Structural Analysis II
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Lecture – 15

Good morning. I am continuing my earlier lecture. Just to review quickly, I was using the method of virtual displacement to generate the equations.

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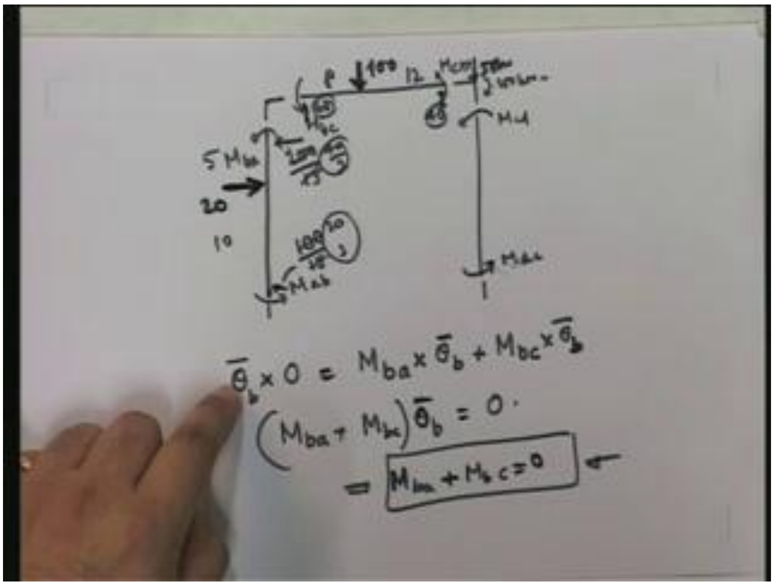

These are all the internal forces and these are the external loads for which I am going to apply it. Note that the support reactions do not matter because the supports never move, so the work done by all the support reactions are always 0. It is only the internal forces and the external joint loads that do any work. First, I put theta_b is equal to arbitrary theta_b.

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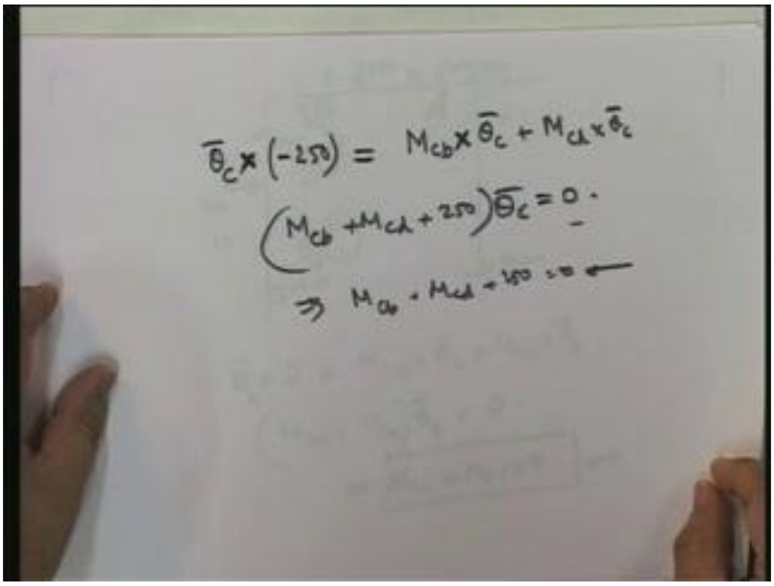

Then, I found out another equation applying an arbitrary theta$_c$. Now, I am going to apply an arbitrary delta, arbitrary delta virtual displacement and let us see what the virtual shape of the structure is.

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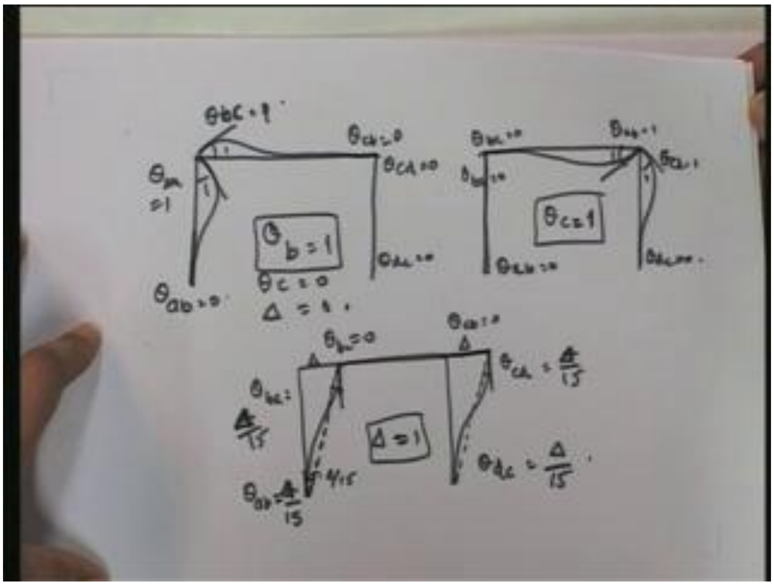

