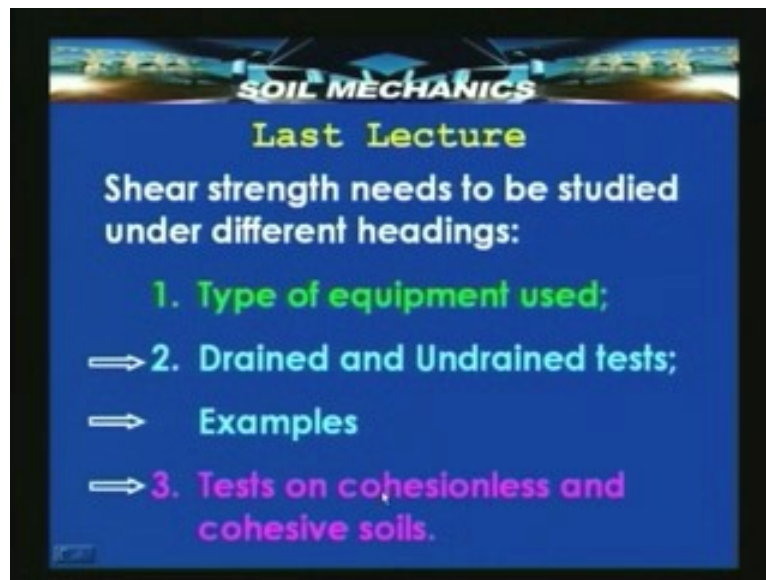


Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
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Lecture – 49
Shear Strength of Soils
Lecture No.7

Students we have gradually come to the very last lecture on the shear strength of soils. We have seen 6 different aspects connected to the shear strength of soils over the last 6 lectures and today is the concluding lecture on whatever items are still remaining with respect to shear strength of soils which a typical student should know. As usual let's take a quick look at whatever we had passed through in the last lecture.

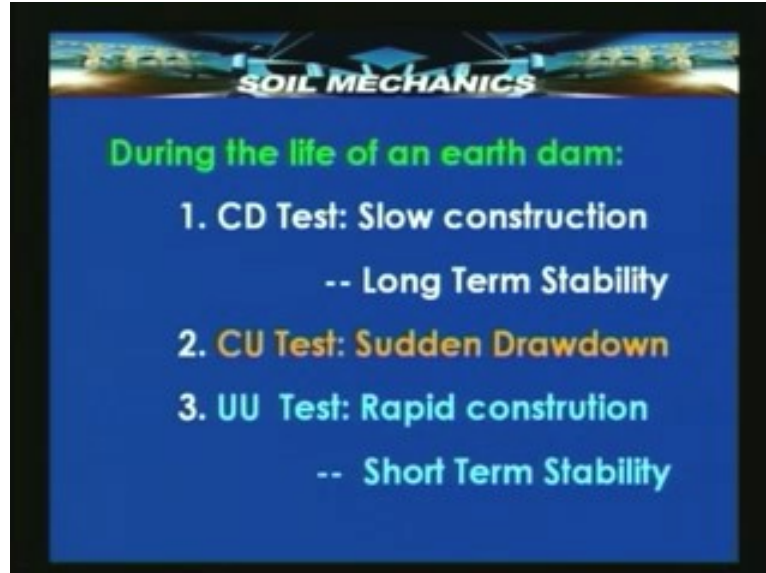
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In the last lecture we saw quite in detail what are the drained test, what are undrained test both direct shear and triaxial shear and where they are used and we even saw some numerical examples. We also saw that it is possible to look at the topic of shear strength in a totally different way in terms of the soil. That is how does cohesionless soil behave under shear and how cohesive soil behaves under shear. We saw in particular the application of the different types of drained and undrained test with a typical example of the earth dam. During the course of life of an earth dam there are several stages it passes through starting from construction to reservoir filling and drawdown to long term stability. And under each conditions the shear parameters which must be used in order to test the safety of the earth dam will vary and the type of that will give the appropriate parameters will also vary.

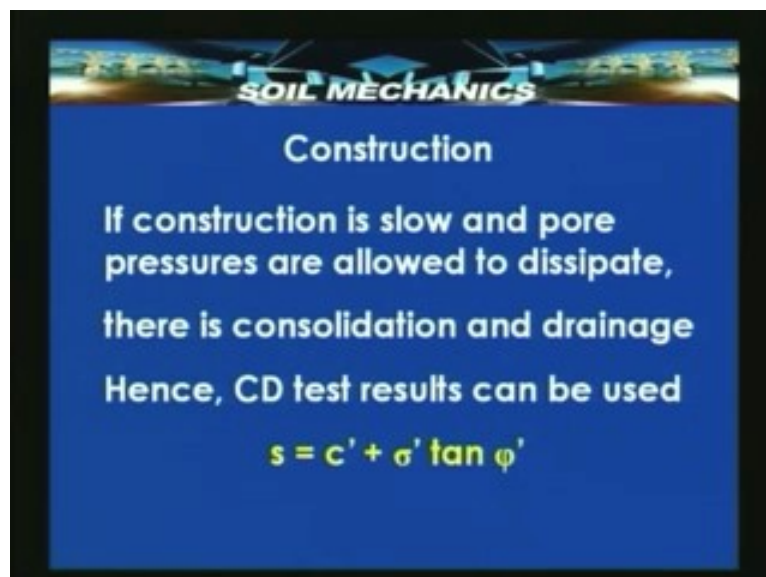
For example the CD test which we saw was good for a slow construction and also for long term stability where in consolidation would have already taken place that mean drainage would have occurred, shear stress also does not produce any pore pressure.

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On the other hand suppose a dam has already been filled, the reservoir is full and the reservoir is emptied somewhat suddenly rapidly relatively for certain purpose. Then the test that is applicable and the corresponding parameter would be the consolidated undrained test because the earth dam has already undergone the consolidation during its life time filling up the reservoir. But now sudden depreciation of reservoir produces shear stresses under undrained condition. And if suppose we are building a dam very rapidly then pore pressures get developed even during construction they do not have time to dissipate and therefore the stability of the dam even during construction has to be kept in mind and that short term stability is best determined under undrained conditions using parameters that derived from a UU test.

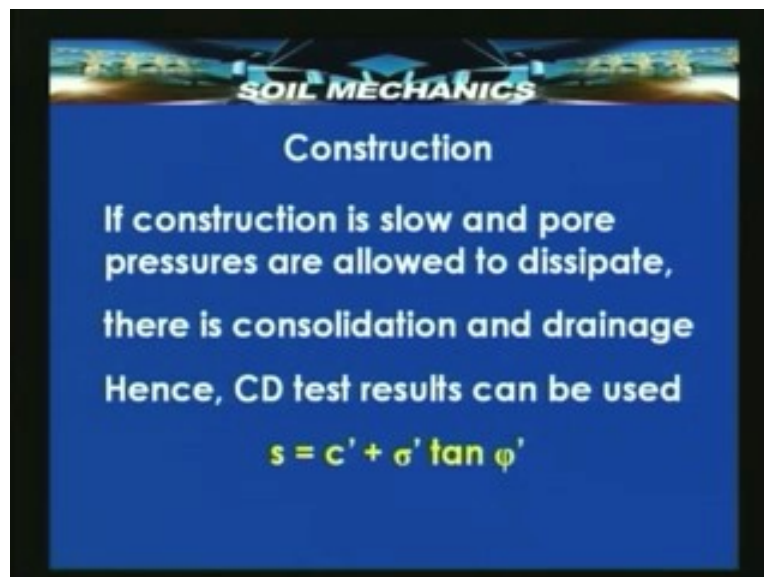
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So if you take construction, if it is slow and pore pressures are allowed to dissipate there is consolidation in drainage, s is equal to c dash plus σ dash tan ϕ dash where all the parameters involved are drained parameters and effective parameters. If the construction is on the other hand rapid, pore pressures are not allowed to dissipate, the shear strength equation becomes s is equal to c_u plus σ tan ϕ_u where c_u and ϕ_u corresponds to undrained conditions. If drawdown is being considered the parameters which will be applicable will be c_{cu} and ϕ_{cu} and the shear strength equation would be s is equal to c_{cu} plus σ tan ϕ_{cu} . Under long term conditions once again drained parameters will apply and s is equal to c dash plus σ dash tan ϕ dash. There are a number of factors which affect shear strength, we must know them and their influence and so we spent quiet sometime in the last lecture to understand the impact of many factors which affect the shear strength.

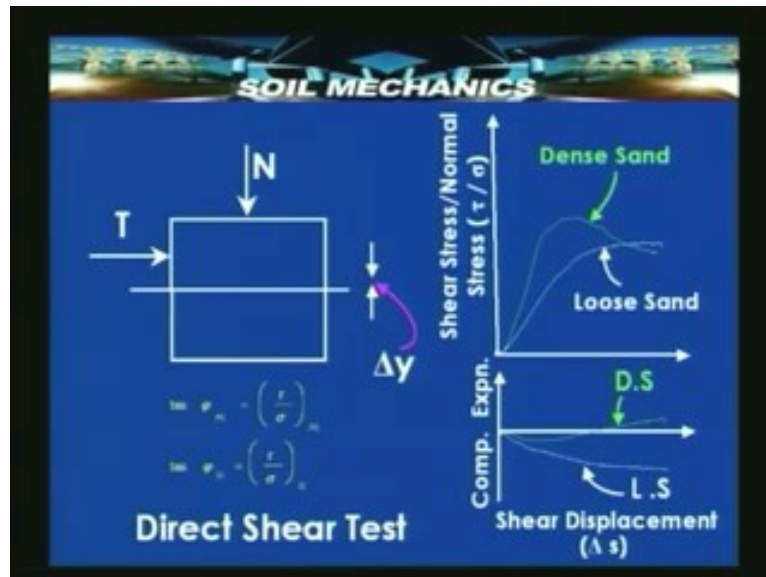
Here is simply a list which I am reproducing to remind what are the factors which influence shear strength. The first three factors shape and gradation of particles, relative density the compaction the denseness with which the soil is packed and the confining pressure that is applied to the soil or the confining pressure under which the soil is at any given depth. These are all parameters which are particularly important from point of view of shear strength of cohesion less soils. On the other hand for cohesive soils what are more important obviously because of their low permeability are drainage conditions, degree of saturation, rate of loading.

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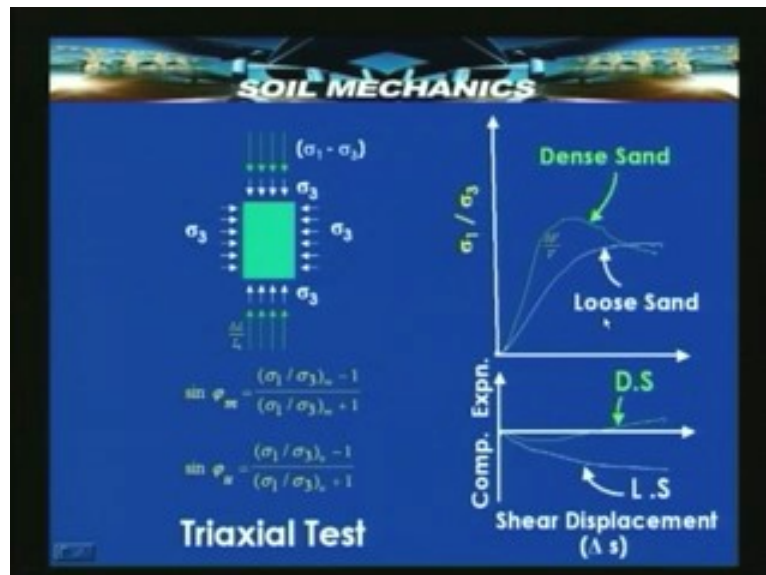
In addition to this there is another important parameter which during the course of today's lecture we will pay some attention to that is the stress history. The history which shows what kind of stress the soil has undergone in the past, this is particularly important for clayey soils. Let us look at the shear strength from point of view of the type of soil. If you have a cohesionless soil and subjected to direct shear test, this is a kind of relationship you will get between shear stress and the shear displacement. If it is a dense soil it will undergo a little bit of compression and then expansion.

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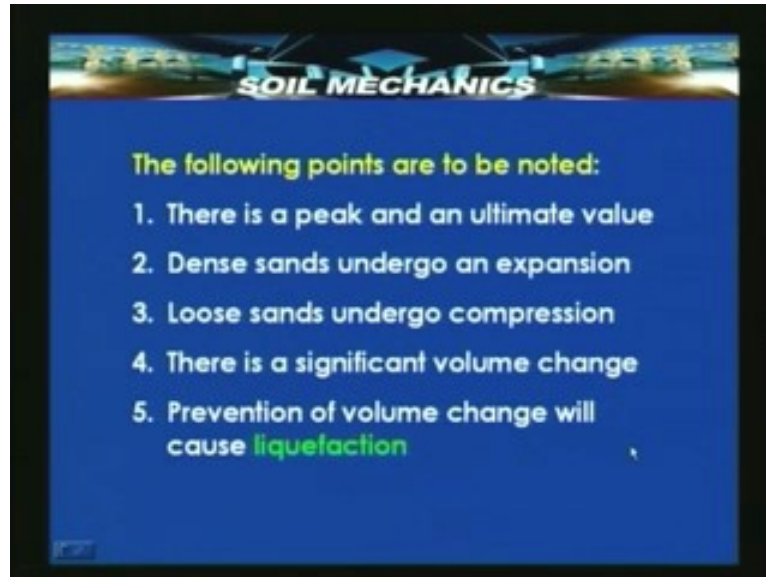
If it is a loose sand it will under compression continuously. There will be a peak shear stress and an ultimate shear stress as the displacement increases. The same thing can be observed in a triaxial test also, for dense sand there is a peak, for loose sand there is no marked peak. Loose sand undergoes compaction or compression, dense sand undergoes a little bit of compression initially but then undergoes expansion subsequently, it becomes loosened.

