**Soil Mechanics**
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**Lecture – 51**
**Earth Pressure Theories – II**

Welcome to lecture number two on earth pressure theories. In the previous lecture we had introduction to earth pressures which is caused by soil and we also discussed in detail about earth pressure at rest and different conditions. In this lecture you will be introduced to earth pressure theories, one of the earth pressure theories which we are going to discuss today is Rankine's earth pressure theory. We discussed in this lecture about active and passive earth pressure states which are postulated by Rankine and some relevant problems and extensions of these Rankine's theory for a different cases in soil mechanics. Before venturing into this let us review what we discuss. We discussed that the earth pressure coefficient $k_0$ which is found to be depend upon relative density of sand, processed by which the deposit was formed and the stress history.

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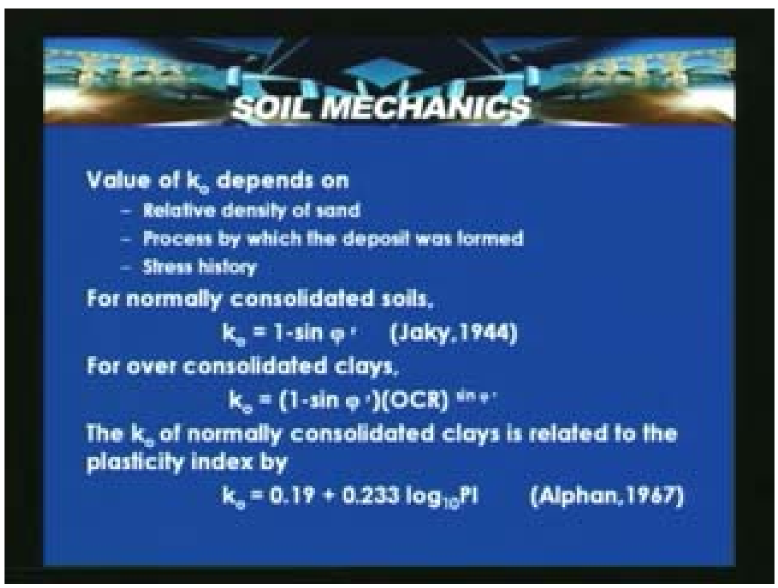


We discussed that it can be estimated by using jaky's formula which is for normally consolidated soils $k_0$ is equal to 1-sin phi dash. For over consolidated clays it is $k_0$ is equal to 1- sin phi dash into OCR raised to sin phi dash. The $k_0$ of normally consolidated clays is related to the plasticity index, according to Alphan 1967 we can also give a correlation between $k_0$ and PI, $k_0$ is equal to 0.19+0.233 log to the base 10 of plasticity index and when we wanted to measure say for example you have got the $k_0$ conditions in a basement wall and which is surrounded by say water in the soil. Then let us see what pressure it creates on the wall due to the earth pressure at rest conditions and because of the water. So here because of the submergence, the unit weight of soil is equal to the submerged unit weight of the soil.

Here the triangular linear distribution is for earth pressure at rest where $P_0$ is the earth pressure thrust at rest that is $P_0$ which is $P_0$ is equal to half $k_0$ gamma dash z square where if z is the say depth of the case under consideration and $k_0$ gamma dash z is the earth pressure at rest. This earth pressure at rest also has a component which is now due to water which is spilled in the voids. So that pressure is equal to $P_w$ is equal to half gamma wz square as water is not having any shear strength, we can say that the ka kp is equal to one so with that $P_w$ is equal to half gamma wz square.

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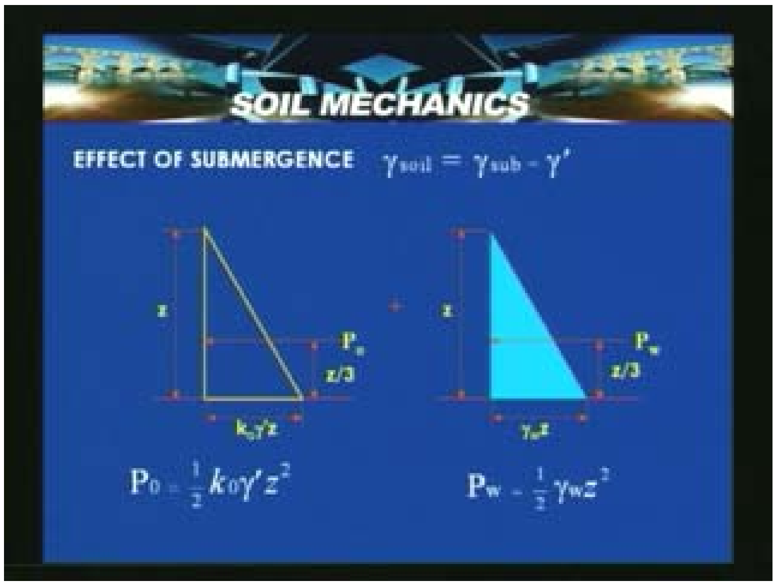


Now let us look into these earth pressure conditions and different possibilities. Let us assume that we have got a soil which is retained by a vertical wall. Let us assume that base and as well the wall interface is smooth. So the base of the wall is assumed to be smooth and the wall is also assumed to be smooth. So these earth pressure problems which are indicated in this case, the soil behind the wall is called backfill and this is wall which is shown here a cross section diagram. So these earth pressure problems are often dealt with plastic equilibrium in soil mass. What is this plastic equilibrium?

A body of soil is said to be in a state of plastic equilibrium, if every part of it is on the verge of failure. So this can be visualized by a perfectly rigid plastic model where with a stress strain relationship if we assume that it is rigid and perfectly plastic. So here a graph which is shown, shear stress versus shear strain and at a zero shear strain itself it actually attains that yield shear stress (Refer Slide Time: 05:44). So here the stress strain behavior of the soil can be represented here by the rigid perfectly plastic idealization.

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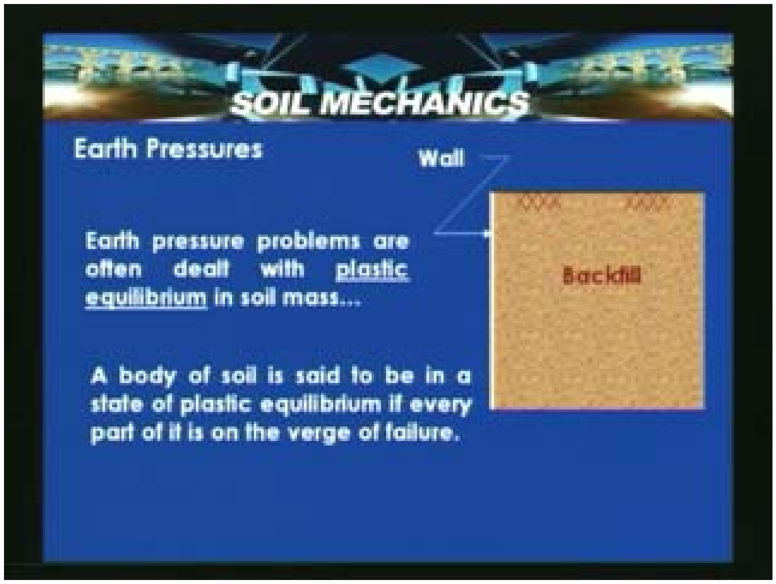


That mean here yielding and shear failure both occur at the same state of stress and unrestricted plastic flow takes place at this stress level itself. So this particular state is actually indicated here by y dash is actually said that it has reached the plastic equilibrium. So soil mass is said to be in a state of a plastic equilibrium if the shear stress at every point within the mass, reaches the value represented by a point y dash. So this plastic equilibrium can arise if there is a movement of the wall either away from the fill or towards the fill. Suppose in case if the wall is at rest then elastic equilibrium conditions prevail, that is what we discuss and we define that as quotient of earth pressure at rest $k_0$.