This is the shape of the structure and let us look at what theta$_{ab}$ is equal to, it is delta by 15. theta$_{ba}$ is equal to delta by 15, theta$_{bc}$ is equal to 0, theta$_{cb}$ is equal to 0, theta$_{cd}$ is equal to delta by 15, then theta$_{cd}$ and theta$_{dc}$ are each equal to delta by 15 and these are for a virtual displacement delta. This will be the work done by all the forces here. Let us see what the work done by the forces is; let us see what the forces are; we have already seen what the forces are; all the forces are noted here. Note that these cannot do any work, the work done by these are represented equivalently by the work done by the reactions to these loads, so it becomes on the internal side because it is an internal force. If I want to write down that equation, I am going to put it equal to…. You have to see the work done by the external forces.

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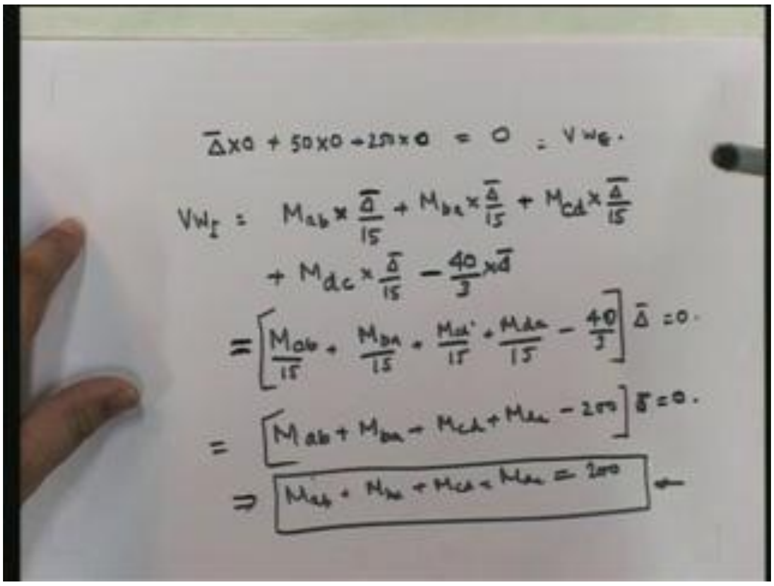

The work done by the external forces is going to be delta prime into 0 plus 50 (that is the force) into 0 because there is no displacement plus 250 into 0 – these are all the external forces; the reactions are 0 anyway, so the right-hand side – the work done by the external forces is equal to 0. Let us see what is the work done by the internal forces; the virtual work done by internal forces is going to be equal to first, M_{ab} into delta prime by 15 (note that both of them are anticlockwise, so it is positive), then work done by M_{ba} into delta bar by 15 – that is the real force undergoing the virtual rotation. What can we say about M_{bc}? $theta_{bc}$ is equal to 0, so that is going to be equal to 0; the work done by M_{cb} into $theta_{cb}$ is equal to 0, so that is going to be 0; then we have M_{cd} undergoing $theta_{cd}$ virtual, so that is M_{cd}; then, we have M_{dc} multiplied by its rotation, so that is the work done; these are all the work done by all the moments. Now, let us find out the work done by all these forces.

What is the work done by these? Let us see. Let us take 20 by 3. What is the work done by this 20 by 3? The displacement of this point is 0, so the 20 by 3 does 0 work. What about the 40 by 3? You will see that that point is displaced by 1 in this direction. The work done by the 40 by 3 is negative, because 40 by 3 and the displacement delta bar are opposed to each other. Now, let us find out the work done by this 60 – there is a reaction also, that is why it is on the internal sides, for 60, the vertical displacement is 0 and work done by the 40, the vertical displacement is equal to 0, so there is no work done by those.

Essentially, what am I left with? I am left with internal virtual work is equal to (M_{ab} by 15 plus M_{ba} by 15 plus M_{cd} by 15 plus M_{dc} by 15 minus 40 by 3) into delta is equal to 0 – this is what we get as our equation. Note one particular thing that we have. This is the equation that we get and now, if I substitute this and put this in…. One of the points that I would like to make over here is that…. This is the equation. Let me just put this into a standard format. (M_{ab} plus M_{ba} plus M_{cd} plus M_{dc} minus 15 into 40 is 600 minus 200) into delta bar is equal to 0 because I have taken the 15 on the other side.