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So with respect to behavior of a soil whether it is cohesionless or cohesive under shear we can see a few special points which are summarized here.

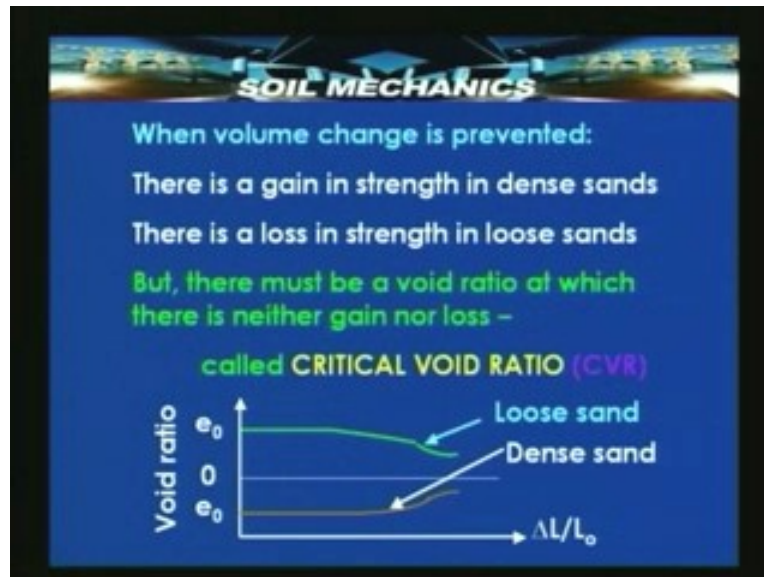
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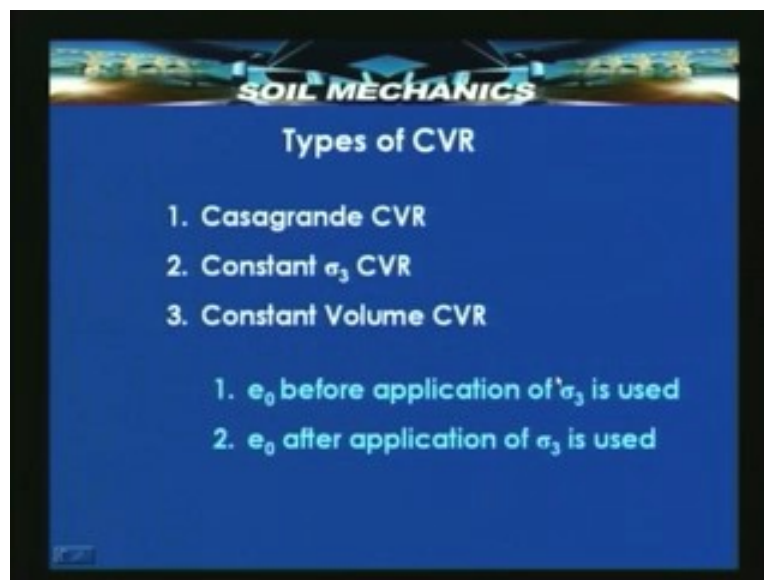
One there is a peak and an ultimate which we saw in the previous diagram. Dense sands undergo an expansion mostly although they undergo a little bit of compression initially, loose sands undergo compression. There is a significant volume change the amount of compression or the amount of expansion is quite significant under shear.

Suppose we prevent this volume change then what will happen is the shear stress that is responsible for causing this volume change will not be borne by the soil and the soil will tend to behave like a liquid and that's known as liquefaction. We also saw last time, having talked about liquefaction what is critical void ratio and how that can be used to decide whether soil will liquefy under a given condition or not. The concept of critical void ratio can be very easily understood from the statement which I made a few minutes back that dense sand undergo expansion, loose sand undergo compression.

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Suppose you have a sand which is neither loose nor dense relatively speaking then if the void ratio remains unchanged throughout shear then that's the soil which will undergo liquefaction because it doesn't undergo volume change during shear and since it is unable to undergo volume change during shear it will liquefy. So what's that void ratio at which the soil will behave like this that can be obtained by plotting void ratio change with axial strain as shown here and finding out which is the void ratio where the volume change will be nullified and that is known as CVR. Depending upon the type of triaxial test we do, there are different values of CVR. The most popularly used one is the Casagrande CVR which is nothing but the void ratio before application

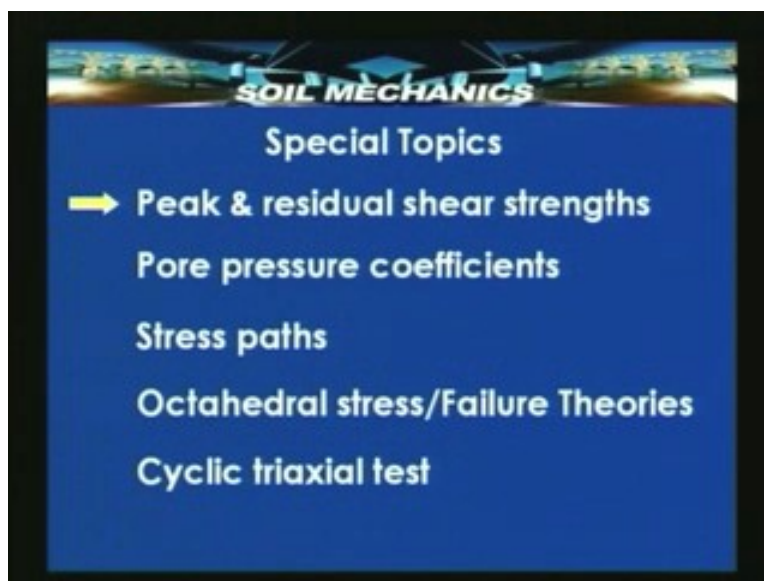
of σ_3 and constant σ_3 CVR is also possible, constant volume CVR is also possible to determine.

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In the second instant where the constant volume CVR test is conducted e_0 after the application of σ_3 is used as the critical void ratio. Now in today's lecture we will see an assortment of topics which I am calling as special topics, all of which are very important. Some of them are slightly advanced and I will try to give an exposure to these topics rather than go into great details. For example let me start with a list of these so called special topics.

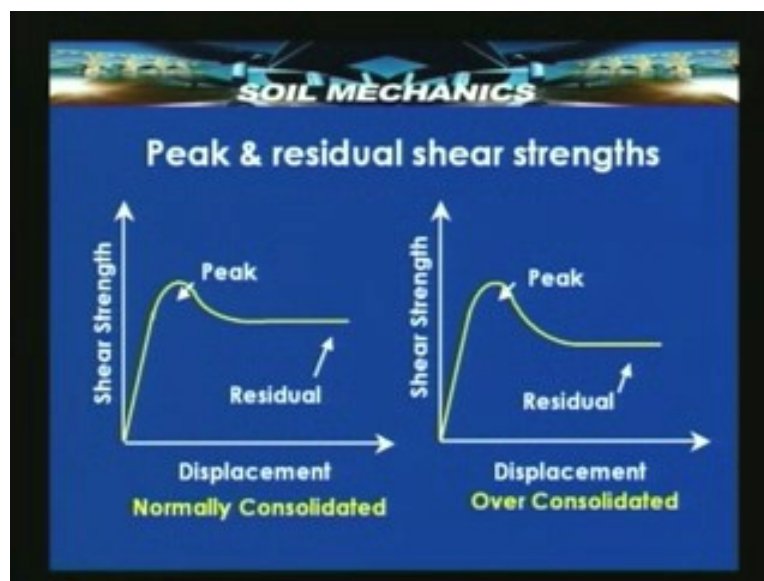
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Peak and residual shear strengths is something which we need to understand very well which is very important from the point of view of particularly slope stability or stability of any structure also. Then we need to understand something known as pore pressure coefficients then what are stress paths? I had mentioned that shear strength of a clayey soil particularly depends upon the stress history. So one needs to know what is the path through which this stresses have changed. Then we need to know something about failure, we have only been seeing so far the Mohr coulomb theory but there are other theories as well because sometime the soils do not behave strictly according to the Mohr coulomb theory although it is universally valid and universally used also. But there are theories which are particularly applicable to clays especially in the plastic deformation stage and those are known better as yield theories. We will have to see those also. And then finally we will see a very important test called cyclic triaxial test. These are days were awareness about earthquakes and their damage has increased like anything because of past recent experiences.

There is one test known as the cyclic triaxial test which is useful in determining the liquefaction strength of a soil. It helps us to predict under what conditions a soil will liquefy and whether the present condition is indicative of a potential to liquefy or not. So site characterization, site assessment from point of view of liquefaction can be carried out using this cyclic triaxial test and it indirectly **obvious** the need for using the concept of CVR. The concept of CVR is now gradually being replaced by another concept based on the cyclic triaxial test. Let us start with peak and the residual shear strength and their utility.

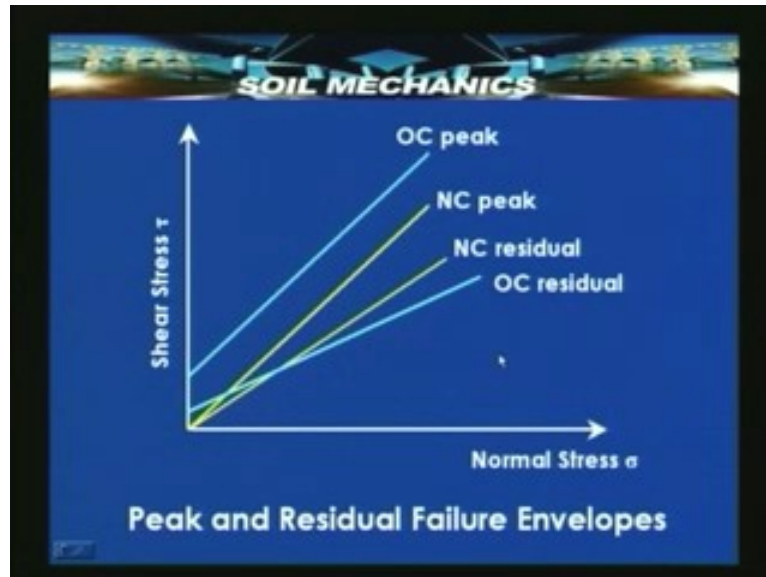
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If you take a normally consolidated soil, you will have a shear strength verses displacement curve which will be something this which shows a peak. Then it comes down and shows a residual value, with large displacements the value goes beyond and then comes down and remains almost constant. So is the case with over consolidated clays also. There is a peak and then a residual value but what is obvious from these two figures is that when the soil is over

consolidated, the fall to the residual is much more. After reaching the peak the drop in shear strength is much more with displacement over consolidated clays.

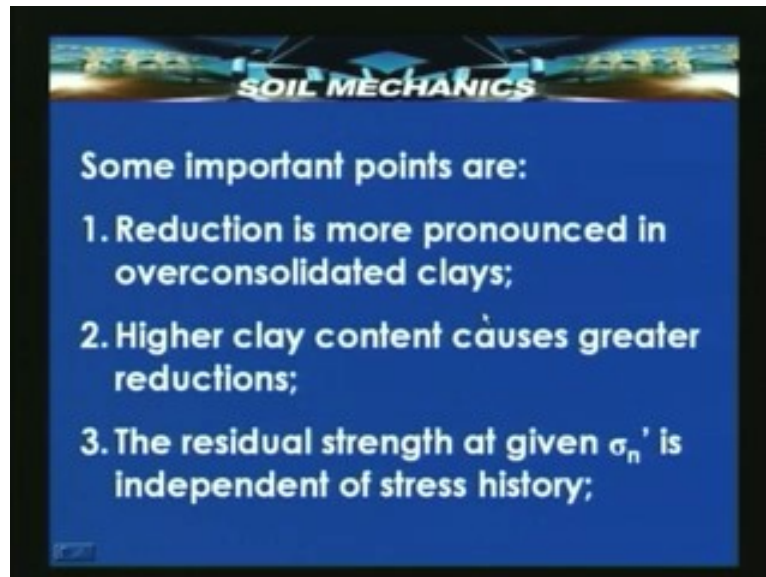
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Now if we were to plot the Mohr envelopes for the normally consolidated clay and the over consolidated clay for peak shear strength as well as for residual shear strength. Then for the normally consolidated clay under drained conditions, the peak shear strength envelop will be depicted by let us say this yellow line. Then the residual friction angle will be at least slightly lesser and that will fall somewhere below like this. Whereas if we have an over consolidated clay, the peak will be much more than in the normally consolidated soil and **let leave** and show a small amount of cohesion intercept.

