Structural Analysis - II
Prof. P. Banerjee
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 12

Good morning. In the last lecture, we found out how to get fixed end moments due to load on the member itself. Today, what I am going to do before I start off looking at problems etc., I am going to give you a few exercises which I would like you to try out on your own using the procedure that I developed in the last lecture. What I am going to do is, I am going to give you a few exercises.

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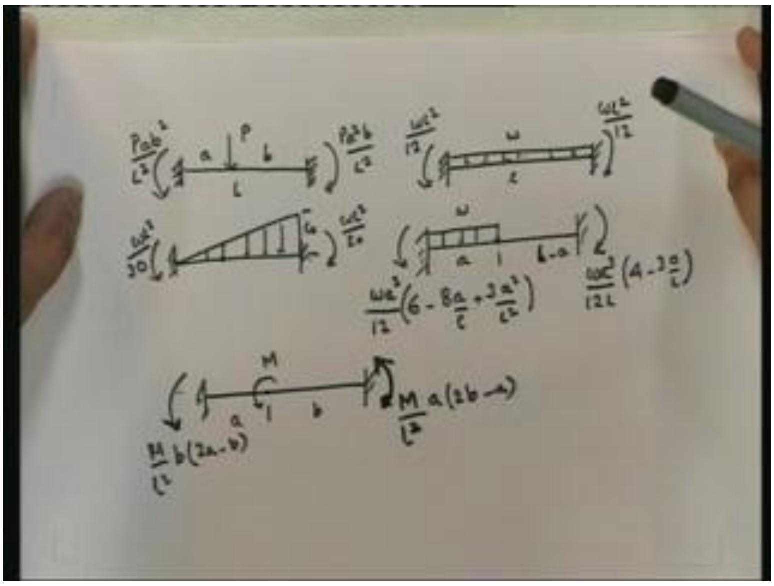

I have already done this for you: this is Pab squared upon L squared, this is Pa squared b upon L squared, this is L and of course, it is a uniform beam. Then, I will give you a few others. This one I have already done; so, using exactly the same procedure, I would like you… w over length l, this is equal to w L square upon 12; this also I have done for you, do some of the more interesting ones. This is a linearly varying intensity of load where w is the intensity at the end. This is a partial uniform loading, uniform loading over part of the…, where this is a and this is l minus a; this is equal to w a squared upon 12 into (6 minus 8a upon l plus 3 a squared upon l squared); and this is w a cubed upon 12 l into (4 minus 3a by l). Satisfy yourself that if you put in a is equal to l, then you are going to get this. This is important because if a is equal to l, then you have to recover this (Refer Slide Time: 05:08) from this.

Finally, I will give you one more. Acting at a distance a and b and then, this is this way, this is this way, this is equal to M upon l squared a into (2b minus a); this is equal to M upon l squared b into (2a minus b). Satisfy yourself that if a is equal to b, then this has to be equal to M upon 4; these two would be M upon 4. I would like you to actually look at all these solutions that I have given you; you should be able to establish the fixed end moments by using the method that you have already developed; so much for all of this. What does this do to the slope deflection equations?

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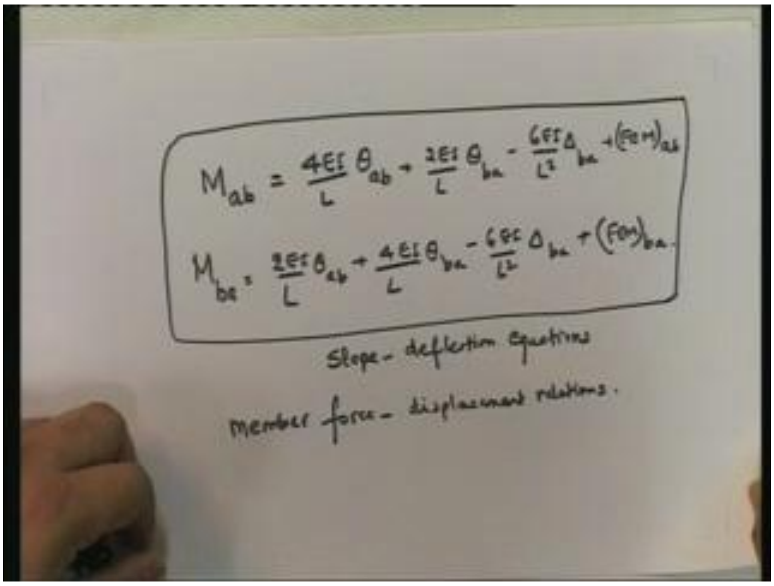

What is the definition of fixed end moment? Let me first write down the entire equation. These are the complete equations; look at this. If theta$_{ab}$ is equal to 0, theta$_{ba}$ is equal to 0 and delta$_{ba}$ is equal to 0 – that is the fixed fixed case, what is M_{ab} equal to? The fixed end moment. That is what it should be. Similarly, this (Refer Slide Time: 08:06). These are the final slope deflection equations which actually present the member force displacement relation; essentially, if you look at this, this is the member force displacement relations that you have. Now, this is the final equation that we are going to be using. Of course, we will see later that there are several things that come out of this, which we have to use, but now we will start looking at problems and see how we can solve those problems to be able to get your solution for equations. Let me give you one thing. What I want to give you is that you should be able to…. Remember the problem that I had given you?

