Structural Analysis II
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Lecture – 10

Good morning everybody. We have spent the last few lectures looking at the force method. Today, I am going to introduce another method. The force method actually lies under a broad class of what are known as compatibility methods, because the equations were essentially based on getting a compatibility equation and the number of compatibility equations is equal to the number of redundant forces that you generate.

Today, we are going to be starting to look at a completely different approach. This is broadly classified as displacement methods just like we have the force method. Initially, what I am going to be doing is, I am going to look at what is known as the classical slope deflection equations, look at beams, and then quickly I will move on to more complex application of the same sort of methods.

We will see that just as we have used the principle of virtual force in the force method, somewhere along the line, we will be using the principle of virtual displacement to write down the displacement equations. Let me just quickly move on to how to use the displacement method and today, I am going to concentrate essentially on what is known as the slope deflection equations and how to use the slope deflection equations to solve a statically indeterminate structure.

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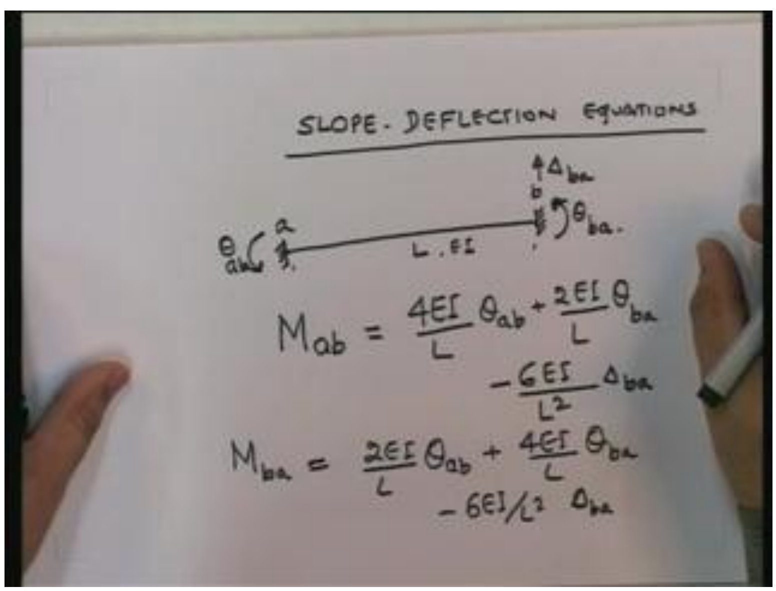

Let us look at what the slope deflections equations are. The slope deflection equations are given. Remember we had said when we had started looking at kinematic indeterminacy or the degrees of freedom, that a kinematically determinant structure is one such as the one that I have over here. In this case, the number of degrees of freedom that you can release, if you look at the release degrees of freedom, we can look at it this way, we can say this rotation we can release. I will call this a, b. I will call this theta$_{ab}$; in other words, it is the rotation at a in the member ab. There is the rotation ba and then there is also the delta$_{ba}$, which is the lateral displacement of b relative to a, please remember that, it is relative to a, that is delta$_{ba}$. Another point that you will slightly notice if you look at standard slope deflection equation books, this theta is defined in this manner. I always define anticlockwise rotation as my proper orientation; I will always have anticlockwise rotation as positive in my notation system.

Note that because I have drawn it in this way, the slope deflection equations become this. I am just writing down the slope deflection equation. If I call this as L, the flexural rigidity at EI is given as 4 EI by L theta$_{ab}$ plus 2 EI by L theta$_{ba}$ minus (6 EI by L squared delta$_{ba}$); and M_{ba} is equal to 2 EI by L theta$_{ab}$ plus 4 EI by L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$).

These are my slope deflection equations. These are the slopes, this is the deflection and essentially, these equations relate the moment at the two ends of the beam and they give it in these terms. Now, these are the slope deflection equations and normally, this is where you start off. However, what I am going to do is this. This goes along with my basis of never writing down an equation without knowing where it comes from. So, we are going to develop these equations of motion so that we know how they are developed, such that if there are any different kinds of situations that we come up with, we can always obtain the slope deflection equations for those kinds of situations. Let us see these slope deflection equations; please look at the slope deflection equations. M_{ab} is equal to 4 EI upon L theta$_{ab}$ plus 2 EI upon L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$); M_{ba} is equal to 2 EI upon L theta$_{ab}$ plus 4 EI upon L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$). These are the classical slope deflection equations. Now, let us see how we can develop this.

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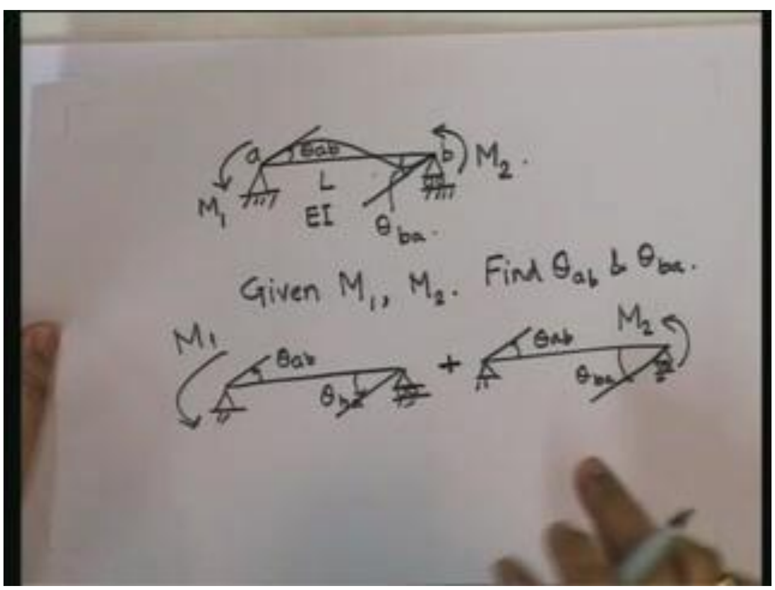

For this, let me go back to my old friend, the simply supported beam. In the simply supported beam, I can apply any kind of load; I am going to apply some specific loads here. What I am going to do is I am going to apply a moment M_1 at this point and I am going to apply a moment M_2. This is my loading on this beam; this is the only loading on this beam, this is a, b; so, I need to find out $theta_{ab}$ and $theta_{ba}$. Under this application, this beam will deform and I want to find out what these rotations are under these loads. This is my question here. How am I going to find this out? I am given the fact that the length here is L and since only flexural deformations are considered, a flexural rigidity is given by EI. My brief is, given M_1 M_2, find $theta_{ab}$ and $theta_{ba}$. Remember that since I am given M_1 and M_2, I can consider this structure to be first subjected to M_1 plus M_2.