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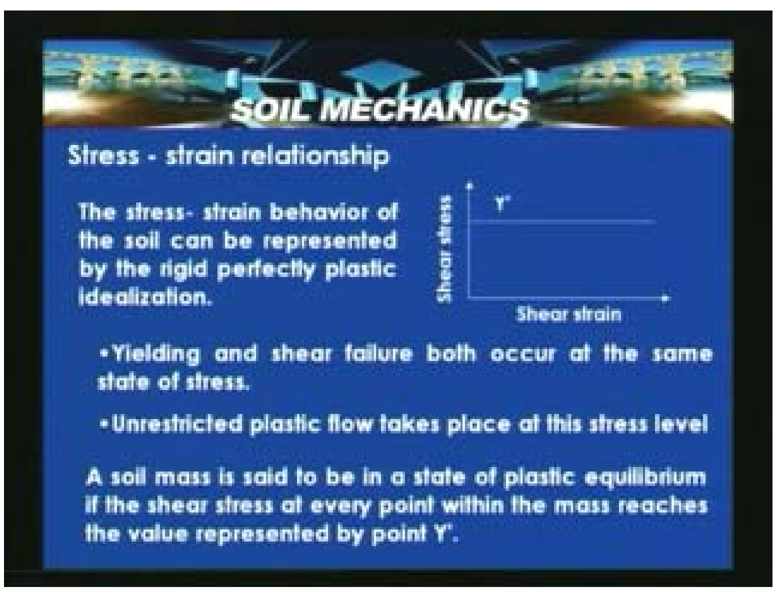

So Rankine in 1857, the Rankine's earth pressure theory is postulated after Rankine in 1857 and he assumed the following way. The soil mass is assumed to be homogeneous and dry and cohesion less that is the first assumption which was put forward by Rankine. Ground surface is plane which may be horizontal or inclined, so the ground surface is usually backfill surface is assumed to plane and then later it has been extended to inclined cases. Face of the wall in contact with the backfill is vertical and smooth that is the back of the wall that is the wall which is in contact with the soil, the interface is assumed to be smooth that is friction less condition which indicates that the wall is having zero shear stress around the interface.

So this particular condition with horizontal backfill and with friction less interface at the soil and wall interface is referred as also as Rankine wall. Another important state which he defined is that the possibility that the wall moves away from the fill and that particular state is actually defined as active state and if the wall moves towards the fill then it is actually defined as a passive state.

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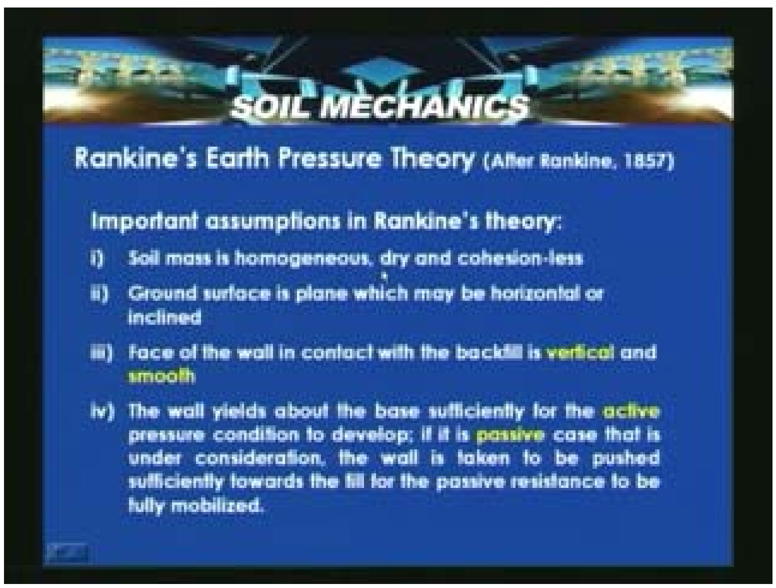


The wall yields about the base sufficiently for the active pressure conditions to develop and if it is passive case that is under consideration the wall is taken to be pushed sufficiently towards the fill for the passive resistance to be mobilized. So which we are actually going to discuss this states, active states and passive states which are actually Rankine’s states of plastic equilibrium conditions which are put forwarded. So one is called active which will actually arise when the wall moves away from the backfill and a passive state can arise if the wall is pushed towards the backfill. Now consider earth pressure states like what has been told in the previous slide like wall moving away from the backfill and wall moving towards the backfill.

Here the wall movement away from the backfill is actually shown in this cross action with backfill here, with a horizontal surface and the friction between the wall interface and the soil is zero that indicates smooth and this is idealized by a Rankine wall and assuming that the base roughness is very smooth so with that you can actually get a uniform movement for the wall. So in this case a wall movement ever from the backfill by a displacement say delta h which moves away from the fill then that actually arises a case, at referred here as active case.

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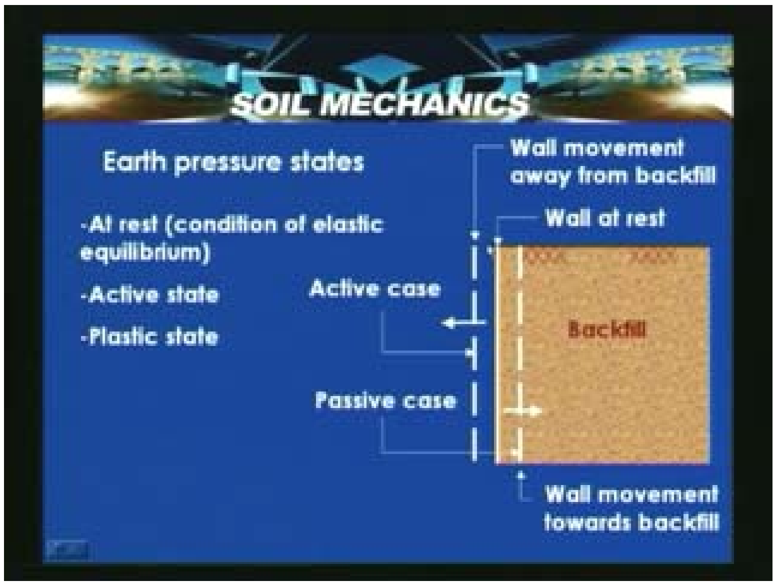


If the wall is pushed towards the back fill then that is referred as passive case here. So earth pressure states here which we define at rest condition that is condition of elastic equilibrium condition where the wall is at rest condition. That is no movement that is zero strain condition which is next to impossible to maintain the state, so possibility of the active and passive states are always there. So in the earth pressure states, now wall movement away from the backfill if you consider, the conditions are same as what we discussed earlier.

Now consider an element here at a depth z which is having vertical stress $\text{sigma}_v$ and horizontal stress $\text{sigma}_h$. These surfaces being horizontal you can see and these stresses can be called as principle stresses and if the $\text{sigma}_v$ happens to be the major then it is called major principle stress and $\text{sigma}_h$ if happens to be minor then it happens to be called minor principle stress. So soil element at any depth z is subjected to a vertical stress $\text{sigma}_v$ and horizontal stress $\text{sigma}_h$ and since there can be no lateral transfer of weight if the surface is horizontal and we can say that the shear stresses are absent here. So no shear stresses exist on the horizontal and vertical planes, so vertical and horizontal stresses are here. The planes on which these vertical and horizontal stresses $\text{sigma}_v$ and $\text{sigma}_h$ are acting can be called as principle planes.

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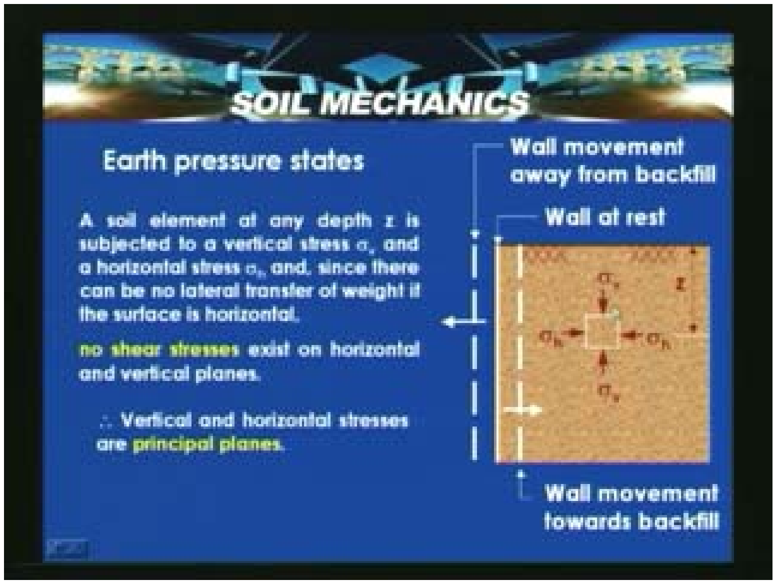


So case where what will happen to these sigma$_h$ and what will happen to the sigma$_v$ when the wall movement away from the backfill and that we will see and discuss in the next slide. So in case say if there is no movement then that condition we referred and then said as at rest condition k$_0$. At rest condition is defined as a state of zero lateral yielding and if there is no lateral strain then epsilon is equal to zero and this condition if you look into the tow sigma plot and a more circle plot, you can see the more circle is at rest more circle is here where sigma$_v$ is represented here and sigma$_h$ is represented here. So this is sigma$_v$ and this is sigma h$_0$. So the k$_0$ which is nothing but sigma h$_0$ by sigma$_v$ which is the coefficient of earth pressure at rest.