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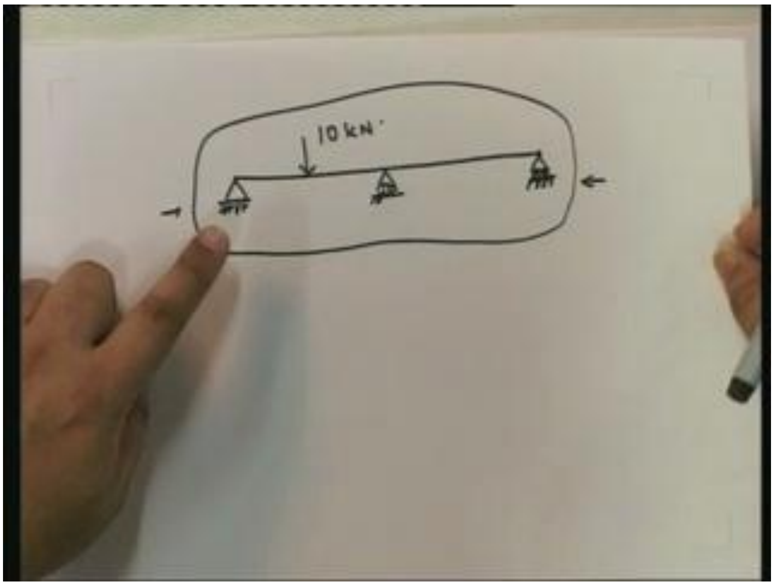

Remember this problem? Try solving this problem and defining the degrees of freedom. Right now, how many degrees of freedom would this have? One, two, three, three degrees of freedom and you can set up three equations etc. I leave it up to you; you can solve this. Let me solve some other problems and I will revisit this problem in a different light, I will talk about it a little bit later.

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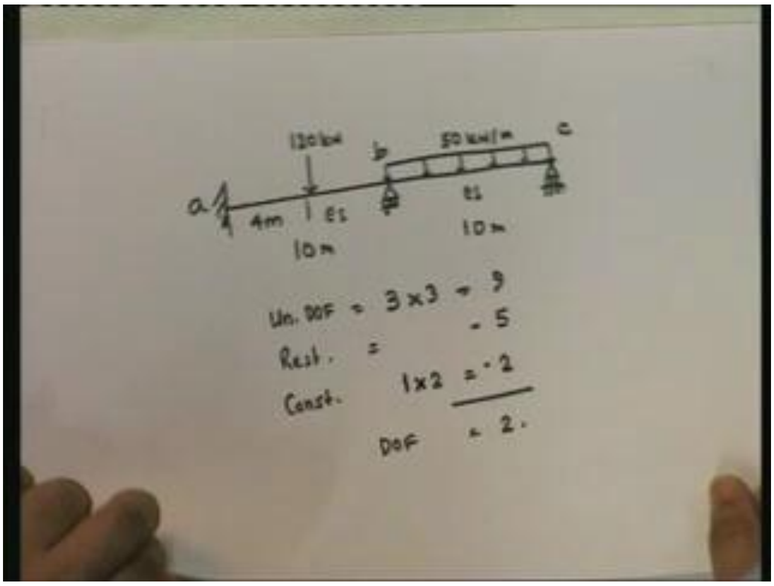

Let me now look at some standard problems that we can solve. The loading here (Refer Slide Time: 10:35), this is 10 meters, 10 meters, this loading is 4 meters, this load is 120 Kilonewton and I have a loading here, this is 50 Kilonewton per meter. You have given the fact that this is EI is the same; the multi-span uniform beam continues over two supports. Let us now do this and I will write this as a, call this b and I will call this c. EI is a constant. Let us look at this particular problem. How many degrees of freedom? One, two, three; so three joints into three is equal to nine. How many restraints? These are the unrestrained degrees of freedom, restraints are three here because it is fully fixed, one here and one here, so that is five. Then, we have constraints. What are the constraints? The constraint is that both the members are actually rigid; if they are actually rigid, then one into two (one constraint per member, there are two members), so two, the total degrees of freedom are two. What are they? They are the rotation here and the rotation here (Refer Slide Time: 12:45). Let me look at what is the possible kind of situation that you are likely to have.

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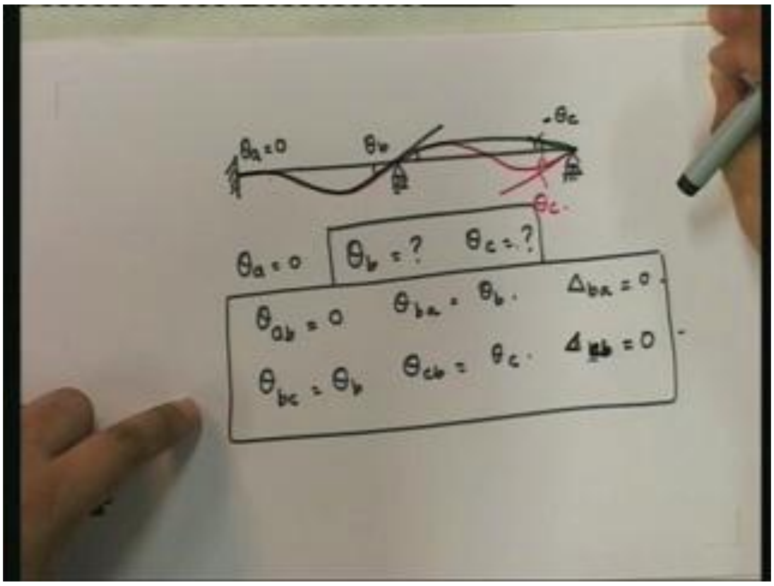