You see, principle of superposition…. If this is my loading, I can say that this loading is equal to M_1 is equal to M_1, M_2 is equal to 0 plus M_1 is equal to 0 and M_2 is equal to M_2; I can always say this. What would be the $theta_{ab}$ due to these two applications? I can find out $theta_{ab}$ and $theta_{ba}$ due to this, I can find out $theta_{ab}$ and $theta_{ba}$ due to this. If I add the $theta_{ab}$s, then I will get these $theta_{ab}$s. Therefore, the question here is very very simple. I want to find out how much….

Note that I define $theta_{ab}$ also as anticlockwise, as positive and also $theta_{ba}$. Similarly here $theta_{ab}$, $theta_{ba}$. Once I find out…. In each case, I add the two up and I get these $theta_{ab}$s that I am interested in. Let me solve this problem first and then I will solve this problem.

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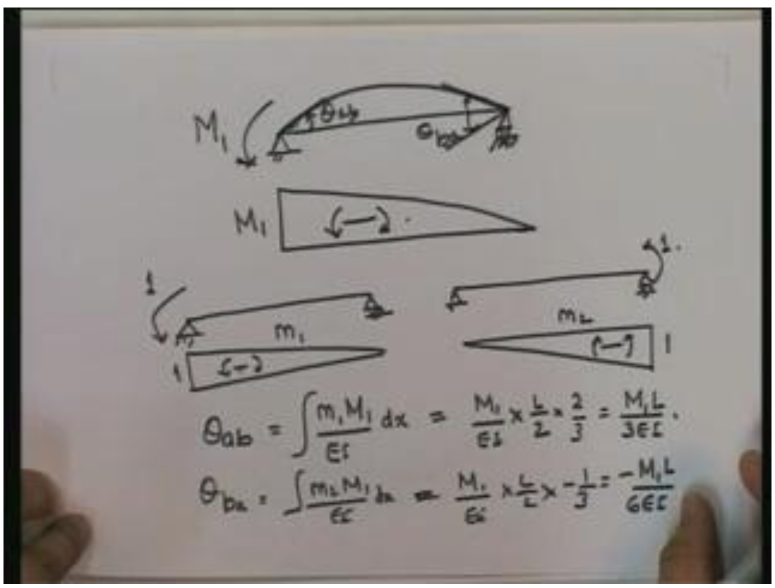

Under this load, how would I find out the rotations here? First and foremost, find out…. This is a standard displacement problem which have you already solved earlier; so, I am going to use the principle of virtual force to solve this problem. First, I am going to apply the real load. When I apply the real load, I get…. This is my bending moment M_1 here, 0 here, linear and this is the bending moment; I am going to leave it up to you to get this. Next, I want to find out the rotation at this point. What do I do? I apply a unit virtual load. What do I get? I will get again linear, 1. If I want to find out this rotation, what do I do? I apply a moment here corresponding to the rotation and draw the bending moment diagram. This is the bending moment diagram. All I need to do is…. To calculate theta_b, I need to do…. 1 into theta_{ab}, that is my external virtual work is equal to.... This is going to be my M_1, this is going to be my M_2, this is going to be equal to m_1 M_1 upon EI dx, which is equal to M_1 upon EI into…. This length is L, so it is L by 2.

This is the area under this curve and the value at this at the centroid of this axis is equal to two-thirds. So, this is equal to M_1 L upon 3 EI. Similarly, theta_{ba} is equal to m_2 M_1 upon EI dx. That is equal to M_1 upon EI into L by 2, that is again the area under this curve, and multiplied by the…. At the centroid, what is the value here? Since the centroid is two-thirds or one-third from this distance, it is going to be equal to one-third and note that it is opposite sign because if this is hogging, this is sagging. This is going to be minus1 upon 3. This is going to be equal to M_1 L by 6 EI; because of this load, the theta_{ab} anticlockwise and anticlockwise theta_{ba} are going to be equal to this. What does theta_{ab} positive and theta_{ba} negative mean? It implies that it actually is something like this, because theta_{ab} is positive but theta_{ba} is negative, which means it is clockwise rotation; clockwise rotation is negative, so this is how it looks. Under this loading, you would expect it to behave in this manner. So, these are the values. Now, let us look at what if I apply M_2.

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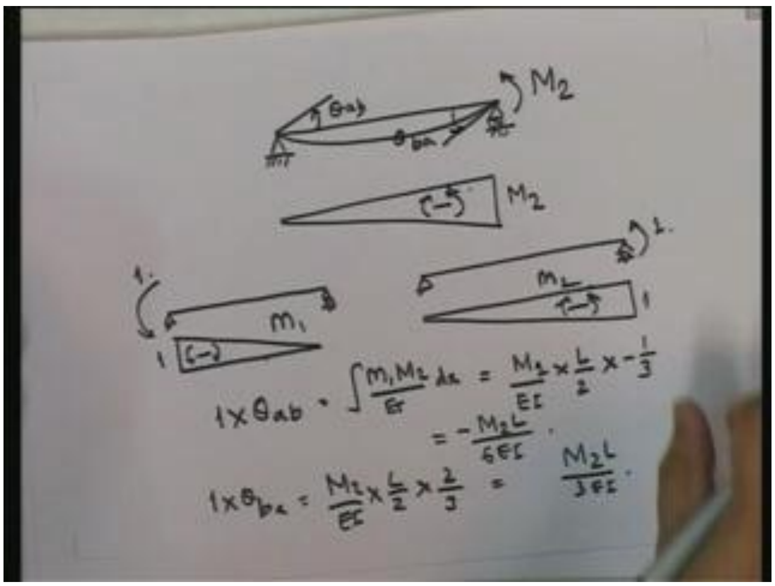

I am going to apply M_2. I need to find out how much theta_{ab} is (that is positive anticlockwise) and how much theta_{ba} is, which is positive anticlockwise. Under this loading, what would be the bending moment diagram? Again, I am not going to be spending time telling you how to do it; you should be able to do it by now and this is going to be this sense that is my M_2. Then, to find out theta_{ab}, I need to apply a moment here and this is going to be equal to 1 this way, and virtual force here to find out this rotation is going to be equal to minus 1 this way. Again, taking the principle of virtual force, external work done to find out theta_{ab} is 1 into theta_{ab}, so 1 into theta_{ab} is equal to…. This is M_2, this is m_1, this is m_2, this is going to be m_1 M_2 upon EI dx. That is equal to m_1 M_2 upon EI. So, this is going to be area under this curve – M_2 upon EI, multiplied by area under this curve – L by 2, multiplied by the value at this centroid. This centroid is one-third from this; if you take one-third from here, you will get one-third. Note the opposite sign, so it will be minus 1 upon 3. This is going to be equal to M_2 L upon 6 EI.

