Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 52
Earth Pressure Theories – III

Welcome to lecture three under earth pressure theories. In the previous lecture we understood active and passive earth pressures and discussed about Rankine's earth pressure theory and then we also discussed about different assumptions which are involved. In this lecture we will try to extend what we learnt with a special cases for computing earth pressures and with some example problems and extension of Rankine's theory for a special cases like wall with a inclined face and backfill soil which is not horizontal which is having a certain inclination with a horizontal.

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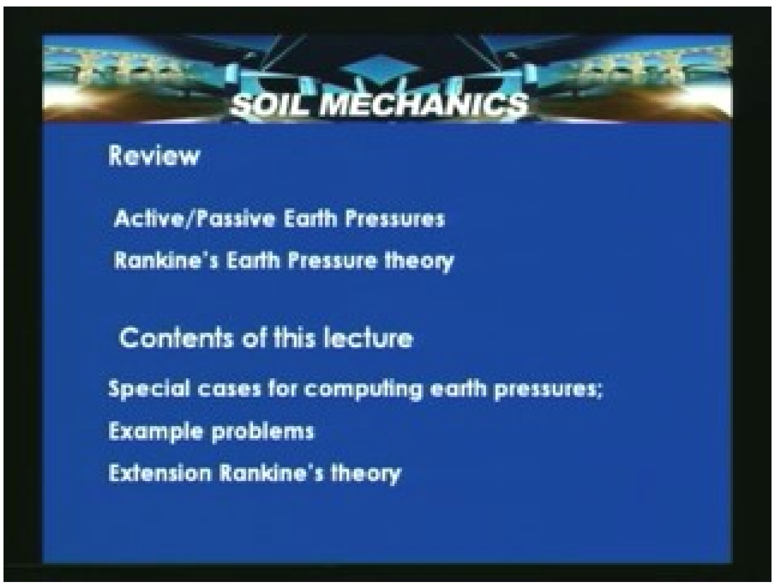

So in the previous lecture we saw the development of these slip lines within the zone of plastic equilibrium through a demonstration experiment which is carried out at IIT Bombay which is with a retaining wall which is nothing but a cantilever wall having sand as a backfill. So you can see the sand in dry state having a relative density about 55% which is constructed by polivating that is air polivation of the sand, like the way the deposit happens in the field and this is the passive soil because the wall movements are not adequate so the passive state of plastic equilibrium was not observed here but you can see here the active state with a movement or rotation about at the top portion, we can see that different series of ruptures planes which actually shows the evidence of what actually we are discussing in these lectures.

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Now we try to derive a universal equation for determining this earth pressures at different depths and this can be extended to say for a homogeneous soils having cohesion less soils or cohesive soils or soils with combination of c and phi in different states are the drained or un drained conditions. So in this slide we are going to discuss about the relationship between $sigma_1$ and $sigma_3$. $Sigma_1$ is major principle stress; $sigma_3$ is minor principle stress and where in soil reaches a state of plastic equilibrium can be derived from the more circle below. So a tow sigma plot is shown here and phi is the friction angle, $sigma_3$ is the minor principle stress. $Sigma_1$ is major principle stress and if this intercept is indicated by say cohesion and phi is the friction angle and we can write from the geometry of the circle, we can write that sin phi is equal to this particular diagram, we can write sin phi is equal to half $sigma_1$ minus $sigma_3$ divided by half $sigma_1$ plus $sigma_3$ plus 2c cot phi.

This deliberation you would have already discussed by learning about shear strength. So by re arranging this term shear, this we can write as $sigma_3$ into 1 plus sin phi is equal to $sigma_1$ into 1- sin phi minus 2c cos phi. So this further simplification in rearranging the terms we can write $sigma_3$ is equal to $sigma_1$ into 1 minus sin phi by 1 plus sin phi minus 2c cos phi by 1 plus sin phi. So this now changes to 1 minus sin phi by 1 plus sin phi this term we can recognize now already which is nothing but k_a.

So $sigma_3$ is equal to $sigma_1$ into 1 minus sin phi by 1 plus sin phi minus 2c cos phi by 1 plus sin phi, this we can write $sigma_3$ is equal to $sigma_1$ into 1 minus sin phi by 1 plus sin phi into minus 2c into 1 minus sin phi by 1 plus sin phi. We have written this by using the trigonometry principle. So now substituting for say in active state of plastic equilibrium say this equation is to be valid then we can say that $sigma_3$ being the minor principle stress in active state of plastic equilibrium, the active earth pressure be minor $sigma_3$ becomes $sigma_a$, $sigma_1$ becomes $sigma_v$ which is actually nothing but a major in case of a active state of plastic equilibrium.

So with these what we get is that sigma a is equal to gamma z k_a minus two c root k_a. So this is an equation for active case with sigma_3 is equal to sigma_a and sigma_1 is equal to sigma_v for an active state of plastic equilibrium condition. So this equation now helps us say for example a sandy soil like the physical modeling experiment were we have seen in a dry state when the cohesion is equal to zero then in that case, the earth pressure equation is nothing but sigma_a equal is equal to k_a gamma z. At z is equal to zero, earth pressure zero at z is equal to H, the earth pressure is k_a gamma H then the total active earth pressure thrust is half k_a gamma H square acting at a distance H by 3 from the base.

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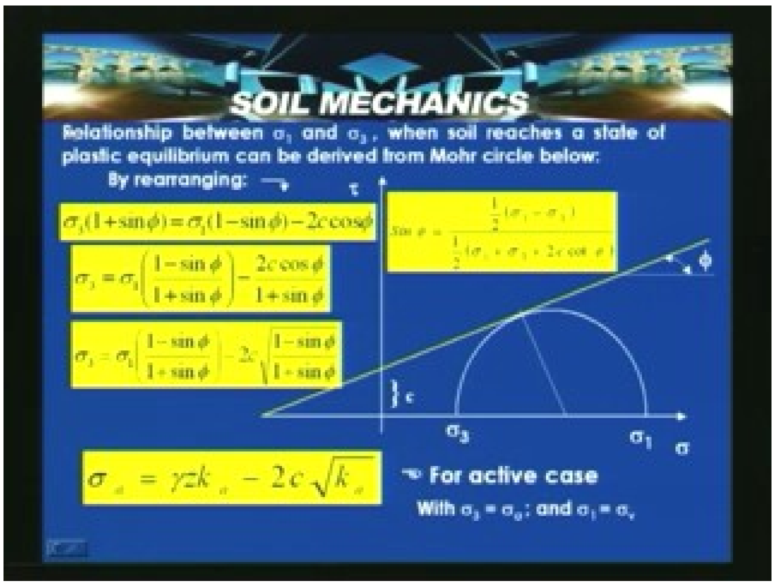

Now in this case suppose if you are having a soil then if you substitute at z is equal to soil which is predominantly say cohesive soil then say at phi is equal to zero, suppose if you are having a soil which is pre dominantly cohesive soil then you can see that sigma_a is equal to minus 2c that is there is a possibility that the soil actually is under tension behind a retaining wall. So when the soil attains its ultimate tensoil stress, it cracks and it actually develops tensile cracks. That is the reason why like some of the retaining walls having say cohesive backfills, the top portions are subjected to tension cracks.

So this particular equation now if you wanted to generalize for cohesive soils or soils having both cohesion and friction angle. Then we can write sigma_z is equal to gamma z k_a minus 2c root k_a and by using the same concepts like, with that when you say for passive state also when we wanted to derive the relationship between sigma_p and sigma_v and sigma cohesion and k_p with that we can arrive the condition for equation for passive case also. In this case the only difference is that the sigma_3 becomes the major principle stress and sigma_1 becomes the minor principle stress which is nothing but sigma_v in passive state of plastic equilibrium.

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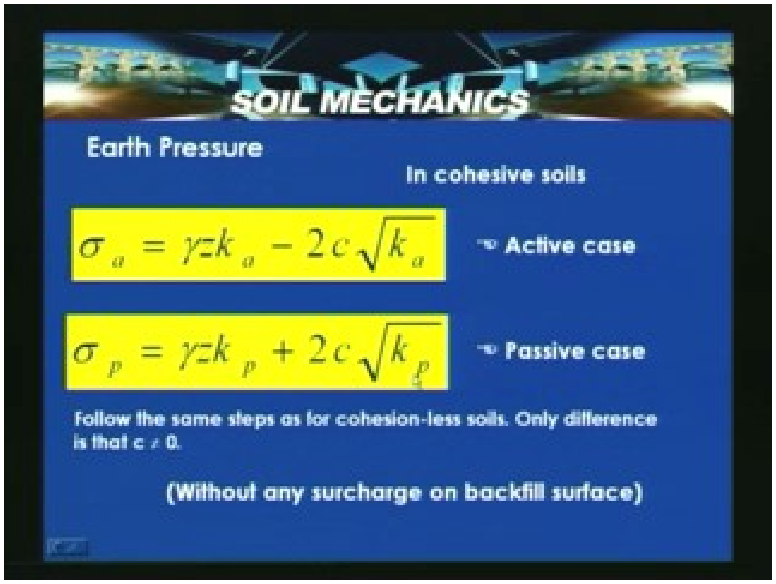

So with this now the generalized equations without any surcharge on the backfill surface if you consider then these are the generalized equations for computing the earth pressure. Then sigma$_a$ is equal to gamma z ka minus 2c root k$_a$ this is the active state and sigma$_p$ is equal to gamma z k$_p$ plus 2c root k$_p$ that is for the passive state. So follow the same steps as cohesion less soils, only difference is that c dash is not equal to zero.