Since this delta prime cannot be 0, we get M_{ab} plus M_{ba} plus M_{cd} plus M_{dc} is equal to 200 – this is my other equation that I generate from the third. Note the advantage of this procedure. Remember how we obtained it the last time? The last time, the way we obtained it was that we actually found out the reactions and then put the reactions equal to the loading that was applied on the structure and based on those, we did the whole procedure and that is how we obtained these equations. Here, just by applying the virtual work equation without actually looking at any kind of equations of equilibrium, we generated this apparent equation equilibrium. Note that this equation is actually a virtual work equation, it is a work equation; the only thing is that it gives you something that looks like a moment equilibrium equation and if you solve these three simultaneously, you will get $theta_{ab}$, $theta_c$ and delta.

So much for all of this and you will see that these equations are identical to what we got last time. This is the concept of the application of the principle of virtual displacement. I just want to go back and review what I have done before I proceed. What I have done is I have introduced a new concept of using a theta. I have used a simpler form of slope deflection equations where all you have are the rotations and fixed end moments. The only point here is that the rotations are defined from the chord rather than from the original position, original tangent, that is from the chord to the displaced tangent of the elastic curve. This is how we define the new thetas and once we define those new thetas, we saw that we got exactly the same equations that we got earlier without considering this new definition – the original definition of theta as well as delta. Finally, I showed you the application of the method of virtual displacement in obtaining the equations.

This method of virtual displacement essentially becomes very important when we have to write down the equations corresponding to a displacement quantity because the only thing that we get directly from the slope deflection equations is actually the moments. All other shear forces etc. are derived quantities and to write down a force equilibrium equation, we have to actually go through a significantly large number of equations to get that one equation corresponding to a displacement. By using the method of virtual displacement and putting external virtual work equal to internal virtual work and noting that the fact that works is a scalar, you can add up the work done by all the forces acting on the individual members and sum them up. We got an equation very easily without having to go through several equations. This is the advantage of the method of virtual displacement.

Now, what I am going to introduce you to is a couple of other problems and I am going to again introduce the concept. One of the very important aspects of using the displacement method is that you saw we have to draw the displaced shapes corresponding to each degree of freedom, so that we can find out the member end rotations are related to the structural degrees of … displacements corresponding to the structure degrees of freedom.

Now, what I am going to do is I am going to introduce you to a couple of problems where all that we do is actually just look at how the displacement pattern comes and see if we can relate the member end rotations to the structure degrees of freedom. Then, you can write down the member end moments in terms of structure degrees of freedom and then you can use virtual displacement to write down the equations. Essentially, this becomes a very simple procedure once you get the kinematics. This is called kinematics because when you want to find out how does the structure displace given a degree of freedom, given a displacement corresponding to a degree of freedom – that is what we are going to concentrate on because once you know that, the displacement method becomes very very simple to apply. Let us now look at a few problems. Let me take you through to some complicated issues, not as simple as they looked last time.

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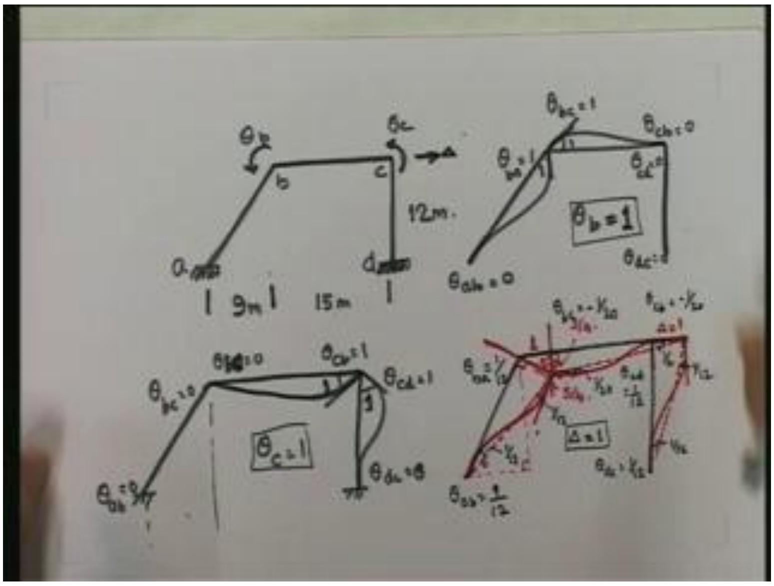

Note that here also, if you assume axial rigidity, there are only three degrees of freedom and the three degrees of freedom are this (Refer Slide Time: 14:28), this and this ; identical. All I have done from the previous problem is I have taken this straight member and I have inclined it. Let us put down some values just to satisfy ourselves. I will make this… this is 3, 4, so I will make this 9 , I am going to make this 12 , make this 15 ; this is a , this is b, this is c and this is d; this is 9 meters, 15 meters, 12 meters. Right now, all I am interested in is what happens when I give this equal to 1, this equal to 1 and this equal to 1 independently; that is all I need to draw; once I know that, given a loading, I can always find out everything else.