As I mentioned, the drop from peak to the residual in the case of overconsolidated soil is much more than in the case of normally consolidated soil and therefore the overconsolidated residual strength envelop will invariably be below the normally consolidated residual strength envelop. The sequence in which the envelop keeps coming down is practically constant and valid for all clays. The over consolidated peak strength is very high, overconsolidated residual strength is very low, the normally consolidated strength lye below between this two line.

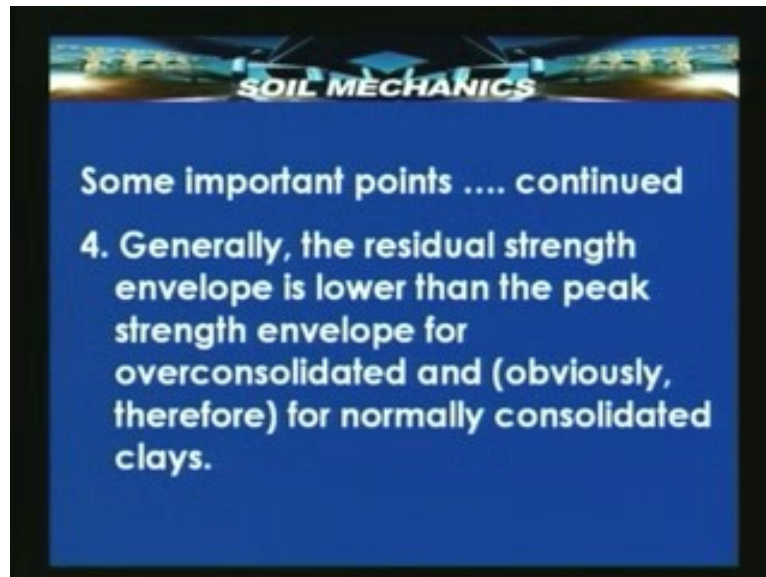
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So some of the points that arise from this discussion are; there is a more pronounced reduction in overconsolidated clays from peak to residual and another point that is worth mentioning is higher the clay content, greater is the reduction from peak to residual. The dropping strength with displacement is more if the clay content is more in the soil. The residual strength indirectly is independent of stress history, it is only proportional to the effective stress that's applied and you can see that in fact that the residual strength remains almost practically constant with displacement and irrespective of what stress history the soil has experienced and it will eventually have a certain residual strength beyond a certain level of displacement.

Generally the residual strength envelop is lower than the peak strength envelop for overconsolidated and therefore also the normally consolidated clays. Now we will pass on to the next special topic, the pore pressure coefficient. One of the important things in a triaxial test is measurement of pore pressure during the application of the confining pressure as well as the deviatoric stress. In fact we have seen in such great length in our previous lectures, how the pore pressure varies during the conduct of the test for consolidated drained, consolidated undrained and unconsolidated undrained tests. We had mapped very systematically as how the confining pressure is applied and the deviatoric stress is applied, how the pore pressure changes and therefore how the total stress and the effective stresses change and what is important therefore is to know how the pore pressure changes during the course of a test as the stresses are applied.

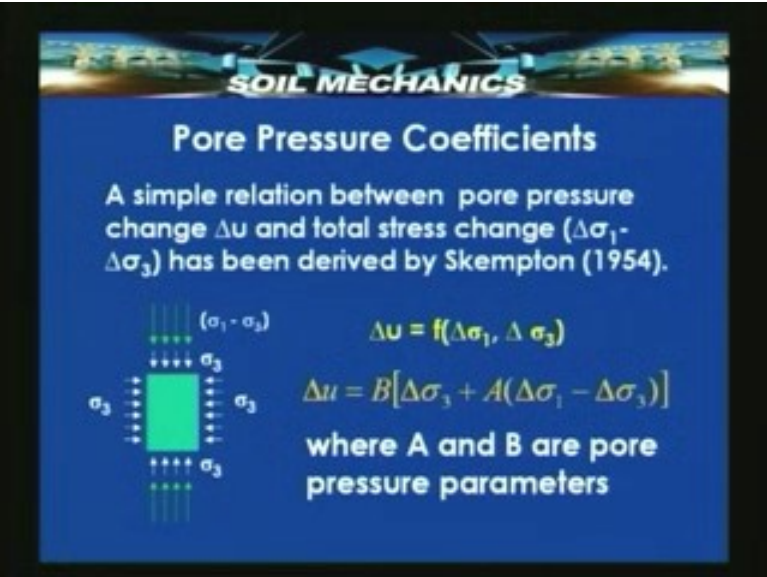
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One other way is to say is; is it possible for us to know what the pore pressure change is likely to be not necessarily only by conducting a test but by using a simple relationship, fundamental relationship. Skempton in 1954 derived a very fundamental relationship from basic consideration for evaluating the change in pore pressure during a triaxial test due to the application of the confining stress σ_3 and then the deviatoric stress. The relationship is extremely simple. ΔU the change in pore pressure is simply a function of $\Delta \sigma_1$ and $\Delta \sigma_3$ which are applied. So here it is, there is a triaxial specimen we first apply σ_3 and then we apply the deviatoric stress. The change in pore pressure, net change that will be produced will obviously be proportional first to the incremental stress in the lateral direction that is $\Delta \sigma_3$ and an increment component of increment due to the deviatoric stress, this obvious relationship between the change in pore pressure and the applied stresses.

Now the constant B and A are what are known as skempton pore pressure parameters and those are the parameters which are very important to know and if you know those parameters for a given clay, then it is possible to make a reasonably good guess about the change in pore pressure due to change in stresses. This is a very important convenience when it comes to solving or addressing practical problems were given a stress change, you will be able to predict what will be the change in pore pressure that is likely to occur. Let us take a look at what these parameters are or what are the likely values. Obviously parameter B, since it depends upon on $\Delta \sigma_3$ it is equal to one for a fully saturated soil.

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SOIL MECHANICS

Pore Pressure Coefficients

A simple relation between pore pressure change Δu and total stress change ($\Delta\sigma_1 - \Delta\sigma_3$) has been derived by Skempton (1954).

Diagram of a soil element under stress: $\sigma_1 - \sigma_3$ (vertical stress difference), σ_3 (horizontal stress).

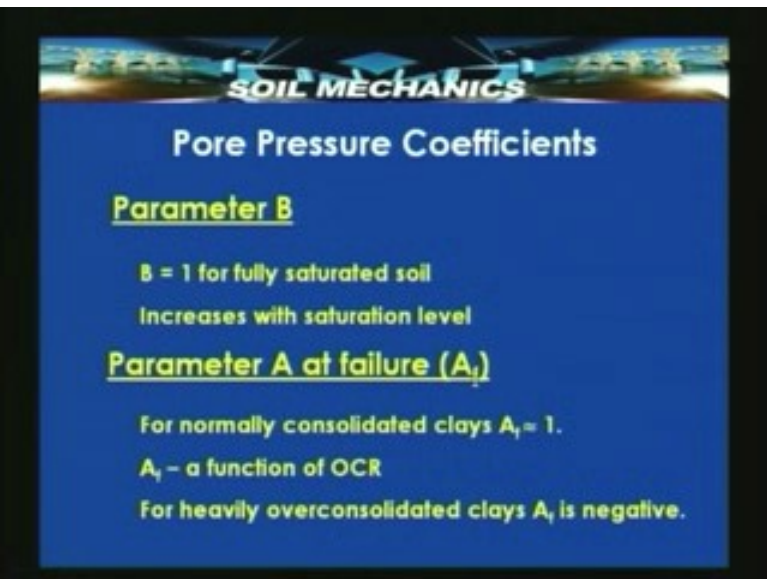
$\Delta u = f(\Delta\sigma_1, \Delta\sigma_3)$

$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$

where A and B are pore pressure parameters

In a fully saturated soil the moment you apply $\Delta\sigma_3$ an equal amount of pore pressure will be generated and therefore B has to be equal to one for a fully saturated soil ideally. Then in general a saturation improves, B will go on increasing until under full saturation it becomes one, under partly saturated conditions B will be less than one. As per the parameter A, the parameter which is of interest is really a value at failure because on the application of the deviatoric stress this $\sigma_1 - \sigma_3$ and bringing the sample to failure, what's the maximum amount of pore pressure that would have got generated and what's the corresponding coefficient A_f at failure that is what of interest.

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SOIL MECHANICS

Pore Pressure Coefficients

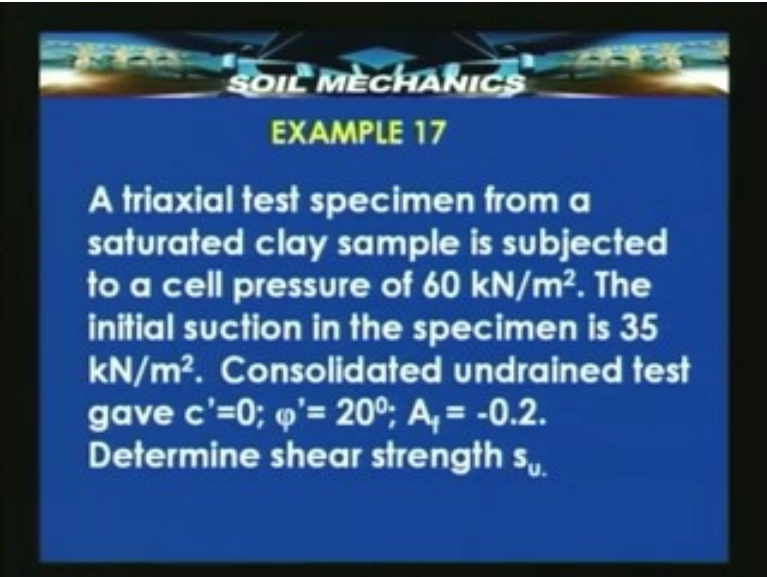
Parameter B

- $B = 1$ for fully saturated soil
- Increases with saturation level

Parameter A at failure (A_f)

- For normally consolidated clays $A_f \approx 1$.
- A_f - a function of OCR
- For heavily overconsolidated clays A_f is negative.

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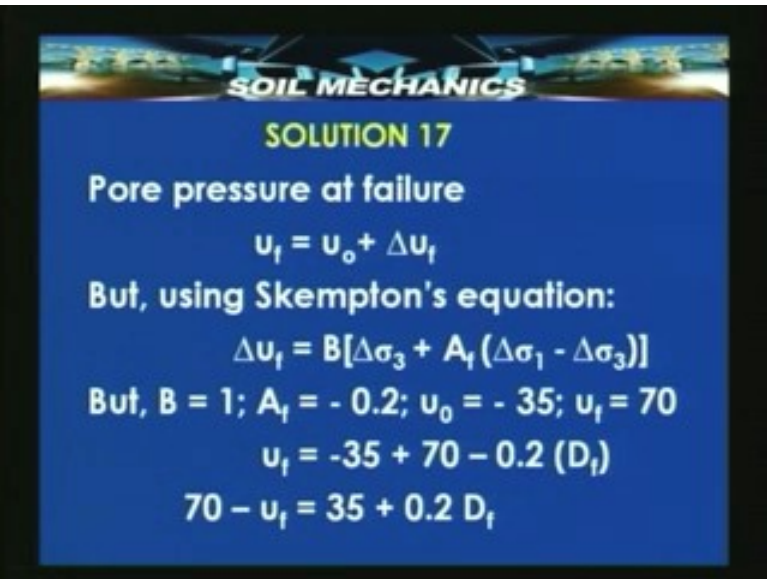
SOIL MECHANICS

EXAMPLE 17

A triaxial test specimen from a saturated clay sample is subjected to a cell pressure of 60 kN/m^2 . The initial suction in the specimen is 35 kN/m^2 . Consolidated undrained test gave $c'=0$; $\phi'=20^\circ$; $A_f = -0.2$. Determine shear strength s_u .

For normally consolidated clays this A_f is approximately equal to one that is whatever deviatoric stress you apply also gets converted into the pore pressure. But if it is an over consolidated soil A_f is a function of OCR and for heavily overconsolidated clays it could even be negative. Typical value is - 0.2 or - 3 and so on. Let us take a simple example of the use of the pore pressure parameter, let me read out the example. A triaxial test specimen from a saturated clay sample is subjected to a cell pressure of 60 kilo newton per meter square. That means a clay sample is taken and a specimen is extracted from that and it is subjected to a cell pressure of 60 kilo Newton per meter square in a triaxial experiment.