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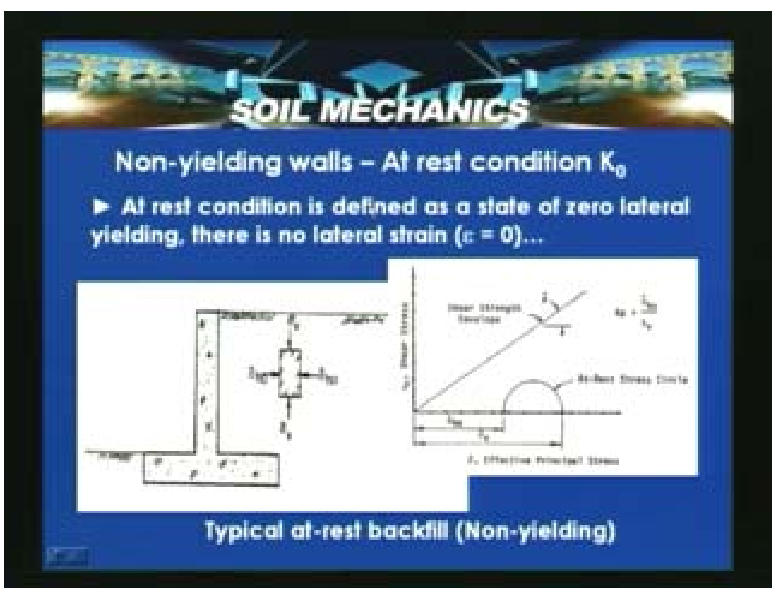

So we can say that if this happens to be the shear strain envelope which is the failure envelope and which indicates that the circle is stable and had to attain any failure. So we can say that is a stable equilibrium condition and that's why we say that at rest condition it actually remains in the elastic equilibrium condition. Now a case where, when the wall movement takes place and then which is away from the backfill. In the previous slide what we saw is a zero strain condition where wall is a non yielding wall and this type of conditions can arise particularly if you have got a basement wall which is propped at the beam connections and if it is having a backfill which is filled then that condition may be very similar to at rest condition.

So at active state in this case consider a wall ab which is again smooth and a base having smooth nature and wall movement occurs away from the backfill. Here again the element which considered is $sigma_v$ and $sigma_h$ at a depth z. Now if there is a movement of the wall away from the soil then $sigma_h$ decreases. Please note down the horizontal stress decreases, as the soil dilates or expand expands outward the decrease in $sigma_h$ being an unknown function of lateral strain of the soil.

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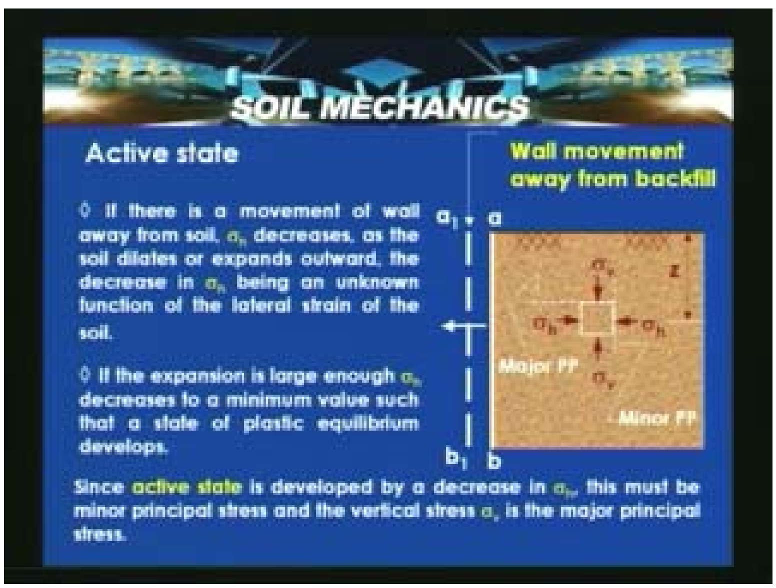


So here as a wall moves away from the backfill the soil dilates or expands outward. So this condition what will happen is that when the wall movement is being taken place then it is actually opposed by the shear resistance offered by the soil. So in this case what will happen, when the wall movement takes place the strain develops. These strains mobilize the shear stresses and with that what will happen, the soil offers its resistance and tries to restrain the movement. So in the process what will happen, the horizontal stress which was they drop down to a minimum value. So the state where it reaches the failure. Then we can say that the active state of failure or active state of plastic equilibrium condition has attained. So if there is a movement of the wall away from the soil $sigma_h$ decreases as the soil dilates or expands outward the decrease in $sigma_h$ being an unknown function of the lateral strain of the soil.

If the expansion is large enough then sigma$_h$ decreases to a minimum value such that the state of plastic equilibrium develops. So since active state is developed by decrease in sigma$_h$, this must be a minor principle stress and the vertical stress sigma$_s$, sigma$_v$ is the major principle stress. So here if you look into here this is the minor principle plane because sigma$_h$ is decreasing. The reason for the decreasing is that because the movement is opposed by the shear resistance of the soil. So because of that there is a decrease in the sigma$_h$ so sigma$_h$ the active state is being developed by decrease in sigma$_h$, the sigma$_h$ becomes minor and sigma$_v$ becomes major. So in this case here with the surface being horizontal what we can say this can be major principle plane and this is minor principle plane. So sigma$_h$ when it reaches to the active state the horizontal stress becomes sigma$_a$ that is the minimum value where it has actually attained a state of plastic equilibrium.

Now let us consider the second case or state what we discuss is passive state. This case wall moves towards the backfill. Again the ground surface is horizontal, ab is the initial position of the wall and $a_1$ $b_1$ is the wall after pushing adequately towards the backfill. So in this case again same element but same depth z sigma$_v$ and sigma h being, the horizontal sigma$_h$ is the horizontal stresses, sigma$_v$ is a vertical stress. So there is a movement of the wall towards the soil, there will be lateral compression of the soil and the value of sigma$_h$ will increase until a state of plastic equilibrium is reached.

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So here there is a movement of the wall towards the soil. There will be a lateral compression of the soil takes place. In the previous case when the wall moves away from of the backfill there is a lateral expansion is taking place. In this case when the wall moves towards the fill there is a lateral compression is taking place. So the value of the sigma$_h$ will increase until a state of plastic equilibrium is reached. Here the wall movement is again is opposed by the shear resistance along the sliding rupture plane. Since the passive state is developed with an increasing sigma$_h$ this must be a major principle stress and vertical stress sigma$_v$ is the minor principle stress.

So sigma$_h$ value increases and even it crosses the sigma$_v$ value, so sigma$_h$ now becomes the major principle stress and sigma$_v$ becomes the minor principle stress but in both active states and passive states at a particular depth, the vertical stress remains constant because the weight of the soil which is involved in the process is same. So at a particular depth if z is a depth. The weight of the soil which is involved in this process of active states and passive states being same. The sigma$_v$ remains same. But only the changes which occur is in the lateral stresses that is horizontal stresses sigma$_h$, in one case what will happen is that when the wall moves away from the fill there is a decrease in the sigma$_h$ and the wall moves towards the fill there is an increase in the sigma$_h$.

So let us consider the state of the elastic equilibrium conditions and then state of plastic equilibrium conditions, when these things can occur and the local states of plastic equilibrium conditions. So in this slide what you are seeing, two diagrams and in case if the wall is at rest condition then we see this type of rupture lines or slip lines which these lines indicates the zone of elastic equilibrium conditions. So entire soil when is at rest condition it actually having $k_0$ is equal to sigma $h_0$ by sigma$_v$ and which maintains this elastic equilibrium conditions.