Right now, we are going to concentrate only on kinematics because you will see that kinematics is important to relate, to get the member end moments due to the displacements corresponding to structure degrees of freedom and it is also important because those become my independent virtual displacement patterns. I am using the same displacement patterns that I generate for each individual degree of freedom because I know that if I choose one degree of freedom equal to 1 and the other degrees of freedom equal to 0, they are each going to be independent because that is the whole definition of independent degrees of freedom. In other words, I could have actually chosen completely different independent deformation patterns for the virtual displacement and been absolutely correct. The only problem is that to figure out two independent displacement patterns is not as easy as it looks.

However, since my degrees of freedom are independent of each other, I know that if I take one displacement equal to 1 and the other displacements equal to 0, I am going to generate a pattern which will be completely independent when I take another degree of freedom equal to 1 and the others equal to 0; those are going to be independent and that is the reason why I choose those patterns as my virtual displacement patterns. For this, there are three degrees of freedom and so, if you could figure out three independent virtual displacement patterns, you could actually write down three independent equations.

I am not that this thing in terms of trying to figure out what would be an independent pattern and therefore, what I tend to do is, I tend to use the simplest independent patterns that I can generate, which are the displacement patterns corresponding to the degrees of freedom themselves. Therefore, you see how important the displacement pattern becomes every time and that is why the kinematics. Here, I am going to first put $theta_b$ is equal to 1 and note that automatically I am assuming $theta_c$ is equal to 0 and delta is equal to 0, I do not need to explicitly state it. This one is actually not a problem; you know the inclined member does nothing to the displaced shape, so, $theta_{ab}$ is equal to 0, $theta_{ba}$ is equal to 1, $theta_{bc}$ is equal to 1, $theta_{cb}$ is equal to 0 and of course $theta_{cd}$ and $theta_{dc}$ are equal to 0; so, I could relate it. I am going to put them down. If this was real, these would be real and if this is virtual, these would be virtual; I mean identical. Then, let me put $theta_c$ is equal to 0. Once I get $theta_c$ is equal to 1, we have to be consistent, so let me put down what…. This is $theta_{bc}$, $theta_{cb}$ is equal to 1, $theta_{cd}$ is equal to 1. Note that these are all thetas being taken from the chord to the tangent – all of them from the chord to the tangent, but remember I told you that when you put thetas, the chords are the undisplaced directions, so there is no change.

Now, for the interesting one and let me draw this; for this one, I will use a different colored pen to highlight to you what actually happens. Now, note that I am going to put displacement delta is equal to 1. What happens? This member, since this is moving perpendicular, there is no change of length, so there is no problem; this is going to remain like this. Let us look at this member. For this member, for it not to change its length, along which direction can it move? It can only move perpendicular. Please note that these are small displacements, so the perpendicular movement does not increase because ideally you will see an arc but when your displacement is small, the arc and the tangent are the same; so, this can move along this direction only.

What about this member? Since this end has gone here, the only way…. This cannot change length, this has already gone by 1 from here to here, so this was the member. How will this member move without increasing its length? Only if was perpendicular. Look at where this point goes. If this member was not there, this point would move here (Refer Slide Time: 23:43), but since this member is there and it is fixed at this point, it can only move along this line, it can only move along this line. This point now has to move because it cannot be here; this point is not on this line, so if I move it only up to here, this would increase this length.

Therefore, this now has to move; when it has to move, how does it move? It moves perpendicular so that this member does not change length. The whole point here is that since axial rigidity is there, nothing can change its length. Where would the point go? The only way is where these two lines intersect; this is the point where it would go. How would it look now? The thing is $theta_b$ is still equal to 0. This point cannot go anywhere, so this would still be perpendicular to this. What about this (Refer Slide Time: 24:47)? Since $theta_b$ is equal to 0, this would still be parallel to this, this would be parallel to this, this would be parallel to this and here, this would be parallel to this. That ensures that $theta_b$ is equal to 0 and $theta_c$ is equal to 0, which is what you need. If this was 1, because delta is equal to 1, how much is this? Let us see what this is.

If you look at this particular one, this is the angle theta, so that is the angle theta, this is perpendicular and this is perpendicular. Which angle is represented over here (Refer Slide Time: 25:46)? If you look at it, by similar triangles, this would be the same as this (Refer Slide Time: 25:53); satisfy yourself. So, if you look at this, this is 12, this is 9, therefore 12 by 9; since this is 90 degrees and this is 90 degrees , tan theta is equal to 12 by 9; here also, tan theta is equal to this by this, so what is this equal to ? This upon this is equal to 12 by 9, so you will see that this is equal to 9 by 12 and 9 by 12 is 3 by 4. If this is 1 and this is 3 by 4, you will see that this is equal to 5 by 4. Once I have drawn the shape, the point is to find out theta$_{ab}$ and theta$_{ba}$ and here, I need to connect….

