Soil Mechanics
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Lecture – 53
Earth Pressure Theories – IV

Welcome to lecture four on earth pressure theories. In the previous lecture we looked into the Rankine's theory and assumptions relevant to Rankine's earth pressure theory and we also discussed about special cases for computing earth pressures and extension of Rankine's theory for a special case like inclined face of a wall and with inclined slope face.

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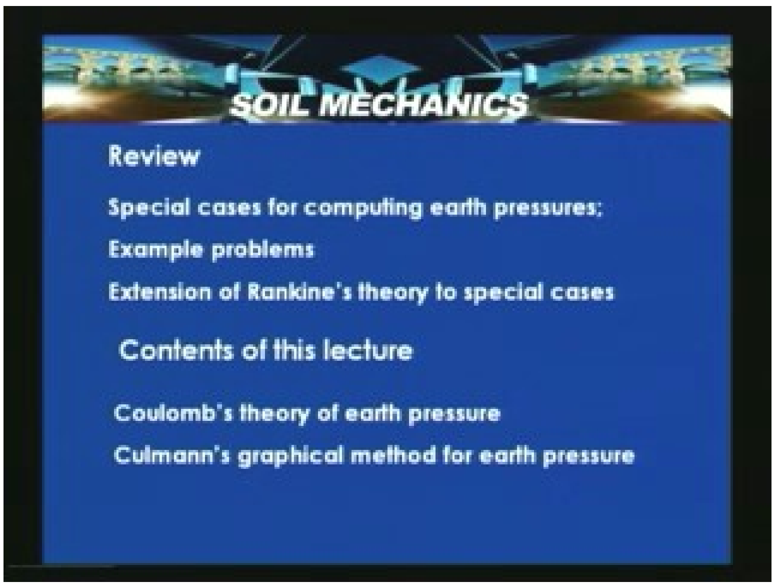

In this lecture we are going to introduce another theory called coulomb's earth pressure theory and subsequently we look into the graphical method for solving or computing earth pressure. This is by using trial wedge method or is also popularly known as culmann's method. So in the previous lecture we have discussed and we have said that two states which are going to be prevalent, one is that active and passive. Active can occur when the wall moves away from the backfill, passive can occur when the wall moves towards the fill. So here a cross section of a shippel wall is shown in which this is the embedded depth level and about this level it can have an active zone. If this wall is rotating about this point and this can have a passive zone.

So here in this case this portion of the soil will be subjected to compression and this portion of the soil is subjected to extension. So the remaining portion of the soil will remain in elastic equilibrium conditions because of the prevailing roughness conditions.

So this we said that 45 plus phi by 2 and this is the failure surface and this is the possible failure surface, in case of passive case and we discuss a failing backfill behind outward yielding wall.

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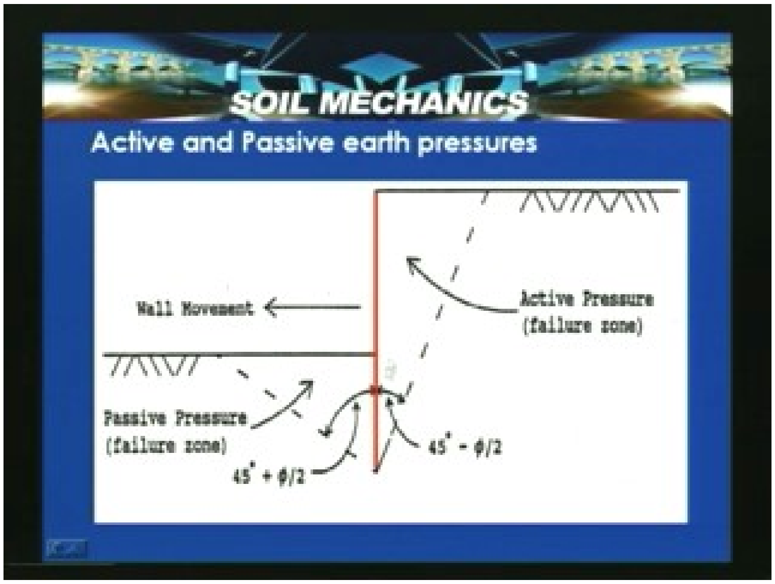

So in this slide a typical cross section where wall yielding away from the backfill soil is shown, we can see along this surface, when a surface which passes through this, a plane which passes through this heal of the wall satisfies the Rankine conditions. So here along this it mobilizes the failure shear stresses and then along this that develops a shear plane. So here a retaining wall when yields away from the backfill and this is the stress state at a particular element along the failure plane.

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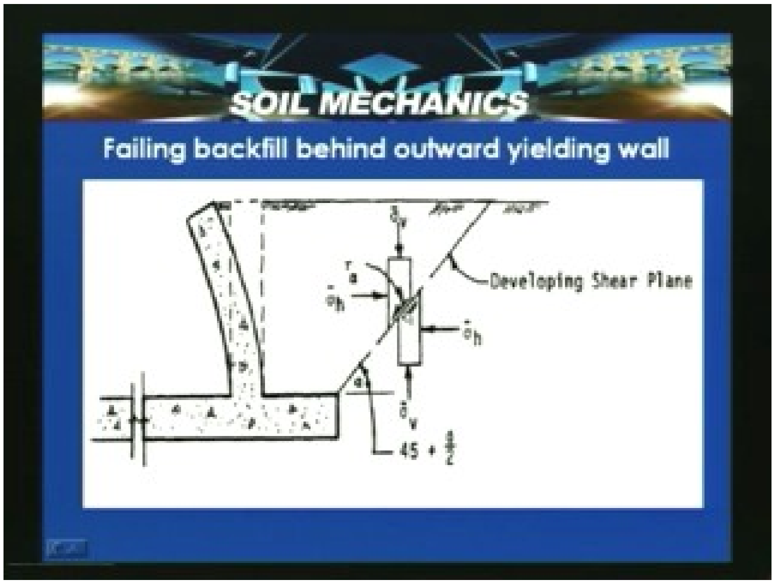

So we also discussed about a Mohr strength diagrams, here the development of the Mohr strength diagram during the progressive outward wall direction is shown here.

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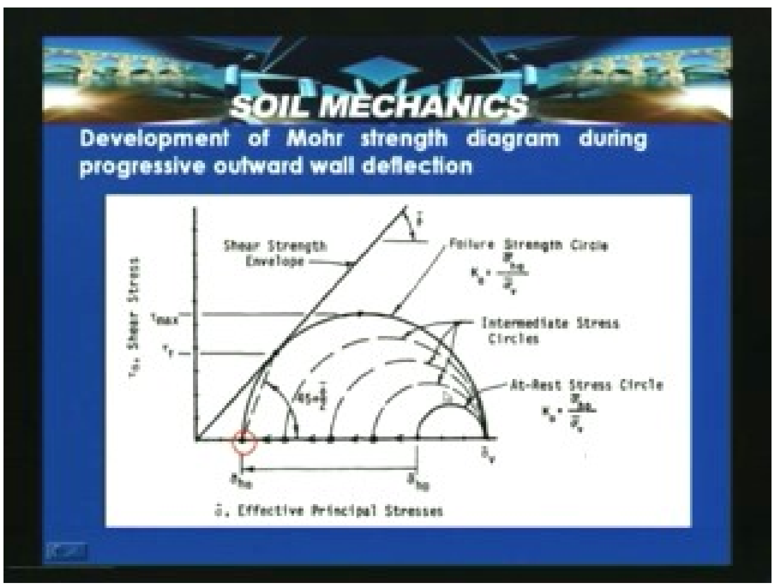

So this is the vertical stress is shear stress and horizontal axis is sigma so initially this is at rest stress circle that is at rest conditions so this is sigma H_0 by sigma v, k_0 is the coefficient of earth pressure at rest. So once wall starts yielding the pressure starts decreasing and it attains a value sigma H_a minimum and then it attains the failure along that shear plane which was shown in the previous slide. So this is how that progressive outward wall movement induces failure to an element under consideration. Similarly when you look in to another case called passive case; in this case a failing backfill behind inward yielding wall is shown.

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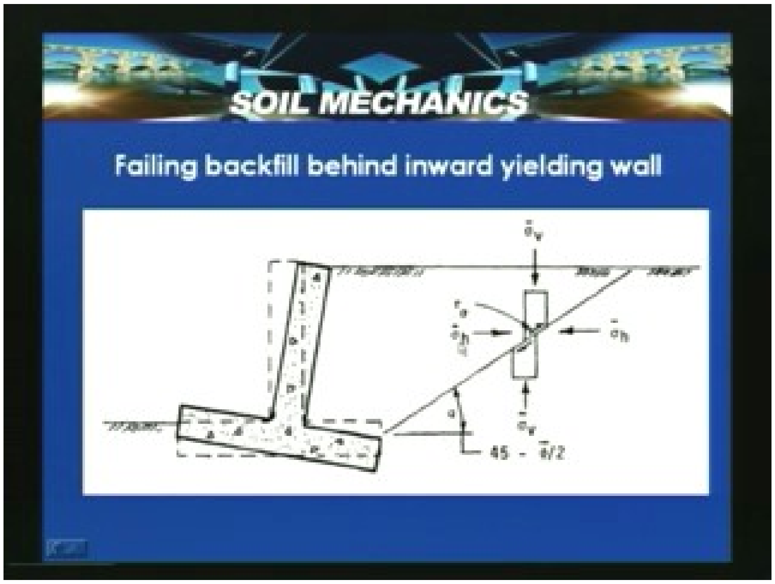

So this retaining wall is yielding towards the backfill so induces a compression in this case where along this shear plane it actually induces tow alpha that is the shear stress which gets mobilized and the wedge of the failure. So sigma$_v$ and with the vertical stress and sigma$_h$ is the horizontal stress. So in this case the sigma$_h$ becomes the major principle stress, sigma$_v$ becomes the minor principle stress.