Similarly, if I do 1 into theta$_{ba}$, you will see that it is the area under this curve, which is M_2 upon EI into L over 2, multiplied by the value at this point. The value at one-third from this point is two-thirds and both of them are the same, so two-third. This is going to be equal to M_2 L upon 3 EI. What does this mean? Theta$_{ba}$ is positive that means it is this way; theta$_{ab}$ is negative, that means it goes in this fashion; and indeed, under this load, you will expect it to move in this fashion; so, these are consistent.

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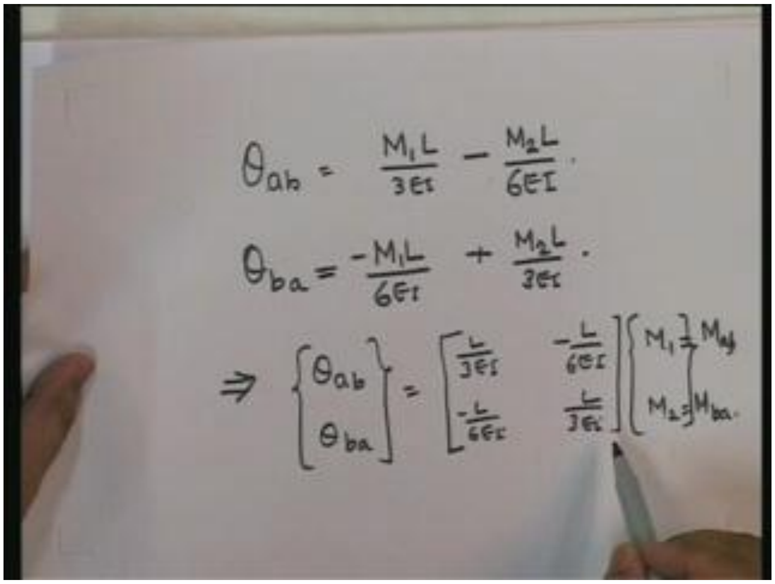

Since I have obtained it for each individually, I am just going to add both of them up. When I add both of them up, what you get is theta$_{ab}$ is equal to M_1 L upon 3 EI minus (M_2 L upon 6 EI), theta$_{ba}$ is equal to minus (M_1 upon 6 EI) plus M_2 L upon 3 EI. I have got the theta$_{ab}$ due to M_1 and application of M_1 and M_2 together. Let us look at this. These are the rotations due to these moments; that means these rotations are related to the moments through these two equations. I can rewrite this in this fashion. I can consider this as a vector, which is equal to L upon 3 EI, minus (L upon 6 EI), minus (L upon 6 EI), L upon 3 EI, multiplied by M_1 and M_2. All I have done is I have written this in terms of a kind of matrix equation.

If you look at this, this implies theta$_{ab}$ is equal to this into this plus this into this, which is what you get here; this is equal to this into this plus this into this, which is this. In other words, given M_1 and M_2 which are external loads, you are able to get theta$_{ab}$ and theta$_{ba}$. You have to understand that I have actually taken loads M_1 and M_2. Now, I can also say, look, those are actually…. What is the bending moment at a in ab due to M_1? You will see that the bending moment at ab due to M_1 is M_1. What is the bending moment at b? It is going to be the same as M_2. This is a fact; you can actually go through it and do it yourself. In other words, what I am saying is that I can also replace M_1 and M_2 by M_{ab} and M_{ba} and that is what I am going to do. I am going to say that M_1 is equal to M_{ab} and M_2 is equal to M_{ba}. If I do that, that means theta$_{ab}$

and theta$_{ba}$ are related to M$_{ab}$ and M$_{ba}$ through this. I can do the flip side, I can then write down M$_{ab}$ and M$_{ba}$ in terms of theta$_{ab}$ and theta$_{ba}$, that is what I am going to do. How would that be?

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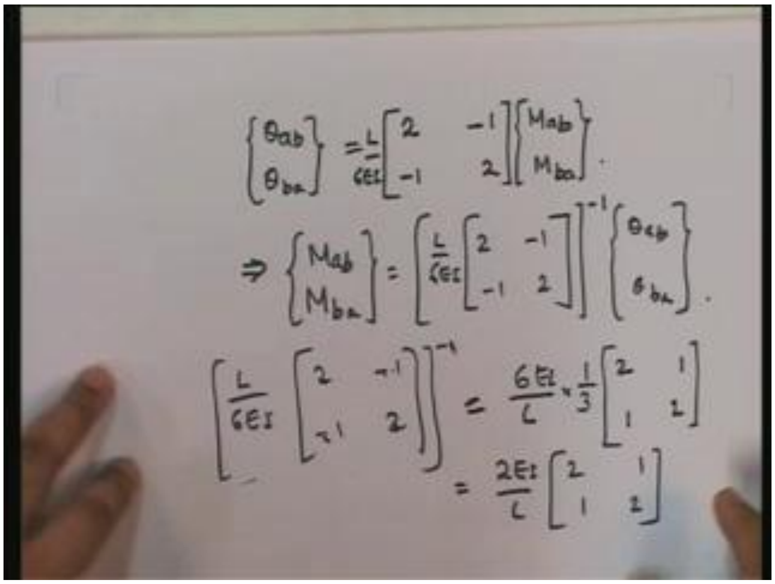

If theta$_{ab}$ and theta$_{ba}$ are equal to…. I am going to now take L upon 6 EI outside, so that this becomes 2, minus 1, minus 1, 2 into M$_{ab}$, M$_{ba}$. This is equally valid because this is what I have got from the previous case. Instead of writing theta$_{ab}$ and theta$_{ba}$ in terms of M$_{ab}$ and M$_{ba}$, I am going to write down M$_{ab}$ and M$_{ba}$ in terms of theta$_{ab}$. What do I need to do? Look at this: if I pre-multiply this by the inverse of this, what would I get? I would get this. The inverse of this multiplied by theta$_{ab}$ is equal to the inverse of this into this, which is going to be a unit matrix, into this. What would be the unit matrix? It would be the same. This implies that M$_{ab}$, M$_{ba}$ is equal to this inverse into theta$_{ab}$, theta$_{ba}$. What is the inverse of this? It is simple.

How do you find out the inverse? You find out the factors and then, if you look at this, the inverse of this is L upon 6 EI, 2, minus 1, minus 1, 2. Please go back and look at it. It will become 6 EI upon L, that is just this flipped, inverse is obviously just this upon this. What is the inverse of a scalar? It is 1 upon the scalar. So, 1 upon this is equal to 6 EI upon L and the inverse of a matrix is 1 upon the determinant; you have to find out the determinant. The determinant is 2 into 2, 4; minus 1 into minus 1, which is 1; this is 3, so it is equal to 1 upon 3 multiplied by the cofactor of this, which is this, so this (Refer Slide Time: 25:26); the cofactor of this is minus of this, so that is that; the cofactor of this is the minus of this, so that is that; and the cofactor of this is this, so that is this. If you look at it, this becomes equal to 2 EI upon L 2, 1, 1, 2.