So if you look into it sigma$_a$ is equal to gamma z k$_a$ minus 2c root k$_a$ and sigma$_p$ is equal to gamma z k$_p$ plus 2c root k$_p$. So this is for the passive case and this is for the active case. Now Rankine's lateral pressure diagram for a cohesive soil. If you look in to it here, what we saw is active case with minus 2c root k$_a$ is the earth pressure at top because k$_a$ gamma z minus 2c root k$_a$ z is equal to zero the pressure is minus 2c root k$_a$ here.

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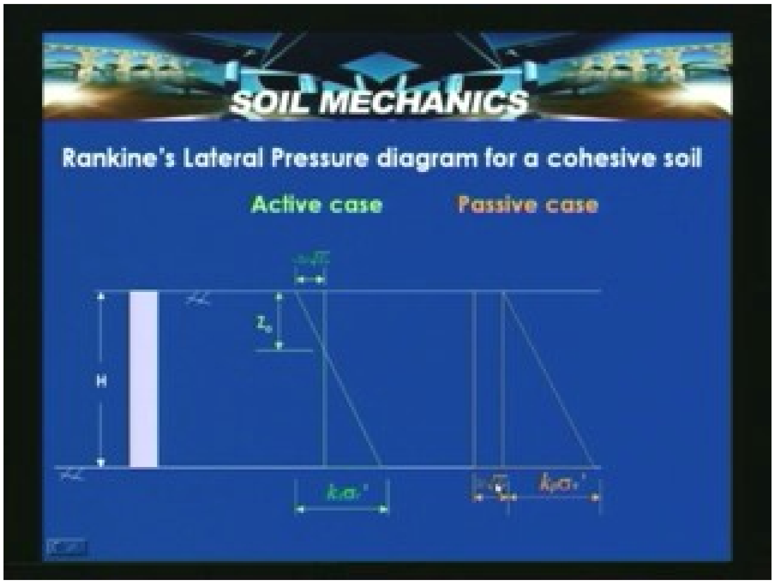

In case of passive case we have 2c root k_p plus k_p sigma$_v$. So this is that equation what we discussed just now and this total length distance is k_a sigma$_v$ dash. So if you analyze further, now we have a lateral pressure diagram for a cohesive soil which we just now have seen. Once again we look into it, if you look into it here at certain depth z_0 the negative pressure changes to positive pressure. So it indicates that for soil there is a certain depth it actually remains un supported, particularly cohesive soils. So that particular depth is called the critical height so beyond which the soil cannot remain un supported particularly cohesive soils.

That is because there will be a negative pressure and positive pressure which actually gets cancelled and then the soil will be in equilibrium and then we can say that it has got safe unsupported height. So this is generally about, for a certain type of soils having cohesive strength or some apparent cohesion up to 4 to 5 meters. So this is with the passive case, the equation which is now shown in a diagrammatic way (Refer Slide Time: 10:28). Now just now we discussed about the cohesive soil and critical height. So if you look into it, this is the equation what we derived sigma$_a$ is equal to gamma z k_a minus 2c root k_a. So z is the depth at z equal to zero here, top of the wall and where the backfill surface is horizontal, H is the height of the wall and wall is smooth that is the Rankine's wall.

Now at z is equal to zero, sigma$_a$ is equal to minus 2c root k_a so this being zero it gets cancelled, sigma$_a$ is equal to minus 2c root k_a and z is equal to H, it is k_a gamma H minus 2c root k_a. So which is nothing but earth pressure at the base. At z is equal to z_0 that is sigma$_a$ is equal to zero that is this point here then that z_0 is nothing but the depth that is 2c gamma root over k_a.

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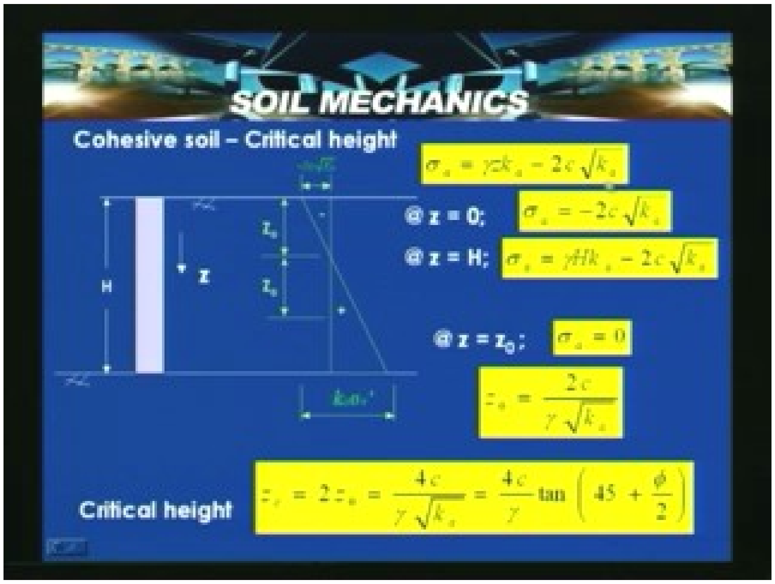

So this particular depth is also referred as depth of the tension crack, so moment the soil cracks then after attaining its ultimate tensile strength. Even with this expression we can even determine the possible depth of a tension crack in a particular site. So z_0 is equal to 2c by gamma root k_a with a negative pressure which is shown here and positive pressure showing, so $2z_0$ which is nothing but the safe that is the critical height of a slope that can actually remain un supported because of the negative lateral pressure and positive lateral pressure. So with that we can say that z is z_c is equal to 2 z_0 is equal to 4c by gamma root k_a which is nothing but 4c by gamma tan into 45 plus phi by 2. Suppose if the soil is say having un drained cohesion, say friction angle is equal to zero. In such situations the critical height of a particular vertical cut can be determined as H_c is equal to z_c is equal to 4cu by gamma where cu is equal to un drained cohesion and gamma is the unit weight of the soil. So in case, soil is having both c and phi then the critical height can be determined by using expression 4c by gamma tan 45 plus phi by 2.

So in this particular slide what we saw, the possible earth pressure distribution what we get when we have a cohesive soil, purely cohesive soil with cohesion pre dominantly and then certain friction angle then we can say that by using this equation we try to derive and we try to derive the depth of the tension crack and that we said as a 2c by gamma root k_a and critical height that is the height to which actually remains unsupported that is 4c by gamma root k_a which is actually given by expression 4c by gamma into tan 45 plus phi by 2.

Now Rankine's lateral pressure diagram with uniform surcharge, if you look into it here suppose if you are having a soil say having unit weight gamma and no water table and then if surcharge is q. So this is possible when you are having a pavement behind a retaining a wall or any other utility which is behind a retaining wall then that exerts a unit surcharge on the wall. How this surcharge can be created surface stress in the… (Refer Slide Time: 14:05). This surface stress can be transferred as an earth pressure to the soil that actually we are now trying to discuss in this slide through a form of a special case where H is the height of the wall which is again a Rankine wall. At top of the wall the earth pressure is zero, at bottom it is k_a gamma H.

