Structural Analysis - II

Prof. P. Banerjee

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture - 02

Good morning. Today is the second lecture in the series of lectures on structural analysis. In the previous lecture we discussed how to calculate the Static Indeterminacy of a structure. Today, we are going to be discussing how to compute the Kinematic Indeterminacy of the structure.

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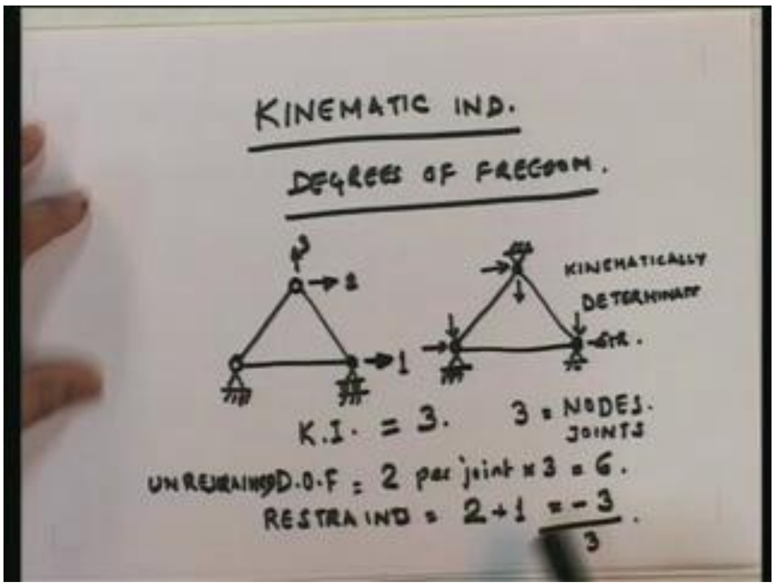

Kinematic Indeterminacy, what do we mean by that? The Static Indeterminacy you saw last time was essentially how to compute the number of redundant forces that you have in the system which, if you removed would make the structure statically determinate and you could then solve and find out all the forces in the structure purely by using equations of equilibrium. Kinematic Indeterminacy is a completely different concept altogether. Although, the word Kinematic Indeterminacy is actually a misnomer, it is more commonly known as the Degrees of Freedom that a structure has. What are Degrees of Freedom? If you were to describe it in words, it is the minimum number of displacement quantities that you require, to know before you can define the displaced geometry of the structure. If you look at this, this is completely different. Although we are using the same word, indeterminacy, it is a completely different concept. The previous one referred to how to find out the forces in the structure, this one is referring to how to find out the displaced geometry of the structure when it is subjected to loads. Note: the Kinematic

Indeterminacy is the minimum number of independent displacement quantities that you need to define to be able to get the complete displaced geometry of the structure.

I will look at two different types of structures. One set of structures which are made up of only actually loaded members if you remember those are trusses. I am going to start off with trusses because even trusses and beams and frames which are essentially structures made up of members which can deform flexurally, the treatment is different. Let us look at a truss and again I go back to the simple truss that I had last time. We want to find out what the Kinematic Indeterminacy of this truss structure is.

Before that I need to define what a Kinematically determinate remember, it is very easy for you to know how to get the statically determinate structure. You know when you look at it what a statically determinate structure looks like. How do you see a Kinematically indeterminate structure? Let us take this same structure; I want to make this Kinematically indeterminate. How will I make it Kinematically indeterminate? This is a Kinematically determinate structure. Why is this Kinematically determinate? Let us look at this structure. Understand what Kinematic Indeterminacy is.

It is the minimum number of displacement quantities that you need to define, the displaced shape of a structure. In this structure, how will it displace under loads? How will it deform? Can this point go anywhere? This point cannot go anywhere because it is restrained both in this direction as well as this direction; it cannot move anywhere on the plane that means this point cannot go anywhere. Can this point go anywhere? Neither can this point go anywhere. Can this point go anywhere?

Again, it cannot, simply because every point has a hinge which restraints the displacements in the plane of the structure. So what would be the displaced shape of the structure if it was subjected to loads? Let us put some loads on the structure. These are the possible loads on a truss structure because a truss structure can only be loaded at the joints. I put all possible loads that this structure can be subjected to. What will be the displaced shape under these loads? Remember, all these loads would get directly transferred to the supports and the structure will not be subjected to any stress. Therefore there is going to be zero strain and since this going to be zero strain there is going to be no displacement. What is a Kinematically Determinate structure? It is a structure that does not displace. Is that interesting? Currently, it is not interesting. Later on, we will see that we need to define a Kinematically Determinate structure before we can use what is known as the displacement method of analyzing statically indeterminate structures.

Once we have a Kinematically determinate structure, it is very easy to look at this structure and see what is going to be the Kinematic Indeterminacy or the Degrees of Freedom of this structure. Let us look at it. Can this point go anywhere? No. Again there is a hinge which is restraining it from moving anywhere. Can this point go anywhere? What is the restraint? The only restraint is it cannot move vertically. But is there any restrain for moving it horizontally? No. So, this is a possible displacement. This node can displace in this direction and therefore this horizontal displacement is a degree of freedom of the structure. Let us look at this point now. Can this point go anywhere? Note that this point can go anywhere on this plane and in a plane how many displacements do you require to define? If you define two orthogonal displacement quantities, they completely define the displacement of this point anywhere in space.

