Structural Analysis - II
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Lecture – 01

Good morning. Today we are going to start off on a course Structural Analysis – II, which is the second in the series on courses of Structural Analysis. During this course, over a period of forty-five odd lectures, we are going to be covering the various methods that are used for analyzing structures which are known as statically indeterminate. Before I start off on the methods, I am going to spend at least two to three lectures on reviewing the basic tenets on which structural analysis are based and the methods that I am going to be using over and over again during this course. Let us start off with what I had called as the basic tenets of structural analysis.

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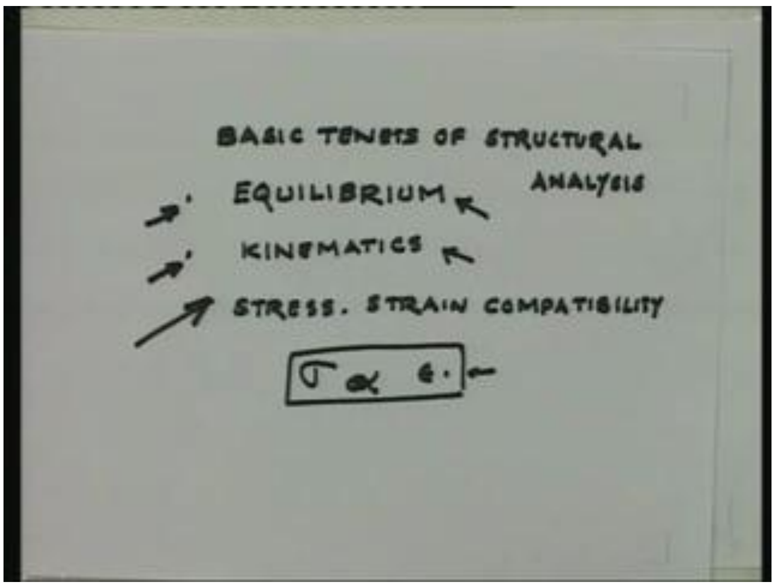

There are three. One - Equilibrium it essentially means that when a structure is subjected to loads, the loads and the internal forces developed in the structure are in equilibrium. In other words, the internal forces equilibrate the external loads. Without equilibrium what you would have is that the structure would start moving and obviously, you do not want a civil engineering structure moving in space. Therefore the body has to satisfy equilibrium. Second - Kinematics. Kinematics is the fact that when a structure is subjected to loads the structure deforms. In other words, under the loads the structure deforms and takes up an equilibrium state. This equilibrium state is different from what is known as the undeformed shape of the structure which is the structure when it was not subjected to any loads. This displaced shape, the displacements in the structure and the strains that are developed due to the deformation in the structure have to be compatible with each other. This is the basic concept of Kinematics.

The third and final tenet is very basic and that is stress strain compatibility. In other words, when you have loads acting on a structure, the body deforms. Due to the deformation of the body strains are set up various points in the structure and because of the strains stresses are developed and these stresses are what are essentially the internal forces developed in the body. The stress and strain have to satisfy the compatibility condition or the stress strain relationship. For example, you know the elastic stress strain relationship which is governed by the Young's Modulus. The stress is given in terms of Young's Modulus into strain. This is the simplest kind of stress stain compatibility that we can discuss. In this course I shall be essentially referring to only linear structures. What do linear structures mean? The relationship between the stress and strain is linear which if you really look at it, is something in this form. Sigma, the stress, is proportional to the strain. This proportionality is linear. This is what I am going to be looking at during this course.

Therefore to review - a structure has to satisfy Equilibrium, it has to satisfy Kinematics and it has to satisfy stress strain compatibility. Without these three your analysis will never be correct. You have to satisfy all three. We shall see later on that when we deal with, what are known as statically indeterminate structures, I will spend a little bit of time, in the next lecture talking about, what do I mean by a statically indeterminate structure, but let us assume that we know what a statically indeterminate structure is. When you want to analyze a statically indeterminate structure you have to satisfy equilibrium, kinematics and the stress strain compatibility.

Stress strain compatibility is fundamental because this is something that is held at the basic tenet or the basic building block on which we develop the entire edifice of structural analysis. This stress strain compatibility can never be violated. However, we shall see later on, that when we talk about statically indeterminate structures, some methods satisfy equilibrium exactly and kinematics in a global sense. Other methods satisfy kinematics exactly and satisfy equilibrium in a global sense. Understand that when I say equilibrium and kinematics, they have to be satisfied everywhere in the structure and when I say that kinematics is exactly satisfied what I mean is kinematics is satisfied everywhere in the structure and when I say equilibrium is satisfied only in a global sense I mean that equilibrium at the local level may not be satisfied, however overall equilibrium of the structure is satisfied.

In other methods they satisfy equilibrium exactly. In other words equilibrium is satisfied at the local level and when I say kinematics at the global level, it means kinematics may be violated at the local level but at the global level the displacements and the overall strengths in the structure are satisfied. There is so much for the basic tenets of structural analysis. I want to discuss a little bit about what I mean by indeterminacy. How do I analyze the structure? Normal structures are always what we know as statically indeterminate. What do we mean by that? Remember that in a statically determinate structure you could satisfy and obtain all the internal forces from the loads by just dealing with equilibrium equations. Remember that in a planar structure for example, a simple truss.

