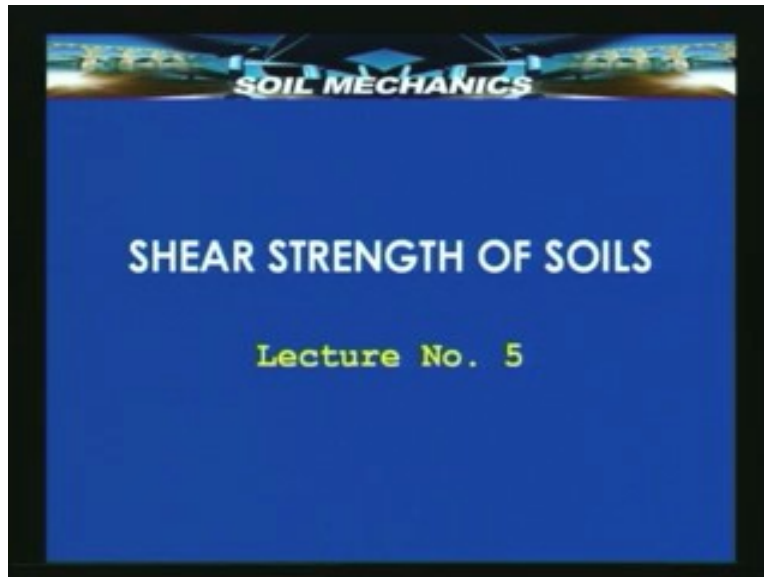


Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
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Lecture – 47
Shear Strength of Soils
Lecture No.5

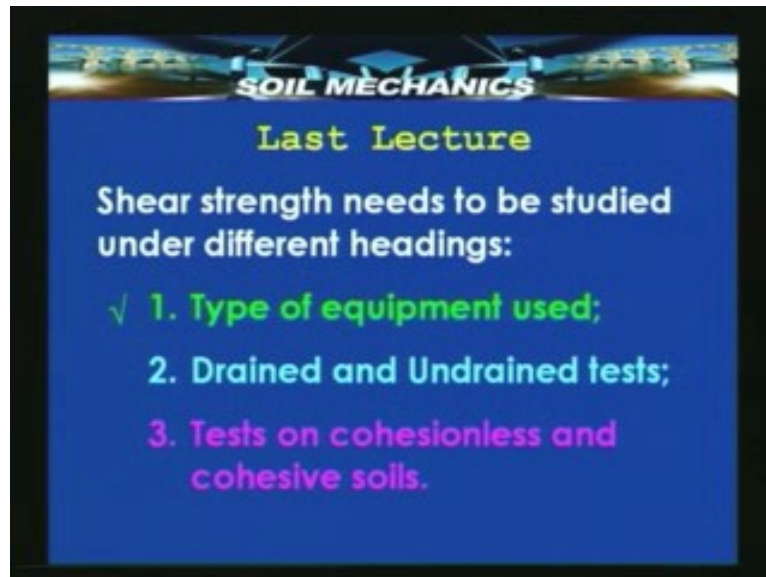
Students we are once again meeting for a lecture on shear strength of soils. We had 4 lectures so far this is the fifth in the series and in this lecture we are going to move forward and see some detailed mechanisms that are going to be involved in the test about which we have already discussed.

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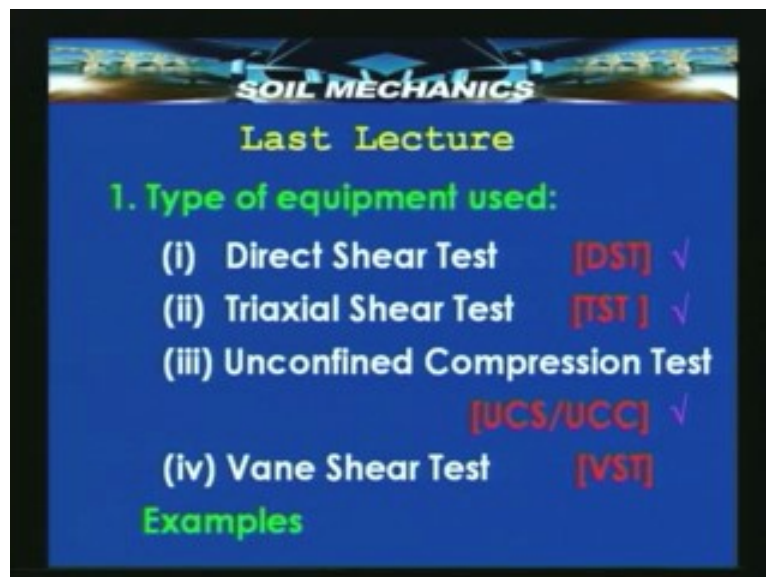
In the previous 4 lectures we had primarily discussed what is Mohr's circle, how stresses are represented by Mohr's circle and how Mohr's circle helps us to identify the planes of failure and the stresses at failure? And to understand in general the failure mechanism which are involved in soils. In this lecture we shall be going a step further now. For example in the last lecture I had mentioned that we have divided this chapter on shear stress into 3 major aspect, one is what are the different types of equipment used and what are the kinds of test that are done with that. Then we have **essuction** (Refer Slide Time: 02:12 min) called drained and undrained test that is in the case of soils drainage is a very important aspect therefore the type of test that are done on soils can be classified according to the drainage that is either provided or absent.

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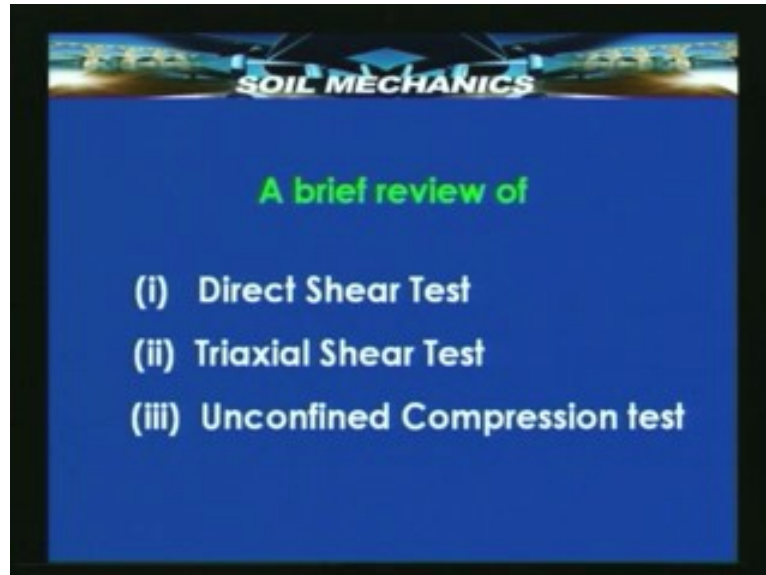
Lastly the nature of the soil is obviously important and therefore we also will see what is shear strength and how it depends upon the nature or soils itself? Now this is the way that I have subdivided the topics and then the type of equipment used and the classification of the tests according to the type of equipment used was also seen in the past few lectures.

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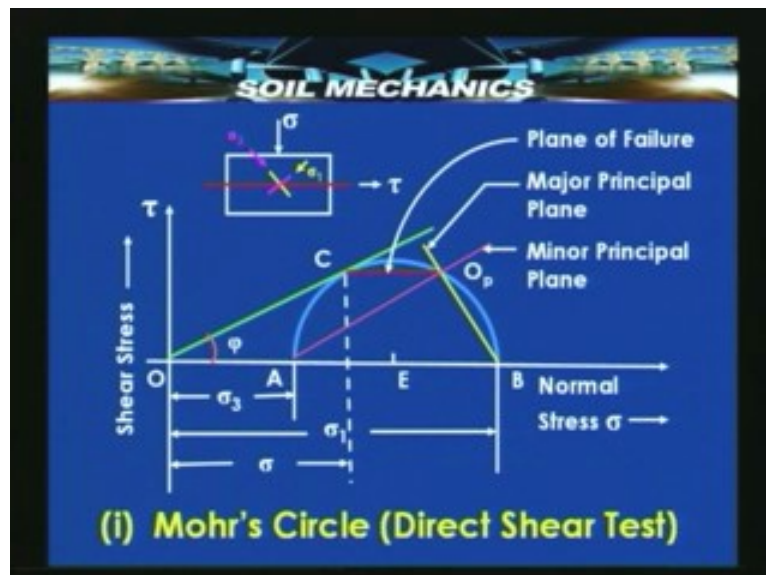
For example we have seen the direct shear triaxial shear and the unconfined compression test in detail. Before proceeding further let us slightly recapitulate what we saw in these 3 sections.

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Let's take a brief review of the direct shear test, the triaxial shear test and the unconfined compression test. First the direct shear test. What is important here is to understand how the stresses vary in the different planes during a direct shear stress which is the failure plane. What's the value of the stress during failure on the plane of failure and how does the failure plane lie in relation to the applied stresses or the major and minor principle stresses.

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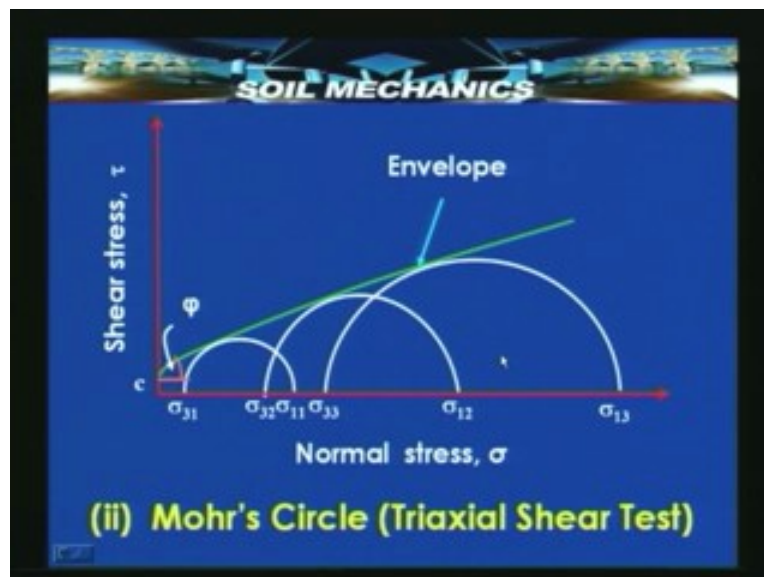


For example if you see this small rectangle this is the sample, there is horizontal plane of failure. In normal stress σ is applied and a corresponding shear stress τ is applied until this fails.

Obviously since this horizontal plane of failure there is a shear stress this cannot be the principle plane. The principle planes will therefore be the plane shown in yellow in this which is experiencing a normal stress equal to the major principle stress σ_1 ; perpendicular to that the magenta colored plane carries a stress σ_3 . Now if we see the depiction of the stress state in the Mohr's circle we find that this A represents this stress σ_3 , B represents the stress σ_1 and the major principle plane and the minor principle plane will both be parallel to the planes of σ_1 and σ_3 meeting at a point Op. Now this point Op is known as the origin of planes is a point such that the line drawn parallel to the known plane where it meets the circle that point has the coordinates which are equal to the stresses on the plane.