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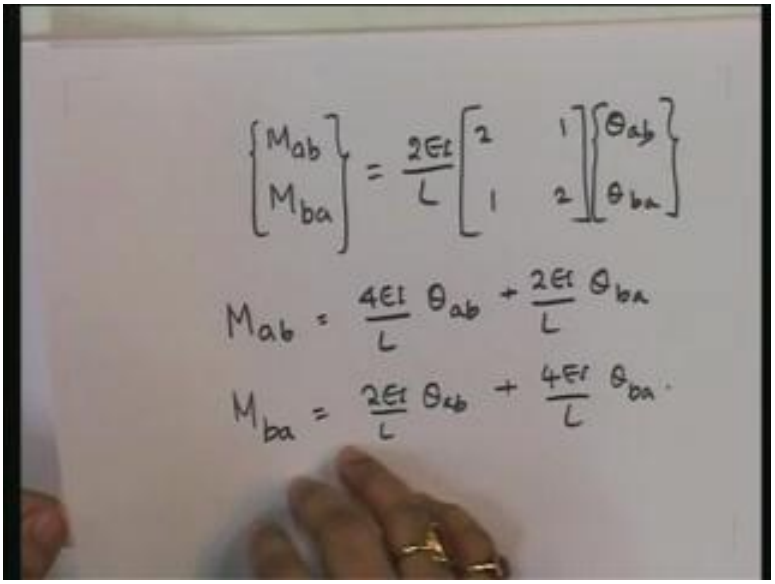

If I rewrite, plug this into here, what I get is M_{ab}, M_{ba} is equal to 2 EI upon L 2, 1, 1, 2 into theta$_{ab}$, theta$_{ba}$. If I write this in a proper format, it will become M_{ab} is equal to 4 EI upon L theta$_{ab}$ plus 2 EI upon L theta$_{ba}$; and M_{ba} is equal to 2 EI upon L theta$_{ab}$ plus 4 EI upon L theta$_{ba}$. Simple. If you look back at the slope deflection equation, what did I write down?

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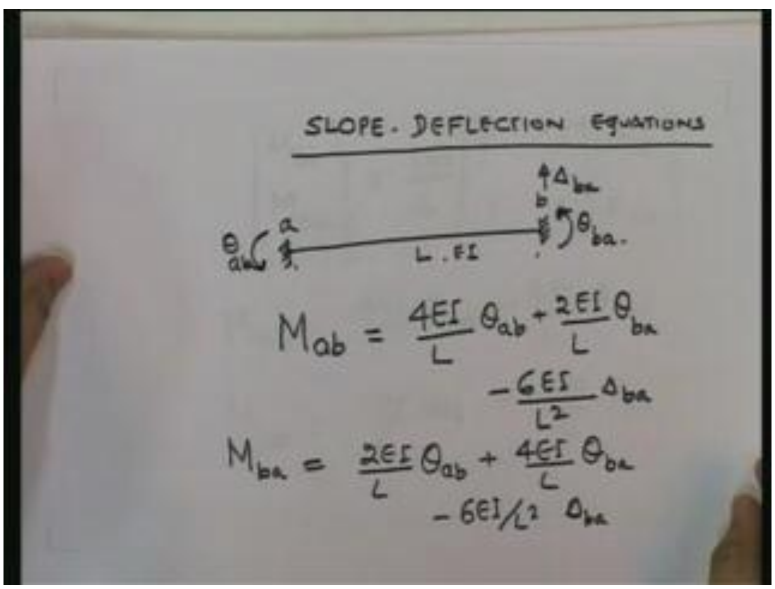

4 EI upon L theta$_{ab}$ plus 2 EI by L theta$_{ba}$. Forget this term for now. These two terms, I have just developed. These terms have not been developed because I did not consider this delta. I only considered theta$_{ab}$ and theta$_{ba}$ to occur due to M_1 and M_2. Now, how do I include this? Simple. Bear with me and I will take you through this. These two terms (Refer Slide Time: 27:30), this

relationship has already been developed here. Now, we are going to develop the deflection part. We have already computed the slope part, now we are going to be computing the displacement part. How do we compute the displacement part? Just bear with me.

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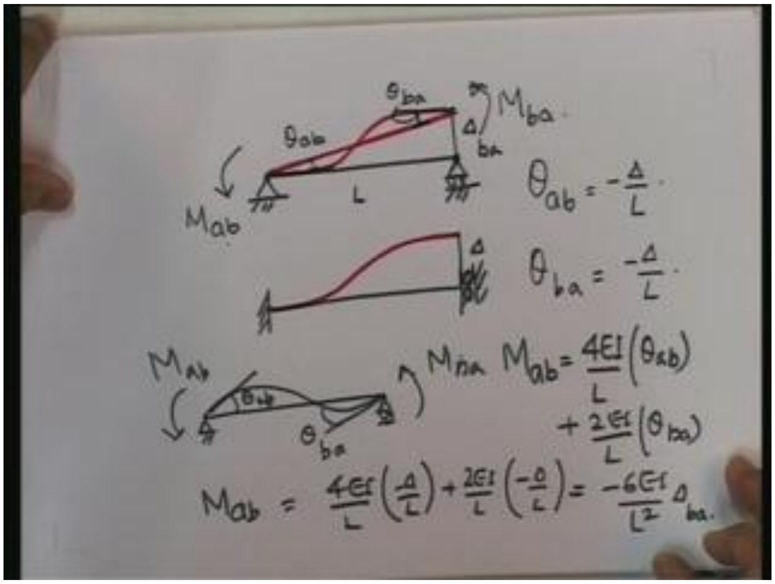

Take this situation; this is a pin here and a pin here. Suppose I were to remove this and let this go up by delta. Note that this delta is small compared to the L and so it is essentially a small displacement. How would this displace? Note that this is free to rotate. So, if you look at the displacement pattern for a simply supported beam it would just be this, it will be a straight line where this angle would be delta upon L. Now, let us look at the system that we were looking at, the original.

How will I get a displacement here? I will just have to release this just like I released that and displace this by L. Note that the fixed end remains. How will this displacement look like? Note that here because they were simply supported and hinged, when I moved it by delta, I got a displacement pattern which was exactly like this; here, because they are both fixed, the displacement pattern, the rotation has to be 0 here and the rotation has to be 0 here and that is the displacement. This and this do not look like each other at all (Refer Slide Time: 29:43). Can I make this behave like this? Sure. Think about it.

