Structural Analysis - II
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Lecture – 24

We have been discussing the moment distribution method over the last few lectures and today, we are going to be considering the moment distribution of a frame with sway included in it. I will explain this. I have given you the background behind this in an earlier lecture and let me illustrate what I meant by actually solving a particular problem. I will start off with a simple problem and then, we will go on to a more difficult problem in the next lecture, so that you understand the basis for the entire procedure.

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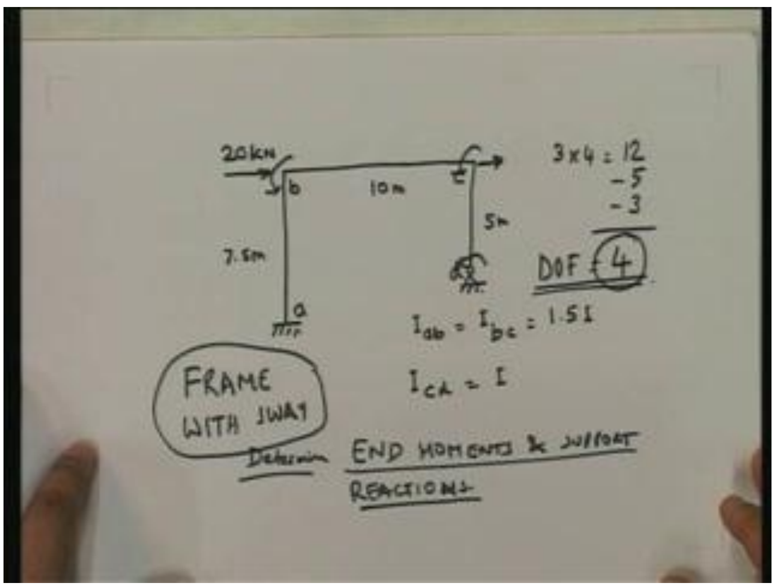

Here the question is to determine the end moments and support reactions – this is the problem statement for you. Let us start the procedure. The first procedure is…. Again, to determine…. I just say this is a frame with a sway, but you have to convince yourself, so you actually need to find out all the degrees of freedom. How many degrees of freedom? One, two, three, four, so 3 into 4, 12 unconstrained degrees of freedom; 3 plus 2, 5 restraints; one, two, three, 3 constraints; degrees of freedom – 12 minus 5 minus 4 is 4 degrees of freedom, so degrees of freedom is equal to 4. What are the four degrees of freedom? It is 1, 2, 3, and 4. So note that in this particular case there are three rotational degrees of freedom and one translation, therefore this is a frame with sway. Let us start. What was the first procedure? The first procedure was to actually restrain the sway.

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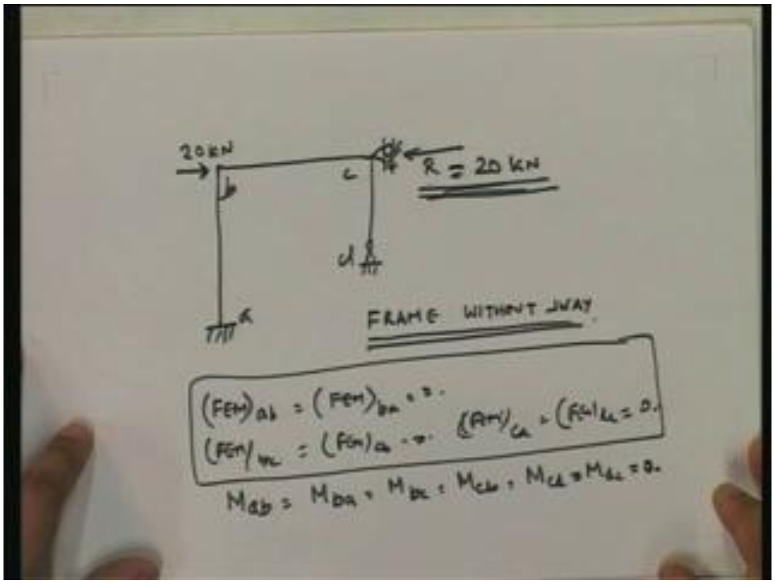

The first problem is going to be…. How do I restrain the sway? By actually doing this (Refer Slide Time: 05:44). Therefore, I am going to get a reaction over here. I am going to get a reaction over here, so this is the frame without sway. Here, what do I do? Well, this is the structure. I find out…. I solve the entire problem – I solve it by finding out first the $(FEM)_{ab}$, $(FEM)_{ba}$ – there is nothing, so this is equal to 0. The $(FEM)_{cb}$ is equal to 0. This is a, b, c, d. The $(FEM)_{cd}$ and $(FEM)_{dc}$ are equal to 0.

