Structural Analysis - II
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Lecture – 22

Good morning. In the last lecture, I introduced you to the concept of moment distribution and the physical reasoning behind how moment distribution works in terms of relatively removing the clamp at the joints – that is how the moments get distributed. Ultimately, we see that it will always converge because as you release the rotations at the joints at which your moment equilibrium has to be maintained, progressively you are going to keep releasing it till you get equilibrium. The entire moment distribution procedure is an iterative procedure that ultimately gives you equilibrium at all the joints – satisfies all the equilibrium equations. Let us look at this. Note that what we are normally considering here is the moment equilibrium – in fact, we are currently only looking at moment equilibrium. Let us now look at certain other aspects today. Remember I talked in the last lecture about how we account for the modified member that we had. Let us look at this.

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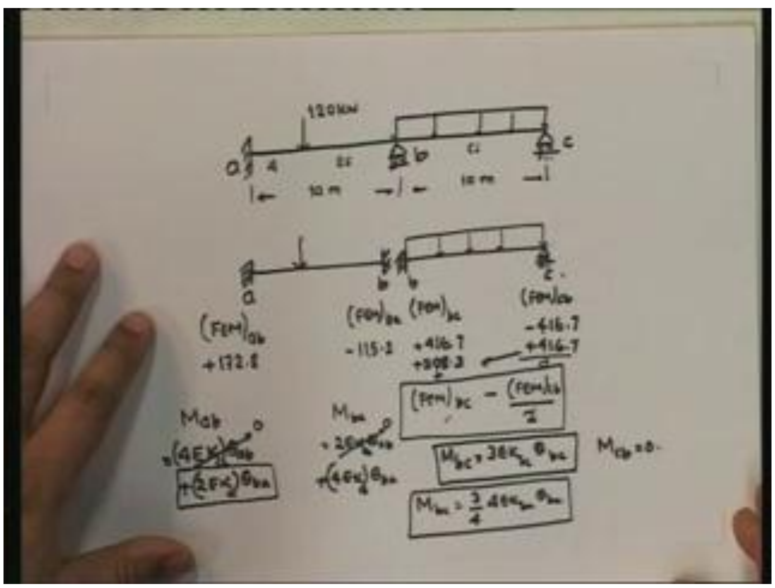

Let us look at this particular problem. It is the same problem, excepting that this time I am going to say that I will consider ab and my clamped clamp, but I am going consider bc as the modified member. If I consider it as the modified member, what happens? For ab, bc, here we have this (Refer Slide Time: 04:32) and here we have this. First and foremost, I can calculate the fixed end moments. The $(FEM)_{ab}$ is equal to plus 172.8, $(FEM)_{ba}$ is equal to minus 115.2; now I am going to calculate the fixed end moments at bc and cb as if they did not exist – as if this is clamped – I am going to compute it that way (it is not, but I am going to compute it). What was the modified fixed end moment here. The modified fixed end moment – remember that we had computed this? Now my point over here is this: is this not the same as my balancing the force here and then carrying over half, which is basically plus 208.3?

This is almost exactly like and once I release this, that is it – this has gone to 0 and it will never come back again because this is a roller. We can do this even in fixed end moments in which we compute the fixed end moment as if this was fixed, then do the balance over here, put it to 0, transfer it over here and then solve the problem, but let us look at what the other things are. These are the fixed end moments. What is M_{ab}? It is equal to 4 EK (I by L – I call it K) theta_{ab} and since this is the fixed end, this goes to 0 plus 2 EK theta_{ba} (which is of course there) plus the fixed end moment; this one has …. M_{ba} is 2 EK theta_{ab} plus 2 EK theta_{ba} – that exists. These are the moment deformation relationships for ab. For bc, what is it? It is given as M_{bc} is equal to 3 EK (I by L is K). This is K_{ab}, K_{ab}, K_{ab}, K_{ab}; this is EI by L for bc – 3 EK_{bc} into theta_{bc}; that is it, there is nothing else. Here, M_{cb} is equal to 0. Can I write this in this fashion? I will write this as M_{bc} is equal to three-fourth of 4 EK_{bc} theta_{bc}. What is M_{ba} plus Mbc?