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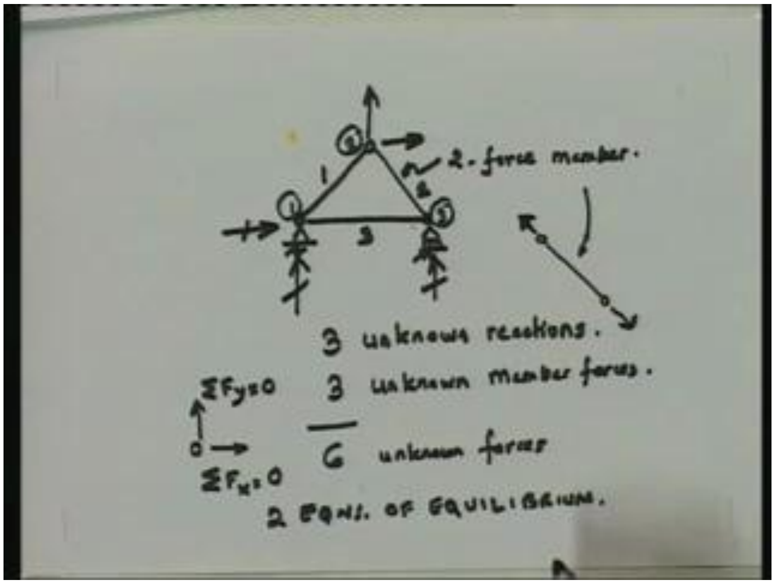

What is a truss? A truss is essentially a structure which is made up of members, where each member is what is known as a two force member. What is a two force member? If I isolate the member, this member will only be subjected to axial forces - that is a two force member. In other words, in a truss, each member is only subjected to axial forces. Therefore, let us look at this particular problem. How many unknowns do I have in this structure? It is a simply supported truss. This is a hinge; a hinge restrains the lateral motion of a structure, however, it does not stop the rotation at the particular point. Since it restrains lateral motion, the movement in the plane, therefore it is possible that it can generate two reactions. These are my unknown reactions. Let us say I am subjecting this structure to some loads. The hinge has two unknown reactions. This is a roller support; a roller support only restraints this direction motion of the structure. Therefore, it can only generate a reaction perpendicular to the movement direction. So we have three unknown reactions. How many members do we have? 1, 2, 3 so three members we have. Each member has only the actual force in it.

In other words, for each member we have one unknown internal force in the member and therefore since we have three members we have three unknown member forces. How many unknowns do you have in this structure? We have what are known as six unknown forces. How many equations of equilibrium do we have? Let us look at it. Remember a point does not have any dimension; a point is in a plane. How many equilibrium equations can we generate? We can generate one which is Sigma F_x equal to 0 and the other one is Sigma F_y equal to 0. Therefore, at a point we can only generate two equilibrium equations Sigma F_x equal to 0 and Sigma F_y equal to 0. Therefore, for each point in the structure we can write two equations of equilibrium for every point in the structure. How many points are there in this particular structure? A point or more commonly, joint, are where two or more members meet or a member meets a support. Therefore how many joints do we have in this structure? 1, 2 and 3 we have three joints and we can write two equilibrium equations for every joint. In other words how many independent equations of equilibrium do we have? 2 power 3 is equal to 6. We have six equations of equilibrium that we can write for this structure and how many unknown forces do we have? 6.

Therefore, just by solving the equations of equilibrium, we can find out the reactions in this structure and we can find out the internal forces in each of the three members. This structure is known as a statically determinate structure. A statically determinate structure is one, in which you can find out all the support reactions and the internal forces in the structure without using anything other than the equations of equilibrium. What happens, if we have a situation, where the number of unknown forces are greater than the number of equations of equilibrium available? Let us look at the simple structure.

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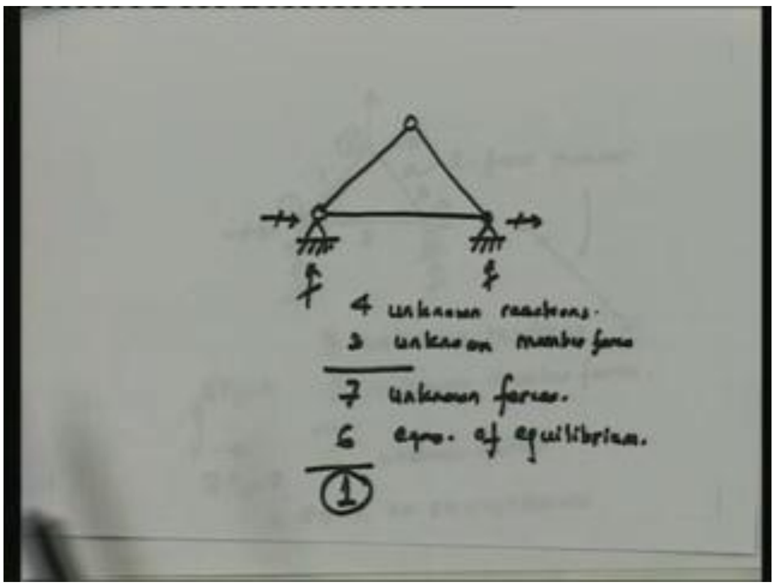

Again, I am looking at the same truss. How many unknown reactions? 1, 2, 3, 4 there are four unknown reactions and of course the same three unknown member forces, so we have total of seven unknown forces. Remember this is exactly the same structure so we only have 6 equations of equilibrium. There is no way that you can find out all the unknown forces with just the equations of equilibrium that you have. So how do you solve this problem? This problem is now what is known as a statically indeterminate structure problem and this course, is where we are going to be looking at all the different methods of being able to solve such a problem.