Whenever I have a displacement, I need to connect the chords and take displacement, rotation from the chord to the tangent. If I do that, what do I get? If you look at it, from the chord to the tangent, what is that? You will see that this displacement is 15 because 12 square by 9 squared, square root is a hypotenuse and that is going to be equal to 15; this is 12, this is 9, this is 15 and 5 by 4 is the displacement here, so 5 by 4 divided by 15 is this rotation, this rotation is equal to 5 by 4 divided by 15 – that is equal to 1 by 12. Similarly, here also, this is 1 by 12. Let us look at this one. This is 3 by 4 and what is the dimension? The length is 15, so 3 by 4 by 15 is equal to this angle is equal to 1 by 20. Similarly, this angle (Refer Slide Time: 28:17) is 1 by 20.

Similarly, over here also, we have to join the chord, the two members, two points of the two ends and join it by a straight line; so, what is this? This is delta and what is this length? 12. This is 1 by 12 and this is 1 by 12. Having written those down…. This is from the chord, so what is theta$_{ab}$? Theta$_{ab}$ from the chord to… is anticlockwise, positive, this is 1 by 12. What is theta$_{ba}$? Theta$_{ba}$ is equal to… from the chord, anticlockwise, so 1 by 12. What is theta$_{bc}$? Let us see. From the chord to the tangent, clockwise, so it is negative 1 by 20. Similarly, theta$_{cb}$ is negative 1 by 20; from the chord clockwise (Refer Slide Time: 29:28), so theta$_{cd}$ is equal to 1 by 12; similarly, theta$_{dc}$ is equal to 1 by 12.

This one is fairly complicated and that is the reason why I have spent so much time actually drawing. Now, you can see that delta is equal to 1. Up till now, what I did was, these were simple problems and so now, if I look inclined, you actually have to spend a lot of time figuring out where each point goes. Once you know where each point goes, you can then draw the displaced shape because delta is equal to 1, theta$_b$ is equal to 0, theta$_c$ is equal to 0 – you know those. From that, you can draw the displaced shape given delta is equal to 1; this is the displaced shape if delta is equal to 1.

Once you have delta is equal to 1 that means this is equal to one-twelfth of delta, one-twelfth of delta, minus delta by 20, minus delta by 20 – that is the overall concept. Then, you can write down theta$_{ab}$ is equal to 0 into theta$_b$ plus 0 into theta$_c$ plus 1 by 12 into delta; that means theta$_{ab}$ is equal to delta by 12 and then, you can substitute in the equation and you can get your expressions for M_{ab}, M_{ba}, M_{bc} and M_{cb}. Once we have this, you can see that this kinematics gives you all the equations for M_{ab} and then, these are also independent virtual displacements, so these can then be used to actually write down your virtual work equations. So much for the equilibrium.

Now, I am going to introduce you to the kinematics of various kinds of members because once you know the kinematics, you can actually write down the equations for any kind of structure. Let me now look at another equation. I am going to take the same thing and I am going to say that this member bc is flexurally rigid also.

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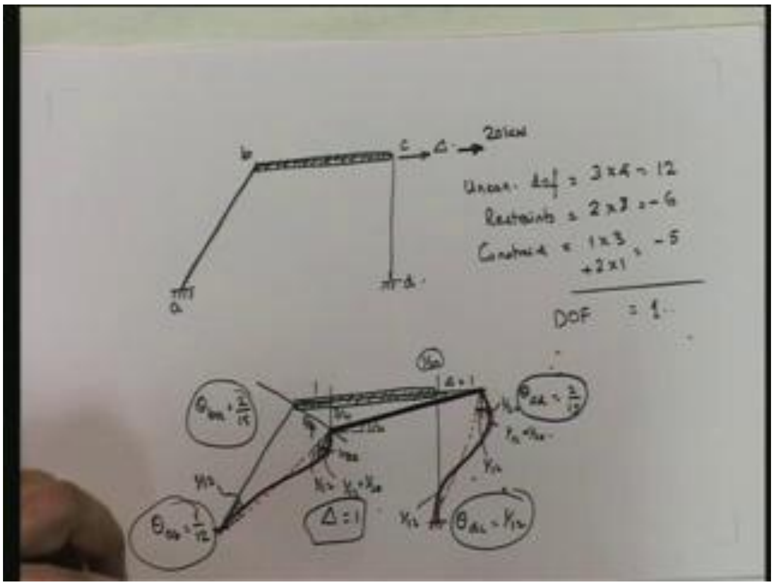