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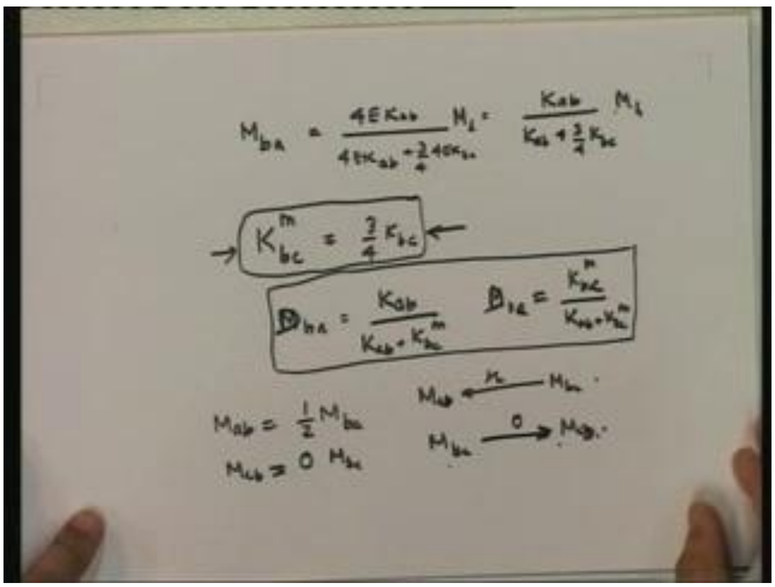

M_{ba} plus M_{bc} is going to be equal to… the moment distribution…. M_{ba} is equal to 4 EK_{ab} upon (4 EK_{ab} plus (3 by 4 into 4 EK_{bc})). Remember that the 4, 4 cancelled and E, E cancelled and that is how we got K_{ab} upon K_{ab} plus K_{bc} in the other case, but here this becomes plus 3 by 4 K_{bc}. Therefore, for a modified member, we can define a K_b modified, which is equal to 3 by 4 K_{bc}. If we do that for the modified member, then M_{ba} is equal to K_{ab} upon (K_{ab} plus K_{bc} modified) and M_{bc} is equal to K_{bc} modified by (K_{ab} plus K_{bc} modified). The only thing that happens is when you have a fixed-hinged, the modified K becomes 3 by 4 into K_{bc}. Is that clear? What about the carryover factor? The carryover factor M_{ab} is still equal to half of M_{ba}, the carryover factor from M_{ab} to M_{ba} is still half; this is fixed, therefore this does not go.

However, what is M_{cb} in terms of M_{bc}? It is 0 because M_{cb} is 0. When you have a modified member, the carryover to the other end is 0 – there is no carryover because moment cannot be carried over. With these modifications, let us see how I can solve this particular problem that we have. In other words, for a modified member, as far as computation of fixed end moments will come, we will see…. Let us not try to make a differential because we will see that this is actually incorporated in the modified, into this thing. As far the fixed end moments are concerned, there is no difference from the original (fixed-fixed); however, for the modified member, the stiffness is modified to 3 by 4; the distribution factors – this is into M_b, M_b, the distribution factors are equal to this (Refer Slide Time: 12:44). If you modify it, then we can continue using the same; the only other thing is that in a modified member, there is no carryover. With this new concept in mind, let us see how we can solve this particular problem, the same problem that we have.

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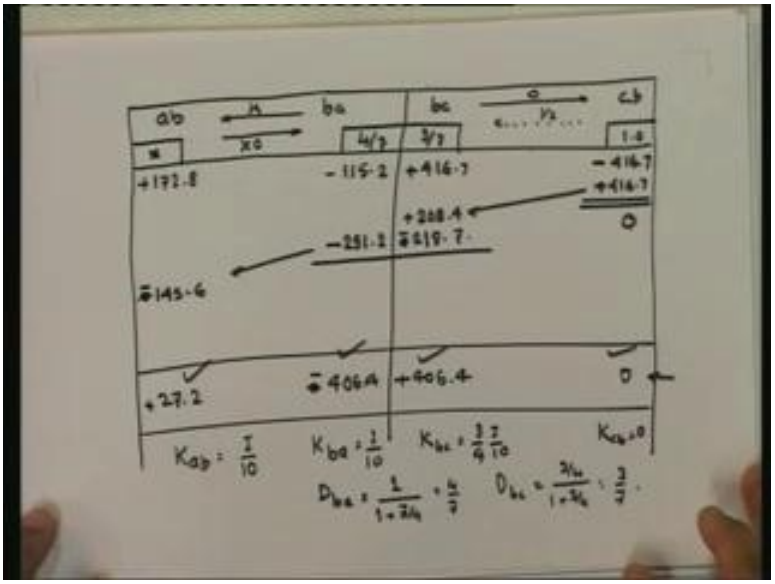

