Structural Analysis - II
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Lecture – 18

Good morning. Today, we are going to be solving some more problems on the applications of the displaced method and I am going to take up a specific problem.

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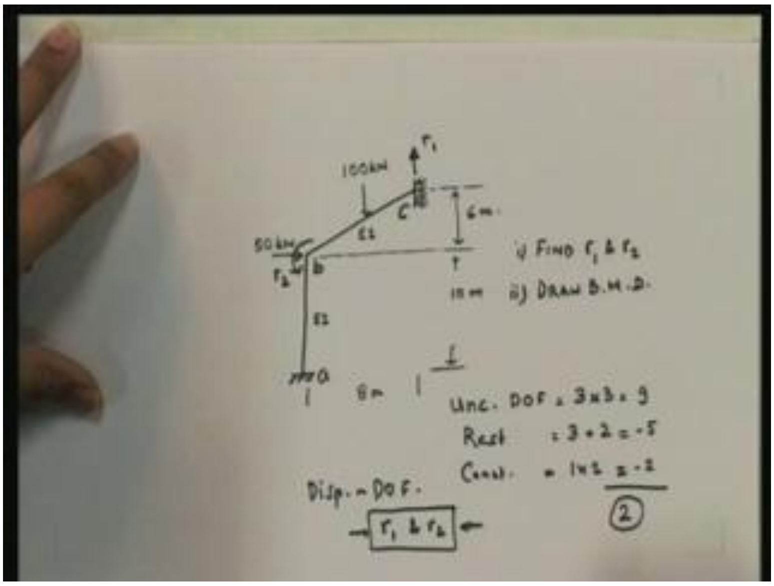

If you look at this particular problem, the various questions are one: find this displacement (Refer Slide Time: 03:19), two: find this rotation and three: find the bending moment. I will call this as r_1 and this as r_2. The question is one: find r_1 and r_2 and two: draw the bending moment diagram – this is the problem statement. Let us see how we can go about solving this problem. First and foremost, .… Let me go back to that problem itself. First, we need to find out the degrees of freedom; this is a, b, c. The unconstrained degrees of freedom are equal to 3 into 3 equal to 9; restraints are 3 here and 2 there, which is equal to minus 5; constraints are 2 members (1 into 2), axial rigidity is minus 2, so this is equal to two degrees of freedom and the two degrees of freedom are r_1 and r_2 – these are the displacements corresponding to the degrees of freedom. What is the next step?

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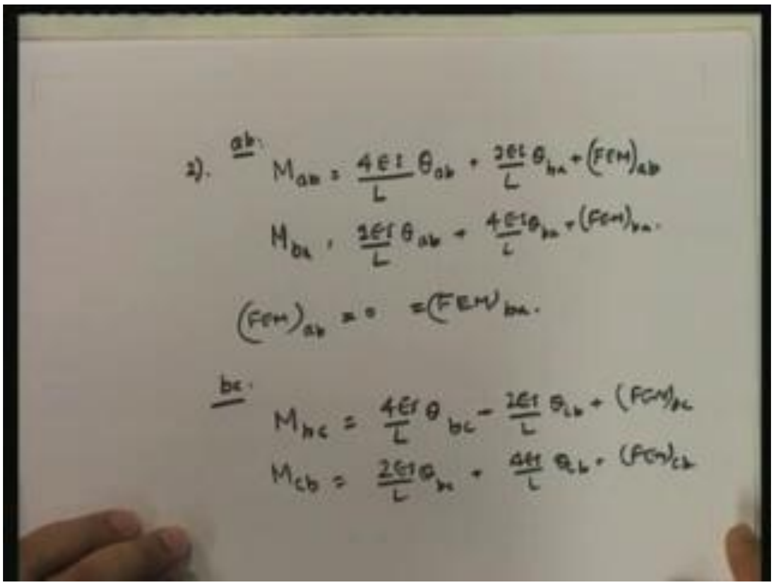

Step 2: for ab, what is the member forced deformation? M_{ab} is equal to 4EI by L $theta_{ab}$ plus 2EI by L $theta_{ba}$ …. We are doing the member deformation relationships and for ab, M_{ab} is equal to 4EI upon L $theta_{ab}$ plus 2EI upon L $theta_{ba}$ plus $(FEM)_{ab}$. Then, M_{ba} is equal to 2EI upon L $theta_{ab}$ plus 4EI upon L $theta_{ba}$ plus $(FEM)_{ba}$. Now, what are the fixed end moments here? $(FEM)_{ab}$ is equal to 0, which is equal to $(FEM)_{ba}$? Why is that? There is no member load on ab; since there is no member load on ab, the fixed end moments are automatically equal to 0. Now, we look at bc. M_{bc} is equal to 4EI by L $theta_{bc}$ plus 2EI by L $theta_{cb}$ plus $(FEM)_{bc}$ and M_{cb} is equal to 2EI by L $theta_{bc}$ plus 4EI by L $theta_{cb}$ plus fixed end moment at cb.