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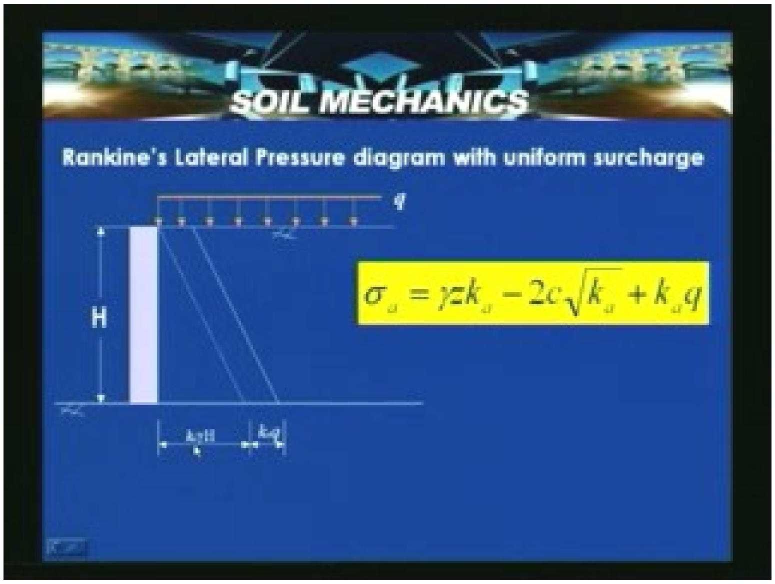

So now when we have surcharge at the top then sigma$_a$ is equal to gamma z k_a minus 2c root k_a and k_aq has to be utilized, so where k_a is the quotient of earth pressure in the active case which is actually has to be multiplied by q. So with that what we add is that, see if you can see a parallel line to this surface. So this is nothing but entire q which is actually transferred with k_a. So that is actually given here as k_aq. So at z is equal to zero the earth pressure diagram in case with surcharge then this is with surcharge, it becomes k_aq and then at the top of the wall and at the bottom that is at depth z is equal to H, it is k_a gamma H plus k_aq. Say sometimes we also have completely submerged soil behind the retaining wall or partially submerged soil. So in such situation how the earth pressure will change?

For example if you have got a wall which is having a sandy soil or a granular soil and say water table is there on both the sides and the liver is same. Then the lateral pressure due to water will get cancelled but if there is say due to some tidal fluctuations, if there is water which actually can cause a differential water pressure which is actually required to be considered in waterfront structures, basically in the bending structures. So Rankine's lateral pressure diagram with partially submerged soil can be discussed. Consider again a cross section of a retaining wall having height H and in this case what we see is a water table location at a depth z_w from the ground surface. Here the ground surface which is horizontal and H dash is equal to H minus H_w, H is the total height of the wall. By using the equation that is sigma$_a$ is equal to, all these examples what we are discussing is for active state the similar concepts can be used for passive case also.

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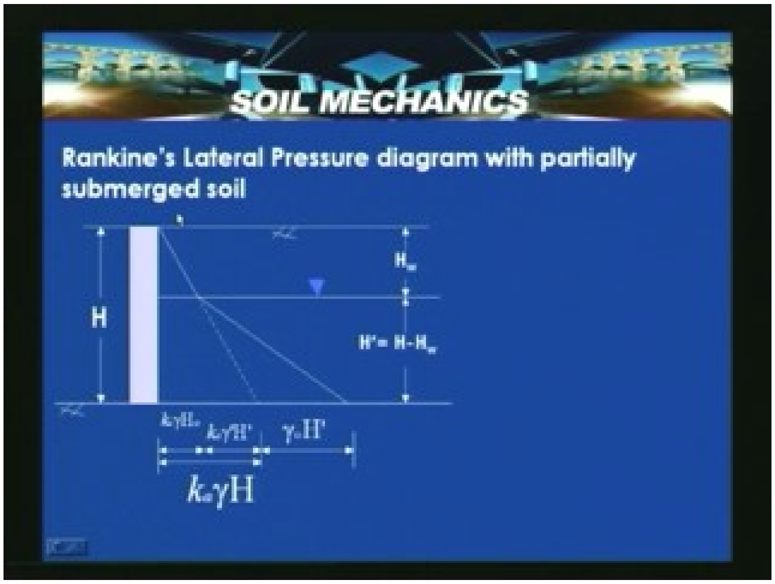

But the Rankine's earth pressure diagram for partially submerged soil for active state. So here will be using a quotient of active earth pressure. So here at z is equal to zero, earth pressure is zero and at z is equal to H_w at this point then you can see that the earth pressure ordinate here is k_a gamma H_w. So k_a gamma H_w which actually gets transferred here. Now below this the soil is completely saturated and having say gamma is equal to gamma dash, here gamma which is equal to gamma sat if you can say then k_a gamma dash H is the lateral earth pressure here and $gamma_w$ w H dash is nothing but a water pressure which is actually exerted by a water of height H minus H_w which is nothing but H dash.

So here the earth pressure is now k_a gamma H plus $gamma_w$ H dash. Suppose this water table is actually up to the top surface then what will happen is that then the water pressure which is the earth pressure diagram which looks like this thing k_a gamma dash H which is actually at the till bottom and then having a water pressure which is $gamma_w$ H.

So presence of the water behind a retaining wall actually causes lateral pressure that is the reason why for designing or constructing retaining walls, we generally use the drainable fills. That is in case if the soil is having inadequate say drainage facility, the drainage facilities have to be provided so that the building up of the water pressure behind the wall can be prevented because designing a wall for the additional water pressure is for a conventional walls is difficult but in such situations where there is a possibility of tidal fluctuations and then differential water pressure is required to be considered.

This situation can be prevented by providing adequate drainage provisions so that the water pressure remains zero. Otherwise if it is required, if the water pressure prevails then this how we have to consider in computing earth pressures. Now let us consider, extend the special case because we all know that the soil is not an homogeneous but we can extend this Rankine's theory for computing the stratified even for earth pressures for the stratified soil deposits. Suppose if you are constructing a retaining wall and assuming that Rankine's conditions prevail then in that case we can extend this theory for determining this earth pressure diagrams. So once we obtain this earth pressure diagram then that can be extended for designing a retaining wall at a particular site.

So here by using the same concept the equation sigma$_a$ is equal to gamma z k$_a$ minus 2c root k$_a$ plus k$_a$q where here the surcharge is not there. So if the surcharge is there for example highway embankment where one of the sides is being retained by a wall and if it is constructed with different soils and if it is subjected to a surcharge. Then that surcharge has to be considered but in this case q is being zero so this particular term vanishes.

So at z is equal to zero earth pressure is zero, so this is the starting point of the earth pressure diagram. In all case actually we are assuming that wall rotates about a tow and it actually yielded sufficiently to produce active state of plastic equilibrium condition. So at z equal to H$_1$ just above A-A so let us assume that in this stratified deposit example, we have got say c is equal to zero and phi$_1$ dash and gamma$_1$ and phi$_2$ dash gamma$_2$, phi$_3$ dash gamma$_3$. So three soils which are considered having thickness as H$_1$ and H$_2$ and H$_3$, H is equal to H$_1$ plus H$_2$ plus H$_3$ and A-A is the interface between these 2 soils that is phi$_1$ and phi$_2$ soils and B is interface between phi$_2$ and phi$_3$ soils and phi$_1$ is greater than phi$_2$ is greater than phi$_3$ and in this case if z is equal to H$_1$ above A-A. So we can see above A-A that means this particular portion then actually it is governed by the soil properties which are in this particular region that is phi$_1$ dash and gamma$_1$.

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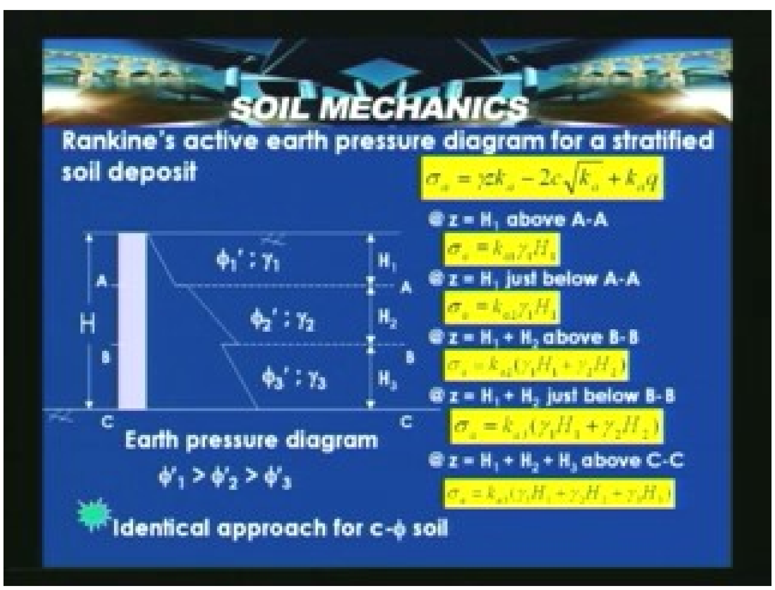

So here sigma$_a$ is equal to k$_{a1}$ gamma$_1$ H$_1$ so observe that coefficient of earth pressure which is actually k$_{a1}$ which is actually obtain from the friction angle phi$_1$. So k$_{a1}$ is equal to 1 minus sin phi$_1$ by 1 plus sin phi$_1$ so with that this gamma$_1$ is the unit weight of the soil in the H$_1$ region. Now the earth pressure ordinate just below A-A that means that we just got this particular point say now we wanted to get this point just below this thing.

Now just below A-A the soil is now same height but the soil is just having properties which is phi_2 soil and which yields k_{a2} is equal to 1 minus sin phi_2 by 1 plus sin phi_2.