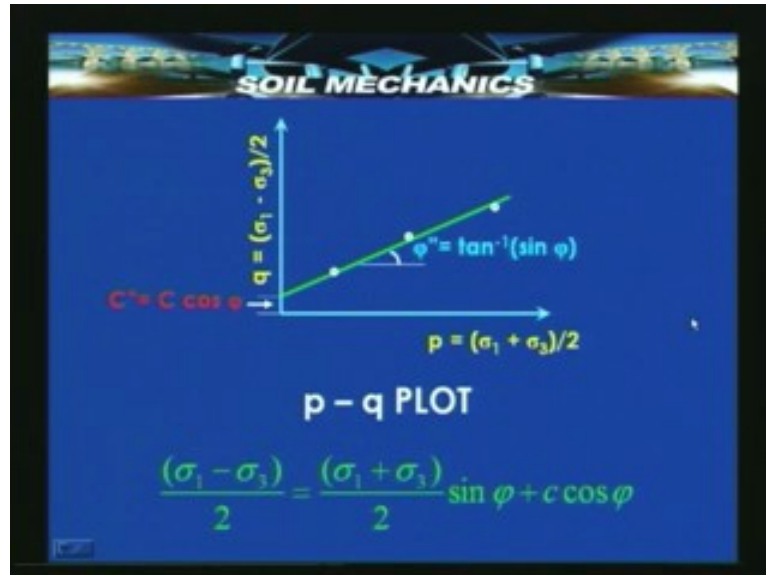
Now we know that the plane of failure is horizontal and therefore Cop must give the origin of planes. If I draw a horizontal line here therefore point C must be the plane of failure which can also be obtained from the tangent OC. Now the point C has stresses equal to σ and τ which are the stresses on the failure plane.

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In the case of Mohr's circle for a triaxial shear test we generally do at least 3 test and plot three Mohr circles and draw a common tangent from which we get the normal and the shear stresses, the shear strength parameters. Now here is an example of three such Mohr's circle for a triaxial test. For convenience the first circle is shown to have minor principle stress equal to σ_{31} and the major principle stress σ_{11} . The second circle corresponding to the second test has a slightly higher minor principle stress σ_{32} and a slightly correspondingly higher major principle stress σ_{12} . The last circle has σ_{33} and σ_{13} as the corresponding stresses. Now if I draw a common tangent to all this then the intercept made by the tangent is the cohesion and the angle which the tangent makes with the horizontal is the angle of friction. This is how the Mohr's circle test data is plotted and the shear strength parameters are obtained.

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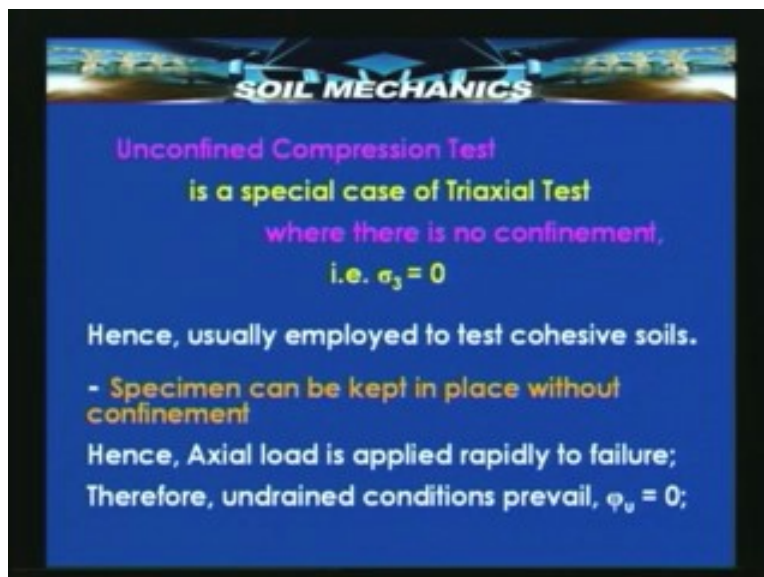
Now it is easy to show that from the Mohr's circle a relationship like this can be devised. At failure it's possible to show that this relationship holds, in fact we have done this or we have seen this relationship holds in one of my earlier lectures. The advantage of representing the Mohr stress state in this form is that we can plot a straight line between the quantities $\frac{\sigma_1 - \sigma_3}{2}$ which represents the deviatoric stress by 2 that is the shear stress and $\frac{\sigma_1 + \sigma_3}{2}$ by 2. That is the stress invariant divided by 2. If I plot this, this will also be a straight line because this relationship is essentially linear and by doing so we now get an intercept and an angle. It's very simple to show that this intercept will be nothing but $c \cos \phi$ that is the constant in the linear equation and the angle will be nothing but an angle ϕ double dash tangent of which must be equal to the $\sin \phi$ which is the slope of this linear equation.

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Thus the triaxial test data can be plotted either in the form shown earlier using three Mohr's circle or in the form of a p q plot. The p q plot is generally preferred because it is always easier to plot a straight line than to draw a common tangent to 3 Mohr's circles. Now we go on to the unconfined compressive strength test.

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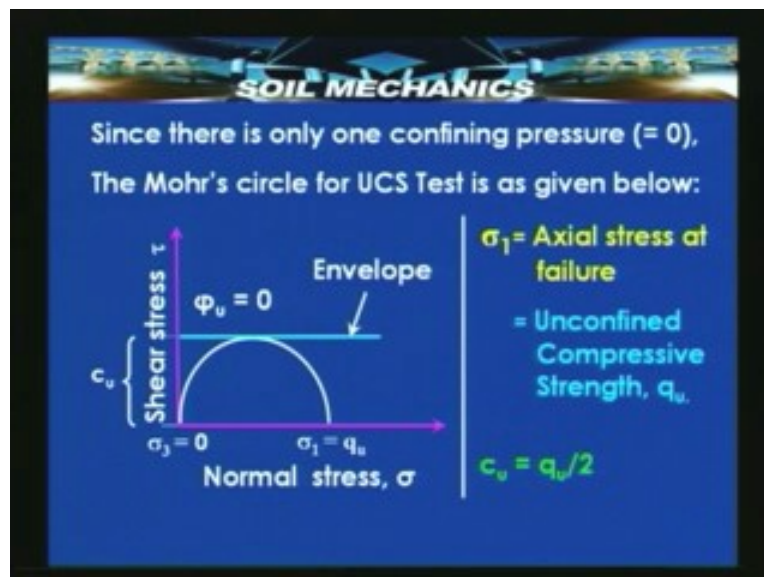


This test as you know is a special case of a triaxial test the difference between the triaxial test conventional triaxial test as it is called and this test is that the confining stress σ_3 is zero. Hence it's necessary that the sample should be able to stand in the apparatus without confinement. Such a soil which will yield a sample in soil which can stand in the apparatus

without a confining pressure is a cohesive soil and therefore this test is usually employed to test only cohesive soils. The specimen can be kept in place without confinement and since it is cohesive and since there is no confinement, the actual load is usually applied rapidly until failure so that the specimen doesn't get dislodged. Such a rapid test obviously provides undrained conditions and therefore friction does not get mobilized and if τ_u undrained equal to zero holds for this test.


And from this test by plotting a Mohr's circle we can get the cohesion as shown in this diagram. Since there is no confining pressure σ_3 equal to zero unlike a conventional triaxial test this will only have one single Mohr's circle corresponding to σ_3 equal to zero which starts from the origin. The axial stress at failure automatically is equal to the compressive strength and half of that becomes the cohesion intercept as you can see from this diagram. So the cohesion is q_u upon 2 as far as this test is concerned.

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What's the importance of this unconfined compression strength test? The importance lies in the fact that it gives a very good indication of the consistence of the soil. For example it is found that as q_u varies from zero to some value more than 380 or 400 kilo pascal. The consistency of the soil varies from a very soft state to a very hard state. Therefore this q_u is a very good measure of the consistency and this table gives you an idea about what is very soft, what is soft medium stiff or very stiff and hard.


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General Relationship Between UCS and Consistency for Clays

Consistency	q_u (kPa)
Very Soft	0 to 24
Soft	24 to 48
Medium	48 to 96
Stiff	96 to 192
Very Stiff	192 to 383
Hard	> 383

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EXAMPLE 12

An Unconfined Compression Test was conducted on a saturated clay specimen 38 mm \varnothing x 76 mm length. The vertical deformation was measured to be 10mm at a failure load of 0.25 kN.

Compute c_u of the clay.

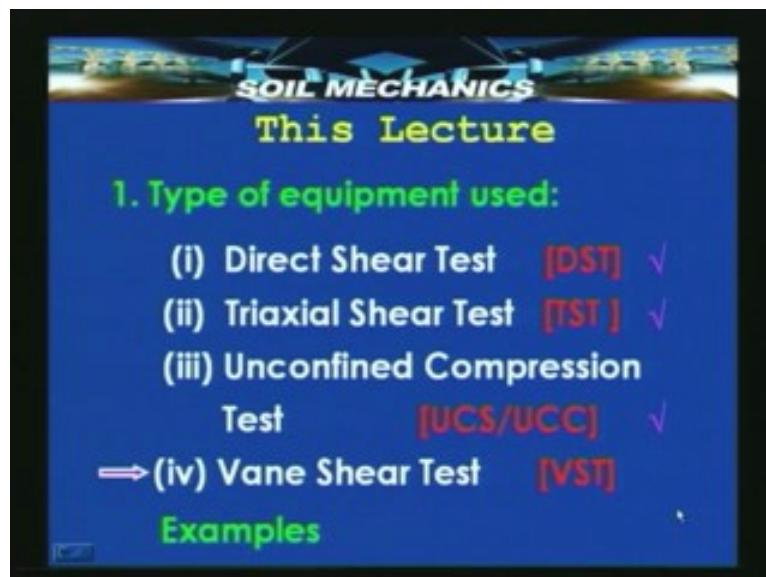
$$A_f = A_0 / (1 - \epsilon) = (\pi (38)^2 / 4) / (1 - (10/76))$$
$$= 13.06 \text{ cm}^2 = 0.0013 \text{ m}^2.$$
$$q_u = 0.25 / 0.0013 = 192.31 \text{ k N/m}^2$$
$$c_u = q_u / 2 = 96.15 \text{ kN/m}^2.$$

Now let's take a very simple example of an unconfined compression test. Obviously we need to know the dimensions of this specimen that's given here 38 millimeter diameter and 76 millimeter length. During the test the vertical deformation was measured and was found to be 10 mm corresponding to a load of 0.25 kilo newtons at which the sample failed. Now what's required is to compute c_u , is of course obvious that the simplest way is to plot a Mohr circle and get the value of c_u or rather we can also do analytically and it is very important to note this first step in the analytical calculation.

The first step is what's the value of area at failure of this specimen because the starting area is given by 38 millimeter diameter. On the other hand since the deformation takes place in the vertical direction as the load is applied due to Poisson effect the horizontal diameter increases and there is a deformation taking place in the horizontal direction which therefore makes the effective area slightly different. So if we start with an original area given by $\pi d^2 / 4$ then that divided by $1 - \text{vertical strain}$ gives you the new area. And compressive strength as to be calculated by dividing the applied load at failure by this new area and that comes out to be 192.31 kilo newton per meter square and half of that becomes the cohesion.

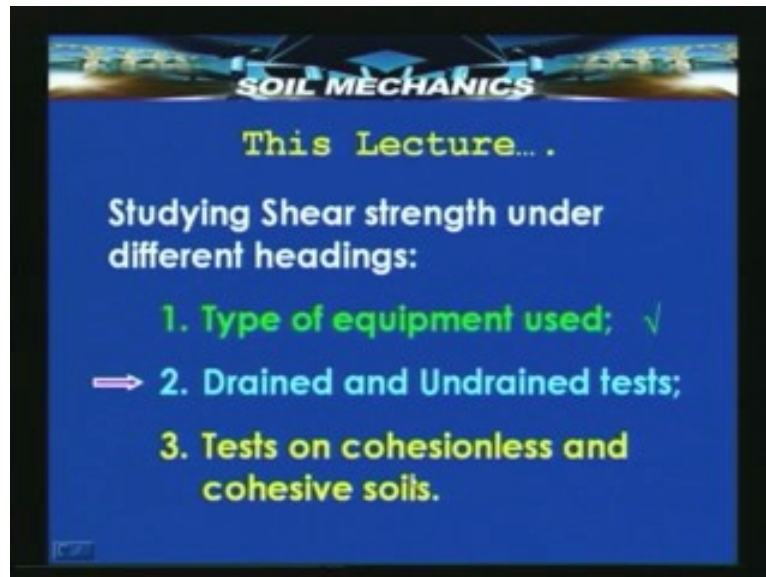
Now all this actually which we had seen in the previous lecture, this was to help you to recapitulate so that there will be a continuity. Now in today's lecture. We will continue and see the forth test that is remaining in this list of equipment and corresponding test, vane shear test namely.