If member bc is flexurally rigid, let us draw this. The same equation and a, b, c, d; now, this member is flexurally rigid. What happens when the member is flexurally rigid? Let us see. We will first find out the degrees of freedom; one, two, three, four, so unconstrained degrees of freedom are equal to 3 into 4, 12. How many restraints? Restraints are equal to 2 into 3 because both of them are fixed, so three constrains per joint, that is minus. Then, how many constraints do we have? One constraint is axial rigidity of all the three members, so that is 1 into 3 members. In addition to that, bc is also constrained to be flexurally rigid. Now remember, I had said how many for flexurally rigid, for each member? One member and two constraints. So how many constraints do I have? 3 plus 2 is equal to 5. How many degrees of freedom? 1. What is the degree of freedom? I am going to take it as this. Note that what has actually happened is that because of this being flexurally rigid, this member cannot rotate because this cannot rotate and here also, it cannot rotate; so, theta$_b$ and theta$_c$ both have disappeared as independent degrees of freedom, because no way can this joint displace. If this joint does displace, then this has to go this way and there is no way this can go, because then, for it to be rigid, this point has to go and violate. You can see that.

Similarly, for theta$_c$, if this rotates and this rotates (Refer Slide Time: 35:24), this has to come down here, and it has to, it will violate this. There is only one degree of freedom and for that one degree of freedom; I have to draw the displaced shape. It is the same, so 9, 9, 12, 15 and 12; rigid, rigid and this is rigid, so it can only deform linearly. Let us see what happens here.
It is very interesting to note that this point is going to go there, this point is going to go there because delta is equal to 1. I have already shown where is point is going to go; this point is going to go here, so this is gone by 1, this is going to come down here, this point is going to be here, this is 1, this is 3 by 4. The points go exactly where they went earlier; the only difference that happens is how are these points connected.

Note, since this member is rigid, this member is rigid, the only way these two points can be connected to each other is in a straight line because this is the only way that member bc can deform, it can only go straight. Now, if it goes straight, what happens? If it goes straight, what happens? Let us see what is the angle here? 3 by 4 by 15, that is 1 by 20. Now, note that that means this also has rotated by 1 by 20. The interesting point here is that since this member has gone this way, there is no way that this tangent can remain in this way. Why? Because that will violate the continuity of the joint. That means that if it rotates by 1 by 20, this also has to rotate by 1 by 20, the tangent at this point has to rotate. So, how will my displacement of this look? This will look like this way and here, again, this was this way; so this also has to rotate in this fashion by 1 by 20. You see, what has happened is if I were to plot this, this is how the thing has rotated due to delta.

Now, let us see what my member end rotations are. For that, I have to draw this again, this one I do not need to draw. These are the chords, so once member bc is rigid, if a member bc is rigid, its slope deflection equations are no longer valid because M_{bc} will be given by 4 EI upon 1 into $theta_{bc}$. But what is $theta_{bc}$? 0, because it cannot rotate. Theta is from the chord to the tangent, that is 0 and when it is rigid, EI is infinity, therefore, essentially what you have is a 0 by 0 situation; therefore, M_{bc} and M_{cb} can only be obtained by equilibrium at the joints, that is all, there is no way…. Therefore, bc does not come into the picture at all. But what about ab and cd? These are flexible members, so these slope deflection equations have to be written. You need to find out from the chord to …. (Refer Slide Time: 40:10). What is this? This is going to be the same as 1 by 20. This is 5 by 4 divided by length, which is 15, so this is going to be 1 by 12.

What about this? What is this angle? From the original to the chord, it is going to be same as 1 by 12. From the chord to the tangent is how much? It is 1 by 12 plus 1 by 20. Here, from the chord to the tangent, what is this? 1 by 12. Here, from the chord to the original undisplaced tangent, it is equal to 1 by 12. What is it from the original tangent to the new tangent because of this rotation? That is 1 by 20. What is the total from the chord to the tangent? It is equal to 1 by 12 plus 1 by 20. Let us see what $theta_{ab}$ is equal to; $theta_{ab}$ is equal to 1 by 12. What is $theta_{ba}$ equal to? 1 by 12 plus 1 by 20 which is 2 by 15. This one (Refer Slide Time: 41:54), I do not need to find out because I cannot write down slope deflection equation for this. $theta_{dc}$ is going to be equal to 1 by 12 and $theta_{cd}$ is equal to 1 by 12 plus 1 by 20, which is 2 by 15. These are the member end rotations in terms of delta.

Here, the point that I was trying to make is that the kinematics of the structure is very very important and once you can draw the displaced shape corresponding to each degree of freedom, you are well on your way to using the displacement method for solving any structure. The advantages in the displacement method as you see it is that it actually makes life much easier for you, in general.

