Structural Analysis II
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Lecture – 14

Good morning. Last time, we looked at a couple of problems introducing the concept of the displacement method for solution of a beam with settlement and then, we solved a frame problem where we saw that the displacement was also an unknown and therefore, we had to write an additional equation to be able to solve for the displacement also.

Today, I am going to take the same problems and I am going to introduce another concept to you. Remember we wrote the slope deflection equations as 4EI upon L theta_{ab} plus 2EI upon L theta_{ba} minus 6EI upon (l square) delta_{ba} plus $(\text{FEM})_{ab}$, that is the moment at ab. Today, I am going to introduce you to a concept where I am going to eliminate the delta. In other words, I am going to redefine my theta_{ab} and theta_{ba} so that you do not need to consider delta at all in the equation.

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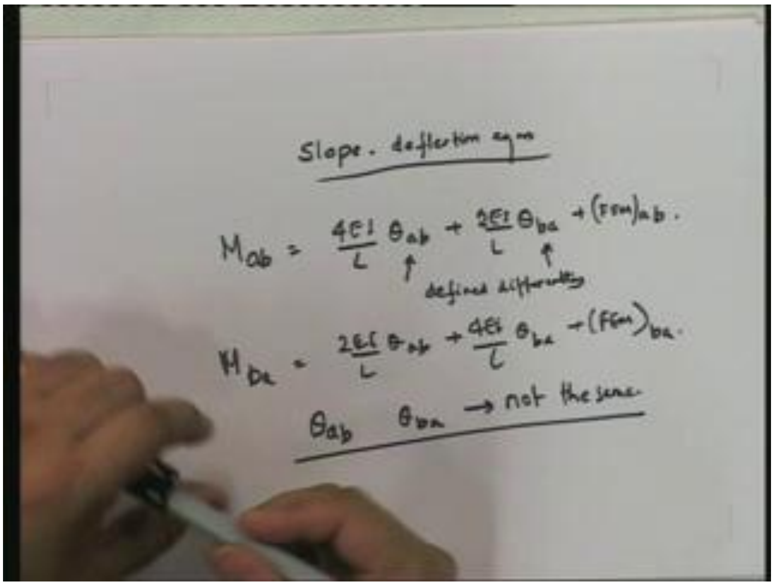

If you look at the slope deflection equations, they look this way. M_{ab} is equal to 4EI upon L theta_{ba} plus 2EI upon L theta_{ba} plus $(\text{FEM})_{ab}$; in other words, I am eliminating… the only thing is that these (Refer Slide Time: 03:18) have to be defined differently. Of course, similarly, M_{ba} is going to be equal to 2EI upon L theta_{ab} plus 4EI upon L theta_{ba} plus $(\text{FEM})_{ba}$; the only thing is that theta_{ab} and theta_{ba} are not the same as those defined in the earlier equations.

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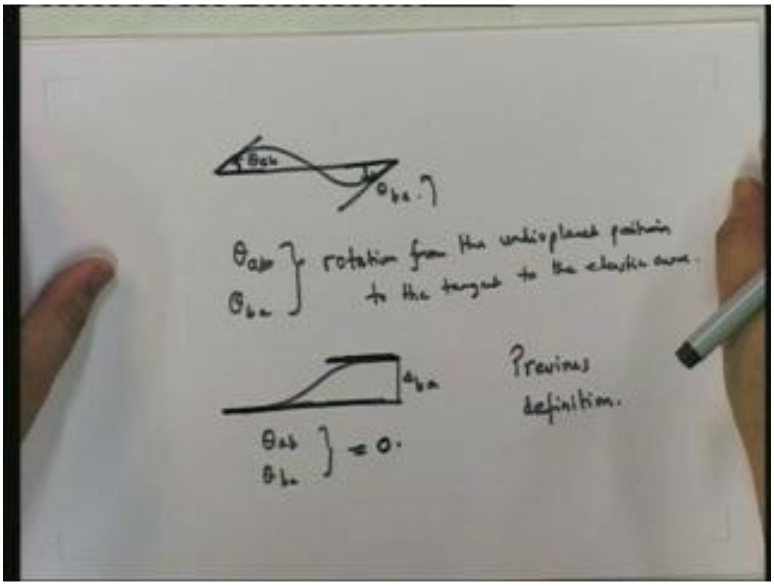

Let us look at how we defined theta$_{ab}$ in the earlier equations. Here, theta$_{ab}$ and theta$_{ba}$ are the rotations from the undisplaced position to the tangent to the elastic curve; so, theta$_{ab}$ and theta$_{ba}$ as defined in the earlier equation were rotations from the undisplaced position to the tangent to the elastic curve – from the original to the tangent, that is how rotation is defined; and I took counterclockwise as positive, so that is this. Let us look at what happens when we have just a displacement. When we have a displacement, what is theta from the original to the tangent? In this case, when you have delta$_{ba}$, what is theta$_{ab}$ and theta$_{ba}$? Both are equal to 0, because the tangent to the elastic curve at both positions from the original displaced position (Refer Slide Time: 06:25), so this is the original displaced theta, this the original, this is the original tangent, final tangent, original tangent, final tangent, original tangent, final tangent, original tangent and final tangent – they are equal to 0. This is the previous definition that we had.

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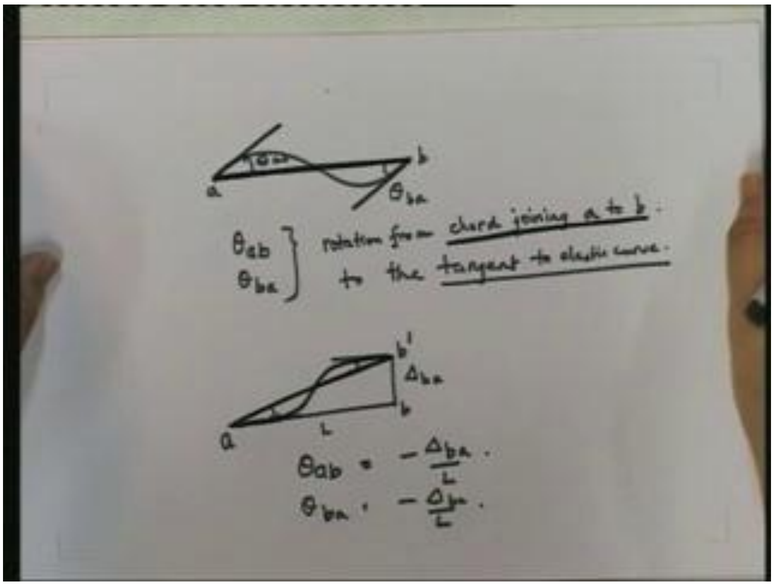

Today, I am going to introduce you to a new definition. The only thing now I say that this is defined differently, this is the chord (Refer Slide Time: 07:24) that goes from a to b. From the chord that joints a to b to the tangent is my theta$_{ab}$ and theta$_{ba}$. What are theta$_{ab}$ and theta$_{ba}$? In this particular case, you may ask how it is different from the original, but the whole definition is different; this is the rotation from chord joining a to b to the tangent to elastic curve.

Therefore, it is now rotation from the chord, it is not rotation from the tangent in the undisplaced position to the tangent of the displaced position, it is rotation from chord joining a to b to the tangent to the elastic curve. When there is no displacement of b with respect to a, they are the same as what I had originally defined, but let us now look at this case where I get delta$_{ba}$; in this case, only delta$_{ba}$.

Now, b has moved here (Refer Slide Time: 09:19), chord joining a to b is this line, chord joining from a to b to the tangent of the elastic curve, so in this particular case… since this is l, what is theta$_{ab}$ equal to? From the chord to the tangent; theta$_{ab}$ is equal to minus delta$_{ba}$ upon L. From the chord to the tangent; since these are clockwise (Refer Slide Time: 10:05), this is my new definition of theta$_{ab}$ from the chord joining a to b to the tangent, so now you see that this becomes my definition of theta$_{ab}$ and theta$_{ba}$; now, the proof of this is to show that both of these give me the same moments.

