Structural Analysis II

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Lecture – 19

In the last lecture, we looked at a particular problem on a frame, which was loaded – we saw how to develop the equations. Then, I spent some time looking at how to solve that particular problem. I promise that today I will start off the lecture by actually giving you the solutions. I hope you have had an opportunity to look at those solutions.

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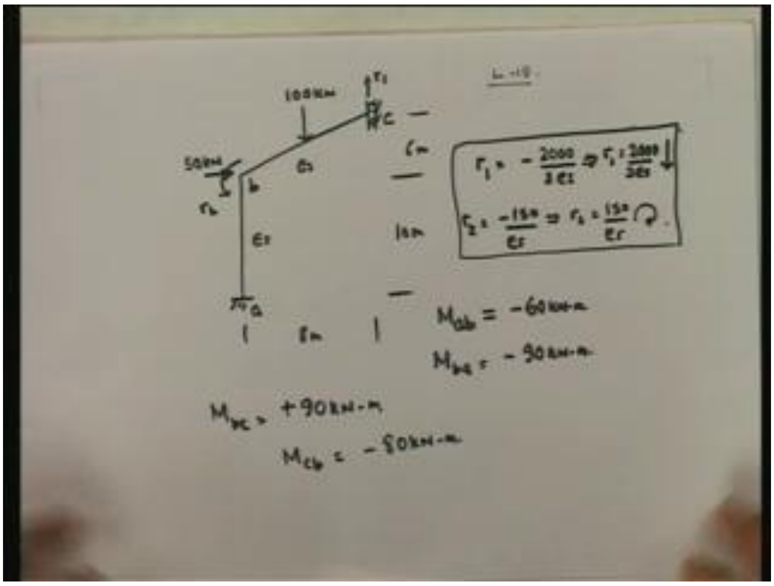

This is the final solution – the equations that I looked at. Remember that this was the abc frame with a 100 Kilonewton load here and 50 Kilonewton load here. r_1 and r_2 were the displacements corresponding to the degrees of freedom and when we solve for r_1 and r_2, we got r_1 is equal to minus (2000 by 3EI), which essentially means that r_1 is 2000 upon 3EI downwards; r_2 was minus (150 upon EI), which essentially means that r_2 is equal to 150 upon EI clockwise – I hope you got these answers too; if you have, pat yourself on the back because you have understood the displacement method reasonably well. Now that you know r_1 and r_2, if you substitute them back into the equations (the member end moment relationships), then you get M_{ab} is equal to minus 60 Kilonewton per meter, which essentially means that M_{ab} is clockwise 60 Kilonewton per meter; M_{ba} is equal to minus 90 Kilonewton per meter, which basically means M_{ba} is clockwise 90; M_{bc} is plus 90, which essentially means it is counterclockwise; and M_{cb} is equal to minus 80, which means it is clockwise – these can be obtained directly by substituting these values into the equations for M_{ab}, M_{ba}, M_{bc} and M_{cb}. What does that mean? Once we have found those out, what do we do? Last time I derived the expressions, I had listed out the expressions for the support reactions in terms of 50 plus H_a is equal to H_c and H_a is equal to minus (M_{ba} plus M_{ba}) upon 10.

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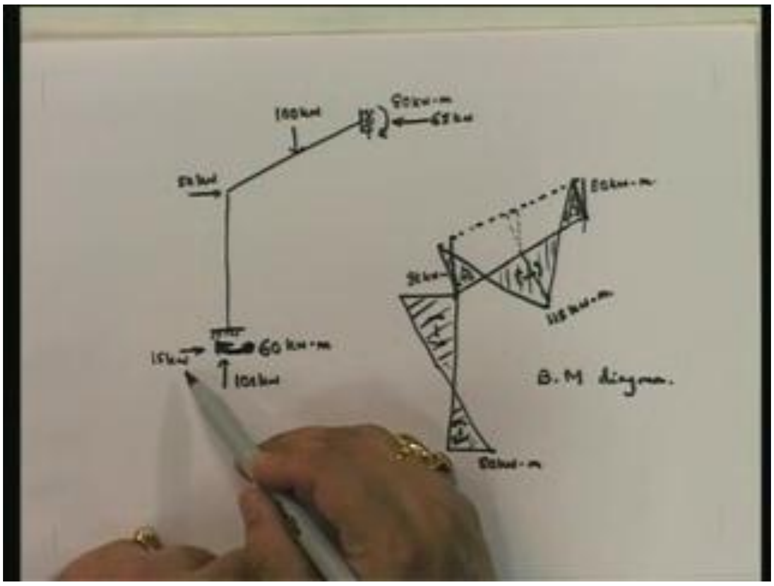

Having substituted M_{ba} and M_{ab} for H_a, we got that H_a is equal to 15. Therefore, H_c is equal to 65 – these are the support reactions. This is a clockwise moment of 60 Kilonewton per meter here, a vertical reaction of 100 Kilonewton, a horizontal reaction of 15 Kilonewton and a horizontal reaction of 65 Kilonewton in this direction, which is clockwise – these are the support reactions. Once you know those, you can always draw the bending moment diagram and this is what the bending moment diagram looks like. It is linear here, continuous here; this is superposed on top of this, so at the center of the beam, this is 115 Kilonewton per meter; and at the end of the beam it is 90 and 80 here and 90 and 60 here.

This is the sense of the bending moment, this is the tension on the top; on the left hand side here, tension is on this side; here, tension is at the top; here, the tension is at the bottom ; here again, the tension is at the top. I hope this is exactly what you have got from your solution. I just gave you the solution to this particular problem. Remember this solution. It is interesting to note here that almost everywhere…. What is the maximum bending moment here? The maximum bending moment is on the right hand side is 60, on the left hand side is 90, on the top it is 90 and on the bottom it is 115. Fairly uniform. What are the displacements? The displacements are 667 upon EI and 150 clockwise, downwards, this is clockwise. Now, let us look at what happens to that particular problem where all that we have done is taken this and made it into a… we have removed the fixity here so that this becomes a hinge roller. By the way, before I end this particular problem, I want to tell you that this actually is the part of a particular problem that we solve in general. This is actually the solution to this problem.