All of them are equal to 0 and then fixed end moments are equal to 0. What would the moment distribution give me? Note that if fixed end moments are 0, at all joints you have the moment equilibrium. Therefore, the moment distribution for the frame without sway in this particular case becomes that we know what the moments are going to be; M_{ab} is equal to M_{bc} is equal to M_{bc} is equal to M_{cb} is equal to M_{cd} is equal to M_{dc} is equal to 0. There are no member end moments. Therefore, if I take equilibrium of this particular case, since moments are equal to 0, you will see directly that this r is equal to 20 Kilonewton meter (Refer Slide Time: 08:03). In this particular case, because of the special load that I have considered here, the computation of this R, the restraint, is very simple and the frame without sway is actually a trivial solution – you do not need to do the moment distribution at all.

Understand that this is a special case because I am introducing you to the concept of frames with sway that is why I have chosen a situation where a frame without sway does not actually require any moment distribution – this is not generally the case. Next time, we are going to be looking at a particular problem, where you will see that you are going to have a situation where the moment distribution, even for the frame without sway is going to give you a non-trivial solution and then you will see that to compute this R, you will have to actually solve a lot of equilibrium equations and get to this particular value of R. Here, it is trivial and therefore I had no problems in computing the value of R. Now, what do I do? What is the next step?

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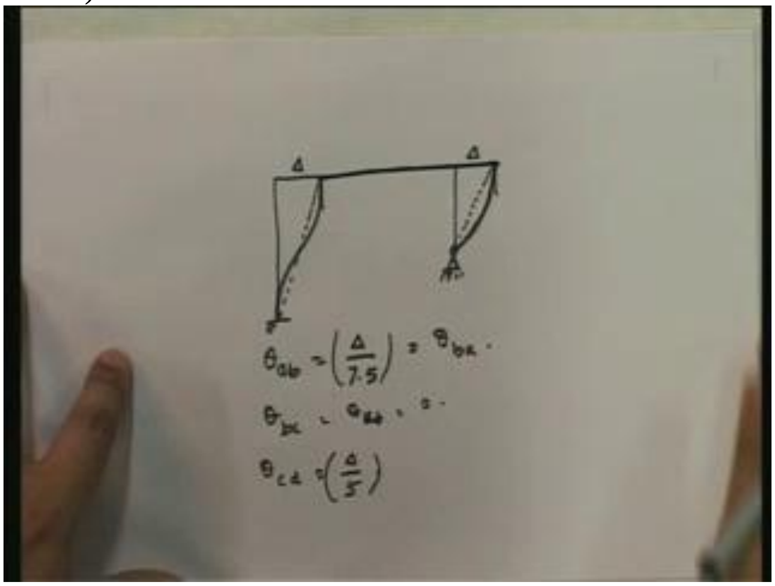

The next step is to release this R and see what happens to the structure when you allow it to move. Let us just say that this is some unknown quantity delta. What is going to happen? Note that when I take this delta, I am assuming that fixed end – everything is… b and c are clamped. If b and c are clamped, how will the rotation look like? You will see that it will look like this (Refer Slide Time: 10:26). Since this point goes here, this point cannot go up – it has to go only this way and the amount it goes by is delta to ensure that bc remains. What are the fixed end moments?

Calculating the fixed end moments is very easy. All I need to do is find out what the rotation from the chord is and plug it in to my equation. If I put that, what is my theta$_{ab}$ equal to? Delta, this is 7.5, so theta$_{ab}$ is equal to delta by 7.5. Is it positive or negative? From the chord to the tangent, anticlockwise **so** positive, so theta$_{ab}$ is equal to theta$_{ba}$ is equal to delta by 7.5. What can we say about theta$_{bc}$ and theta$_{cb}$? Both are equal to 0. What about theta$_{cd}$? It is equal to delta by 5 and this is also anticlockwise, so it is delta by 5. Can I compute the fixed end moments? All I need to do is I need to plug it into my equation and if you plug it into my equation.