Let us go through the steps. We have all ready calculated the fixed end moments. Again, ab, ba – this is member ab, this is bc, cb. Let us see what the distribution factors turn out to be equal to. Let me just do the distribution factors at this point. What is K_{ab}? That is equal to I by 10. What is K_{ba}? It is equal to I by 10. What is K_{bc}? Three-fourths into I by 10. What is K_{cb} equal to? 0. Therefore, the (distribution factor)$_{ba}$ is equal to 1 over (1 plus 3 by 4), which is equal to 4 by 7 and D_{bc} is equal to 3 by 4 upon (1 plus 3 by 4), which is equal to 3 by 7. The distribution factors… since I am considering bc as modified, this is 4 over 7, this is 3 over 7, and this one – no distribution because it is a fixed end and here, distribution is 1. There is no carryover in this direction; however, I am going to say that only for the release of the fixed end moment is there a half carryover, here there is a half carryover, there is no carryover in this direction because the fixed end is not to be released. Let us put down the values: minus 115.2, plus 416.7, minus 416.7.

The first thing we do is put plus 416.7, put this equal to 0, carry over here (Refer Slide Time: 16:56**),** this becomes plus 208.4 and this one is a first-off case **so** this has gone to 0; this is **not to** be considered again because this is a modified beam – the fixed end moment alone is released. Understand that this is identical to doing (FEM)$_{bc}$ minus (FEM)$_{cb}$ by 2 – we could have done that, we would have got the same thing. Once we have done that, this is the unbalanced. What is the unbalanced? Plus 625.1 minus 115.2 – this is the unbalanced, so this needs to be balanced now.

Balancing will be done as 4 upon 7 and 3 over 7, this is positive, so this is negative, so negative 291.2 minus 218.7 – that is a net unbalanced; once I have done that, that is balanced, then I have a carryover here (Refer Slide Time: 18:21) – the carryover becomes minus 145.6. Now, let us look at it. This is the first level of carryover. Any unbalanced left here? No, this is 0. Here, we balanced it and we have carried it over – there is nothing else; we have ended it, we have finished it off; and so this one turns out to be equal to plus 27.2, this turns out to be equal to minus 406.4, this turns out to be plus 406.4, this turns out to be 0 – this is what we get; this is only one iteration. Why is there only one iteration? Since this is released, this is the only one clamped; as soon as we release it, we have got balance here and that is it, there is nothing else, theta_{cb} can go any way it decides to – that is the reason why this is spectacular. We had nine steps to get to convergence and by using the modified beam we just did it in one iteration and reached the solution. It is never going to be this spectacular, but this illustrates how using the modified element actually helps in speeding up the moment distribution convergence process.

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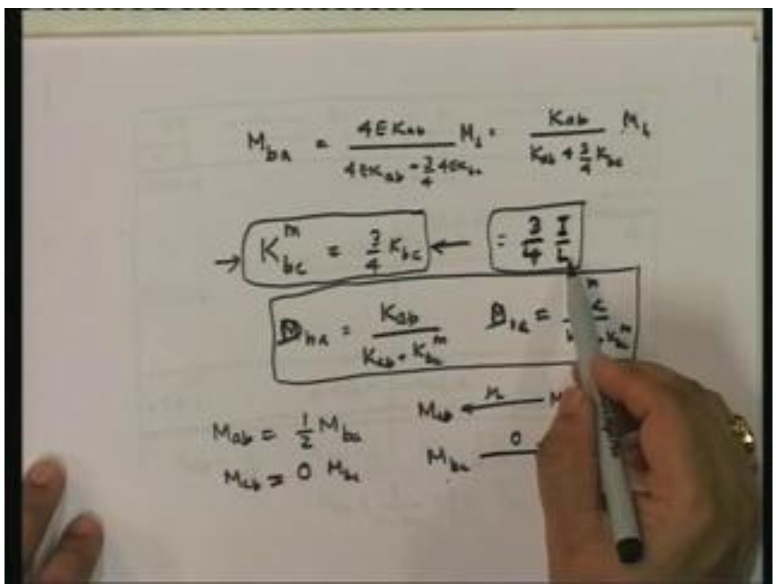