Now, I am going to actually take this particular problem, this problem and I am going to actually put some loads on it and see what happens in reality. Let me take a very simple load initially; let us not complicate the issue too much; let me say that this structure is only subjected to 20 Kilonewton and due to this 20 Kilonewton, I need to draw the bending moment diagram for this particular structure. I have got these. From these, what do I get? This is a single degree of freedom, so what do I get from this? I will write down the equation.

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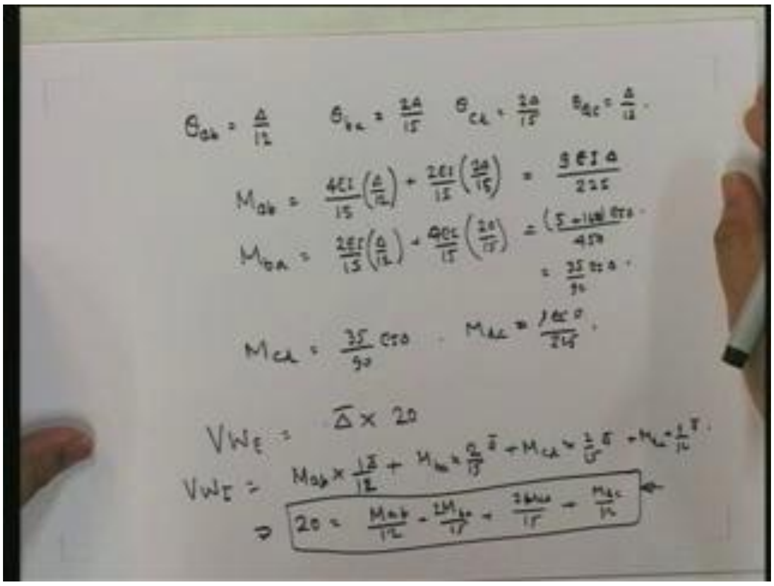

Theta$_{ab}$ is equal to delta by 12, theta$_{ba}$ is equal to 2 delta by 15, theta$_{cd}$ is equal to 2 delta by 15. Wait a minute. I made one mistake and that is that we did not find out what the values were. Let us go back here. From the chord to the tangent, anticlockwise; from the chord to the tangent, anticlockwise; from the chord to… anticlockwise – all of them are positive and I can actually write them down. Theta$_{dc}$ is equal to delta by 12. I am going to substitute this into my equation. What are my fixed end moments? Since there are no loads in the members, fixed end moment is equal to 0. What is my M$_{ab}$? All my fixed end moments are 0, so what is M$_{ab}$ equal to? M$_{ab}$ is equal to 4 EI and length of ab is 15 times delta by 12 plus 2 EI by 15 into (2 delta by 15). What does this become? This becomes one-third, this is EI upon 45; this is going to be equal to 5, this is going to be equal to 45, so 5, 5 plus 9, so this going to be 9 EI delta by 225. M$_{ba}$ is equal to 2 EI by15 into (delta by 12) plus 4 EI upon 15 into (2 delta by 15). Here, I have 6 and it will go to 450, this is going to be equal to 5 and this going to be equal to 160, so this is going to be (5 plus 160) into EI by delta. If I divide throughout by 5, I get 35 by 90 EI delta.

In the same way, I can find out M$_{cd}$. You will see that M$_{cd}$ will be equal to 35 by 90 EI delta and M$_{dc}$ is going to be equal to 9 EI delta upon 225. I have obtained these equations in terms of the delta and then, I need to write down the virtual work equation. How will I write down the virtual work equation? Find out all the work done by all the internal forces undergoing the displacement. What is the external virtual work? External virtual work is delta prime into 20.

What is the internal virtual work? Internal virtual work is going to be equal to M$_{ab}$ into 1 by 12 plus M$_{ba}$ into 2 by15, of course, delta remains, this is going to be delta prime, this is going to be delta prime plus M$_{cd}$ into 2 by15 delta prime plus M$_{dc}$ into 1 by12 delta prime. This is equal to this. From this, if I put it in, I am going to get that this implies that 20 is equal to M$_{ab}$ by 12 plus 2 M$_{ba}$ by15 plus 2 M$_{cd}$ by15 plus M$_{dc}$ by12 – see how easily we got the equation. If you had to actually do it in the other way, you would have to find out all the moments, you would have to actually solve everything, find out all the reactions and from the shear and the actual force, you would then get the horizontal and then equate the horizontal reaction that these two equal to 20 – it will take you a few hours to do that. Satisfy yourself; try it in that way and you will get this equation.