Remember that when I had this under an application of M_{ab} and M_{ba}, what happened? It is going to be this where I got $theta_{ba}$ and $theta_{ab}$ and I know the relationship between $theta_{ab}$ and $theta_{ba}$. Let me take this situation. What I am going to do is…. You see, this is a small displacement, so it does not really matter. I am going to now apply on this a moment M_{ab} and moment M_{ba}; I am going to apply these two moments.

Of course, I am applying them here and I am just showing it over here because these have displaced it. Remember that these displacements are always very small. I just exaggerate them to show them clearly. If I apply moments here, what would happen? Think about it. Could I apply a moment M_{ab} and M_{ba}? Under M_{ab}, this happens (Refer Slide Time: 31:13). So, can I apply a moment M_{ab} such that this is the displacement pattern about this? Remember that this is also length L because this is small. This is length L, this happens. In other words, I apply a moment M_{ab} and M_{ba} on the simply supported beam to ensure that this displacement pattern essentially displaces in this way, so that it mimics this displacement pattern.

What are my rotations? Let us look at my rotations. What is my rotation? This is the undisplaced shape. Note displacement is this way. This is my $theta_{ab}$ and this is my $theta_{ba}$. What are my $theta_{ab}$ and $theta_{ba}$? Let us look at this. What is this angle? Delta by L. Is this clockwise or anticlockwise? From the undisplaced to the tangent, that is equal to …. $theta_{ab}$ is going to be equal to minus (delta upon L). Why minus (delta upon L)? delta upon L is the magnitude of this rotation, that is for sure; and the fact that it is clockwise makes it minus (delta upon L). Similarly, what is $theta_{ba}$ equal to? From here to here, this is clockwise; so, $theta_{ba}$ is also minus (delta by L). Can I find out an M_{ab} and an M_{ba} which would give rise to these? Sure, I can, because I have already developed this.

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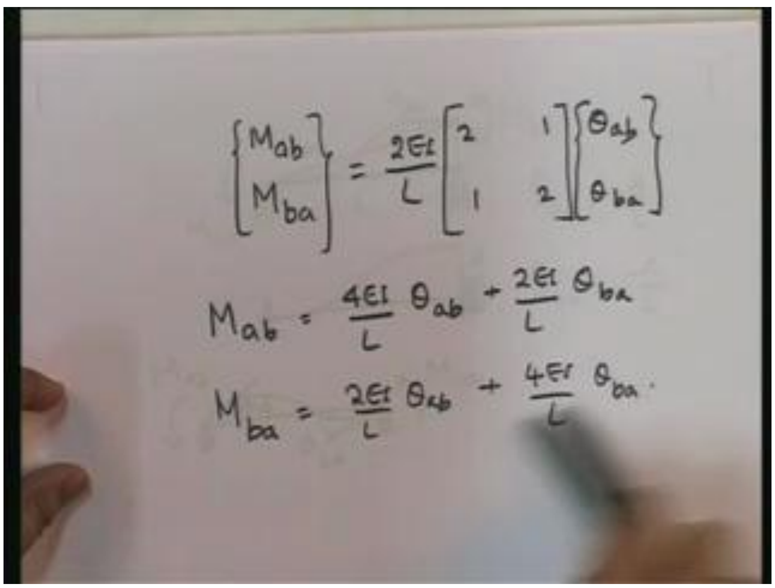

I can find out M_{ab} and M_{ba} given $theta_{ab}$ and $theta_{ba}$ just by applying this equation which I have developed; so, I am going to use that; let me use that. What happens? M_{ab} is equal to 4 EI by L $theta_{ab}$ plus 2 EI by L $theta_{ba}$. Similarly, M_{ba} is equal to 4 EI upon L $theta_{ba}$ plus 2 EI by L $theta_{ab}$. That we have already got there; just substitute the values of $theta_{ab}$ and $theta_{ba}$ in here. You will see that M_{ab} is equal to 4 EI upon L into minus (delta by L) (note that $theta_{ab}$ is minus (delta by L)) plus 2 EI upon L. $theta_{ba}$ is also minus (delta upon L). Plug that in. What do you get? You will see that this will become minus (4 EI upon L square delta) and minus (2 EI upon L square delta) and this becomes minus (6 EI upon L square delta). By definition, what is this delta? I have defined this as $delta_{ba}$, so I have got M_{ab} in turn. You can similarly get M_{ba}, you will get it equal to this.

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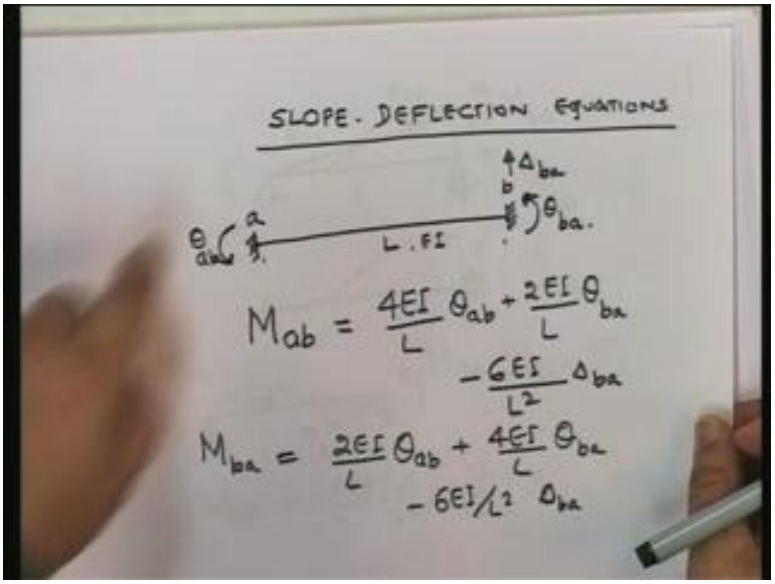

That means if I substitute all these factors which is a theta$_{ab}$, a theta$_{ba}$, and a delta, if we put all those factors in, then what would be my M_{ab}? I just add all of them up. When I add all of them up, what do I get?