Let us review. We are saying that a statically determinate structure is one, in which the number of unknown forces that are in the structure (member forces, support reactions etc.) are equal to the number of independent equations of equilibrium. Remember, it is always very easy to develop equations of equilibrium and you will find sometimes for example, if in a plane you take moments at two different points on the plane are they independent, they will give you two different equations are they independent? No. Why not? Because moment everywhere on a plane, you should remember, moment is a free vector and therefore the two moment equations that you write are not independent of each other. So, be very careful; when we say equations of equilibrium we actually mean independent equations of equilibrium. Therefore, if the number of unknown forces is equal to the number of independent equations of equilibrium that you can write for this structure that structure is known as a statically determinate structure.

If, however, the number of unknown forces in the structure is more than the number of independent equations of equilibrium that you can develop, then, it is a statically indeterminate structure. How do you solve such a problem? We are going to spend the next forty-odd lectures looking at that. Let us not try to solve that problem today itself.

Let us just look at what do I mean by Degree of Static Indeterminacy. You must have come across this; you have done it in school, you have also been introduced to the concept in your strength of materials or solid mechanics course and you have seen at many a time, degree of Static Indeterminacy. We shall see later that for certain methods we will be looking at, the Degree of Static Indeterminacy in the structure determines the complexity of the problem in using those methods. This is a very important concept.

What do we mean by Degree of Static Indeterminacy? Let us go back to this problem. How many unknown forces? 7. How many equations of equilibrium? 6. What is the static indeterminacy? Subtract the equations of equilibrium from the number of unknown forces and you have the Degree of Static Indeterminacy. What is the static indeterminacy of this particular structure? 1. It is what is known as a single Degree of Static Indeterminacy. The single Degree of Static Indeterminacy is 1. This is very simple for truss type structures to determine the static determinacy. How do you determine the static indeterminacy? Let me define this for a truss problem. How many unknown forces?

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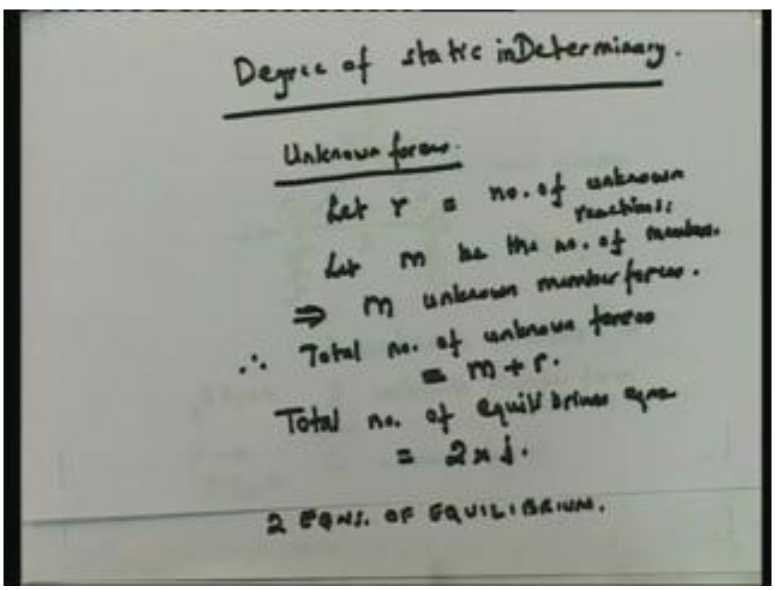

Let r be equal to the number of unknown reactions in the structure at the supports. Let m be the number of truss members in the structure. Note, if m is the number of members, each member has one unknown member force. Therefore when you have m number of members it implies that you have m unknown member forces. Therefore, total number of unknown forces is equal to m plus r. In a truss- planar truss (I am discussing only planar truss), what does a planar truss mean? Let us look at it.

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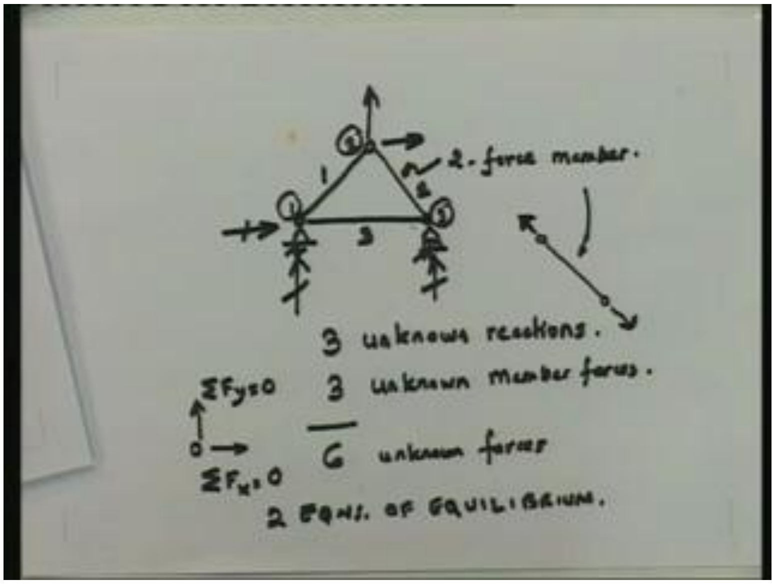