So with that $sigma_a$ is equal to k_{a2} $gamma_1$ H_1, this $gamma_1$ H_1 is now for one simple methodology which one should understand here is that for computing earth pressures just consider the top soil as a surcharge which is lying. Suppose if I assume that my retaining wall starts from here, now in this case you can observe at depth say at this level A-A you can see how will you calculate. Suppose $gamma_1$ H_1 is the say surcharge which is acting on this particular level then we calculate that $sigma_a$ is equal to k_{a2} gamma H_1. So just consider that as a top soil you consider that as a surcharge and then transfer that as a lateral pressure.

So here at z equal to H_1 above A-A we said that ordinate is here and z equal to H_1 just below A-A which is k_{a2} $gamma_1$ H_1. At depth z_1 H_1 plus H_2 that is z equal to H_1 plus H_2 that is at level B and just above BB. So in this case $sigma_a$ is equal to k_a to $gamma_1$ H_1 plus $gamma_2$ H_2. Now at depth z is equal to H_1 plus H_2 just below BB that is this particular portion then now the third soil which is now phi_3 soil which is k_{a3} which is obtained by 1 minus sin phi_3 by 1 plus sin phi_3 which is actually $sigma_a$ is equal to k_{a3} into $gamma_1$ H_1 plus $gamma_2$ H_2. Now above cc if z is equal to H_1 plus H_2 plus H_3 is there then $sigma_a$ is equal to k_{a3} into $gamma_1$ H_1 plus $gamma_2$ H_2 plus $gamma_3$ H_3.

Suppose if the soil is also having cohesion then we can use the same concept of example where using this equation $sigma_a$ is equal to k_a gamma z minus 2c k_a plus k_aq which we can determine the earth pressures in a stratified soil deposit also. So is the identical approach for the c phi soil also can be adopted for with different stratified, c phi soil with a stratified soil deposits we can use for computing earth pressures. So here what we need to consider is that the here at just above AA and just below BB, you try to get this ordinates. So when you try to obtain the resultant pressure and that is actually gives the resultant earth pressure. that means the resultant of all these, this particular shape seesaw type this which is obtained saw type diagram and that is nothing but earth pressure diagram in case of a stratified soils.

This ordinate movement this side and that side depends upon the properties of the soil with the phi_1 is less than phi_2 is more. So based on that it changes so with that we can obtain earth pressure diagram and by obtaining the resultant we can actually calculate the active thrust. In case if you are determining for the passive case then we can determine the total passive thrust. So the variation of active and passive lateral earth pressures with depth which are shown here. So in this case we are having a soil, vertical wall which is actually soil having water up to the top surface and now as we discuss in this case, this is the earth pressure diagram.

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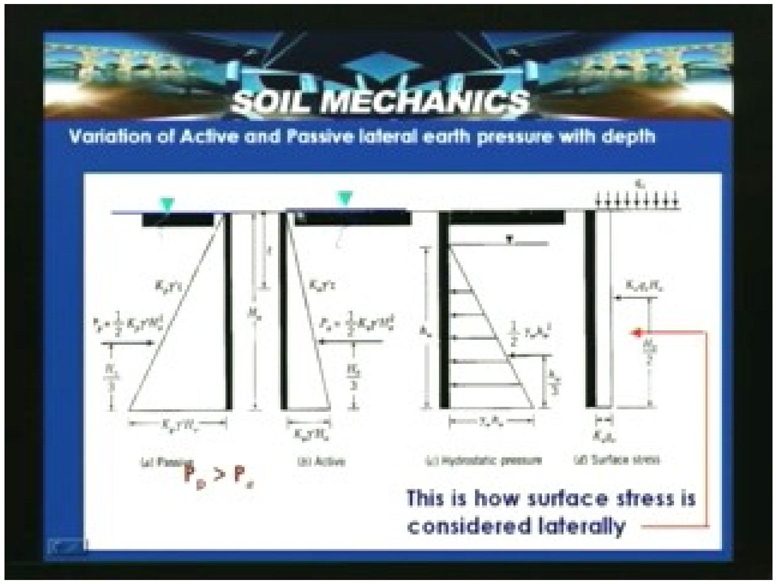

The equation of this particular line is k_a gamma dash z, at z is equal to zero the lateral earth pressure is zero, at z equal to H the lateral earth pressure is say z is equal to H_0 in this diagram H is indicated by H_0 say k gamma dash H_0. So p_a is nothing but half k_a gamma dash H_0 square that is the lateral earth pressure plus this has got a additional component which is nothing but if the water is there up to this level then that actually acts as a hydro static pressure which is half $gamma_w$ for this case if the water is up to this level it acts like a half $gamma_w$ H_0 square.

But if the water table is there on the both sides of this thing case then what will happen is that say the wall is submerged, say this part here and the left hand side of the wall there is a water and right hand side of the wall there is water. Then this case that hydro static pressure gets cancelled but the effect of water we need to consider by calculating the submerged weight of the soil and then lateral pressure due to the submerged soil mass. Similarly if you look into here, if that condition prevails here and the wall move towards the fill then this is the equation k_p gamma dash z and for the earth pressure diagram and k_p gamma dash H_0 is the ordinate at the base then p_p is equal to half k_p gamma dash H_0 square is the passive earth pressure thrust. So if you look into it k_p is greater than k_a and with that what will happen is that the earth pressure thrust in passive case is much more than the passive case.

Incase if you are having a partially submerged soil state then you have got this hydrostatic pressure or if the water table is there on one side only then actually it exerts hydro static pressure that is considered and if so wall is actually, the surface stress is there, this is how the surface stress is considered laterally. So if Q_s is the surface stress which is actually acting then $k_a Q_s$ into H_0 which is actually here gets transferred and $k_a Q_s$ H_0 is the resultant thrust due to surface stress acting at a certain point. If suppose if I have got a different complicated lateral stresses then it will be considered in the subsequent design.

So basically in this slide variation of the active and passive lateral pressures which are shown and by knowing this, we can actually obtain this equation of this particular line and then earth pressure diagrams. Once you obtain the earth pressure diagrams then you can calculate the active thrust and then passive thrust and then by considering the equilibrium conditions we can actually design the wall for equilibrium. Now let us consider some examples with whatever we have discussed it just now.

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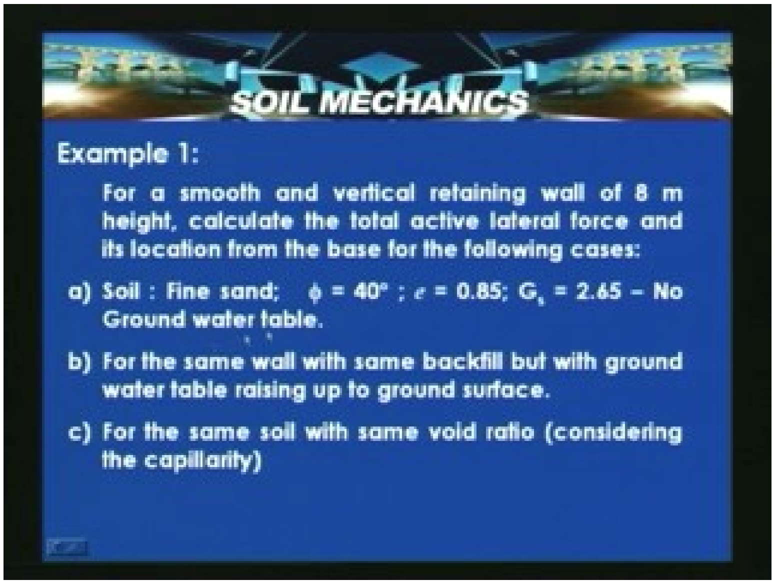

In the example one, let us consider for a smooth and vertical retaining wall of height 8 meters. We need to calculate the total active lateral force, so active lateral force that means the wall moves away from the soil and its location from the base in the following cases. The soil is basically fine sand having friction angle is going to 40 degrees, void ratio 0.85 and in specific gravity of the soil this is 2.65 and no ground water table anywhere and for the same soil with the same backfill but with a ground water table raising up to the ground surface and for the same soil with a same void ratio considering the capillarity. So we have the three different conditions then we will see that how the influence of, suppose if the ground water table raises behind the retaining wall what will happen to the active earth pressure. So that will actually tell us the influence of the raising ground water table on the lateral earth pressure magnitude.

So in this case the case a if you look into it 8 meter wall, it's a smooth wall Rankine's wall and the earth pressure diagram is now k_a gamma h so using sigma_a is equal to gamma z k_a minus 2c root k_a plus $k_a q$, $k_a q$ is equal to zero so q that is because q being zero and where c is equal to zero with that k_a is equal to 1 minus sin phi dash by 1 plus sin phi dash with that we will get coefficient of active earth pressure 0.2174 and for the unit weight of the soil which is obtained from the given parameters 14.3 kilo newton per mere cube.