So let us see how the development of the Mohr strength diagram can be there during the progressive inward wall deflection, when the wall moves towards the backfill. So here again, at rest the circles are shown here. So this sigma$_{h0}$ changes and surpasses sigma$_v$ and for a given sigma$_v$ it develops sigma$_{hp}$ finally attains this is the passive earth pressure and attains failure. This is how the development of the progressive inward wall deflection can be shown in a Mohr strength diagram with systematic Mohr circles which are shown here in this slide.

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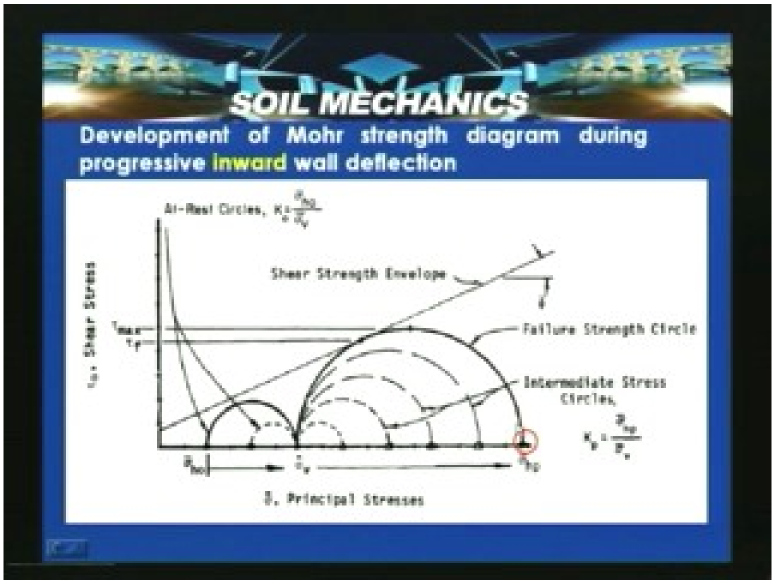

Now before discussing coulomb's theory in the last class we discussed about a vertical smooth wall with inclined slope surface, ideal Rankine wall is with a vertical wall which is smooth in nature and horizontal surface which is the backfill surface is horizontal but a case where it can be extended this Rankine's theory can be extended by assuming that the stresses which are sigma$_a$ parallel to the slope surface.

So from the discussions which we have made in the previous lecture we can solve this problem, a vertical smooth wall of 6 meter high and retains a soil with a slope of 20 degrees that is the slope of this surface is 20 degrees with horizontal which is shown here. Determine the total active thrust on the wall, the shear strength parameters given as cohesion is equal to zero and friction angle which is drained 40 degrees and unit weight of the soil is 18 kilonewton meter per cube, the height of the wall is 6 meters.

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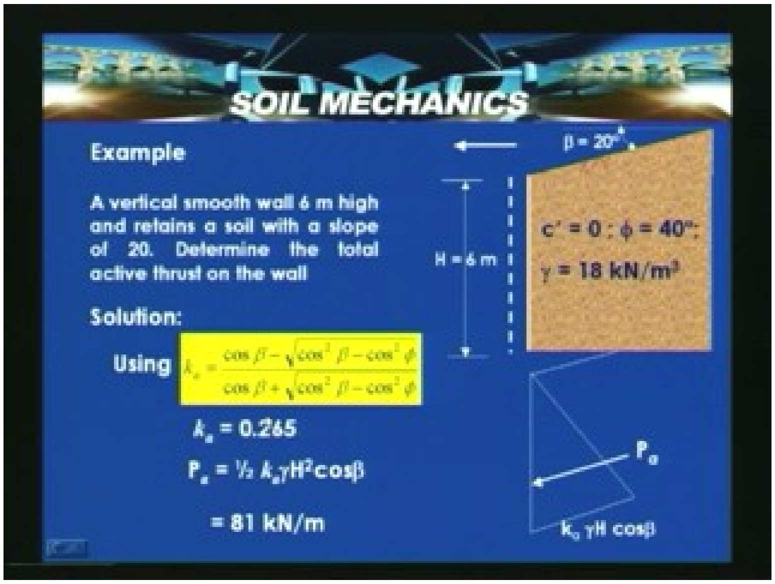

So using the previous class we discussed about k_a is equal to cos beta minus root over cos square beta minus cos square phi divided by cos beta plus root over cos square beta minus cos square phi, by using this we can get k_a is equal to 0.265 by using p_a is equal to half k_a gamma H square cos beta will be able to get the active thrust on the wall; this is how you can solve analytically. The same solution can also be done by using by plotting Mohr strength diagram which we will see in the next slide but here the earth pressure diagram which is shown here this ordinate is nothing but k_a gamma H cos beta at depth z equal to H and this is the P_a which is acting at H by 3 vertically from the base and P_a is equal to which is shown here half k_a gamma H square cos square cos beta.

So for the given condition in this problem the lateral thrust which is acting parallel to the direction of the slope surface that is 20 degrees with horizontal is 81 kilo newton per meter at a height of 2 meters from the base that is H by 3, H is equal to 6 meters so 2 meters from the base. How this can be solved graphically? Let us look into next slide. So the continuation of the previous example beta is equal to 20 degrees that is the surface and friction angle is 40 degrees. So let us plot this on a tow sigma plot, so on a tow sigma plot, first failure envelope is known to us.

The failure envelope is inclined at 40 degrees with horizontal so plot this on the Mohr circle then the ground surface is 20 degrees with horizontal. So plot ground surface so on this ground surface along this direction remember that the stresses are acting. So now at a depth of 6 meters that is at z equal to H is equal to 6 meters, sigma_z is equal to gamma into H cos beta which will give us 102 kilo Pascal's or 102 kilo newton per meter square.

So with that set 101 kilo pascal to a scale and draw and represent along this particular 20 degrees line that is this surface line. So OA is equal to 102 kilo Pascals. So represent with say 1 centimeter is equal to 10 kilo Pascals or some units and represent on the Mohr circle. So from here if you draw a line, the point where it cuts that is the pole which is drawn generally from the known stress on the Mohr circle. So this is a point B so from the point where pole is there, from there if you draw a line to failure envelope, if it cuts at F so by seeing that it passes through F and it passes through A and pass through B. If you draw a Mohr circle you can construct Mohr circle by satisfying this conditions.

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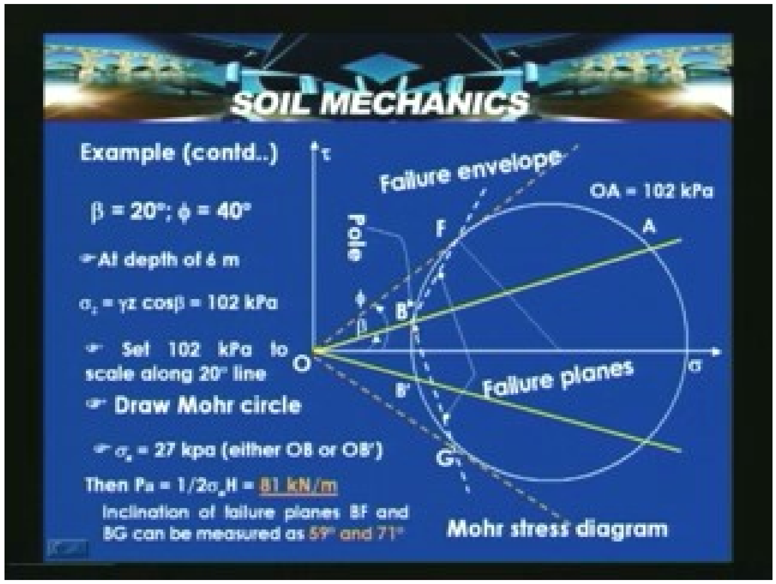

So once we draw a Mohr circle so sigma_a is equal to 27 kilo pascals or that is nothing but either OB or OB dash both are same. So with this we will be able to get that is active earth pressure ordinate that is 27 kilo Pascal's which is either OB or OB dash. So then P_a that is active earth pressure thrust is equal to half that is sigma_a which is the base and H which yields to about 81 kilo newton per meter. So inclination of the failure planes BF and BG can be measured so BF is 59 degrees with horizontal and BG which is the another failure plane which is 71 degrees with the horizontal.

So by using these concepts now we can even also by Mohr strength concepts we can also solve this problem graphically and compute active thrust. In this problem what we did is that we keeping the soil properties under consideration and geometry of the slope surface. We calculated earth pressure quotients from there we try to solve by computing the earth pressure ordinate at a depth z is equal to h is equal to 6 meters and then active thrust has been calculated, the same problem we try to solve by using Mohr strength diagram. Now having discussed Rankine's theory but we knew that many situations, most of the retaining walls which exists they are not rough in, they are not smooth in nature. In such situations the coulomb theory of earth pressure which was put forwarded almost one century prior to Rankine's theory but we discuss Rankine theory first and then coulomb's theory later.

Because the only difference the chief difference between coulomb's theory and Rankine's theory is that in coulomb's theory, coulomb considers the friction between the wall interface and soil. That is very important where it has got lot of effect on the earth pressures which will act on the wall. So as we discussed just now the back of every real retaining walls is rough to some extent so the boundary conditions for the validity of Rankine's theory are seldom satisfied.

So the ideal Rankine's conditions are seldom satisfied like smooth vertical wall or with a fill surface horizontal. In such situations we have to resort to coulomb's theory. So earth pressure computations usually involve an appreciable error if we able to deal with Rankine's theory with the assumption of negligence of the friction between the interface, wall soil interface.