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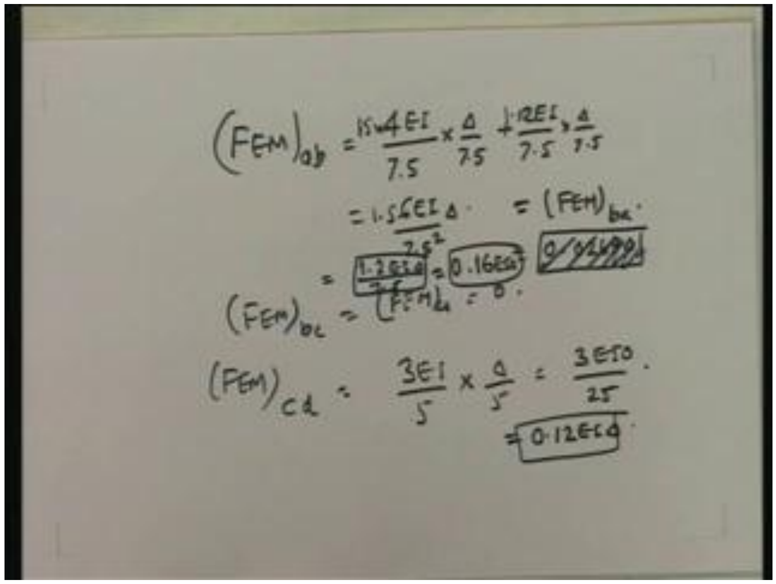

What do I get?

I get that the $(FEM)_{ab}$ is equal to 4EI by L – L is 7.5 – multiplied by $theta_{ab}$ which is delta by 7.5 plus 2EI upon L multiplied by delta upon 7.5 – this is equal to 6 EI upon 7.5 square and this is equal to $(FEM)_{ba}$. This is the fixed end moment at ab and ba. What are the fixed end moments at bc and cb? They are equal to 0. What is the $(FEM)_{cd}$? It is equal to 3EI upon delta multiplied by delta by 5 so this is equal to 3EI delta upon 25. These are my fixed end moments. The only problem over here is that I do not know what my values are. If you look at it, we had the situation that I_{ab} and I_{bc} were 1.5, so I have to multiply by 1.5 actually, so this becomes 4EI into 1.5 and 1.5 here; this becomes 1.5 here; this one turns out to be equal to 0.0267 delta. (Refer Slide Time: 15:13) let me just compute this directly – this will be equal to 6, so this is going to be 1.2EI delta upon 7.5; this is going to be equal to 0.1, so this is going to be 0.12EI delta; this becomes 3, 40, 30 by 4, so 30 by 4 becomes 4.8 upon 30, so this becomes 0.1, 4.8 upon 30 is equal to 0.16EI delta.

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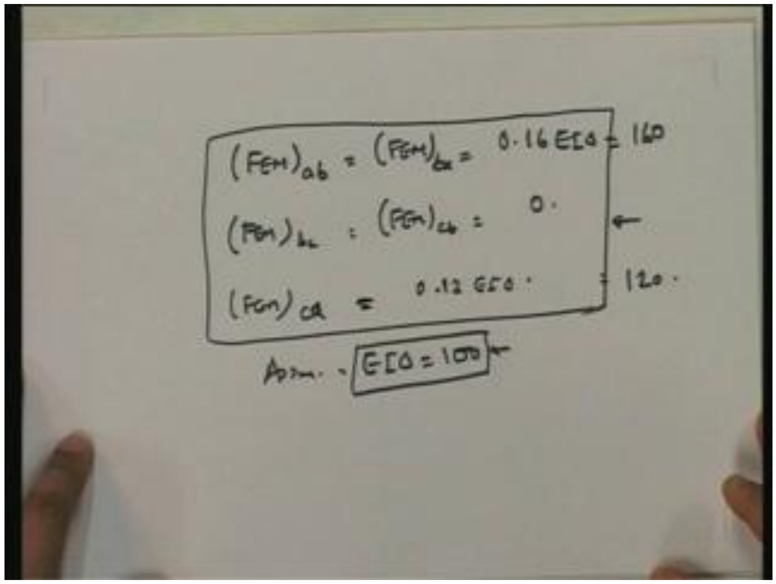

Now if you look at it, what you get essentially is that the fixed end moments turn out to be in this fashion; 0.16 EI delta, 0, $(FEM)_{cd}$ is equal to 0.12EI delta. Note that the fixed end moments depend on delta. Here, what I am going to do is I am going to actually assume a certain value of a delta. If I assume a particular value of delta, then what happens? I am just going to say that let us assume that EI delta is equal to 100 – I am going to assume it and then we will see what happens. In that particular case, you will see that this will be equal to 160, this becomes equal to 120. Now I have numbers with which I can do a moment distribution. We have done the fixed end moments, now let us look at the distribution factors.