If you look at the end, this is fixed, therefore, ab is normal. In bc, c is a fixed roller and when it is fixed, there is a moment; since there is a moment, we have to use the original; if this had been a hinge, then of course we would have used the modified, but in this particular case, it is not and therefore, we are going to continue in this way. What about the fixed end moment at bc? We have to find that out; let us find that out. The next step is… The fixed end moment at bc and cb are going to be there because there is a load in member bc, so we have to find out what that is. How do we find that out?

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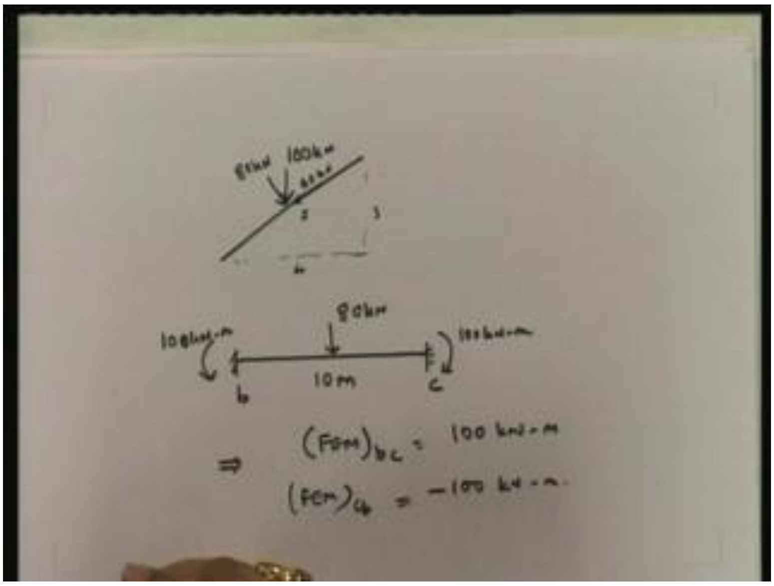

Note that there is a slight difference here: the member is inclined and the loading is this way, so I can actually write this load of 100 Kilonewton. This is a 3, 4, 5 triangle and therefore, if I write down the vertical component and then the horizontal component, the vertical component of this is going to be 80 Kilonewton and the horizontal component is going to be 60 Kilonewton.

Now, the horizontal component, since the member is axially rigid, there is absolutely nothing; therefore, this problem essentially becomes one where you have a 10 meters with 80 Kilonewton and we have to find out what the fixed end moments are for b and c. We know that it is equal to PL by 8; this becomes 100 Kilonewton meter, 100 Kilonewton meter; this implies that $(FEM)_{bc}$ is equal to 100 Kilonewton meter and the fixed end moment at cb is equal to minus 100 Kilonewton meter – this is something that we know as of now. What is the next step? We have written the member end deformations and we have found out the fixed end moments. The next step is to find out the kinematic relationship for each degree of freedom – the relationship between the member end deformations and the structural degrees of freedom. How do we do it? Well, kinematics.

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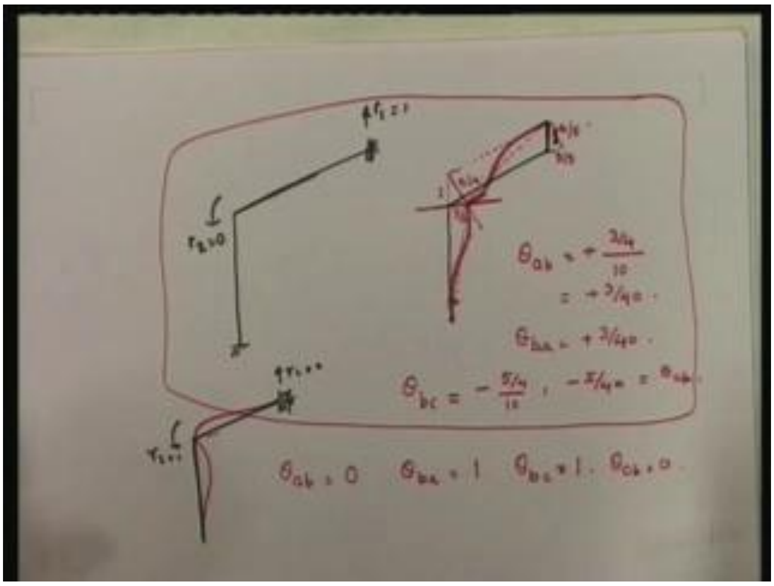

The first step is this: give r_1 is equal to 1 and r_2 is equal to 0. If you give that, what happens? This point is to move up by 1; this point can only move up because that is the only direction that it can go, so this goes up. How do I find out where this point goes? This is important; this is what I mean by kinematics. Therefore, what we do is, we let that go up by 1 – that is the only way it can go. We know where this point is (Refer Slide Time: 11:52). This point is fixed and therefore, it is not going to go anywhere.