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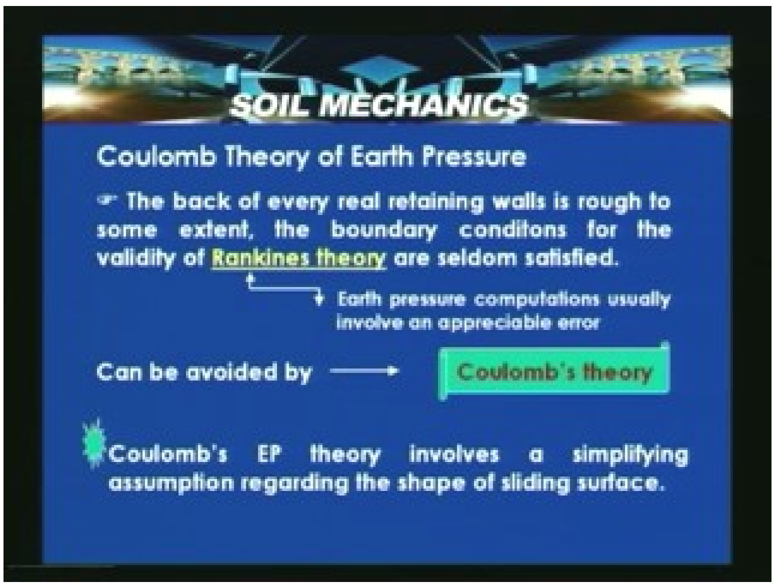

So this can be avoided by adopting coulomb's theory so coulomb's earth pressure theory involves a simplifying assumption regarding the shape of the sliding surface. It has got an assumption which is called a sliding surface is assumed in this theory but though despite of this limitation, another limitation is that on the failure wedge the forces were not under equilibrium but despite of these demerits the theory has got a merit over Rankine's theory. So the coulomb's theory of earth pressure always superior because the real retaining walls always have a friction between wall along the wall interface and the soil. So this can be avoided and the error which can cause by neglecting that wall friction can be avoided by adopting coulomb's theory. So coulomb's pressure theory after coulomb's 1776 so the coulomb's theory is similar to Rankine's theory except there is friction between the wall and soil and takes this into account by using a soil wall friction angle equal to delta.

Note that delta ranges from 50% of the friction angle to about 66% of the friction angle that is two thirds of the friction angle and delta is equal to two third that is two times, two by three times the friction angle is commonly used, delta is equal to two by three times phi is commonly used. that is the friction between the wall and the soil and takes this into account by using the soil wall internal friction angle of delta that is the fundamental difference. Then soil behind the wall is considered as a whole instead of an element.

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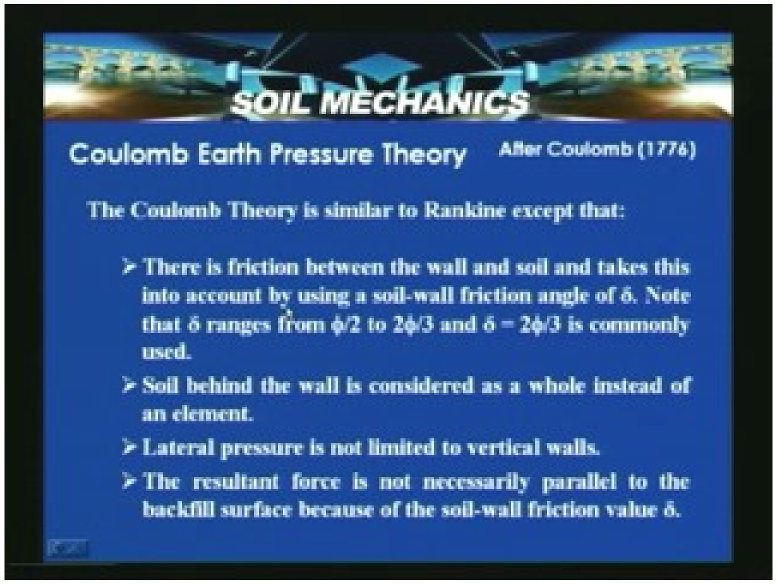

Lateral pressure is not limited to vertical walls so you can have wall with any inclination and also the backfill surface can be having like slope surface, need not necessarily be horizontal and resultant force is not necessarily parallel to the backfill surface because of the soil wall friction value delta. So the resultant force is not necessarily parallel to the backfill surface because of the soil wall friction value delta. So in this slide what we saw is that the chief differences between coulomb's theory and Rankine's theory we saw and the fundamental difference what we said is that the friction between the wall and soil which is actually considered in coulomb's theory which was absent in Rankine's theory.

Now let us look in to some assumptions which are involved before discussing about coulomb's theory. The assumptions are the backfill surface or backfill soil is considered to be dry, homogeneous and isotropic and it is elastically undeformable but breakable. The granular material possessing internal friction but no cohesion. So it is basically for cohesion less soils it is postulated, the backfill soil is considered to be dry homogeneous and isotropic and it is elastically un deformable but breakable and granular material possessing internal friction but no cohesion. Rupture surface is assumed to be a plane for the sake of convenience in the analysis. So this has got some limitations which we will discuss subsequently; rupture surface is assumed to be plane for the sake of convenience in the analysis. The sliding wedge acts as a rigid body, it actually once the failure it represents as if that with the portion of the soil got detached from the rest of the soil mass, the sliding wedge acts as a rigid body.

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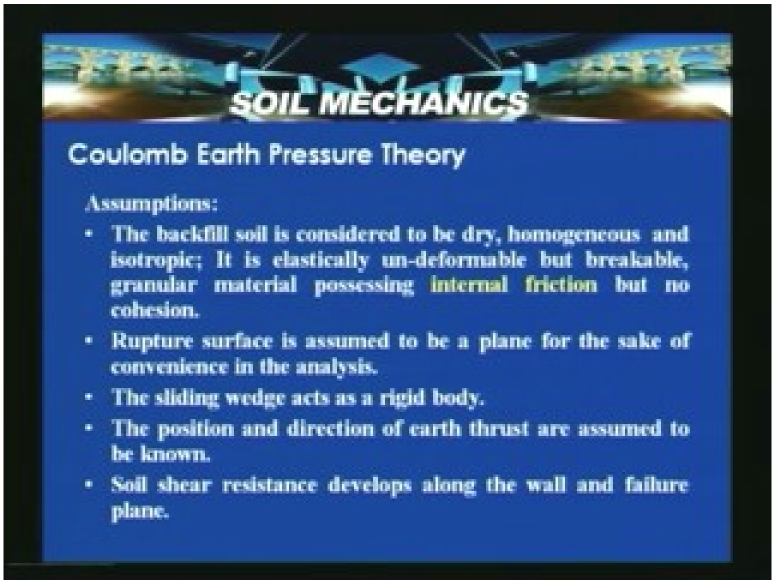

The position and direction of earth thrust are assumed to be known. Generally the position which is in a given failure wedge, if the weight of the wedge is at the centroid and at the centroid location if a line is passing through the failure surface and wherever it touches the surface or a vertical face, suppose if the wall is vertical and if it cuts at that particular point and that is where it is assumed that the active thrust or passive thrust is acting. That means the position and direction of the earth thrust are assumed to be known in this theory and soil shear resistance develops along the wall and failure plane. It is assumed that the soil resistance or the friction is assumed to be developed uniformly along the wall and the failure planes.

Another assumption which was assumptions which are continued in this slide when the soil wedge is impending that is either moving away from the backfill or towards the backfill, the theory gives two limiting values of earth pressure like in Rankine, the conditions are similar, one is active another one is passive compatible with equilibrium. So when the soil wedge is impending the theory gives two limiting values of earth pressure active and passive and compatible with the equilibrium.

If the wall yields and the rupture of the backfill soil takes place a soil wedge is assumed to torn off from the rest of the soil mass. So this is as we discussed in the previous slide if the wall yields and rupture of the backfill soil takes place, a soil wedge is assumed to torn off from the rest of the soil mass and friction is assumed to distributed uniformly and back face of the wall is a plane. So the back face of the wall is a plane that is another assumption which we made in this. Then we also discussed in the previous slide while discussing assumptions, the failure surface which is assumed to be a plane. So in effect due to deviation from the actual plane of failure. Here a wall cross section is shown and here according to coulomb's plane of failure this is indicated with broken lines, here this is in case of active and this is the active wedge and this is a coulomb's failure surface and this is coulomb's a failure wedge.

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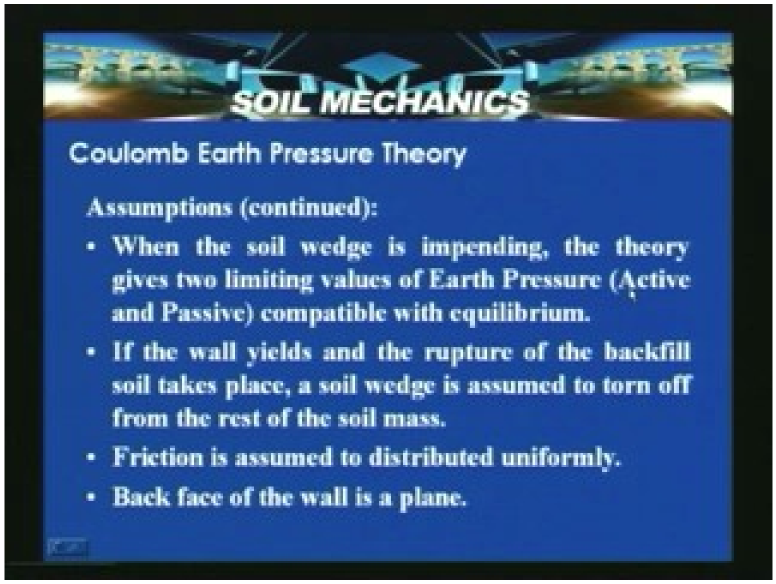

But if you look into this figure (Refer Slide Time: 19:29) in this slide actual failure plane for active case is this one which is approximated by the log spiral theory and if you approximate in case of a passive case there is a much deviation and this passive wedge which is actually having log spiral based on the log spiral theory and this is the actual plane of plane of failure. Now there is a deviation the deviation is very clear from this figure it is minimal in case of active case but it is maximum, the difference cannot be ignored, may not be acceptable in case of a passive case.