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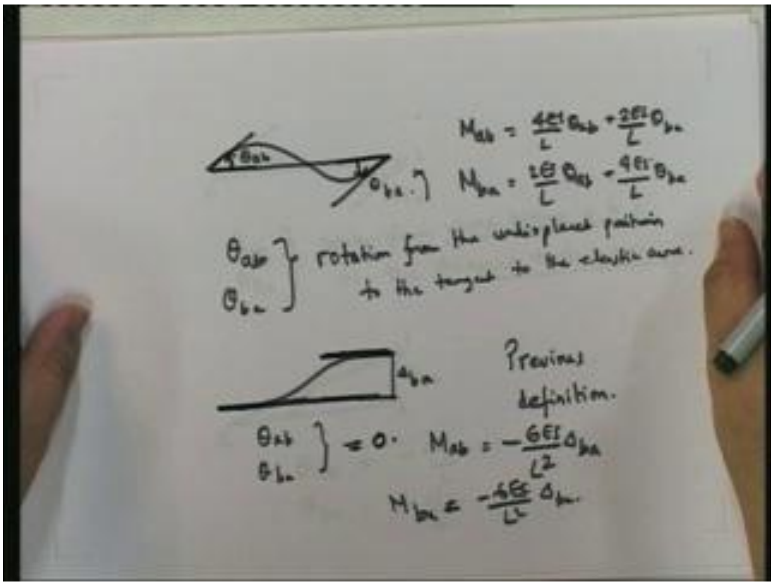

In this particular case, what is M$_{ab}$ equal to? M$_{ab}$ is equal to 4EI upon L theta$_{ab}$ plus 2EI by L theta$_{ba}$ and M$_{ba}$ is equal to 2EI by L theta$_{ab}$ plus 4EI by L theta$_{ba}$. Here, what would M$_{ab}$ be equal to? M$_{ab}$ would be equal to minus (6EI upon L square delta$_{ba}$) and M$_{ab}$ would be equal to minus (6EI upon L square delta$_{ba}$) – these would be the moments at the end.

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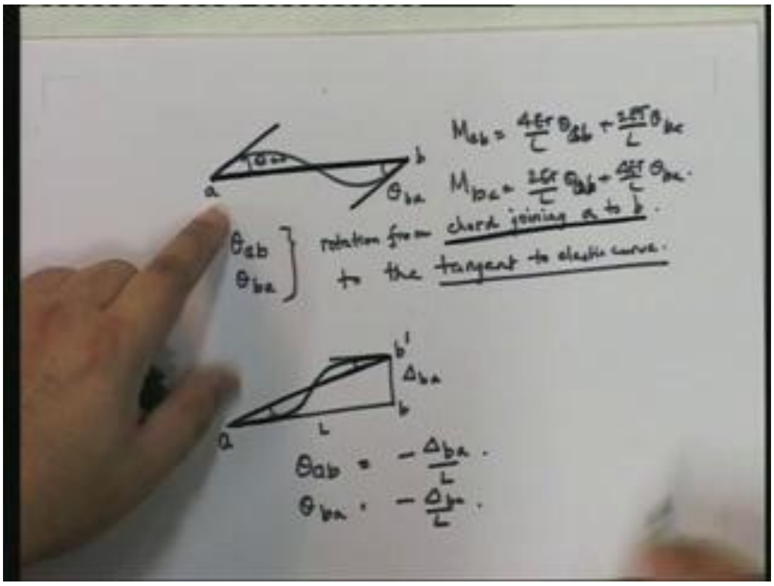

Let us see whether we get the same M_{ab} and M_{ba} here. There is no difference, so M_{ab} would be equal to 4EI upon L $theta_{ab}$ plus 2EI by L $theta_{ab}$ and M_{ba} would be equal 2EI by L $theta_{ab}$ plus 4EI upon L $theta_{ba}$. Now, let us look at this – see there is no difference because $theta_{ab}$ and $theta_{ba}$ are also the same.

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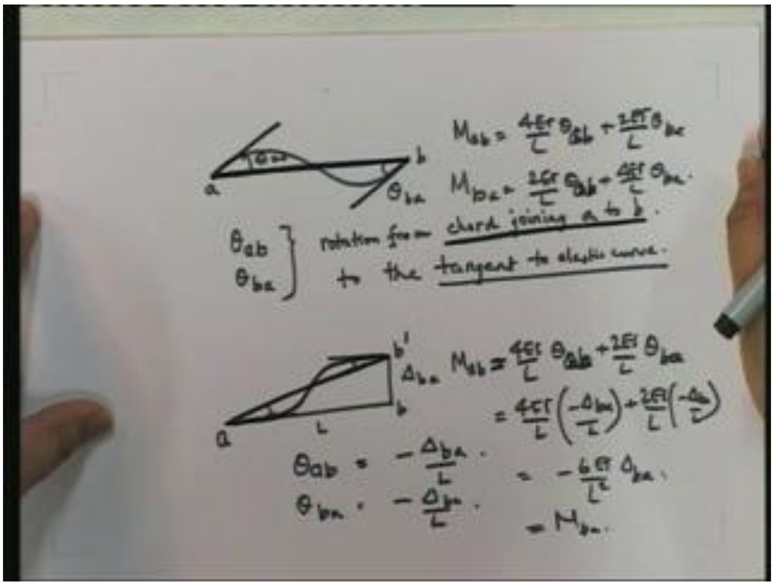

Let us now look at this one. Here, M_{ab} was equal to this. Now, M_{ab} is going to be equal to 4EI by L $theta_{ab}$ plus 2EI by L $theta_{ba}$; now, let me plug this in (Refer Slide Time: 12:33), 4EI by L $theta_{ab}$ into minus ($delta_{ba}$ upon L) plus 2EI by L $theta_{ba}$ into also minus ($delta_{ba}$ upon L). Substituting this, you will get minus (6EI upon L square $delta_{ba}$) and you will get a similar M_{ba} also. The point to note is that we have got exactly the same M_{ab} and M_{ba} as we had got using the earlier definition and therefore, there is no difference in the entire method.

The only point here is that we have negated the effect of trying to take delta$_{ba}$ in the equation; we have simplified our slope deflection equation to just containing three terms – one containing theta$_{ab}$, one containing theta$_{ba}$ and one containing fixed end moment. You may ask what is the reason behind this; we will slowly come to what the reason behind this is.

The point that I am trying to make is that this definition of theta$_{ab}$ and theta$_{ba}$ from the chord to the tangent of the elastic curve is completely equivalent to the previous definition. The only thing is that in the previous definition, you had theta$_{ab}$, theta$_{ba}$, delta$_{ba}$ – all these as terms, whereas here, you only have theta$_{ab}$ and theta$_{ba}$ and the advantage of this will become obvious a little bit later on. Let us now see whether we can use this concept to solve the problem that we had solved last time. We will look at the problems and see if we get the same answers, I think that is the key thing.

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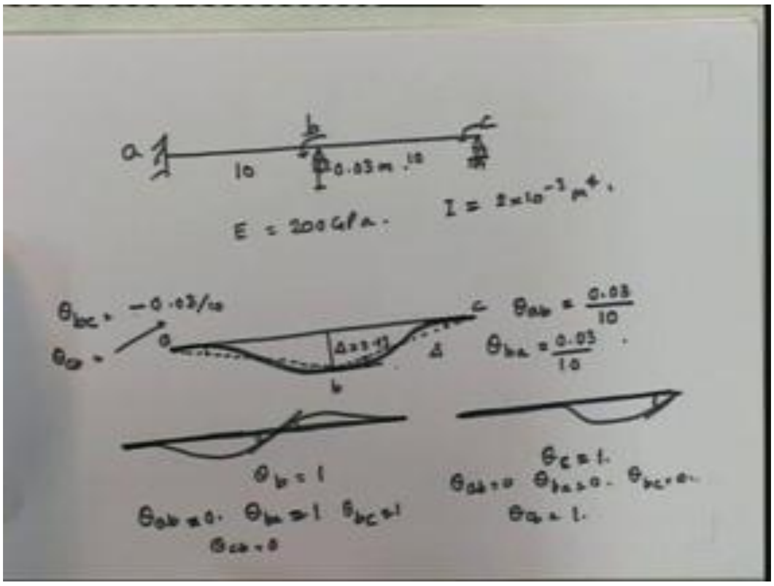

Here, we are given that this is a (Refer Slide Time: 14:53), this is b, this is c and this goes down by 0.03 meters, you were given that E is equal to 200 Gpa, I is equal to 2 into 10 to the power of minus 3 meter fourth. This is what you were given and essentially, what you were asked to find out is the bending moment diagram due to the settlement alone, only b settles by 0.03 meters.