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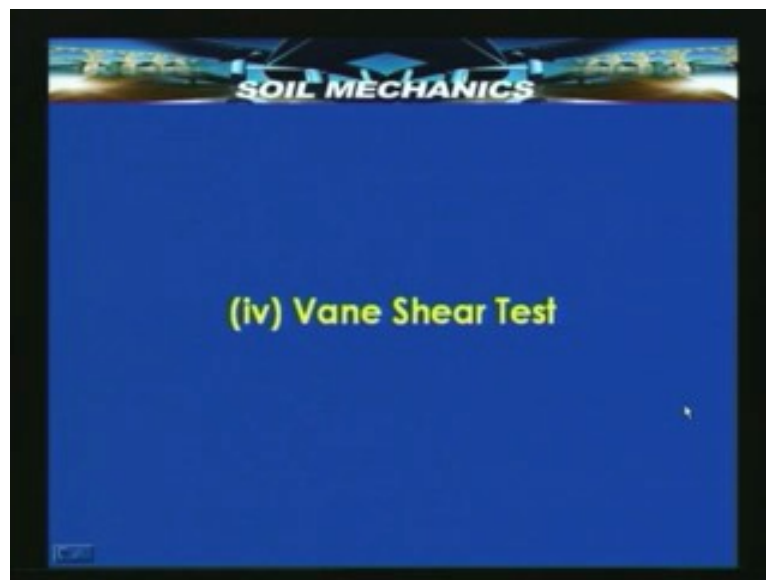


In this lecture we will also be seeing the second aspect of my classification of the entire lectures on shear strength. That is the second aspect classifying the shear strength studies into those corresponding to drained state those corresponding to undrained state.

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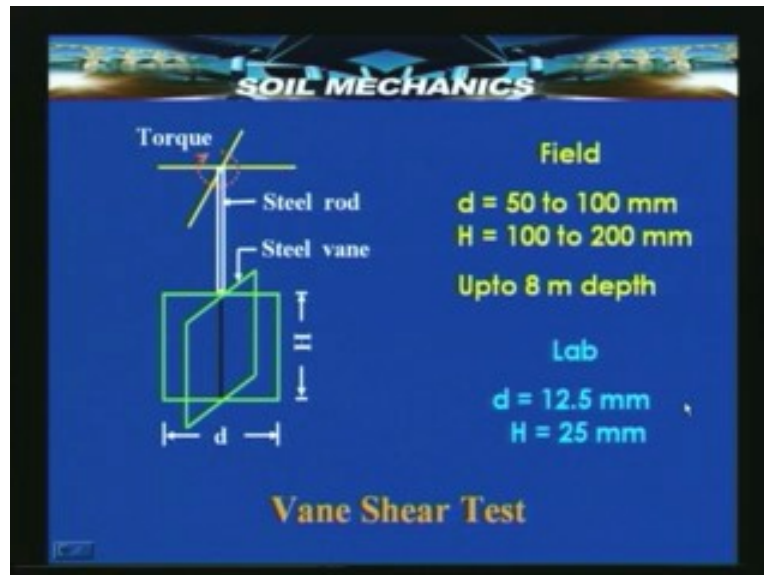


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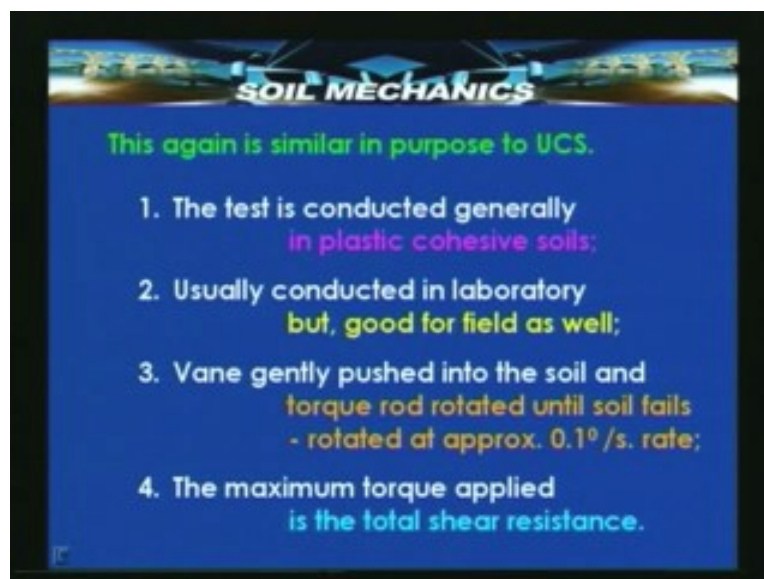


Let us begin with vane shear test. This vane shear stress makes use of an apparatus called a vane. This vane consists of a rod a set of vanes, 2 vanes to be precise this has got a width equal to d and height equal to capital H . There is a steel rod and there is a handle to apply a torque. Now this vane is pushed into the soil gently and rotated until the soil fails. This particular test is such that it can be either done in the field or in the lab. The equipment that is used in the field is slightly large in dimension, diameter is 50 to 100 millimeter, height is 100 to 200 millimeter and it can go upto 8 meters depth in vey soft soil. On the other hand if we want to do this in a laboratory sample, the equipment has to be much smaller in dimensions, d is usually around 12.5 mm and the height is double that 25 mm.

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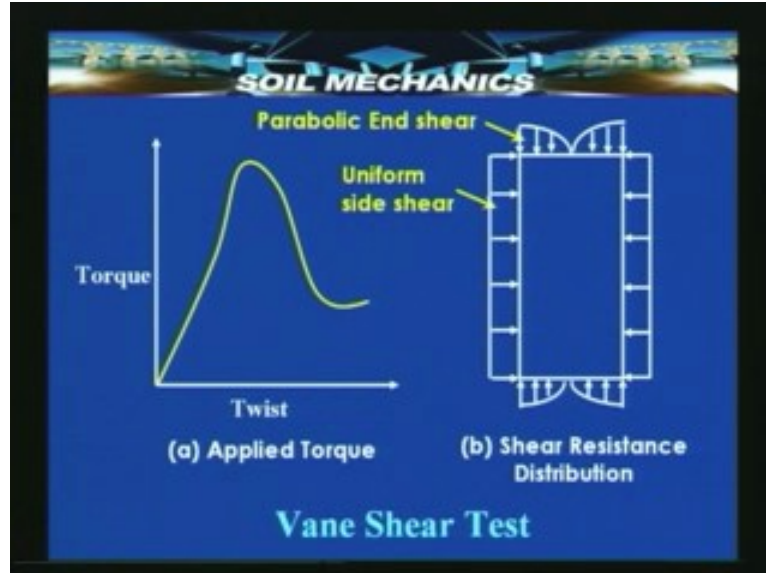


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This test is again very similar to the unconfined compressive strength test in the sense that it is valid mainly only for plastic cohesive soils. It is usually conducted in the laboratory but it is good for field as well. The vane is usually very gently pushed into the soil so that it does not disturb the soil and then a torque is applied to the handle or the rotating rod which is at the top until slowly the vane rotates and shears the soil and the soil fails in shear. Rotation at a slow speed is very important. Usually the rotation is done at approximately 0.1 degree per second rate. The maximum torque applied is obviously the total shear resistance.

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Now we can make computations or derive expression from which we can compute the total torque applied and the corresponding shear resistance. How do we do that? If we take the torque versus the rotation, usually the relationship is like shown in this figure. Its take a value which corresponds to the peak here and then after the soil fails the torque comes down. The torque goes on increasing until failure and then it comes down. The vane itself experiences stresses as it cuts through the soil. It experiences shear stress along the phases on the sides as well as at the top and the bottom. Usually the shear resistance distribution is taken as uniform on the two sides and at the top it's taken as either uniform and so is the case with the bottom. What is shown here is a more generalized distribution than a uniform one that is a parabolic vane shear distribution whether at top end or at the bottom. The shear resistance offered by the soil is assumed to be having parabolic distribution.

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SOIL MECHANICS

VANE SHEAR TEST

Shear resistance along cylindrical surface
 $= \pi d H \tau_f$

Shear resistance along top and bottom surfaces
 $= 2 \int_0^{d/2} (2 \pi r) dr \tau_f$

$$T = (\pi d l l) \tau_f \frac{d}{2} + 2 \int_0^{d/2} (2 \pi r dr \tau_f) r = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6} \right]$$

**This assumes uniform shear -
 at top and bottom ends.**

If $H/d > 3$, end correction is negligible

Now how do we arrive at an expression at the time of failure? The shear resistance along a cylindrical surface is $\pi d H \tau_{wf}$. Why do we consider a cylindrical surface? It is very easy to imagine, as the torque is applied and the vane rotates it cuts a cylindrical surface through the soil and therefore the shear resistance of the soil is mobilized along this cylindrical surface. If τ_{wf} is the unit shear resistance then $\pi d H$ which is a surface area into τ_{wf} becomes the total shear resistance along the side cylindrical surface, whereas the shear resistance along the top and the bottom surfaces together will be twice zero to d by 2 . Because d by 2 is equal to the radius and the shear distribution is from the centre to the outer edge, $2 \pi r$ into $dr \tau_{wf}$ gives you the total shear resistance over a small elemental length dr at the ends.

Now when this is integrated we will get a final expression for T the total torque which is the sum of the shear resistance along the cylindrical surface and the shear resistance along the top and the bottom surfaces and one can easily do this integration and see that the final expression for this torque is $\pi d^2 \tau_{wf} \left[\frac{H}{2} + \frac{d}{6} \right]$. Here we have assumed as I mentioned sometime back uniform shear at top and bottom ends.

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SOIL MECHANICS
VANE SHEAR TEST

If, however, it varies non – uniformly, say, parabolically or triangularly, then

$$T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{ad}{4} \right]$$

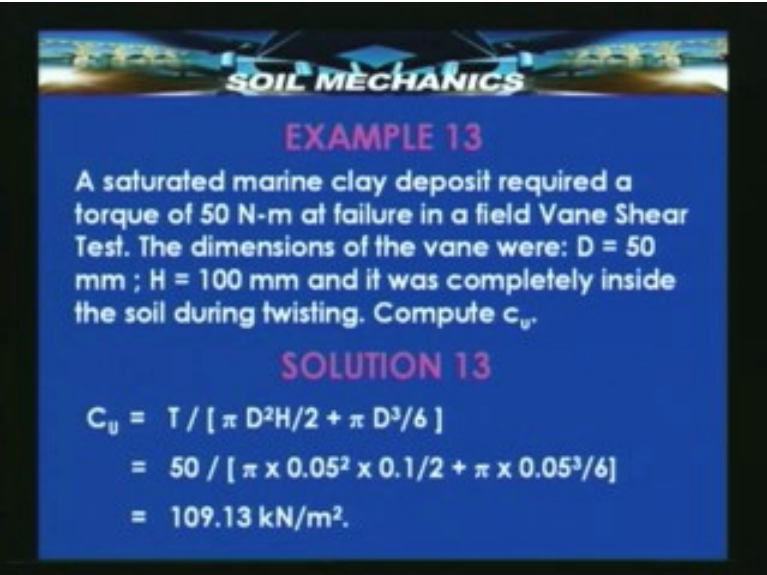
Where, $a = 0.67, 0.6$, respectively.