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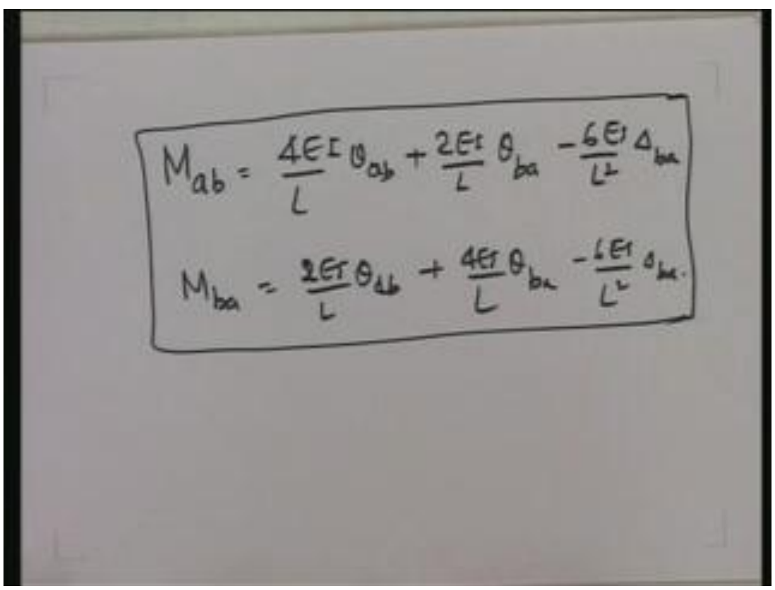

M_{ab} is equal to 4 EI upon L theta$_{ab}$ plus 2 EI upon L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$). M_{ba} is equal to 2 EI upon L theta$_{ab}$ plus 4 EI upon L theta$_{ba}$ minus (6 EI upon L squared delta$_{ba}$). We have just developed the slope deflection equations from first principles. This is what I want you to do. You have to understand where they come from and then, once you know those, you can actually develop these equations for any kind of member. Here, we considered that EI was a constant over the entire member. It is not necessary that the member has to be uniform all the time, you could have a non-uniform member also. How did we evaluate this? We first applied loads M_1 and M_2 and found out $theta_{ab}$ and $theta_{ba}$.

Once we found those out, we knew that M_1 and M_2 are equal to the bending moments of that point, which is M_{ab} and M_{ba}. Once we did that, we took the inverse. We got $theta_{ab}$ and $theta_{ba}$ in terms of M_{ab} and M_{ba}, then we inversed it and we got M_{ab} and M_{ba} in terms of $theta_{ab}$ and $theta_{ba}$. In essence, that is your slope deflection equations. Then, we got the deflection part just by being innovative and seeing how to relate a $delta_{ba}$ in terms of a spurious $theta_{ab}$ and $theta_{ba}$. So, this is the slope deflection equation. What can we do with the slope deflection equation? Let us look at a simple structure that we have. Let me take a very very simple structure.

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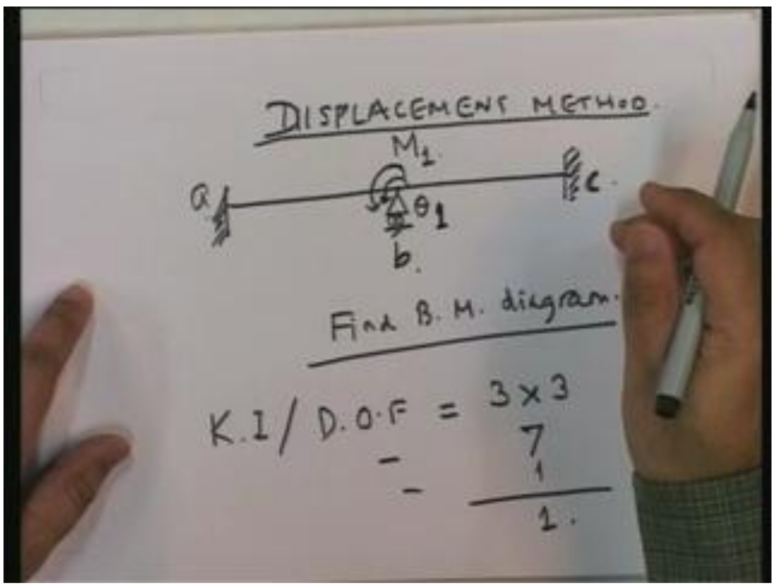

I am going to take a very simple structure and then, I am going to apply a moment here. I am going to take a very very simple structure just to illustrate how to use the slope deflection equations. I have a, b and c. There is only one load and that is the moment. How did this moment come? Maybe it is because of some applied load over here, I am not bothered about that. Is this a statically indeterminate structure? Of course, it is. What is the static indeterminacy of this? You have 1, 2, 3, 4. You will see that static indeterminacy is 4 here, but in this application of slope deflection equation, you do not worry about what is the static indeterminacy. I found out the static indeterminacy just to illustrate to you that this is a statically indeterminate beam and there is no way you can find out the bending moment diagram for a static indeterminate beam. My point here is to find the bending moment diagram.

How do I apply the slope deflection equations to get this bending moment diagram? Instead of finding out the static indeterminacy, what you have to find out is the kinematic indeterminacy of this structure. Remember how we developed that? Kinematic indeterminacy or I also called it as degrees of freedom – how do I find that out? Let us go through this. How many joints? a, b, c, 3; so, 3 into the 3 degrees of freedom per joint. Then, we have to subtract the restraints. How many restraints? Three here. There is translation here, there is translation here and there is rotation here; they are 0. This is also restrained; 3 plus 3 and there is 1 degree, so that is 7. Plus, I have one constraint. What is that constraint? The actual force in this member is 0, so that is 1. How many degrees of freedom? 9 minus 7 minus 1, that is 1. What is the degree of freedom? You will see that the degree of freedom is essentially this rotation. What I want to say is that the whole point in this exercise is to find out $theta_1$. This is unknown displacement. All I need to do is find out $theta_1$ and hopefully, if I find out $theta_1$, I should be able to find out the bending moments given $theta_1$. That is the reason why…. Since we are trying to find out rotations and displacements, this method classically is called as the displacement method of analysis. Let us see how this….

Think about it. In the force method, what did you do? You found the static indeterminacy. Once you find the static indeterminacy, you identified redundant forces; once you found out redundant forces, you actually wrote down compatibility equations corresponding to the redundant forces and that is how you solved it. In this particular case, it is just the opposite. In the force method, actually, the redundant forces…. The only thing about the redundant forces is that when you put redundant forces equal to 0, you need to have a stable statically determinant structure; that is all that is there. Therefore, you know there is uncertainty about which redundant force. In other words, there are so many redundant force systems that can create a stable statically determinant structure that you and I may not identify the same redundant forces ever and still solve this problem correctly.

Whereas, if you look at the displacement method, degrees of freedom, once you have the degrees of freedom, can any of you tell me that this is kinematic indeterminacy or degree of freedom is one? Can you identify a degree of freedom which is different from this rotation? No. This is restrained completely, this is restrained completely, this is not allowed to go this way because of axial rigidity. This is not allowed to go this way (Refer Slide Time: 43:39), so the only thing that can happen is this. In other words, in the displacement method, the degree of freedom identification is unique. You and I and everyone else would choose the same degree of freedom; there is no other degree of freedom to choose in this particular case. There is a slight amount of uniqueness. In other words, once you identify the degree of freedom, you or I or anybody else cannot choose any degree of freedom. It has to be a unique degree of freedom.