Look at this. What is this? This by definition is theta$_b$ and the way I have shown it, this is actually minus theta$_c$ because it is clockwise. I can define it in this fashion. Let me actually do it in that way since that will make it easier for us to understand. I will draw this and then do this (Refer Slide Time: 14:03) so that by definition, this is theta$_c$. That means what we have are two unknowns. Here, the theta$_a$ is equal to 0, we do not know theta$_b$ and theta$_c$; what we do not know are theta$_b$ and theta$_c$, but for the member boundary conditions, we know theta; theta$_{ab}$ is equal to 0, theta$_{ba}$ is equal to theta$_b$, theta$_{bc}$ is equal to theta$_b$ and theta$_{cb}$ is equal to theta$_c$. What is delta$_{ba}$? Since there is no deflection, delta$_{cb}$ is equal to 0. Always note that the delta is of the right end relative to the left end and that is why it is always done in this fashion. These are our boundary conditions and compatibility conditions where the unknown displacements are these. Therefore, the overall thing is that we have to use the slope deflection equations to be able to find these values; once we find these values, if we find these two (Refer Slide Time: 15:51), remember that we can find these and then we can find out the member end moments and that becomes a statically determinate structure. Let us look at how to solve the problem. The whole point here is that once we have written these compatibility equations, we write the slope deflection equations for each member. What would be the slope deflection equations?

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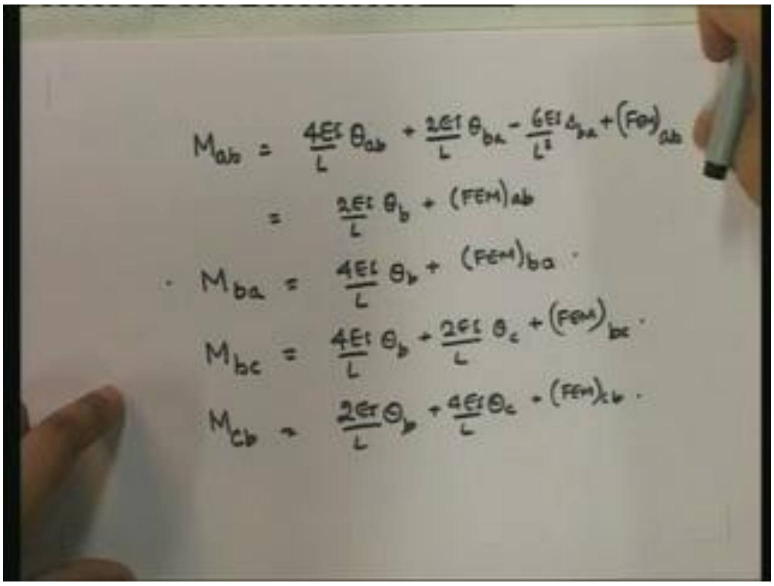

Slope deflection equations would be M$_{ab}$ is equal to 4EI by L theta$_{ab}$ plus 2EI by L theta$_{ba}$ minus (6EI by L squared delta$_{ba}$) plus the fixed end moment due to any load on the member ab. If I substitute the values of theta$_{ab}$ and theta$_{ba}$ that I have written down here (this is my theta$_{ab}$, theta$_{ba}$ and delta$_{ba}$, if I substitute those in, what do I get? I get 2EI by L theta$_{b}$ plus fixed end moment at ab. I am not going to write down this equation, I can write down the same equation. Similarly, when we put it in, we will get 4EI upon L theta$_{b}$ plus fixed end moment at ba. So, you see M$_{ab}$ and M$_{ba}$ are in terms of theta$_{b}$. I will find out these fixed end moments a little bit later. Let me write down the equations for M$_{bc}$ by writing down the equations of motion, slope deflection equations and substituting that theta$_{bc}$ is equal to theta$_{b}$, theta$_{cb}$ is equal to theta$_{c}$, and delta$_{cb}$ is equal to 0, what I get is the following: 4EI upon L theta$_{b}$ plus 2EI upon L theta$_{c}$ plus fixed end moment at bc; similarly, M$_{cb}$ is equal to 2EI by L theta$_{b}$ plus 4EI by L theta$_{c}$ plus fixed end moment at cb. We have got these equations, but we need to find out what these are for the given loadings. What is the loading on ab? Note that if there is no loading on member ab, then these (Refer Slide Time: 19:41) will be automatically 0, but let us see what are the loads on the member.

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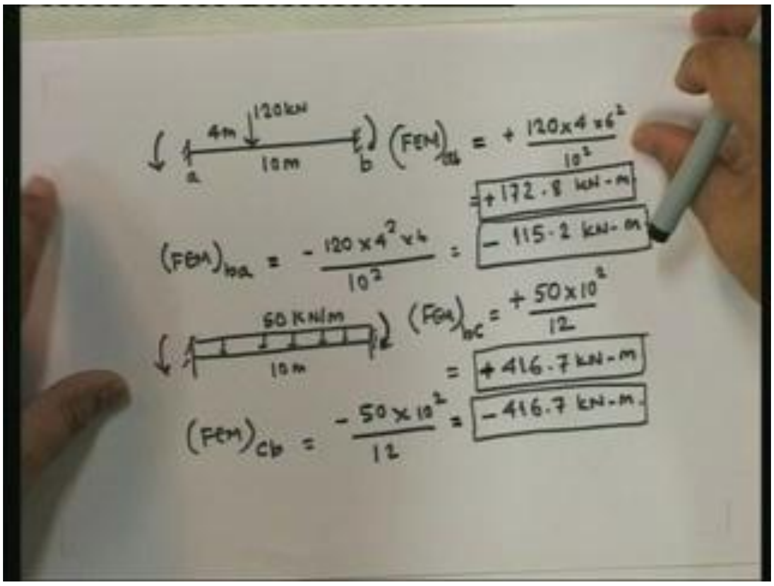