Now, if you look at this structure, a simple truss has three nodes: one node completely restrained and cannot go anywhere, this node vertically restraint and can move horizontally, this node is free to displace in any direction in the plane. We can define two displacement quantities which will define the motion of this point anywhere in space. Now how many Degrees of Freedom or what is the Kinematic Indeterminacy of this structure? 1, 2, 3 the Kinematic Indeterminacy (K.I) of this structure is equal to 3. Therefore if you have a truss, any joint in the truss can move in the plane and what you need to look at is what the restraints are. Let me redefine this problem. I am not going to do an algorithmic way of getting the Kinematic Indeterminacy. How many nodes does this structure have? It has three nodes. Now for the movement of a node or a joint in a plane is given by two independent displacement quantities. In other words, if I know those two displacements quantities I know where the joint is in the space. Let us look at it this way. How many joints? 3. How many Degrees of Freedom per joint? 2. Therefore if this plane truss were to have no supports what will be the Degrees of Freedom? It would be 2 into the number of joints so let it be 6. The Degrees of Freedom 2 per joint multiplied by 3 is equal to 6. 6 Degrees of Freedom since this truss has three joints. This is the unrestrained Degrees of Freedom.

In other words, the truss did not have any support, then the number of degree of freedom would be 6. However, you do have two supports in the structure. What are the supports? At one joint, you have a hinge which has two restraints; it stops it from moving vertically and it also stops it from moving horizontally. It does not allow this joint to go anywhere so it gives two restraints to this joint. What about this joint? This is a kind of roller joint. How many restraints does it give? It gives exactly 1 it restrains the vertical motion of this joint. 1 meaning 2, plus 1. The total number of restraints in this is 3. What is the restrained structure? How many Degrees of Freedom does it have? 3.

Kinematic Indeterminacy by finding out displacements and also the algorithmic way: Find out the number of joints in a truss, multiply that by 2. That gives you the unrestrained Degrees of Freedom. You find out the number of restraints that the body has and then subtract those restraints and you have the Degrees of Freedom or the Kinematic Indeterminacy of the truss. Let us look at one another problem. Let us look at the slightly more complicated truss.

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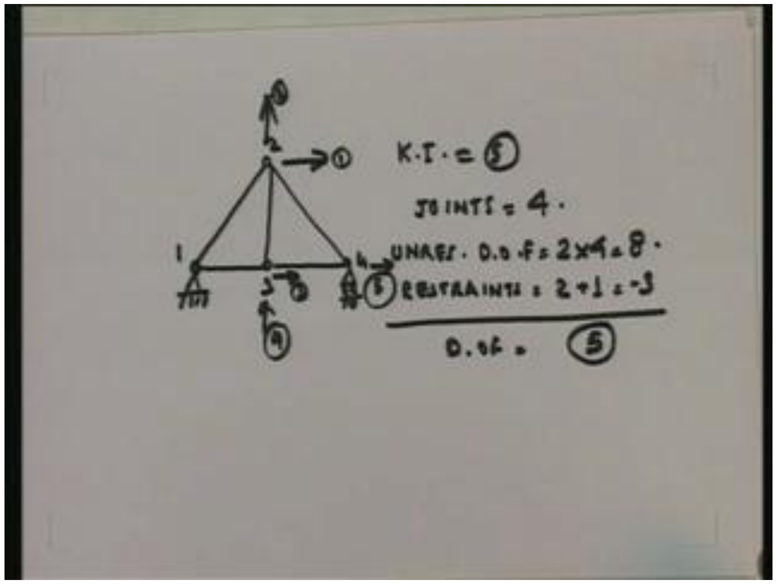

This is the truss; we have to find out the Kinematic Indeterminacy. How would I go about it? We are going to use only the algorithmic way of getting it, and then I will show you which are those displacements. How many joints? 4. What is the number of Degrees of Freedom? Note that Kinematic Indeterminacy and Degrees of Freedom are used interchangeably. What are the Degrees of Freedom per joint for a planar truss? 2 Degrees of Freedom. What is the total number of unrestrained Degrees of Freedom? 2 into 4 is equal to 8. We need to define what the restraints are? Restraints here are one hinge support, two restraints - one roller support and one restraint. So a total of 3. The total actual Degrees of Freedom is 8 minus 3 is 5. The Kinematic Indeterminacy is 5, for this structure. Once I have defined the kinematic Degrees of Freedom, you need to understand what is Kinematic Indeterminacy, it is the minimum number of displacement quantities, that you need to define, to be able to get the displaced geometry of the structure. I need to find out and define those displacement Degrees of Freedom. Note that I should have only 5 independent displacement Degrees of Freedom. Let us look at this. Are there any Degrees of Freedom at this node? No. Both of them are restrained. Let us go to joint 2; restraint none. What are the Degrees of Freedom? 1 2. Let us go to 3. Any restraint? No.

What are the Degrees of Freedom 1 2 the vertical displacement and the horizontal displacement. What about 4? It is restrained in this direction and is not restrained in this direction, so how many do I have? I have 1 2 3 4 and 5. 5 Degrees of Freedom; 5 Kinematic Indeterminacy. It should be fairly easy for you now, to get the Kinematic Indeterminacy of a truss type structure. Planar truss - 2 Degrees of Freedom, space truss - 3 Degrees of Freedom because you need 3 displacement quantities to define the position of a point in space. 3 independent: x displacement, y displacement and z displacement. What will be the unrestrained Degrees of Freedom for a space truss? 3 into the number of joints; that will be the unrestrained and then you find out the number of restraints that you have which are the support conditions or the supports. Subtract the restraints and you got the Degrees of Freedom for a truss and that defines the Kinematic Indeterminacy for a truss structure. Let us move on to that for a beam.