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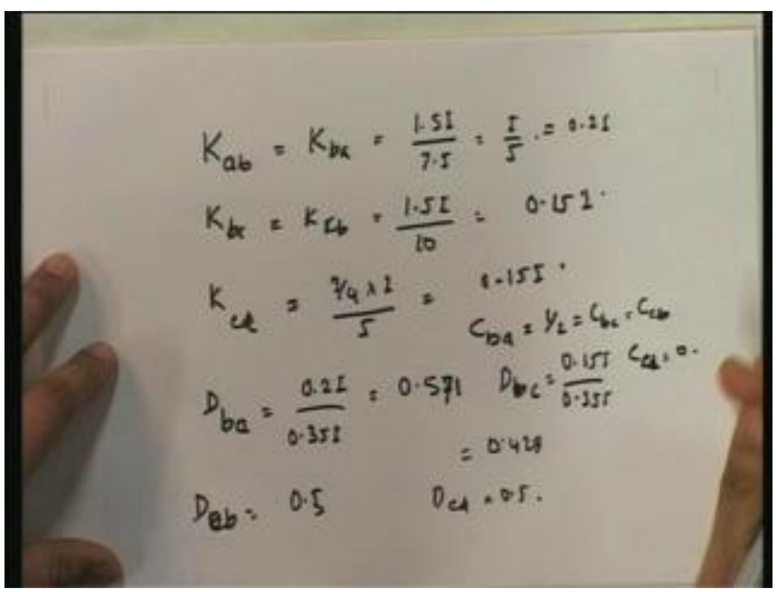

For the distribution factors I need to know that K_{ab} and K_{ba} are equal to 1.5 I upon 7.5, this is I by 5. K_{bc} is equal to K_{cb} is equal to 1.5 I upon 10. This is equal to 0.2 I, this is equal to 0.15 I and finally we have K_{cd} which is equal to three-fourths I upon 5, so this turns out to be 0.15 I. Once we have this particular thing now we can find out the distribution factors. How many joints? There are two joints, so (distribution factor)$_{ba}$ is equal to 0.2 I upon the summation of this (Refer Slide Time: 19:50) which is 0.35 I, so you essentially have 0.571; and D_{bc} is equal to 0.15 I upon 0.35 I which is equal to 0.429. Then finally at c we have D_{cb} 0.15, 0.15, so it is 0.5 and D_{cd} is equal to 0.5. What can we say about carryover factors? Let me put down the carryover factors here itself. C_{ba} is going to be half, which is going to be equal to C_{bc}. However, $C_{(c\text{ to }d)}$ is going to be equal to 0. Having put all of that in, let us now do the moment distribution.

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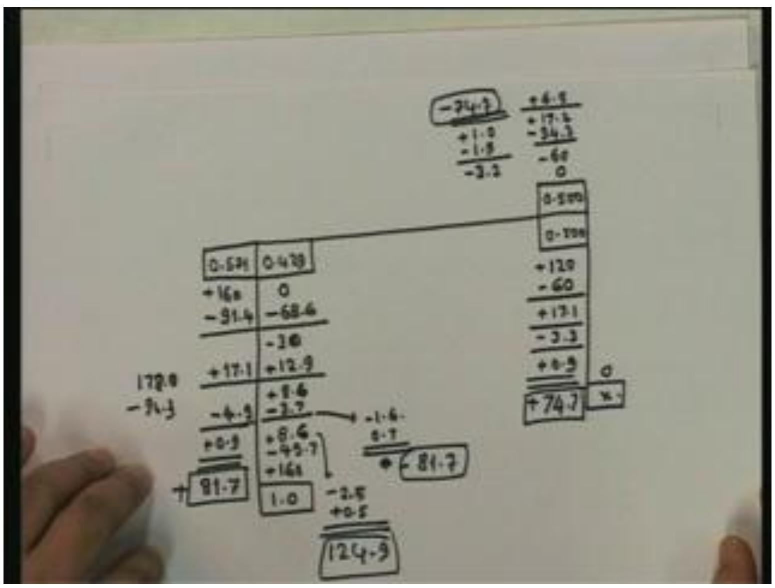

Remember left side for this (Refer Slide Time: 21:21), right side for this, bottom for this, top for this, left side for this and right side for this. Here, I have 1.0 – whatever comes in, distributes here; here, I have 0.571, here I have 0.429, here I have 0.500 and 0.500; and this here, since there is no moment. What are the values of the moments that I need to put in? (FEM)$_{ab}$ was equal to plus 160, this is also plus 160. What was the fixed end moment here? 0. What is the fixed end moment here? 0. What is the fixed end moment here? Plus 120. What is the fixed end moment here? 0. These are my values. I am going to now start doing the moment distribution and let us just go through the process.