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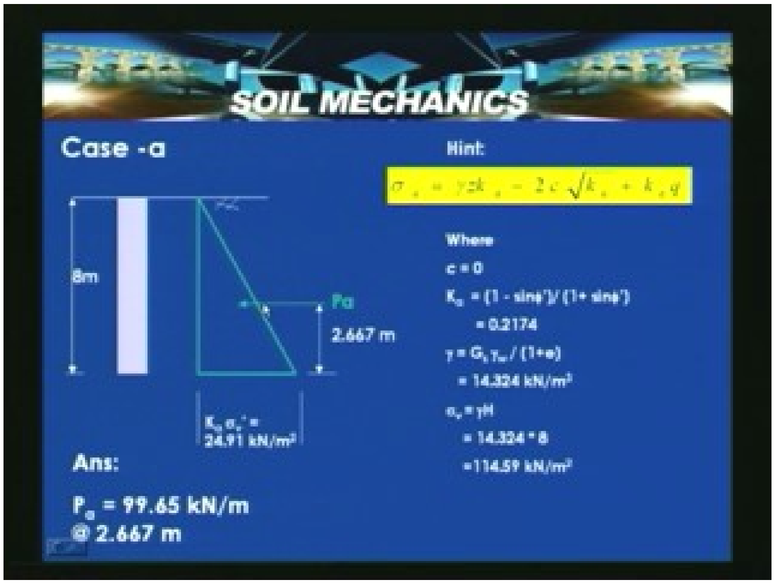

So vertical stress is about 114.6 kilonewton per meter square so at a depth of 8 meters the vertical stress is this much, here is zero. So sigma$_a$ is equal to at depth, z equal to 8 meters soil being uniform which is k$_a$ sigma$_v$ dash which is equal to 24.91 kilo newton per meter square. So these retaining walls are generally considered as per meter walls, the active earth pressure thrust units are kilo newton per meter. Suppose if the wall say of about 5 meters length then total earth pressure thrust acting on the wall in active case can be calculated for the given length.

So but generally these are considered as a per meter length walls being a continuous and then being a plain strain structures we get finally the thrust in per meter length. So here in this case please note p$_a$ is equal to 99.65 kilonewton per meter and its acting at a H by 3 from the base that is 2.67 meters. Similarly case b now the water surface is at the ground that is at the ground surface here you can see at the top of the wall, so this is k$_a$ gamma dash H now being because of the submerged unit weight which we actually obtain by using this deliberation. Then we calculate water pressure now we can see that tremendous amount of water pressure which actually acts on the wall.

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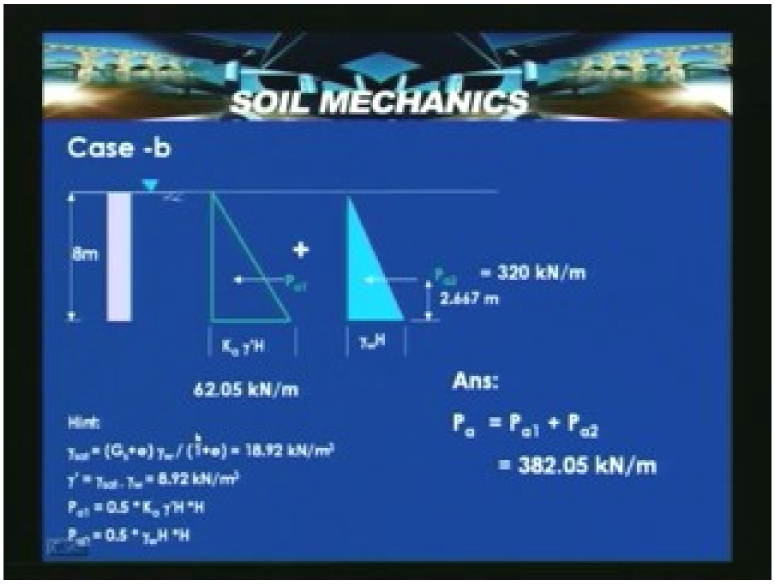

So as it actually rises from the bottom so actually induces instability to the wall, if it is not designed if it is not drained properly. So now the net pressure is now 382.05 that is about 3 to 4 times there is a increase in the lateral active thrust on the wall with an raising water table. So the effect of the ground water surface on the active thrust can be noted here. So a case where suppose you have got a soil and then which is actually prone for say capillarity saturation.

Let us assume that we have got a complete, the saturation is possible in this particular zone, entire capillarity fringes develop here. So this is 3 meters is the zone and 5 meters is which is saturated with water, this is the ground water surface. Then in this case here the earth pressure is, the total stress is here and then pore water pressure for this case is pore water pressure which is minus 3 gamma_w because gamma_w is say 10 minus 30 kilo meter per square and 50 kilometer per square. So this is the raising water table above this is because of the capillarity nature. So the effective stress is sigma dash is equal to sigma minus u with that we will get this particular distribution for the effective stress. So this is the stress distribution diagram for the total stress and pore water and effective stress.

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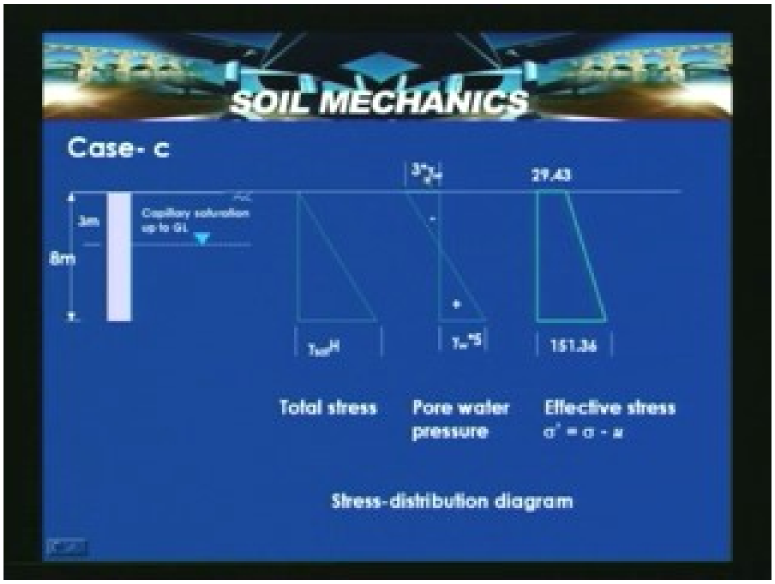

So this is that vertical stress, now that one if you convert into for calculating the lateral thrust that is now k_a into 29.43 because k_a which is actually obtained from that for the soil and now this particular portion of the water induces hydro static pressure on the wall so that is gamma$_w$H that H is equal to 5 meters. So with that what will happen is that the p_a is now 279.81 kilonewton per meter that is this particular situation is that earth pressure diagram changes and which is actually having k_a into 29.43 at the top and k_a into 151.36 ordinate at the bottom and the resultant of this trapezium actually is nothing but the total active thrust which for example which is given here is simplified way, p_a is equal to 279.81 kilonewton per meter acting at a 2.47 meters from the base.

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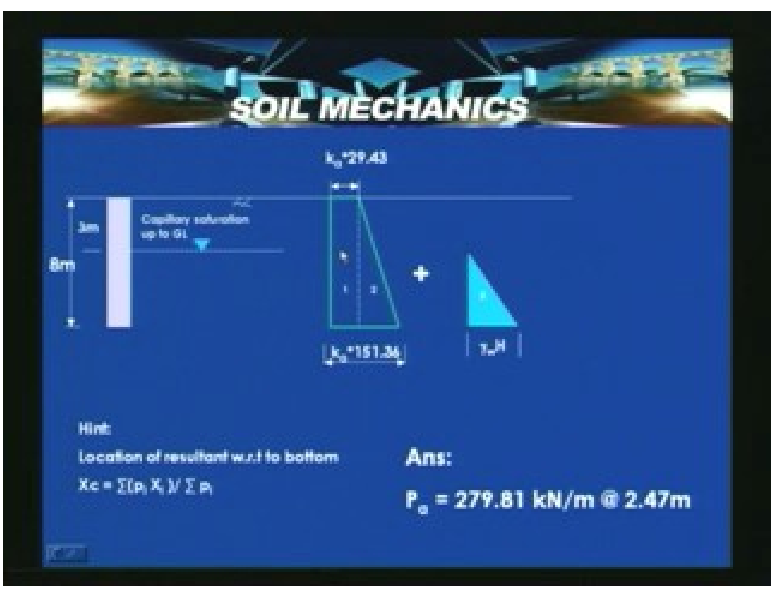

So the location of the resultant with respect to the bottom can be calculated by individual cg. For example the cg of this particular portion or rectangle located here at H by 2 from the base and this particular portion is located H by 3 in the bottom and then cg of these particular water pressure diagram, they combine you will able to get the total active thrust and then acting at a location 2.47 meters from the base. So in this example what we saw first is that a case where the dry soil and then raising ground water table and a soil subjected to capillary saturation. So these conditions actually affect the active earth pressure and then can induce lateral pressure on the soil. So that is the reason why we generally use drainable or granular soils as a backfill material. In case the situation is that you are prone to use low quality fills then adequate measures have to be taken to drain the water behind the wall.