Member ab has a load here, which is equal to 120 Kilonewton. This is a (Refer Slide Time: 20:03), this is b, this is 4 meter and this is 10 meter. I need to find out these fixed end moments by plugging it into the formulae that we have already developed. Note that my definition of fixed end moment is positive if it is anticlockwise, so this is going to be this way and this is going to be this way. This $(FEM)_{ab}$ is positive, it is (plus 120 into 4 into 6 squared) upon L squared. What is that equal to? That is equal to plus 172.8 Kilonewton meter.

What about fixed end moment at ba? If you look at fixed end moment at ba, that is going to be negative (120 into 4 squared into 6) upon 10 squared, this is going to be equal to minus 115.2 Kilonewton meter. These are the fixed end moments at ab and ba in member ab due to this 120. Similarly, I can find out what is the loading on member bc. The loading is 50 Kilonewton per meter; this is 10 meter, so these are going to be this way. My fixed end moment at bc is equal to plus (w L squared) by 12 – which is equal to plus 416 Kilonewton meter, and fixed end moment at cb is equal to minus (50 into 10 squared), minus because I am assuming anticlockwise to be positive – please note that and please note my sign convention that I assume anticlockwise to be positive, others may choose clockwise as positive. There are various ways of solving this problem but understand that as long as you use one particular method, then you have to be consistent and you cannot keep changing your values. This is my $(FEM)_{ab}$, this is my fixed end moment at ba, this is fixed end moment at bc and this is my fixed end moment at cb. Let me plug those into my equations. What do I get?

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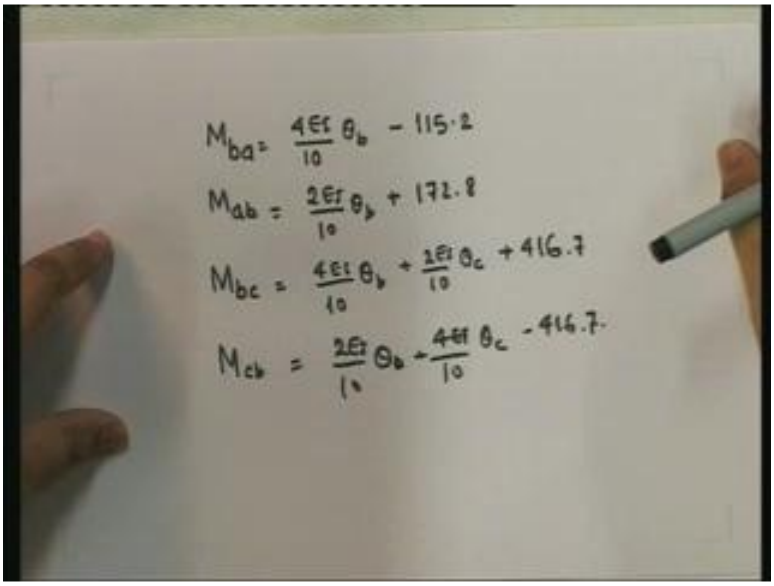

I get that M_{ba} is equal to 4EI and now I am going to put in the value of 10, so it is 4EI by 10 θ_b minus 115.2; M_{ab} is equal to 2EI upon 10 θ_b plus 172.8; M_{bc} is equal to 4EI upon 10 θ_b plus 2EI by 10 θ_c plus 416.7; M_{cb} is equal to 2EI by 10 θ_b plus 4EI by 10 θ_c minus 416.7. These are my member end moments. Let me look at what I have to do now. I have to actually write down two equilibrium equations so that I can solve for θ_b and θ_c. Let us see what those equilibrium equations are.

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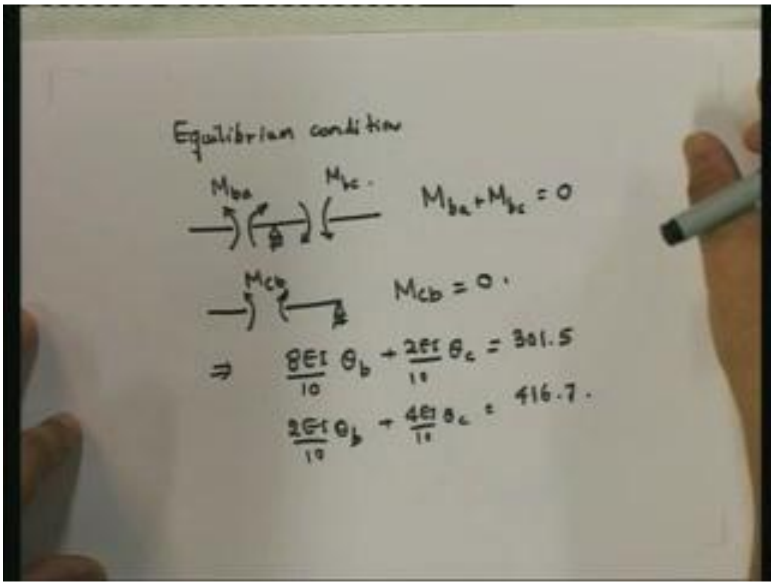

Let us look at the equilibrium conditions here. At joint a, you do not have any equilibrium. The equilibrium says that M_{ab} is equal to the moment at a; that does not give you anything. Let us look at joint b. At joint b, what do we have? From ab, we have M_{ba}, then we have the joint and then from cb, we have M_{bc}. If I look at the joint itself, what do I have? M_{ab} and M_{bc}. Then, if I take the moment equilibrium at the joint, what do I get? I get M_{ba} plus M_{bc} is equal to 0.