In this particular case, I am going to do the distribution together, so over here, I get minus 60, minus 60 distribution over here and this is going to be 4 over 7, so this is going to be 640 upon 7, which is equal to minus 91.4 and this is going to be 480, so this is going to be minus 68.6. Now the next step is the carryover, so from here (Refer Slide Time: 24:12) I have carryover to here, so this becomes minus 45.7; I have carryover from here to here so this is minus 34.3; here I get minus 30 and no carryover here.

The next step is distribution. This needs to be distributed, so this becomes plus 17.2 – distribution done, plus 17.1 – distribution done, here the 30 has to be distributed, so what we have is… here, it becomes 17.1, this becomes plus 12.9, so this distribution done. Now, we do the carryover process: 17.1 over here will become plus 8.6, here, the distribution will go as plus 6.5 and here, there is no distribution.

Now I need the distribution from here (Refer Slide Time: 26:18) to here, so that 17.2 distribution becomes plus 8.6. Now, I need to distribute this and if I continue with my distribution process, I am going to come over here, so that I can keep adding and here, I am going to just go in this direction, so this is going to be distributed over here, it is going to be minus 3.2; this is going to be minus 3.3, so this is distribution done, distribution done; here, plus 8.6, when we go 4 by 7, so that becomes 34.4, 34.4 becomes minus 4.9 and this becomes minus 3.7, block here, block here.
Now, let us put this together over here. I am going to have… This comes to this point, so this will go in this direction and this will come here, so this becomes minus 2.5, there is a distribution here, the minus 3.7 goes over here, it becomes minus 1.9 and the distribution from this turns out to be minus 1.6. I am going to do my final distribution. because I want to go down. If I do this distribution, I will get it equal to plus 1.0 – that is the distribution, this one turns out to be plus 0.9. When I distribute this, I get plus 0.9 here and plus 0.7 here. Note that I am not going to carry over any of these because these have gone down in to the less than 1 percent level.

Once I have done that, I can add these up. Note the plus 9 needs to come over here as plus 0.5, so here also, this goes (Refer Slide Time: 29:42); this also goes this way, this also goes this way. All I need to do now is add these up and when I add these up, what do I get? I get 178.0 minus 96.3 so I get plus 81.7. If I go through this, I get minus 81.7 here, here I get plus 169.1, this becomes minus 2 minus 2 minus 43.7 becomes 116.3, 124.9. Let us see what I get over here: plus 60 plus 77.1 plus 77.9 plus 78 plus 78, plus 78 minus 3.3 is plus 74.7, this also turns out to be minus 74.7 and we have our member end moments. Once we have the member end moments, let us see what we get in terms of the values themselves.

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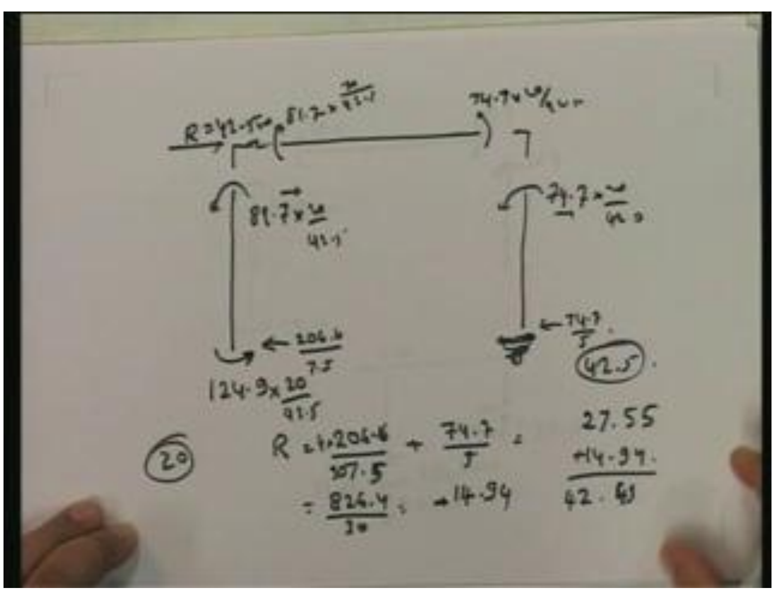