The second example, a retaining wall with a smooth vertical back is 10 meters retains at 2 layer sand backfill with the following properties. So assume that you have got zero to 5 meters, that is c dash is equal to zero, phi dash is equal to 30, gamma is 18 kilo newton meter per cube, below 5 meters that is a wall of total 10 meters. So below 5 meters the c dash is equal to zero, phi dash equal to 34, say 2 soils are there now with gamma is equal to 20 kilo newton meter per cube.

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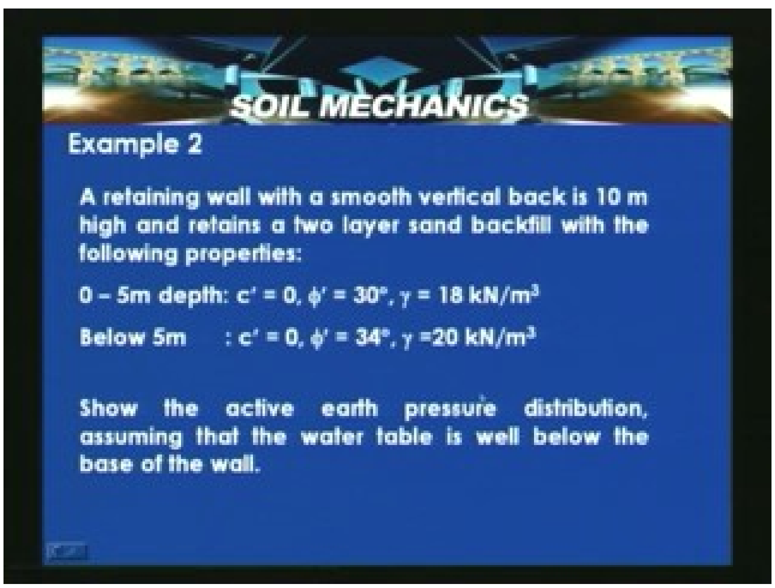

So basically we need to show the active earth pressure distribution assuming that the water table is well below the base of the wall and here we have 2 types of soils and up to 5 meters and below 5 meters there is another type of soil. So this particular case is illustrated here so c dash is equal to zero, phi dash is equal to 30 dash, k_{a1} is obtained 0.33 from 30 degrees friction angle 1 minus sin 30 by 1 plus sin 30. So 1 minus sin 34 by 1 plus sin 34 that is for k_{a2} it is 0.283.

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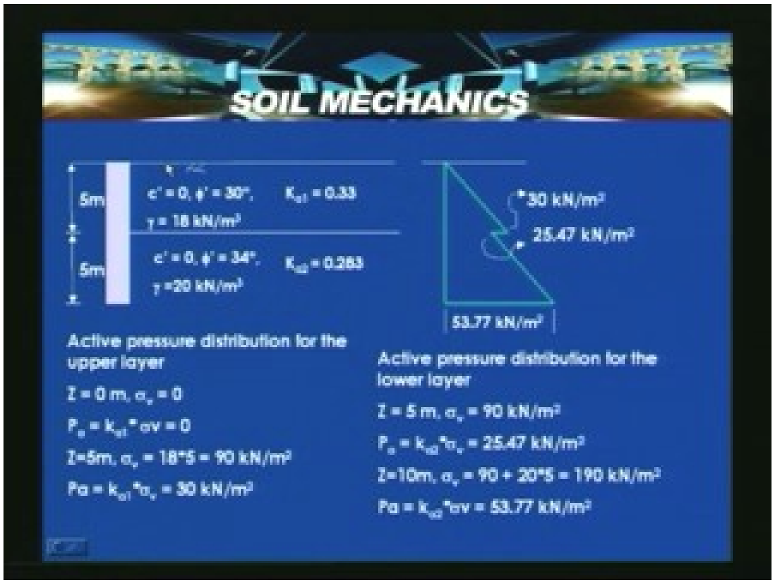

Now by using sigma$_a$ is equal to k$_a$ gamma z minus 2c root k$_a$ plus k$_{aq}$ that is the universal equation for determining earth pressure distribution for the active case. So active pressure distribution for the upper layer at z is equal to zero that is at this point sigma$_v$ is equal to zero so lateral pressure is equal to zero, at z is equal to 5 meters just above this point then we need to use the soil properties that is k$_{a1}$ 0.33 into this 90 kilo newton meter per square which actually is 30 kilonewton meter square which actually yields this ordinate. At z is equal to 5 meters but just below this line which actually is governed by k$_{a2}$ that is phi is equal to phi dash is equal to 34 degrees which k$_{a2}$ is equal to 0.283 which yields now k$_{a2}$ into sigma$_v$ that is same vertical stress but with a different soil properties. With that what will happen is that this ordinate that we computed now at z is equal to 5 meters just below this line which is 25.47 kilonewton meter per square.

Now at z is equal to 10 meters we have vertical stress plus the 90 kilonewton meter per square plus 20 into 5. So this 20 is that unit weight now, please note that unit weight is also changed here. So with that 190, now entire this is governed by which is something like surcharge on this level where k$_{a2}$ which is nothing but 0.283 into 190 which actually yields 53.77 that is ordinate. So the resultant of this actually gives the total active thrust exerted on this particular type of 10 meter wall. So that is how we use whatever we have learnt to determine this earth pressure distribution and then active earth pressure at thrust on the wall can be determined.

So third example where retaining wall 6 meter high with a smooth vertical back say in this case we should be able to push against a soil mass where this wall is actually move towards the soil and c dash is equal to 40 kilonewton meter per square and phi dash is equal to 15 degrees and gamma is 19.40 kilonewton per cube and what is the total Rankine pressure, if the horizontal soil surface carries a uniform load of 50 kilo newton meter per square. There is a surcharge which is acting on the top. What is the point of application of the resultant thrust? This is the problem under cohesion.

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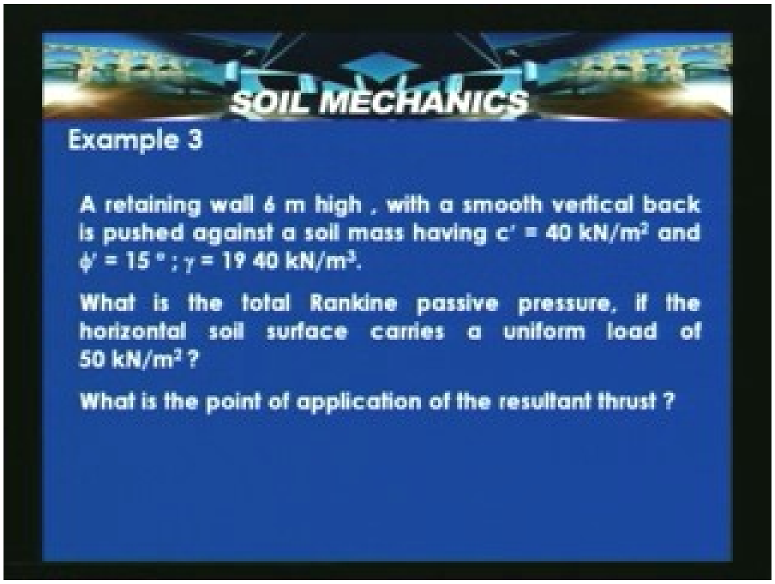

So what we can do is that surcharge is 50 kilonewton meter per square, wall is height of 6 meters Rankine case but wall moves towards the backfill. So it indicates the passive case now, $sigma_p$ is equal to k_p gamma z plus k_p q plus 2c root k_p.

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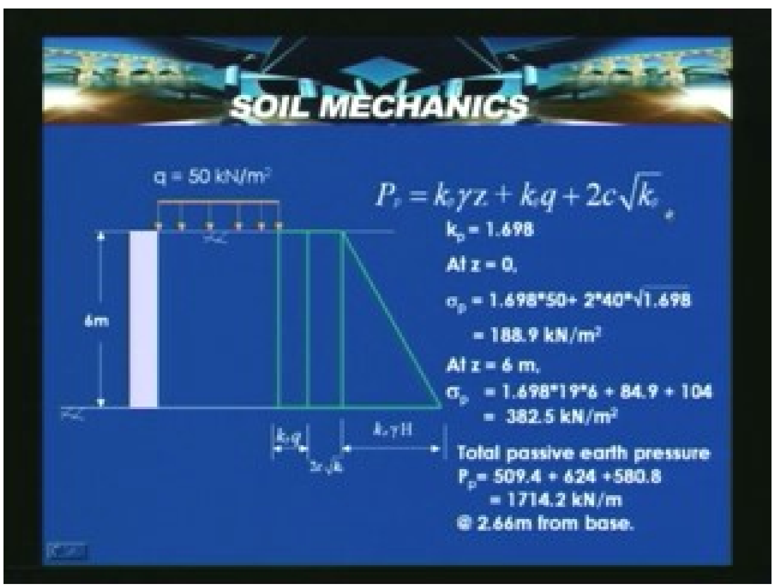

So with that what will happen is that k_p for a given friction angle, we can obtain at z is equal to zero, $sigma_p$ is equal to 1.698 into 50 plus 2 in to 40 into root over 1.698 which actually yields 188.9 kilo newton meter per square that is at z is equal to zero because this is a passive case wall moves towards the backfill so 1.698 into 50.