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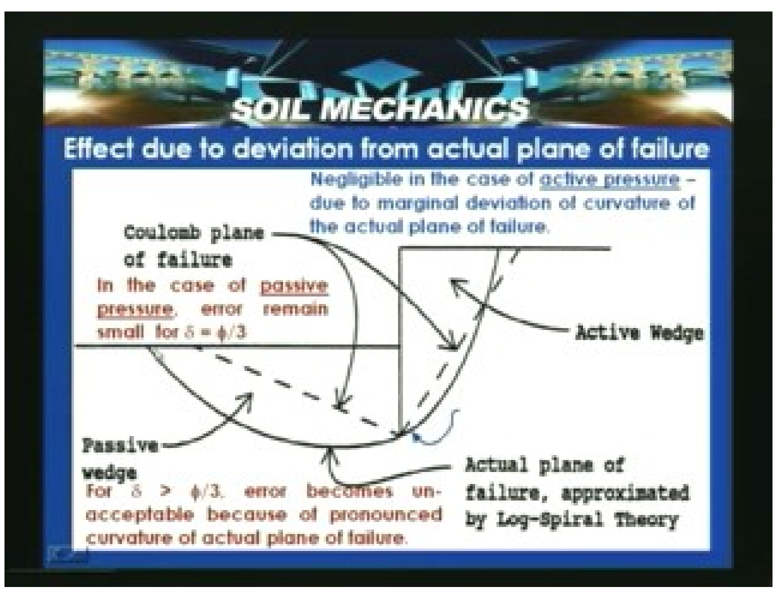

So if you look in to this, this particular assumption of failure plane this is the actual failure plane and this is what it has been assumed as a plane for the sake of convenience in the analysis by coulomb. So negligible in the case of active pressure due to the marginal deviation of the curvature of the actual plane of failure. So because there is a marginal deviation from the actual plane of failure so there is negligible as far as the active state is concerned but where as in case of passive case the error remains small as long as the interface friction angle is about one third of the friction angle of the soil. That is if delta is equal to within phi by 3 then you know the difference in the case of passive pressure the error remains small but for cases where delta greater than phi by 3 that is the wall interface friction angle delta is more than the one third of the friction angle of the soil. The error becomes un expectable and because of the pronounced curvature of the actual plane of failure.

So the deviation from the actual plane of failure and the resumed plane of failure by coulomb increases for the values of delta greater than phi by 3. Now let us look as before derive deriving coulomb’s active earth pressure cohesions and passive earth pressure cohesions. Let us try to look into different possible types of wall frictions and how they effect and how they can exist in the field with typical cases. So types of wall friction let us discuss active case. In an active case we have two types of wall frictions positive wall frictions and negative wall friction, similarly in case of a passive case we have positive wall friction and negative wall friction. Let us look positive wall friction this is for the majority of the active cases in case of coulomb theory. So this is a cross section of the wall which is shown here and this is the backfill surface which is inclined and this is the one of the failure assumed failure surface.

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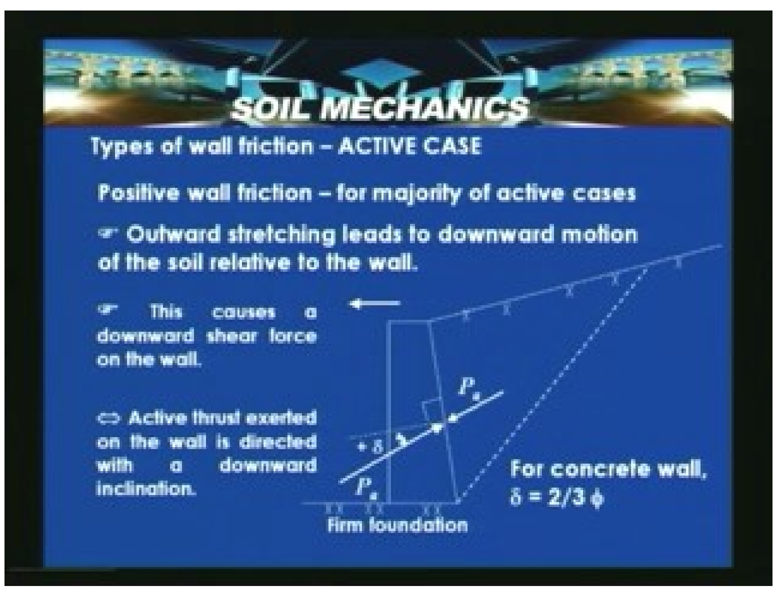

So outward stretching lead to downward motion of the soil relative to the wall. As the wall moves away from the backfill and also the wall is assumed to be rest on a firm foundation.

In such situation outward stretching lead to the downward movement of the soil relative to the wall. In such situation this causes a downward shear force on the wall. So active thrust exerted on the wall is directed with a downward inclination because of this downward drag what happens is that the active thrust is exerted in the downward direction so that's how the P_a which is the active thrust is shown here. Now this positive wall friction is indicated and this is the plane perpendicular to the wall surface. So this angle is 90 degrees so that delta which makes with this particular surface is the direction of the thrust exerted by the wall on to the soil that is P_a. That is P_a which is indicated here and this case is referred as positive wall friction active case. Mostly for concrete walls the friction angle is 2 by 3 times the friction angle of the soil. For concrete walls the wall interface friction angle is 2 by 3 times the friction angle of soil. So in this we have seen positive wall friction active case where the direction which is shown here it acts below this line which is indicated by positive delta.

Let us see the next slide. Assume that the wall is resting on a soft foundation; in the previous case in the positive wall friction case in active case the wall is assumed to be rest on a firm foundation. In this case wall is assumed to rest on a soft foundation or at the crest there is a heavy load has been applied. So negative wall friction which causes settlement of the wall which may result due to soft foundation or due to load on its crest. So settlement of the wall can arise because of the soft foundation or due to heavy load on the crest. So in case when the active thrust occurs that is active wall movement is away from the backfill, the active thrust exerted on the wall will be directed in an upward direction because here wall is moving downwards to oppose that there will be an upward drag. So that is a reason why the P_a is acting upward direction on the wall surface as shown here. So this is the again surface this is the line which is perpendicular to the wall face, here which is the wall face which is shown here and if this is above this particular line then that is indicated as minus delta that is call negative wall friction angle.

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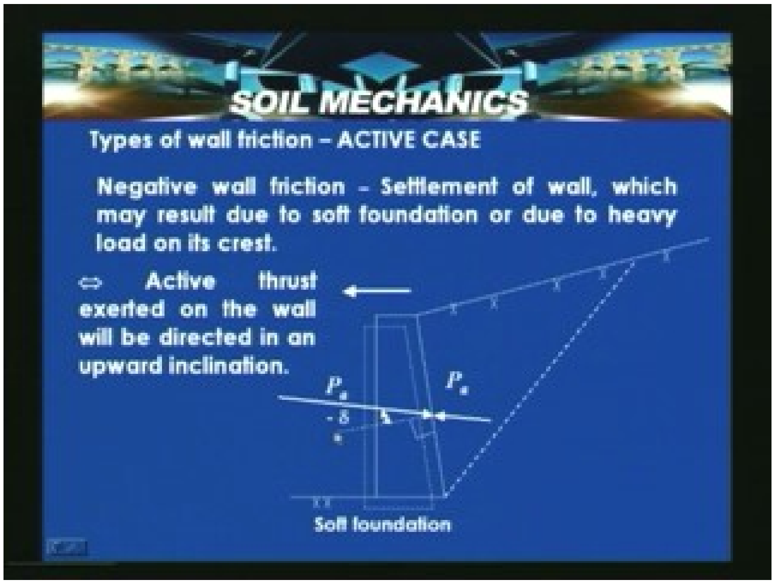

So the negative wall friction angle case can arise if the wall is resting on a soft foundation or being settled because of the heavy load which exist on the crest of the wall. In such situation the moving soil, when the wall moves outward it exerts on an upward drag so because of this the negative wall friction case arises. Similarly now the types of the wall friction for the passive case. In previous slides we have seen active case, one is positive wall friction and negative wall friction, in the passive case also there is two cases are possible one is the positive wall friction this is again same valid for the majority for the passive cases.

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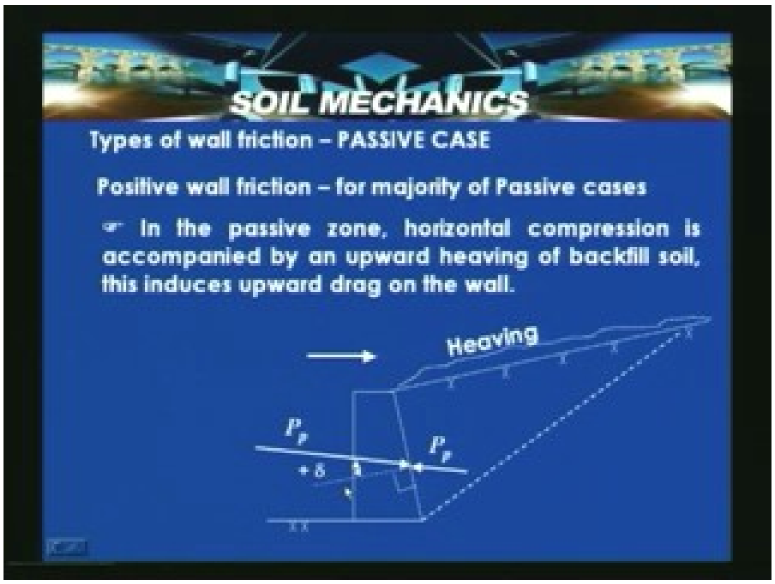

So in the passive zone the horizontal compression is accompanied by upward heaving of the backfill soil and this induces upward drag on the wall. So when the wall is exerted or move towards the backfill and that increases causes a heaving of the soil. So this heaving which is shown in this diagram like this and this is that failure surface and this is the original ground surface and this is the heaved surface. So in the passive zone the horizontal compression is accompanied when upward heaving of the backfill soil. So because of this the upward heaving of the soil so oppose that this thrust which is actually acts in the upward direction here.