Let us try to solve this problem using the new approach that we have. If I were to take just this displacement (Refer Slide Time: 15:42), what would it look like? This is fixed, this is displacement, this is displacement that is due to 0.03. Then, let us see what happens when I have theta$_{ba}$. We know that the degrees of freedom are this and this, so I have this and then I have this, this is theta$_{b}$ is equal to 1, this is delta is equal to 0.03, this is theta$_{ba}$ is equal to 1 and finally, we have theta$_{c}$ is equal to 1.

So, this is delta (Refer Slide Time: 17:07), this is theta$_{ba}$ and we can find out all these things that come out of it. Let us see. Due to all of these, what is theta$_{ba}$? Note that theta$_{ab}$ is now defined from the chord to the tangent; from the chord, so here, the chord would be this – a with b; this would be this way; here, the chord is this, here, the chord is this. Let us look at what theta$_{ab}$ is; theta$_{ab}$ here is equal to 0, so theta$_{ab}$ in terms of theta$_{b}$ is 0, theta$_{ab}$ in terms of theta$_c$ is 0, and in terms of delta, it is going to be equal to… from the chord to the tangent (Refer Slide Time: 18:17), it is positive, so theta$_{ab}$ in this case is equal to 0.03 by L, where L is 10; remember this was 10 and this was 10 (Refer Slide Time: 18:39), theta$_{ab}$ is 0.03 by 10. What about theta$_{ba}$ here? We will see theta$_{ba}$ is also 0.03 by 10.

Let us see what theta$_{bc}$ is from the chord to the tangent, from the chord to the tangent, from the chord to the tangent (Refer Slide Time: 19:09), clockwise; so, theta$_{bc}$ is equal to minus (0.03 by 10) and theta$_{cb}$ is the same; for theta$_b$ is equal to 1, theta$_{ab}$ is equal to 0, theta$_{ba}$ from the chord to the tangent is 1, from the chord to the tangent is theta$_{bc}$ is equal to 1 and theta$_{cb}$ is equal to 0. Here, theta$_{ab}$ is equal to 0, theta$_{ba}$ is equal to 0, theta$_{bc}$ is equal to 0 and theta$_{cb}$ is equal to 1.

Note one thing: when you only have a rotation, the new definition of theta and the old definition of theta remain the same, the only difference is when you have a displacement, the new definitions of theta$_{ab}$ and theta$_{ba}$ are different from the old one; also note that here (Refer Slide Time: 20:23), I do not have any delta anywhere.

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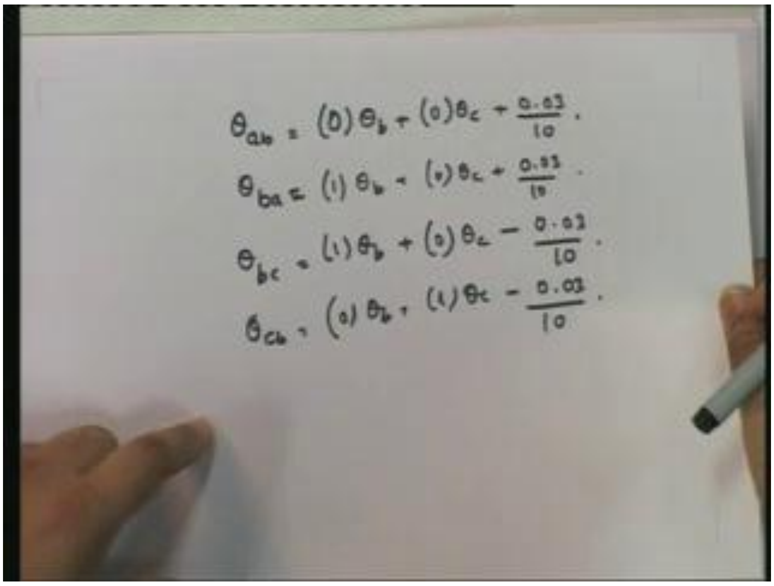

Once we have that, I can write down theta$_{ab}$ as equal to 0 into theta$_b$, so, 0 times theta$_b$ plus 0 times theta$_c$ plus 0.03 upon 10; theta$_{ba}$ is equal to 1 times theta$_b$ plus 0 times theta$_c$ plus 0.03 upon 10; theta$_{bc}$ is equal to 1 into theta$_b$ plus 0 into theta$_c$ minus 0.03 by 10; and theta$_{cb}$ is equal to 0 times theta$_b$ plus 1 times theta$_c$ minus 0.03 upon 10 – these are my equations and now, I substitute these into my slope deflection equations.

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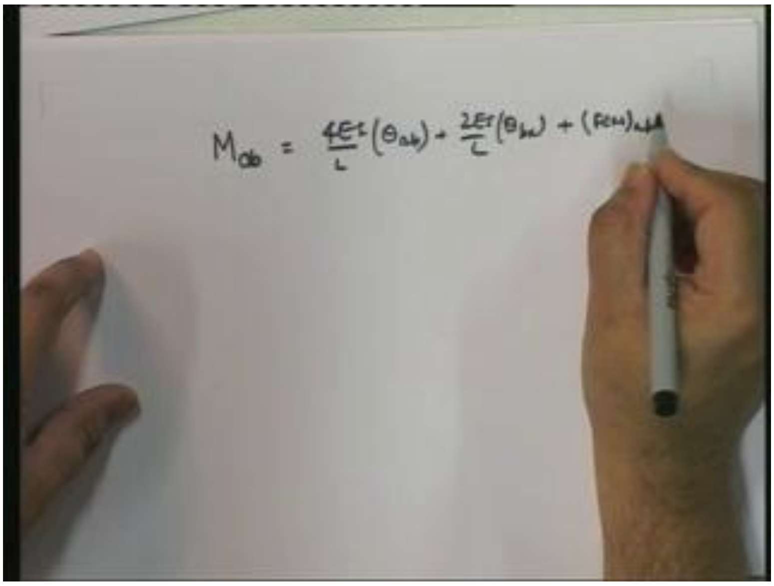

What do my slope deflection equations look like? M_{ab} is equal to 4EI upon L theta_{ab} plus 2EI upon L theta_{ba} plus $(FEM)_{ab}$.

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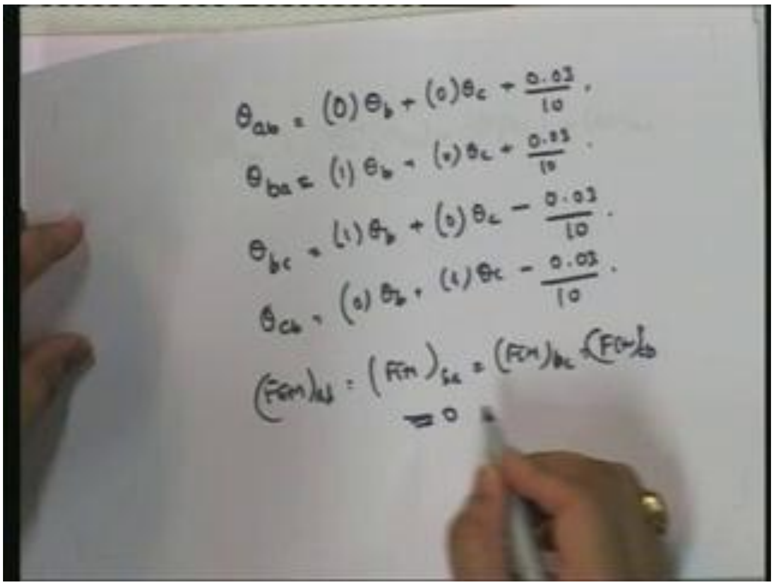

Note that in this particular case, since there are no loads, $(FEM)_{ab}$ is equal to $(FEM)_{ba}$ is equal to $(FEM)_{bc}$ is equal to $(FEM)_{cb}$ and all of them are equal to 0.