This, you will see later, makes the displacement method particularly amenable to use in computers because the computer is not going to be confused. In the redundant force, the application of force method, how is the computer going to determine the redundant forces? You have to identify for it, but degrees of freedom, the computer knows which degree of freedom. Given a structure, it can always identify the degrees of freedom very very easily. You can develop an algorithmic way which can be implemented on a computer using the displacement method and this is the reason why the displacement method is the more popular method for analyzing structures in general. Now, let us look at how to solve this problem.

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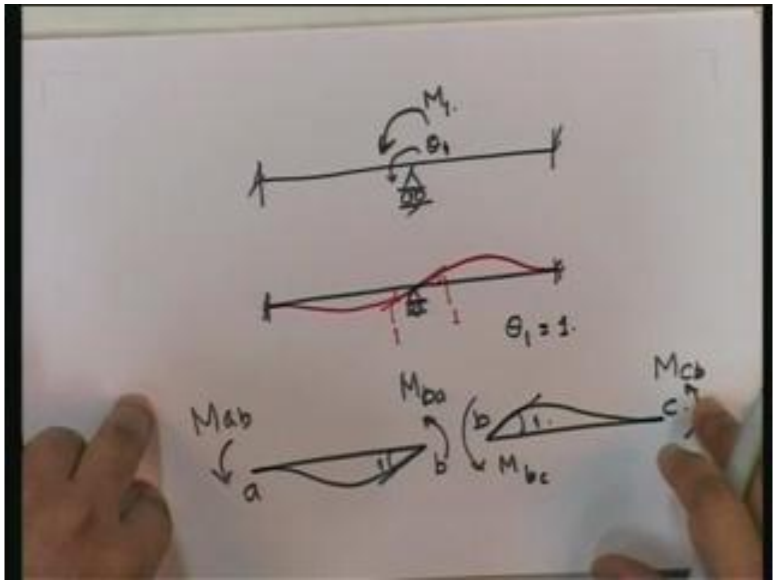

This is my problem. This is my degree of freedom and I have to find out this degree of freedom given this moment and hopefully if I find out the degree of freedom, then I can get the bending moment diagram. Ultimately, never forget that the overall factor here is to find out the bending moment diagram. What are the steps? The first step: give $theta_1$ equal to 1. What will happen? Think about it. $theta_1$ is equal to 1 implies that at this point, the joint rotates by 1. If it rotates by 1, since this cannot rotate, it is going to take up something like this. This is the displacement pattern under $theta_1$ is equal to 1. Note that when I say $theta_1$ is equal to 1, understand it is not one radian that I am saying. All I am saying is that $theta_1$ is equal to 1, it is very small. You can you can take 1 into 10 to the power of 4, who cares?

The whole point is that it is unit rotation; however, the rotation still remains small. All displacements that we are going to consider in this case are going to be small. This is my rotation pattern. What next? I take this to a member level; I take member ab and I take a member bc. Under this rotation, this thing looks like this and bc looks like this. Note I have a situation where in a member, I know the rotations at the two ends and the displacement, if any, of b relative to a. Since I know these, can I find out the bending moments at the two ends, which are M_{ab}, M_{ba} here? Here, it is going to be M_{cb} and this is going to be M_{bc}. Can I find these out in terms of these rotations? Sure. Apply the slope deflection equations. What do they give me? $theta_{ab}$ is equal to 0, $theta_{ba}$ is equal to 1 and $delta_{ba}$ equal to 0.

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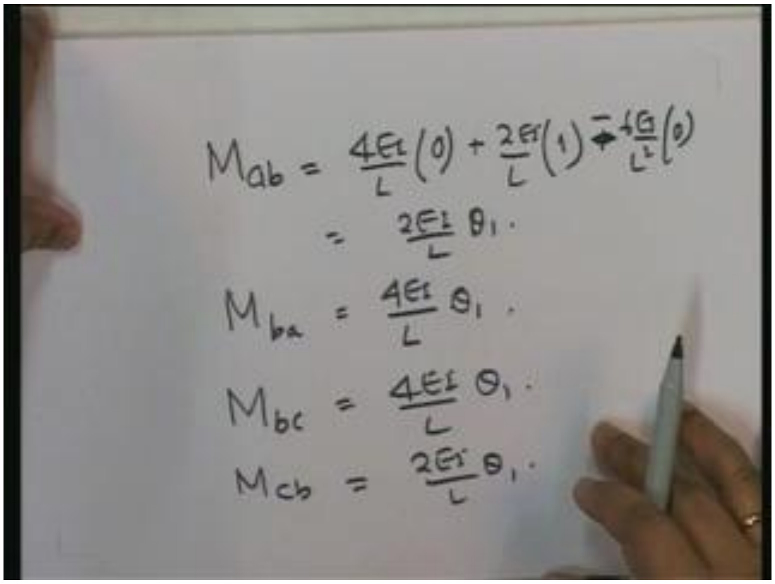

I just need to plug that into the slope deflection equations and my M_{ab} turns out to be equal to 4 EI upon L $theta_{ab}$, which is 0, plus 2 EI upon L $theta_{ba}$, which is 1, minus (6 EI upon L squared) into 0. This is equal to 2 EI upon L. Similarly substituting, you get M_{ba} is equal to 4 EI upon L or I can put this in this terms. These are the moments given. $Theta_1$ is equal to 1. If $theta_1$ was actually $theta_1$, which is an unknown, then what would be M_{ab} and M_{ba}? Just multiply by $theta_1$. This is linearity. We can just find out M_{ab} in terms of theta. Similarly, for bc, if I apply it, you will see that M_{bc} (I am not going to go into the details, you can apply it yourself), you will get it equal to 4 EI by L $theta_1$ and M_{cb} is equal to 2 EI by L $theta_1$. I have got the member in moments now. Once I have got the member in moments, what do I do? Let us look at it; let me draw it.