Similarly, if I look at joint c, what does it tell me? This is M_{cb} and by definition, since the moment is 0, M_{cb} is equal to 0 – these are my two equilibrium equations. Now, I substitute M_{ba} and M_{bc} that I have written down earlier and if I put those in by substituting these that I evaluated into these equations, what I get is 8 EI upon 10 theta$_b$ plus 2EI by 10 theta$_c$ is equal to 301.5 and the other equation is 2EI 10 theta$_b$ plus 4EI upon 10 theta$_c$ is equal to 416.6. So, I have two equations in theta$_b$ and theta$_c$ and I can actually solve this; the easiest way to solve this is by writing it in matrix form and solving it.

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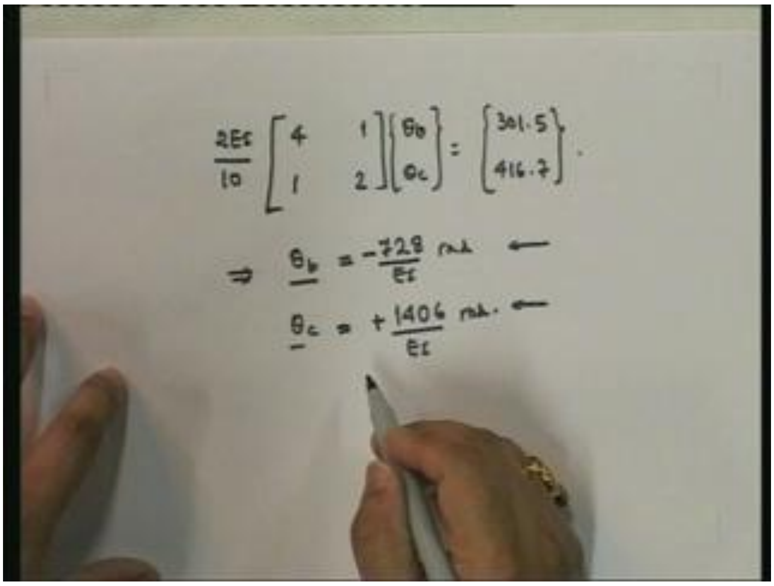

I am going to put 2EI by 10 outside, so inside what I have is 4, 1, 1, 2 into theta$_b$, theta$_c$ and this is equal to 301.5 and 416.7. Then, what do I get? If I solve for this, I get theta$_b$ is equal to (728 upon EI) radians, theta$_c$ is equal to (minus (1406 by EI)) radians. I have solved for theta$_b$ and theta$_c$ and I can actually draw the deflected shape now, because if you look at the deflected shape that we had drawn earlier, now I know what theta$_b$ is and I know what theta$_c$ is.

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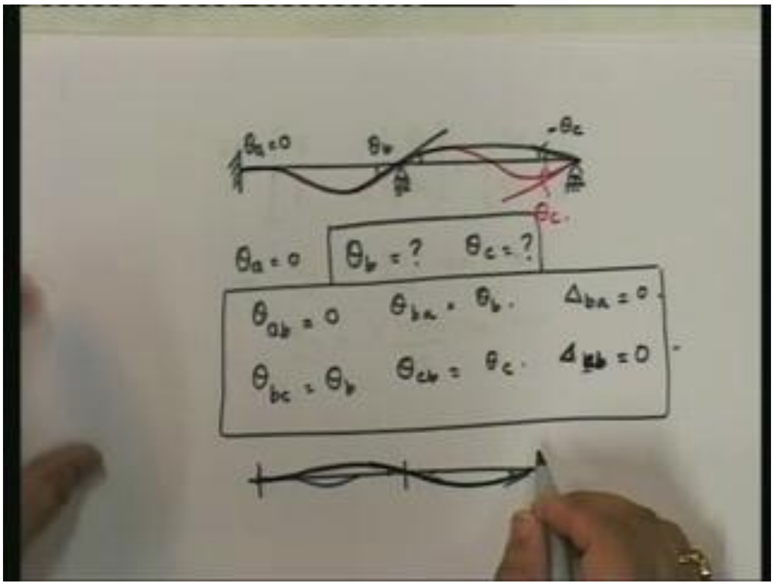

Essentially what this means is that the actual deflected shape goes in this fashion, this is negative and this is positive. Anyway, that does not really matter; what was my interest? My interest was in the bending moment diagram. Now, when I have these and I put that into the equation that I have here and since I know $theta_b$ and $theta_c$, I can put those in and get my M_{ab}.