Suppose if the wall moves away from the backfill and say if ab is the original position of the wall and it moves to $a_1$ $b_1$ new position say by constant distance $b_1$ one across the depth. Then what will happen, entire soil mass reaches this plastic equilibrium state provided, if there is a smoothness that is if the surfaces are smooth at the base as well as at the interface. So in such situations that is possible but it is next impossible to have that condition like in such conditions like in walls or retaining walls resting on soil because of the roughness at the base there is a possibility of local states of plastic equilibrium to develop. So a certain zone in which actually the plastic equilibrium or a sliding verge or a sliding entire failure verge develops, the remaining portion of the soil remains in a state of elastic equilibrium condition.

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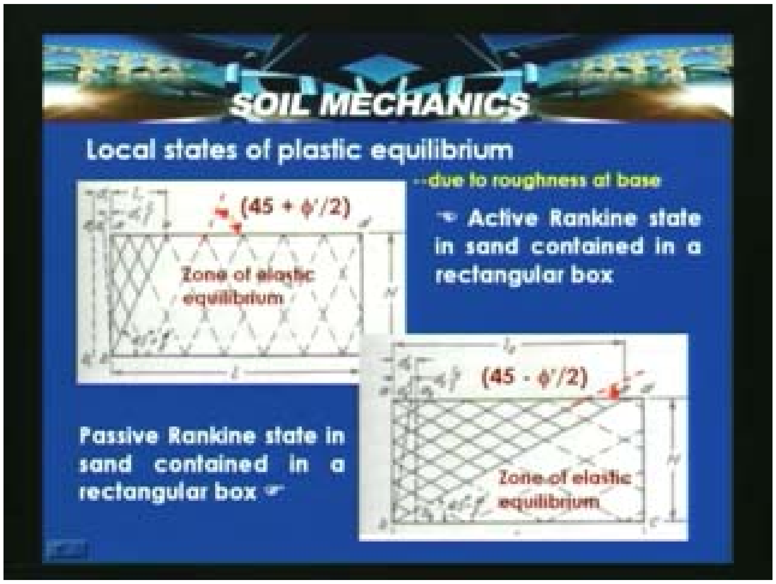

So here the top diagram which is shown which is active Rankine state in sand contained in a rectangular box. So what you see here this particular portion which actually is nothing but the failure verge and these are nothing but the slip lines, what you are seeing these are slip lines and this slip lines because soil being homogeneous we can have slip lines in both the directions. So you can see that in these directions, slip lines are indicated and this line inclination which is actually indicated by 45 plus phi dash by 2. So due to the roughness at the base when possible that here the lateral displacement is zero at the tow of the wall and then at the crest actually it can move by a certain distance.

So in such situation when the wall yields about its base that the condition can prevail if there is a roughness at the base in that situation there is a possibility of local states of plastic equilibrium to develop. So active Rankine state in sand contained in a rectangle box with a roughness at the base and another case if the wall moves towards the surface. If the wall moves towards the soil that is backfill in such situation the passive Rankine state contain in sand in a rectangular box. Here the wall actually moved adequately so that this condition occurs. So here you can see that the failure or rupture planes or sliding verge is actually inclined at 45 minus phi divided by 2 with the horizontal that is indicated and shown here 45 minus phi dash by 2 and here wall is actually because of the roughness at the base whose displacement is zero here and is actually having a movement or rotation towards the backfill.

In such situation then this is the portion which actually has undergone the failure and then this is the called also a failure verge and these are slip lines in both the directions. So in passive Rankine state which is shown here with a local state of plastic equilibrium and as well as in the active state with a local state of plastic equilibrium. Now a case again let us consider active and passive earth pressures. Here in this case in this slide which is shown here, in cohesion less soils basically for granular soils if the wall actually moves towards the backfill, you can see that here in this zone say particularly for element A, which is going to come here which actually observed to experience lateral expansion.

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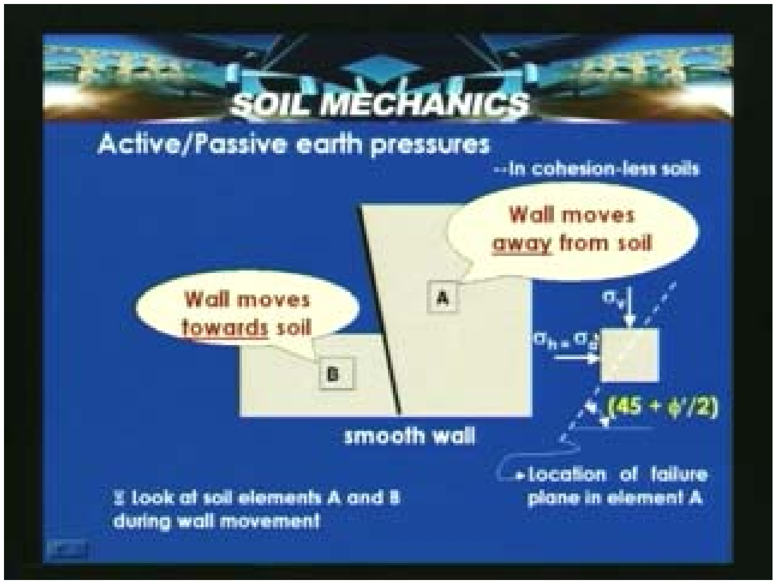


So here wall moves away from the soil here and which actually has got the deflection here which indicates that the wall moves away from the soil. In this case at this part of the zone and this actually wall moves towards the soil. This particular type of wall which is referred as cantilever sheet piles which we are going to discuss later but here at point A and point B if you observe, here in this case the wall moves towards the soil and is actually subjected, this is called a passive zone and in this case if it is rotated about the tow then actually here this subjected to wall moves away from the soil.

Once again we look into this. So this is the smooth wall with wall at the interface and subjected to deflection. Now we can see that the expansion which was taken place and which induces a lateral expansion and the $\text{sigma}_h$ now tends to become $\text{sigma}_a$ and that is actually called active earth pressure and here in this case the wall moves towards the soil where the passive condition is represented here. So this active states and passive states can occur in say in cantilever sheet pile walls, suppose if you are retaining the soil and that you can occur behind the wall it is subject to active state but at the below its verge level, it can be subjected to passive state. These two when they are in equilibrium condition they actually maintain the stability of the wall.

So element A if you consider, location of the failure plane is at 45 plus phi dash by 2, this you would have already acquainted from the shear strength lectures. So $\text{sigma}_v$ is the major principle stress and this is the major principle plane and $\text{sigma}_h$ equal to $\text{sigma}_a$ which is nothing but active earth pressure and this is the minor principle plane (Refer Slide Time: 24:58). So here in this slide we tried to demonstrate the active and passive states, the categories of lateral earth pressures. Now if you summarize first condition is that at rest case that is no movement condition that is zero strain condition then it actually induces $k_0$ is equal to $\text{sigma}_{h0}$ by $\text{sigma}_v$ that is at rest condition.

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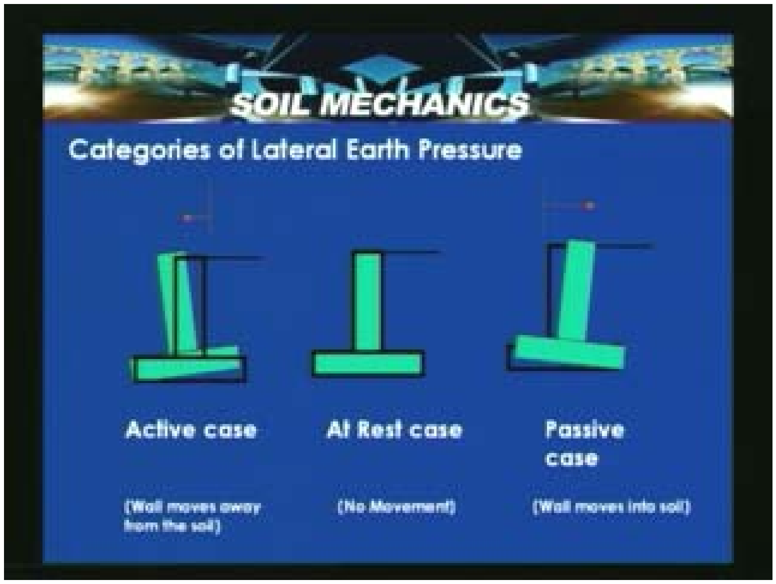