If you look at this, a planar truss is one where the truss and the loading on the truss are all in one plane. In this particular case it is in the plane of the paper. So that is a planar truss. Now I am looking at planar trusses, so you have the total number of unknown forces equal to m plus r. What is the total number of equilibrium equations? They have to be equal to 2 into the number of joints that you have in the structure. I am defining j as the number of joints that you have in the structure. The total number of equilibrium, since you have two independent equations of equilibrium at each joint, the total number of independent equilibrium equation is 2j. Then what is the Degree of Static Indeterminacy? Let me write this down very carefully:

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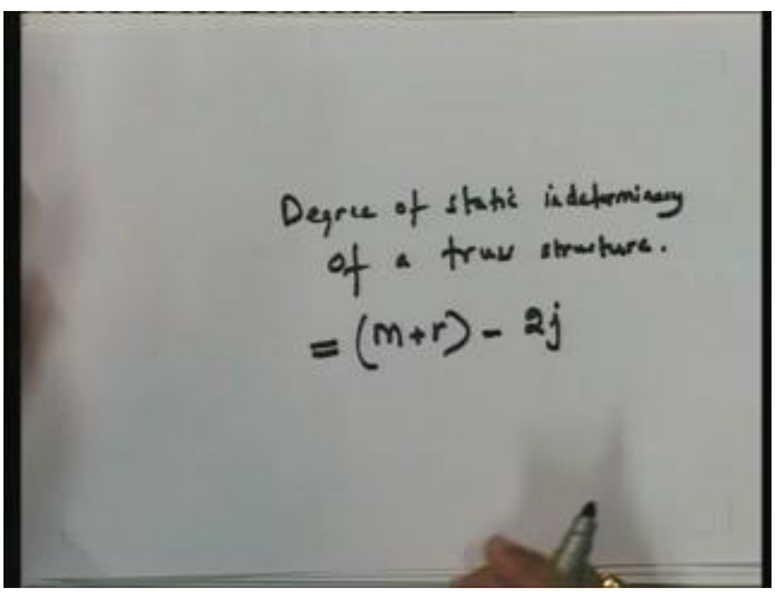

This is fundamental: the Degree of Static Indeterminacy of a truss structure is equal to m plus r which are the total number of unknown forces in the structure minus the number of independent equilibrium equations which are given by 2j i.e. This is the formula for determining the static indeterminacy of a planar truss; m plus r 'm' is the number of members r are the total number of reaction forces. (m plus r) minus 2j, where j is the number of joints in the structure. This is the way that you can determine the static indeterminacy of a planar truss structure. Now let us see how we determine the static indeterminacy of other structures.

What other kind of structures do you have? A truss structure, remember, is a structure where each member is subjected to just one unknown member force, an axial force. You have already been exposed to a beam. What is a beam? A beam is a member which is subjected to both axial deformations as well as flexure.

Of course, we know these more commonly as beam columns because a beam is one which is primarily subjected to flexure. What is flexure? It is bending whereas a column is one, which is primarily subjected to axial deformations. There is a difference between a column and a truss member because a truss member is only subjected to axial forces; it cannot be subjected to any other kind of forces and it only deforms axially. Whereas, a column is primarily subjected to axial deformations that means that a column can be subjected to bending. However it primarily is subjected to axial deformation and axial forces whereas, a beam can be subjected to axial forces and axial deformation but primarily it carries the loads through bending or flexure. Let us look at a structure that is made up of beam columns. What kind of structure is made out of beam columns? A planar frame.

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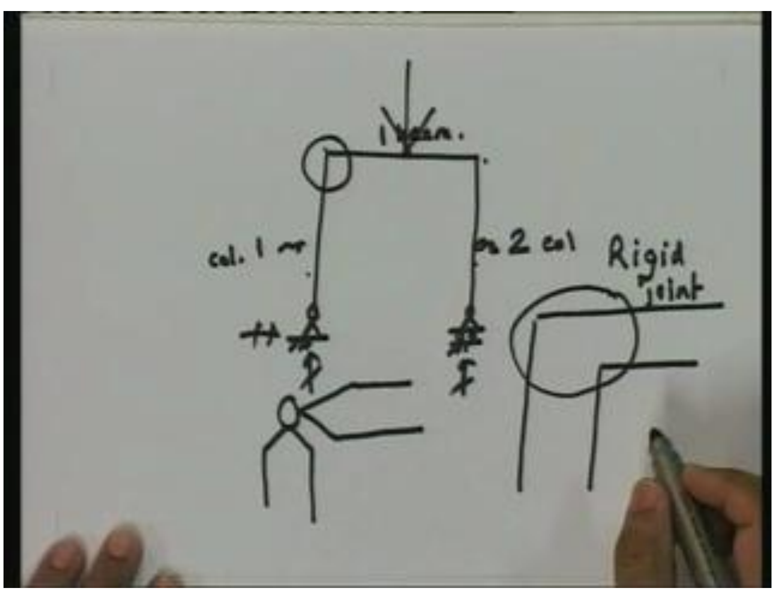