Therefore, to review, I just want to say that for a modified member, you take…. For a regular member, K is equal to I upon L; for a modified member, K is three-fourths I by L – only at the continuous end. Is that clear? You can only apply the modified if the other end is a support – at the support, you never have another member, so that does not come into the picture; even if you have, note that here you have a hinge support at b also but since there is a continuity, you cannot take the moment. You know that, you have already seen that and I do not want to belabor that point again and again. All you have to do is for the modified member at the continuous end, you just make the stiffness 3 by 4 and you use the modified stiffness in the distribution factor computation. Secondly, there is no carryover in a modified member – ever. This in essence is the application of the modification or the modified element in the moment distribution method. Let us look at a kind of a situation…. Again, I am going to take the same problem in the earlier case where we had this situation.

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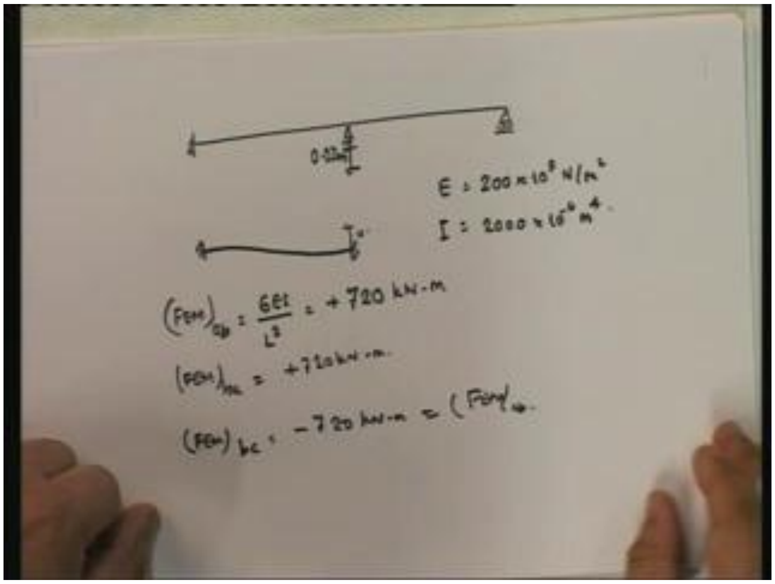

This support went down by 0.03 meters – 3 centimeters and we had to find out the moments at this point. Here, what are the fixed end moments? Let us see. Due to this 0.03, we are going to have the $(FEM)_{ab}$ is going to be equal to 6 EI upon L square. I am solving the same problem. We were given E is equal to 200 Newton per meter squared and I was given as 2000 into 10 to the power of minus 6 meter fourth. Substituting these in, we got that the fixed end moment here was plus 720 Kilonewton meter. Similarly, $(FEM)_{ba}$ was 720 Kilonewton meter. At the other end, what did we have? At the other end, I am going to consider bc the same, so it is going to be just the opposite, so you are going to have minus720 Kilonewton meter and this is equal to the $(FEM)_{cb}$. Now, I am going to use the modified members.

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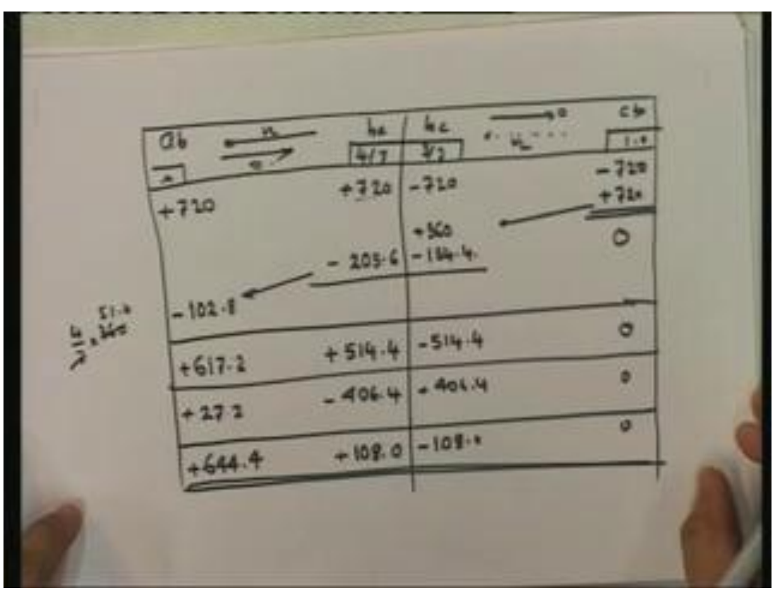