If the upper surface doesn't take part in offering shear resistance :

$$T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{12} \right], \text{ where } \tau_f = c_u.$$

And this is quite valid as long as H by d is greater than 3 but if that is not so if H by d is not greater than 3 then we may apply or we have to apply a small correction and then this expression will be written in terms of a correction factor a and d by 4. The correction factor is 0.67, 0.6 depending upon whether we have a parabolic or triangular end shear distribution. Now if it is uniform this ad by 4 becomes d by 6 and a becomes one third in fact. If the upper surface doesn't take part in offering shear resistance that is if the vane is not pushed completely into the soil. Then T the torque will only be d by 12 because the contribution from the bottom surface only will be there, contribution of the top surface will not be there. And in all this τ_f is the unknown and that's what we determine from knowing the torque and this expression and all the geometry of the vane and that's what is equal to the undrained cohesion of the soil. The shearing of the soil is done relatively rapidly so that we get an undrained cohesion value in this test.

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SOIL MECHANICS

EXAMPLE 13

A saturated marine clay deposit required a torque of 50 N-m at failure in a field Vane Shear Test. The dimensions of the vane were: D = 50 mm ; H = 100 mm and it was completely inside the soil during twisting. Compute c_u .

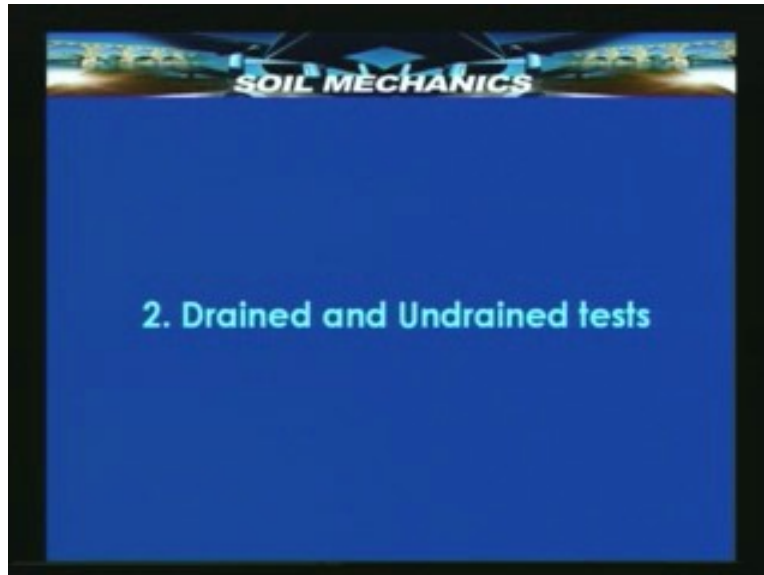
SOLUTION 13

$$C_u = T / [\pi D^2 H / 2 + \pi D^3 / 6]$$
$$= 50 / [\pi \times 0.05^2 \times 0.1 / 2 + \pi \times 0.05^3 / 6]$$
$$= 109.13 \text{ kN/m}^2.$$

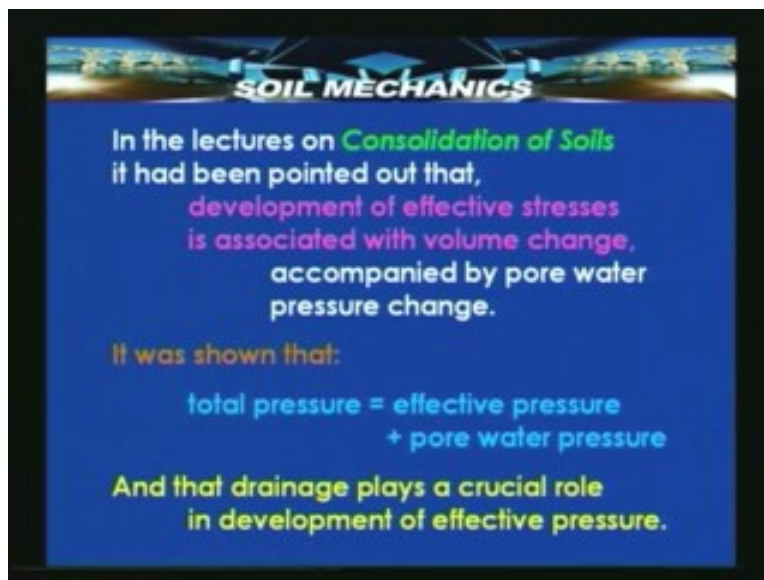
Now having seen the theory behind the vane shear test and the expression that is used for computing the torque in terms of a c_{uf} value, let us see an example of how this is used. There is a typical torque the value of which is around 15 newton per meter for a typical soil. The dimension of the vane are also known, this particular example which I want to discuss is that of a field vane shear test. Knowing the geometry of the vane and that a typical value of 50 newton meter torque is obtained at failure, we will calculate c_u . So the statement of the problem is a saturated marine clay deposit that mean this test is done in clay as I have being saying, required a torque of 50 newton meter at failure. The dimensions of the vane were 50 and 100 mm it is also given that the vane was completely inside the soil which means both the top and bottom surfaces participated in offering shear resistance. Compute the value of c_u , obviously c_u can be obtained by dividing capital T by this expression which we have derived earlier and this will give us a c_u value of 109.13 newton per meter square. Now that completes the vane shear test and its usefulness and its application.

Now in this particular lecture we will go now forward and take the second subdivision that is drained and undrained tests. Now it's very well-known, we have discussed it sufficiently often frequently in both my lectures on consolidation as well as in this. That when a soil is saturated and when a load is applied, be it compressive or be it sheared, whenever a load is applied if the drainage is permitted then the load gets applied to the transferred to the soil grains as intergranular stresses. Otherwise they remain as neutral stresses in the soil and in the form of excess pore pressure until the water drains off and the stresses are transmitted to the grains. In the case of shear as well, when the shear force is applied to the soil in the test for example when the shear force is applied depending upon whether it is a drainage or not, the applied stress will get transmitted first to the water and then to the soil grains. And therefore it is very important to know what is the condition with respect to drainage in a typical shear strength test. Let's see the next slide drained and untrained tests.

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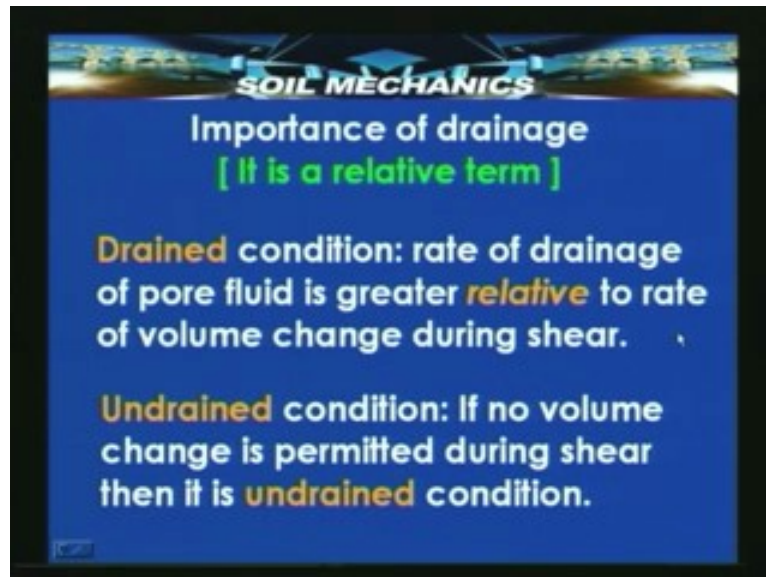


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In my earlier lecture I have told that development of effective stresses is associated with volume change and by pore water pressure change. Therefore if we apply a shear force and do not allow volume change then pore pressure will be developed and the equation, well known equation total pressure equal to the effective pressure plus pore water pressure will be valid. Thus drainage plays a very important role.

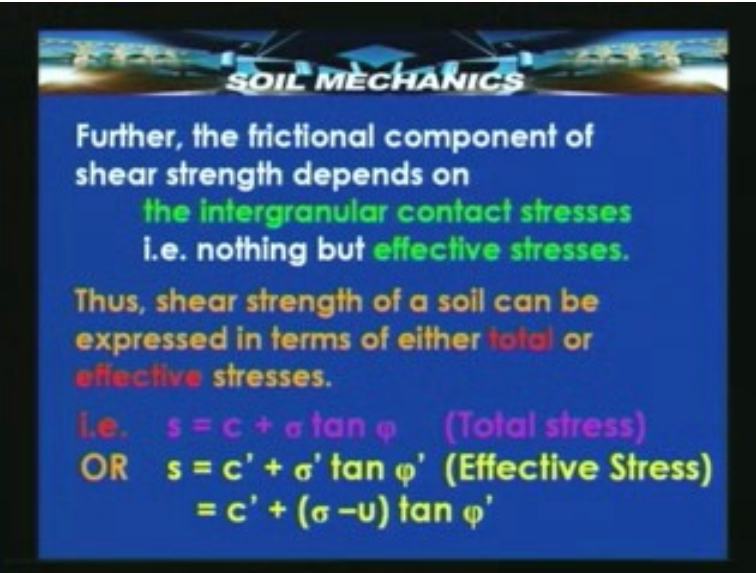
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And what is this drainage, why it is important? It is important because actually it is a relative term. When I say there is drainage provision during a test, what it means is that the rate of shearing is such that water has enough time to flow out of the soil. On the other hand if I say that the test is conducted under undrained conditions it could mean either that there is no passage for water to go out or it could mean that the shearing is done so rapidly that water does not get time to get a free passage out.

Now this is important because whereas in a laboratory test we can create conditions of complete drainage or complete no drainage. In the field it's not possible, in the field there could be either completely drained conditions for example if the soil is a granular soil, sand then soil is highly pervious and complete drainage conditions could exist. On the other hand if the soil is clayey, complete drainage conditions will never exist there will be always partial drainage and depending upon the rate of shear water will be either be allowed to flow out or not and therefore the rate of shear relative to the rate of flow of water decides whether or not the conditions is drained or undrained. So it's a relative term.

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SOIL MECHANICS

Further, the frictional component of shear strength depends on the intergranular contact stresses i.e. nothing but effective stresses.

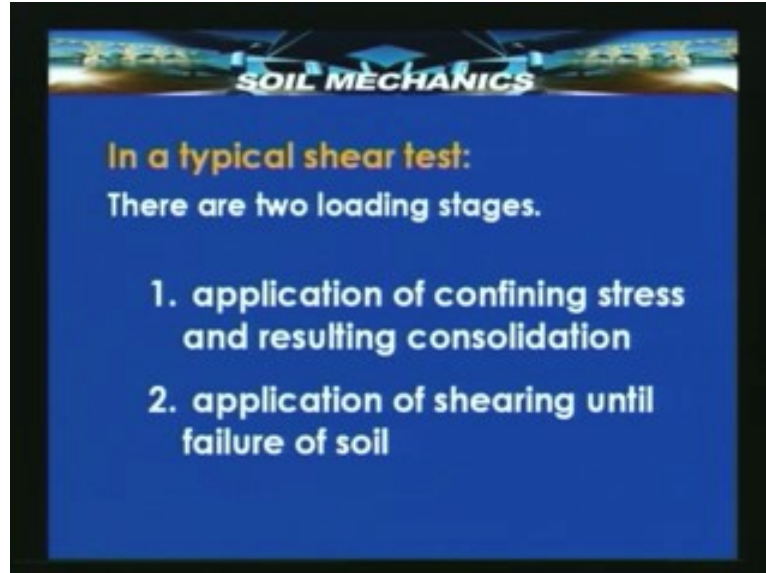
Thus, shear strength of a soil can be expressed in terms of either total or effective stresses.

i.e. $s = c + \sigma \tan \phi$ (Total stress)
OR $s = c' + \sigma' \tan \phi'$ (Effective Stress)
 $\sigma' = \sigma - u$

Now if you see the next slide it shows very clearly that if we use total stresses then we will have shear strength given by the expression $c + \sigma \tan \phi$. If we measure the pore pressure and if we calculate the effective stress σ' then the shear strength can be expressed in terms of the effective strength parameter c' and ϕ' and this stress σ' which is the effective stress will depend upon the total stress applied and the measured pore pressure u . So depending upon the drainage either we will have total stress or effective stress.