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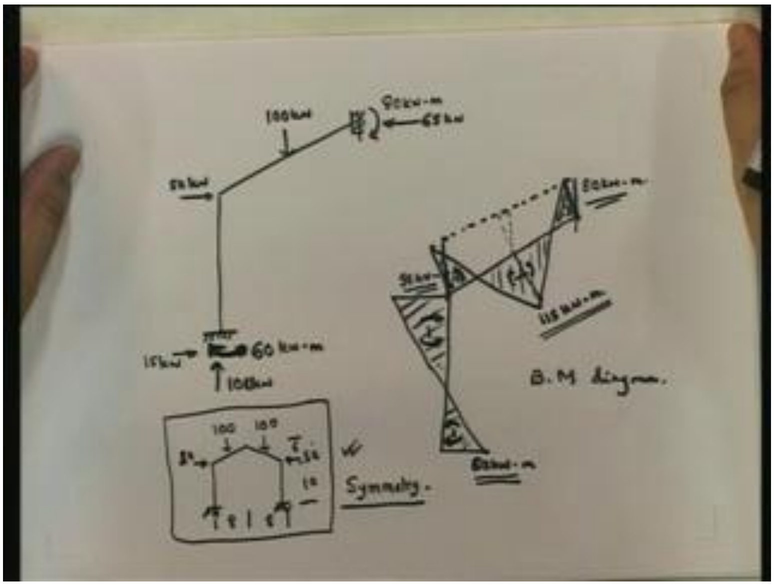

This portal frame problem with vertical loads of 100 uniformly distributed and an equal and opposite lateral load of 50, 50, essentially using the fact that this structure…. Here again, this would be 8, 8, this would be 10 and this would be 6. This is a portal frame and this portal frame by using symmetry can simplify to this particular problem. Although this particular problem that I am solving does not look like a real problem, it is actually a very real problem. This is the solution to this problem using symmetry. I will take an opportunity to discuss this particular topic of how to use symmetry to make structures simpler so that they have lesser number of degrees of freedom a little bit later in this particular course. Remember that this problem is actually not a problem that is constructed just like that – it is actually a real problem.

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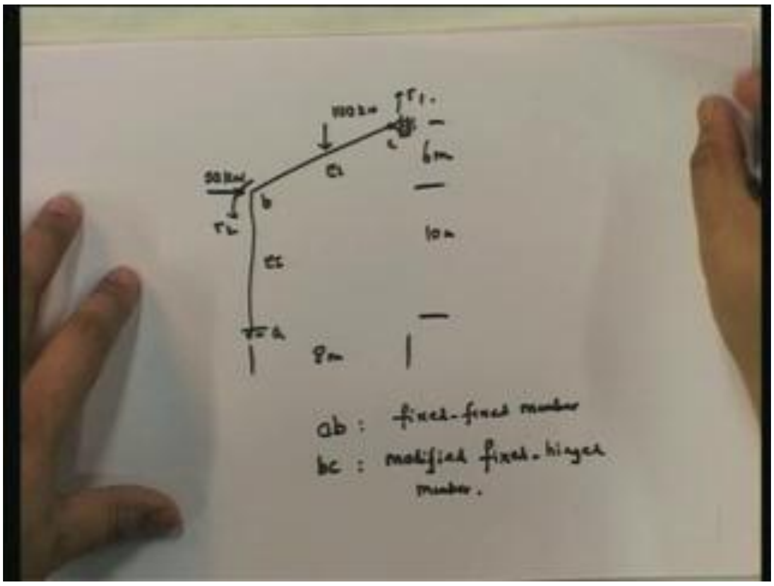

Now, we come to the next problem that we have. If you look at the next problem, there are identical loads but the only thing that we have done is, we have removed the fixity over here so that the moment at this point is 0, so that it becomes essentially a hinge roller support; fixed support, hinge roller support. In this particular case, what is the difference in the solution process? Member ab continues to be a fixed-fixed member because it is fixed at one end and continuous at another end. However, we made bc into a modified fixed-hinged member because it is continuous here, so it is fixed; this is an end support here – we know that the moment at this particular point is equal to 0, so that makes bc a modified fixed-hinged member. Let us see how we proceed with that.

We already know that this has two degrees of freedom but there is a third degree of freedom which is the rotation at this point but since we are considering the fact that the moment at this point is 0, this is not an essential degree of freedom and so we are left with two essential degrees of freedom in this particular structure. Having identified the two degrees of freedom, the first step is to write down the member force deformation relationships.

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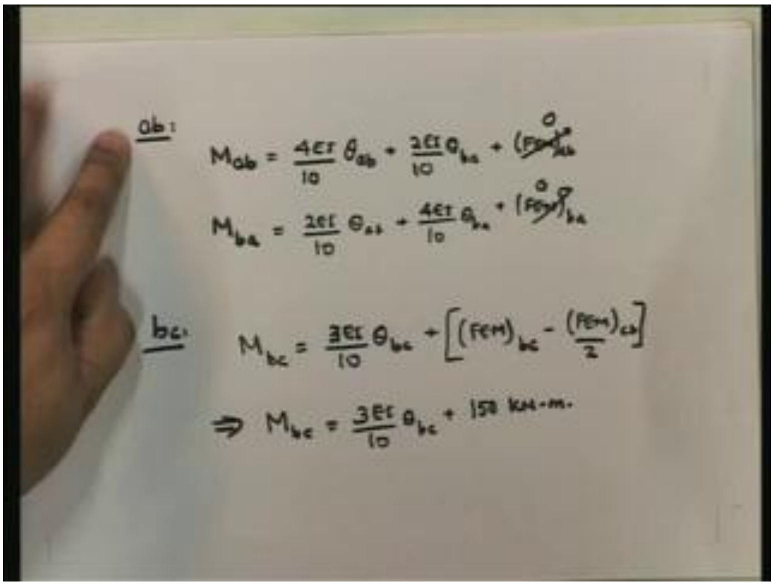

Here ab is a fixed-fixed member, so M_{ab} is equal to 4EI by 10 (since L is equal to 10) into theta$_{ab}$ plus 2EI upon L into theta$_{ba}$ plus fixed end moment at ab. Since there is no load in member ab, obviously the fixed end moment is 0. Then, M_{ba} is equal to 2EI by 10 theta$_{ab}$ plus 4EI by 10 theta$_{ba}$ plus fixed end moment at ba, which is also equal to 0 – that is the force member relationship for M_{ba}; note that the way we have defined it in this particular equation actually turns out to be…. What are theta$_{ab}$ and theta$_{ba}$? Remember that these are the rotations of the tangent from the chord joining ab – remember that; theta$_{ab}$ and theta$_{ba}$ are rotations of the tangent at a and b, respectively, considering the rotation from the chord joining a and b; remember that; I just wanted to tell you that this is the solution.