All that happens due to support settlement is that I use the same one that I have just used; I am not going to go through the same points, I am just going to write them down. This is 4 by 7, 3 by 7, this is 1, this way 0, only to release the fixed end moment we have half this way, this way we have half, this is the fixed end, this way 0 because it is a fixed end (we do not carry over from fixed end) and this is plus 720, this is plus 720, this is minus 720, minus 720 – these are the fixed moments. First, balance out at cb, put it equal to 0, bring it over here, plus 360. The next step is…. What is the unbalance here? You will see that these two balances each other, the only left unbalanced is this – this has to be divided as 4 upon 360.

What you get is 200 and plus 360, so 4 upon 7 of 360 – this goes 5 and 51.4, so this becomes minus 205.6 and this becomes minus 154.4 – this is the balance; then you have half here, so this becomes minus 102.8 . That is it – that is done, so this is it. This alone is going to be plus 617.2, this is going to be equal to plus 514.4 and this is going to be equal to minus 514.4, this is going to be 0. Now, this is due to the settlement alone. If I superpose, all I need to do is add the ones I have got from before, which was plus 27.2, minus 406.4, plus 406.4, 0. If both the force and the rotation occur together, what we get is plus 644.4, here we get plus 108.0, here we get minus 108.0 and 0. If both the settlement and the loads are together, you do it separately or you can do them together – I do not care, you are going to get the same thing. Understand that all that a known support settlement does is that it introduces fixed moments and we do the distribution exactly in the same manner as before. Now let us look at another problem, just to go through the steps. Up till now, we have only looked at two members; let us look at what happens when you have multiple members coming into the picture. I am just going to now take up a realistic problem.

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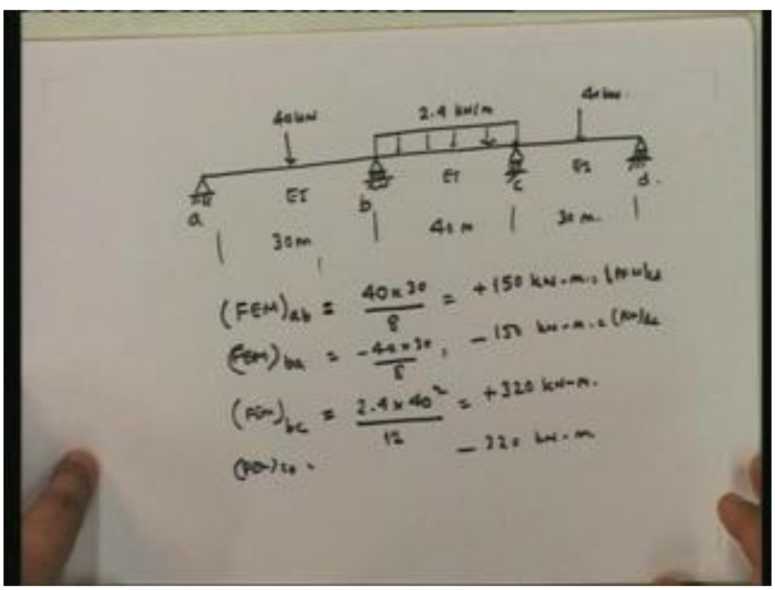

Do not go by what I have drawn; take down the numbers that I have: 30 meter, 40 meter, EI is a constant, so this is EI, EI, EI, then on this, we have 2.4 Kilonewton per meter, at the center point, we have a load 40 Kilonewton meter and here at the center point, we have 40 Kilonewton meter. For this one, we need to find out the end moments and then the bending moment. Once we have the end moments, the bending moment diagrams are obvious. This is a, b, c, d. First and foremost, compute…. I am just algorithmically going through the steps that you do in applying the moment distribution method to a beam problem.