Let me draw a planar frame; the simplest kind of planar frame that we can have. It has two columns and one beam. This is a plane frame, where if you look at it the connection or the joint between the beam and column, if I draw it in reality it looks like this. This is the column, this is the beam and this joint is a rigid joint. In a truss what does this joint look like? It looks like this, a pin joint and that is the reason why in a truss, since this joint cannot transfer any flexural forces the members become axial members whereas, in a plane frame this joint is a rigid continuous joint and because of that you can transfer both forces as well as movements at the joint. We were discussing the Degree of Static Indeterminacy. How do I determine the Degree of Static Indeterminacy of a planar frame structure? Let us go back and let us look at a simple plane frame member and I am going to draw a generalized member.

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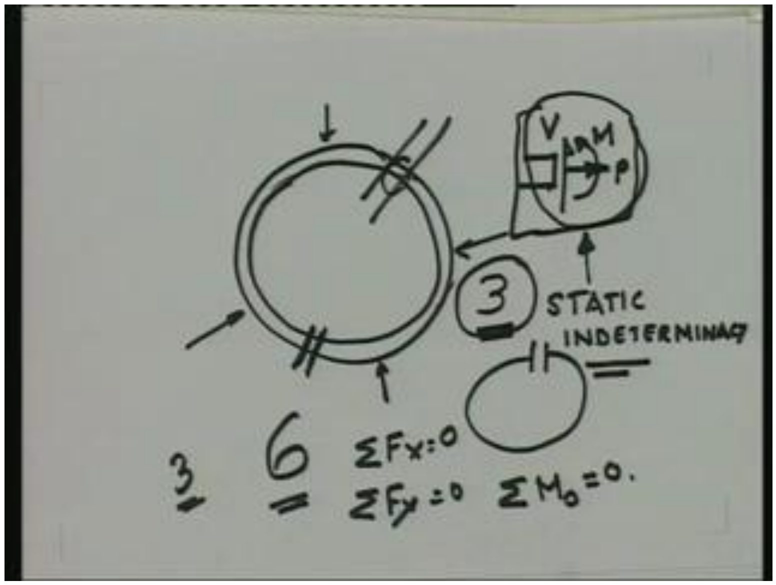

Let me draw a ring. Let us say that this is subjected to loads because this ring is a structure it is subjected to loads it deforms, because of the deformation it is subjected to internal forces. What are the internal forces? I can find out what the internal forces are by making a cut and see what happens in this zone. If we look at this, if I make a cut here what are the forces that I develop at this point? If, I look at this at the cut, I am taking now the cut out here, making the cut out first and foremost I can generate an axial force, I can generate a shear force and I can generate a bending movement. This is the notation that I am going to be using during this course: P - actual force, V - shear force, M - bending movement.

In other words, if I look at this, these are my internal forces developed at a cut. Note that in this particular problem, when I make this cut and I generate these three, can I find them out? I cannot find them out. These three are unknowns which I cannot use any equilibrium equation. Take any free bodies that you wish to and we can make two cuts and separate out another free body this is a free body too. Every time I make a cut how many forces do I release? 3. If I make a cut and I take this free body I have three unknown forces here. So how many forces do I have? 6. 6 unknown forces and for one free body in a plane how many equations of equilibrium can you generate? 3. What are they?

Sigma F_x equal to 0, Sigma F_y equal to 0, and Sigma M_o equal to 0 (M_o is movement about any point). These are the three equations of equilibrium. As soon as I release this and I take a free body we have six unknowns and three equations. What is the Degree of Static Indeterminacy? It is the number of unknowns minus the number of equations of equilibrium. What is the static indeterminacy of this ring? It is 3. Therefore, this is fundamental. This is how we generate it. A ring has three degrees of static indeterminacy. This is what I am going to be using, to find out the static indeterminacy of a plane frame structure. How do I create a ring? Can I take a structure and make it into a ring? Actually I cannot. I have to think a little bit and do it. Let us come back to the problem that I was discussing a little bit earlier.

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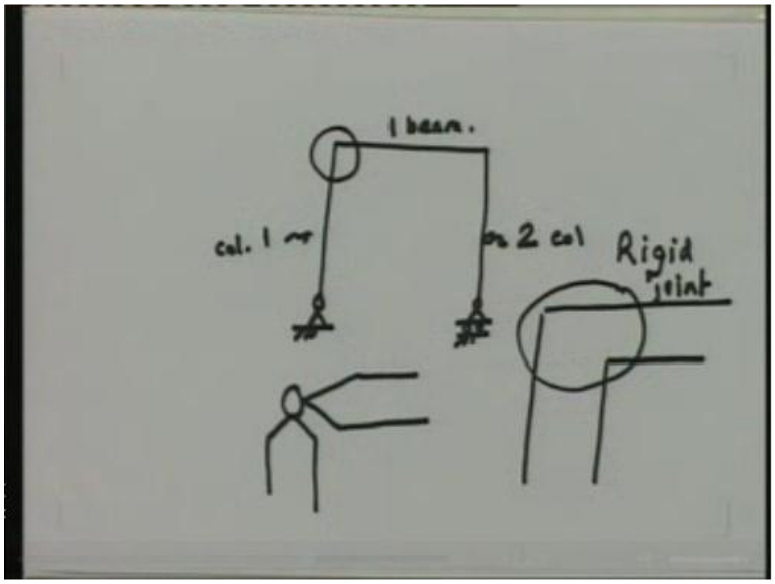