Now for bc, since bc is a modified member, M_{bc} is equal to 3EI upon L and L is 10 (because 8 and 6, 10) into theta$_{bc}$ plus (fixed end moment at bc minus (fixed end moment at cb upon 2)). Remember this particular thing? Here, what are these fixed end moments? These are the fixed end moments in a fixed-fixed beam such that when the load is…. Remember that we had evaluated these two quantities last time. This turned out to be plus 100 Kilonewton per meter and this had turned out to be minus 100 Kilonewton per meter. When you substitute that into this equation, you get 150 Kilonewton meter. Therefore, M_{bc} becomes equal to 3EI upon 10 theta$_{bc}$ plus 150. We know that M_{cb} is equal to 0 and we do not need to know theta$_{cb}$ because we have condensed out theta$_{cb}$. Remember that theta$_{cb}$ is not equal to 0; theta$_{cb}$ is not equal to 0 – remember that. In the modified fixed-hinged member, theta$_{cb}$ is not equal to 0. However, we consider the fact that M_{cb} is equal to 0 and so theta$_{cb}$ can be statically condensed out and we are only left with theta$_{bc}$; theta$_{cb}$ is not an essential deformation quantity required to define the force member relationship in bc. We have got the relationship for M_{ab}, M_{ba} and M_{bc} in terms of its member end rotations. Remember that this theta$_{bc}$ is also from the chord to the tangent. What is the next step? The next step is to find out the kinematic relationship.

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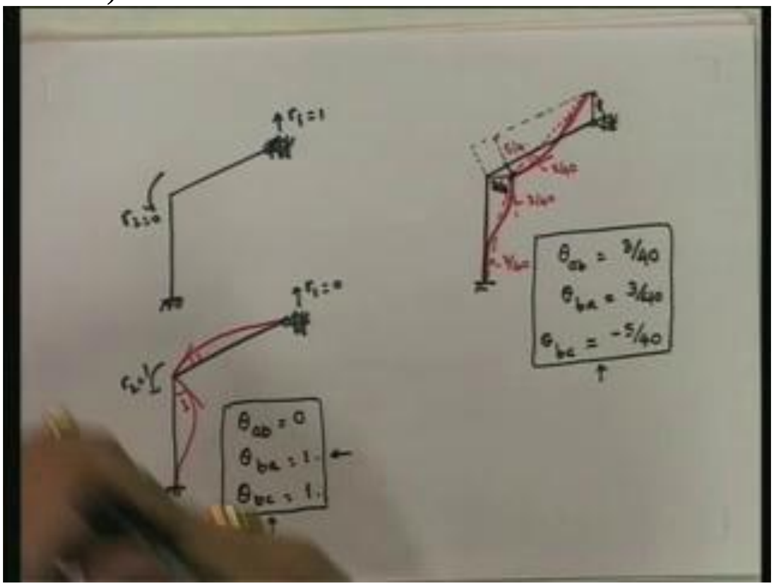

First, we put r_1 is equal to 1 and r_2 is equal to 0 and second, we put r_2 is equal to 1 and r_1 is equal to 0. First and foremost, for this, this is simple and it turns out to be this way and this is this way, this is equal to 1 and this is equal to 1. Therefore, what we have here is that for r_2 is equal to 1, theta$_{ab}$ is equal to 0, theta$_{ba}$ is equal to 1 and theta$_{bc}$ is equal to 1 – these are the only essential degrees of freedom that we are interested in and we have evaluated them. Note that theta$_{cb}$ does not have to be equal to 0 because this is a hinge and we acknowledge the fact that the moment at this point has to be 0 and we do not need to find this out. If you look at this one, note that the displacements of the points remain exactly the same as we had computed last time, so this is 3 by 4 and the displaced shape looks this way. Remember that here, r_2 has to be equal to 0, so the tangent at this point has to remain straight. Similarly, the tangent over here has to remain straight so this goes, but over here, it does not have to remain straight because it is a hinge. This is 1, this is 3 upon 4 and you can compute it – remember last time we had done it – if this is 3 by 4, then this is going to be 9 upon 20 and this is going to be 4 upon 5, so that is 16 by 20; 9 by 20 plus 16 upon 20 – we had evaluated that this was equal to 5 upon 4.

If I draw the chords, because ultimately it is the tangent from the chord, my chord goes this way. This is my angle – this angle is equal to 5 upon 4 divided by 10, so it is 5 upon 40; this angle is equal to 3 by 40 and this angle is equal to 3 upon 40 – these are our rotations. Therefore, theta$_{ab}$ is equal to plus 3 over 40 (from the chord to the tangent, it is anticlockwise), theta$_{ba}$ is equal to plus 3 upon 40 (from the chord to the tangent anticlockwise, so positive) and theta$_{bc}$ is equal to minus (5 upon 40) because from the chord to the tangent, it is anticlockwise, so it is minus.

Note something very interesting: if you look at these values and if you look at these values, the fact that this point has become a hinged roller and not a fixed roller as earlier has done

nothing to the shapes, excepting for the fact that at this point, you do not have 0 rotation but instead, you have 0 moment. In other words, in the earlier case, it had gone like this and gone like this because this cannot rotate. Here also, this would go like this and then come like this because it cannot rotate, but as far as these values are concerned, they do not change at all because the tangent and the chord remain exactly the same. Therefore, these things are not determined closely by the fact that this fixed roller has become a hinged roller.
The point that I would like to make is that the kinematics definitely depends on the support conditions. However, if we change a fixed support condition to a hinge support condition and keep the geometry the same, it has no effect on the kinematics, on the overall kinematics. Of course, as I said, the displaced shape may look different, because if it was fixed, it looks like this and would look like this. However, as far as the kinematics is concerned (that is the relationship of the member and rotations with the degrees of freedom of the structure), they are no different from what we had evaluated last time. Thus, this is an important point to note that kinematics is essentially driven by geometry rather than by changes in support conditions.

