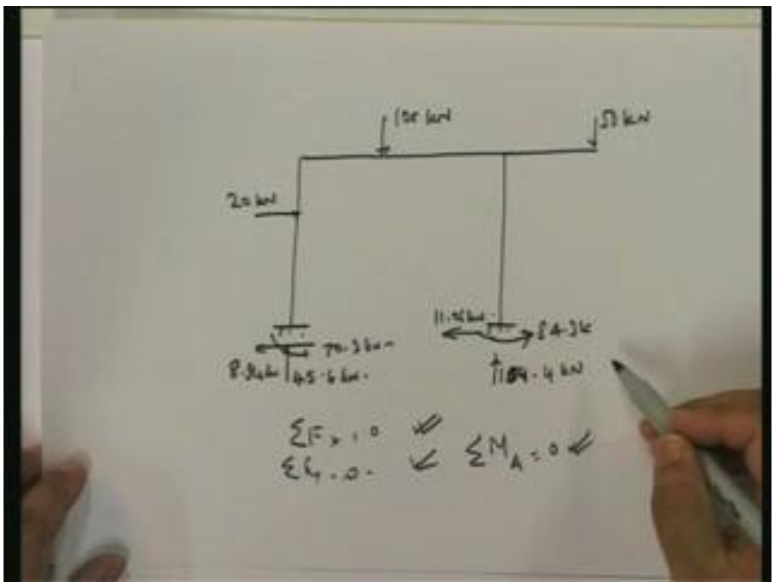

Ultimately, if I were to look at the support reactions, this is going to be equal to 60 and this one if you look at, it is going to be equal to…. let me do this over here. This is going to be equal to 4.7, so this is going to be 295.3 upon 20, this is 60, so this is going to be equal to 60 upon… here, you have 14.8, so 14.8, 60 is going to be equal to upwards 45.6; this is going to be upwards 54.4. Over here, what do you have? The support reaction is 45.6 Kilonewtons, reaction: here, it is going to be 8.94 Kilonewtons, moment is going to be equal to 70.3 Kilonewtons; over here, the moment is going to be 84.3 Kilonewtons, this horizontal force is going to be 11.06 Kilonewtons and the vertical force is going to be equal to 104.4 Kilonewton.

This balances everything out. Ultimately, if you look at the total sigma F_x equal to 0, you will see 80.94 plus 11.06 minus 20 – it is checked; sigma F_y is equal to 0 – 45.6 plus 104.4 is going to be 150 minus 100 minus 50 is 0 – check; take moment about A and you will see that you will get it equal to 0 – check. This in essence finally tells you that you have a situation in which you do have to solve a no-sway moment distribution; by solving the moment no-sway moment distribution, you get the no-sway moments; from the no-sway moments you compute the horizontal reactions and you compute the total reaction required to ensure that there is no sway. Then, the next step is to assume that there is sway and compute the load required to give you the sway that you get; now that sway, that reaction, the total sum of the two reactions has to be equal to 0 and because of that when you put it all together, you finally get both; you add the sway and non-sway moments.

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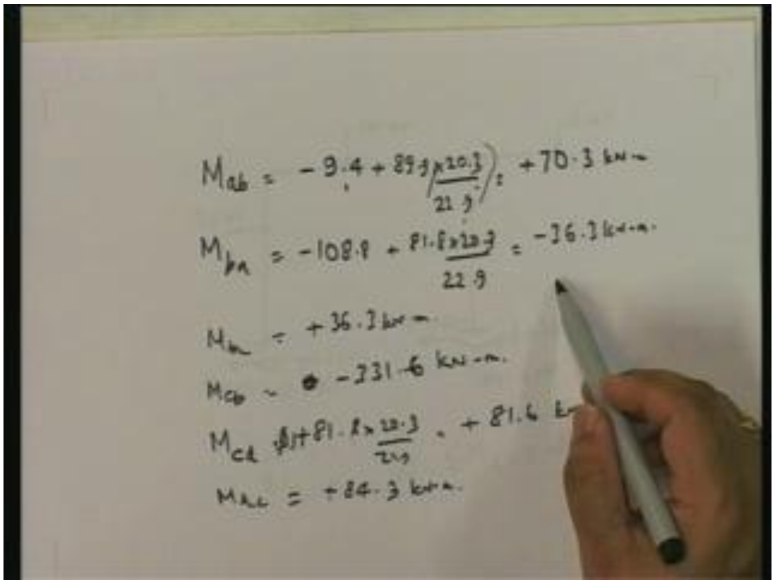