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SOIL MECHANICS

SOLUTION 17

Pore pressure at failure

$$u_f = u_o + \Delta u_f$$

But, using Skempton's equation:

$$\Delta u_f = B[\Delta \sigma_3 + A_f (\Delta \sigma_1 - \Delta \sigma_3)]$$

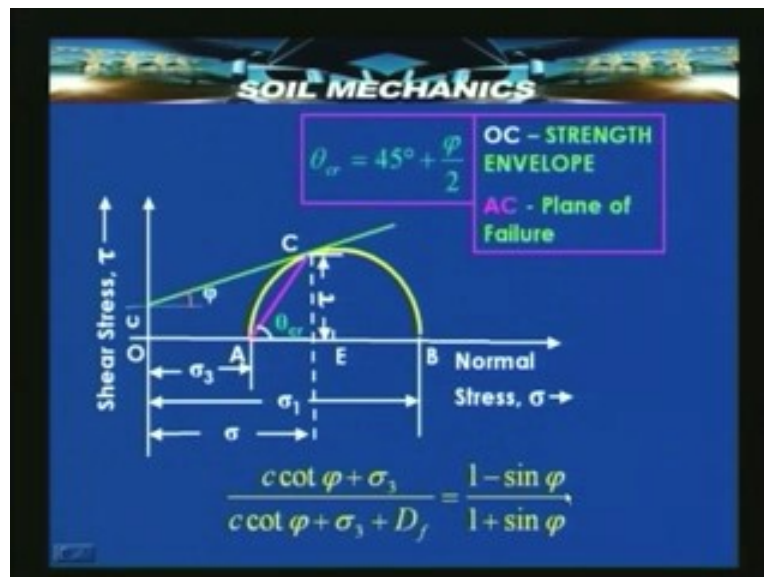
But, $B = 1$; $A_f = -0.2$; $u_o = -35$; $u_f = 70$

$$u_f = -35 + 70 - 0.2 (D_f)$$
$$70 - u_f = 35 + 0.2 D_f$$

The initial suction pressure in the specimen is 35 kilo Newton per meter square before it is saturated and then after saturation and conducting the triaxial test particularly, a consolidated undrained test in this case c dash found to be zero, ϕ dash is found to be 20 degrees and the parameter A_f at failure is found to be minus 0.2. What is of interest here is what was the suction pressure that was originally present in the soil before it was saturated and tested in the consolidated undrained test. It's very easy to compute this. Now we know that pore pressure at failure U_f is the initial pore pressure U_0 whatever was there in the sample which is not known. U_0 is what is not known and that is what is required to be determined. The pore pressure at failure will be U_f is equal to U_0 plus ΔU_f , whatever the change in pore pressure is taking place during the test. Now we know that the change in pore pressure that takes place during the test can be expressed in terms of the pore pressure coefficients.


So using skempton equations, ΔU_f will be equal to the parameter B into $\Delta \sigma_3$ which is applied plus the parameter A_f into the deviatoric stress ($\Delta \sigma_1 - \Delta \sigma_3$). Now this being a saturated sample when the test is actually started, the sample is saturated and therefore B can be taken as equal to one and A_f is given to be minus 0.2 and the initial suction pressure is -35 and the final pore pressure is 70.

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Let's take a look at the problem again before we go further. So we know that U_f has to be equal to this initial pore pressure -35 + 70 - 0.2 into the deviatoric stress D_f or 70 - the pore pressure is 35 + 0.2 D_f . Now you may recall that from simple considerations of Mohr's circle, we can drive a relationship shown here $c \cot \phi + \sigma_3$ by $c \cot \phi + \sigma_3 + \text{deviatoric stress}$ equal to $1 - \sin \phi$ by $1 + \sin \phi$. This is very useful for us in determining the final pore pressure.

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$$\frac{c \cot \varphi + \sigma_3}{c \cot \varphi + \sigma_3 + D_f} = \frac{1 - \sin \varphi}{1 + \sin \varphi}$$

Applying the above equation in terms of effective stresses:

$c' = 0; \varphi' = 20^\circ; \sigma_3' = 70 - u_f$


$$\frac{(70 - u_f)}{(70 - u_f + D_f)} = \frac{(35 + 0.2 D_f)}{(35 + 0.2 D_f + D_f)}$$

$$= 0.49$$

Hence, $D_f = 45.77 \text{ kN/m}^2$; $s_u = 22.9 \text{ kN/m}^2$.

If I apply this above equation I know c' is zero, φ' is 20, σ_3' we have just seen is 70 minus final pore pressure. So if I substitute that here, I will have c being zero, $c \cot \varphi$ becomes zero and σ_3 is nothing but in terms of effective stress $70 - U_f$ and the denominator will become $70 - U_f$ plus the deviatoric stress D_f and this must be equal to $1 - \sin \varphi$ by $1 + \sin \varphi$ and φ is 20 degrees which gives us that the right hand side is equal to 0.49. Now from this we find that D_f is 45.77 kilo newton per meter square and the shear strength undrained shear strength which we were interested in is 22.9 kilo newton per meter square.

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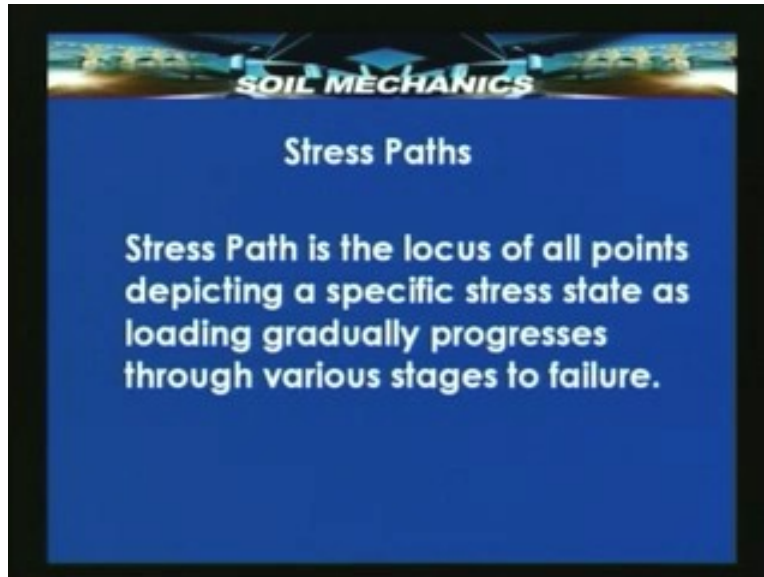


Special Topics

- Peak & residual shear strengths
- Pore pressure coefficients
- Stress paths
- Octahedral stress/Failure Theories
- Cyclic triaxial test

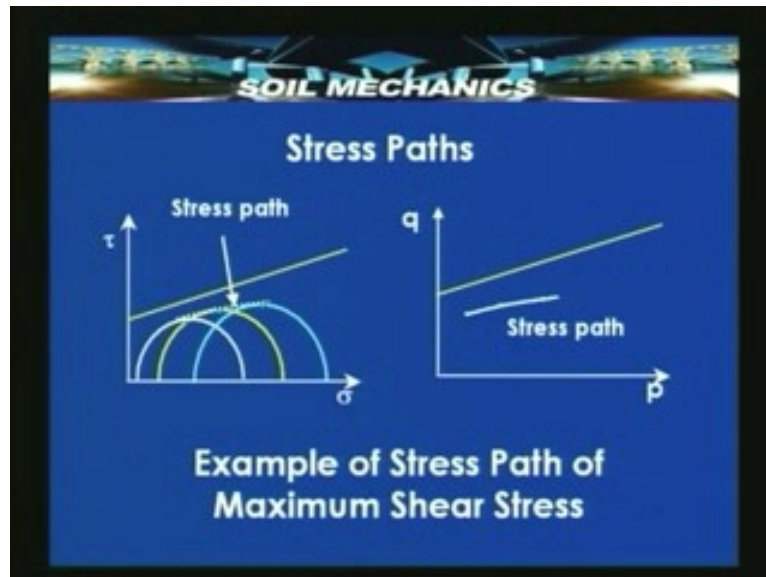
So this illustrated the use of the pore pressure parameter in determining the final pore pressure and hence the strength of the soil. Now we pass on to another important topic known as stress pulse.

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What is a stress path? Let us take a look at the basic definition. It is the locus of all points that is it's a line joining a certain points which depict a specific stress state as loading gradually progresses through various stages and causes finally failure. Suppose we are testing a sample in say triaxial test, it is subjected to loading at different stages, different amounts of loading at different stages. The stage through which the loading goes and leads to failure is what is known as the stress path. Let us take an example.

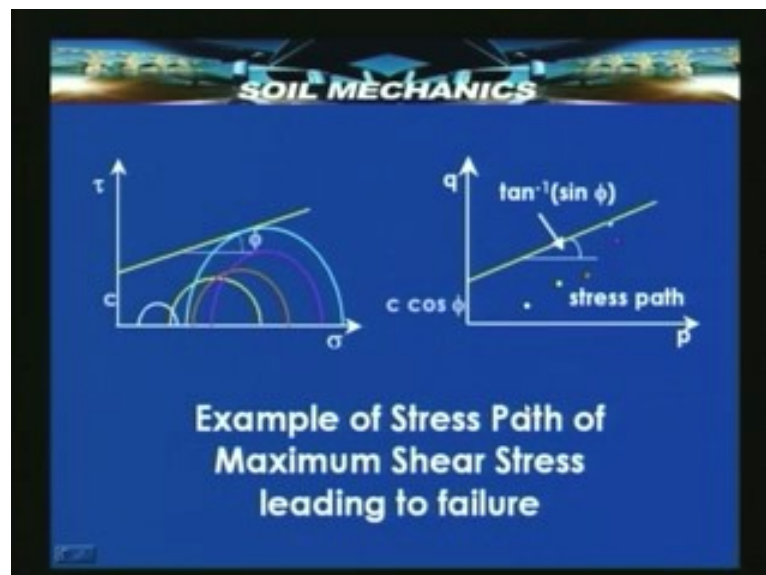
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Here is an example of a stress path which is showing how the maximum shear stress is varying as loading changes in typical triaxial test. Here is the sigma tau plot and here is the corresponding p q plot. From the sigma plot we know very well that if 3 tests are conducted 3 triaxial test, 3 successive Mohr's circles can be obtained and if I take the stress state corresponding to the maximum shear stress that will obviously be the point at the top of the semicircle here.

Then as the loading has changed during the test with every Mohr's circle, the position of maximum shear stress point has changed from here to here to here and in a p q graph we can see the same path through which maximum shear stress has changed with loading. It is this what is known as the stress path, this stress path is for maximum shear stress and how it varies with loading. Now here no failure is involved at the moment.

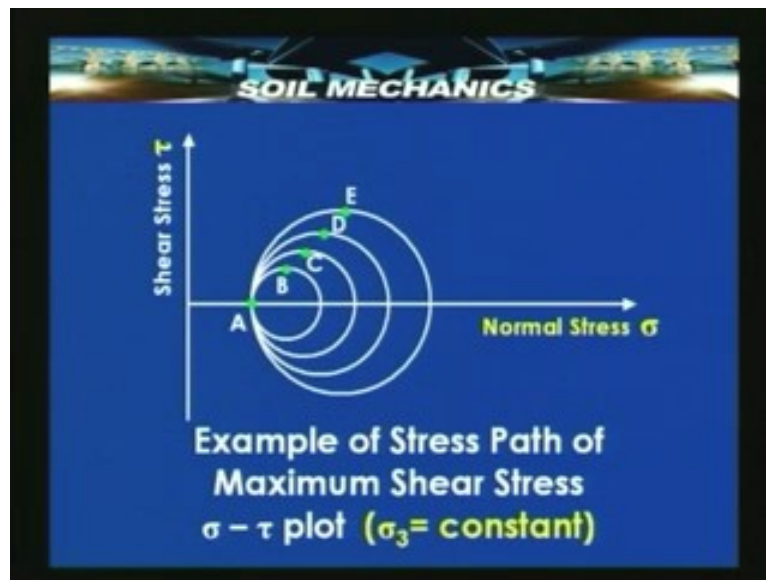
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Suppose I continue the test and a few more tests are conducted, few more specimens are tested and ultimately we reach a set of σ_3 , σ_1 combination which makes the soil fail that means the last Mohr circle shown here is touching the failure envelop. Same thing is true here in the p q graph. Now if I take the points corresponding to maximum shear stress from each one these Mohr's circles and plot, you will find the progression of the maximum shear during the test from a low value here, gradually to a point which sits on the envelop indicating that is the point of failure.

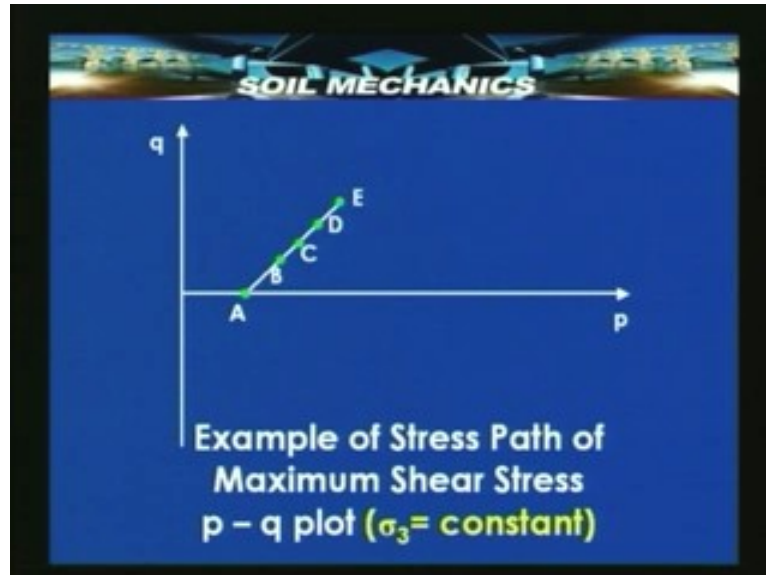
So this is an example of stress path for maximum shear stress upto failure. If one knows the stress path, one knows what kind of stress history the soil has gone through and what has led to failure and that's very important because that throws insight into the behavior of a soil subjected to loading and how it gradually tends to move towards a state of failure.