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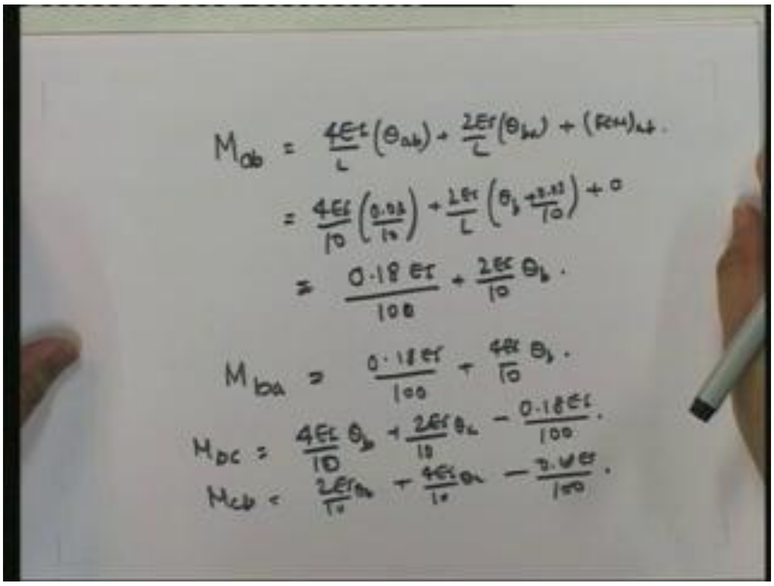

I am going to substitute these in. I am going to get 4EI upon 10, $theta_{ab}$ is 0.03 by 10 plus 2EI upon L, $theta_b$ is $theta_b$ plus 0.03 by 10 plus 0, which is going to be equal to 0.12 plus 2, so 0.16, it is going to be 0.18 EI upon 100 plus 2EI upon 10 $theta_b$. Similarly, if you substitute everything in, you will get M_{ba} is equal to 0.18 EI plus 100 plus 4EI upon 10 $theta_b$; M_{bc} is going to be equal to 4EI upon 10 $theta_b$ plus 2EI upon 10 $theta_c$ minus 0.18 EI upon 100; and M_{cb} is going to be equal to 2EI upon 0 $theta_b$ plus 4EI upon 10 $theta_c$ minus 0.18 EI upon 100. These are going to be the equations.

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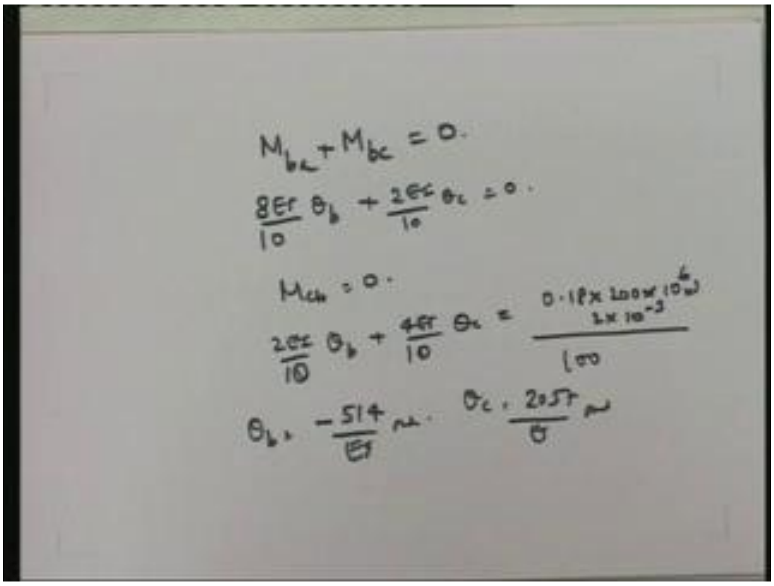

Again, going into the fact that equilibrium equations give us… I am not going in, I have already done this problem earlier, so I am just substituting them in. This one gives me 8 EI by 10 $theta_b$ plus 2EI upon 10 $theta_c$ is equal to 0; the other equation was M_{cb} is equal to 0 and that gives me 2EI by 10 $theta_b$ plus 4EI by 10 $theta_c$ is equal to 0.18, EI is 200 into (I am doing it in Kilonewton), so it going to be 200 into 6 into 2 into 10 to the power of minus 3 upon 100.

After that, you are going to see that once you substitute these equations, this is going to land up being equal to… when I substitute these equations, you will see that these equations look identical to what I had obtained earlier and you will see that theta$_b$ is equal to minus (514 upon EI) radians, theta$_c$ is equal to 2057 EI radians and when you substitute, you will get exactly the same equations. The most important thing is that once I have utilized this procedure, the solution process remains identical; I have just proved that to you with one problem. What is the advantage of this definition? I will shortly come to that by actually taking up the other problem that I had discussed last time – that was the frame problem.

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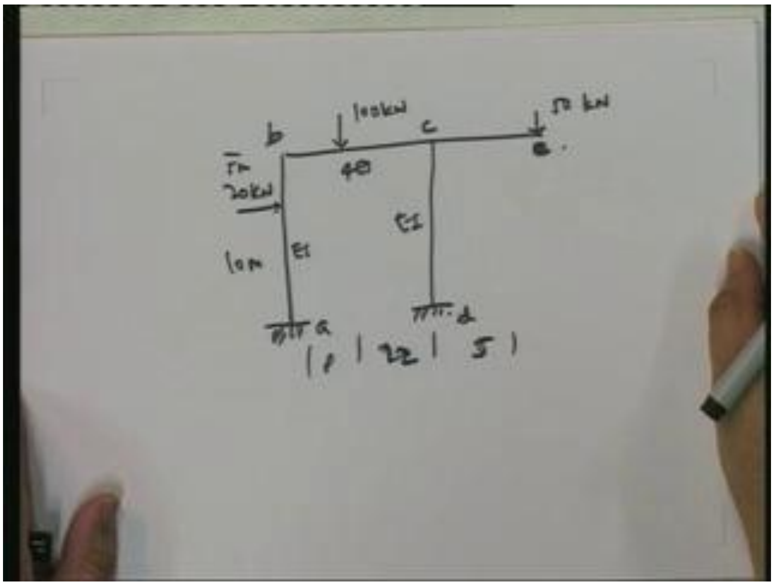

Let us just revisit that problem and I will introduce you to the concept of…. Remember I said that when you were using…. What virtual work method did we use for the force method? We used the method of virtual force. I had said that I was going to use the method of virtual displacement when I came to using the displacement method – that is what I am going to illustrate to you – and this is where it will become obvious as to why I was using this new definition that I had today. This is 12 (Refer Slide Time: 27:53) and this is 5. I am just restating the problem that we had done last time. Today, what I am going to introduce to you is the concept of the principle of virtual displacement and we will see how ….

Last time, if you remember, to be able to generate the third equation, we had to go through quite a bit of involved computation, where we computed shears etc. Once I give you slightly more complicated problems, especially with inclined members, you will see that this is going to become a very very messy affair because not only will you have to compute shears, you will have to compute axial forces and then resolve them, find out the vertical reaction, horizontal reaction – all kinds of things.

Just to be able to get the third equation, you will probably have to solve ten different equations to ultimately get to the third equation; it gets quite messy. I will illustrate this concept to you as we go along; there are several lectures that I am going to spend on actually solving example problems for you. But, I will introduce the concept in today's lecture so that the concept remains clear in our mind while we are solving all the particular problems. I am going to introduce the concept using this example problem which we had already solved in the last lecture.