Here, you will see that all these are in terms of delta, so I will substitute these values in. I will get only one equation in delta and I can solve for delta. Once I solve for delta, I can immediately plug it back in here and I get M_{ab}, M_{ba}, M_{cd} and M_{dc}. In other words, once I solve for it, I can get M_{ab}. Now, I know M_{ab}. I am not solving them but I know M_{ab}, M_{ba}, M_{cd} and M_{dc}.

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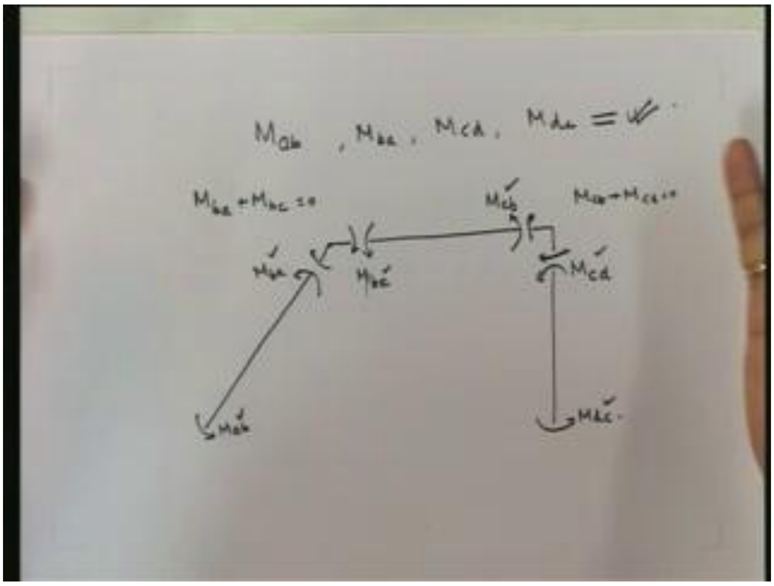

These can be evaluated by plugging it into that equation, solving for delta and then plugging them back in here. I have only found out M_{ab}. How do I draw the bending moment diagram? This is where equilibrium comes in. You cannot forget equilibrium. So, what you have to do is, you have to do this. You cannot forget equilibrium because it cannot be violated. Therefore, this is M_{ab}, this is M_{ba}, this is M_{bc} ; we did not evaluate M_{bc} because bc was rigid, but nonetheless even in a rigid member, you are going to have bending moments, remember that; this is M_{cd} and this is M_{dc} . These are my bending moments where I know this, I know this (Refer Slide Time: 51:58), I know this and I know this. Can I find out M_{bc}? Sure, I can. How? Equilibrium says that M_{ba} plus M_{bc} is equal to 0, since there is no moment applied. Similarly, over here (Refer Slide Time: 52:21), you will have M_{cb} plus M_{cd} is equal to 0. You can actually evaluate this and this.

Once you know these, since there are no bending moments, there are no other loads on the members, just the bending moments themselves, you can find out the reactions and draw the bending moment for the structure. The whole point here is just to go back. You can draw the bending moment; I am going to leave that to you. I have already found out M_{bc}. There are no member end moments, so you can always find out the reactions and then, the bending moment diagram will look like this (Refer Slide Time: 53:06), straight line here, continue straight line here, come here, continue straight line here (Refer Slide Time: 53:11) – this is your line and you have got the bending moment diagram for the load of 20 Kilonewton force.
The point that I am trying to make here is this: what are the important aspects to the entire thing? The important aspects to this are a) finding out the number of degrees of freedom, identifying…. For the degree of freedom, let us come down here. You will see that for the degree of freedom, you get the displaced shape of the structure; the displaced shape of the

structure can be obtained through kinematics. Please keep practicing this; you have to get kinematics.

Once you get the kinematics, you have got all the thetas and member end deformations. Once you have got the member end deformations, you can get the member and then apply the same delta as my virtual displacement pattern and so, I can write down my virtual work equation; from the virtual work equation, I can actually substitute these and get an equation for delta prime, from where I can solve delta prime. Once I solve delta prime, I plug those back, get M_{ab}, M_{bc} and then use equilibrium to obtain your bending moment diagram.

I hope over the last couple of lectures I have been able to impress upon you how the method of virtual displacement becomes a very powerful tool in simply analyzing a frame by the displacement method. I shall continue by actually solving a few more problems, so that you can gain confidence of how to actually use the displacement method and the method of virtual displacement to…. In fact, the method of virtual displacement is used to generate the equation, it is not a separate method; the method is the displacement method in which you generate the equations to solve for the displacement using the method of virtual work, virtual displacements. Once you solve them, you can get the member end moments; once you know the member end moments in any member, it becomes a statically determinate structure and you can draw the bending moment diagram.

Thank you very much. I look forward to continuing with you on using the displacement method.