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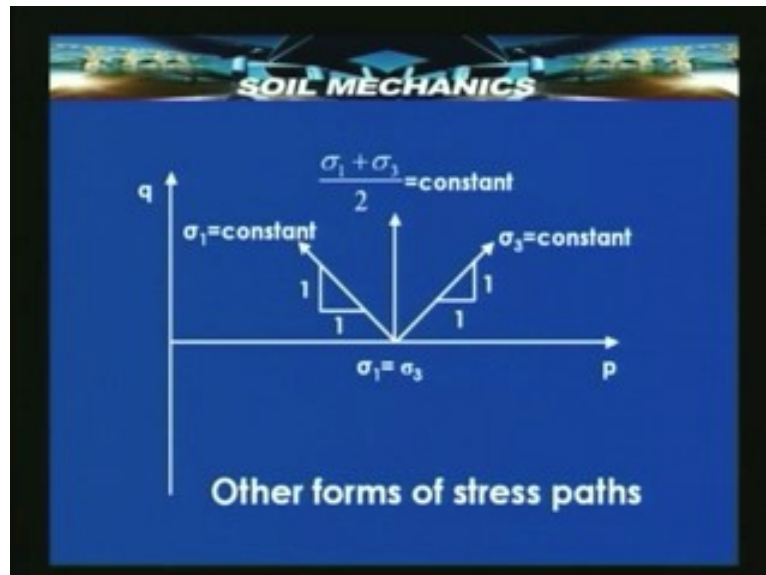
Here is another example of stress paths. We do a lot of tests continuously one after the other on a same specimen but with constant σ_3 and only σ_1 goes on changing and again let us take the maximum shear stress point those will be at constant σ_3 A B C D E.

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If you now plot them on the p q graph, you get a straight line at 45 degrees and this is nothing but a maximum shear stress path corresponding to σ_3 constant state.

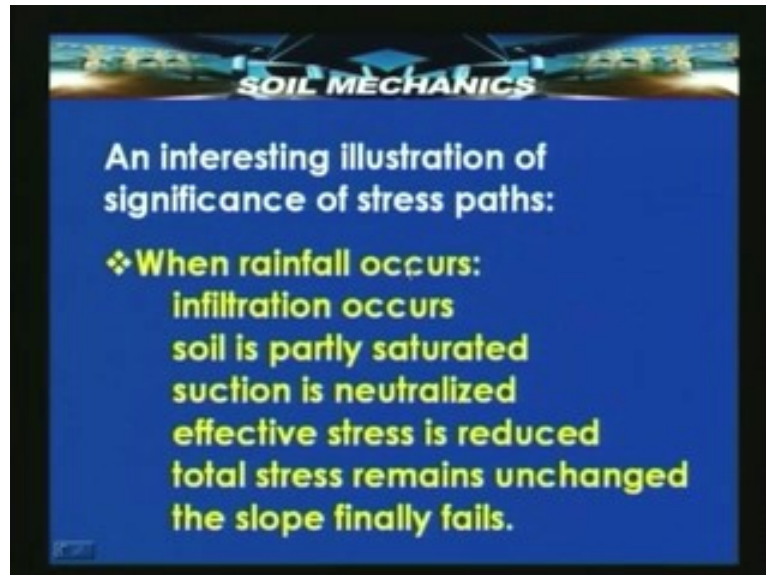
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Now here is another diagram, a p q plot which shows different stress path other different forms of stress paths under different other conditions. We just saw that at σ_3 equal to constant we have a stress path which is at 45 degrees, if on the other hand σ_1 is kept constant, the stress path would have been as shown here on the left of this diagram. If on the other hand we had kept $\sigma_1 + \sigma_3$ by 2 as constant and conducted the test and plotted the points of maximum shear

stress then we would have found that those would have led to a stress path which is going straight upward.

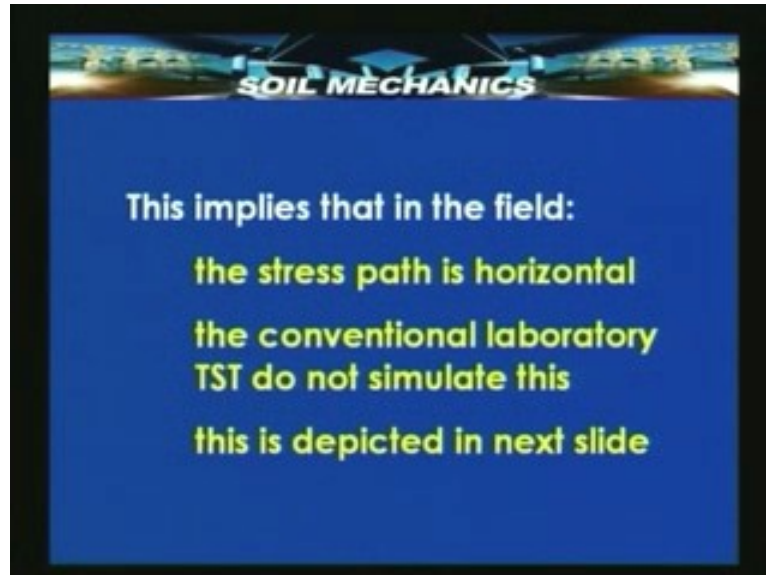
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Let me now give you a very interesting illustration of the application of this stress path and how it has led to a better understanding of the effect of rainfall on slope and subsequent failure and landslides. So an interesting illustration of significant stress path can be quoted as the effect of rainfall. When rain occurs on a slope, infiltration is caused. The rain water gradually flows into the earth. Soil is initially partly saturated and therefore as infiltration takes place, gradually the suction pressure which is there in the partly saturated soils is neutralized. Positive pore pressure gradually start developing and because of this the initial tension which was binding the particles in the partly saturated conditions, capillary tension which was holding the particles together during the partly saturated state now gradually gets nullified and the effective stress therefore gets gradually reduced.

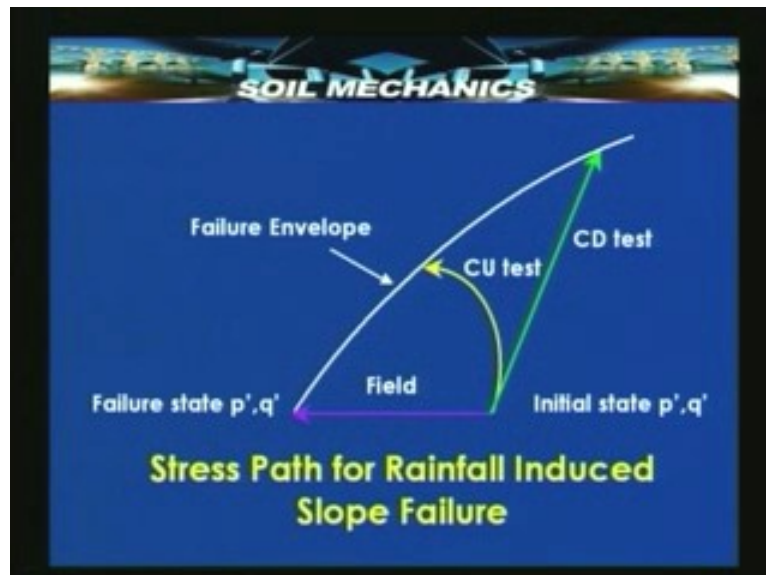
You know what will happen if effective stress goes getting reduced, the shear resistance goes decreasing and therefore ultimately a stage is reached where the effective stress is reduced to such an extent that the slope fails finally. But during all this process we must remember that the total stress in the system remain unchanged, neither an external stress is applied nor an internal stress is disturbed, just the pore pressure goes on changing with total pressure remaining the same. This is a very interesting concept that has been developed by Brand to explain the phenomenon of rainfall induced landslides. So this implies that total stress remaining same, the slope is gradually coming to failure because of increase in pore pressure that means the stress path has to be horizontal that we will see in the next slide.

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The conventional laboratory test unfortunately does not help you to simulate this horizontal stress path and therefore we will have to devise a special testing technique, cyclic triaxial test could probably be an alternative. In the next slide we will see how the stress really varies, what kind of stress path we get in our conventional test and what should be the real stress path that should be simulated.

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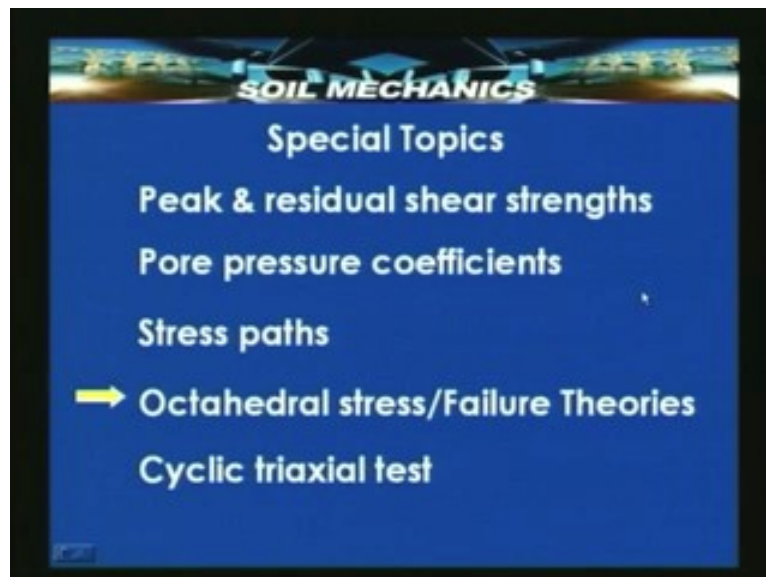
Here we have the stress path which actually exists during rainfall. This arrow going from right to left shows how the initial values of p' and q gradually change to the final failure state p' and q with total stress remaining same. So this is the true stress path in the field. The soil

undergoes change in stress according to this path. Now suppose this is the failure envelop, the soil reaches failure or the slope in question reaches failure through this horizontal stress path with total stress remaining same and it touches failure envelop finally.

If on the other hand you do a consolidated undrained test or a consolidated drained test because the slope has been an existence for sometime consolidation has taken place and now suddenly rainfalls occur and the slope fails. So consolidated drained or consolidated undrained are the appropriate tests depending upon the rate of infiltration of water or the rate of rainfall. Usually the rain storms are so heavy, consolidated undrained conditions are more preferred as the representative ones rather than the consolidated drains.

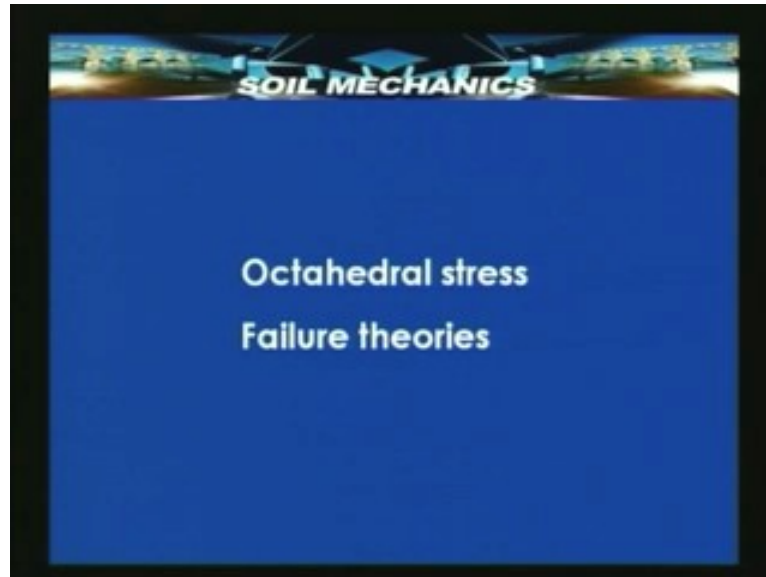
So if you see here the stress paths for consolidated drained test is given by this green line and that goes and meet the failure envelop like this. This is the path that will be followed whereas if it is consolidated undrained this is the path that will be followed to failure and none of these paths unfortunately depict what is really taking place in the field. Therefore we need to know and have a method by which we can simulate this, if you really want to determine the correct shear strength parameters for a slope which has failed due to rainfall.

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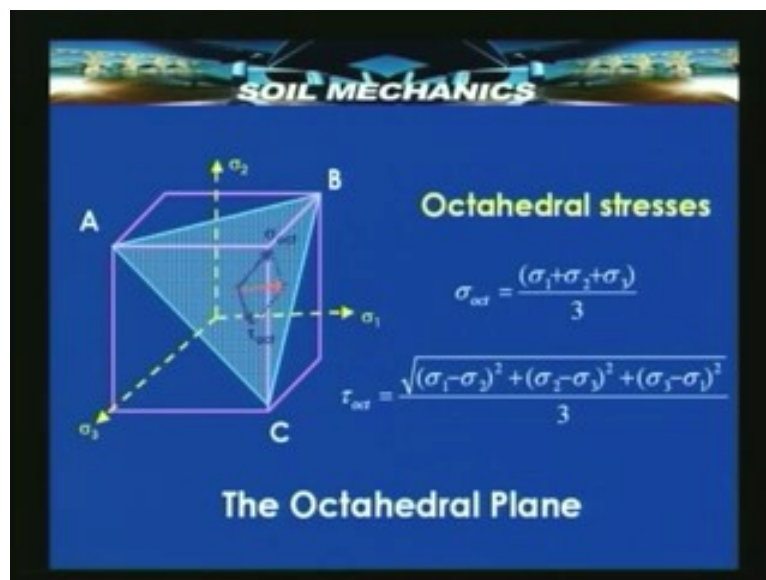


Now we pass on to an important topic from point of view of failure of soils since we are now taking about failure, octahedral stress failure theories. What are octahedral stresses and what are failure theories associated with that?