So the passive thrust which is positive wall friction which is actually again shown 90 degrees with the wall face is above this line which is indicated which is shown here in this diagram (Refer Slide Time:27:25). So this is the case of positive wall friction for passive cases. So in the passive case the horizontal compression is accompanied by the heaving so because of this it induces an upward drag on the wall. Similarly a case where types of wall friction passive case but negative wall friction that is friction angle which is at the wall interface is negative. So this may arise due to settlement of the backfill soil and appears as if the wall is going up.

If this is an existing wall and this is that failure surface and this is the original ground surface, when the wall is moving towards the backfill surface because of the settlement of the backfill soil it appears, it actually induces the drag on the wall in the downward direction. So because of this, this exerts a negative wall friction and qualifies for a passive case. So this is the case for types of wall friction passive case where a negative wall friction can occur because of the settlement of the backfill soil and appears in reality as if the wall is going up.

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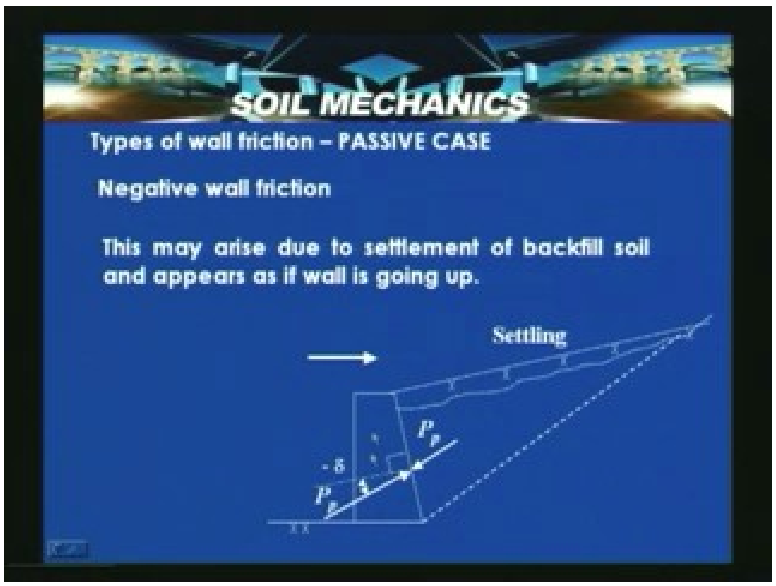

Now let us consider we look into the analytical expressions which are derived by coulomb for satisfying equilibrium conditions. So let us look into first active earth pressure of cohesion less soil. So let us assume that a wall of height H this AB is the wall face which is having friction so let us assume the case where here positive wall friction assuming that the wall is resting on a firm foundation and AC is a potential failure surface or a shear plane where the failure can takes place or the surface where it gives the maximum active earth pressure on the wall.

So beta is the slope inclination with the horizontal that is the slope surface inclination, alpha is the inclination of the wall face with the horizontal and delta is the wall interface friction, P_a is the active thrust acting on the soil block. So as we discuss ABC is the triangular wedge and which is oppose now by a soil resistance R which acts again with a normal with the failure surface AC and this line is perpendicular to AC and then this is that friction angle phi and it's with vertical it is theta minus phi and this failure surface AC which is the one which gives the maximum active thrust on the wall say assume that inclined at angle theta with the horizontal. So the same conditions and the weight of the entire wedge per meter length, again in the case of coulomb's theory also this conditions are solved by assuming the 2 d conditions are valid. So the weight of the soil within the wedge ABC per meter of the wall can be computed and that can be said as w.

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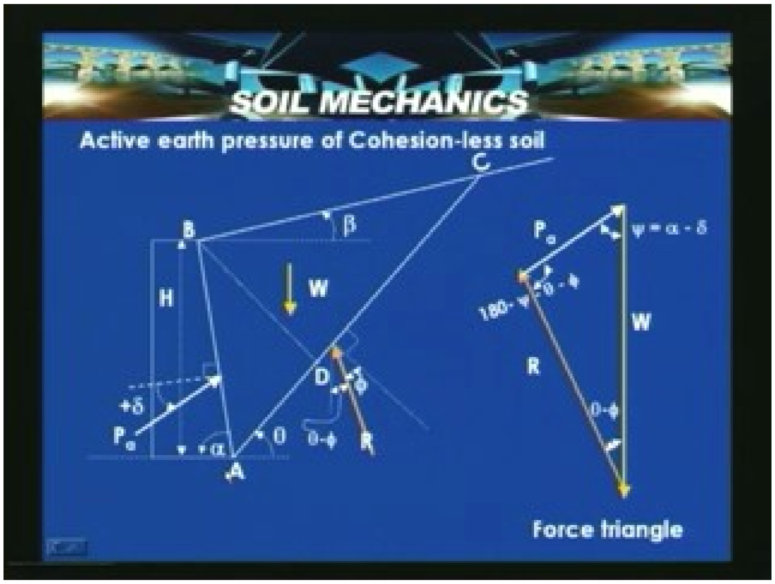

So this position of the active thrust which is assumed like this, if this is the cg and if this located at this point, a line passing through this say failure surface which is inclined at theta which gives the location of the active thrust that is at this point. So the same forces like W and this R and P_a which are acting on this triangular wedge ABC for the active case when the wall moves away from the backfill can be represented in the force triangle like this.

So here W which is shown vertical and P_a which is parallel to this that is which is shown here then R which is the soil resistance along the failure surface is assumed to be completely the friction assumed to be completely developed uniformly. So this R is the soil resistance along the failure surface for the movement so this inclination is 180 minus psi minus theta minus phi and this is psi is equal to alpha minus delta where alpha is this inclination of the wall face with the horizontal and this inclination between R and W which is shown here theta minus phi. So inclination between P_a and W is psi is equal to alpha minus delta.

So this is by solving this equilibrium and as we discuss despite of one of the demerits is that the equilibrium conditions are not satisfied that means when these forces are acting all these three are not non concurrent, at least two forces will try to meet and third force will deviate from that point. So despite of these differences because of the merit in which the theory has been formulated has got an edge over other theories. Though it has got this demerit of not satisfying the equilibrium conditions completely but it is widely used.

The continuation of previous slide let us now try to compute the weight of the triangular wedge ABC. So that can be given by W is equal to gamma into area of ABC into one meter that is the per meter length of the wedge. So this can be written as W is equal to gamma into H square by 2 psi square alpha into sin theta plus alpha into sin alpha plus beta divided by sin theta minus beta.

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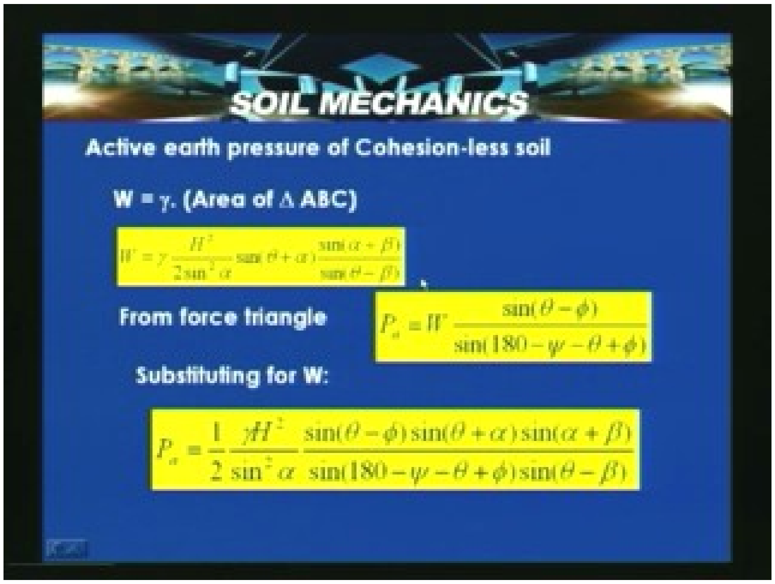

This is one expression we have. Now from the force triangle like we have shown in the previous slide, here the force triangle is shown P_a active thrust this angle is psi is equal to alpha minus delta and this is W that is weight of this wedge, R is the soil resistance along the failure surface, P_a is the active thrust exerted by the wall on to the block. So from this force triangle now we can write P_a is equal to W into sin theta minus phi divided by sin 180 minus psi minus theta plus phi.

Now substituting for W in this, we can get an expression P_a is equal to half gamma H square divided by sin square alpha into sin theta minus phi into sin theta plus alpha into sin alpha plus beta divided by sin 180 minus psi minus theta plus phi into sin theta minus beta. So this we can now try to differentiate with respect to theta so theta is an inclination of the failure surface with the horizontal which ever the surface which gives the maximum active thrust on the wall is computed here as active thrust.

So let us consider the same case, a failure wedge ABC and let us assume that this is the theta. So we need to find out the surface which actually gives the maximum active thrust. So initially we do not know these particular surfaces so we need to take the innumerable surfaces and try to get the… Otherwise we can do like for maximum P_a we can differentiate this expression which derived in the previous slide with respect to theta and then try to get angle theta and from there we can calculate what is P_a.

So here the same force triangle is shown this is W, this is the resistance R so in the resistance R has got 2 components, one is that acts along the surface t and other one is normal to the surface that is n that is n acts here like this, that is through which that angle phi is making and t acts along this surface. So R is the resultant resistance offered by the soil along the failure surface and P_a which is along this which is also has got components like this and horizontal and vertical components.