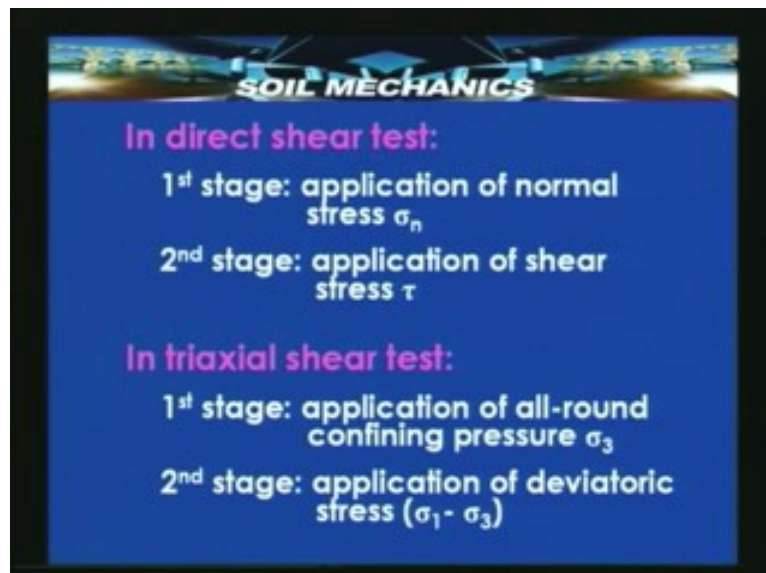
Accordingly as you will be seeing it later in some test we will only be getting total stress shear strength parameters and in some test we will be getting effective stress shear strength parameter. There are two stages of loading in any typical shear strength test. First is application of a confining stress and the second stage is application of the shearing force.

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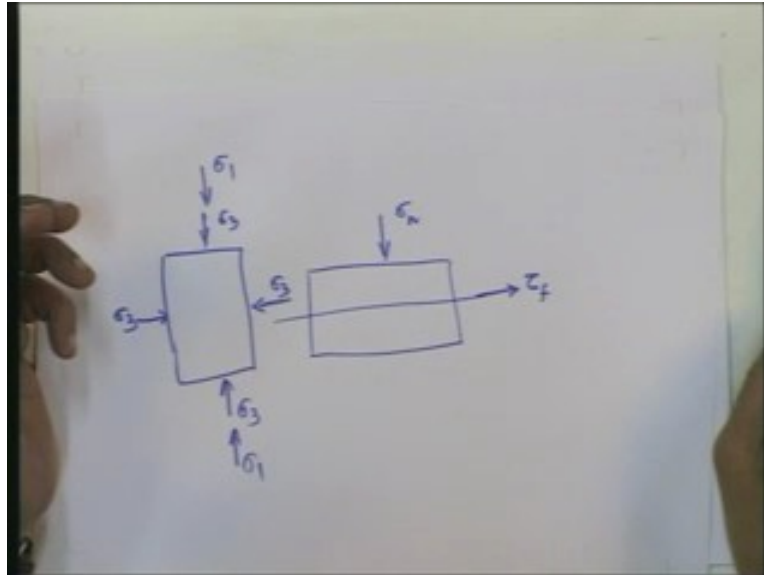
There are two major types of test which are commonly used in the laboratory the direct shear test and triaxial shear strength test. In the direct shear test, the first stage of the so called confinement corresponds to the application of the normal load. If I have a direct shear sample, this is the failure plane. Application of the normal load σ_n corresponds to the confining stress and the application of the shear stress until failure corresponds to the second stage of loading. On the other hand we have triaxial test specimen then application of the lateral confining pressure σ_3 corresponds to the first stage of loading.

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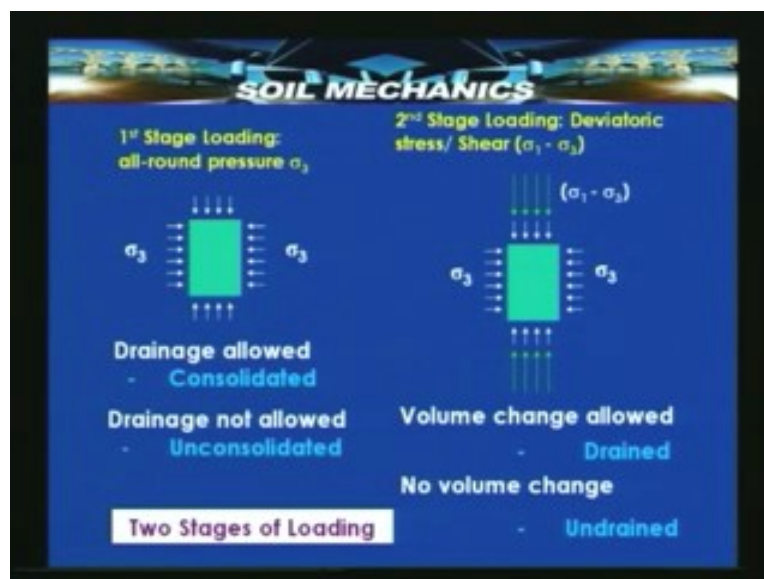
And as you know σ_3 will get applied equally at the top and bottom as well because the specimen is kept in a Perspex container carrying water. And then subsequently increasing σ_3 to its final value σ_1 that corresponds to the application of the second stage of loading that is the deviatoric stress $\sigma_1 - \sigma_3$ half of which corresponds to the maximum shear stress.

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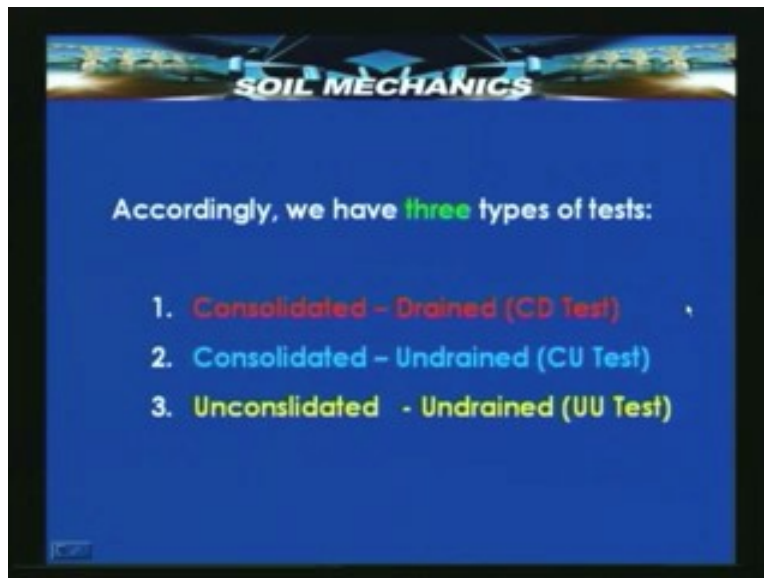
This diagram shows the various stages in loading. The first stage of loading corresponds to the application of σ_3 , while applying σ_3 we could either allow drainage and allow consolidation under this pressure σ_3 or not allow drainage and not permit consolidation.

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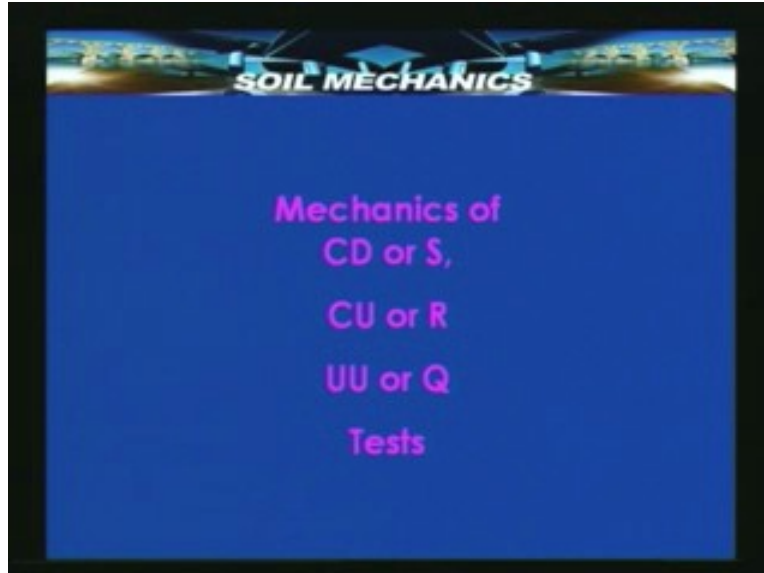
And in the second stage of loading when we apply the deviatoric stress, either we could allow volume change in which case drainage is permitted. That's the implication or we may prevent volume change which means no drainage is allowed. So the first stage has two possibilities. The second stage also have two possibilities and combining the two we have therefore 4 different combination out of which one is not a realistic combination and therefore we normally use 3 combination of the first and second stage loadings.

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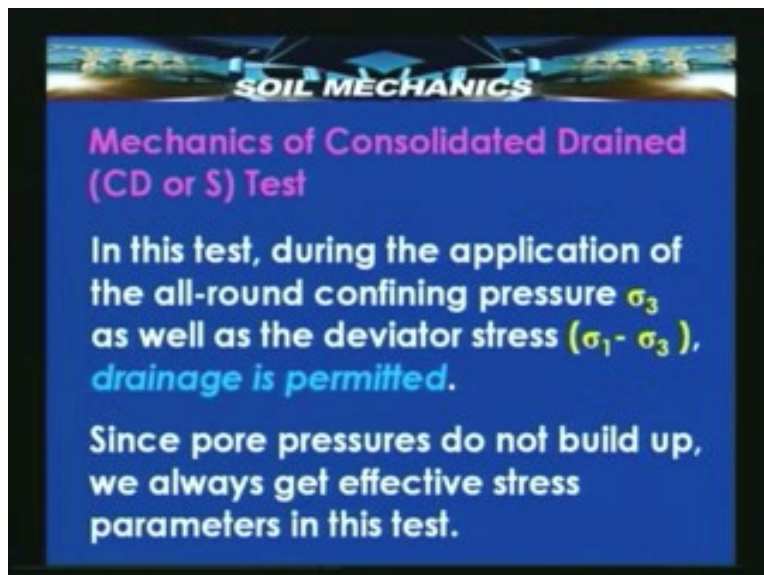
Accordingly we have 3 types of tests, first one is consolidated drained test, second one is consolidated undrained test. The last one is unconsolidated undrained test. We do not use the unconsolidated drained test that combination because it is not a realistic one, it doesn't normally happen in the field. The names which are normally used for these tests are CD, CU and UU. Now let's see the mechanics of these tests.

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And unless we understand the mechanism involved in these tests how the stresses are changing we will never be able to understand what is the kind of shear strength parameters which we are getting from these tests. Are we getting total stress parameters? Are we getting effective strength parameters? If we are getting the total strength parameters, how to get the effective strength parameters from them and so on. Let us take the consolidated drained test first.