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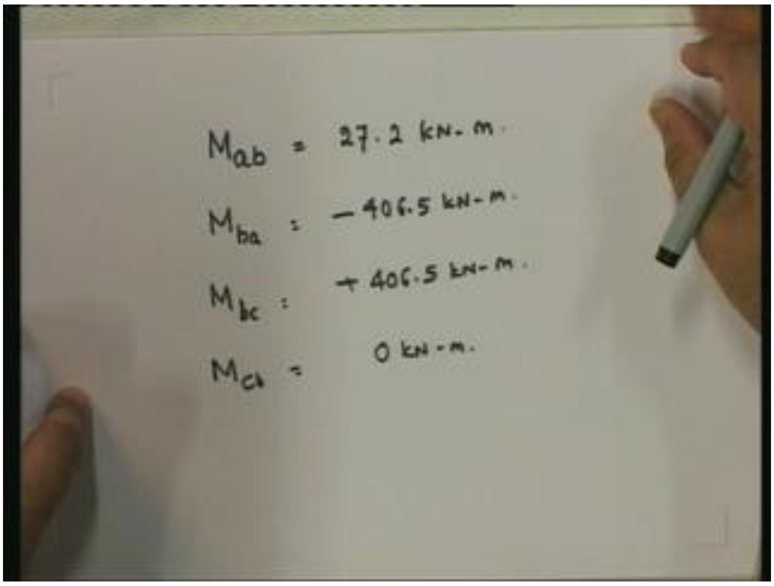

My M_{ab} is equal to 27.2 Kilonewton meter, M_{ba} is equal to minus 406 Kilonewton meter, M_{bc} is equal to plus 406 Kilonewton meter and M_{cb} is going to be equal to 0 Kilonewton meter. If I look at this, what am I getting? I am getting the following.

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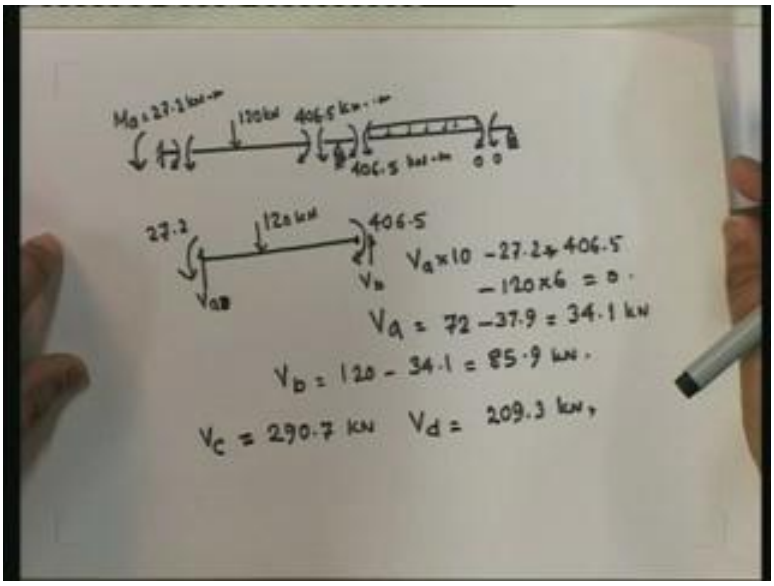

Let me now draw this. I am going to break… whatever I have broken these up, this is my moment at the fixed end. This is equal to …. To balance this (Refer Slide Time: 33:12), I need something like this and to balance this, I need M_{ab}. So, this M_{ab} is equal to 27.2 Kilonewton meter. Then, I have this load 120 Kilonewton then I have minus 406? What does minus 406 mean? Minus 406 means it is clockwise, so clockwise and this way. Then, I have this, this will be going this way and what is this value? This is 406. Note at bc also, I have 406, anticlockwise, so this is correct, it goes this way and then on this side, what do we have? 0. We have this and now, we have this. These are my member end moments that I have obtained over there; I have put them in here and now, I need to find out the shear forces also. How do you evaluate that?

Take member ab. Member ab at this end has 27.2. I am just writing down the values, I am not writing down the dimensions because we already know what the dimensions are. In addition to that, at 4 meters, you have 120. How do I obtain the shear force at a? To find out the shear force, I can take moments about this end because I have shear force over here too. What would this shear force be equal to? Take moments about this end. What you will have is V_a into 10, that is clockwise, then this is anticlockwise, so minus 27.2, this is clockwise, so it is plus 406, and this is anticlockwise (Refer Slide Time: 36:23), so this is minus 120 into 6 is equal to 0. This gives me that the shear force is equal to… 720, so it is going to be 72 and then minus, this is going to give you 379.3, I will just put that in, divided by 10, so it is going to give minus 37.9 (I am dropping the 3, of course) and so V_a is equal to 34.1 Kilonewton. Let me look at this one here. Obviously, the shear at b, V_b is going to be 120 minus 34.1 and that is equal to 85.9 Kilonewton meter. This is the shear at a and similarly for this (Refer Slide Time: 37:41), with the loading of 50 Kilonewton meter, you will see that V at c turns out to be equal to 290.7 Kilonewton meter – that essentially comes from 406.5 divided by 10 plus 250, and V at d turns out to be equal to 209.3 Kilonewton meter. In one place, you have additional and in one case, you have less. These are our shears. Now, let us draw the bending moment diagram. How does it look? The bending moment diagram would look in this fashion.