How can this point go? This point can actually go on this. Now, let us look at this carefully. Since I have taken this up by 1, if you look at the components of this one, this is vertical (Refer Slide Time: 12:27), along the member and perpendicular along the member; these are the two things that we have to find out; you can find that out with the 3, 4, 5 triangle; please do the…; you will see that this is 3 by 5 and this is 4 by 5. In other words, what has actually happened is…. Suppose this point was not fixed, this could have gone up also with it by the same amount.

Here, the point to be noted is that in actual fact, what has happened is that it has gone along this way and this way, but you know that this point cannot go there; that means this member has become something like this if we displace it, but this cannot go there because this point can only move along this line. So, what has to happen? Since this member is this way, the only way this member can move is along this and when we take it along this, this is the point at which it is going to get the cut and that is the point where this member is going to be. What is that point going to be? Let us look at that. This is 1 and this is a 3, 4, 5 triangle, so if 4 is 1, that means this is going to be equal to 3 by 4 and this total distance is going to be 5 by 4. Once I know this point, I can now draw my displaced shape. Note that this cannot rotate, so this remains straight and this also has to remain parallel to the line; this will go in and here also, it cannot rotate, so this is our rotation.

If we look at this, from the chord, this angle gives θ_{ab} and what is that angle equal to? Theta_{ab} from the chord to the tangent is anticlockwise, so plus 3 by 4 (the displacement) upon 10, which is going to be 3 upon 40. What about theta_{ba}? It is the same thing 3 by 40 and positive (from the chord to the tangent), so plus (3 upon 40).

What about this one? The total displacement of this is 5 by 4, this is moved by 5 by 4 and the length is 10. Therefore, looking from the chord to the tangent, it is minus because it is clockwise from the chord. Theta$_{bc}$ is going to be minus (5 by 4 divided by 10), which is going to be minus (5 by 40) and theta$_{cb}$ is the same. We have found out theta$_{ab}$, theta$_{ba}$, theta$_{bc}$ and theta$_{cb}$ in terms of r_1; when r_1 is equal to 1; these are the values. Now, r_2 is much easier. All these computations are for r_1. For r_2 is equal to 1 and r_1 is equal to 0, the displacement pattern is going to be very very simple; it is going to be this. If we put this, what do we get as theta$_{ab}$? Theta$_{ab}$ is equal to 0. Theta$_{ba}$ is equal to plus 1 (from the chord to the tangent that is anticlockwise); similarly, from the chord to the tangent, so theta$_{bc}$ is equal to 1; and theta$_{cb}$ is equal to 0. We have done the kinematics and this kinematics is going to be important even when we do virtual displacement; so remember this kinematics.

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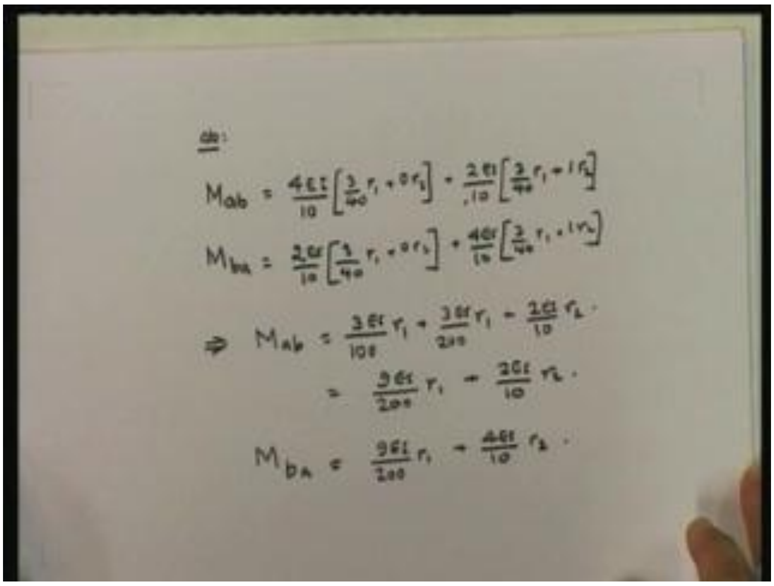