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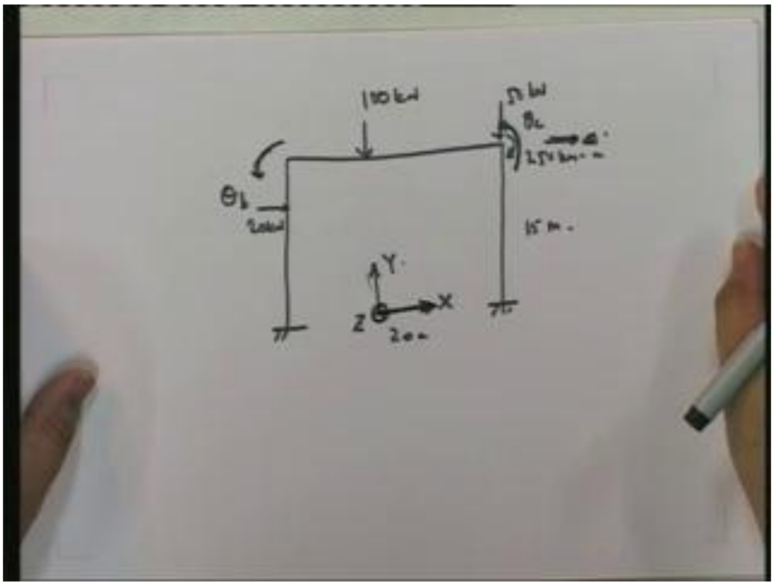

Let us just put one or two things down and then go back to the concept that we are going to have. Remember that I am not going to go through the steps of trying to define what everything is; I am just going to put down my new definition. These are my degrees of freedom: theta_b (Refer Slide Time: 29:57), theta_c, and the translation of this is delta. Then, I have this load here, here I have a load of 50 Kilonewton and I have a moment of 250 Kilonewton meter, I have a 100 Kilonewton load here and a 20 Kilonewton load here, this is 20 meters and this is 15 meters.

These are the degrees of freedom: theta_c (Refer Slide Time: 30:37), theta_b and delta. One thing you must note is that always, my structure degrees of freedom are implicitly assuming this as my structural Z coming out (Refer Slide Time: 30:51); so you will see that if I have a displacement in this direction, it is taken positive in this direction, a rotation is taken positive in this direction (Refer Slide Time: 31:07) so that it comes out of the paper, which is in the positive Z direction. If I ever have any displacement in the Y direction, I will have it positive in that. My definition is always completely related to the idea that positive displacements are along the positive axis of the structure.

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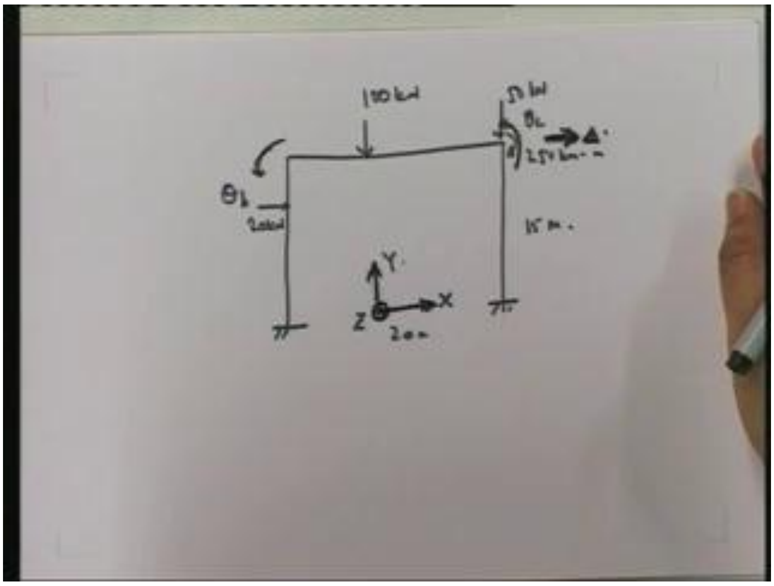

That is the reason why I always define anticlockwise moment as positive (Refer Slide Time: 31:41) because that then gives me the concept of positive; it can be generated very easily because in positive degree of displacement, the corresponding degree of freedom is aligned along the direction. Now, let us look at this. First and foremost, we have to compute the fixed end moments. There is no problem in computing the fixed end moments; the fixed end moments are identical to what we had the last time.

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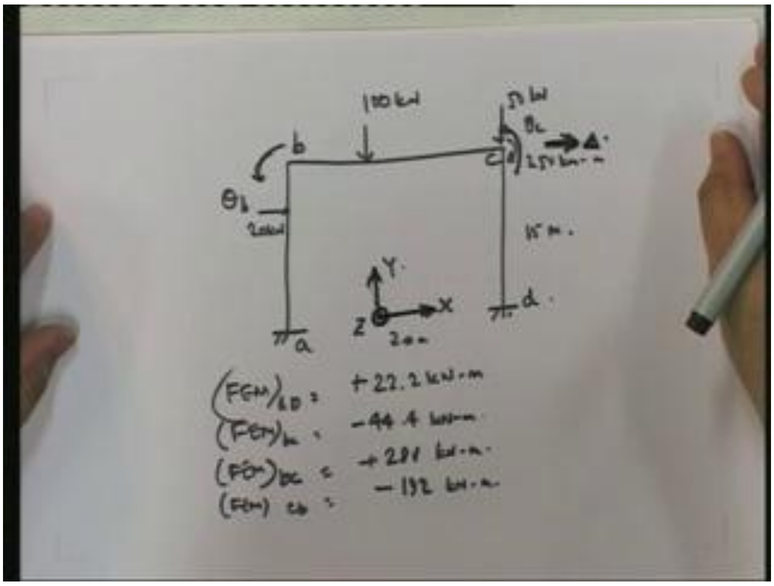

Let me just go through and quickly get you the fixed end moments. Here, this is a (Refer Slide Time: 32:25), this is b, this is c, this is d. The $(FEM)_{ab}$ is equal to plus 22.2, $(FEM)_{ba}$ was equal to minus 44.4, $(FEM)_{bc}$ was equal to plus 288 Kilonewton meter and $(FEM)_{cb}$ is equal to minus 192 Kilonewton meter. In the computation of the fixed end moments, you do not have any specific problem; the only thing is that we need to actually go and define the displacements corresponding to the degrees of freedom and find out how the structure moves.

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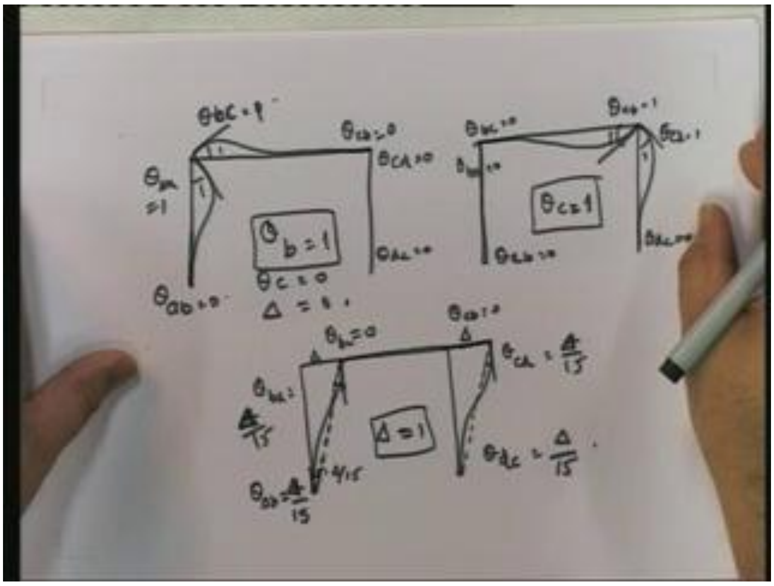

Now I have theta$_{ba}$ is equal to 1, so theta$_{ba}$ is equal to1 is going to give me this (Refer Slide Time: 33:41), this and that is it; this is 1, this is 1, this is theta$_{b}$ is equal to 1 and theta$_{b}$ is equal to 1 automatically assumes that theta$_{c}$ is equal to 0 and delta is equal to 0; then, I am going to put theta$_{c}$ is equal to 1. Remember that for rotations, the definitions do not change. Let us put delta, delta give me this (Refer Slide Time: 34:31), this is delta, delta, so this is delta is equal to 1.