Of course, one thing is there: in this particular case, it has not changed. Why? Because the support condition change has actually introduced a degree of freedom; however, because we have used the modified method, we have the same number of degrees of freedom and for those degrees of freedom, the kinematic relationships remain the same. However, if we had not used the modified beam, then we would have had another additional degree of freedom and then of course, we would have to do the kinematics of that degree of freedom; remember that. That is the point that I am trying to make to you. Having done that, let us now write down the relationship.

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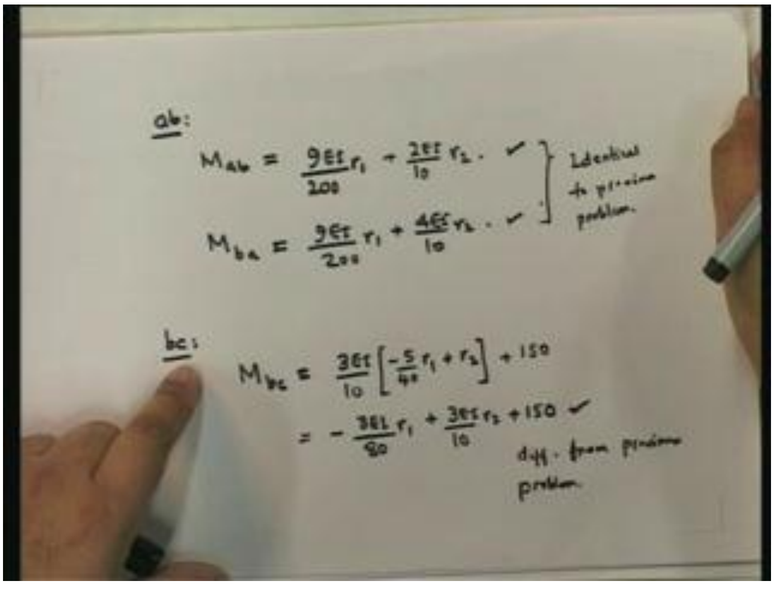

For ab, since the kinematic relations are the same and the member force deformation relationships are the same, M_{ab} and M_{ba} are identical to what we had computed when the c support was fixed. It has no bearing on the member end force deformation relationship in ab and therefore, I have just written down the value of M_{ab}, which of course incorporates that theta$_{ab}$ is equal to 3 by 40 into r_1 plus 0 into r_2 and that theta$_{ba}$ is equal to 3 by 40 into r_1 plus 1 theta into r_2. If you substitute those into the forced deformation relationships, you get M_{ab} in terms of r_1 and r_2 in this manner. Similarly, if you substitute again theta_{ab} and theta_{ba} in terms of r_1 and r_2 into the member end deformations, you get this M_{ba}. This is identical to the previous problem; note it.

Let us look at bc now. Because of the fact that c is a hinge, we had to write a modified member end force relationship which only relates the moment at bc with the theta at bc, because we had to incorporate the fact that M_{cb} is equal to 0. I have substituted theta_{bc}; if you look at the kinematics, theta_{bc} is equal to minus (5 by 40) into r_1 plus 1 into r_2; if I substitute that into this and rewrite it, I get M_{bc} is equal to minus (3EI by 80 into r_1) plus 3EI by 10 into r_2 plus 150 – these are our relationships. Note that this relationship (Refer Slide Time: 24:35) is different from the previous problem. Why? Because bc in the previous problem was a fixed-fixed member whereas in this particular problem, it has become a modified fixed hinge. We have written M_{ab}, M_{ba}, M_{bc} in terms of r_1 and r_2. What is our next step? Our next step is to write down the equilibrium equations and for that, we will take help of our virtual work principle – the principle of virtual displacement and get the first equations.

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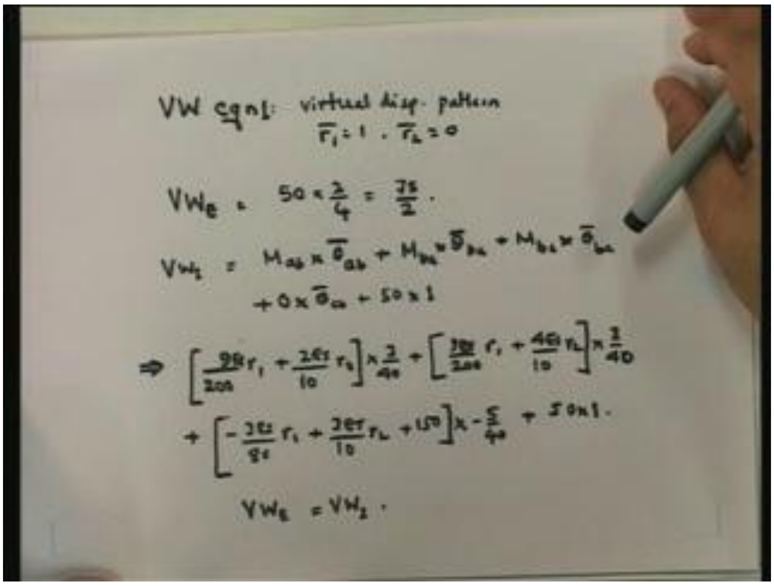

We have two unknown displacements and therefore we essentially need to write down two equations. The first equation, the virtual displacement pattern is r_1 is equal to 1 and r_2 is equal to 0; the bar actually gives the fact that they are virtual; this is a virtual displacement pattern. If I substitute those in, you will see that the external virtual work is 50 Kilonewton is undergoing a 3 by 4 virtual displacement, so real force into virtual displacement is the virtual work – that is the external virtual work. The internal virtual work is going to be equal to M_{ab} into $(\text{theta bar})_{ab}$ plus M_{ba} into $(\text{theta bar})_{ba}$ plus M_{bc} into $(\text{theta bar})_{bc}$ plus now M_{cb} is 0, so 0 into $(\text{theta bar})_{cb}$ plus 50 into 1; that is the reason why we have no interest in finding out what $(\text{theta bar})_{cb}$ is; it is because we are going to multiply it with 0.