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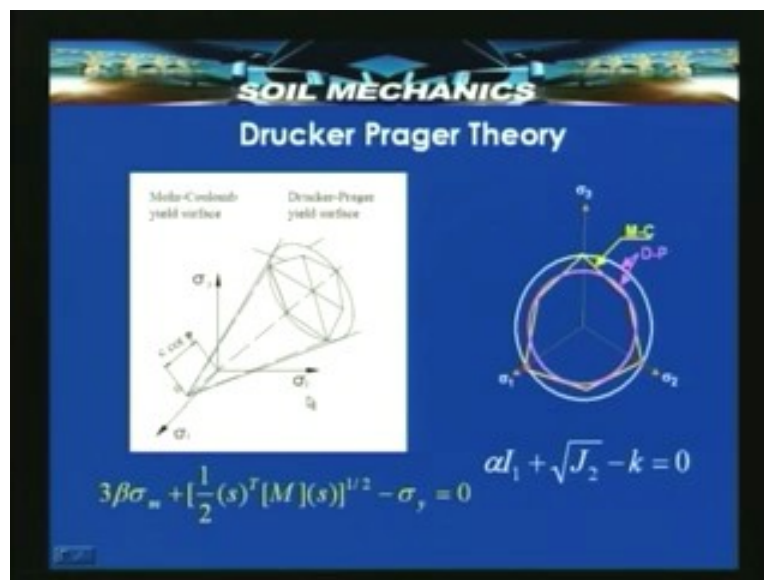
Here is a very simple illustration of three dimensional stress state σ_1 , σ_2 , σ_3 . This is a stress cube which shows what are the stresses on different planes. We are interested in one particular plane depicted here as ABC. What is special about this plane is that this is a plane on which σ_1 , σ_2 and σ_3 are equal. If you take a perpendicular to this plane that's a line showing hydrostatic stress state, the perpendicular to this plane will have σ_1 equal to σ_2 equal to σ_3 along it. Such a plane is a very special plane known as the octahedral plane and the octahedral normal shear stress on this particular plane are given by these two equations. What is special about this again is this is total stress in terms of octahedral stresses and if you consider

pore pressure then pore pressure will get subtracted from each one of these stresses and therefore effective sigma octahedral would simply be this minus U.

Whereas effective shear stress octahedral will not change, it will be the same as the total octahedral shear stress. The octahedral stress state is a very important stress state to know because this plane ABC is often very close to the actual failure observed in the soil under different stress state and many of the failure theories make use of the octahedral stress which is known and which can be determined and from that they derive their yield theories and predict whether or not a soil will yield.

We take a look now at one important failure theory other than the Mohr coulomb theory. This is known as the Drucker Prager theory and it is quite applicable to soils. We know that in the Mohr's strength theory, the failure envelop is a straight line in two dimensional stress state. In three dimensional stress state it's not a straight line. If you look at this figure for example this is a three dimensional stress state figure sigma₁ sigma₂ sigma₃ are shown axis.

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Suppose you plot all those points which corresponds to failure as predicted by the Mohr coulomb theory then what you will get be in three dimensions, a hexagon like this. A hexagonal hollow surface that's what you will get, in two dimensions it looks as a straight line. What Drucker Prager theory says is that Mohr coulomb theory does not predict failure appropriately in certain conditions and they have modified the Mohr coulomb theory particularly to account for behaviors such as strain hardening with displacement and they have given a slightly modified equation for predicting the strength or yield.

Here is one equation and here is another; they are actually similar they are merely two different forms of writing the same failure condition as per the Drucker Prager theory. Both Mohr coulomb surface and Drucker Prager surface are shown in this figure. The hexagonal surface corresponds to Mohr coulomb yield theory whereas the conical surface refers to Drucker Prager

yield theory. In cross section it will look like this, the hexagon and the Drucker Prager circle. Now do these theories match at all, how deviant from each other. The Drucker Prager theory can be fitted to the hexagon of the Mohr coulomb theory either by fitting a circle which circumscribes the hexagon or by inscribing a circle within the hexagon.

Accordingly we will have some equivalence between Mohr coulomb theory and the Drucker Prager theory, the equivalence is of course will vary depending upon whether we are using outer circle or the inner circle. In this two equations which are given here, k is a material parameter, alpha is a material parameter so is beta and J_2 , M_s , these are matrices involving the mean stress or the stress invariance.

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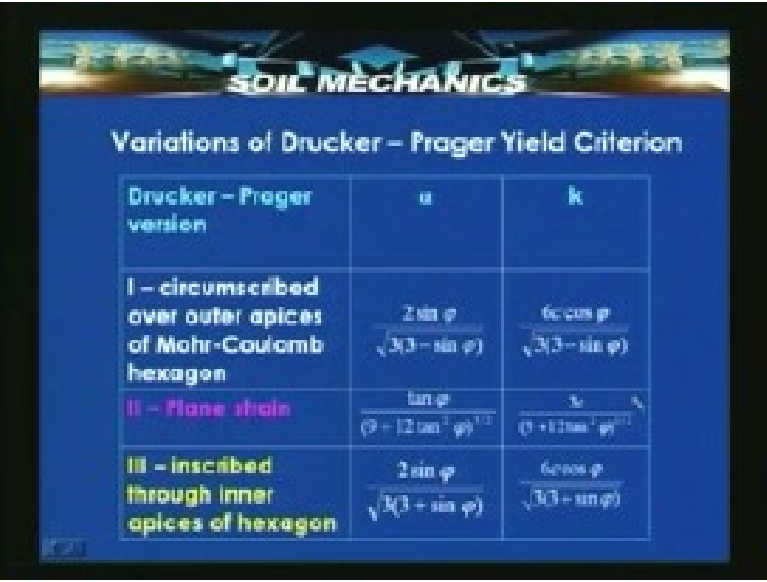


Variations of Drucker – Prager Yield Criterion

Drucker – Prager version	α	k
I – circumscribed over outer apices of Mohr-Coulomb hexagon	$\frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}}$	$\frac{6c \cos \phi}{\sqrt{3(3 - \sin \phi)}}$
II – Plane strain	$\frac{\tan \phi}{(9 + 12 \tan^2 \phi)^{1/2}}$	$\frac{3c}{(9 + 12 \tan^2 \phi)^{1/2}}$
III – inscribed through inner apices of hexagon	$\frac{2 \sin \phi}{\sqrt{3(3 + \sin \phi)}}$	$\frac{6c \cos \phi}{\sqrt{3(3 + \sin \phi)}}$

We shall not go into very great detail about what is stress invariant, what are mean stresses, suffice it to know that there is a theory as the Drucker Prager theory and it is quiet widely applied in order to predict yield in soils particularly clayey soils. Now if I use the circumscribed circle over the outer **abscises** (Refer Slide Time: 40:23 min) of the Mohr coulomb hexagon.

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SOIL MECHANICS

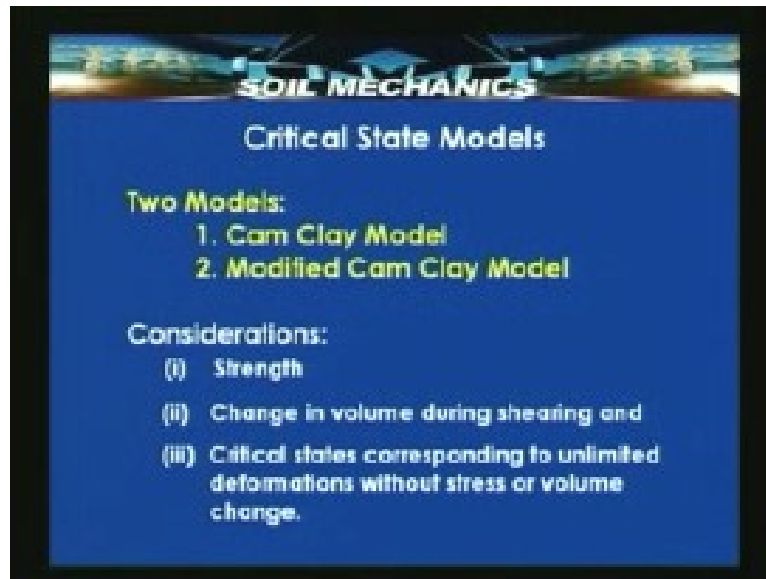
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III – inscribed through inner apices of hexagon	$\frac{2 \sin \phi}{\sqrt{3(3 + \sin \phi)}}$	$\frac{6c \cos \phi}{\sqrt{3(3 + \sin \phi)}}$

Then the values of α and k that will need to be determined and used in the Drucker Prager theory will be given by this formula. Obviously if you know the friction angle for the soil, you can determine the parameters that will go in to the Drucker Prager theory, α and k . The parameters that will go into the Drucker Prager theory depend upon the conditions under which the soil is subjected to various different types of stresses. Suppose if the soil is subjected to three dimensional stress state, α will be given by $2 \sin \phi$ by root of 3 into 3 minus $\sin \phi$ and k by another similar equation.

If a plane strain condition prevails then this $\tan \phi$ by $9 + 12 \tan^2 \phi$ to the power of half is the corresponding applicable value of α and if on the other hand we use the inscribed circle which is within the hexagon then α and k will become values shown in the last two columns of the table here. Depending upon whether we are analyzing a plane strain problem or whether we want an upper bound or a lower bound, we can choose an appropriate values of α and k in a typical analysis of say a tunnel or a slope using a finite element technique for example there are other model as well although it is beyond the scope of this lecture to go into great details about these other models.

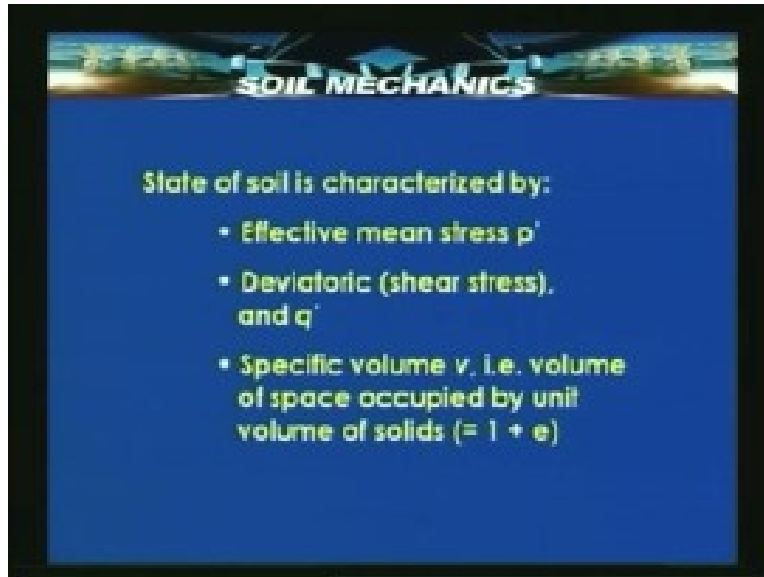
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One model is of special interest and it is worth making a mention about it although not much in detail. This is known as the critical state model. There are two different models proposed under this, one is known the cam clay model another is known as modified cam clay model. These were proposed by Andrew Schofield in Cambridge.

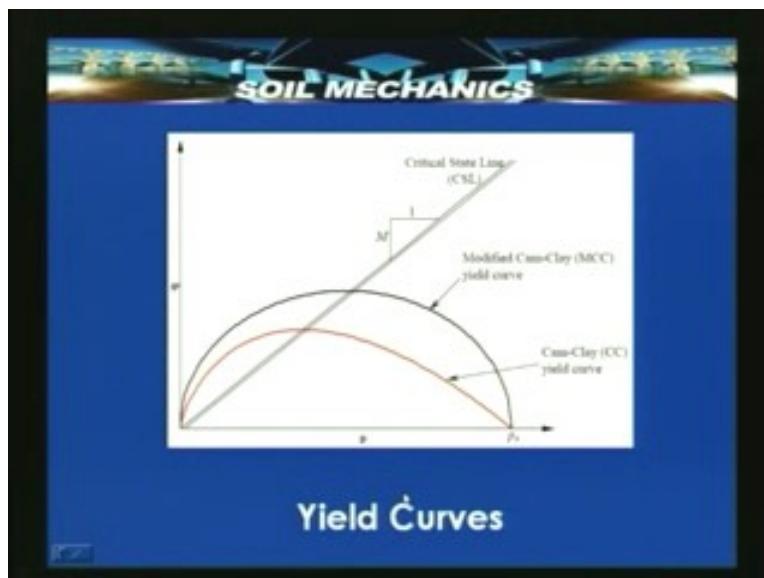
The basic considerations in development of these models were what is the strength of the soil, the change in volume that takes place during shear then what are those critical states corresponding to large deformation without stress change or volume change. If you think of liquefaction it is very easy to comprehend the meaning of the statement, liquefaction is nothing but a case of large deformation at constant stress or constant volume. Therefore the cam clay model attempts to explain these large deformations which may takes under constant stress or constant volume conditions. The state of the soil is characterized by three parameters.