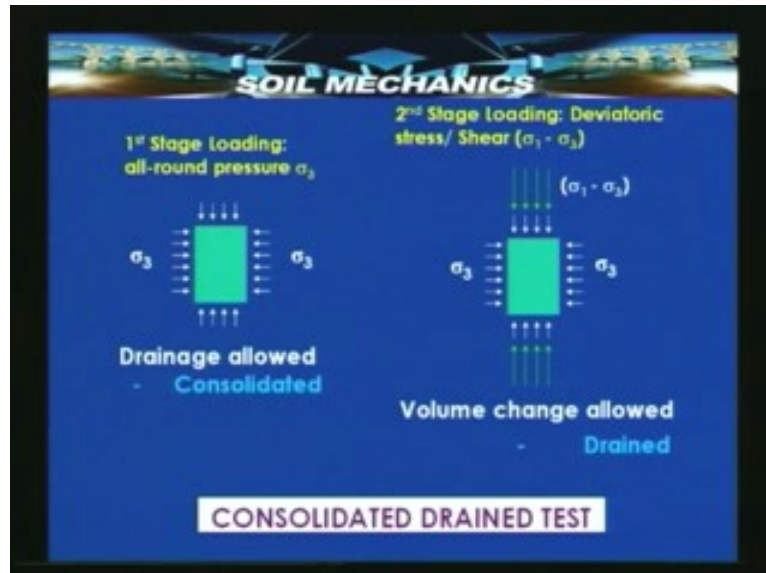
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The mechanics of the consolidated drained test is very simple. In the first stage of loading that is the application of confinement, drainage is permitted and then the second stage when shear stress is applied volume change is permitted that means in effect drainage is allowed that's why this

permits consolidation to take place during the first stage of loading and drainage to take place during the second stage of loading and that's why it is called consolidated drained test.

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This implies that the loading is slow enough to allow water to expel and not build any pore pressure. With a help of the diagrams which we had seen a little while earlier, we can say that in the first stage drainage is allowed and consolidation is permitted and in the second stage volume change is allowed and drainage is permitted.

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The table, titled "Mechanics of Consolidated Drained (CD) Test", summarizes the stress conditions during the test stages.

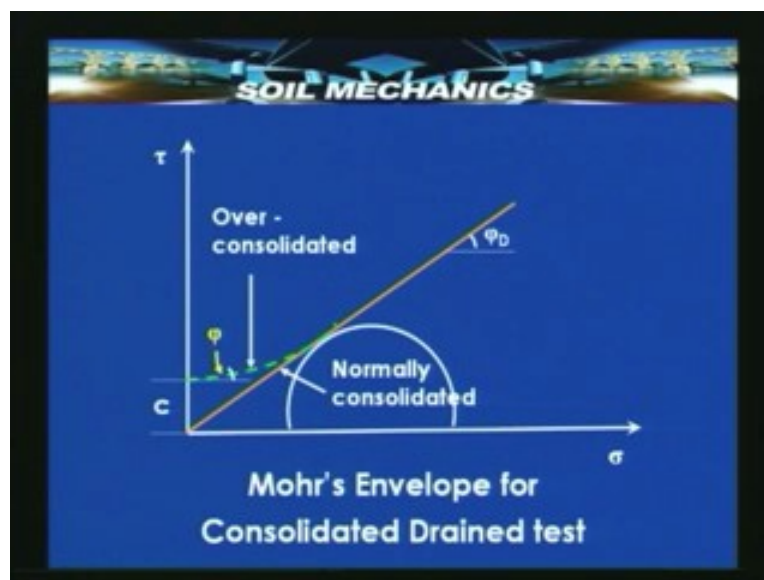
Loading	Total stress	Neutral stress	Effective stress
Confining Pressure $\Delta\sigma_3$	$\sigma_1 = \sigma_3$ $\sigma_3 = \sigma_3$	$u = 0$ $u = 0$	$\sigma_1' = \sigma_3$ $\sigma_3' = \sigma_3$
Axial Load $\Delta\sigma_1$	$\sigma_1 = \sigma_3 + \Delta\sigma_1$ $\sigma_3 = \sigma_3$	$u = 0$ $u = 0$	$\sigma_1' = \sigma_3 + \Delta\sigma_1$ $\sigma_3' = \sigma_3$

We now come to the most important aspect of this test that's the way the stresses change during the test. Let us take a look at this table. There are 4 items here the type of loading that is applied. The total stresses, the stress in the water neutral stress and the effective stress that's transmitted to the soil grains. The first stage of loading is application of the confining pressure. Let's call this $\Delta \sigma_3$. When a pressure $\Delta \sigma_3$ is applied horizontally or since we are applying a fixed value of confining pressure we may as well call it as σ_3 and not as an incremental stress $\Delta \sigma_3$. Then whatever stress we apply immediately becomes a total stress, its taken by water. And the total stress value will therefore be lateral pressure σ_3 equal to the applied pressure σ_3 . The vertical pressure is also equal to the applied pressure σ_3 because the sample is surrounded by water on all sides. Corresponding pressure in water will depend upon whether drainage is permitted or not.

Now this is a consolidated drained test and therefore drainage permitted and consolidation is permitted which means that the water does not take any stress, neutral stress is zero, u equal to zero both in the horizontal lateral direction as well as in the vertical direction. And whatever stress we apply therefore become effective stresses σ_3 dash the effective stress in the lateral direction becomes equal to the applied stress σ_3 so also the vertical pressure σ_1 dash becomes equal to the applied stress σ_3 .

Now comes the second part of loading that is application of the incremental vertical stress. Let us say the vertical stress applied is $\Delta \sigma_1$ then the total stresses will obviously be the original total stress plus $\Delta \sigma_1$. Since $\Delta \sigma_1$ is applied only in the axial direction, σ_1 becomes σ_3 plus $\Delta \sigma_1$ whereas σ_3 remains as σ_3 . Once again during the application of this axial load, during shear we permit drainage, we permit volume change we do not restrict it. Therefore u is again zero, there is no pore pressure developed which means that the effective stress is applied pressure once again, σ_1 dash plus is σ_3 plus $\Delta \sigma_1$, σ_3 dash is equal to σ_3 that means all the stress that we applied during this test get converted into effective stresses. Let's see next.

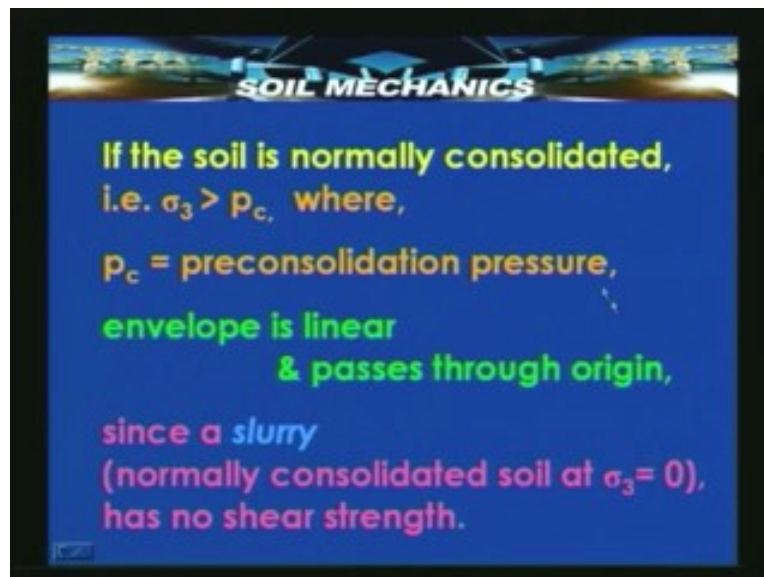
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Therefore the typical Mohr diagram for this will involve sigma tow plot with a Mohr circle and a corresponding envelop which will give us an effective value of cohesion and an effective value of friction angle and these are usually called as c and ϕ_D corresponding to the drained condition. Now here it's very important to remember that in nature soil undergoes consolidation over millions of years. There is always a possibility that the soil which we test has already undergone some preconsolidation. So if we have an overconsolidated soil then the confining pressure we apply may not necessarily be the pressure corresponding to the pressure which has already experienced in the past.

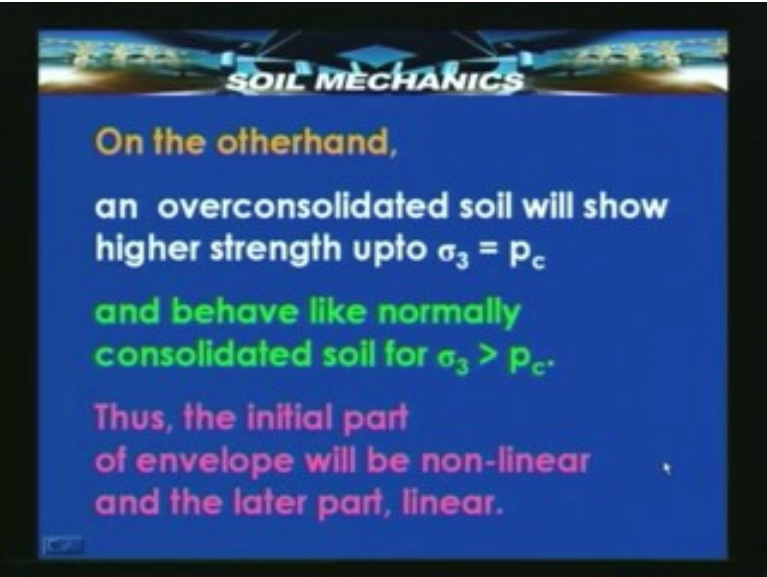
If the pressure we apply is less than the pressure to which it has been subjected earlier then the soil will behave like an overconsolidated soil. On the other hand it will behave like a normally soil. So if it's a normally consolidated soil we will get a straight line envelop. If on the other hand it's an overconsolidated soil, it will show a higher strength as given by this green dotted line until the applied normal stress becomes equal to the preconsolidation pressure. And beyond that once again the Mohr circle of the overconsolidated soil follows the envelop of the normally consolidated soil. That means after the preconsolidation pressure is exceeded for that higher pressure, the soil even the so called overconsolidated soil is actually relatively a normally consolidated soil.

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So if the soil is normally consolidated, if σ_3 is greater than p_c where p_c is a preconsolidation pressure then envelop is linear and it will pass through origin. Now why should the drained test envelop pass through the origin. It should pass through the origin because the origin corresponds to a pressure of σ_3 equal to zero. A soil with σ_3 equal to zero, saturated soil corresponds to a slurry and a slurry does not have a shear strength, it's a liquid. And therefore at σ_3 equal to zero there will be no shear stress, no shear strength in a totally drained test. Therefore the envelop should pass through the origin. On the other hand if the soil is overconsolidated it will not pass through the origin. That's what the next slide shows, the initial part of the envelop will be nonlinear.

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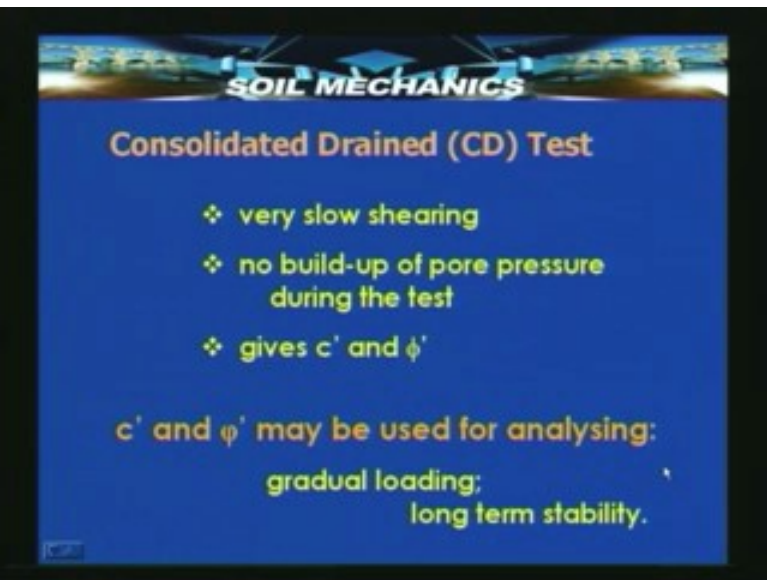


SOIL MECHANICS

On the otherhand,
an overconsolidated soil will show
higher strength upto $\sigma_3 = p_c$
and behave like normally
consolidated soil for $\sigma_3 > p_c$.