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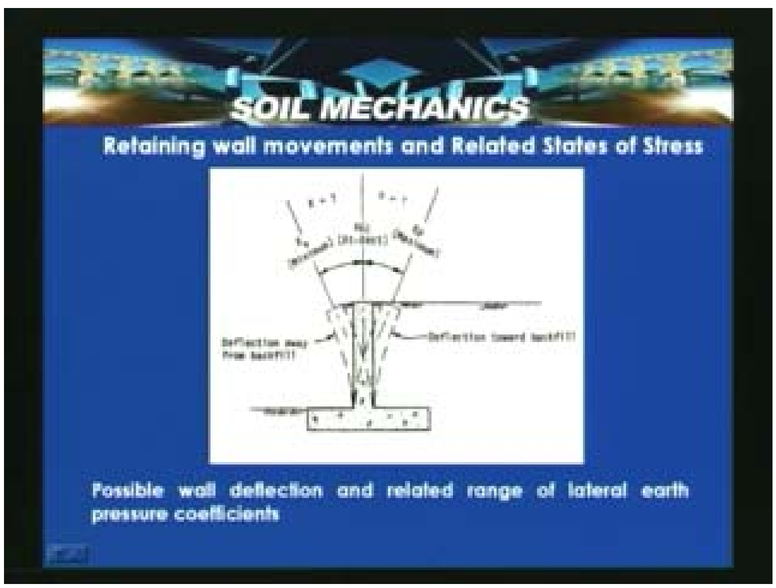


In another case which we have discussed is active case, wall moves away from the soil. In this case wall moves towards the soil that means wall moves into the soil or towards the soil that is nothing but a passive state. So now we need to look into it, retaining wall movements and related states of stress. So if you look into here, this first condition what we represented is at rest condition that is state condition and that zero strain condition and here this from $k_0$ to $k_a$, now that coefficient of earth pressure at rest can no longer be coefficient of earth pressure at rest. It changes to $k_a$ and that is referred as coefficient of active earth pressure $k_a$ and that $k_a$ which is formulated is a function of say a friction angle and conditions like wall surface horizontal and smooth interface and $k_p$ is the coefficient of passive earth pressure.

So this is the wall deflection towards the backfill so if you are retaining a soil wall again along a certain high way and then if the wall actually tends to move away from that and actually it can represent active state and this is the example where how, when the wall moves towards the backfill can induce a passive condition. So possible wall deflection related range of lateral earth pressure coefficients are shown here. Suppose if there is a wall notation takes place, in this case active case and here at this case if there is a soil compression is taking place then they are the passive state which can be mobilized.

In active earth pressure and in cohesion less states, particularly for granular soils, so what we have seen here is that $\text{sigma}_v$ dash that is at the particular depth z and $\text{sigma}_h$ dash that is a horizontal stress which is $\text{sigma}_h$ dash is equal to $k_0$ $\text{sigma}_v$ dash which is nothing but $k_o$ gamma z because z and gamma is the unit weight of soil. Now as the wall moves away from the soil $\text{sigma}_v$ remains the same that we discuss because the weight remains the same or the soil which is actually participating whose weight is same. So with that $\text{sigma}_h$ decreases till failure occurs. So what we have seen is that in active state the $\text{sigma}_h$ decreases till failure occurs.

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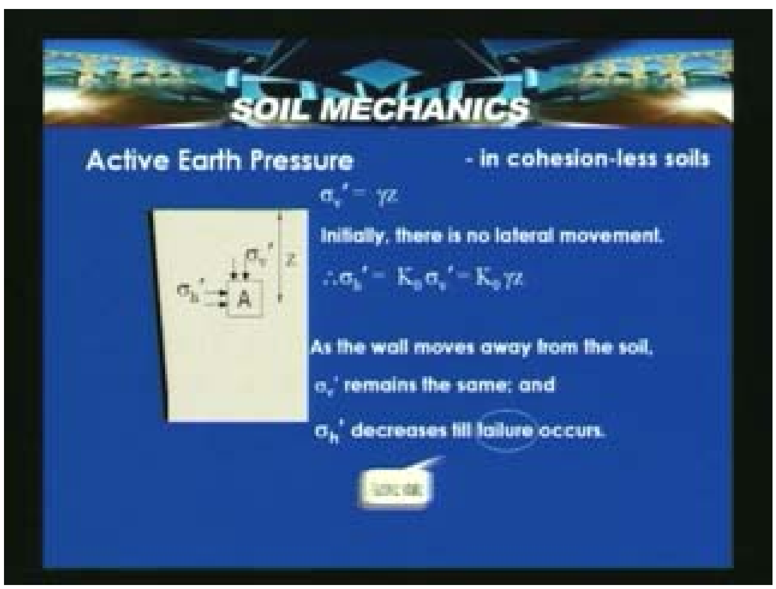


So now let us represent the state in active earth pressure condition but particularly in tow sigma condition with more circles, as the wall moves away from the soil. Initially this condition is called $k_0$ state that is coefficient of earth pressure at rest case but as $\text{sigma}_h$ decreases, if this happens to the failure envelope for a soil having the friction angle phi dash then we can say that the moment it actually comes in contact at tangential with this failure envelope, the soil attains failure that means that it has reached the plastic equilibrium state. So in this case when you look into here the decreasing $\text{sigma}_h$ dash taking place from here to here and moment it attains here that $sigma_h$ dash becomes to $sigma_a$ that is the minimum value that is the active earth pressure condition.

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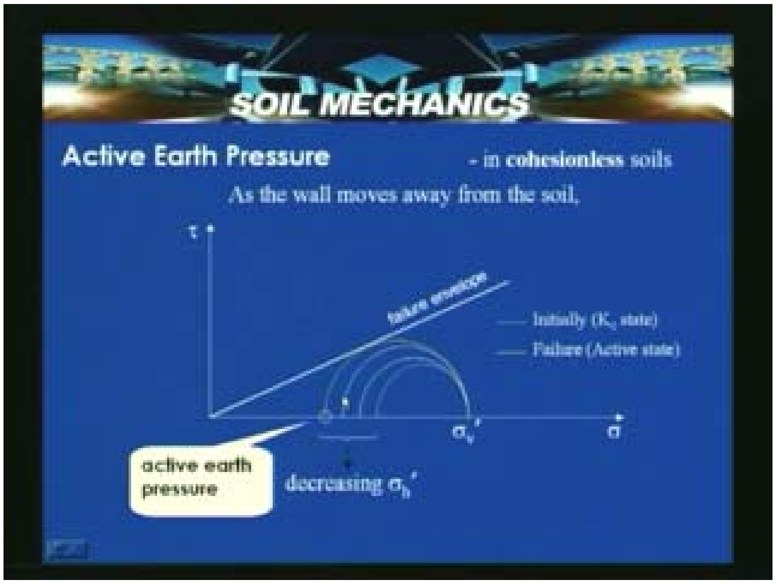


So here this condition, series of more circles are possible with $sigma_v$ dash as constant with that what will happen is that gradually the wall attains failure and the moment it comes in contact. Similarly if you have got a c five soil then what will happen? The failure envelope will be there like this and that becomes the concepts are same but we are actually being introduced for a cohesion less soil or granular soils but if it is a c five soil the failure envelope will be here and then the state will be initially $k_0$ state circle, Mohr circle will be here and then the moment it comes in contact with the failure envelope, it attains failure.

So I will set to be in the active state. Once again let us look into it so this is the active the Mohr circle tow sigma plot which is shown here and this is the $sigma_v$ which is the vertical stress which we know. So from the Mohr circle fundamental which we are studied in shear strain, from the known state of stress on a Mohr circle when you draw a extended line to the other part of the Mohr circle and that is we can locate that point as a pole. In this case the pole is on the left hand side here which is for the active state here and this is a failure plan for a c phi soil having c and phi. That is cohesion and friction (Refer Slide Time: 30:11) intercept here, the c dash and friction angle is phi dash.

So this is when the $sigma_h$ decreasing that means the initial Mohr circle somewhere here and then it actually change it to or $sigma_h$ dash decreasing up to $sigma_a$ and attains failure plane. So this more circle is developed, this is nothing but a failure Mohr circle at the active state so Rankine's theory of earth pressure in the active state when you represented Mohr circles its how looks like this.