These are the non-sway moments (Refer Slide Time: 49:39), these are the sway moments corrected for the actual values. Then, once you get that, you have got the actual moments due to both the sway and actual structure and ultimately if you look at this solution that I have given to you, it is identical to the one that we computed using the displacement method; so essentially, the moment distribution method, if applied using sway and non-sway distributions together, you will get the final solution. Now, let me give a particular problem; I will give you a specific problem and also give you the answers so that you can actually compute what you are going to have.

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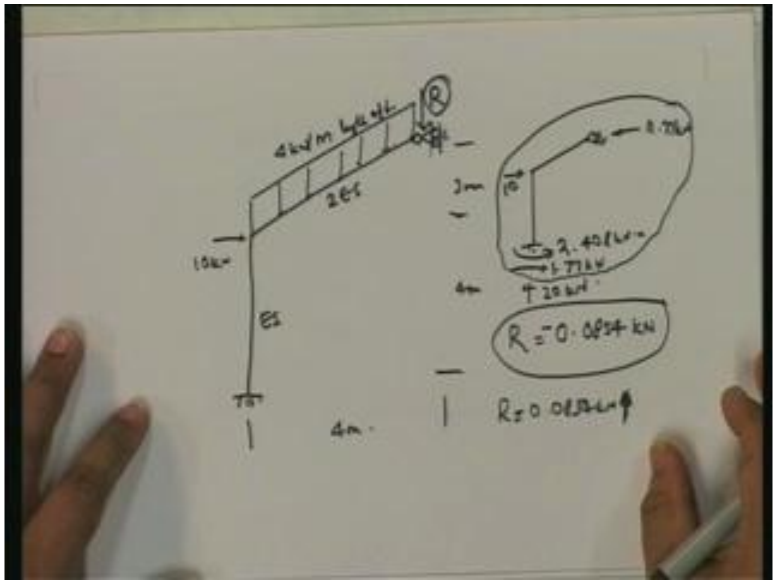

This is an old problem that we have looked at. It is just a little bit of a different loading that I am putting on it. Here, I have 10 Kilonewton and I have a UDL; this UDL is 4 Kilonewton per meter length of bar. Here, this is 2EI, this is EI, this is 3 meters up to here (Refer Slide Time: 51:25). Here, let me tell you that what you have to do is restrain this so that there is no sway and this is the R that you consider for the no-sway. In other words, for the no-sway you make this into a hinge.

I will give you the final answers to this particular problem.

When you solve it using the no-sway case, you get R equal to 0.0854 Kilonewtons and it is minus. In other words, it is actually R equal to 0.0854 Kilonewton going up. Therefore, when you do the no-sway case, I am going to give you the final answers that I have and find out if you can solve it. If you can solve it, you know that you have done it correctly. You get 2.408 Kilonewton meter here (Refer Slide Time: 52:59), you get 1.77 Kilonewton here, 20 Kilonewton here, 11.77 Kilonewton here, the load is 10 Kilonewtons here and that is it – that is the solution that I want you to get. Is that clear? I hope you will be able to solve this particular problem. That brings me to the end of the moment distribution method as applied to beams and frames – beams and planar frames.

Thank you very much. From next time onwards, I am going to be starting off on mid-fix methods.