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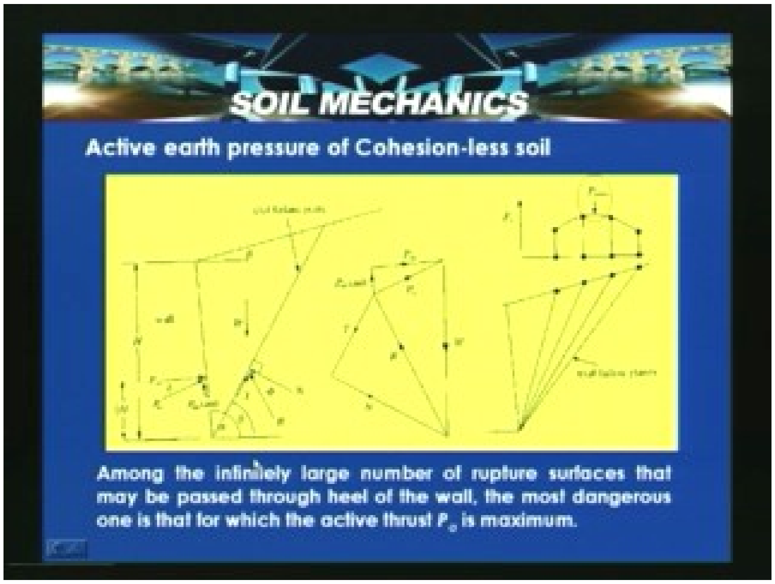

Now when you have got number of surfaces, the one the trial surface or a surface which gives that maximum active thrust and that is taken as a surface or a potential failure surface which can actually give the maximum active thrust. So among the infinitely large number of rupture surfaces that may be passed through the heal of the wall, so heal of the wall is somewhere here and the most dangerous one is that for which the active thrust P_a is maximum. So by using this condition by differentiating with respect to theta we can calculate the maximum active thrust. So by using this p_a versus this number of surfaces we can calculate and we will be able to calculate the p_{max}. This envelope is shown here the one which surface which gives that maximum is somewhere lies at certain angle theta with the horizontal.

So for maximum p_a, dow p_a by dow theta is equal to zero, with that we will be able to get P_a is equal to half gamma H square into sin square alpha plus phi divided by sin square alpha into sin alpha minus delta into one plus root over sin phi plus delta into sin phi minus beta divided by sin alpha minus delta into sin alpha plus beta whole bracket square. So this particular P_a is equal to half gamma H square into k_{ac}. So this k_{ac} is written this particular portion which actually indicated as a coulomb's active earth pressure cohesion where here alpha which is the inclination of the wall face with the horizontal and beta which is nothing but inclination of the slope surface or a backfill surface with the horizontal then we have wall friction angle which can be positive wall friction or negative wall friction.

So here active earth pressure of the cohesion less soil means we need to calculate P_a is equal to half gamma H square k_{ac}. Suppose if you know the delta is equal to say 20 degrees and beta is equal to some 20 degrees and phi is equal to 40 degrees based on that substituting in this first we need to compute the coulomb's active earth pressure cohesion and by substituting for a given height of wall, we can calculate what is the active thrust exerted by the wall on to the soil or soil on to the wall.

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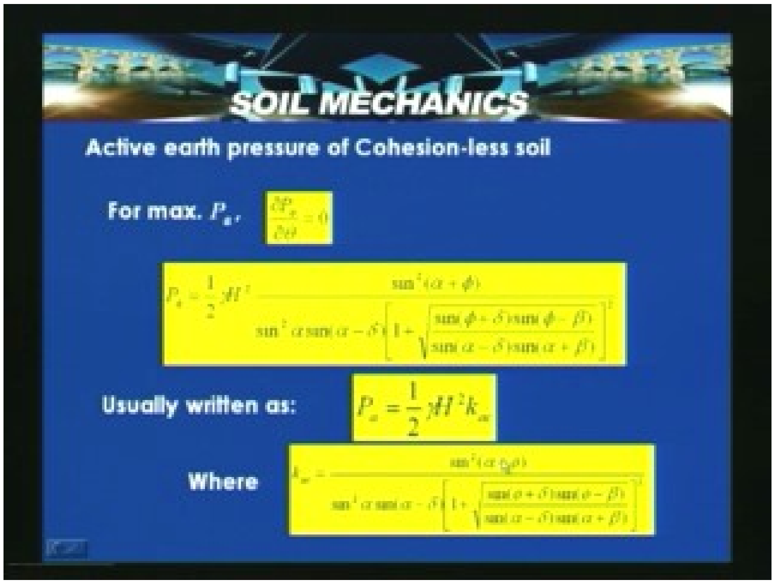

Similarly after having done for active case let us try to look for the passive case. So passive earth pressure of cohesion less soil so basically this theory is developed for a cohesion less soils. So let us assume that consider a wall height H and this is a positive wall friction angle and which is 90 degrees to this wall face and this actually adds a delta positive delta above this line.

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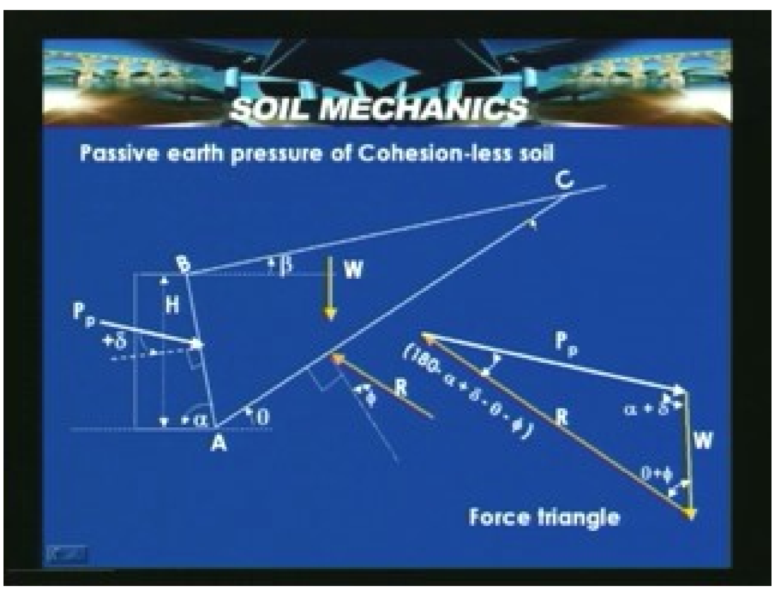

Now beta is the backfill surface inclination and theta is AC is the inclination of the potential surface where here in this case passive case where the one which gives the minimum value that need to be considered.

So here this is the force triangle, W is the weigh of this triangular wedge ABC then P_p is the passive thrust parallel to this and R is the resistance offered by the soil along the failure surface AC that is R and this inclinations are 180 minus alpha plus delta minus theta minus phi and P_p and W the inclination is alpha plus delta between R and W the angle is theta plus phi. So now to continue in the similar way, we can calculate by adopting the same procedure we can calculate for the minimum P_p then we can calculate P_p that passive earth pressure thrust which can be calculated as P_p is equal to half gamma H square into this particular entire expression will come and this is referred as k_{pc} that is coulomb's quotient of passive earth pressure.

So P_p is equal to passive earth pressure thrust is equal to half k_{pc} gamma H square where k_{pc} is indicated here. In the similar way we can calculate active and passive earth pressure cohesions for the coulomb's case. Cases can be sometimes delta is equal to zero and sometimes you have got beta is equal to zero then you can deduce Rankine's cases again and these when you put delta is equal to zero that is wall friction angle and beta is equal to zero and alpha is equal to zero then it changes into k_{ac} is equal to k_a is equal to one minus sin phi by one plus sin phi and k_{pc} is equal to k_p is equal to one plus sin phi by one minus sin phi, it changes into this Rankine solution.

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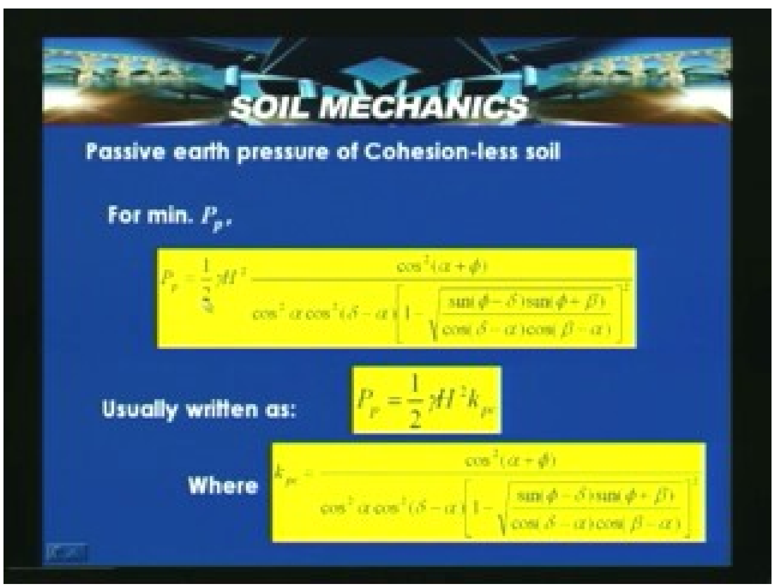

So this coulomb's solution can be converted into Rankine solution by adopting the suitable boundary conditions. So active and passive pressure rough wall that is with this particular interface for alpha is equal to beta is equal to zero, these are the conditions active case and for passive case. The similar case the difference is that as I said for alpha is equal to beta is equal to delta is equal to zero the solution converts to Rankine condition and fulfills that ideal Rankine's case. So in this case this particular portion is completely plastified where the plastic equilibrium prevails the rest of the portion is under elastic equilibrium conditions.