Now, our definition is from the chord delta to the tangent; so if you look at this particular point, what is theta$_{ab}$ equal to? 0. What is theta$_{ba}$ equal to? 1. theta$_{bc}$ is equal to 1, theta$_{cb}$ is equal to 0, theta$_{cd}$ is equal to 0, theta$_{dc}$ is equal to 0. Here, theta$_{ab}$ is equal to 0, theta$_{ba}$ is equal to 0, theta$_{bc}$ is equal to 0, theta$_{cb}$ is equal to 1, theta$_{cd}$ is equal to 1, theta$_{dc}$ is equal to 0; no difference; remember that whenever you have a rotation, there is absolutely no difference in the computation, I mean with respect to last time and this time.

Here, once you have a displacement (Refer Slide Time: 35:42), it all changes because it is from the chord (the chord is the one that connects the displaced positions of…) to the tangent, from the chord to the tangent (Refer Slide Time: 36:05), so, what is theta$_{ab}$ equal to? Delta by 15; if you look at this angle, it is delta by 15 so, theta$_{ab}$, look at it from the chord to the tangent it is anticlockwise, so it is positive delta by 15. What about theta$_{ba}$? Similarly, from the chord to the tangent, it is anticlockwise (Refer Slide Time: 36:32), it is delta by 15. What about theta$_{bc}$? From the chord to the tangent, it is 0, from the chord to the tangent, theta$_{cb}$ is equal to 0 (Refer Slide Time: 36:45) and from the chord to the tangent, anticlockwise, positive; theta$_{cd}$ is equal to delta by 15 and theta$_{dc}$ is delta by 15. We have figured out everything in terms of the displacements.

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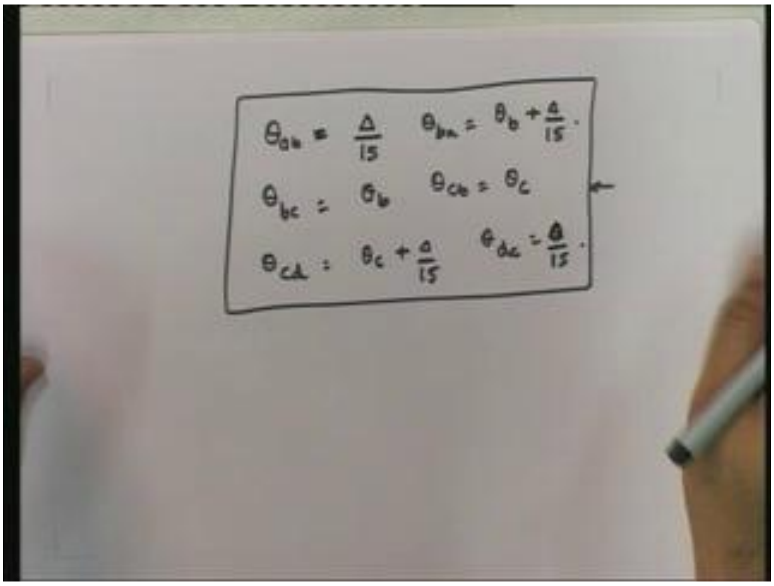

Therefore, I can write down theta$_{ab}$ is equal to 0 into theta$_b$ plus 0 into theta$_c$ plus delta EI by 15, so this is just delta by 15; theta$_{ba}$ is equal to 1 times theta$_b$ plus 0 times theta$_c$ plus delta by 15; theta$_{bc}$ is equal to 1 into theta$_b$ plus 0 into theta$_c$ plus 0 into delta; theta$_{cb}$ is equal to 0 into theta$_b$ plus 1 into theta$_c$ plus 0 into delta; theta$_{cd}$ is equal to 0 into theta$_b$ plus 1 into theta$_c$ plus delta by 15; theta$_{dc}$ is equal to 0 into theta$_b$ plus 0 into theta$_c$ plus delta upon 5; these are my rotations.

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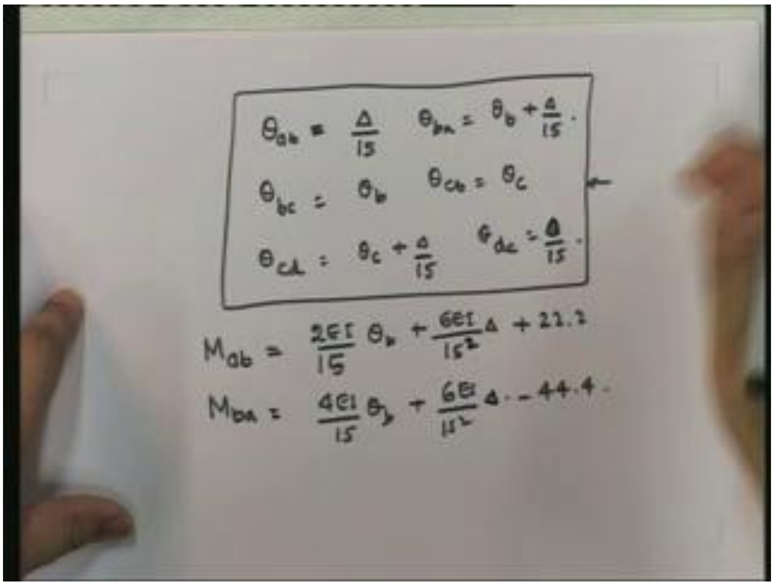

Now, all I need to do is find out the moments and so, I am just putting them down, I am not going to actually calculate. By now, just by substituting into the equations, you will get M$_{ab}$ is equal to 2EI by 15 theta$_b$ plus (6EI by 15 squared) into delta (just plug this in (Refer Slide Time: 39:26) and you will get 6EI delta) and M$_{ba}$ is equal to 4EI upon 15 theta$_b$ plus (6EI upon 15 square) into delta (when you plug this in (Refer Slide Time: 39:46), you will get 4EI into theta plus 2EI etc). You will get all this by substituting these into the equations. Then, you have the fixed end moments, so this going to be equal to plus 22.2 (Refer Slide Time: 40:06) and this is going to be minus 44.4. This way, by substituting bc and cb, you can find out M_{bc} and M_{cb} and you can find out M_{cd} and M_{dc} by substituting them in; I am not going to write those down.

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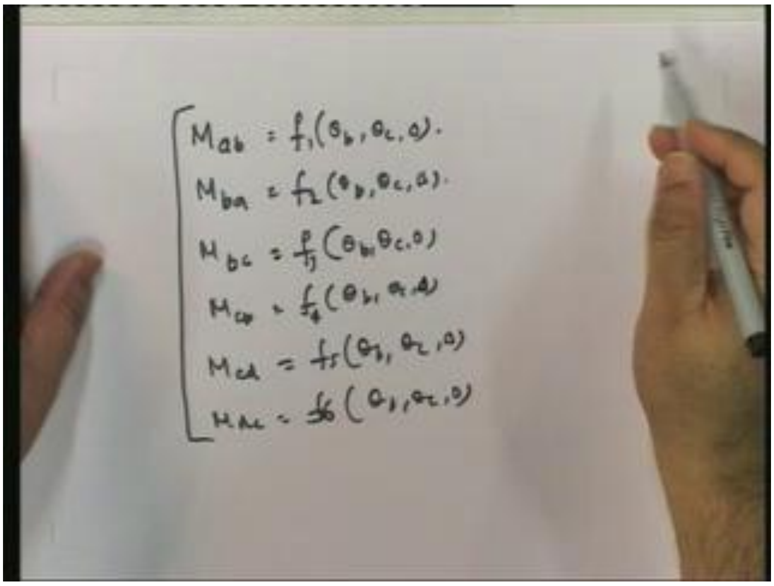

Let me put forward the thought process to you. Essentially, I know that M_{ab} is a function of theta$_b$, theta$_c$, delta – all of them. Of course, it is not necessary that all of them will be a function; for example, M_{ab} does not have theta$_c$ in it, but I am just putting down in general that these will be functions of this. M_{cb} is function… some other functions – f_1, f_2, f_3, f_4 and M_{cd} is going to be equal to f_5 of (theta$_b$, theta$_c$, and delta) and M_{dc} is going to be f_6 (theta$_b$, theta$_c$, delta). All are different functions, but essentially they are functions and then, there are some fixed end moments. These are my real moments and these happen due to the loading that we have on the structure. Now, I am going to use the principle of virtual displacement. What am I going to do? I am going to say that I am going to apply three different independent virtual displacement patterns.