By substituting the kinematics into the equations, what do we get for ab? Let us put down values. M_{ab} is equal to 4EI upon L (which is 10) into theta$_{ab}$, which is (3 by 40 into r_1 plus 0 into r_2) plus 2EI upon 10 into (3 by 40 into r_1 plus 1 into r_2) and FEM is 0. M_{ba} is equal to 2EI upon 10 into (3 upon 40 into r_1 plus 0 into r_2) plus 4EI upon 10 into (3 by 40 into r_1 plus 1 into r_2). This implies that M_{ab} is equal to 3EI by 100 r_1 plus 3EI by 200 r_1 plus 2EI by 10 r_2, which becomes equal to 9EI upon 200 into r_1 plus 2EI upon 10 into r_2 – that is M_{ab}. Similarly, M_{ba} is equal to 3 by 200 r_1 (Refer Slide Time: 20:38) plus 3 by 100 (Refer Slide Time: 20:39), so we get exactly the same thing: 9 EI upon 200 into r_1 plus 4EI by 10 into r_2. These are M_{ab} and M_{ba} in terms of r_1 and r_2. Similarly, we can write down for bc.

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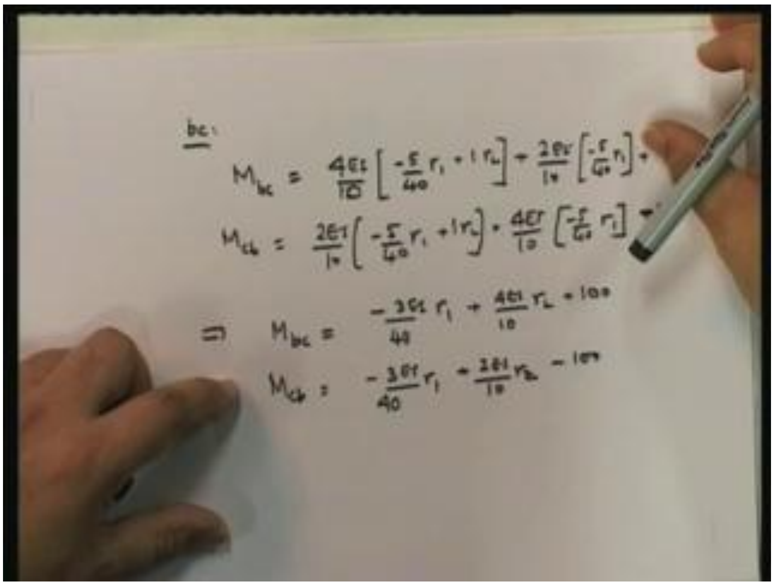

We get M_{bc} is equal to 4EI by L (which is 10) into theta$_{bc}$, which is minus (5 by 40 into r_1 plus 1 into r_2) plus 2EI upon 10 into theta$_{cb}$, which is minus (5 upon 40 into r_1) plus fixed end moment at bc, which is equal to plus100. Similarly, M_{cb} is equal to 2EI upon 10 into minus (5 upon 40 into r_1 plus 1 into r_2) plus 4EI by 10 into minus (5 by 40 r_1) minus 100. This implies that M_{bc} is equal to.… This is going to be minus 5 by 100 and then minus 5 by 200; 5 by 100 is 1 upon 20; 1 upon 20 and this is 1 upon 40; so this is going to be equal to minus 3EI upon 40 into r_1) plus (4EI upon 10 into r_2) plus 100. M_{cb} is equal to minus (3EI by 40 into r_1) plus (2EI upon 10 r_2) minus 100 – this is M_{cb}. I have got my M_{ab}, M_{ba}, M_{bc} and M_{cb}. Next, what I have to do is, I have to find out the work done by the loads, so I am going to write down the virtual work equation. The first virtual work equation is going to be written down for this as virtual displacement; if I put this as virtual displacement, then what do I get?

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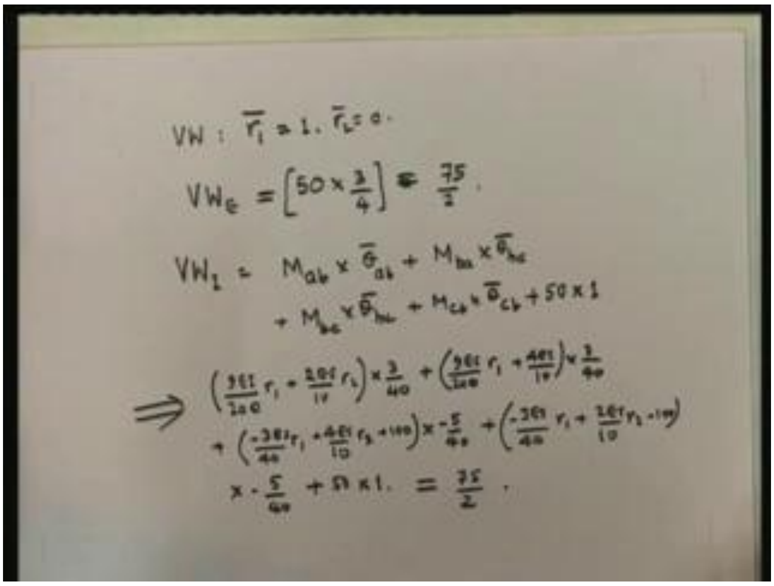

I am going to put the first virtual work equation as r_1 is equal to 1; since it is in an arbitrary displacement pattern, the first displacement pattern is going to be this pattern. My external work will be the load into 1. What is the external load? There is only one external load, which is 50 tons because that is applied at the joint. How much displacement does that undergo?