This is the plane frame. How do I find out the static indeterminacy of the plane frame? Can I use (m plus r) minus 2j? What does that mean? Let us look at how many reactions I have; 1, 2, 3 so three unknown reactions and how many members? It is 1, 2, 3 so three members. How many unknown equations do I have? m plus r is 3, r is 3, m is 3 so m plus r is 6. How many joints 1, 2, 3, 4. So, if I use my formula (m plus r) minus 2j where j is equal to 4 m plus r is equal to 6 what does that give me. The Degree of Static Indeterminacy is minus 2. Does that make sense? Obviously it does not. What do you mean by Degree of Static Indeterminacy? Actually, whenever the Degree of Static Indeterminacy is less than zero it essentially means that the structure is unstable in a mechanism. Is this structure unstable in a mechanism? It is not. It is a perfectly valid frame. If I subjected to loads; let's say I subjected to load here, is this structure going to move or collapse? No it is not. Obviously, the formula that I generated for a truss structure (m plus r) minus 2j is not valid for plane frame structures. Please understand this. This equation is valid only for a truss structure not a plane frame structure. Then, how do I find out the Degree of Static Indeterminacy of a plane frame structure? Let us go back to the ring. The ring if you remember had three degrees of static indeterminacy. Can I take any plane frame structure and make it into a ring? Actually it turns out I can.

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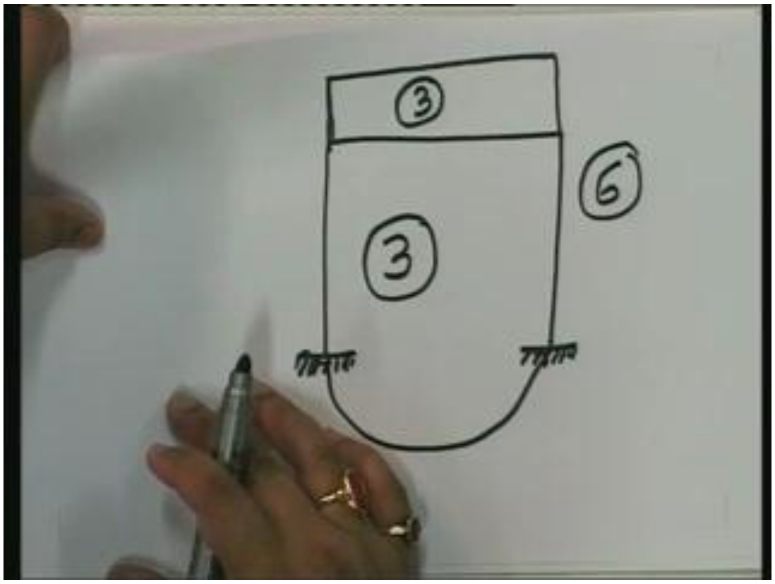

Let us look at the single storey, single bay plane frame and now instead of having a hinge and a roller support I am going to fix it. This in reality is the most common single bay single storey plane frame and it is fixed at the foundation. This is the most standard thing. If you look at this particular structure think about it, it is fixed over here, it is fixed over here. For all practical purposes, I could have a situation where this could have something like this because it is fixed here fixed here so I could have a member like this here. Now, look at this, what does this resemble? Does it not resemble your ring? What is the static indeterminacy of this structure? Since it is a ring it is 3. 3 is the Degree of Static Indeterminacy. Therefore a fixed, single bay single storey frame has a static indeterminacy of three.

What happens if I put another storey on top of it? How many rings do you have in this structure? You have one ring here and a second ring here. Each ring has three degrees of Indeterminacy so a two storey single bay frame has, how many Degrees of Indeterminacy? 6. Now, do I always have to take a fixed support? No. What do you mean by reducing the indeterminacy? How do you reduce the indeterminacy? I have this single bay single frame which is fixed.

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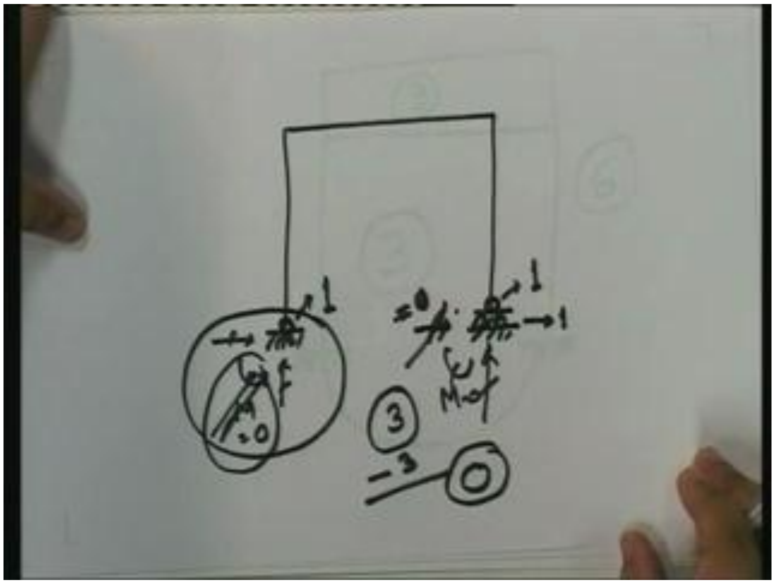