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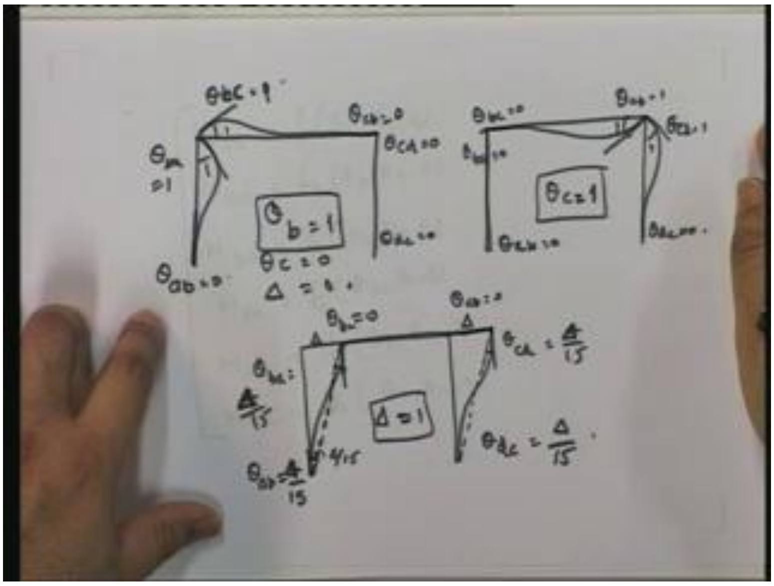
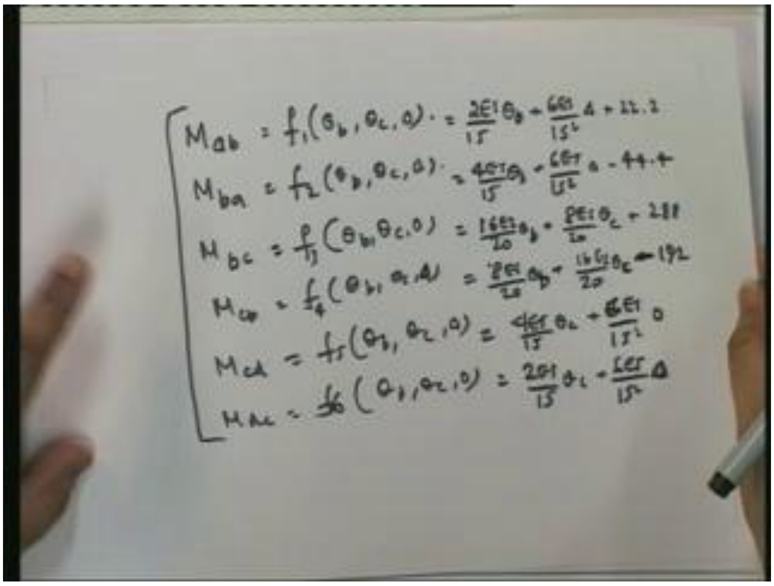


What are my three independent virtual displacement patterns? One is this (Refer Slide Time: 42:11), one is this, one is this and note that these are now virtual displacement patterns; I am going to find out the work done by all the forces subjected to these virtual displacements. The work done by the real forces subjected to these virtual displacements is going to give me the virtual work done in the structure.

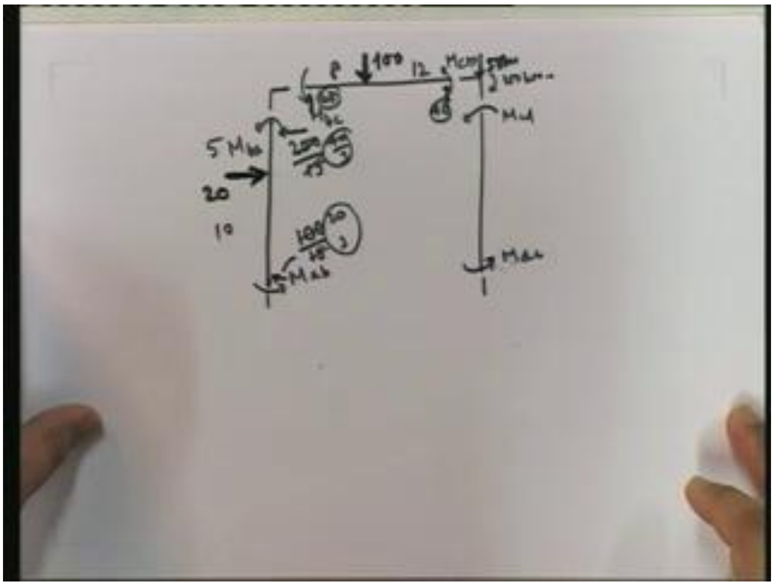
Once I have the virtual work done in the structure, what can I do? I can write virtual work done by the external loads equal to the virtual work done by the internal loads and once I put that, all the total virtual work is equal to 0 and from that, I get an equation. What will be the equation in terms of? Equation will be in terms of $theta_b$, $theta_c$ because these are the internal forces and I am going to find out for each displacement pattern what are the thetas are; once I have those thetas, I can just say what the work done by the internal forces is. If this is my virtual displacement (Refer Slide Time: 43:29) times, it is going to be $theta_{ab}$ into M_{ab} plus $theta_{ba}$ into M_{ba} etc... and in that way, we can continue doing. So, that is my virtual work equation and that virtual work equation is what is going to give me the three independent equations and I can solve for the displacements. I have three equations, three unknowns, I can solve for them. That is in essence the method of virtual displacement.

(Refer Slide Time: 44:22)



Now, what I am going to do is, I am actually going to put it into practice and actually take this particular problem. Let me just put them down and you will see that this (Refer Slide Time: 44:20) is equal to 2EI by 15 theta$_b$ plus (6EI upon 15 square into delta) plus 22.2; this is equal to 4EI by 15 theta$_b$ plus (6EI upon 15 square into delta) minus 44.4; this is equal to 16EI upon 20 theta$_b$ plus 8 EI upon 20 theta$_c$ plus 288; this is equal to 8 EI upon 20 theta$_b$ plus 16EI upon 20 theta$_c$ minus 192; this is equal to 4EI upon 15 theta$_c$ plus (6EI by 15 square into delta); and this is equal to 2EI by 15 theta$_c$ plus (6EI upon 15 square into delta). These are the moments which you can get. Then, what is the next step? The next step is to find out the work done by all the forces.

(Refer Slide Time: 46:16)



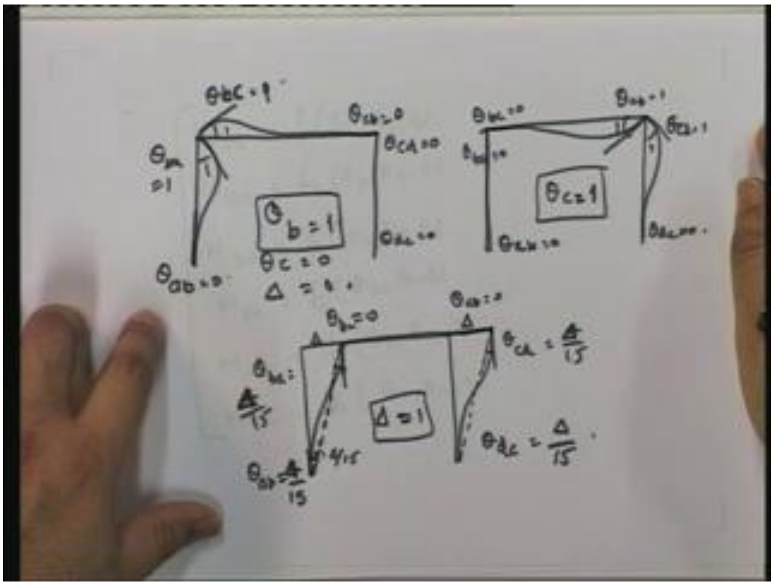
Let us look at what happens. I am breaking it up because I need to know what is the work done by all the forces. Here, you will have M_{ab} (Refer Slide Time: 46:37), here you will have M_{ba}, here you will have M_{bc} (Refer Slide Time: 46:49), here you will have M_{cb}, here you have the load 50 and a moment 250; then, you have the moment M_{cd} and then you have the moment M_{dc}. These are all the moments and note that I am just putting down all the loads; this is 100 and here, you have 20 (Refer Slide Time: 46:21).