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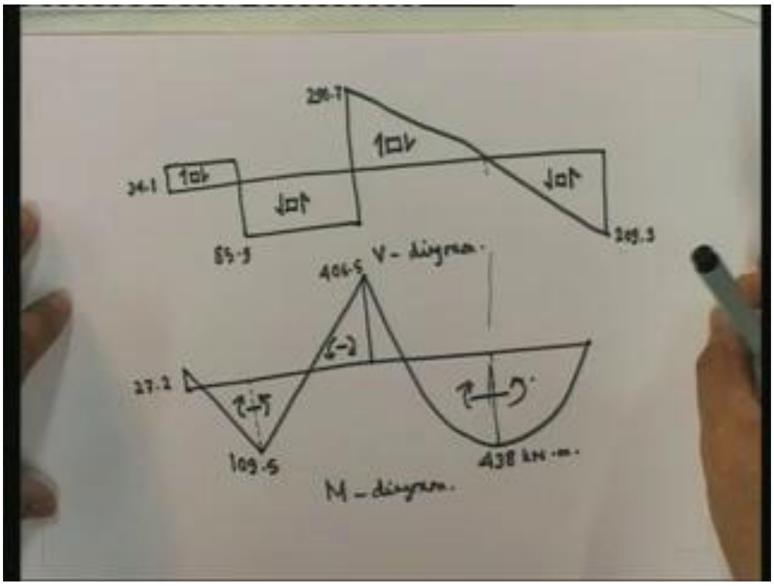

Let us draw the shear force diagram first. You know that the shear force diagram is a simple one; I am going to draw it in one direction and be consistent, I am drawing it in this direction. What is the value? 34.1. What is the direction? It is this way up, this way down. That is a constant till we hit 4 meters; from 4 meters, it goes to minus 85.9; this minus does really matter, plus, minus is really irrelevant for us. This is going to be this way. In reality, I always consider this as positive but it does not matter, I am just drawing it. Then, what do we have? It has to go to 290.7 on the other side; so, here, it is going to go up, it is going to be 290.7; from there, because it is linear, this is going to be 209.3 and because it is UDL, it is going to go like this (Refer Slide Time: 40:30), this is going to be this way, this is going to be this way and this is my shear.

Now, what is the reaction at b? The reaction at b is going to be 290 plus 85 is equal to 376.6 – that is the upward reaction at b. What is the reaction at c? The reaction at c is going to be 209.3. That is my shear force diagram and now let us look at our bending moment diagram. This is going to be equal to 27.2, then it is going to go down (because this is constant) to this value, which is going to be equal to 109.5 Kilonewton meter and from there, it is going to go up at this point to 406.5 – that is the bending moment at b. Note that this goes this way, this goes this way, ultimately, this goes this way and then 0; at this point, it just turns out to be 438 Kilonewton meter. This is our M diagram and this is again this way (Refer Slide Time: 43:10). This is your bending moment diagram and shear force diagram and that is the total analysis of this particular problem.

This, in essence, gives you what the displacement method is. In the displacement method, what did we first do? We first identified the degrees of freedom. Once we had the degrees of freedom, what was our next step? The next step was to evaluate the fixed end moments in each member; the fixed end moments will only exist if there is a load on the member; if there is no load on the member, then it does not exist. You find out the fixed end moments; now, I have already told you that you should be able to find out the fixed end moments given any loading from first principles – that is what I did in my last lecture; you should be finding out the fixed end moments.

Once you know your fixed end moments, you write down the member force displacement relationship which is given by the slope deflection equations. Substitute everything in. Next, write down the equilibrium conditions; the number of equilibrium conditions will always be equal to the number of degrees of freedom that you have. Essentially, the equations are in terms of the unknown displacements or in this particular case for the beam, they are the rotations. You find the rotations by solving those equations and once you find out the rotations or the displacements, what do you do next? You then substitute back into the member force displacement relationships, which are the slope deflection equations. Now, you can find out the member end moments; once you know you the member end moments, then you can take each member and find out its reactions, its shear force, bending moment, everything; then, you put it all together and this is what you have. I am just going back to the original problem – this is the original problem.

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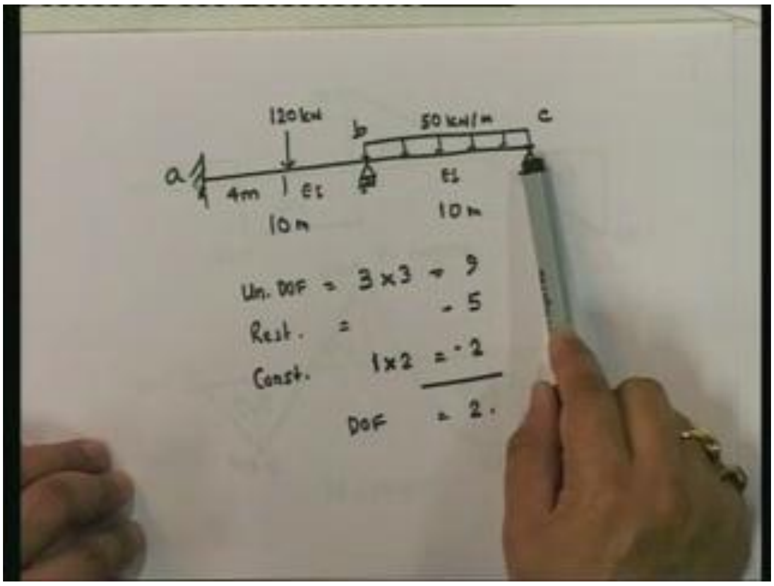

Given this loading on this multi-span beam, we use the displacement method to generate the shear force and bending moment diagram. To take it forward, let us look at another aspect of this particular problem and that is, what do we do when we have support settlement? Remember that I talked about support settlement when we looking at the force method. Let me take this particular problem itself.