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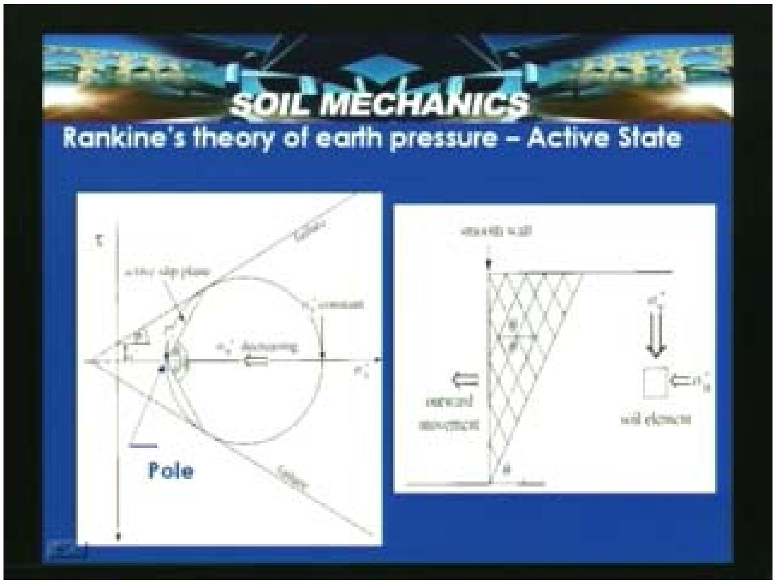


That means here these elements, each and every element here attains failure and attains plastic equilibrium condition means that they attain failure and reach the active state of plastic equilibrium conditions. So here this condition is represented so this failure plane which is when you draw a line from here to this point to the point where this point cuts the failure envelope and join this, this inclination is nothing but theta which is nothing but 45 plus phi by 2 that is 45 plus phi dash is the friction angle. So as the soil is homogeneous it can have a failure plan like this, it can have a failure plan like this so this failure plane actually represents this failure plan which is here with this (Refer Slide Time: 31:29). Now let us try to look into how to you know arrive at this magnitude of active earth pressure. We have introduced the concept of the Rankine's active earth pressure when it actually attains the state of plastic equilibrium condition.

Now consider like in the previous case what we discussed is that this is a Mohr circle in a tow sigma plot, it's a failure Mohr circle. So here for upper part as well as lower part of the Mohr circle is shown. So in the next slide we have considered only in a tow sigma plot where only the upper part of the Mohr circle is shown and here this is again a failure envelope with phi dash being the friction angle, angle of internal friction and sigma$_v$ dash is the vertical stress at a depth z which is nothing but a gamma z and sigma$_h$ dash, the active is nothing but sigma$_a$ which is active earth pressure.

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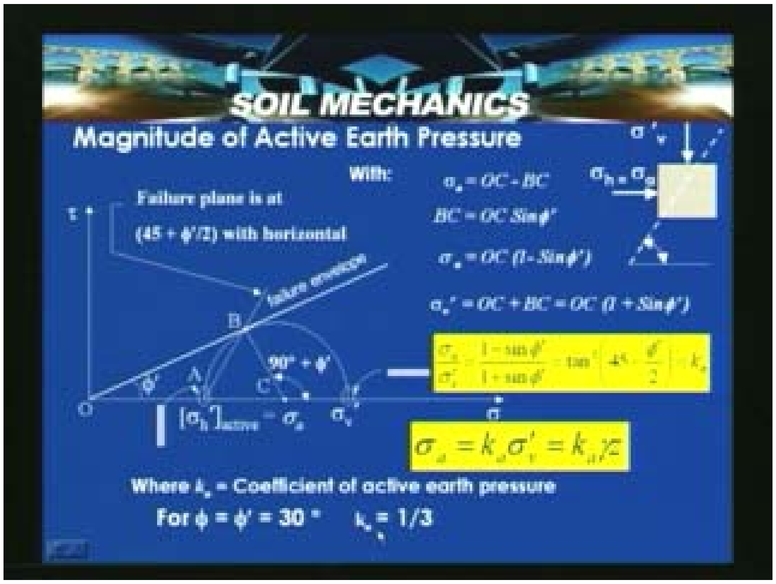


So when this Mohr circle is having points intersects at sigma axis at A and at point B at the failure envelope and at point c at the center of the Mohr circle. So from the geometric conditions, we can say the sigma$_a$ is equal to oc minus bc which is nothing but this distance. But from this condition, from this triangle obc we can see that bc is equal to oc sin phi dash. So now substitute here for bc is equal to oc sin phi dash that is sigma$_a$ is equal to oc into 1 minus sin phi dash and similarly sigma$_v$ dash we can write this stress is nothing but oc plus bc. Now bc is equal to oc sin phi dash so with that we can say that sigma$_v$ dash is equal to oc plus bc which is nothing but oc sin phi dash which we can be written as oc into 1+ sin phi dash.

So now the ratio of sigma a by sigma$_v$ dash which is nothing but oc minus bc divided by oc plus bc. So oc and oc will get canceled so what we get is that sigma$_a$ by sigma$_v$ is equal to 1 minus sin phi dash by 1 plus sin phi dash which is nothing but tan square 45 minus phi dash by 2 which is actually indicated by k$_a$ which is nothing but a coefficient of active earth pressure. Now we can write sigma$_a$ is equal to k$_a$ into sigma$_v$ dash because sigma$_a$ by sigma$_v$ dash is equal k$_a$ so sigma$_a$ is equal to k$_a$ and sigma$_v$ dash so sigma$_v$ dash is nothing but at given depth gamma z. So at depth z is equal to zero then it becomes sigma$_a$ is equal to zero, at depth h, say z equal to h then sigma$_a$ is equal to k$_a$ gamma h.

So where the coefficient of active earth pressure is indicated by k$_a$ say for phi is equal to phi dash is equal to 30 dash with k$_a$ is equal to 1 minus sin phi dash by 1 plus sin phi dash we get k$_a$ is equal to 1 by 3 that is nothing but 0.333. Similarly for the passive state if you consider this is initially k$_0$ state and this is the failure envelope and here in this case the wall moves towards the soil. So you can see that when the wall is moved towards the soil it tries to attain and reach the failure surface but in this case the sigma$_{h0}$ which is initially coefficient of horizontal earth pressure at rest which is actually changes to sigma$_p$ by increasing. So it happens like this, with the same sigma$_v$ dash these Mohr circles propagate towards the right and attains the failure.

So initially the sigma$_h$ dash now which is changed on to sigma with increasing sigma$_h$ dash reaches to sigma$_p$ that is nothing but a passive earth pressure. So here the failure condition which is wrongly shown here which is failure passive state which is actually indicated by passive earth pressure. Now the condition like similarly like what we discuss in case of active condition, similar way we can derive also for passive earth pressure conditions in cohesion less soils.

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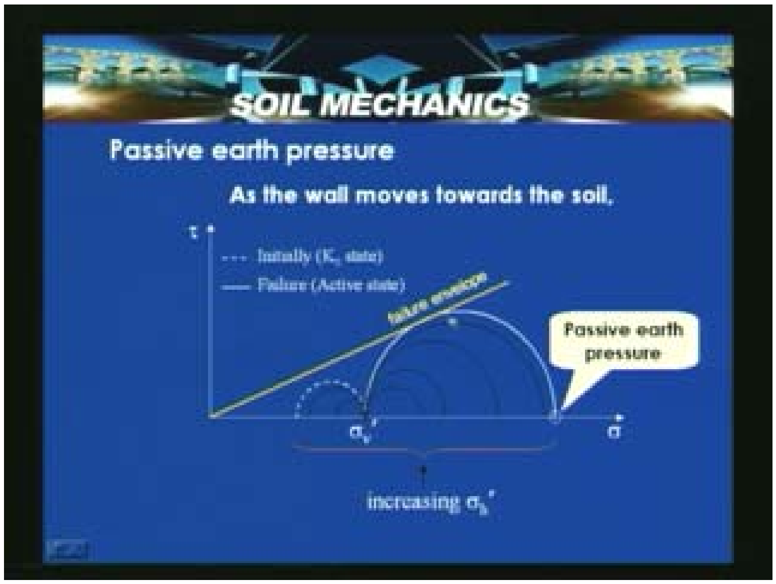


So here the same element is taken sigma$_v$ dash, now sigma$_v$ dash which is the vertical stress, sigma$_v$ or the v dash the vertical stress and sigma$_p$ is the horizontal stress which is at passive state that is the passive earth pressure. Now this is that Mohr circle at failure so sigma$_v$ dash is here and which is minor principle stress and the sigma$_h$ dash passive is equal to sigma$_p$ is the major principle stress and with this now, this becomes the major principle plane that is this is the major principle plane which is actually shown here, this is the major principle plane and this is the minor principle plane, this horizontal is the minor principle plane so sigma$_v$ dash vertical stress is acting on the minor principle plane.