Today, we are only going to be looking at beam problems; from the next time onwards, we will see how to consider a frame. Let us look at a beam. The first step is calculate the fixed end moments and when you take fixed end moments, you do not worry about whether it is a modified member or not – you calculate fixed end moments as if everything was fixed.

For this one, it is going to be equal to 40 into 30 divided by 8 (Refer Slide Time: 31:56). The $(FEM)_{ab}$ is going to be equal to plus 150 Kilonewton meter; the $(FEM)_{ba}$ was equal to minus (40 into 30 upon 8), so minus150 Kilonewton meter; the $(FEM)_{bc}$ is equal to 2.4 into 40 squared upon 12 – this is equal to 1600 into 0.2, that is plus 320; the $(\text{fixed end moment})_{cb}$ is going to be equal to minus 620. This is equal to $(FEM)_{cd}$ and this is equal to (FEM)dc because they are the same thing. Let me then compute the distribution.

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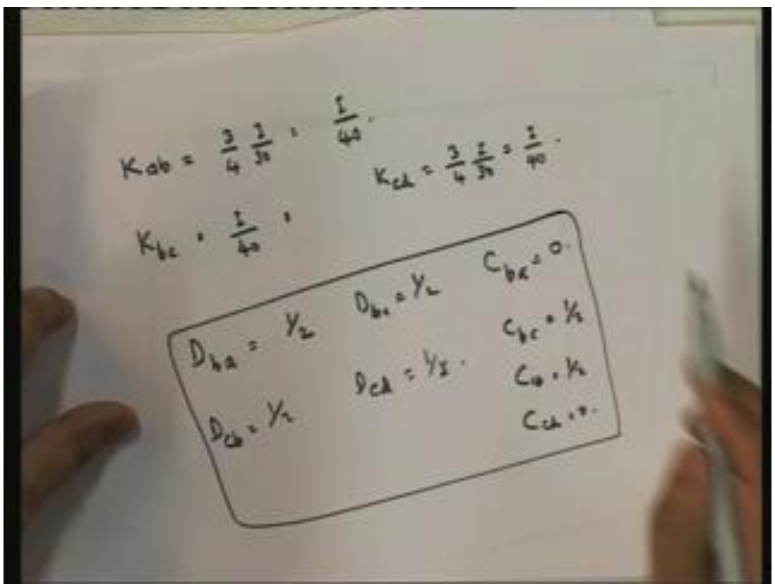

Now, ab is a modified member, so K_{ab} is equal to… (this is for the member); it is going to be three-fourths I upon 30 – this is equal to I by 40. Since bc is continuous, K_{bc} is going to be I by 40; K_{cd} is going to be 3 by 4 because d is a roller, so it is going to be modified (c is going to be the continuous end, fixed end and d is going to be the roller); in ab, a is going to be the hinged end and b is going to be the roller. If you look at it, distribution of ba is going to be K_b plus K_{bc} upon the summation, which is going to be half; D_{bc} is going to be half; at c, it is going to be I upon 40, I upon 40, so D_{cb} is going to be half; and D_{cd} is going to be half. Having done that, let us see what the whole thing comes out to be equal to; let us put it.

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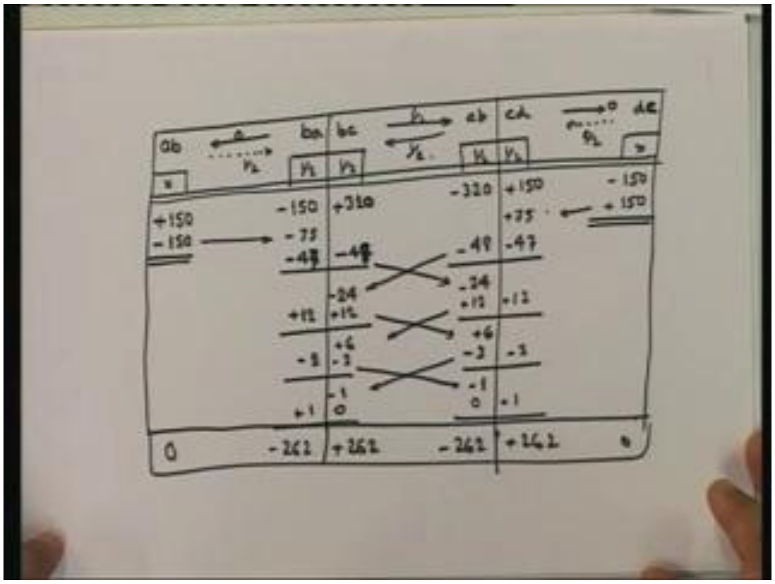