This plus 124.9 is going to be in this direction, then I have plus 81.7, then I have this, I have this member as equal to 81.7 and this is going to be 74.7, this is 0, this is 74.7. Now because of these, we can find out what the shear forces are, so due to this, I am going to get a shear force in this direction, this is going to go in this way, so ultimately 124, this is going to be 206.6, so this is going to be 206.6 divided by 7.5, this is going to be equal to this way, this is also going to be in this direction and this is going to be 74.7 by 5. If you really look at it, this force R that is going to require doing this is going to be equal to 206.6 upon 7.5 plus 74.7 upon 5. This is equal to 3 by 4, 30 by 4**,** so this is going to be equal to 826.4 upon 30; this is equal to 14.94 and this part is going to be equal to 27.55 plus 14.94 – this is going to be equal to 42.49, let us say 42.5. That means to generate these moments, I require 42.5 but what is the actual R? It is 20. What are the final moments? They are going to be equal to 20 upon 42.5 multiplied by all the moments.

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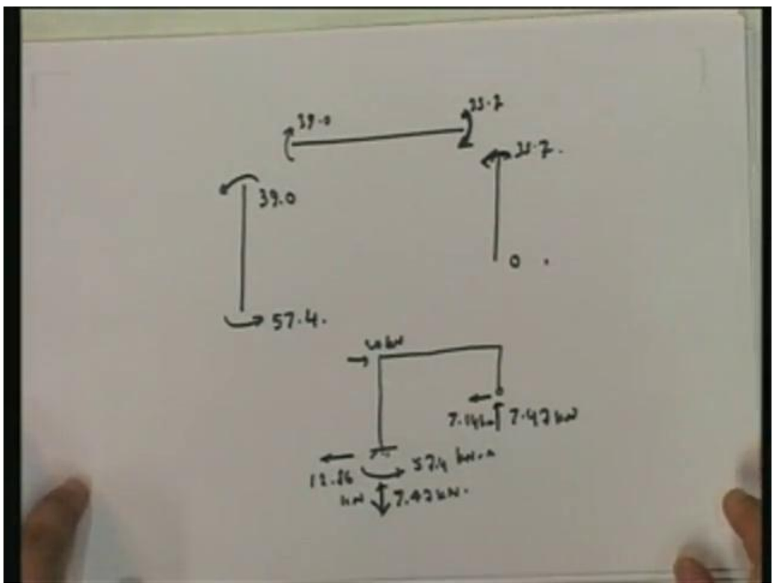

If you take those, you will see that this turns out to be equal to 57.4, this is going to be equal to 39, this is of course going to be 39, this is going to be 35.7, this is going to be this way and this is going to be in this way, 35.7, here, this is 0. So if you look at the support reactions, the support reactions become something like this. At this point, I have a moment of 57.4 Kilonewton meter, a 7.47 reaction here (Refer Slide Time: 37:31) and 12.86 Kilonewton moment here; over here, I have… this one is downwards, this one is upwards 7.47 and there is going to be a force of 7.14 and this is the load 20 Kilonewton.

Therefore, the point that I want to go back and point out to you is that we did not know what the delta was. We know that if delta was there, this would be 0.16EI delta and this would be 0.12EI delta – we knew this. You cannot do this, so I am just saying 'assume'; we do not know what the delta value is, so I am just saying assume EI delta; all of them have EI delta in them, so I assume that EI delta equal to some value – I have assumed EI equal to 100 but you could have actually taken it to be some other value.

For example, one way that a lot of people work with is take the largest value to be given by 100; I would have said that 0.16 EI delta is equal to 100; I could say that is equal to 100 but note that as soon as I say that it is equal to 100, then I know what the EI value is and I can substitute that. If you look at it, that if you have taken this to be 100 (Refer Slide Time: 39:26), this would be 75. The only point that I am trying to make is it does not matter what you take – you need to be consistent that these values need to be taken together and therefore in this particular case, I have taken 160, 120 but you might equally have taken it to be 100, 75 – it is all relative.

Once you have done that, the fixed end moments – you solve through the fixed end moments; and once you get… you do the moment distribution. The moment distribution procedure, if you note is identical to that of beam again. Once you have fixed end moments, the moment distribution procedure is unique. You have to compute the stiffnesses and once you compute the stiffness, you can compute the distribution factors, you can calculate the carryover factors and then once you put those in, you go through the moment distribution process. This time, I actually did it so that I am doing it simultaneously – I am releasing this and this – distribution; then, do all the carryovers.