We only need to know theta_{ab}, theta_{ba} and theta_{bc} – that we know; in terms of r_1, that is 3 by 40, 3 by 40 and minus (5 upon 40). I substitute M_{ab} from the previous equation and multiply it by this plus this plus M_{bc}, which is this multiplied by minus (5 by 40) plus 50 into 1. This is the support reaction due to the member load of 100 Kilonewtons – that, if you remember I had told you, is something that you have to consider; that is the support reaction that goes up by 1, so therefore the virtual work is 1. Obviously, the virtual work equation says the external virtual work is equal to internal virtual work, so all we do is we take this and equate it to this. If we do that, what happens? I have actually gone and multiplied all of those.

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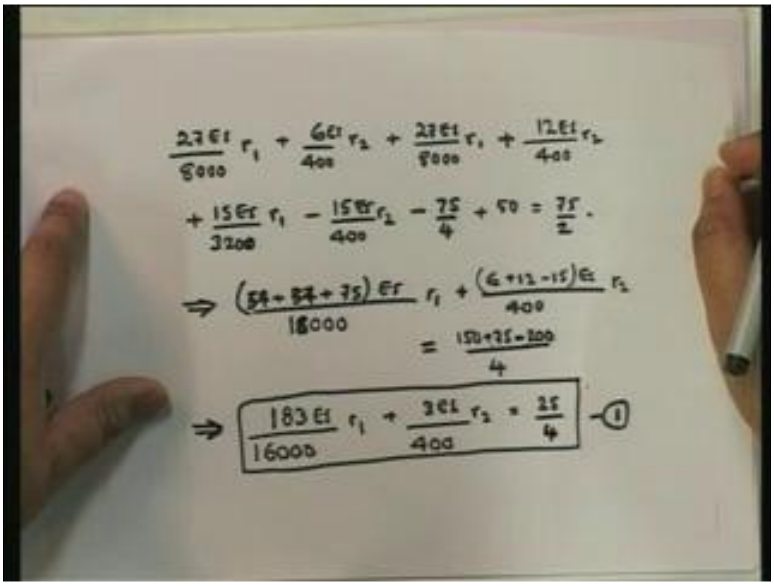

This is the work done by M_{ab}, this is the work done by M_{ba}, this is the work done by M_{bc} and so this is the external virtual work and this is my equation. If I plug in these, this is 8000, 8000 and 3200, so LCM is 16000; if I put that, this becomes 54 plus 54 plus 75 – that is r_1; then, 6 plus 12 minus 15 upon 400 EI into r_2 is equal to … this is 75 by 2, so by 4, it will become 150; this minus comes here, plus; and this 50 goes here, becomes minus 200. Substituting all of those, what I get is 183EI upon 16000 into r_1 plus 3EI upon 400 into r_2 is equal to 25 by 4.

Please note that these are simple numbers and that is the reason why I am actually going into this problem. Otherwise, as far as you are concerned, you can actually take out these and make them into fractions and do the solution; you will get the same thing, only thing is that these will be in decimal points; it does not matter. I have done it this way because I could do it; in some other particular problem, since it gets too complicated, I might actually just calculate on the calculator itself and put down in decimal engineering digits. I have kept it this way but I leave it up to you to do it any which way you want to. That is the first equation.

Now, for the second equation, we need a completely independent virtual displacement pattern so that we can get an independent virtual work equation. Note that I cannot highlight enough that to get two independent equations, you require two independent virtual displacement patterns. Since getting independent virtual displacement patterns is hard, what we tend to do is we know two independent virtual displacement patterns automatically. They are first, the one where r_1 is equal to 1 and r_2 is equal to 0 and the second one where r_1 is equal to 0 and r_2 is equal to 1 – we know that these are independent patterns. As I had told you earlier, you could take anything else and get, as long as you use two independent patterns, because if you use two independent patterns, then and only then will you get two independent equations, which you can use to solve for r_1 and r_2. I use obvious independent patterns and that is why I am using r_1 is equal to 1, r_2 is equal to 0 and the other one r_1 is equal to 0 and r_2 is equal to 1.

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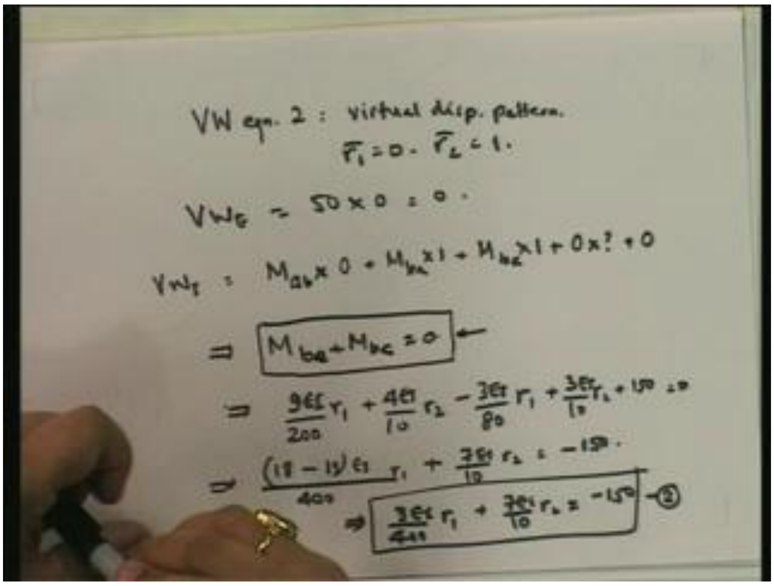

For r_2 is equal to 1, the 50 Kilonewton does not undergo any load, so the work external virtual work is equal to 0. Internal virtual work is M_{ab} into theta$_{ab}$, which is 0, plus M_{ba} into theta$_{ba}$, which is 1, plus M_{bc} into theta$_{bc}$, which is 1, plus 0 into this value – we do not know what it is but since 0 into anything is 0, we do not care – plus this is the work done by the 50 Kilonewton, because it does not undergo any load. If you put this in, you get this equation. Remember I told you that if you use the rotational kinematics, you essentially get back the equation that you would get by taking the equilibrium of joint b. Substituting for ba and bc into this and then putting the terms together, what we get is 3EI upon 400 into r_1 plus 7 EI upon 10 into r_2 is equal to minus 150. This is my second equation and I know that this is independent because my virtual displacement pattern is independent of the other. Now I write down the two equations.