Since the load is applied here and it is undergoing three-fourth in the same direction, the external work done is 50 into 3 by 4. If you look at everything else, what is it going to be? Think about it. There is no other external load that does work; this is the only one that does work and so this is going to be equal to 75 by 2 – this is the external work in Kilonewton. The internal virtual work is done by all the moments undergoing the rotations: one moment is M_{ab} into the rotation that it undergoes, which is (theta bar)$_{ab}$ plus M_{ba} multiplied by (theta bar)$_{ba}$ plus M_{bc} multiplied by (theta bar)$_{bc}$ plus M_{cb} undergoing (theta bar)$_{cb}$. These are all the moments and they also take care of the work done by the shears. The only thing left behind is the load 100, which gives rise to reactions at the supports.

The reaction that I get is actually equal to 50 and 50 – vertical in both the directions. I just have to find out what is the work done by each vertical reaction. Is this point moving vertically? If you look at it, it is not, it is only moving horizontally, so this one does not do any work; however, this vertical reaction does work. How much work does it do? It does into 1. This is going to be equal to plus 50 into 1; this is my internal virtual work. Having written that, I can put internal virtual work equal to external virtual work and if I substitute all the terms that I have, you will see that this essentially becomes the following: (9EI upon 200 into r_1 plus 2EI upon 10 into r_2) multiplied by (theta bar)$_{ab}$, which in terms of r_1 is equal to 3 by 40 plus M_{ba}, which is equal to (9EI upon 200 into r_1 plus 4EI upon 10) into 3 upon 40 plus M_{bc}, which is equal to (minus (3EI upon 40) into r_1 plus 4EI by 10 into r_2 plus 100) multiplied by minus (5 by 40) (note that this work done is multiplied by the rotation; rotation being clockwise, it is negative work) plus M_{cb}, which is equal to (minus (3EI upon 40 into r_1) plus 2EI by 10 into r_2 minus 100) multiplied by minus (5 upon 40) – that is all the moments – plus 50 into 1 is equal to 75 by 2. That is my first equation; it is fairly complex, but I can always simplify it and write it in terms of r_1 and r_2 and a number.

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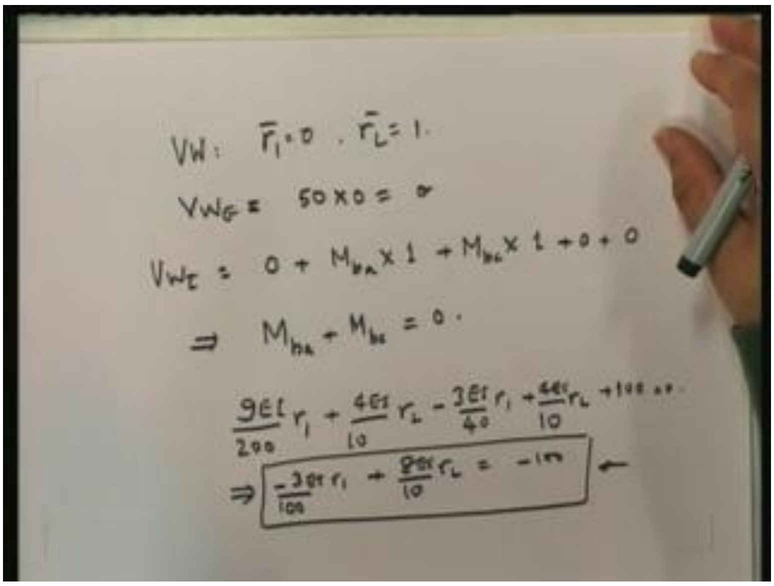