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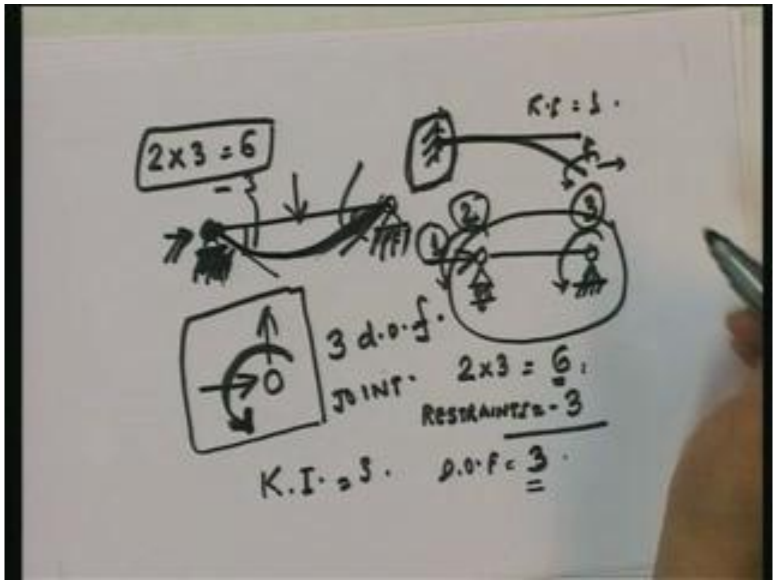

This is a structure that you are all familiar with; a simply supported beam. What is the Static Indeterminacy of the simply supported beam? 0. You know that it is a statically determinate structure but now we are not looking at Static Indeterminacy we are looking at Kinematic Indeterminacy. Again there are 2 joints 2 nodes in the structure and you can ask why we do not use the same method as a truss. For every node to move in the plane, there will be two Degrees of Freedom. 2 nodes, 2 Degrees of Freedom and three restraints. This is kinematical so if you use the truss model 2 into 2 so 2 nodes 2 Degrees of Freedom per node that is 4 and there are 3 restraints; 2 here and 1 here one degree of freedom. What is the one degree of freedom this one (Refer Slide Time: 20:43). Is that correct? Is this single kinematically indeterminate structure single degree of freedom? No.

Understand that members in a beam and frame behaved differently from a truss; a truss is only an actually loaded member. All we need to know is where the two joints at the end of a truss member is and that gives us elongation of the truss and from that we can find out the force in the truss. Can we do that for a beam? No, because the behaviour of a beam is in the way it is loaded, it goes like this. The entire concept of displacement, Degrees of Freedom for a beam and a frame would be different from a truss and that is why we need to look at it differently. How do we look at it differently? An important point is that a joint that connects beam members or frame members in addition to the two Degrees of Freedom which define its movement also has another additional degree of freedom. What does that do?

That degree of freedom actually defines the movement of the members, relative to each other. What is that? That degree of freedom is the rotation degree of freedom. Whenever you look at a beam or a frame every joint when it is unrestrained has 3 Degrees of Freedom. So a joint has 3 Degrees of Freedom. What does this degree of freedom do? If I fix this and this, this disappears because this cannot go up or down.

Remember, that if I have a Kinematically determinate structure I can define the displaced geometry of the structure completely. If I restrain this and I restrain this, both the displacements are restrained which means no displacement. But is that true? No. Under this load it displaces in this manner. Understand that I required additional displacement quantities to define the displaced shape of the structure and what are those? Look at this rotation and this rotation; these two rotations gives me the displaced shape of the structure. That is that additional degree of freedom that I need to define. Whenever you have a beam or a frame you have 3 Degrees of Freedom per joint. Let us come back to this structure; the simply supported beam. How many Degrees of Freedom does it have? How many joints? 2. How many unrestrained Degrees of Freedom? 3 per joint, so a total of 6. Note that this hinge support still restraints only 2 which is this displacement and this displacement. It has 2 restraints and this one. What was the total number of restraints? 3. What are the Degrees of Freedom? 3. This simply supported beam has 3 Degrees of Freedom or its Kinematic Indeterminacy is equal to 3.

What is Kinematic Indeterminacy? Number of unknown displacement quantities. Let us find those out for this structure just like we did in the truss. Note that it restraints this but allows this, so this is my first degree of freedom. This is a pin roller; it allows free rotation so this is another degree of freedom. This joint restraints both the motions but allows it to rotate and that is my third degree of freedom. In other words, this simply supported beam, if I knew the horizontal displacement of this point, the rotation at this point and the rotation at this point, I would be able to give you how this structure has displaced. If I knew the displacement quantities I could draw the displaced geometry of the structure. This, in essence, defines the Kinematic Indeterminacy for beams and frames.

I want to introduce one more joint that you are all familiar with; another statically determinate structure; this is a cantilever beam. What is the Kinematic Indeterminacy of this cantilever beam? How many joints? 1 and 2 so 2 joints. How many unrestrained Degrees of Freedom? 6. You need to find out the number of restraints. How many restraints? Let us look at this joint, this is a fixed support; a fixed support is one which restrains the point from displacing in any direction and it also restrains the rotation of that point. How will this displace? This will displace something like this. At this point, the slope would be 0, the displacements both vertically and horizontally would be 0, so completely restrained. At this point, it is not restrained; it does not have any support. What does this restrain? It restrains vertical, horizontal and rotation, so there are three restraints. So the Kinematic Indeterminacy is 3. What are those 3? It is 1, 2, 3. So, for a beam you should be able to get the Kinematic Indeterminacy or the number of Degrees of Freedom for the structure. Let us move on to Kinematic Indeterminacy for a frame.