Once you do the carryovers, you look at what is the net moment unbalanced – balance that and you keep doing it and as I said, this procedure is going to be convergent and we can get the final values. Note these values have no meaning – if you had taken this to be 100 and this to be 75, these values will be different; these values per se do not have any meaning, but once you have these values, these would be the values if you had a certain value of delta; since you do not know the value of delta, these values have no meaning.

However once you have these values, these are the member end moments; once you have the member end moments, you actually have to go and put them and compute. You know what force is required to get these kinds of moments. I computed this and this (Refer Slide Time: 41:54), I know that this plus this is equal to this. So all I found out R by adding this and this and I got it to be 42.5. Now I know my R value is 20 Kilonewtons. Therefore, what do I have to do? All I have to do is, I have got the member end moments; for a force of 42.5, these are the moments. What would it be for 20? Linear system – you can actually scale. What does the scaling mode imply? For 42.5, you have this (Refer Slide Time: 42:33), so for 20 the value would be 124.9 multiplied by 20 upon 42.5, multiplied by 20 upon 42.5, multiplied by 20 upon 42.5, multiplied by 20 upon 42.5.

You will see that R is equal to 20, which means all I need to do is scale all these values by 20 into 42.5. Once I scale the values, this is what I get. Once I get this, this is the ultimate solution. I have got the reactions at the supports, I have got the member end moments – so my problem is solved. In essence, I just want to go back and review that a frame which has sway – how do you tackle that problem? First, you take frame without sway. Now in this particular case, when you took frame without sway, because there were no member loads, the fixed end moments were 0 and I could compute the restraint reaction directly. Once I calculate the restraint reaction, I can actually apply an opposite force to get delta. Now I take as if the frame was swayed and so let me just write down the procedure for you, so that you can get.

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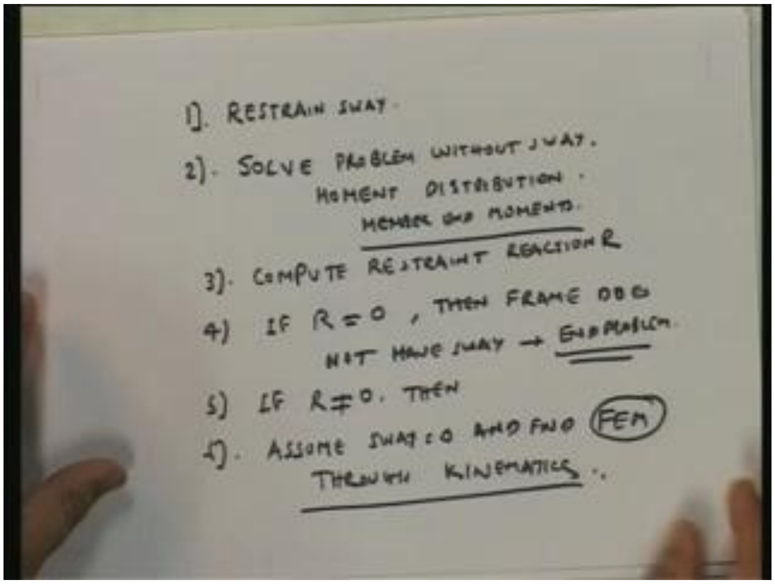

This is moment distribution for a frame with sway. First: restrain sway. Two: solve the problem without sway, because once you restrain sway, you can solve problem without sway; you do the moment distribution – solve the problem without sway is moment distribution; once you do moment distribution and you get the member end moments, you can…. Three: compute restraint reaction. Four: if reaction R is equal to 0, then frame does not have sway, so end the problem – whatever member end moments you get at the end of the problem without sway, that is your member end moments; however if R is not equal to 0, then what do you do? Six: assume sway equal to delta and find fixed end moments. How do you find out fixed end moments? Through kinematics. You find out the fixed end moments through kinematics.

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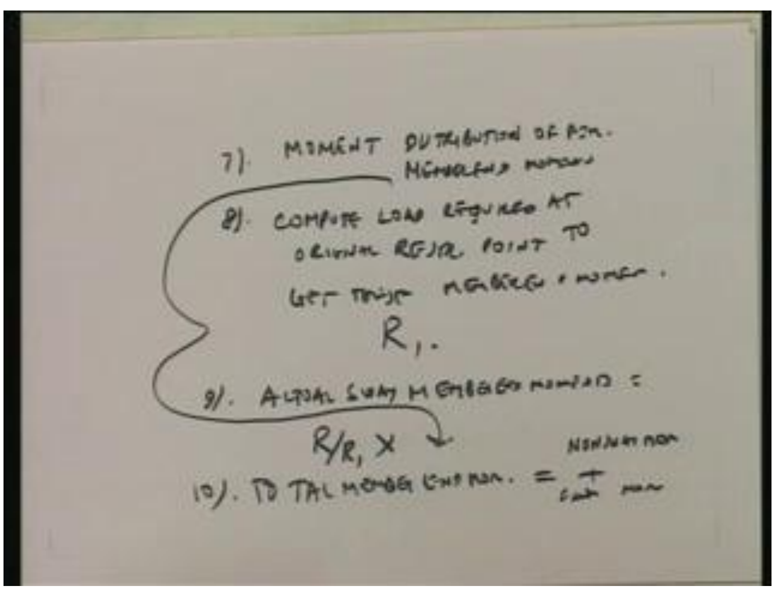