What about the second one? In the second virtual work equation, r_1 is equal to 0 and r_2 is equal to 1 – this will give me independent. What is the external virtual work done? You will see that since that point does not displace, it is going to be equal to 50 into 0, which is 0. Internal virtual work is going to be again M_{ab} into theta$_b$ etc., M_{ab} into theta$_b$ is 0 plus M_{ba} into theta$_{ba}$, which is 1, plus M_{bc} into theta$_{cb}$, which is 1, plus M_{cb} into theta$_{cb}$, which is 0, plus the work done by the reactions, which is 0. Therefore, the second equation becomes our most common M_{ba} plus M_{bc} is equal to 0, which we know already; we got it from virtual work. Therefore, if you notice, when you have a rotation, the virtual work boils down to the fact of it being the equation of joint b and that is what we are interested in. We have written the two equations and note that once I substitute for M_{ba} and M_{bc} (I will just put them down), you will get that 9EI by 200 into r_1 plus 4EI upon 10 into r_2 minus 3EI by 40 into r_1 plus 4EI by 10 into r_2 plus 100 is equal to 0. This is easy to do, so what you get is 5, 15 minus 6 by 200; minus 6 by 200 becomes minus 3 by 100EI into r_1 plus 8EI by 10 into $r_2$is equal to minus 100 – that becomes my second equation. Let us go through the steps of the first equation.

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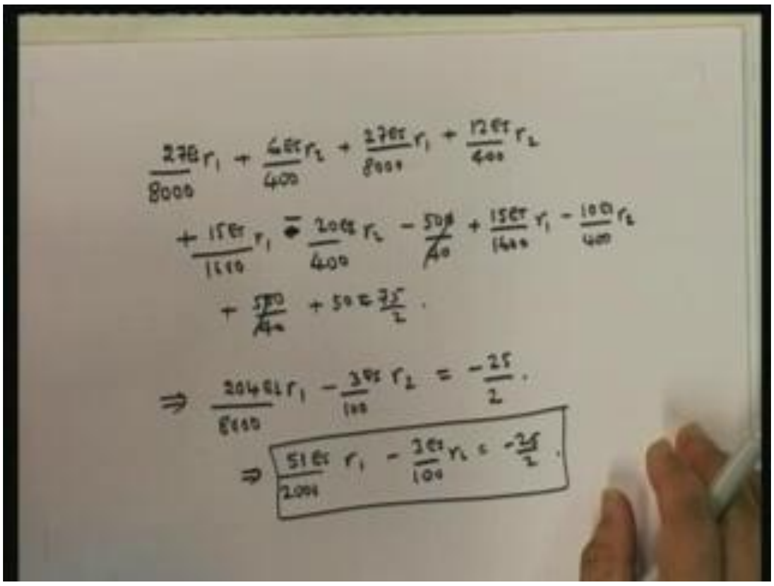

I am going to actually go through this. Let me put down all the things that we are going to get. This equation is simple and if I expand the first equation, it is going to be equal to 27EI by 8000 into r_1 plus 6EI upon 400 into r_2 plus 27EI upon 8000 into r_1 plus 12EI upon 400 into r_2; then you have minus and minus, so it is plus 15EI upon 1600 into r_1 minus 20EI upon 400 into r_2 minus 500 upon 40 plus 15EI upon 1600 into r_1 minus 10EI upon 400 into r_2 plus 500 by 40 plus 50 is equal to 75 by 2. If I put in all the r_1s together, you will see that if I put 8000, 8000, this will become 75, 75, 150; 150 plus 54, so that is 204EI upon 8000 into r_1; all of them are 400 in r_2, so it is 6 plus 12, 18 minus 20 is minus 2, minus 12 upon 400, so minus 3EI upon 100 into r_2; this cancels this, plus, minus; minus goes on the other side; so this becomes minus (25 by 2). We can simplify this, we can divide by 4, so we get 51 upon 2000 EI into r_1 minus 3EI upon 100 into r_2 is equal to minus 25 that is one equation. Let me write down those two equations and you will see that those two equations land up being this way.

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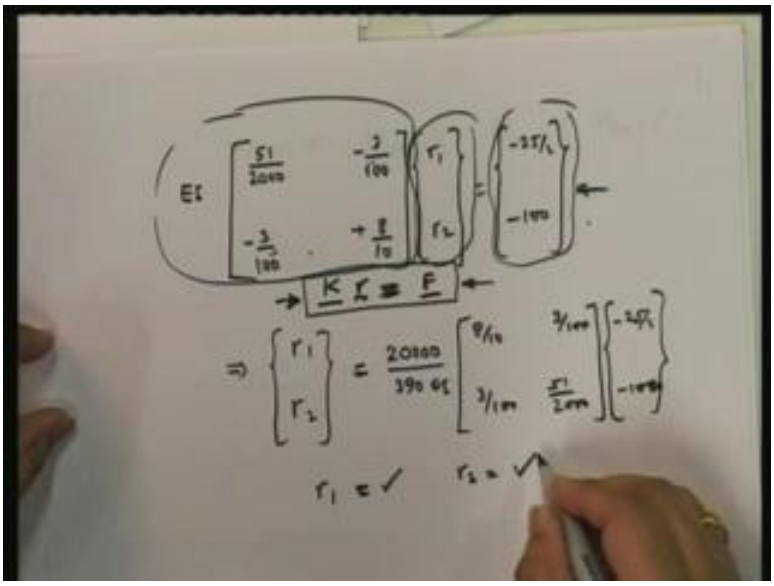