Thus, the initial part
of envelope will be non-linear
and the later part, linear.

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SOIL MECHANICS

Consolidated Drained (CD) Test

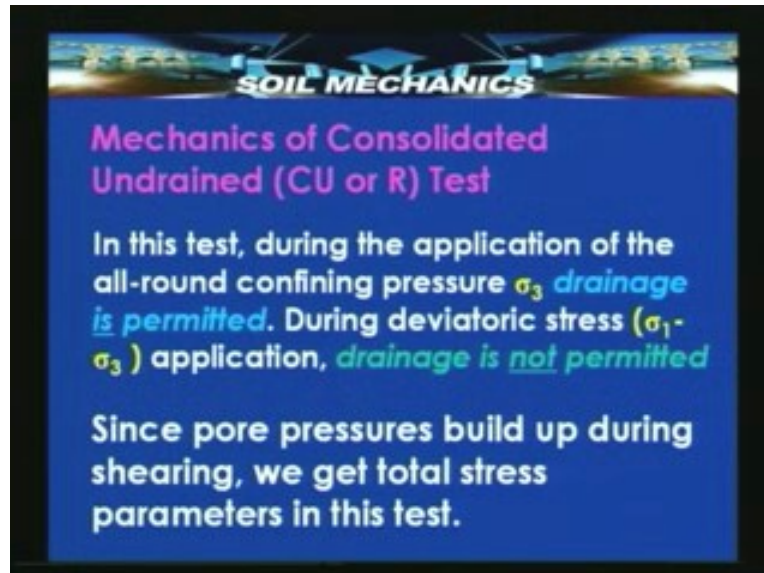
- ❖ very slow shearing
- ❖ no build-up of pore pressure during the test
- ❖ gives c' and ϕ'

c' and ϕ' may be used for analysing:
gradual loading;
long term stability.

A consolidated drained test is a very slow test, shearing has to be done at such a slow rate that pore pressure are not allowed to build up and what it gives are shear strength parameters c' and ϕ' . Now where do we use this c' and ϕ' , what's the application of this test? The application of this test lies in the fact that it gives us effective strength parameters and they are very useful wherever this test condition replicates the field condition. In the field if we have gradual loading and if volume change is permitted and drainage is allowed and that's what happens for example in an embankment which has been in consistent for several years, for a very long term. It has already gone undergone consolidation and therefore if it undergoes slow shear, it will exhibit an effective strength parameter c' and ϕ' . The c' and ϕ' can

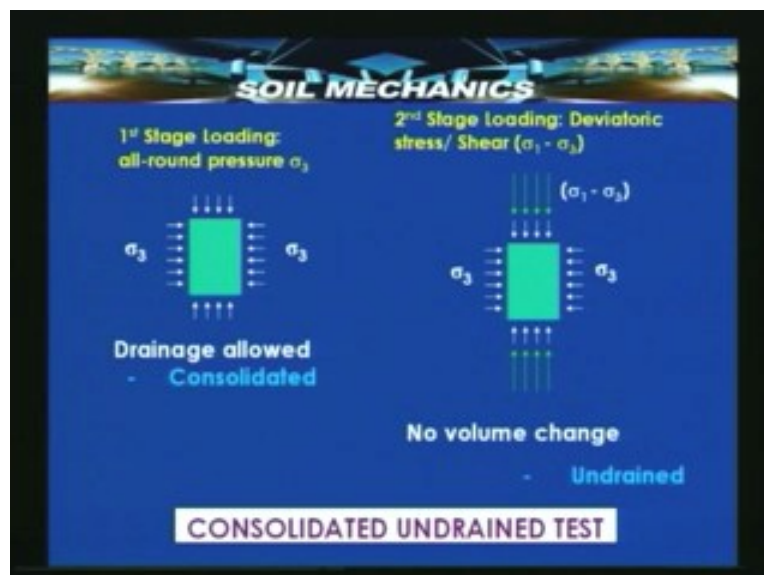
therefore be used for analyzing the effect of gradual loading or long term stability of an embankment.

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Now let us take the next test that is the consolidated undrained or R test, it is also called the R test. The difference between this and the previous test is here consolidation is permitted in the first stage of loading but in the second stage of loading drainage is not permitted, volume change is not allowed.

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Therefore we get total stress parameter in this because unless the drainage is permitted the stresses don't become effective. Like last time if we take a look at this diagram, we find that in the first stage drainage is allowed, consolidation is permitted whereas in the second stage it is undrained during the application of the deviatoric test.

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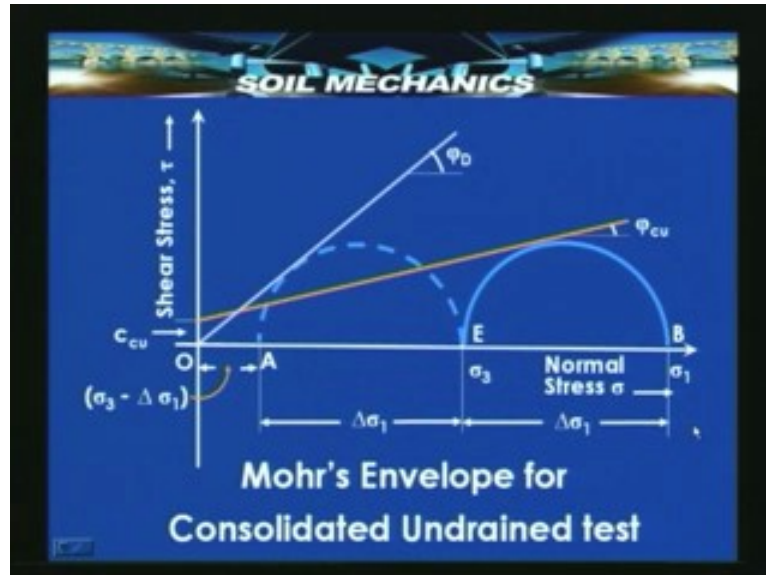


Loading	Total stress	Neutral stress	Effective stress
Confining Pressure $\Delta\sigma_3$	$\sigma_1 = \sigma_3$ $\sigma_3 = \sigma_3$	$u = 0$ $u = 0$	$\sigma_1' = \sigma_3$ $\sigma_3' = \sigma_3$
Axial Load $\Delta\sigma_1$	$\sigma_1 = \sigma_3 + \Delta\sigma_1$ $\sigma_3 = \sigma_3$	$u = \Delta\sigma_1$ $u = \Delta\sigma_1$	$\sigma_1' = \sigma_3$ $\sigma_3' = \sigma_3 - \Delta\sigma_1$

This table is now important, mechanics of the consolidated undrained test like the consolidated drained test we have here a column for loading. Then three columns for total neutral and effective stress. In loading first stage is that of application of $\Delta\sigma_3$, second stage is application of $\Delta\sigma_1$. Since we are actually applying a finite value of lateral pressure σ_3 this so called applied increment in loading can be considered to be nothing but σ_3 . The total stresses will obviously be the applied stresses and therefore σ_1 equal to σ_3 , σ_3 equal to σ_3 and since consolidation is permitted u is zero, effective stresses will be the applied stresses. This part of loading and the corresponding stresses are identical to the earlier test. When the axial load is now applied then comes the difference.

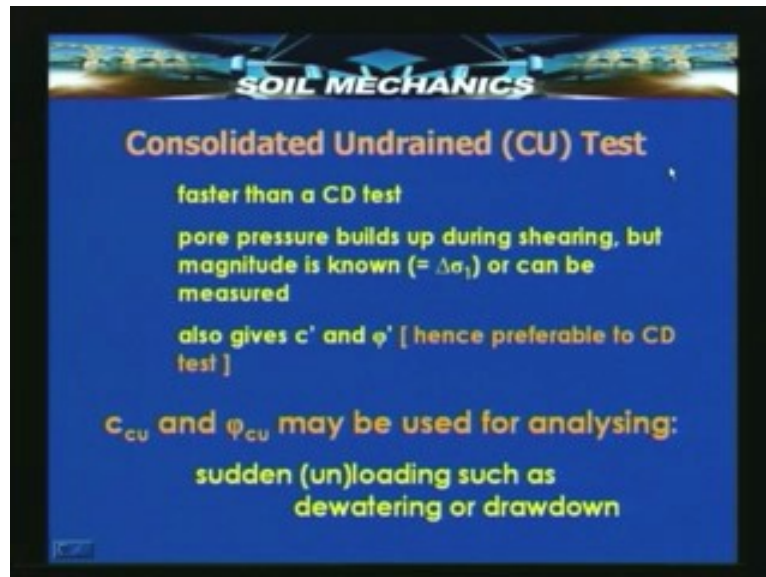
The total stresses remain the same because the axial load gets applied as total stress. Now the water is not free of stress because undrained condition prevail, u in water is $\Delta\sigma_1$ the applied axial load both in the horizontal direction and in the vertical direction because the specimen is surrounded by water in a Perspex cylinder. But the effective stress will be σ_3 plus $\Delta\sigma_1$ minus $\Delta\sigma_1$ that is σ_3 whereas σ_3' will be σ_3 minus $\Delta\sigma_1$. Thus the effective stresses are not equal to the applied stresses but slightly less. And where do we use this test this kind of a test?

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Before that let us see what a Mohr's circle looks like for this. Since this is a consolidated undrained test the pore pressure developed, the applied load $\Delta\sigma_1$ will both be equal irrespective of what the value of σ_3 is. So if we were to draw a Mohr's circle, if point E corresponds to σ_3 and point B corresponds to σ_1 , the diameter $\Delta\sigma_1$ is the applied stress under drained condition and the total stresses will therefore be σ_1 and σ_3 as shown in this figure. And to get the effective stresses we have to subtract the pore pressure which is equal to $\Delta\sigma_1$. If I subtract $\Delta\sigma_1$ I get this dotted circle whose point on the left A corresponds to the effective stress σ_3 minus $\Delta\sigma_1$. The point E for the left circle has a value of σ_3 which is nothing but effective stress as far as this is concerned. The Mohr's envelope if it is plotted for the total stresses then that will give an intercept c_{cu} .

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Because this test is called consolidated undrained, the intercept is called c_{cu} and the angle is called ϕ_{cu} for the envelop. On the other hand if we plot the effective stress Mohr's diagram, obviously as per the reasoning we gave a little earlier, the envelop should give the drained friction angle and it should pass through origin O as long as this soil is a normally consolidated soil. In this test the pore pressure build up is allowed during the application of shearing, this can give c dash and ϕ dash because we know what will be the magnitude of the pore pressure. We don't have to really measure though we do measure in a typical test for validation but we know that the pore pressure is equal to the applied vertical stress and therefore c dash and ϕ dash can be easily calculated. And c_{cu} and ϕ_{cu} is of course can be calculated from the total stress readings which are directly applied from the test.

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SOIL MECHANICS

Mechanics of Unconsolidated Undrained (UU or Q) Test

In this test, during the application of the all-round confining pressure σ_3 as well as during deviatoric stress ($\sigma_1 - \sigma_3$) application, drainage is not permitted

Since pore pressures build up during shearing, we get total stress parameters in this test.