Note that carryover ba is equal to 0, carryover bc is half, carryover cb is half, carryover cd is 0 and everything else is automatically 0. With these kinds of things, putting them in… ab, ba, bc, cb, cd, dc. What happens over here is ab is a hinged end – there is nothing that happens; there is only one half, that is only once; this way it is 0 carryover. Here, we have half, half; here, we have half, half; here this is 0, the only thing that we have is half for one, this way it is 0 and here, this is half, half. What are the fixed end moments? This ab is plus 150, minus 150; this is plus 320, (Refer Slide Time: 37:39) this is minus 320 and if you look at cd, this is plus 150, minus 150.

The first step is to release these two, so I am going to put minus 150, plus 150. These are finished and all that we do is take this here (Refer Slide Time: 38:18) and put minus 75 here and plus 75. This is the one that we need to distribute, so if you look at it 225 and this is 320, so that net unbalanced is plus 95, so minus 95 has to be distributed, so we do minus 48, minus 48.

Similarly, over here, we do the same. Now here, this is the point that I was trying to make and this is the problem: I have taken up this problem to show you how we do not do it one after the other – we look at every joint and distribute whatever is the unbalanced moment at that point in one shot. Again, here at the same point, I am not going to be doing the carryover; first, I am going to look at the unbalanced.

Look at what we have here: this is 47. I will make this 47 and this 48. Here also, I will make this 48 and this 47. You will see that the same unbalanced moments come up and what we have done is; we have done both of them together. Now we do the carryover: there is no carryover here, no carry over here. How was this carryover here and here? We have minus 24, minus 24. Now we unbalance these, so this is going to be plus 12, plus 12, plus 12, plus 12; now balance: no carryover here, only carryover here and here, this is going to be plus 6, plus 6. This is going to be minus 3, minus 3, minus 3, minus 3 – that is what it is; then we do again carryover: carryover is going to be minus 1, minus 1.

For all practical purposes, when I distribute that, I am going distribute plus 1, 0, 0, plus 1 and that is it – it is over, no other carryover, nothing else, so we have reached the end. Add all of them up, 0; minus 225 and this is going to be equal to 225 plus 47 is 272, minus 272 plus 12 is minus 260, minus 263; this is 225 and 47 is 272**,** and this thing is equal to here minus 260 and minus 3 is minus 263 so it is minus 262; this is going to be plus 262, this is going to be minus 262, this is going to be plus 262 (Refer Slide Time: 42:31) and this is going to be equal to 0.

Once you have this, you have got the member end moments and you can do the distribution. The point that I am trying to make over here is that note that I did this problem just to show you that in reality, you do not do the procedure as release one, clamp the other one and so on. Once we have established it, now you cannot tell what is happening here. I am going to be simultaneously releasing both and then looking at the effects, then releasing, taking effect, so physically understanding this is very difficult.

Physically, this does not make sense, but you have to understand that in the moment distribution the steps are the following. First, compute the fixed end moments. Even if you have a modified member, you compute the fixed end moments as if it was a fixed-fixed member; the fixed end moments are computed as if every member was fixed. Then the next step is actually computing the stiffness of each member.

In this particular case, when you compute the stiffnesses the first step that you do is you actually go ahead and compute the stiffnesses based on whether a member is the original member or the modified member. Therefore in this particular case (Refer Slide Time: 44:29); when we computed the fixed end moments, we computed the fixed end moments as if ab, bc, cd were all original members; so fixed-fixed, fixed-fixed, fixed-fixed and we computed the fixed end moments. Actually, accept the fact that ab is a modified member, bc is the original member and cd is a modified member.

While computing the stiffnesses, that is where we take into effect that ab is a modified member; so K_{ba} is 3 by 4 of I upon L; bc is a normal member, so it is I upon L; cd is the modified member; then the distribution factors are computed based on the modified members; the carryover factors of course depend on whether it is a modified member or a regular member. Then, the first thing that we do is we have computed the fixed end moments based on fixed-fixed, so the first thing that we do is actually release the two end releases and carry that over once only and that is it; then these two are finished because now nothing goes back to them.