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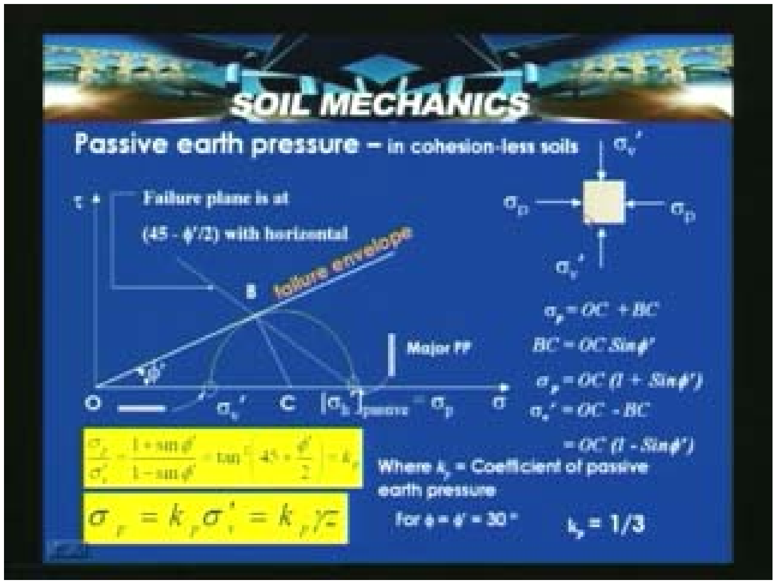


Now this is the known point for us and when you draw from the known state of stress to a point on the Mohr circle and this becomes the pole in case of passive state. So here in the case of active state what we did is we drawn from here to here and tried to represent this and these being major and this actually is now major principle plane and this being minor and this actually is a minor principle plane here. The failure plane is indicated here 45 plus phi by 2, the point where it coin touches with failure envelope. The failure plane is at 45 plus phi by 2 with horizontal plane.

Similarly here which is shown for passive earth pressure condition and attains passive state of plastic equilibrium with increasing sigma$_h$, sigma$_h$ dash changes to sigma$_p$. Again with the similar conditions like sigma$_p$ is equal to we can say oc plus bc that is oc plus bc here. The bc is equal to oc sin phi dash with that sigma$_p$ is equal to oc into 1 plus sin phi dash, sigma$_v$ dash is equal to oc minus bc with that oc into 1 minus sin phi dash. So the ratio of sigma$_p$ by sigma$_v$ dash is equal to 1 plus sin phi by 1 minus phi is equal to tan square 45 plus phi dash by 2 is equal to $k_p$ where $k_p$ is the coefficient of passive earth pressure. Now passive earth pressure which is now indicated as sigma$_p$ is equal to $k_p$ sigma$_v$ dash where sigma$_v$ dash is equal to gamma z at a depth z from the ground surface.

So in case of passive state, $k_p$ is defined as coefficient of passive earth pressure say for phi is equal to phi dash is equal to 30 degrees, $k_p$ is equal to 3. This is wrongly indicated (Refer Slide Time: 39:13) which is 3. So Mohr circle at rest, active and passive states which are combinely represented in a tow sigma plot. So this is the same soil which is having a failure envelope and the angle of internal friction is 30 degrees and this is the failure Mohr circle and this is the initial Mohr circle that is at rest state and once attains this failure which actually changes to active state Mohr circle.

From here sigma$_h$ z dash is that vertical stress at a depth z so from this point when you draw a line to the other end in the Mohr circle and that point becomes the pole for active state which is shown here with a highlighted portion and cd is the failure plan, inclination which is nothing but 45 plus phi dash by 2 here. So sigma$_x$ dash is equal to sigma$_3$ dash nothing but the minor principle stress is equal to sigma$_a$ dash is nothing but k$_a$ times sigma$_z$ dash which is nothing but coefficient of active earth pressure times sigma$_z$ dash.

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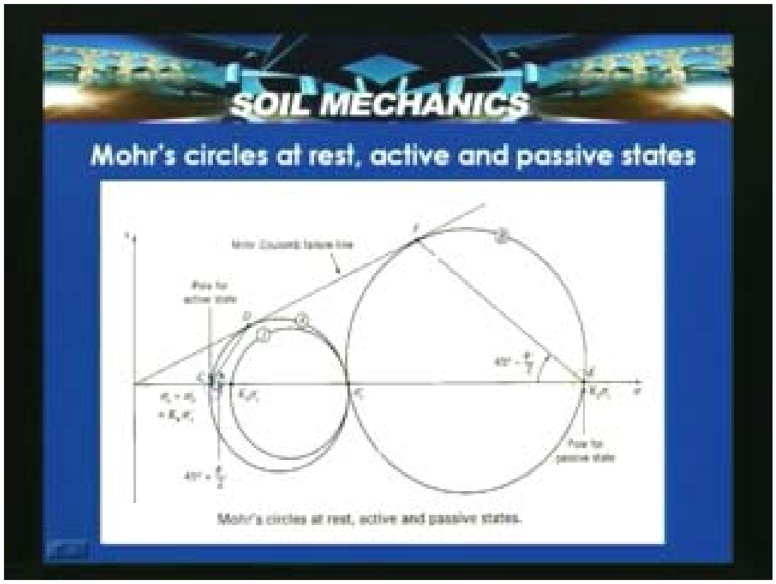


So if you compare the water and solid, water and soil has got some resemblance like water it exerts lateral earth pressure. So as we discussed in the previous lecture so here we can say that which is nothing but k$_a$ times sigma z dash, it actually transpose where k$_a$ is the coefficient of earth pressure in active state. Similarly Mohr circle which is shown here is for passive state where sigma$_z$ dash is the vertical stress and this is the Mohr column failure line and when you draw a line to the other end of the Mohr circle, we can say that we can locate the pole.

The pole for the passive state from here, from e if you draw a line to the failure envelope then it intersects at f then this nothing but the angle of inclination of a failure plane, in case of passive state that is 45 minus phi dash by . So in this case also if failure plane can be possible if the soil is homogenous.

So here in this slide combinely in Mohr circles at rest and active and passive states. So this Mohr circle is for at rest condition and this Mohr circle is for active state condition and this Mohr circle is for the passive state condition. Now we have discussed two states, active states and passive states and when you look into it k$_0$ is actually greater than k$_a$. So then we should ask ourselves, shouldn't we design retaining walls to resist than active case earth pressure, since the thrust on the walls is greater than the k instant. That means do we need to design walls for earth pressure at rest conditions.

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The next forth coming slides will answer our question. So our question is that should we design retaining walls to resist at rest than active earth pressures since the thrust on the walls is greater in the $k_0$ state because if you look into it the $k_0$ is actually greater than k which indicates that at a particular thrust if you compute with p p that is $p_0$ that is the active, the thrust at the rest condition which is half $k_0$ gamma h square it will be more than half $k_a$ gamma h square which is the thrust due to in active state. Now if you look into it, this is in cohesion less soils $sigma_h$ dash which is in the $k_0$ state but with a little movement, as the wall moves away from the soil $sigma_h$ decreases and till failure occurs. It is next impossible to maintain these conditions; also it will become unconservative to actually design for these conditions.

So except in some special cases like a basement walls when they are subjected and such type of conditions where $k_0$ conditions assure that they prevail then they can be designed for that Mohr pressure but otherwise when the cantilever wall or is a cantilever sheet pile wall, a retaining the soil and in this case they have to be designed for active state when they are subjected to… For attaining this active state, from the next slide we can see that a minimum movement sufficient to actually reach that active state. So it is seldom impossible to actually maintain that $k_o$ conditions so which is actually it's not wise enough to actually design and have a conservative design.

So it's actually design for active state only so as the wall moves away from the soil $sigma_h$ decreases till failure occurs. So this is a $k_0$ state and with wall movement as you can see that it actually attains an active state and it's maintained.