That is surcharge at this particular point plus this particular portion is due to plus 2c root k_p that is actually this portion here. At z is equal to 6 meters we get $k_p q$ plus 2c root k_p plus k_p gamma dash that is actually this particular portion when z is equal to H which actually is a pressure here. So at $sigma_p$ is equal to 1.698 into 19 into 6 plus 84.9 plus 104 yields 382.54 kilo newton per square. So total passive earth pressure is this much kilonewton per meter acting at 2.66 meters from the base. So that is how we actually use for active case as well as passive case to determine the active thrust or passive thrust.

Now this particular case of Rankine theory, all we discuss can be extended for special cases like, when you have got a backfill but vertical wall initially let us consider that wall surface is vertical but it is smooth and in such situations we can actually extend this theory for calculating earth pressures. So in this particular slide what you are seeing for an extension of the Rankine theory for a sloping soil surface with a smooth vertical retaining wall. So this particular surface, the wall initially is at this particular position and when the wall moves away from the backfill, when the wall movement is this side then is an active case. So here that fill inclination with the horizontal is beta that is actually shown here and which is having a soil which is shown in this element.

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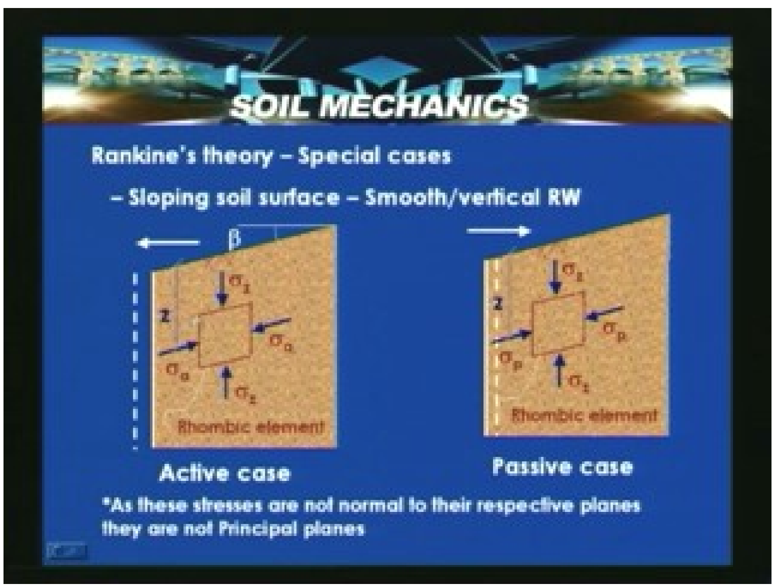

So here in this passive case the elements are now, one thing which is to be noted here is that the surface is not horizontal. In the previous case we have actually assumed that typical ideal Rankine wall having a smooth interface and as well as backfill surface is horizontal but in this case, you have got a smooth surface but the backfill surface is having a certain inclination. So this is a case where passive case which is shown, the wall portion is initially here and the movement of the wall is towards the backfill then what will happen is that there is wall movement. This is the wall position which is shown after pushing adequately inside to create a passive state of plastic equilibrium. Then at point of plastic state of equilibrium a rhombic element is shown here.

So in this case these stresses can no longer be called as a principle stresses because these surfaces are not horizontal. So they carry some shear stresses so but these particular stresses what you are showing this is sigma$_z$ at a depth z, this z is measured here as shown here and sigma$_a$ is the active earth pressure and sigma$_z$ at a depth z and sigma$_z$ is vertical stress here. So this is a typical rhombic element in case of active case and this is the case in the passive case where you have got sigma$_p$ as the passive earth pressure and sigma$_z$ is the vertical stress.

So as these stresses are not normal to their respective planes, they are not principle planes, that has to be noted. Now assumptions involved in this case in extending this theory to this particular case. This existence of the conjugate relationship between the vertical pressures and lateral pressures on vertical planes within the soil adjacent to a retaining wall.

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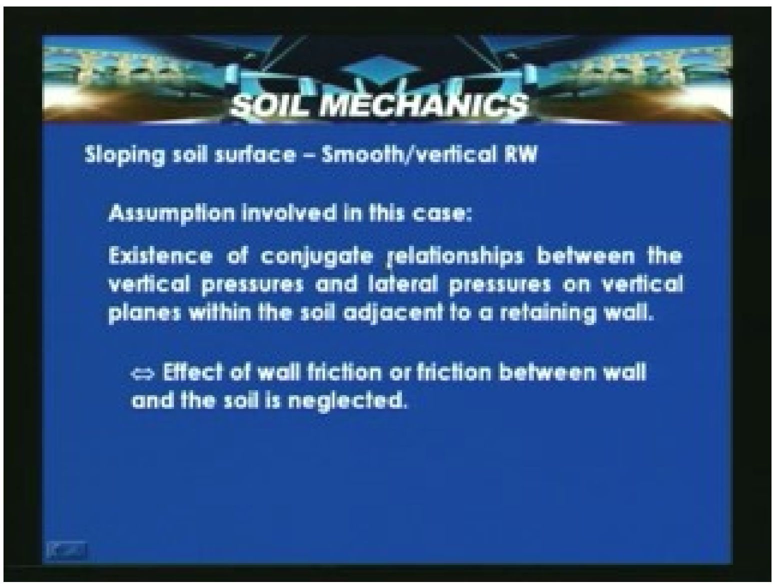

So existence of conjugate relationship between the vertical pressures and lateral pressures on the vertical planes within the soil adjacent to a retaining wall. That means these are the vertical planes which are there. So what is being shown is that existence of conjugate relationship between the vertical pressures and lateral pressures on vertical planes within the soil adjacent to a retaining wall. The effect of wall friction and friction wall, the soil is neglected. So with this assumption what it means is that the effect of wall friction or that means or friction between the wall and the soil is neglected. So with this assumption the Rankine action theory is extended to calculate the earth pressures for a case.

Now what we need to do is that to extend this theory, sloping the soil surface with smooth vertical retaining wall. So consider this particular horizontal length is actually having a unit length. Now weight of the column soil above the horizontal width is gamma z that is at depth z as shown in that rhombic element. Area of the parallelogram is z into 1 that is z is the depth and one is that unit width.

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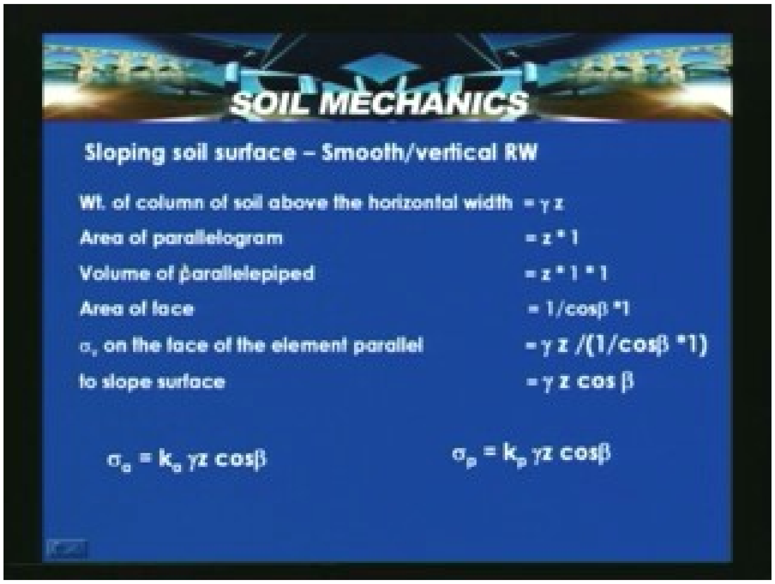

Then volume of the parallelepiped is z is equal to 1 into 1, the area of the face is 1 by cos beta into 1. So sigma on the face of the element parallel to the slope surface is gamma z by 1 by cos beta into 1 which actually gets sigma_z that is gamma z cos beta that is the sigma_v on the face of the element parallel to the slope surface. That is on that inclined which is parallel to the slope surface means which is having an inclination beta with that sigma_a is equal to now we can obtain k_a gamma z cos beta. Suppose if cos beta is equal to zero then it actually changes to sigma_a is equal to k_a gamma z with some inclination beta what we get is sigma_a is equal to k_a gamma z. In case if it is for passive case then sigma_p is equal to k_p gamma z cos beta.