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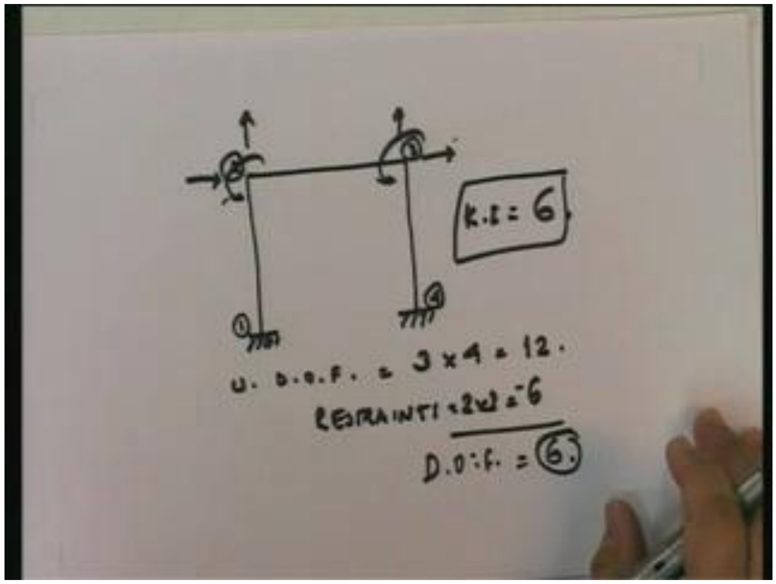

Note I just said that, the Kinematic Indeterminacy for both a beam and a frame are computed in exactly the same fashion. I can use the same algorithm that I developed for the beam. Look at this; how many nodes 1, 2, 3, 4 there are four joints in this structure. Following my procedure, each joint unrestrained has three Degrees of Freedom. The number of unrestrained Degrees of Freedom equal to 3 into 4 is equal to 12. Unrestrained Degrees of Freedom is 12. Now, we need to find out what are the restraints in the structure. How many supports? Two supports, 1 here and 1 here. What are they? fixed supports. How many restraints does the fixed support give? 3. How many fixed support? 2 What are the restraints 6 is equal to 2 into 3. Two supports with 3 restrains each. How many degrees of freedom? 6. Kinematic Indeterminacy, remember, is a same as Degrees of Freedom 6. I should be able to draw them 1, 2, 3, 4, 5, 6. The vertical, horizontal and rotation of this joint and the vertical, horizontal and rotation of this joint; 3 per node; total six Degrees of Freedom. If I know these 6 displacement quantities I can draw the displaced shape of this frame.

Now, I want to bring in the concept of constraints. Typically what we do, when we consider beams and frames we tend to neglect, the axial deformations because by and large in a beam and a frame, the main way that the loads are transferred is through flexural deformations. In other words, the structure bends to take the loads. While flexural deformations are very large the axial deformation are not zero, but are very small. We tend to neglect the axial deformations. What happens then?

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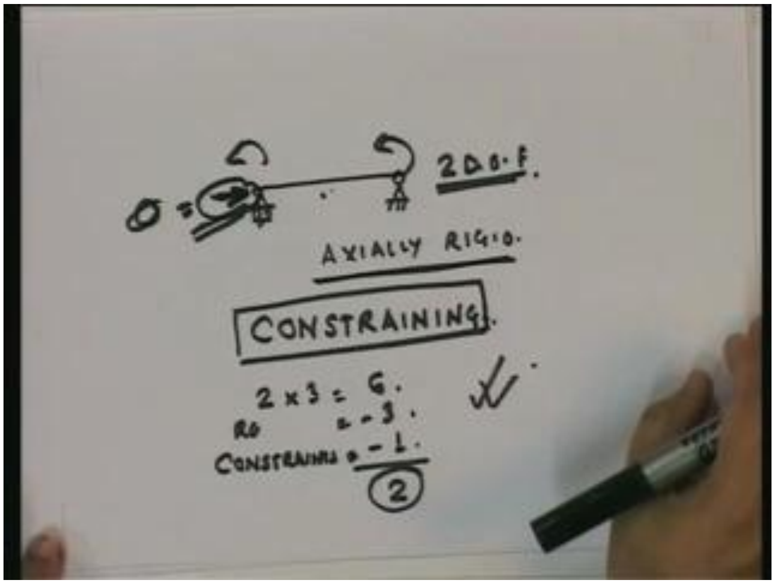

Let me go back to a simply supported beam; I neglect the axial deformation of this member, therefore I am saying that it is axially rigid. Note, in general, for a statically determinate beam, the forces that you normally find out are the sheer force bending movement not the axial force. Implicitly, without knowing, you are actually neglecting axial deformation; you never explicitly mention that. When you are analyzing a frame or a beam the member is axially rigid. What happens when you have a member which is axially rigid? What happens here is that you are constraining the structure. It has no effect on the Static Indeterminacy of the structure.

Remember, that it has a tremendous effect on the Kinematic Indeterminacy or the number of Degrees of Freedom that you get from a structure. You are constraining a structure into one actually rigid. When I say that this member is actually rigid, this cannot go anywhere. It can move horizontally. Forget about the rotational Degrees of Freedom; I am looking only at the axial deformation. It can go this way so this goes this way. What effectively happens to this member when this goes this way? This joint cannot go anywhere, this joint can move horizontally. The member is either axially shortened or if the displacement is in this direction it is elongated. In other words, this degree of freedom actually corresponds to the axial deformation of the body. If this member is axially rigid, since this point is not going anywhere; it cannot deform axially. That means this displacement has to be zero.

By making the structure axially rigid I have eliminated a degree of freedom. If I say that this beam is axially rigid how many Degrees of Freedom does it have? It has 2 rotations. It is a 2 degree of freedom structure. How do I put it into my algorithm? I truly believe that unless you have an algorithm way of computing Static Indeterminacy and Kinematic Indeterminacy, you are always going to flounder because you would not know how many indeterminacies there are in the structure both static and kinematic. Let us go back to algorithm way. How do I include this effect in the algorithmic way? What we define is two joints unrestrained Degrees of Freedom: 2

into 3 is equal to 6, restraints -3. Now, what have I done by making it axially rigid? I am constraining the structure.