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As in the Drucker Prager theory we had 2 parameters, here we have 3 parameters. The Mohr coulomb theory has again only 2 parameters c and ϕ . The cam clay model on other hand uses p dash and of course q dash but it also uses the specific volume v .

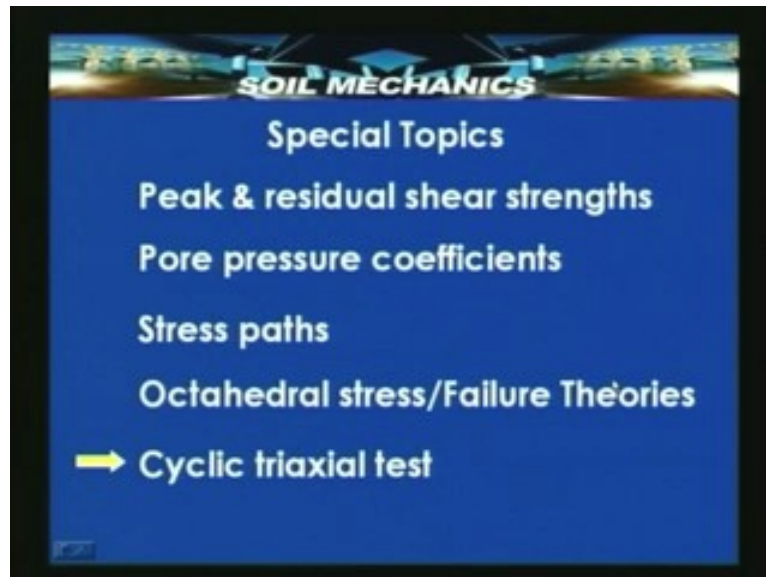
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Here are the yield curves which will result if we use the cam clay model. Here is the yield surface on the p q plot for cam clay, this is for the modified cam clay and this is the line which tells us which are those critical stress state at which a soil will either according to cam clay model or according to the modified clay model corresponds to a situation where there is yield or there is a large deformation.

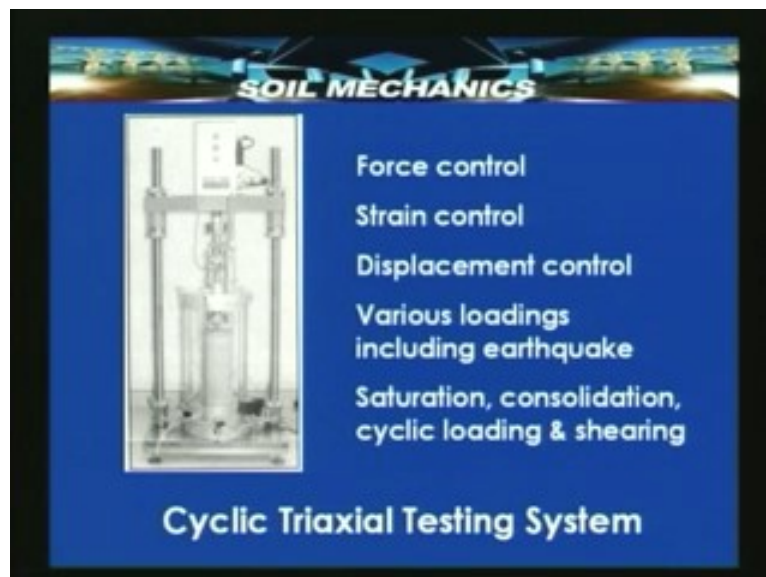
So this point or this point will represent, the point either this or this will represent a situation of large unlimited deformation as in the case of liquefaction. This can be predicted reasonably well using the cam clay or the modified cam clay model.

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Now lastly we come to the test called cyclic triaxial test about which I mentioned sometime back which is a very useful triaxial test and is a very innovative relatively recent development and completely digital usually and is of interest to know.

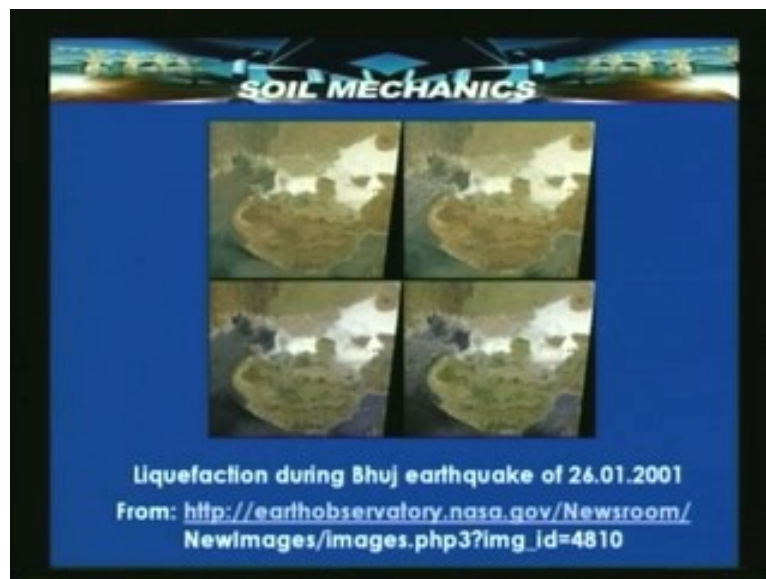
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Here is a simple illustration of a cyclic triaxial equipment. This has the facility to apply cyclic loading and that's why it is called cyclic triaxial test. It is a conventional triaxial test except that the load that can be applied can be cycled. Suppose you take earthquake as an example then the load due an earthquake can be considered to be somewhat cyclic in nature. It is repetitive and therefore if we want to really understand the effects of the earthquake on shear strength of soil we must conduct the triaxial test under the conditions of cycled loading, cyclic loading must be possible. So this equipment permits precisely that, you can apply load in terms of forces and have control over the force applied, you can apply the load indirectly by inducing a strain. A direct shear example is a test where we induce strain and produce the shear forces appropriately. So this could be strain controlled, it could be displacement control also. A different type of loadings is permitted because this is usually digitally computer control.

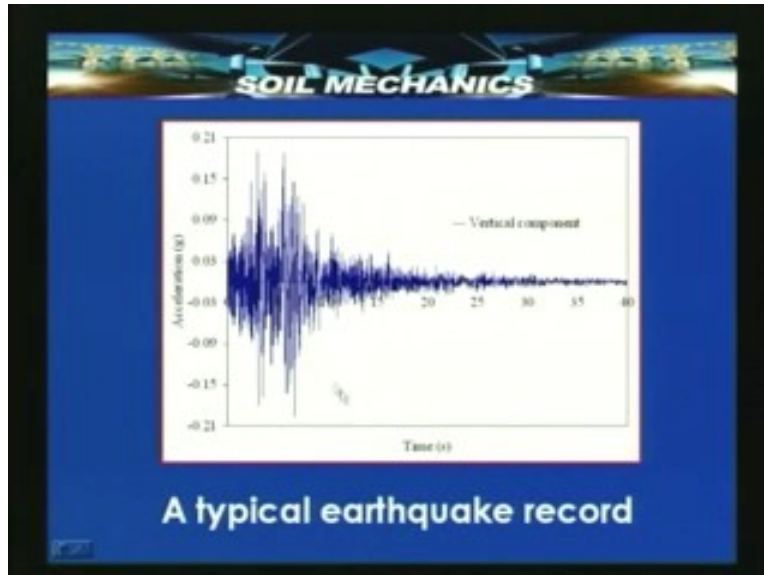
The different types of loading that can be applied; it can also be such as to simulate a given earthquake. A given earthquake loading if you simulate and conduct the test under those conditions, the parameters which you get from this triaxial test will be the best that one can get in order to truly analyze the behavior of the soil under earthquake conditions. It permits, the equipment has the facility to saturate the sample to consolidate the sample and to apply cyclic loading and then finally to shear the sample. That's what the cyclic triaxial testing system is capable of doing. Now this can be used to understand and predict liquefaction in a very ingenious way. This is a typical scenario that was found during the Bhuj earthquake on 26 January 2001.

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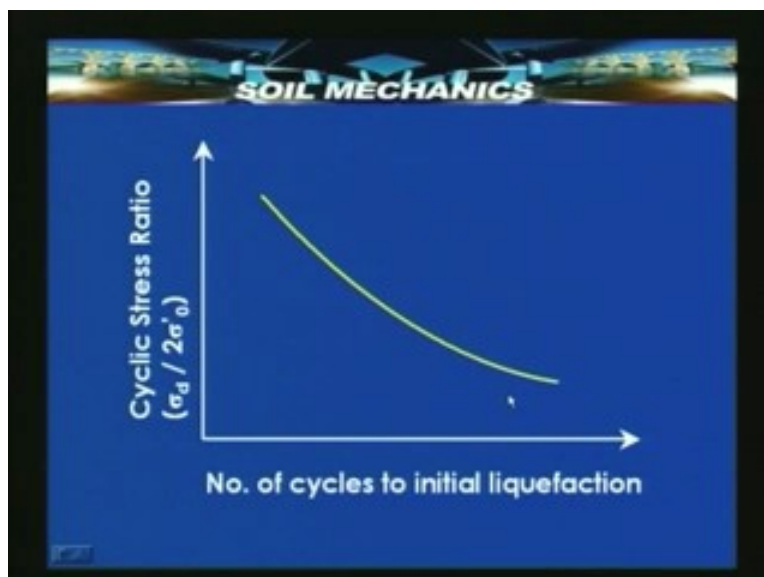
This shows large scale flow of soil and this is exactly what is known as liquefaction. This is the earthquake spectrum which produced this kind of liquefaction. Suppose you know the spectrum, suppose you want to identify the potential for liquefaction in a given soil then this cyclic triaxial test is a wonderful test for this purpose.

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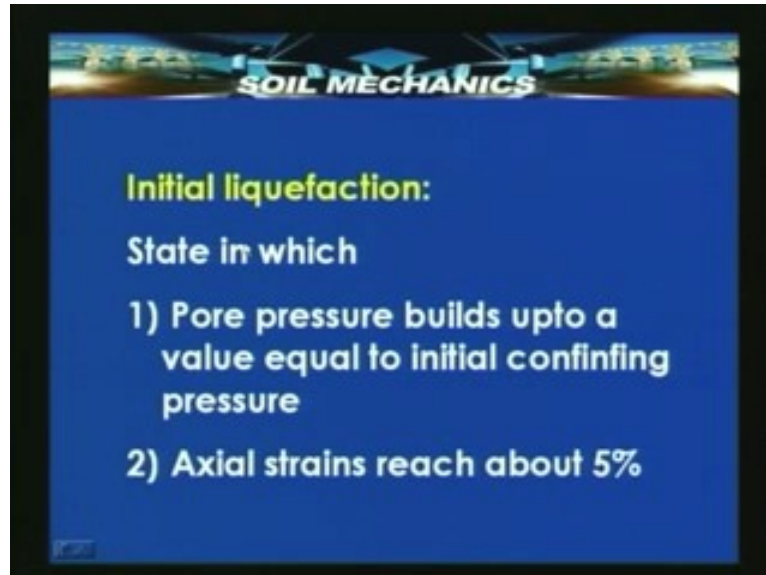


Here is a typical plot that you might obtain from typical cyclic triaxial test. You apply a number of cycles and find out when the liquefaction starts then plot the number of cycles which causes initially liquefaction against a ratio known as the cyclic stress ratio $\sigma_d / 2\sigma'_0$. Let's see in the next slide what this represents.

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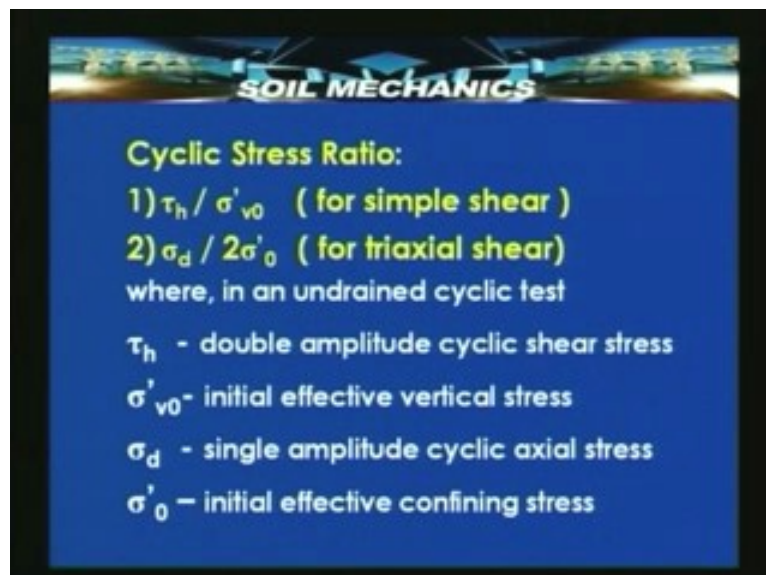


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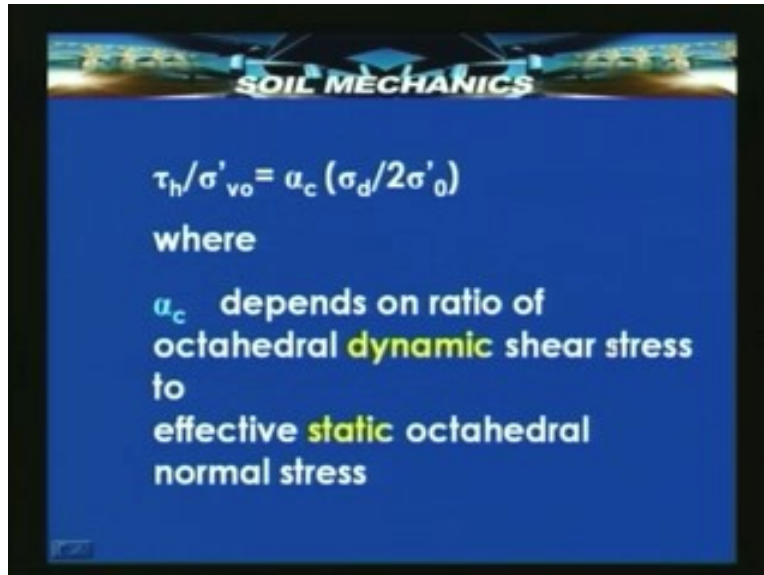


Initial liquefaction is the state in which pore pressure builds up to a value equal to the initial confining pressure that means it completely neutralizes the initial confining pressure and therefore the soil starts liquefying or where strains reach very high values for example 5%. Now if you do a cyclic triaxial test and identify the number of cycles at which one of these conditions is satisfied and liquefaction takes place and then plot the cyclic stress ratio against it, you get a relationship from which you will know whether or not liquefaction can take place. The cyclic stress ratio for simple shear stress is given by this parameter.