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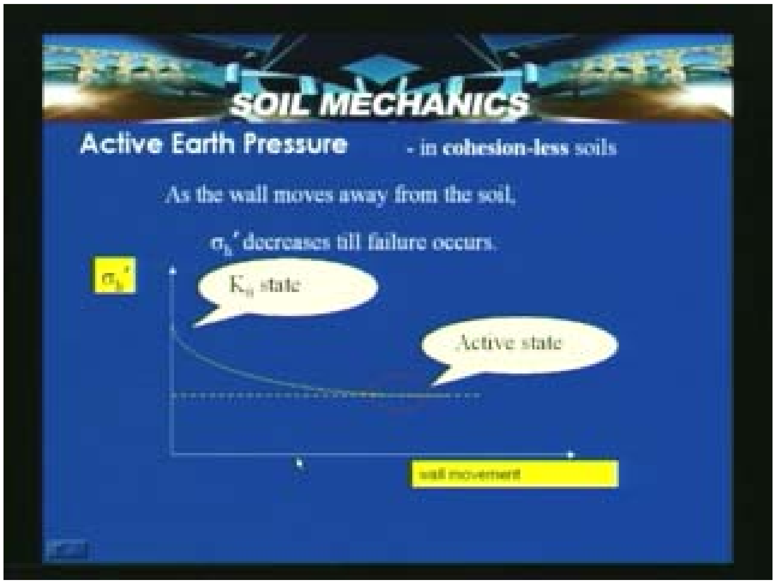


The relation between lateral earth pressure and movement of the soil if you look into it. For example if $h_0$ is the height which is undergoing say lateral movement here, say k condition which is shown and the wall actually is at rest condition that means here this is the portion. Now the active condition can occur when wall actually rotates about the tow that means if rotation is nothing but delta x by $H_0$ say gradually the wall yielded away from the backfill or rotated about the backfill.

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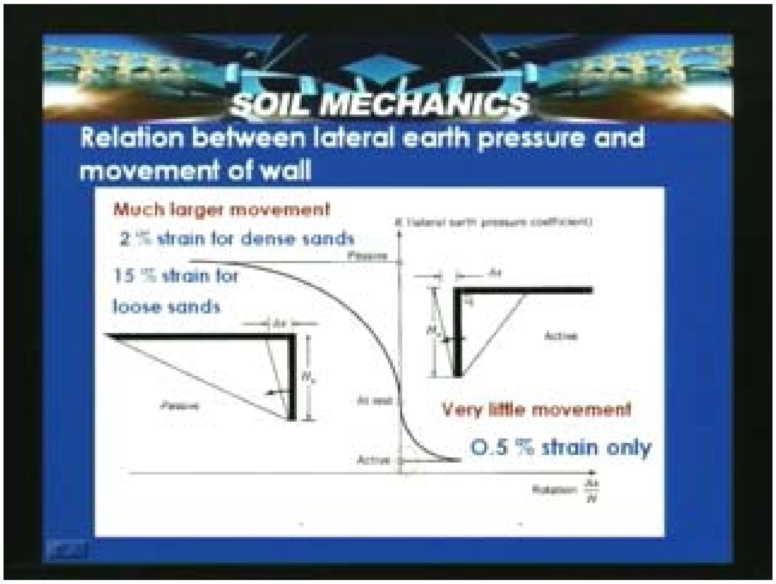


So in such situation you can see that the active condition actually occur with a 0.5% strain only with very low strains but whereas in case of a passive condition much larger movements are required. So that is in case it is also a function of type of soil like dense sand or loose sand, like in the soil states particularly 2% strain for dense sands and 50% strain for loose sands. So if the compressive strain if it actually reaches 15% or so for loose sands then actually it will actually attain the passive condition whereas in case of say dense sand it actually required only 2% strain. So keep in this in point of you, actually design the walls for active state condition only. So here in this diagram what you are seeing is a delta x by h and k that is lateral earth pressure coefficient. So here much larger movements are required for passive condition to be developed and here very little movements are required to actually attain these active conditions.

So keeping this point of you, actually we design the walls for the active pressures and in case if there is a passive state arises like bridge apartments when they are apartments when they are push it towards the retain soil then the condition of passive state occurs there in that particular side. These type of conditions like what we discuss like development of this rupture planes, all this things can be also demonstrated through physical model experiments. So in this place a photograph, a demonstration experiment which was carried out in the physical modeling experiment high gravities at IIT Bombay is shown. Here what you can see is that this is a cantilever retaining wall having a active straight at the top wall movement away from the backfill and this is the passive state which is actually embedded here.

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So here is a model cantilever wall and this is a soil which is nothing but a sandy soil in dry state and having relative density about 55% then wall yielded about certain point here and moved away from the backfill. So in the process what you can see the development of this, these lines are nothing but sand marker lines.

So these lines you can see that they cut into number of planes and these planes indicate that this rupture planes which are developed, they are nothing but these which we discussed in the previous case as the wall moves away from the fill, you can see that the soil followed the wall and settled in this case and induced the rupture planes. So here in this you can see that because of the condition here represent at this particular state is actually is a plastic equilibrium state, the rest of the state is here the elastic equilibrium still remain. So these are high gravity experiments which are carried out at IIT Bombay by using national (jet melo interference) (Refer Slide Time: 48:11) facility available at IIT. So in this case the soil is not yet reached the passive state so the sand marked lines remains intact.

So this is another view of the same model, this is the deflected wall, you can see the wall deflection and you can see here the formation of slip lines. In this zones it indicates that the plastic equilibrium has developed, the soil has actually attained the failure and active state has reached and this is the outward movement of the wall, the wall was initially here and moved in this direction, so in the process what will happen you can see this cut like, the bench like formations or deformations at the top they indicate that the movement or a settlement of the wall, settlement of the backfill soil. So these are shown just to see how the demonstration experiments can be carried out and also to understand the concepts what we discussed just now like active states and passive states.

So when the wall which is say a smooth and a backfill surface is horizontal then we refer that as a active wall and with that actually were two states are possible one is active state and another one is passive state. The passive state can occur when wall movement takes place towards the soil and active state can occur wall movement takes place away the soil. So in the process when the wall moves away from the backfill, with that there is a lateral expansion takes place, the $\sigma_h$ dash which was initially at rest horizontal pressure reduces to $\sigma_a$.

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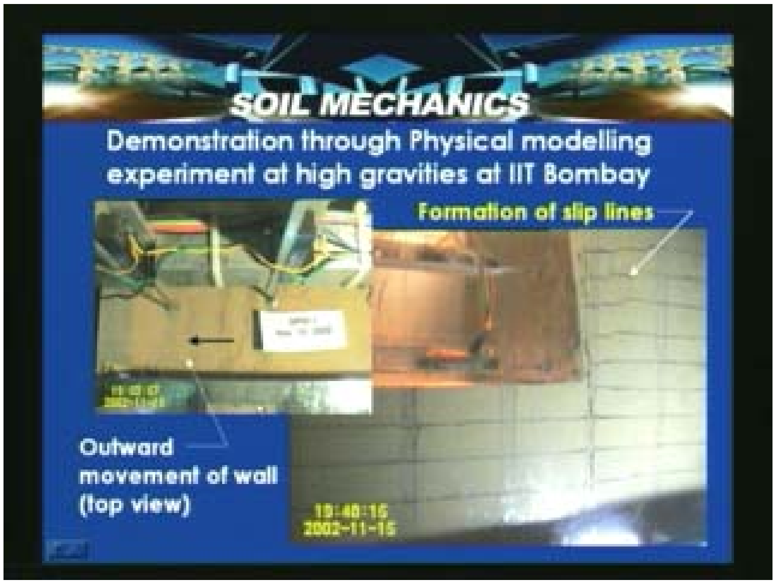


So the reduction is actually due to resistance offered by the soil against the wall movement and in this case of passive state when the wall moves towards the soil, so such situation again the soil offers the resistant that is the resistance or the strength in the soil offers against the movement. So with that the $\text{sigma}_h$ dash has to mobilize more so it reaches to the passive state which is nothing but a passive state of plastic equilibrium and that pressure is called $\text{sigma}_p$ which is coefficient of $\text{sigma}_p$ is the passive earth pressure.

So now if you have got a wall of height h, when the wall movement takes place away from the backfill and at top the earth pressure is zero, at bottom the earth pressure is $k_a$ gamma h and in the case where if it is dry condition and the lateral active thrust is half k gamma h square, acting at h by 3 from the base. In the case of passive if the wall is actually having height h and the movement takes place towards the soil then the total passive thrust is half $k_p$ gamma h square and $k_p$ gamma h being the passive earth pressure at the base and zero at the top, with that what will happen is that the passive earth pressure thrust is half $k_p$ gamma h square acting at h by 3 from the base.

Now we can extend these theories to number of cases like wall in submerged case, with surcharge without surcharge and walls which are having cohesive soils and combination of layered soils. So with that we can actually use and calculate earth pressures and once we get these earth pressures and these earth pressures can be used to design the retaining structures. So these are the inputs what we are studying which is actually helpful in arriving at in designing the retaining structures.