If I take out the EI and write it in matrix form, this will become 51 upon 2000, this is going to be minus (3 upon 100), here I am going to have minus (3 upon 100), plus 8 upon 10 multiplied by r_1, r_2 is equal to minus (25 by 2), minus 100. Note something interesting; you will see that this is equal to this. In fact, that is also always true that if you write it down properly, this and this are equal to each other and it is symmetric. It turns out that we can solve for r_1 and r_2 by taking the inverse of this; the inverse of this is going to be equal to 20000, this is going to be 100000, this is going to be minus 18 upon 20000 and this is going to be 408; so 408 minus 18 is going to be 390, so this is going to be equal to 20000 upon (390EI) multiplied by the inverse; this is 100; so I can find out r_1 and r_2. Once I know r_1 and r_2, then I substitute these back into the original equations.

The original equations being M_{ab} is equal to this into r_1 and M_{ba} is equal to this into r_1 and r_2. For bc, M_{bc} is given in r_1, r_2 and M_{cb} is given in r_1, r_2, so you can find out M_{bc} and M_{cb}. Now, I have found out r_1 and r_2. There are several interesting points to note over here. One is that this can be written in terms of stiffness matrix into r is equal to some force; we will see later when we use them. Right now, this is not matrix method; I am just writing down two equations, which I am solving using matrix algebra; I am just solving two simultaneous equations using matrix algebra. Right now, I have not yet introduced you to the concept of the stiffness method; understand I am still using the displacement method excepting that I am slowly beginning to introduce the concept of K r is equal to f; this term is K, this is the r vector and this is like the equivalent load vector which represents all the effects of all the loads that come in. Let us just assume that I have been able to find out M_{ab}, M_{ba} and M_{bc}. Now, where do I go from there?

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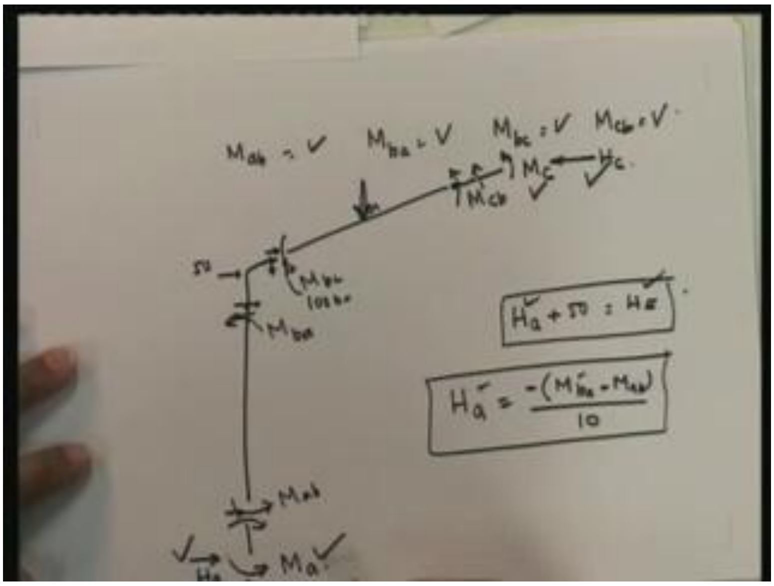

I have found out M_{ab} – this is known; M_{ba} – this is known; M_{bc} – this is known; $M_{cb,}$ – this is known. The point that I would like to make is that I have not written down the values of r_1, r_2, M_{ab}, M_{ba}, M_{bc} and M_{cb} just because I want you to actually solve this problem yourself; when you solve this problem yourself, you are going to get the values. Just before I start my next lecture, I am going to give you these values so that you can check whether you have done the problem correctly. Now, I am just giving you the steps that you have to go through; I will give you the solution at the beginning of the next lecture so that you have an opportunity to actually try and solve this problem. Let me just go back to what I was trying to state. Ultimately, how you get your bending moment diagram is this way.

You know M_{ab}, you know M_{ba}, you know M_{bc} and you know M_{cb}. Over here, what do you have? You have this load and over here, what are the things that you have? Note that at this point, I know that there is not going to be any vertical reaction. This is because it is a roller. So, what is the only force that is going to come? This is what I am trying to say: be innovative, incorporate, do not try to solve problems just by themselves. Let me just put it.