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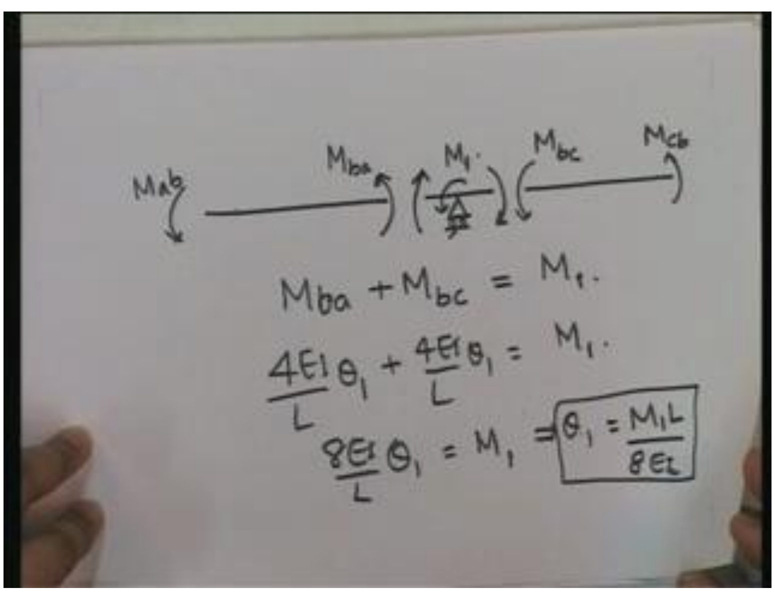

I am now drawing the original structure; the original structure is this way. What do I have? I know M_{ab}, M_{ba}, M_{bc}, M_{cb}; I have got these in terms of $theta_1$. By equilibrium, let us look at a few things. By equilibrium, what is the moment at this point? The moment at this point is in this fashion, that is equal to M_{ab} and the moment at this point is equal to M_{bc} and remember that at this point, I had applied a moment M_1. If I take moment equilibrium of this, I am only taking moment equilibrium of this, what do I get? I get M_{ab} plus M_{bc} minus M_1 is equal to 0. Therefore, M_{ba} plus M_{bc} is equal to M_1 – this is my equilibrium equation. Now, what is M_{ba} equal to? I you look back, M_{ba} is equal to 4 EI upon L $theta_1$? What is M_{bc} equal to? Also 4 EI upon L $theta_1$ is equal to M_1. That means 8 EI upon L $theta_1$ is equal to M_1, which implies that $theta_1$ is equal to M_1 L upon 8 EI. I have found out my unknown rotation. Once I have found out my unknown rotation, can I find out my M_{ab}, M_{ba} and M_{bc}?

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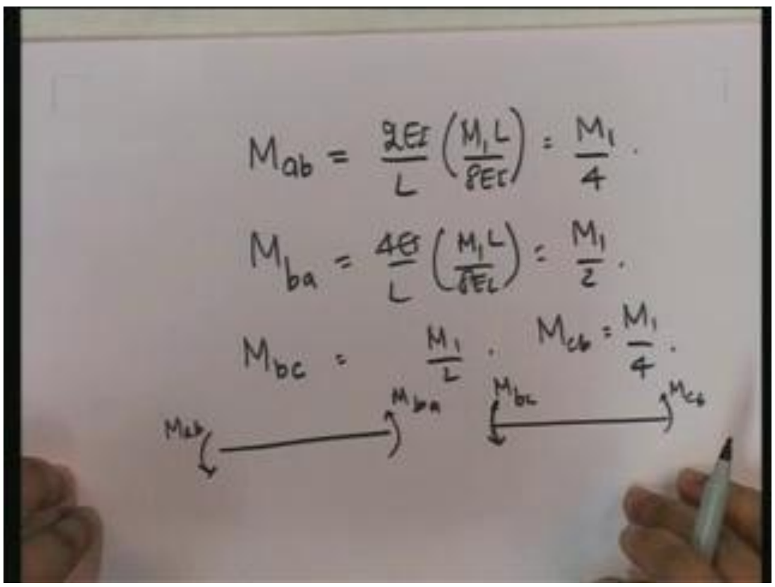

Since I have got $theta_1$, my M_{ab} is equal to 2 EI upon L into $theta_1$ ($theta_1$ is M_1 L upon 8 EI), which will become M_1 upon 4; M_{ba}, which is 4 EI upon L into $theta_1$, is going to be M_1 upon 2. Similarly, M_{bc} will be M_1 upon 2 and M_{cb} will be M_1 upon 4. I have M_{ab}, M_{ba}, M_{bc} and M_{cb} and since I know all of these, I can draw the bending moment diagram because this is now a statically determinate structure where I know the member and moments.

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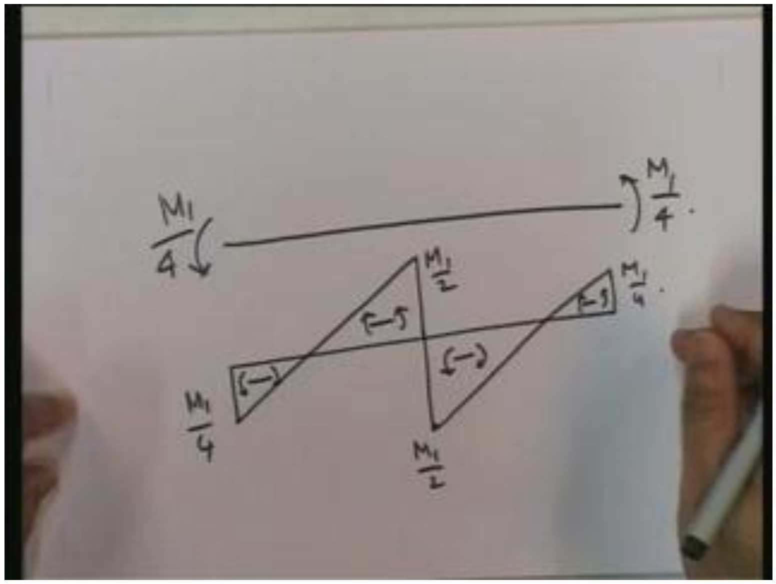

If I were to show you the member and bending moment diagram, this is M_1 upon 4, this one is M_1 upon 4. If I were to draw the bending moment diagram, it would look like this. I will leave that as an exercise for you where this one is M_1 upon 2, this one is M_1 upon 2, this is this way and this is this way. Why is there a discontinuity at this point? What is the discontinuity? $M_{1.}$ What is the applied moment over here? M_1. So, this is the discontinuity.

How did we go about applying the displacement method? We actually found out the kinematic indeterminacy then found out the degree of freedom. Then the next step is, given a unit displacement corresponding to the degree of freedom find out the displaced shape of the structure. Once you find out the displaced shape of the structure, you can find out the displaced shape of each member. For each member then, once you know the displaced shape, you know the member and rotations and the displacement of one end relative to the other. You can apply that into your slope deflection equations to get the bending moments at the two ends.

Once you get the bending moments at the two ends, you can then apply equilibrium equations to relate the unknowns and find out the value of $theta_1$ in this particular case. Once you know the value of the rotations, then you can always go back, substitute and get the member and moments exactly. Once you know the member and moments, then you have a statically determinate structure for which you can find out the bending moment diagram. These are the steps in what is known as the displacement method.

Do not worry if you have not been able to understand the quickness with which I have gone through. I have just established the procedure for you today. I have taken a very simple question and gone ahead with it. This I am going to expound on…. Since I know that this is a topic that you have not covered earlier, I am introducing the topic to you today; later on, you are going to see how this method is developed over the next many lectures.
Thank you very much.