So, a new thing: constraints. Constraints, also reduce the number of Degrees of Freedom in the structure. In this case how many constraints? It is axially rigid; whatever I have done I have made this into 0. How many constraints do you think axial rigidity in a member brings in? 1. Because a single degree of freedom defines the axial deformation of a member and when I make it axially rigid that is the additional constraint that I am giving; that this body cannot deform axially. I am making one constraint per member. How many members are there in this structure? There is 1. How many Degrees of Freedom? 2. I have already shown you the Degrees of Freedom in the structure. How about the algorithm for getting the Kinematic Indeterminacy of a beam or a frame? Start off by finding out how many joints there are in the structure. Multiply the number of joints by 3 that gives you the total number of unrestrained Degrees of Freedom. Then find out the restraints. What are the restraints that are provided always by supports? You go to every support and find out what kind of restraint it gives add up all the restraints from all the supports. Those are the total number of restraints those are to be subtracted from the unrestrained Degrees of Freedom. In addition to that, you need to look at the constraints. If, a member is axially rigid how many constraints? Single; it is 1 per member. So, you subtract the total number of constraints and you got your Degrees of Freedom or the Kinematic Indeterminacy of the structure. Let us go to the frame.

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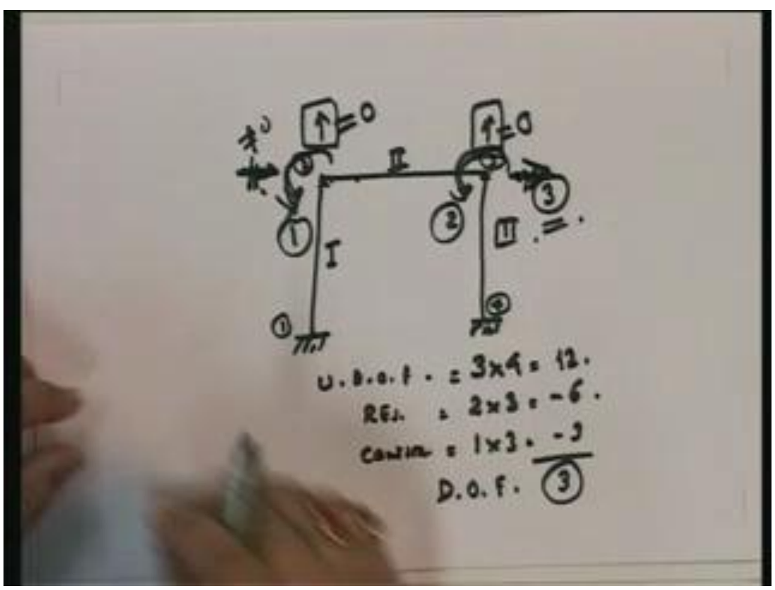

In the frame that we were looking at, all the members are axially rigid members. Let us now see how I compute the Degrees of Freedom. How many joints? 1, 2, 3, 4. How many members? 1, 2, 3. Unrestrained Degrees of Freedom: 4 joint 3 into 4 is equal to 12. Restraints: 2 fixed supports, so 2 into 3 is 6 restraints. Constraints: how many members? 1; all axially rigid so constraints -3. So, how many Degrees of Freedom does this structure have? 3. So a frame where all the members are axially rigid has three Degrees of Freedom. What are those three Degrees of Freedom? What did I have in the previous case? Let us compare the previous case to this case.

When I did not have the constraint of axial rigidity, these were the 6 Degrees of Freedom that I had. Now let us look at what happens. 3 of those have to disappear because my Kinematic Indeterminacy for this structure is 3; so I have to make three displacements disappear. Remember, it is not 3 displacements that I am making disappear or equal to zero.

It may so happen that two displacements are equal to each other in which case it is only one degree of freedom because remember the Kinematic Indeterminacy is the minimum number of independent displacement quantities that I need to define to be able to give the displaced geometry. If one degree of freedom is equal to another degree of freedom then what happens? I need only one of them. Let us look at this degree of freedom. What does this entail? This joint is not going anywhere vertically. If this joint moves vertically what happens? It means this member is being axially deformed but it is axially rigid. So what happens? I know that this displacement is equal to zero. Similarly, I know that this displacement equal to zero because this displacement would entail this column deform. What else do I know? This member cannot axially deform. What if this joint displaces by 1? How much will this joint displace by horizontally? Since this member is axially rigid if this displaces horizontally by one so will this.

That means that these two displacement quantities are the same; they are going to have equal value. So, I need to only consider one of them. Remember, I am not saying that this is equal to zero. No. It is not equal to zero. However, I am eliminating this degree of freedom because this and this are the same. I could do either one of them; either this or this you could eliminate. Then what are my independent Degrees of Freedom for this axially rigid frame? 1 rotation, 2 rotations and 3 horizontal displacement. Those are my three Degrees of Freedom for this axially rigid frame. So you see it is fairly simple. You should now have no problems, in computing the Kinematic Indeterminacy of a planar structure, be it a truss, a beam or a frame. How do you find out the number of Degrees of Freedom per joint depending on whether you have a truss structure or a beam structure?

If it is a truss structure, number of Degrees of Freedom per joint is two. Find out the number of joints multiply it by two, then find out the restraints and you have got it. Remember that an axial member cannot be constrained. Because if you stop the axial deformation of a truss member, what else is left? Nothing; it cannot deform at all. Typically, in a truss structure you never have constraints; you only have restraints provided by the supports. When you come to a beam and a frame, if you do not stop any of the deformations, then you have three Degrees of Freedom per joint. Number of joints gives you the total unconstrained Degrees of Freedom. You find out the restraints and subtract them. Furthermore, if you want to neglect the deformation of a particular member then you have to consider that member as a constraint. If you neglect the axial deformation for each member, you have one constraint and if you make all of them then you have that many numbers of constraints. This is relatively simple. I hope that at the end of this tr… you should be able to find out the Kinematic Indeterminacy and Static Indeterminacy of frames. I am now going to show one more constraint where this beam is totally rigid.