One aspect that is very important is that when you have loads acting on the member, what you need to do is you need to find out the reactions at this (Refer Slide Time: 47:45). Think of this as a simply supported beam with M_{ab} and M_{ba}. What I need to compute for getting the virtual work properly is, I need to find out the reactions at the end of the member due to this. Since I am considering it to be a simply supported beam, what are the reactions going to be equal to? This is 5, this is 10, so if I find out the reaction, I am only finding out the reaction due to the load only, remember that, not due to the unbalanced moments etc.; only the load.
What is the reaction at this point due to this load (Refer Slide Time: 48:23)? If I take moments about that point, it is going to have 5 into 20, so this is going to be anticlockwise and if I put this in this direction, this is going to be clockwise, so this into 15 is equal to 20 into 5; this is going to be equal to 100 upon 15 and this is going to be 200 upon 15.

Similarly, here, we are going to have 8 (Refer Slide Time: 49:01) and 12, so just due to the load alone, you will see that this is going to be equal to 1200 (Refer Slide Time: 49:08), this is going to be 60 and this going to be 40; if I substitute, then this is going to 20 upon 3 and this is going to be 40 3. What have I done? On this member, there is nothing. Note that these are applied at the nodes, so do not I need to find out, it is only for the members I need to find out the reactions.

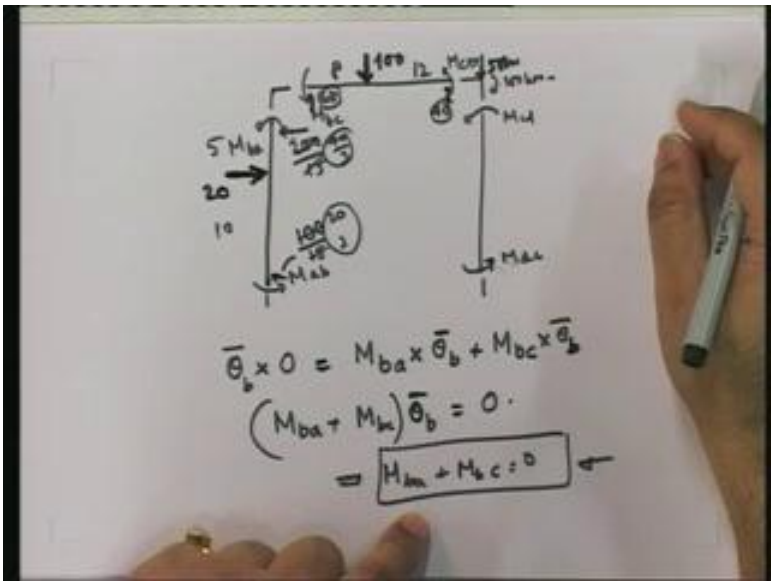
I have just found out the reactions, so what is my virtual work equation? I am going to put theta$_b$ is equal to 1. This is my virtual displacement pattern, so theta$_b$ into the moment at b (which is 0) – this is the external work done and I am calculating all the external. Note that these I have translated (Refer Slide Time: 50:16), so these are no longer…. Member loads are never external, it is only joint loads which are external. How much is this joint (Refer Slide Time: 50:25) displaced by under theta$_b$? Let me look back at the equation that I have drawn.

(Refer Slide Time: 50:35)



This is my theta_b, I am giving this virtual displacement, so I am going to have that this into 1, nothing, 0; this is not going up (Refer Slide Time: 50:47), neither is it rotating, so these two do not do any work.

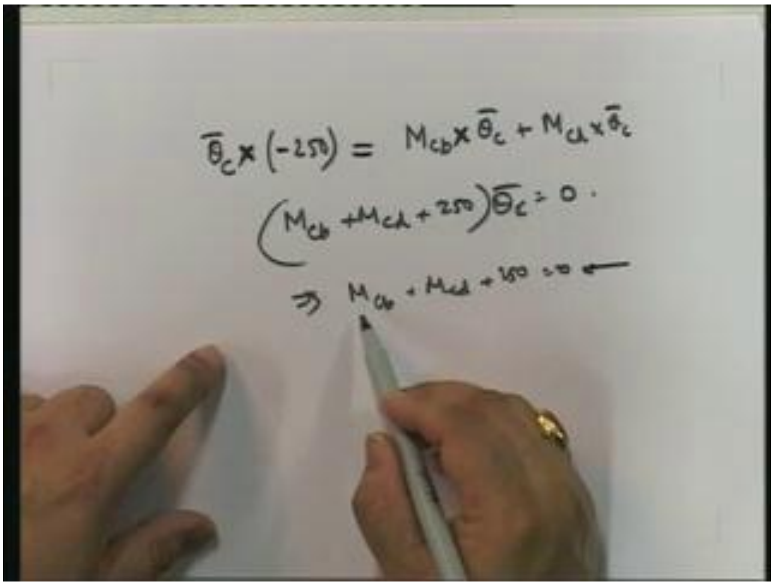
(Refer Slide Time: 50:55)



The external virtual work is equal to this. What is the internal virtual work? The M_{ba} does work, so it is going to be M_{ba} into theta_b because it is 1 into theta_b (note that the moment is this way (Refer Slide Time: 51:17), the rotation is this way, so it is positive work) plus M_{bc} into theta_b. Then, let us look at all the work done by this and this; these do not undergo any displacements, neither do those and neither does that, so this is my equation.

This is the external virtual work (Refer Slide Time: 51:46), this is internal virtual work. This is going to give me M_{ba} plus M_{bc} into theta_b is equal to 0 – this is true for any arbitrary theta_b; this implies that M_{ba} plus M_{bc} is equal to 0. Note that is exactly the equation that we get when we put equilibrium conditions in the previous case; the only thing that we did was we computed the virtual work done. Remember I had said that the method of virtual displacement essentially replaces the equilibrium conditions and I am showing this here – I am getting the same equilibrium conditions. Let me put theta_c and I have the same equation; let me put the arbitrary equation theta_c is equal to 1.

(Refer Slide Time: 52:55)



Then, the work done will be theta$_c$; theta$_c$ is in this direction (Refer Slide Time: 53:02), so what is the moment at that point? You will see that is clockwise moment, so you are going to get 250 into theta$_c$ – this is the external virtual work. All the other external forces, including the support reaction and the joints, do not undergo any displacement; all of them are going to be 0 and so the internal virtual work is going to be M$_{ab}$ into theta$_{ab}$ (which is 0) this theta is 0, all the M$_{ba}$, theta$_b$, 0 etc.

You will see that the only term that exists will be M$_{cb}$ into theta$_c$ plus M$_{cd}$ into theta$_c$ and none of the others do any work; so this one is going to be (M$_{cb}$ plus M$_{cd}$ plus 250) into theta$_c$ is equal to 0. Now, this has to be true for all arbitrary theta$_{cb}$, so I get M$_{cb}$ plus M$_{cd}$ plus 250 is equal to 0. These look like equilibrium equations but they are not, they are actually weak solutions, they are actually the work done by this (Refer Slide Time: 54:31).

I have shown to you that when I put theta$_b$ is equal to 1 and theta$_c$ is equal to 1 as the virtual displacement patterns, I get back my M$_{ba}$ plus M$_{bc}$ is equal to 0, which was the simple equation that we had got earlier. I also got that M$_{cb}$ plus M$_{cd}$ plus 250 is equal to 0. In other words, when I put the theta$_b$ and theta$_c$ virtual displacements, I essentially get back the simple equations that I had developed by taking equilibrium of the joints, joint b and joint c, in the earlier case.

I am going to stop over here because I have come to the end of my lecture. In the next lecture, I am going to show to you that by taking the virtual displacement delta is equal to 1, how I can generate a third equilibrium equation easily without having to go through a whole host of equilibrium conditions for each individual member.
So, on to the next lecture I will show you the actual power of applying the method of virtual displacement. Thank you.