We just discussed that this particular frame has three degrees of static indeterminacy. Now let me start releasing some of them. What does the fixed end have? A fixed end stops this point from moving in this direction. It has two unknown support reactions plus at this point the joint cannot rotate. In other words it can generate a movement. So a fixed support has three unknown reactions. Now, what I am going to do is, I am going to release the restraint against rotation. What happens?. When I release the restraint against rotation m has to disappear this m has to be here. As soon as I have taken a fixed end and released its rotational capability what does it become? It becomes exactly the same as a hinge support. Let me do something more; let me do the same thing over here. Now I have made two releases in the three Degrees of Indeterminacy, one I have released the hinge and two, I have released the rotation. I have made two releases and I am going to make a third release. I am going to allow it to move in this direction, which means I have taken this restraint and put it equal to 0. In other words, how many restraints am I left with at this point? It is a single one.

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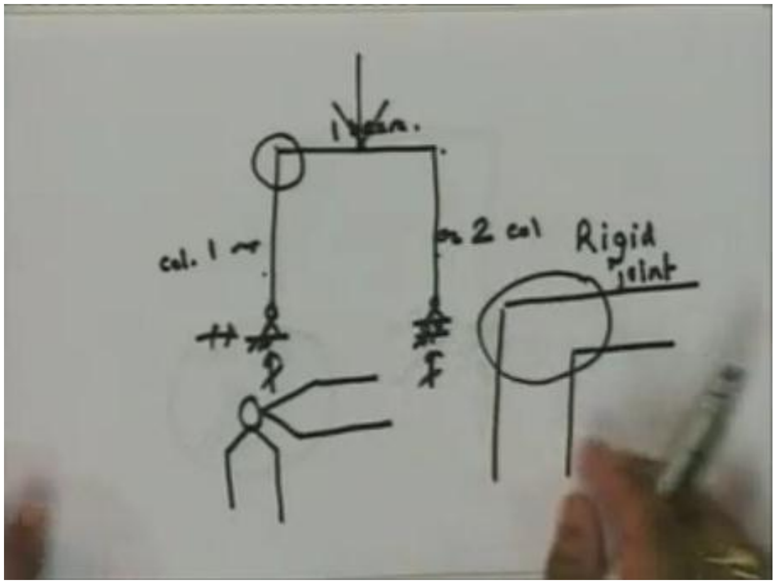

Look at the structure. What does this resemble? It resembles this (Refer Slide Time: 46:04) hinge supported at the left end roller supported at the right end.

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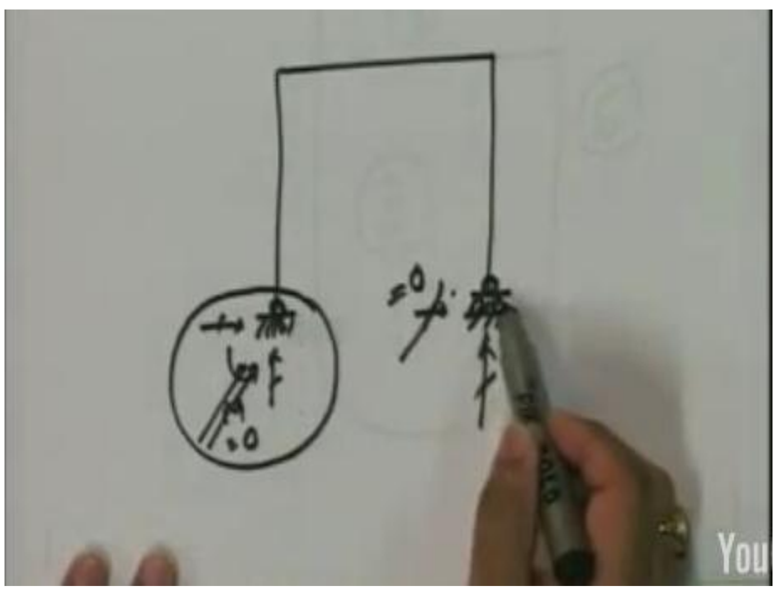

How many releases did I make in the fixed structure to get to the hinge roller? I made one release here, the rotational release here, one release here which is the rotational release here and one release here which is the horizontal movement here. How many releases? There are three. When you take a three Degree of Static Indeterminacy structure and put three of the forces equal to 0, that is what I have done, here I have taken this moment put it equal to zero, taken this put it equal to zero, taken this moment put it equal to zero. What have I done? I have removed three unknown forces. When you remove three unknown forces what do you do? You essentially make it into a 0 Degree of Static Indeterminacy. What is the 0 Degree of Static Indeterminacy? A statically determinate structure. Therefore what do you do in planar frames? You essentially start off with the ring. Try to create a ring. Let us now look at another structure, you might say well you are always taking a plane frame structure and trying to make it into a ring.