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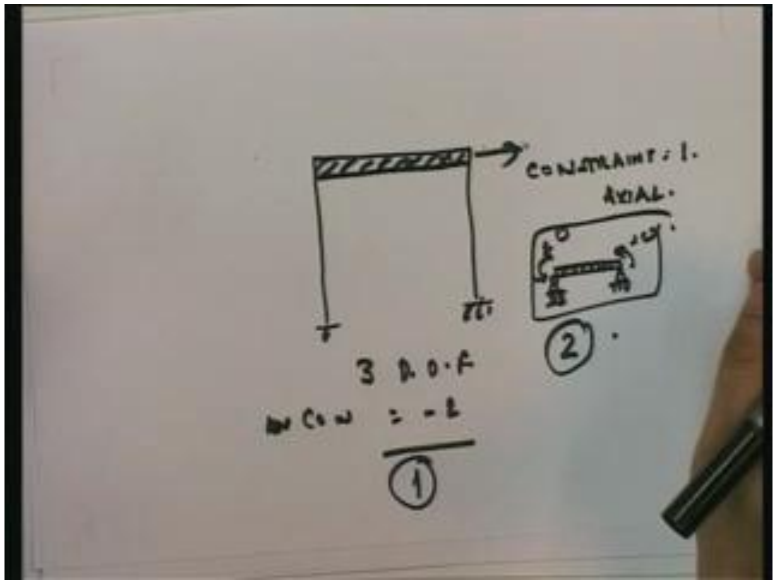

This beam cannot deform axially or flexurally. What happens in such a situation? Axial constraint I have already shown it as 1. How many for flexural constraint? You are making it flexurally rigid also now. Let us look back at this simple beam. How many Degrees of Freedom does this beam have? It is axially rigid, so how many of Degrees of Freedom is 1 2. Now let me make this beam totally, flexurally rigid. It cannot bend. If it cannot bend, this is 0 and this is 0. If you have flexural rigidity then how many constraints per member are there? 2. When this was not flexurally rigid we had 3 Degrees of Freedom. I am going to start from there; three Degrees of Freedom. How many constraints? Additional constraints due to flexural rigidity is 2. How many Degrees of Freedom? 1. Now I want to show you some structures and we will go through and compute the Kinematic and Static Indeterminacy of these frames.

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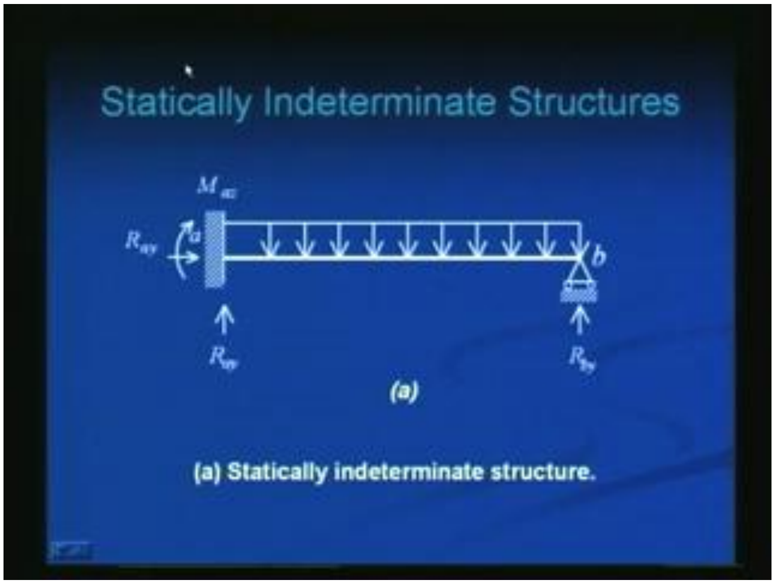

This is a propped cantilever. It is a statically indeterminate structure, because it has 4 support reactions and 3 equations of equilibrium. It is statically indeterminate structure.

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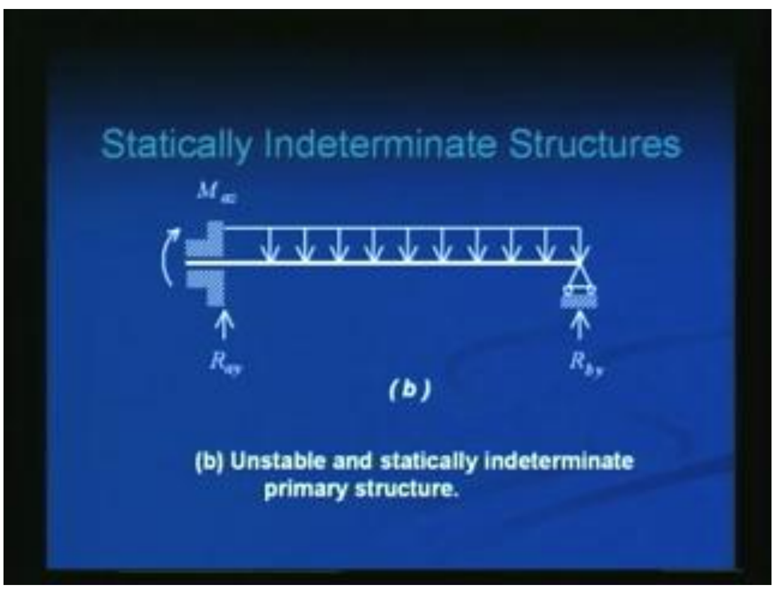

Let us look at another one. Here, what you have is this left joint which you have allowed to move horizontally. Suppose, I apply a horizontal load to this beam what is going to happen? This beam is going to move because there is no horizontal restraint. It only has two vertical restraints and a movement restraint to the left hand. This structure is unstable. Therefore understand one thing, it is very important before you start looking at Static Indeterminacy, you need to know whether it is unstable or stable and you can only find out the indeterminacy of stable structures. Look at this. This is stable, it is a cantilever.