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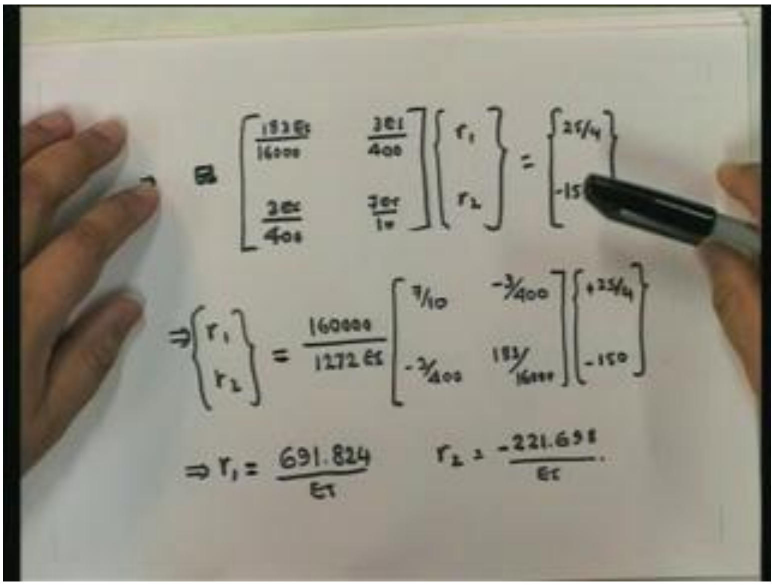

This is what you get; look at this. This is my first equation, which is 183EI by 16000 into r_1 plus 3EI by 400 into r_2 is equal to 25 by 4; the second one is 3EI upon 400 into r_1 plus 7EI upon 10 into r_2 is equal to minus 150. I have written these equations in matrix form just to be able to solve it. Therefore, r_1 and r_2 will be the inverse of this into this. The inverse of this is written in terms of 1 upon the determinant; 1 upon the determinant is this; the determinant of this is 1272EI upon 160000, so inverse is this; the cofactor of this is this (Refer Slide Time: 33:11), the cofactor of this is this, the cofactor of this is the minus of this, the cofactor of this is minus of this.

Again, like last time, this matrix is symmetric. The reason for it to be symmetric is the force at a particular point that gives the displacement at another point. By the Maxwell-Betti reciprocity theorem, you know that the force load at one point that gives a displacement at another point is the same as the load at the second point giving a same displacement at the first point; and this is essentially because of that reason. Remember when we are using the force method, we got that the flexibility coefficients were the same – these are known as stiffness coefficients. What are stiffness coefficients? Stiffness coefficients are the force to produce a unit displacement – that is stiffness. What is flexibility? The displacement due to a unit load. This is the load required to get unit displacement. You can see that flexibility and stiffness are actually the opposite of each other and if you take a single spring, the flexibility of the spring is equal to 1 upon the stiffness of the spring – the stiffness constant, so the flexibility constant is equal to 1 upon the stiffness constant. This goes that they have to be the same; if you do not get them to be the same, you have done something wrong somewhere. Having got this and putting this in, I get r_1 is equal to 691.824 upon EI and r_2 is equal to minus (221.698 upon EI). Let us compare that to the situation where you have the fixed case.

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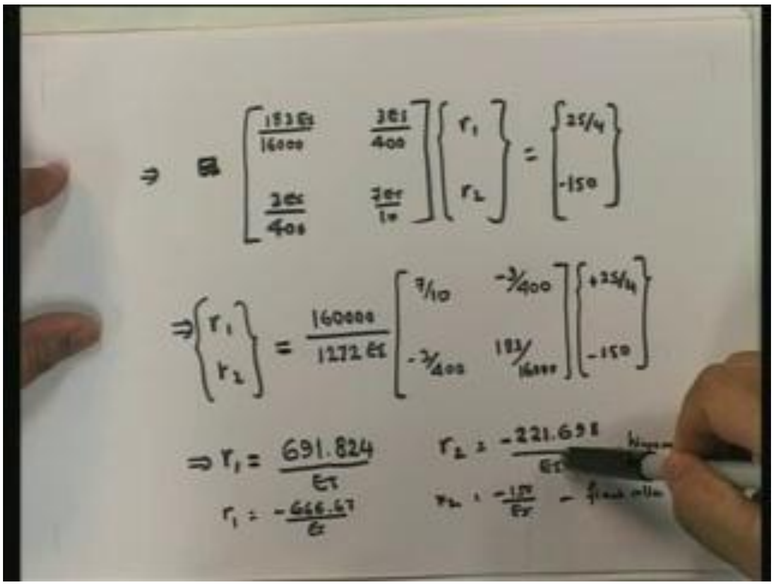

In the fixed case, r_1 was equal to minus (666.67 upon EI) and r_2 was equal to minus (150 upon EI) – that is for fixed roller; this is for hinge roller. Let us look at the effects of removing the fixity at this point to make it into a hinge, in other words, eliminating the rotational restraint – what that has done to the actual displacement of the structure under the same loads; the loads for both the structures are identical, the geometry is identical, the only difference between the two structures is that in the previous case at c, you had a fixed roller and in the second case that we are dealing with right now, it is a hinge roller. All we have removed is the restraint of rotation and see what that has done to the displacement shape. It has had a significant effect. Look at the rotation. The rotation has gone up significantly. More interestingly, you see that r_1 was downwards when it was fixed and now that you have removed the restraint, it is actually upwards. Is it not very interesting? You would think that you have actually just taken a support at one end and all I have done is removed the rotational restraint but the whole displacement pattern now changes. The support at c – instead of moving down, it is actually moving up and the value is much larger.

This brings us to a particular point: when I removed the restraint, what did I do to the structure? Think about it. When I put more restraints on the structure, what do I do to the structure? When I add restraints to a structure, I actually make the structure stiffer. When I make a structure stiffer for the same load, what would you expect the displacements to be? Less, is it not? Look at that, it is less. Now, The point is when I remove the restraint, I made the structure more flexible and therefore, the displacements have to go up and they have gone up. Therefore, the behavior of the structure is actually reflected in the results that you get – this is very very important. The displacements have gone up and now I can put these displacements into the equations for the moments. These are the equations where M_{ab}, M_{ba} and M_{bc} are in terms of r_1 and r_2 and since I know r_1 and r_2, I can substitute into that equation.