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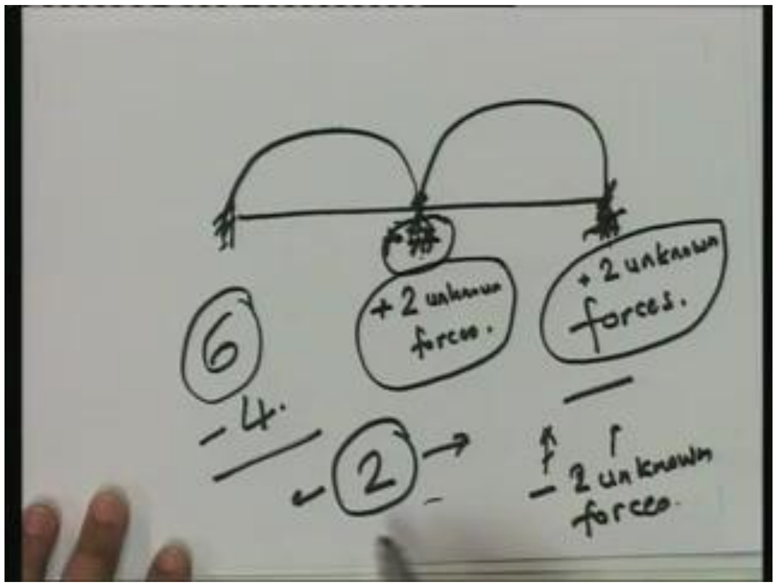

Give me a multi-span beam and show me how you make it into a ring. This is a typical multi span beam. What is the Degree of Static Indeterminacy? The first thing I am going to do is to start fixing. This is a roller support; I am going to add two restraints to this. If I add two restraints to a roller support I am going to stop it moving in this direction. What happens? It develops an unknown reaction here. I am also going to stop the rotation here. So, what have I added? I have added two unknown forces to the structure. Remember that I have added two unknown forces in the structure. Furthermore, I am going to stop this also. I am going to fix this so I have added two more unknown forces. So I have added four unknown forces.

In other words, I had a structure which was fixed at this end roller here and roller here. Now, to make it into a fixed I have added two unknown forces here and I have added two unknown forces here. Let us look at this problem. How many rings does it have? This is one ring, this is a second independent rings. How many Degrees of Indeterminacy does this fixed structure have? Obviously two rings 6. Now, to generate these two rings, how many unknown forces did I add? It is 4 that is 2 plus 2 is equal to 4. What was the original structure how many Degrees of Indeterminacy does it have? I have to take off these two unknown forces because these are fictitious unknown forces that I added to make them into rings. I have to subtract 4 and how many indeterminacies do I get? 2.

Let me see if I can get this same in another fashion. Suppose I remove this one unknown force that was this, and also remove this. How many unknown forces I have removed? I have removed two unknown forces and what does it become? A cantilever. And we know that a cantilever is a statically indeterminate structure. So, by removing two unknown forces I have made it into a statically indeterminate structure. So what is the static indeterminacy? 2. Either way you get that this is a 2 degree of statically indeterminate structure. Today I have discussed how to generate degrees of static indeterminacy and how to determine the Degree of Static Indeterminacy in a structure.

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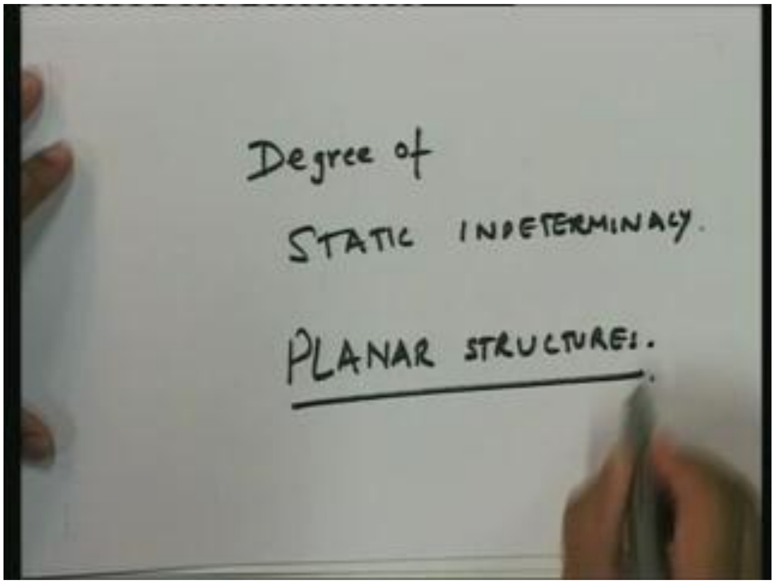

Of course I have talked only of planar structures. In this particular course I am going to be only talking about planar structures because once you can solve for planar structures you can solve for space structures also. So, today I have looked at how to determine static indeterminacy of planar structures be it trusses: (m plus r) minus 2j where m number of members r number of reactions j number of joints. For a plane frame structure, add forces to make them into ring and then you always remember that the number of rings that you have in a structure, the Degrees of Indeterminacy per ring is three so suppose you have two rings, 2 power 3 is 6. How many unknown forces did you add to make them rings? Subtract that and the answer will always give you the static indeterminacy.

Thank you very much for today. Tomorrow I am going to be taking up on another kind of indeterminacy that is Kinematic Indeterminacy.