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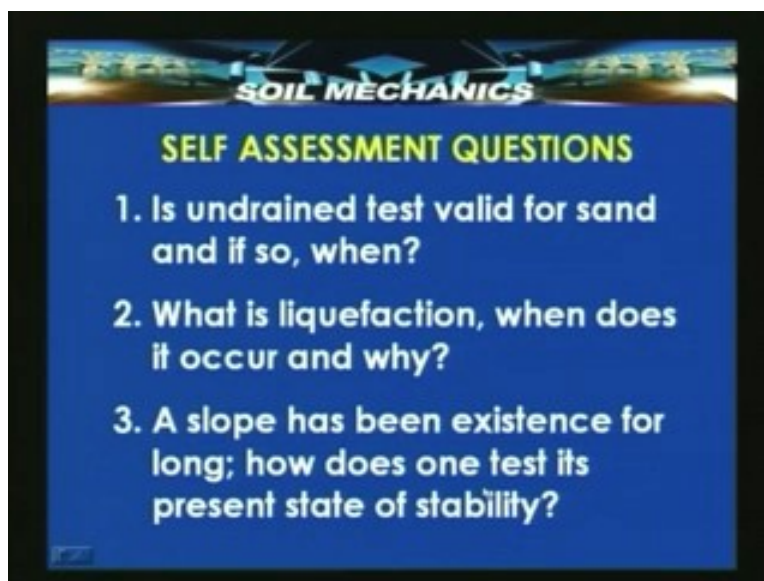


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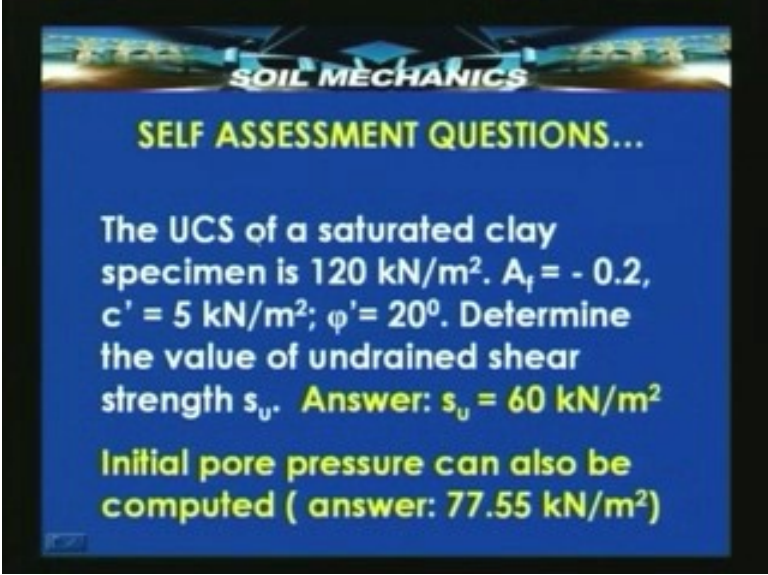
For triaxial shear stress it is given by σ_d by $2\sigma'_0$ where each one of these symbols represent different shear and vertical stresses under different amplitude conditions as listed here. Suppose you want to relate the simple shear tests results with triaxial test results that's possible by relating the cyclic stress ratio through a parameter α_c and incidentally this α_c depends upon octahedral dynamic shear stress divided by effective static octahedral normal stress. So this shows how in yield theories, the octahedral shear stress plays an important role.

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Now we come to the end of the chapter of shear strength of soils and here are a few questions which you are well advised to try to answer. Suppose you go back through this lectures then you will find an answer to a question, is undrained test valid for sand and if so when. What is liquefaction and when does it occur and why? A slope has been an existence let us say for quiet sometime. Now what kind of test will you do in order to get the strength parameters which are required to analyze its present state of health? Try to answer these questions. The answers have been given by me during the course of my lecture. They are available in the slides which had been displayed some time back. Try to also solve simple problem for example here.

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SOIL MECHANICS

SELF ASSESSMENT QUESTIONS...

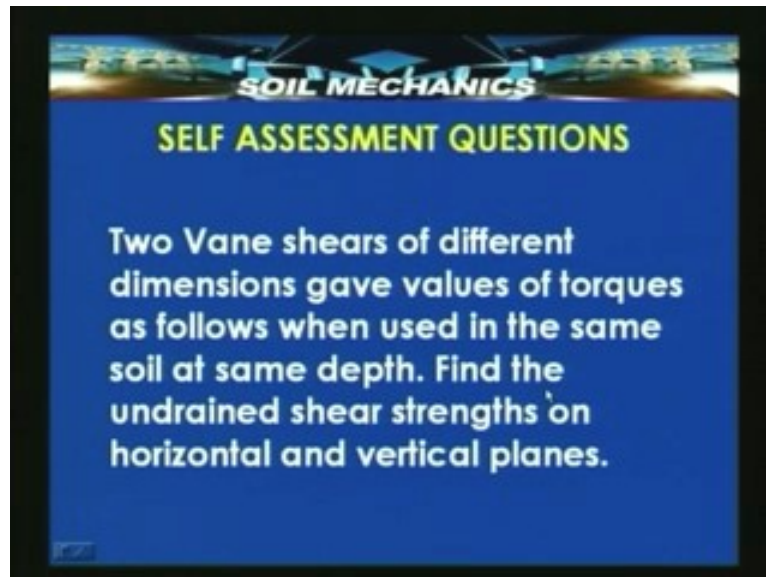
The UCS of a saturated clay specimen is 120 kN/m². $A_f = - 0.2$, $c' = 5$ kN/m²; $\phi' = 20^\circ$. Determine the value of undrained shear strength s_u . **Answer: $s_u = 60$ kN/m²**

Initial pore pressure can also be computed (answer: 77.55 kN/m²)

The unconfined compressive strength of a saturated clay is given and when it is subjected to a triaxial shear test, the pore pressure coefficient A_f is found to be -0.2, c' and ϕ' determined from the test are 5 and 20. Now try to find out the undrained shear strength. Now incidentally, in this particular case although the values of c' and ϕ' are given, A_f is given. It's very obvious right from the statement of the question that it's the compressive strength 120 divided by 2 which give you the shear strength. Therefore the undrained shear stress is readily available here as half of the compressive strength 60 kilo newton per meter square. You can under this better if you go back to my illustration and explanation of the unconsolidated undrained test.

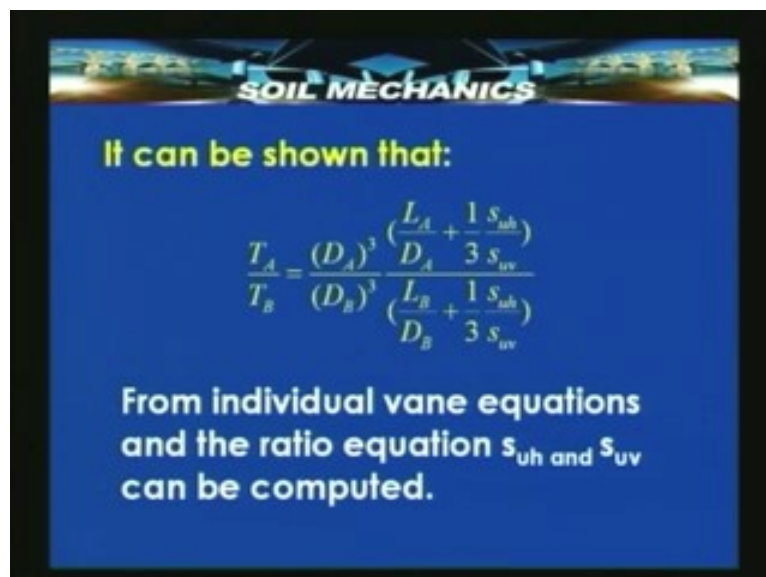
Now the other data that is given in this particular problem is useful for determining the initial pore pressure and you can determine this in a manner similar to the recent problem which we saw in this lecture that is problem number 17, using the similar expression you can find out that initial pore pressure is 77.55 kilo newton per meter square in this particular case.

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Now suppose if you have 2 different vanes and you test the same soil for strength with these 2 different vanes, how do we get the shear strength of the soil in the horizontal and the vertical direction planes? This can be done by going back to fundamental equations of torque generated in a vane shear test.

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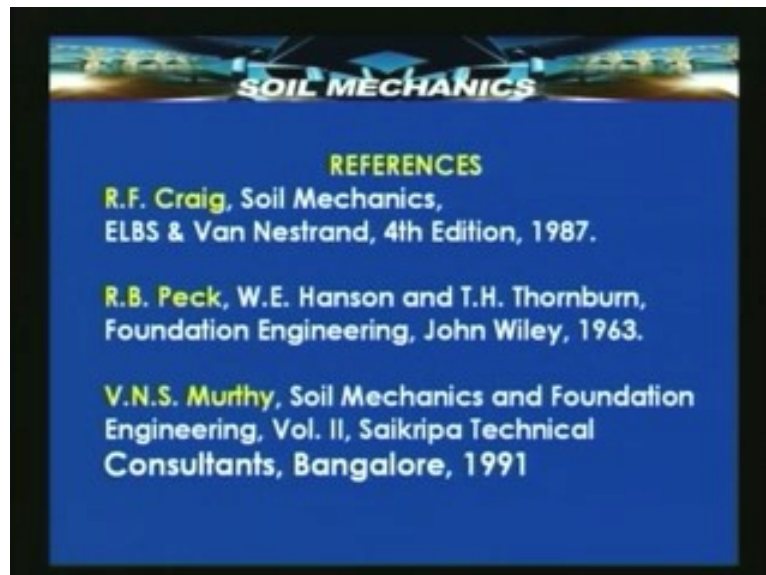
By using the individual equations for torque for each vane and the ratio of the torques and by knowing the geometry of the vanes, one can easily determine s_{uh} and s_{uv} which are the required shear strengths in the horizontal and the vertical directions.

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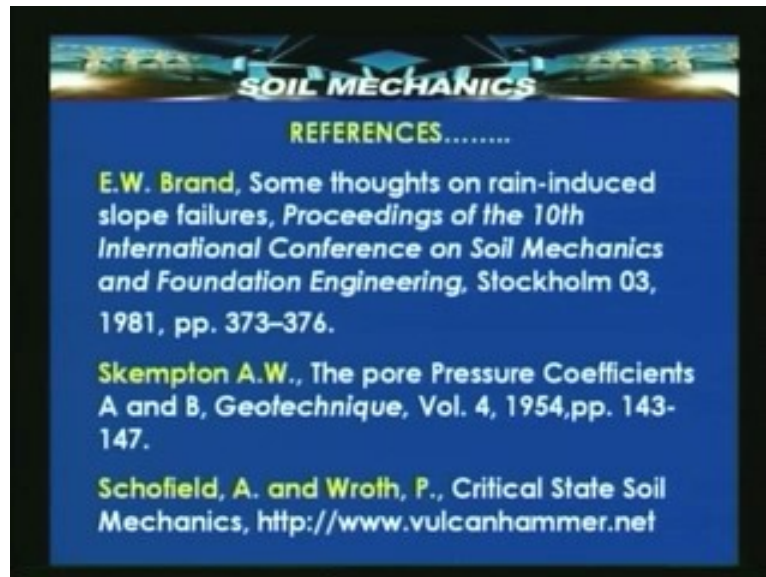
We have used a number of terminology, you are advised to go through complete list of terminology and understand their real meaning and significance in practice. The terminology are strain and stress controlled test, constant σ_3 test, CD, CU and UU test, critical void ratio and liquefaction, peak and residual strengths, stress paths, cyclic stress ratio, octahedral stresses and yield theories.

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Here is a list of books which will be useful to you for going through and understanding in greater detail whatever I have attempted to cover in these lectures.

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Here are a few more references, with this we come to the end.
Thank you.