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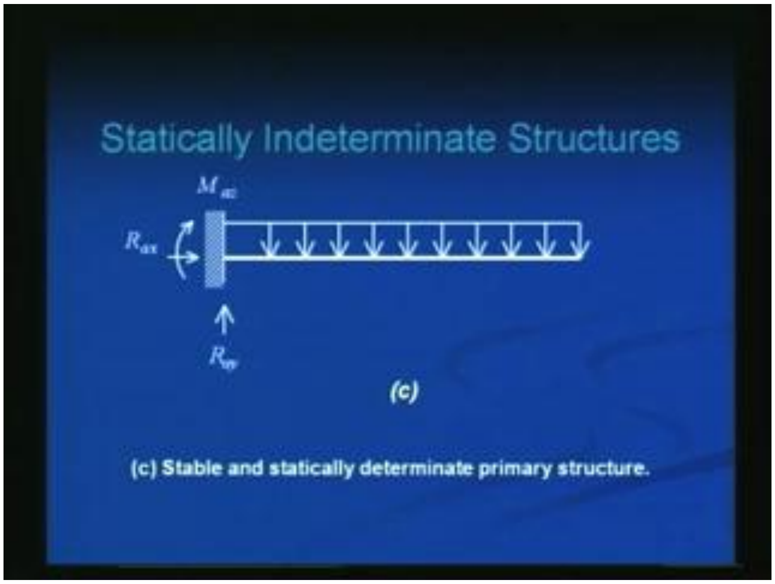

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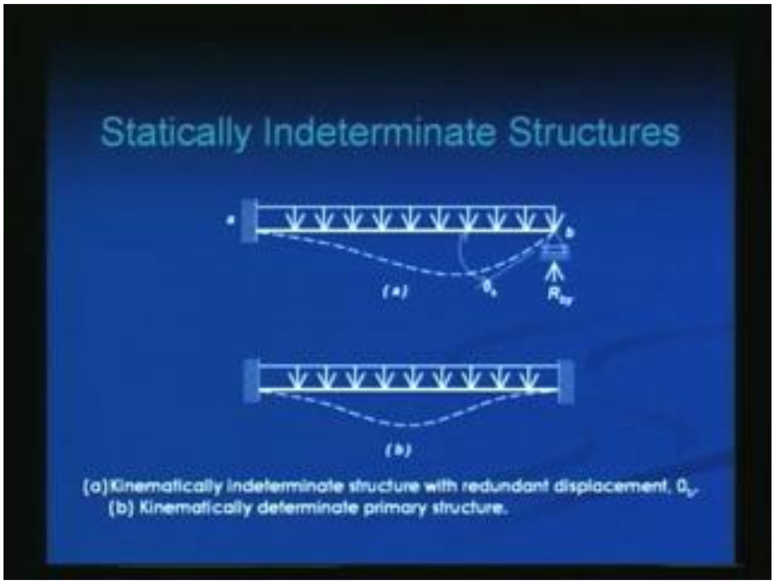

Now let us look at this particular member. Look at the top. How many Degrees of Freedom do you think it has? There is one frame and one member how many joints? 2 joints. There are 2 joints here; one here and one here. How many unconstrained Degrees of Freedom? Two joints so you have 2 into 3 is equal to 6. You have six unconstrained Degrees of Freedom. Let us look at the restraints. This joint is fixed, how many restraints? 3. What about this restraint? 1. So how many restraints in total? 4. That means unconstrained 6 minus 4 is equal to 2 Degrees of Freedom for this structure. What are the two Degrees of Freedom? One is this displacement and one is this rotation. In addition to that I am going to say that this is actually rigid. This is implicit, I do not want to state it. Typically, for a beam, unless you explicitly allow axial deformation, you assume it to be axially rigid. So if it is axially rigid constraint is 1, Degrees of Freedom is 1 which is this rotation. So Kinematic Indeterminacy is 1. How do I find out Static Indeterminacy? 3 plus 1 is equal to 4. How many equations of equilibrium? 3. So what is that Static Indeterminacy? 1. This structure has a single Kinematic Indeterminacy and a single Static Indeterminacy.

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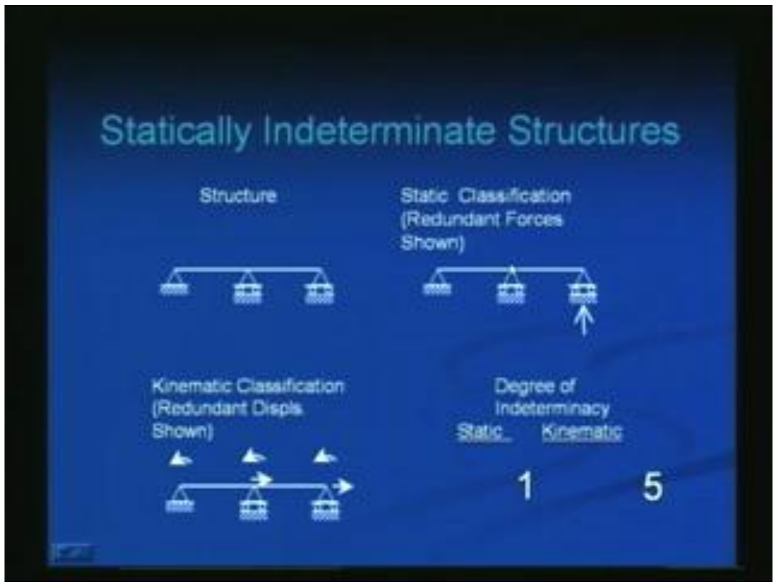

Let us look at a few other structures. This is a multispan beam. I need to find out its number of Degrees of Freedom and the Static Indeterminacy. If I remove this it becomes statically determinate so there is only one Static Indeterminacy. How many kinematic? It is 1, 2, 3 so 3 into 3 is 9. How many restraints? It is 1, 2, 3, 4 so totally 4 restraints. Then 9 minus 4 is 5. Static Indeterminacy is 1 and Kinematic Indeterminacy is 5.