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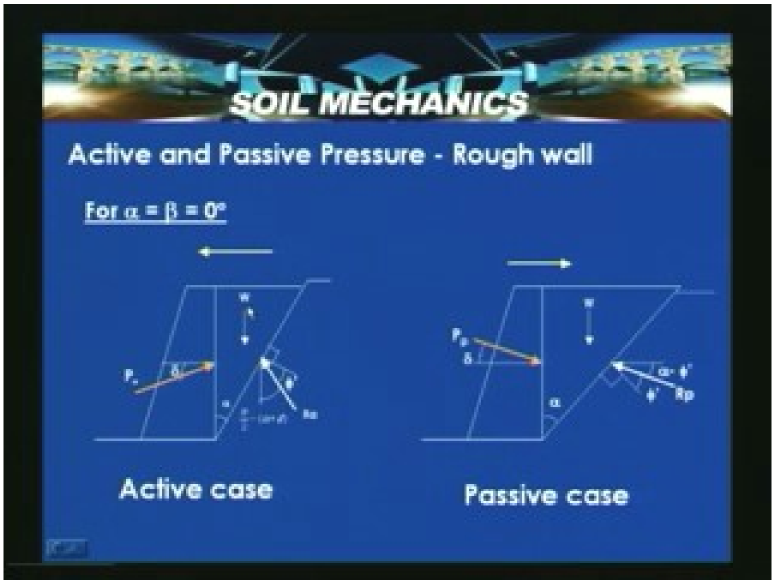

So P_a that is here its horizontal which can be found out again by substituting this they change into this corresponding Rankine's cohesions. For a special case where alpha is equal to beta is equal to zero but with delta wall interface then these are the coulomb's earth pressure cohesions which are given here, phi dash is indicated as a drain interface friction angle, drain friction angle. Then for the effect of all friction on lateral earth pressure you can see for this is a wall friction angle in case of $delta_a$ with a positive and negative wall friction angle and here in this case with k_{pc} that is for when $delta_p$ is equal to zero these are nothing but the Rankine's cohesions that is coulomb's earth pressure cohesions are equal to Rankine's cohesions that is k_a.

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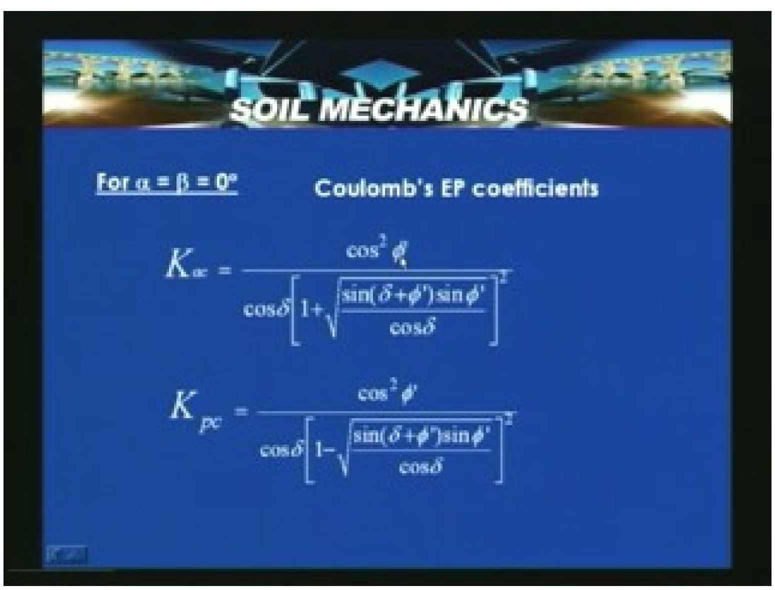

$$K_{ac} = \frac{\cos^2\phi'}{\cos\delta\left[1+\sqrt{\frac{\sin(\delta+\phi')\sin\phi'}{\cos\delta}}\right]^2}$$

$$K_{pc} = \frac{\cos^2\phi'}{\cos\delta\left[1-\sqrt{\frac{\sin(\delta+\phi')\sin\phi'}{\cos\delta}}\right]^2}$$

For example here for phi is equal to 30, $delta_a$ is equal to zero as you can see that for phi is equal to 30 $delta_a$ is equal to zero it is nothing but one minus sin 30 by one plus sin 30 and which gives 0.333. Similarly for phi is equal to 30, for k_{pc} change to k_p, delta p is equal to zero it is nothing but 3. So you can see that as the wall interface friction angle is varying then there is the variation in earth pressure cohesions which are shown in this table form for reference.

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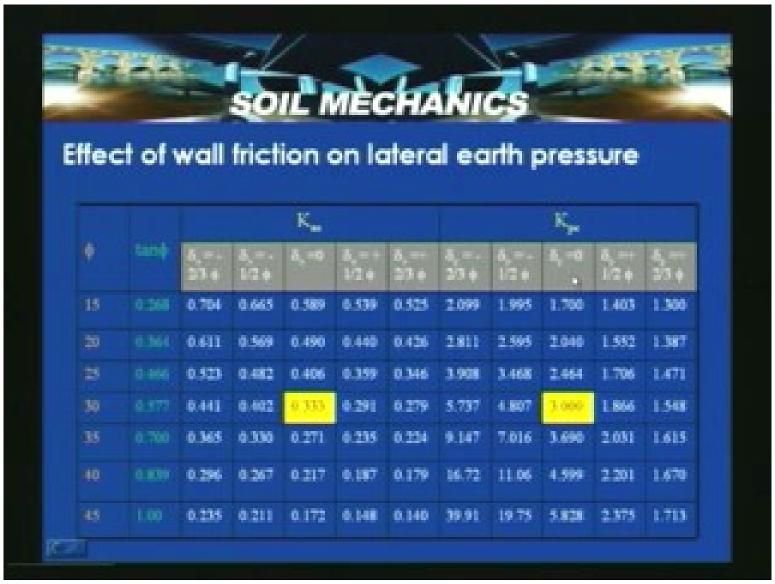

ϕ	$\tan\phi$	K_{ac} $\delta_a = -2/3\,\phi$	$\delta_a = -1/2\,\phi$	$\delta_a = 0$	$\delta_a = +1/2\,\phi$	$\delta_a = +2/3\,\phi$	K_{pc} $\delta_p = -2/3\,\phi$	$\delta_p = -1/2\,\phi$	$\delta_p = 0$	$\delta_p = +1/2\,\phi$	$\delta_p = +2/3\,\phi$
15	0.268	0.704	0.665	0.589	0.539	0.525	2.099	1.995	1.700	1.403	1.300
20	0.364	0.611	0.569	0.490	0.440	0.426	2.811	2.595	2.040	1.552	1.387
25	0.466	0.523	0.482	0.406	0.359	0.346	3.908	3.468	2.464	1.706	1.471
30	0.577	0.441	0.402	0.333	0.291	0.279	5.737	4.807	3.000	1.866	1.548
35	0.700	0.365	0.330	0.271	0.235	0.224	9.147	7.016	3.690	2.031	1.615
40	0.839	0.296	0.267	0.217	0.187	0.179	16.72	11.06	4.599	2.201	1.670
45	1.00	0.235	0.211	0.172	0.148	0.140	39.91	19.75	5.828	2.375	1.713

Now let us see having look in to this coulomb's earth pressure but there are also graphical methods for arriving at this earth pressure; many graphical methods are there but the famous one is culmann's method. The culmann's method involves what you need to do is that you have to represent the wall dimension to a scale on a graph paper on a millimeter graph paper that means the selected scale has to be made and the wall has to be with proper inclination and this wall has to be drawn on to the graph paper with proper scale.

Then one has to draw slopes line that is inclination with horizontal phi and along that line we also need to draw the slope surface or a backfill surface. So backfill surface is inclined with horizontal or sometimes beta is equal to zero so with that we will be able to, so by drawing a number of trial failure lines passing through the heal of the wall draw the number of failure lines. So in case if you have got say uneven surfaces or if there is a berm (Refer Slide Time: 44:05) which is need to be considered in the analysis for calculating active thrust this is possible this graphic method which is very robust in adopting and calculating earth pressures.

So let us look in to this procedure and will try to look into the problems how the external loads can be calculated and how we can do in case of active and how we can do this in case of passive case. So this involves as we look into this, this is for the case of a positive wall friction that is the wall resting on a firm foundation, AB is the wall surface which is considered as vertical in this case.

Now this is that surface that is ground surface is drawn and this is the horizontal and this is the heal of the wall, now draw weight line or a slope line at an inclination phi dash with the horizontal. So first draw the weight line or a slope line with an inclination phi dash to the horizontal. As it represents the natural slope of the backfill so this is drawn like this.

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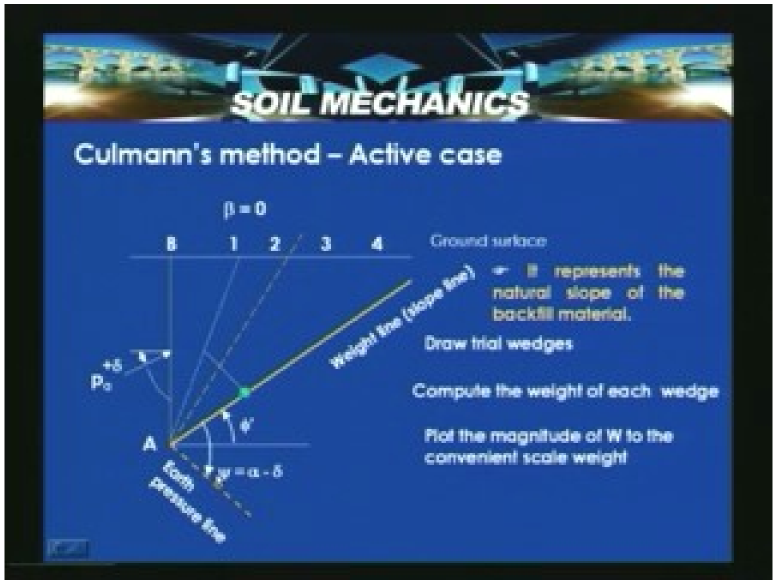

Now draw trial wedges like this which is the first trial wedge, we do not know whether this qualifies for the potential failure wedge or not. So we need to draw a number of trial wedge surfaces so this A1 is the first wedge surface, AB1 is the first wedge. Similarly once you have drawn AB1 as the wedge, determine the weight of this wedge that is per meter width per meter length AB1 gamma is known to you. You calculate what is the weight and that weight is plotted on a suitable scale on the weight line or a slope line.