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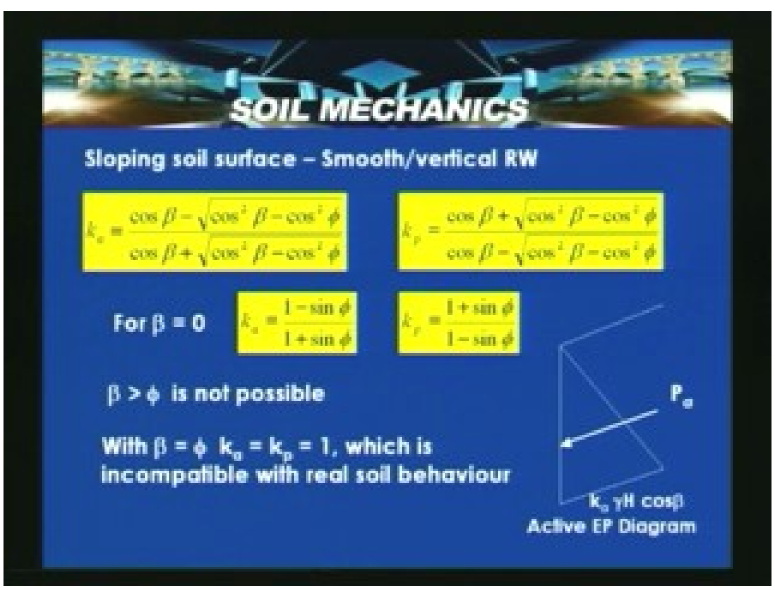

Now by further simplifications and deliberations we can actually obtain quotient of active earth pressures for this case where the fill surface is not horizontal, is inclined with having a beta with horizontal. For a smooth vertical retaining wall with vertical face we can actually obtain k_a is equal to cos beta minus root over cos square beta minus cos square phi divided by cos beta plus root over cos square beta minus cos square phi where if you set beta is equal to zero it actually changes to k_a is equal to 1 minus sin phi by 1 plus sin phi. This is the typical ideal retaining wall case where smooth interface and backfill surface is horizontal. Similarly for passive quotient of earth pressure can be obtained k_p is equal to cos beta plus root over cos square beta minus cos square phi by cos beta minus cos square beta minus cos square phi with that what will happen is that k_p quotient of passive earth pressure can be obtained.

So when we substitute beta is equal to zero, k_p also changes out to one plus sin phi to one minus sin phi when beta is greater than phi is not possible because the phi is suppose if it is more than its angle of repose then there is which is not possible and beta is equal to phi, k_a is equal to k_p is equal to one which is incompatible with a real soil behavior. So with the beta is equal to phi, k_a is equal to, it actually these coefficients change to k_a is equal to k_p is equal to 1 which is in incompatible with a real soil behavior. Suppose now you have got a case where there is a vertical wall and there is say vertical wall of height 8 meters and there is a slope inclination that is of the fill behind the wall is say 20 degrees that is beta is equal to 20 degrees and soil is having a unit weight gamma then you can determine the active thrust like this.

Suppose if you have got a wall of 8 meters then if the beta is equal to say 20 degrees then now the condition is that wall is smooth that is the (Not understandable) (Refer Slide Time: 46:56) case. So we can't use one minus sin phi by one plus sin phi but we need to use this particular quotient of active earth pressure say k_a is equal to for the sloping soil surface but still Rankine case, with that we can by substituting for beta is equal to 20 degrees and say phi is equal to some slope inclination say 40 degrees with phi is equal to angle of internal friction say for example for 40 degrees then we will be able to get k_a is equal to cos beta minus root over cos squared beta minus cos square phi by cos beta plus root over cos square beta minus cos square phi with that we will be able to get the quotient of active earth pressure for this case.

By substituting in sigma_a is equal to k_a gamma z cos beta will be able to get z is equal to H, we will get the earth pressure ordinate here parallel to the slope surface of the fill then k_a gamma H cos beta. The resultant thrust is actually determined by p_a that is which is indicated here. This is how one of the case which we can be extended like the ideal Rankine wall case two, a case where sloping soil surface and in this case what you need to do is that use appropriate expressions for quotients of active earth pressure and passive earth pressure and determine those quotients of active earth pressure and passive earth pressure correctly and then use the expression for sigma z is equal to k_a gamma z cos beta where beta is the sloping relation of a backfill surface. With that you will be able to get the earth pressure ordinate or a equation for the earth pressure so from their we will be able to calculate the active earth pressure thrust due to soil, having a inclined backfill surface.

Suppose we have say retaining wall which is smooth but inclined retaining wall. Suppose initially let us consider, suppose you have got a wall face, all the time we are actually assuming vertical and smooth and backfill surface is horizontal. Suppose you have got a wall say typical mesentery wall say assume that it is smooth, in such situation the earth pressure is the resultant of the weight of the soil in that wedge portion and the lateral thrust exerted by the soil. So in this case suppose if you are having an inclined wall surface but a smooth interface but with a backfill surface inclined in this case. How you need to determine is that, we need to determine first the weight of this wedge which is w and then the resultant active thrust along this face, now this face which actually is the frictionless interface and with that what will happen is that you will be able to get p_a. The resultant of w and p_a which actually is somewhere here that is nothing but the resultant active thrust which is needed to be considered in designing a wall which is having a inclined slope face and having smooth interface behind the wall.

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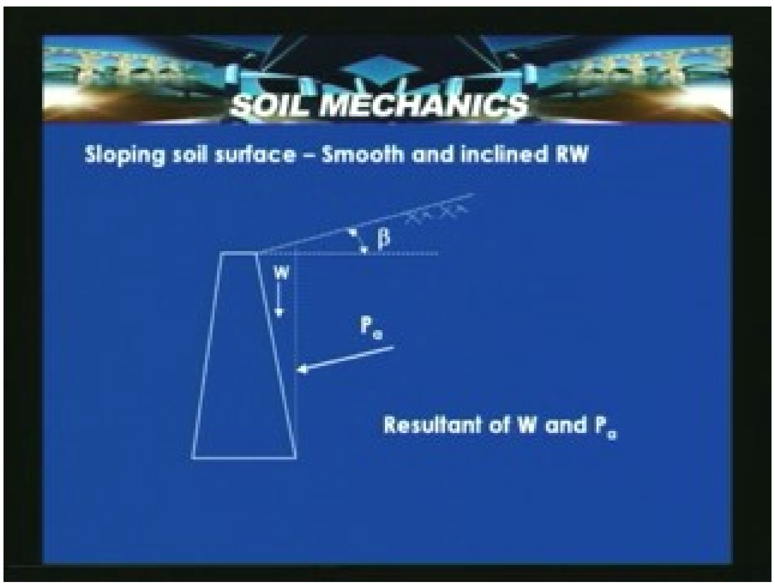

So in this lectures what we try to understand is that how the Rankine's theory can be used for computing earth pressures and we also discussed the problems like a simple homogeneous soil deposit and a cases like when we have got a partially submerged soil behind the retaining wall and what will be the influence of raising ground water table on lateral earth pressure, especially for the active case and then we try to also discuss extension of this problem for a stratified soil deposits then what we need to do is to follow the equation sigma_a is equal to k_a gamma z minus 2c root k_a plus k_aq for an active case.

In case of passive case sigma_p is equal to k_p gamma z plus 2c root k_p plus k_pq in case of a passive case and follow the case like when you got a interface say two soils are separated by say interface AA then consider the soil properties for that particular depth above that interface and the below that interface and try to get the net pressure diagram for the case.

So when we wanted to calculate say for a cantilever sheet pile wall then you need to calculate the net earth pressures from the active side and passive side and those things have to be considered in the design as a net earth pressure diagrams. Further we extended this theory for determining say a case like having inclined wall face but smooth in nature but having horizontal surface then in that case what you need to do is that you need to determine the weight of the soil in that triangular portion, wedge portion and then active earth pressure due to the horizontal surface, the convention one so that is the resultant.

For example if you have got a wall which is having a vertical face with a inclined surface then we actually discussed about the quotient of active earth pressure and passive earth pressure with new expressions but when the beta is equal to zero they again change over to k_a is equal 1 minus sin phi by 1 plus sin phi and k_p is equal to 1 plus sin phi by 1 minus sin phi were passive case and we used these things to calculate the active earth pressure thrust and passive earth pressure thrust.

In the next lecture we will be introducing the Coulomb's theory of earth pressure and then graphical methods for determining the earth pressure and thereafter we will be discussing examples relevant to Coulomb's and kalmonds methods.