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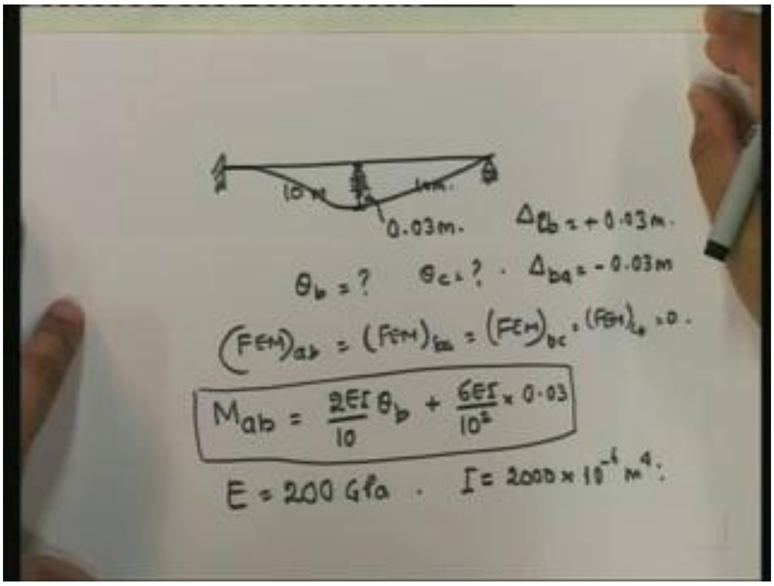

This is 10 meters, 10 meters; no loading on the structure; remember I said that there is no loading on the structure. Why? Because we can always consider the loading to be done separately and we can then consider other conditions. Remember that the principle of superposition is always valid in all the problems that we do because we are only considering linear elastic structures; when you have linear elastic structures, you always can superpose. Let us just say that due to the loading that we considered in the previous case, this support settles. We have already found out what effect the load has, now we want to find out what effect this settlement has; let us just assume a particular value and it goes down by 0.03 meters. Then, what do we have? This is going to become something like this (Refer Slide Time: 47:51). Again, I have $theta_b$ unknown and $theta_c$ unknown. Only thing is, since there is no loading, fixed end moments are all 0; they are all 0 because there is no loading.

Just quickly, you can go back to my equations, I am just writing down one of them. Substitute these values in and we have the same compatibility conditions. M_{ab} is equal to 2EI by 10 $theta_b$. Do we have a $delta_{ba}$? Note that we do have a $delta_{ba}$. This is gone down relative to this. I had always said that $delta_{ba}$ is positive going up, so when it is going down, it is negative. Here, you have $delta_{ba}$ as minus 0.03 meters. What about $delta_{cb}$? If you look at $delta_{cb}$, ba has gone down relative to c, which means c has gone up and so, it is positive, plus 0.03 meters. If I substitute this in, this becomes plus 6EI upon L squared multiplied by 0.03 meters; this is the way that you have to solve it.

What I will do is, I am going to spend the next lecture looking at how you can solve this particular problem in more detail. Now, let us just put down some values, because I will require the values of E and I to be able to get estimates on everything that we have here. I am going to put down some values of El: E is 200 gigapascals and I is equal to 2000 into 10 to the power of minus 6 meter fourth. These are some of the values that you have been given and so, once you know E and I, you will be able to solve for this. More about this when I go into my next lecture. Just to review, what is the procedure for the displacement method?

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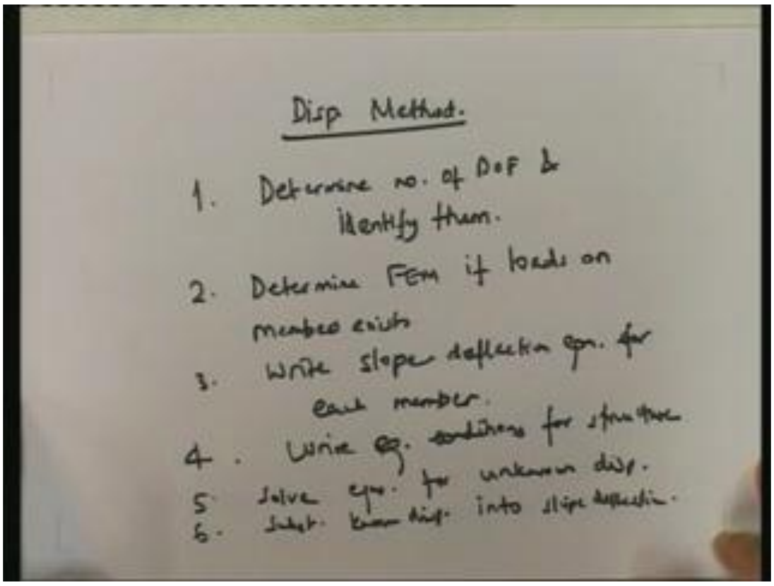

One: determine the number of degrees of freedom and identify them; two: determine the fixed end moment if loads on members exist; three: write the slope deflection equations for each member; four: write the equilibrium conditions for the structure; five: solve the equations for unknown displacements; and six: substitute the known displacements into the slope deflection equations so that you can get your member end moments and once you get your member end moments, you should be able to solve it.

I am going to stop here and in the next lecture, I am going to take up this particular problem and show you how to solve this problem for support settlement using the displacement method. Thank you very much.