So AB1 weight is indicated as say some point A, A dash. Now at an inclination of psi is equal to alpha minus delta which is this angle is nothing but angle between W and P_a active thrust and W. We have seen this in the previous slide, this can be seen in this slide where in this culmann's graphical method this is that force triangle P_a R resistance so psi is equal to alpha minus delta, the same problem we are trying to solve by using graphical method.

So similar thing we have done by using analytical method and we try to obtain coulomb's earth pressure cohesions in active case and passive case. So similar problem but in case this particular method is very easy to consider even uneven surfaces or to consist the berm or in case if you are having a railway track and there is a line load acting at one point and other point and effect of these line loads and then the net active thrust can be calculated very easily.

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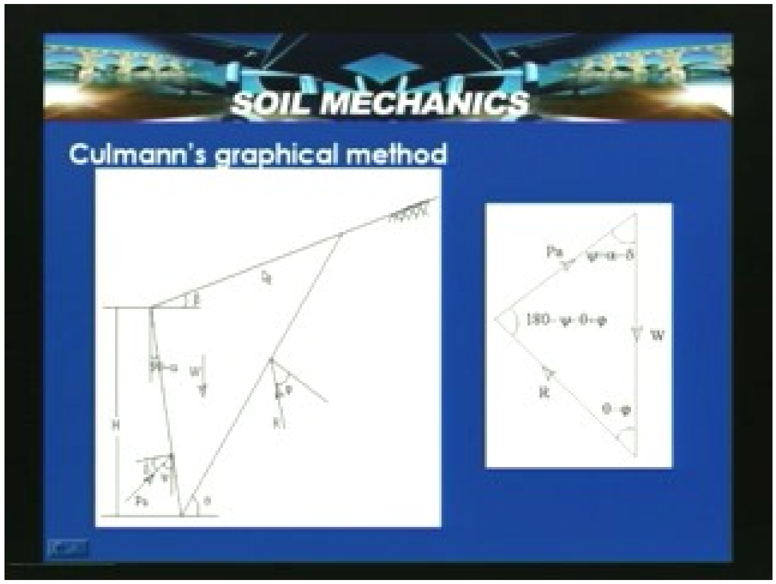

So that we look into the procedure, now once again draw this earth pressure line. This earth pressure line has to be drawn at psi which is equal to alpha minus delta inclination from the slope line; please note the earth pressure line has to have inclination of psi equal to which is actually has to be from alpha minus delta. Now having drawn trail wedges, number of trail wedges. Now AB2 is the next trail wedge, AB3 is third trail wedge, AB4 is fourth trail wedge like that you can have trail wedge. If at all you have got trail wedge or in a surface which actually changes elevation and all we need to take the wedge passing through that particular point.

So after having obtained what we need to do is that this particular line is nothing but weight of AB1 represented to a scale on this line and these two distances between these two points is nothing but the weight of the wedge AB2 and distance between these two points nothing but the weight of the wedge AB 3. Similarly this point is nothing but the weight of the wedge AB4. Now from these points draw a line parallel to the earth pressure line that is this line so a draw line, see where it cuts that is a point where it cuts. Then this point where it cuts that is line A2 and this is the point where it cuts A3 similarly this is the point where it cuts A4.

Now what will you get is that joining these we get the culmann's envelope this is nothing but the earth pressure locus. Now we can draw a line parallel to this slope line and wherever it is tangential and that is a point where it gives that maximum P_a that is the maximum P_a and the line passing to that which was shown before itself which generally will not know, we will be knowing only in the end and this is that possible failure wedge potential failure wedge which actually gives that maximum P_a.

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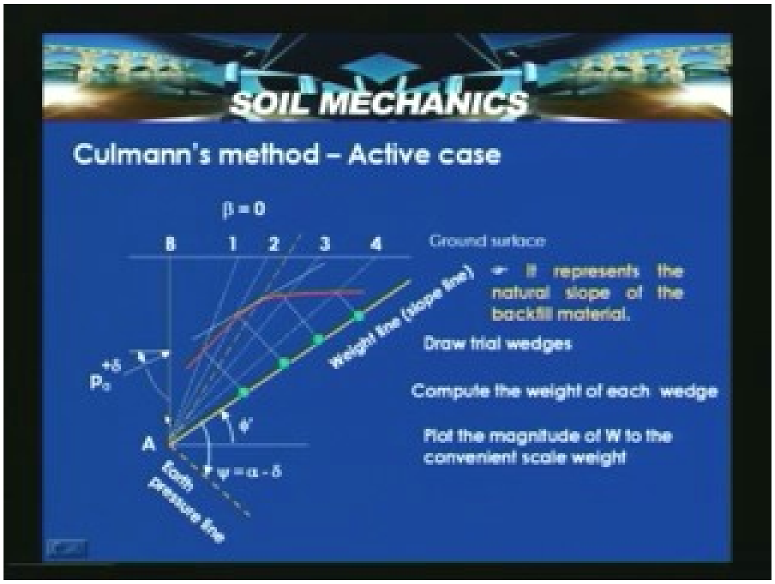

So here that P_a which is actually shown here and this ordinate now you have to measure and then to the weight scale it has to be given as that active earth pressure, active thrust P_a. The similar problem now if you have got say q kilonewton per meter say railway track load which is actually at this point say AB is the wall surface. Now if we assume that this is that surface which is having is un even surface the backfill surface is not having any inclination so very difficult to consider in an analytical method but in a graphical method its very easy to consider. Suppose AC1 is horizontal C_1, C_2 has got certain slopes C_2, C_3 has got certain slopes so it has been considered like this but only difference is that here there is a line load or a boundary wall is there which exerts a load.

So now this is the similar procedure draw the slope line with that phi and draw the pressure line. So this is for extending external line load on the backfill surface. So through the heal of the wall draw innumerable number of surfaces but make sure that your wedge line passes through the point C_1 and C_2 is the location for example if there is a load at C_3 suppose q another load q_2 is there and that also has to be considered. So like this or if there is another load acting at this point and that has to be considered. So similar procedure by plotting the weights of this wedges on the slope line and drawing a line parallel to the surface will be able to get this culmann's envelope and this hump which is actually cause because up to here, you need not consider the weight of this extra load additional load caused due to external load but moment you reach this slide that is this wedge you need to consider the weight of that particular load which is coming at that particular point. So in this case because of these, the envelope gets projected and then it goes. So these envelope which gives is the envelope with line load and this is the same envelope without any line load. So again drawing a line parallel to this and parallel to this we will be able to get the P_a which is nothing but active earth pressure thrust which can be obtained graphically here by measuring E_{max} is equal to E_a is equal to P_a converting in to the scale we will be able to get what is that P_a with line load and without line load.

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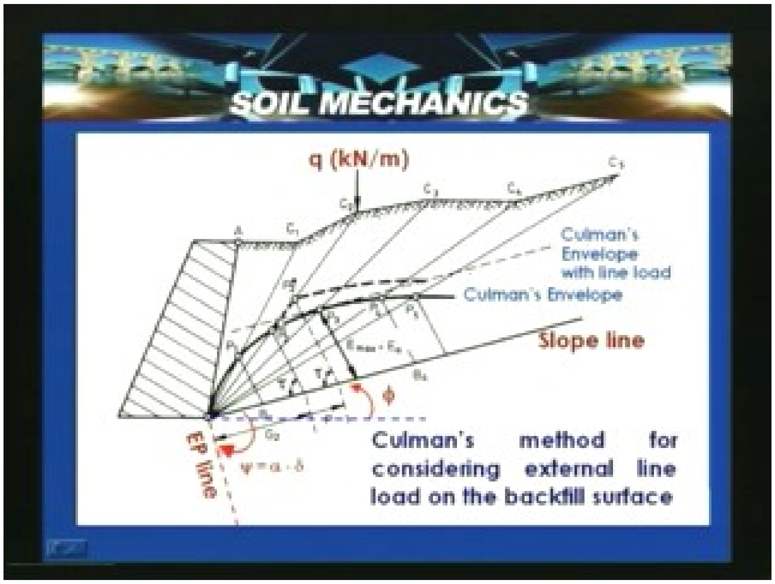

So like this we will be able do this by representing this into the scale and then trying to solve this problems for either active case or passive case this can be adopted for calculating the earth pressure and here what we need to do is that we need to draw the geometry of the wall, like whatever that wall which is there and this geometry has to be represented and this surface, ground surface has to be drawn and first the slope line has to be drawn with angle phi which is above the horizontal and earth pressure line has to be drawn which is psi is equal to alpha minus delta.

Then in this case an average angle alpha can be considered and then we can plot this weights of this wedges on the slope line and then draw lines parallel to the earth pressure line which is not exactly parallel in this figure (Refer Slide Time: 53:05) but when you draw you will be able to see that it cuts this potential, the trial wedge surface lines at point P_1, P_2, P_3, P_4 and P_5 and P at point where this load is there and that point there that hump will be caused that is because of the accounting for that load.

So subsequently when you are considering, you need to consider that load which is coming that but for a wedge or for example here we need not consider the effect of this particular load only at this point when you consider and then subsequently when you do the envelope changes. So by drawing a line parallel to this surface, we will get the culmann’s envelope here and then we can calculate that active earth pressure thrust resultant thrust acing at a point H by 3 at certain location which is known location can be find out.

So in this lecture what we try to see is that we looked into the coulomb’s theory and assumptions we discuss and then we also discussed that the limitations which are with coulomb’s theory especially from the failure surface. The failure surface assumed to be plane but in case in reality it is because of the wall friction it is having estimated as log spiral theory which actually deviates for the passive case.

In case of active case the deviation is minimal and then even in case of passive case for the delta which is wall friction angle is more than one third of the internal friction of the soil then we said that this theory gives un expectable error which is very high. So in the next class we will be discussing about application of the Rankine’s method or in a case were we can solve a typical problem with culmann’s method and we try to look into this extend this to anchored bulkheads and then try to see the stability of this walls in different conditions with and without water conditions.