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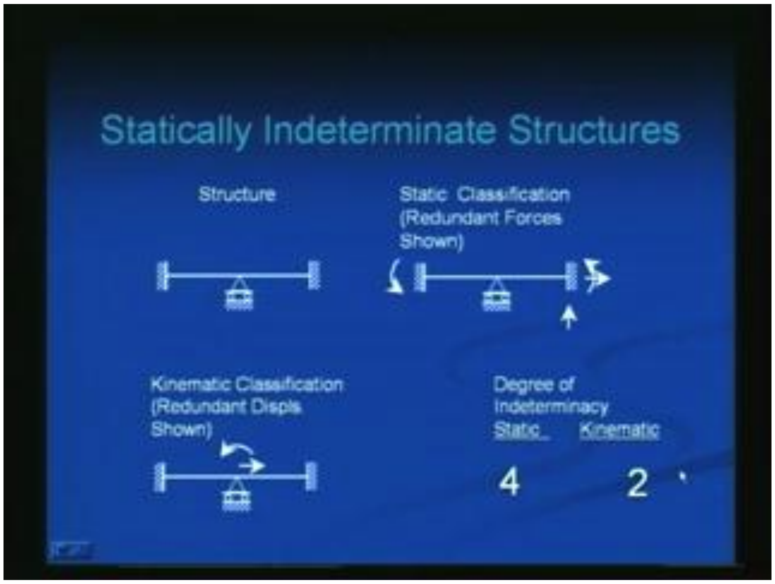

Let us look at another structure. In this structure how many redundant forces? I am not going to go into this; if I am release this and I release this, totally I will get a simply supported beam. It has 4 redundant forces, so Static Indeterminacy is 4. Kinematic: it has three joints 1, 2, 3 so 3 into 3 is equal to 9. Restraints: 3 here, 3 here and 1 here. So 2 Degrees of Freedom and you have shown the 2 Degrees of Freedom.

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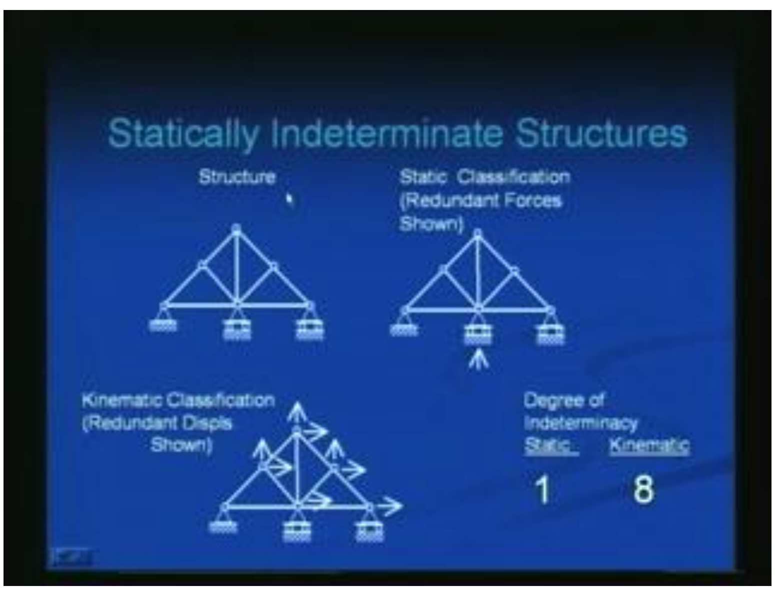

For a truss, remember that each one is triangulated so it is internally determinate and if I remove one of these it becomes 3 and becomes externally determinate also. This is the only redundant force. The Static Indeterminacy is 1 and for the kinematic you have 1, 2, 3, 4, 5, 6: 6 into 2 so 12. 12 - 4 restraints 2 1 1: 4 restrains gives you Kinematic Indeterminacy 8.

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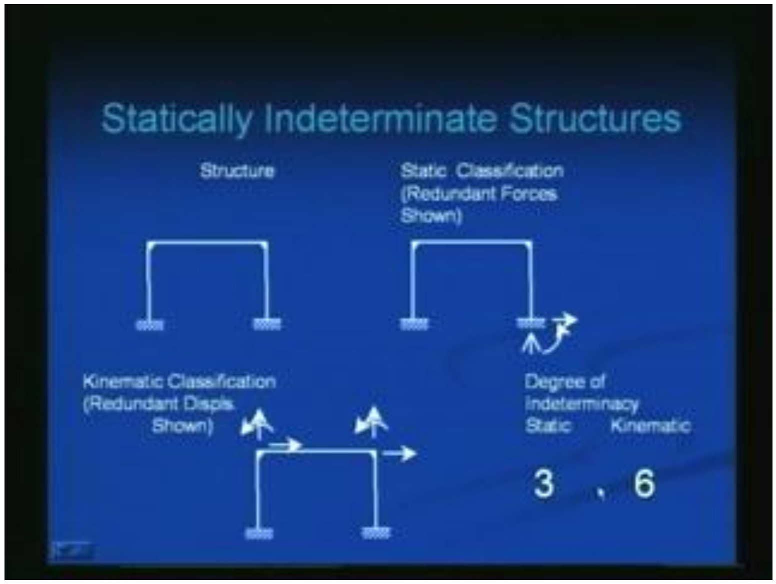

Look at the frame structure; we have already seen this. It is 3 and 6. Static Indeterminacy is 3, Kinematic Indeterminacy is 6 and that brings us to the end of this lecture. At the end of this lecture, I am going to assume from here that given any structure you should be able to find out its Static Indeterminacy and Kinematic Indeterminacy or Kinematic Indeterminacy I replace sometimes by degrees of freedom.

Thank you. We will continue next time by looking at some methods, from which we can use to do our structural analysis.