Once you have found out the fixed end moments through kinematics, the next step is moment distribution of fixed end moments; moment distribution will give you member end moments. Eight: once you have a member end, you can compute the load required at original restraint point to get these member end moments. Let this be equal to R_1. Then, the actual sway member end moments are equal to R upon R_1 into these member end moments. Ten: total member end moments are equal to non-sway moments plus sway moments. I just wanted to list out the overall details. Now, I think you should be able to solve a problem with sway. I am going to take up some more problems with single sway in the next lecture. I just want to briefly tell you how to do it suppose we have two sways – let me take that problem.

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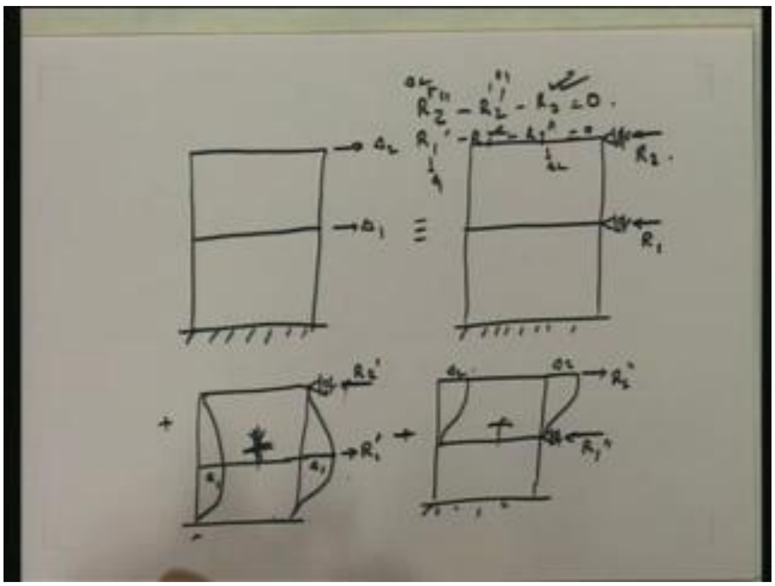

In this particular case, you can find out that you have two sways: delta_2, delta_1. How would you solve this problem? This problem is solved in this way. The no-sway case becomes this (Refer Slide Time: 50:17)**.** Compute R_1 and R_2 – the reactions at the two supports; you can do that because once you get the member end moments, you can always compute what these reactions are. Then, plus … compute this load which is going to give you a delta – compute this; and you can get the reaction at this point plus … now you note that this is the no-sway case, this is the first sway case, this is the second sway case. You can find out the fixed end moments, do the moment distribution, compute the member end moments and compute these.

Ultimately, you will see that this (Refer Slide Time: 52:13) plus this, plus this so you will see that R_2 double prime minus R_2 prime minus R_2 is equal to 0; we also have R_1 prime minus R_1 minus R_1 double prime equal to 0. Note that this is a function of delta_2 this is a function of delta_2, this is known, this is known, this is a function of delta_1, this is a function of delta_1. You have two equations which are a function of delta_1 and delta_2 – you can actually find out what those delta_1 and delta_2 are and then add the revised values of this and the revised values of this to this to get the final moment. When you have double sway, you have to solve equations in the two sways together. I just wanted to introduce the concept to you – you do not have to solve a particular problem; you can look at it yourself. Finally, just to state that when you have frames, there is a likelihood of sway displacements; then the moment distributions procedure requires you to solve multiple moment distribution problems to be able to get the final member end moments. I am going to take up another problem in the next lecture, which will illustrate to you all these procedures over and over again. I hope you have understood how to do the moment distribution method for both beams as well as frames and hopefully by the next lecture, which will be my last lecture on moments distribution, you should be able to understand the moment distribution procedure completely.

Thank you very much.