What I have over here is M_a, V_a, H_a and then I have this over here; here, I am going to have M_c and the reaction is going to be just H_c because vertical cannot be there; since this is going to be this way, you might as well apply this, apply this; the fact that this is going to be like this…; this is going to be H_c and it is only going to be vertical; since this is going to be vertical, I know that over here, I am going to get both the horizontal and vertical. Note that these are not the shear and this not the axial force. Once I evaluate these, I can actually evaluate all the support reactions. What is this going to be equal to? This is going to be equal to 100 Kilonewton, then this is going to be 100, this is going to be 100, this is going to be 100 and over here, this is going to be 100.

Here, I have this, this is going to be here, this is going to be here, this is going to be this way, this is going to be here and here, I have 50. One of the other equations that I get is H_a plus 50 is equal to H_c. Can I find out H_a? Sure, I can find out H_a. This is this way and over here also, it is this way. H_a is equal to minus (M_{ba} plus M_{ab}) upon 10, minus because both of them give the same; that is my H_a.

Now, these two I know, so I can find out H_a; once I know H_a, I can find out H_c. I have got my horizontal – I know this; I know this, I know this, I know this, I have found out this. Once I have all of those, I know all of these, I can draw the bending moment diagram; I leave it up to you; I will draw the bending moment diagram next time.

Now, what I want to do is, I want to bring to you a concept and I would like you to solve this problem yourself; I will come back next time and solve the problem. I have done enough problems using the displacement method and now, I am going to propose problems for you to do; once you have done the problem, I am going to go back, look at my next lecture and you will get an opportunity to find out how to do the problem correctly; but I recommend before you look at the next lecture, I would suggest that you solve this completely. I am going to give you the answers next time and I would like you to look at one other problem; let us look at the problem that I would like you to look at.

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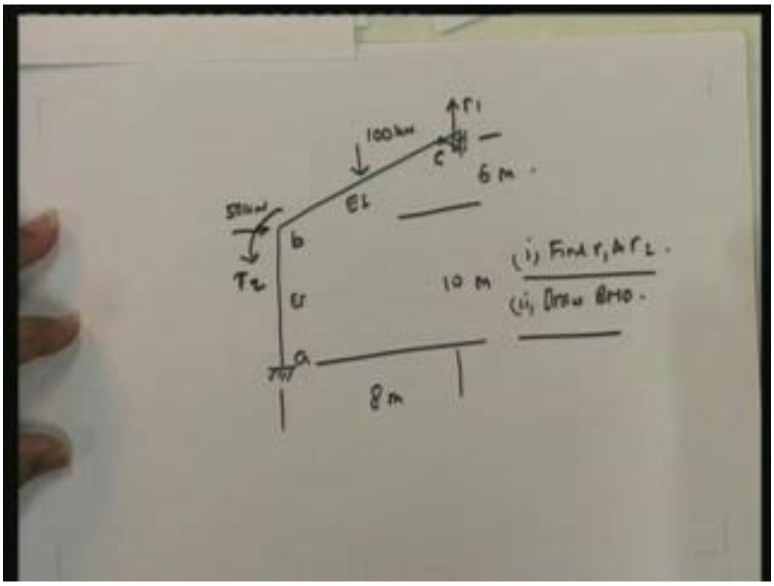

This looks suspiciously like the problem that we solved today, does it not? Excepting that I have made a small change. This is a, this is b, this is c. What I have done is, I have made this fixed roller into a hinge roller support and I am still going to say that you have to solve this problem using these two displacements as your degrees of freedom. Actually in this particular case, you have a third degree of freedom but remember that this is an end hinge, so I can actually treat bc as a modified member and keep ab as an original member; bc is a modified member with a fixed at this end and a roller at this end. What I would like you to do is, given this problem, one: find r_1 and r_2 and two: draw the bending moment diagram. This is exactly the same problem as I have done.

I am leaving you at the end of this lecture with two things to do. This is the first assignment that I would like you to do in the displacement method; please go ahead and solve the problems; first problem: having done everything, find out r_1 and r_2 and then draw the bending moment diagram using the procedure that I have outlined; second: take this problem, solve this problem through and find out r_1 and r_2 and draw the bending moment diagram. I am going to give you the answer to this and solve this next time. Another interesting point that I am going to bring out and this is of course what I tend to do always; and that is not just the ability to solve problems, but you see, what I have do over here is I have changed a fixed into a hinge.

Next time, I am going to actually look at what effect does changing support conditions have on the displacements and the bending moment diagram in the structure at all times. Therefore, one aspect that I would like you to look at when you are doing this assignment is not just get the numbers, but once you get the numbers, you also try to see what are the effects of changing this fixed to hinge, because I have got exactly the same member with exactly the same load and since I have the same member and the same load, the only difference here is this support condition; this difference between the two answers is going to give you the effect of changing this fixed to hinge and that is something that I am going to concentrate on the next time. Thank you very much. Do please solve the problems and if you want to know how the problems and their answers are, wait for my next lecture.
Thank you.