Now what we have to do is, we look at both the members. In other words, how many ever joints, continuous joints we have, we look at their distribution to the unbalanced forces at one shot – we do all the distributions at one shot; so you will see this, this, if there are more members, all would be distributed. Then, the next phase is the carryover phase: you carry over everything that you have to; in this particular case, the carryover is only here because there is no carryover on the other side. Once we have finished the carryover, we look at all the unbalanced and do the distribution, so you have fixed.

The first it is the modified member carryovers, then distribution, carryover, distribution, carryover, distribution, carryover, distribution till we hit a situation where the carryover is 0.

Ultimately, the distribution is ended and that is your final member force. This in essence is your overall moment distribution method as we see it for beams. If I want to review what I have done today, I have looked at the moment distribution method as applied to beams, we have looked at how modified members can be incorporated into the moment distribution, we have seen the physical understanding behind the moment distribution method and then we have applied it, where we actually do it over in a distribution phase, carryover phase, distribution phase, carryover phase. We also saw how to consider support settlements into the entire consideration.

The other point I would like to make is that even if you have temperature effects, the temperature effects are again local effects, which only give rise to fixed end forces. Everything else, the moment distribution procedure – it is not different whether you have a load in a member or whether you have temperature in a member. In other words, at the end of this lecture, you should be actually able to solve any problem that you have for a beam – you should be able to do the moment distribution method. What I want to do is, I want to give you a problem so that you can solve it yourself. Please take a note of a problem. Let me give that problem and you have to do the problem yourself.

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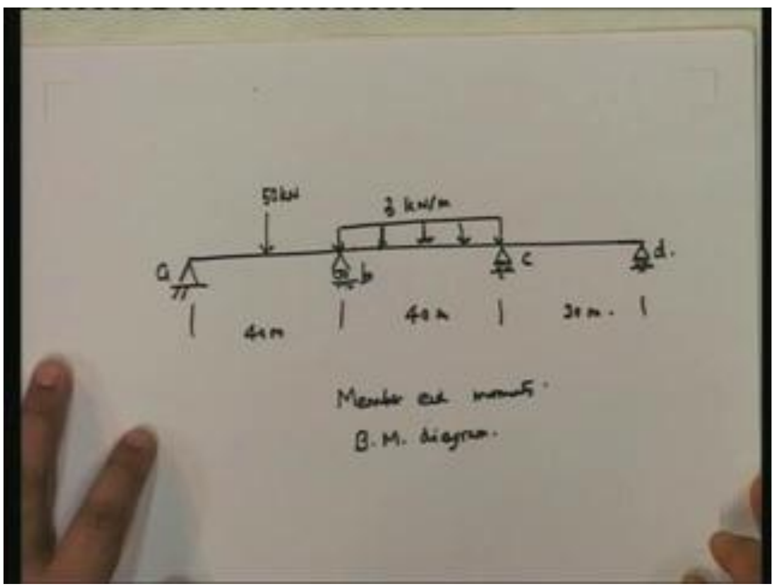

This is the structure and I want you to solve this particular problem and get the member end moments. Once you get the member end moments, you can draw the bending moment diagram – I ultimately want you to get the member end moments and automatically the bending moments diagram, using all modified stiffness and distribution factors that you can use in this particular case.

After today, I hope you will be able to apply the moment distribution method to beams for any kind of situations. I have given you this problem as an exercise – you have to do it and satisfy yourself that you understand the moment distribution method. From tomorrow, I am going to spend a few lectures looking at how to apply the moment distribution to frames.

When we looked at the displacement method, we saw that whether it was a frame or a beam, the method was really the same; the only thing is that in a frame, you might have to write down certain equilibrium equations related to displacements but otherwise, the procedure was identical. In the moment distribution method, you will see that in frames that have displacement degrees of freedom the entire procedure is very very different. I will first start off by looking at a frame and see how we can do the moment distribution where it does not have a displacement degree of freedom.

We will look at that and then we will go on to see how to modify the moment distribution method so that you can consider displacements or what is known as sway in a frame. Thank you very much. I hope you have understood the moment distribution. Please do solve this problem that I have given you and come back later and with confidence.