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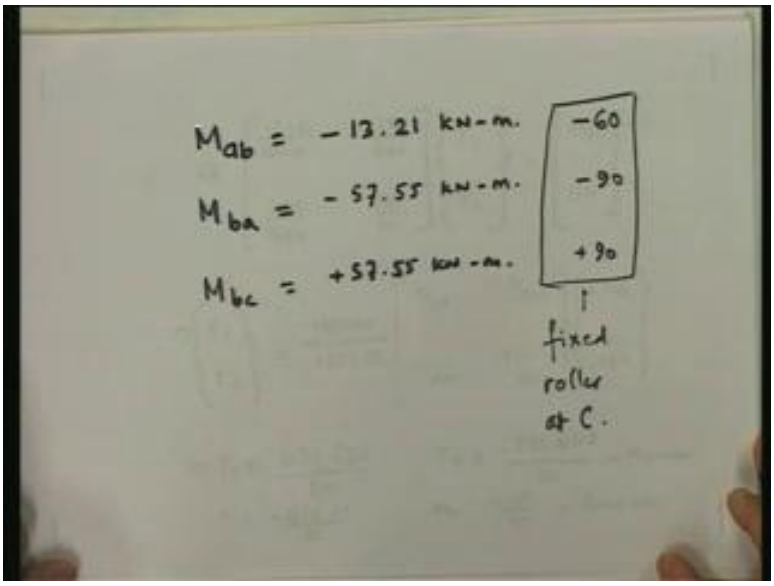

When I substitute into that equation, what I get is this: M_{ab} is minus 13.21, M_{bc} is minus 57 and M_{bc} is plus 57. What were they when it was fixed? This was minus 60, this was minus 90 and this was plus90. What has happened to the member end moments? They have gone down. Why? When I made the fixed roller into a hinge roller, what did I do? I increased the flexibility of the structure. Here is where I make the points that I have tried to make: when you make a statically indeterminate structure more flexible, a, you increase the displacements – we have already seen that the displacements are significantly larger than in the fixed roller case; and b, you make the member end moments smaller. Why? Because the structure is more flexible. When a structure is more flexible, it can actually displace to relieve the stresses in the structure; when it can deform more to release the stresses, the stresses (or in this particular case, the member end moments) reduce. Therefore, for a stiffer structure, what do you do? When you make a statically indeterminate structure stiffer, you reduce the displacements but you increase the member end moments. When you make it more flexible…. Why the member end? Because when you make it stiffer, it is less able to deflect and therefore, the deflection is small and the loads are essentially transferred through forces. When the structure is more flexible, it can deform and relieve the stresses. Therefore, the member end stresses are less. Does that mean that if you do not have a serviceability criteria…. This is where I come into the design aspects. You have two design aspects: one is to design for strength and two is to design for serviceability. Serviceability always puts a restriction on your displacements and the strength is so that it can resist the forces that it is subjected to. When you make a structure more flexible, what happens? Displacements go up. If you do not have a serviceability problem, then you might think that you get the displacements go up and the moments to go down and that you can actually make the structure sleeker because it needs to resist less forces, but you know this is incomplete information. Let us look at what happens to the bending moment diagram due to these – that is the next step that we are interested in.

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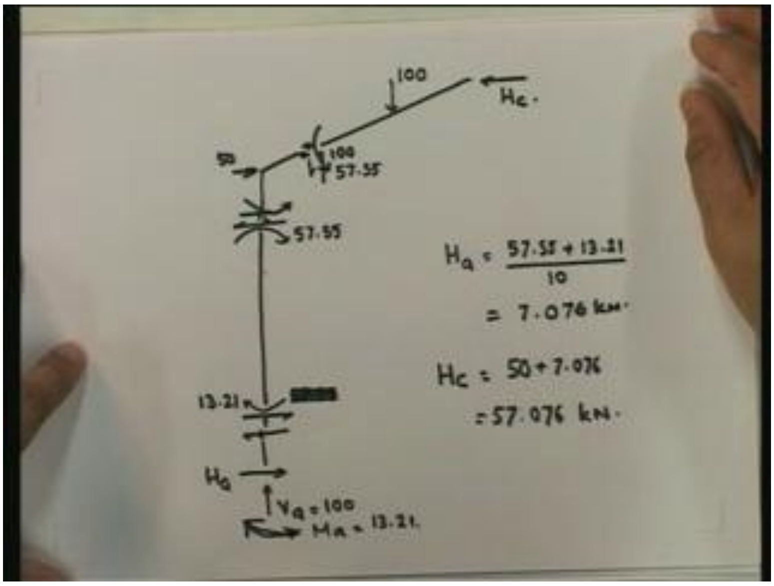

We have got the member end moments. Therefore in this particular case, here, the member end moment is minus 13.21 since it is clockwise, minus 57.5 since it is clockwise, plus57.35 is anticlockwise – these are the member end moments. This is the joint at b, this is the joint at a and this is the joint at c. From this if I take M_{ab}, since it does not have any loading, the shears at this point are going to be equal to this into this (Refer Slide Time: 43:47) is equal to this plus this. The couple generated by the shear is going to negate this moment and from that, we get H_a is equal to this plus this upon 10, that is equal to 7.076; that means H_a is in this direction and the value is 7.076. In the previous case, what was this? 15. H_c becomes 57.076. What was it before? 65. We seem to be going in the same direction, meaning that the member end moments and the support reactions seem to be all less. Still we think that things are going to improve, but now since I have got this, I can actually find out the support reactions and this is ultimately what the structure looks like.

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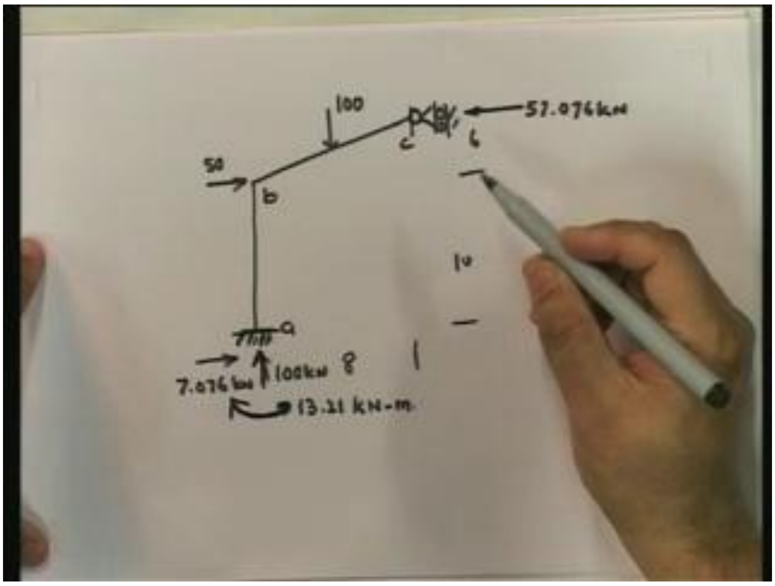

Once I have the support reactions, this becomes a statically determinate structure and I can draw the shear force bending moment diagram for this. If I do that, this is clockwise 13.21, this is 7.076, 100, since this is a roller, the entire 100 comes here and here, since it is a hinge, there is no moment, there is only 57. Till now, the structure is more flexible, the support reactions are less, we are seemingly…. In other words, if your serviceability is not a criterion, making the structure more flexible by removing a restraint of support seems to make the structure much better. However, let us look at the bending moment diagram. I leave it up to you to do it.

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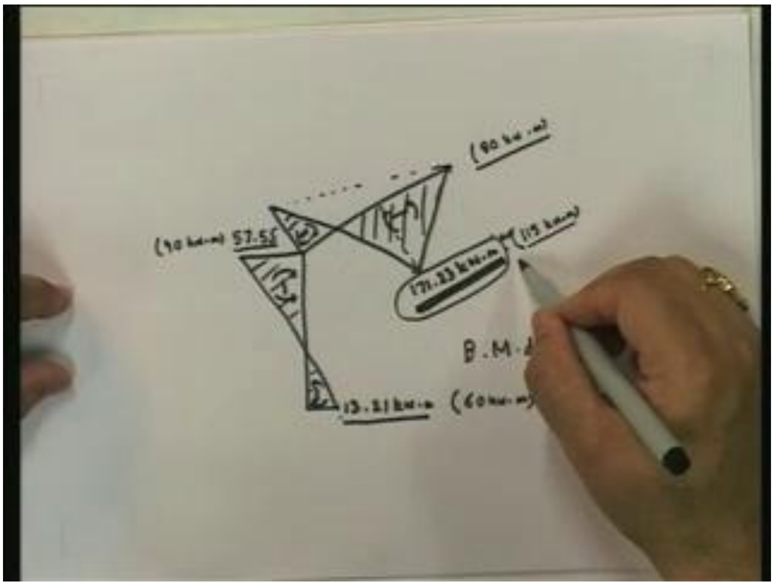

Remember I had told you that I am not going to be doing equilibrium for you. Once I have got the support reactions, you should be able to generate the bending moment diagram, shear force diagram for any member. I have drawn the bending moment diagram; this is the bending moment diagram. The bending moment over here has to be 13.21 because that is the support reaction; the bending moment over here has to be 0 because at this point, you have a hinge, so you cannot have a moment; and if you look at this, what happens here? What was this? 60 Kilonewton meter, 90 Kilonewton meter, this was 115 Kilonewton meter, and this one, which is 0 now, was 80 Kilonewton meter. Let us look at it from a design point of view: is flexibility good? As far as this member ab is concerned, it is good; it has actually reduced the bending moments significantly from the previous case. However, in this case, what has happened? In the previous case, this was 115 but now this is 171.23 – significantly more, significantly more. In other words, the member bc….

Until now, we were saying make it flexible, it is going to get much much better as long as you do not have a serviceability limit. In other words, the rotation and displacement can be anything that it wishes to be. Then, we thought that introducing flexibility was being wonderful because it was reducing all kinds of bending moments in the structure, but you have a problem: it has not. This particular one is more than a one and a half times and this member bc has to be designed for this. In other words, what has happened here is that making the structure more flexible does indeed increase the displacements and reduce the member end moments.

However, if any structure has a load on a member, then this is not a good idea because the maximum bending moment in the member goes up significantly if you have a member loading. Now, would you say that increasing the flexibility of the structure makes it better? No. In fact, by restraining the rotation here and making the structure stiffer, you may have increased the moments, however, you have made the moments much more even. The difference between this, this and this and this, this and this – the lowest moment here is 60 Kilonewton meter and the highest is 115; here, the lowest is 13.21, highest 171. There is a tremendous amount of bending moment variability and it implies that curvature goes up and you might actually have cracking problems, etc., if you have a reinforced concrete.

Therefore, by and large, increasing restraints in the structure is actually a good idea, of course as long as you do not keep increasing it to a point at which what happens is, the displacement goes down and almost the entire forces are transmitted. At that particular point, if you take restraints beyond a certain point, you reduce the effect of the loads, but the member end moments go up significantly higher and you no longer reap any benefits of making the structure stiffer. The point that I was trying to make is that there is an optimum stiffness for a particular type of loading that good designers always look for. Ultimately, that was my take on the behavior of structures but as far as getting the solution is concerned, you can see that if you have a structure, then the displacement method for a statically indeterminate structure… I am now ending the entire discourse on using displacement method for a statically indeterminate structure. I want to end it by going back through the steps.

The first step: determine the number of degrees of freedom of a structure, given all the restraints and constraints that you have – that is number one. Number two: once you have determined the number of degrees of freedom, you have to define the displacements corresponding to the degrees of freedom. Three: determine what each member is and write down the member end force deformation relationships. Four: determine the kinematic relationship between the displacements corresponding to the degrees of freedom and the member end deformations – that is where the kinematics comes in. Five: once you have got the kinematics, you can write down the member end moments in terms of the displacements corresponding to the degrees of freedom. Six: use the virtual work principle, specifically the virtual work displacements, to write down independent equations corresponding to each degree of freedom. Seven: solve those equations for finding out the displacements corresponding to a particular loading system. Eight: substitute those displacements into the member end displacement relationships to get the member end moments. Nine: once you have got the member end moments, do equilibrium and find out the support reactions and the bending moment diagram, shear force diagram, whatever you have to do – that is your analysis completed.

The next lecture is going to be my last lecture on the displacement method. Till now, I have only looked at loading in the member. Now I am going to look at other situations – other kinds of member loads. Up till now, I have only looked at flexural deformations and flexural member end force deformation relationships. These are not valid in trusses, so I shall look at trusses briefly so that you can apply the displacement method for all types of planar trusses and planar frames. Thank you very much.