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SOIL MECHANICS

1st Stage Loading: all-round pressure σ_3

2nd Stage Loading: Deviatoric stress/ Shear ($\sigma_1 - \sigma_3$)

Drainage not allowed - Unconsolidated

No volume change - Undrained

UNCONSOLIDATED UNDRAINED TEST

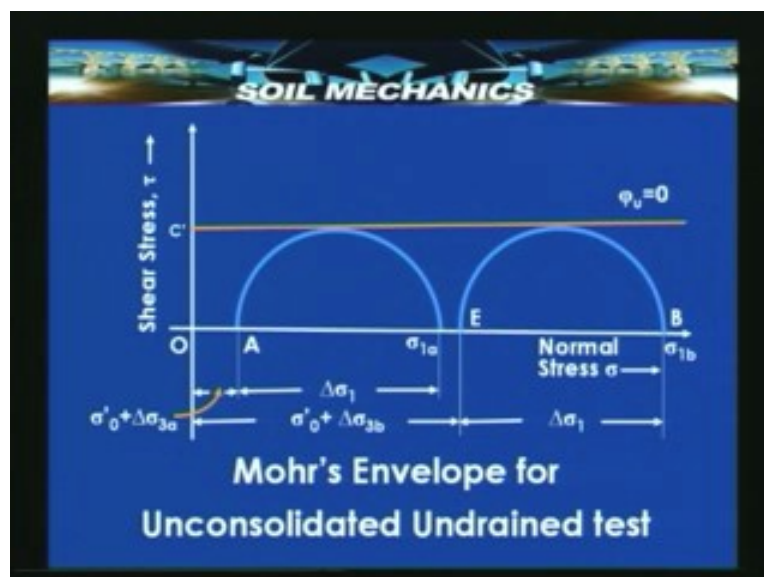
Where do we apply this test? This test is good for situations where an embankment for example has spent a little bit of its life and now it's undergoing a sudden unloading. In such a case c_{cu} or ϕ_{cu} will apply, examples are dewatering, sudden draw down. Now mechanics of unconsolidated undrained test, the third kind of test. Here drainage is not permitted either during the first stage of loading or through the second of loading. As per this diagram therefore both in the first stage and in the second stage drainage will not be allowed and there soil is neither consolidated nor is volume change permitted. The mechanics now will be slightly different. Let us take the loading column and the stress columns.

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SOIL MECHANICS			
Mechanics of Unconsolidated Undrained (UU or Q) Test			
Loading	Total stress	Neutral stress	Effective stress
Overburden	$\sigma_1 = \sigma_0'$ $\sigma_3 = \sigma_0'$	$u = 0$ $u = 0$	$\sigma_1' = \sigma_0'$ $\sigma_3' = \sigma_0'$
Confining Pressure $\Delta\sigma_3$	$\sigma_1 = \sigma_0' + \Delta\sigma_3$ $\sigma_3 = \sigma_0' + \Delta\sigma_3$	$u = \Delta\sigma_3$ $u = \Delta\sigma_3$	$\sigma_1' = \sigma_0'$ $\sigma_3' = \sigma_0'$
Axial Load $\Delta\sigma_1$	$\sigma_1 = \sigma_0' + \Delta\sigma_3 + \Delta\sigma_1$ $\sigma_3 = \sigma_0' + \Delta\sigma_3$	$u = \Delta\sigma_3 + \Delta\sigma_1$ $u = \Delta\sigma_3 + \Delta\sigma_1$	$\sigma_1' = \sigma_0'$ $\sigma_3' = \sigma_0' - \Delta\sigma_1$

Under the loading column although we are applying a confining pressure and an axial load, in this case as you will be seeing during the course of discussion we need to add another loading which is due to the initial over burden. The reason is not for to seek because this is an unconsolidated undrained test, the confining pressure that we apply to the sample does not get transmitted to the soil grains and it remains as the total pressure and therefore when you shear this sample, soil should show no strength at all but on the other hand the soil does shows some strength because even before applying the confining pressure $\Delta\sigma_3$ the soil already has some pressure due to the overburden.

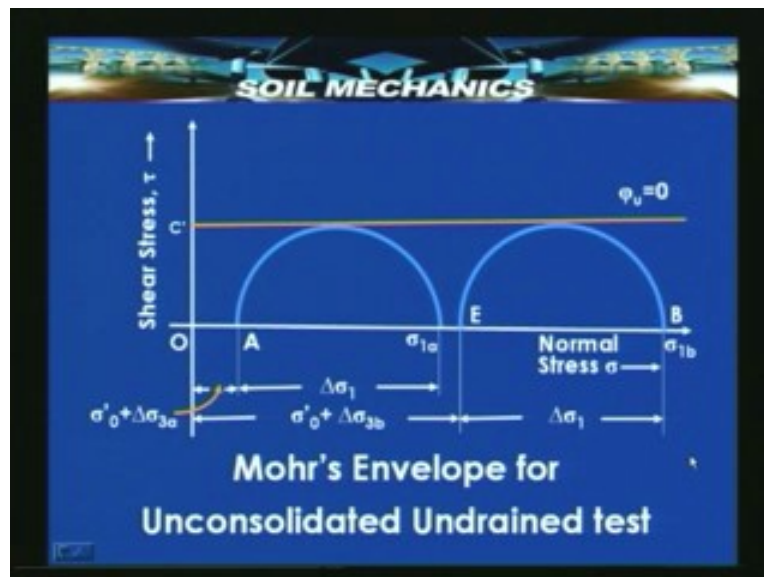
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So we must begin with the overburden loading in order to see the effect of the undrained confining pressure and undrained axial load applied. The overburden pressure will be an effective pressure because sufficient time has passed in history during the formation of the soil and σ_1 and σ_3 will both be equal to the overburden pressure σ_0 which is taken as the effective stress. Pore pressure will of course be zero under the normal natural conditions and the effective stresses will also be equal to σ_1 and σ_3 both equal to σ_0 . Now when you apply the confining pressure, the pressure that we apply additionally gets added to the total stresses and so we have σ_1 equal to σ_0 plus $\Delta\sigma_3$ and so is σ_3 and because undrained conditions prevail.

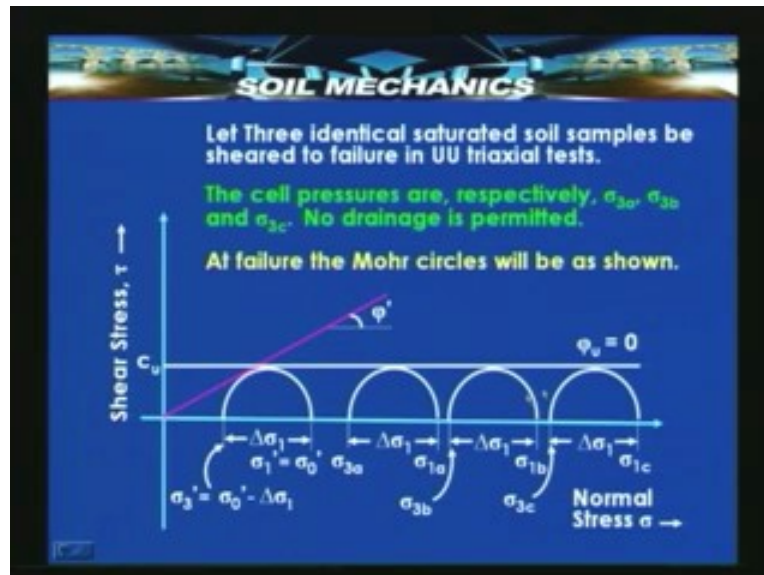
Pore pressure is not zero now, it is equal to the applied pressure, incremental pressure $\Delta\sigma_3$. The effective stresses will be σ_1 equal to σ_0 minus $\Delta\sigma_3$ equal to σ_0 meaning no effective stress is produced during the application of the confining pressure. Similar situation prevails when you apply the axial load total pressure gets increased by $\Delta\sigma_1$ in the vertical direction, pore pressure gets increased by $\Delta\sigma_1$ both in the vertical and also the horizontal and the effective stress gets modified into σ_0 minus $\Delta\sigma_1$ in the horizontal direction.

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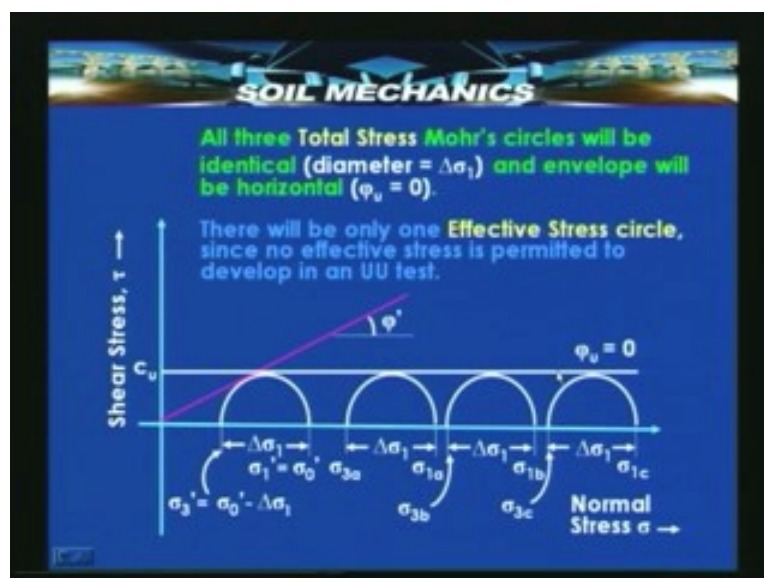
The Mohr's circle for this will involve similar Mohr's circles. Suppose I consider two Mohr's circles with two different confining pressures σ_0 plus $\Delta\sigma_3$ a or σ_0 plus $\Delta\sigma_3$ b. Since undrained condition prevails, they will both have the same value of the deviatoric stress equal to $\Delta\sigma_1$ and the Mohr envelope will be a common tangent which will be horizontal because due to the total undrained conditions that are prevailing, friction will not get mobilized in this particular test, ϕ_u will be equal to zero. Here is another example of the same.

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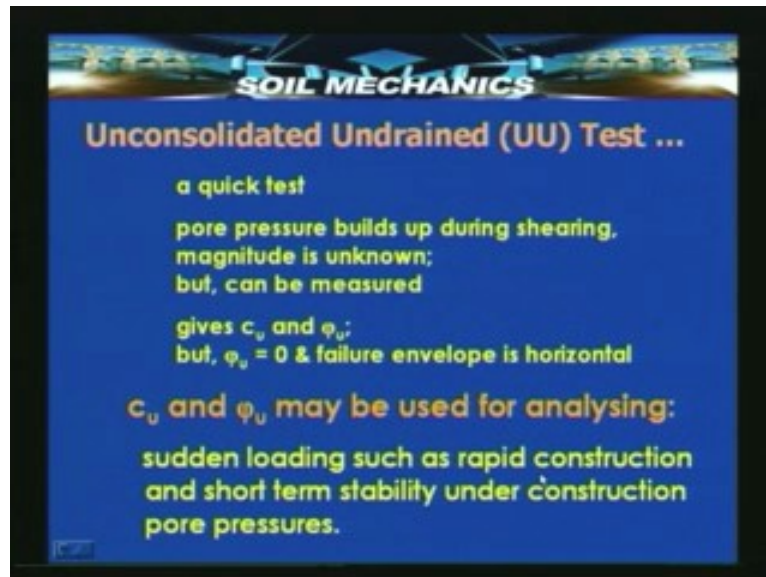
If I conduct three different tests with different values of σ_3 a, σ_3 b, σ_3 c and correspondingly also calculate the major principle stresses and plot Mohr diagram. The common tangent will give me an undrained cohesion c_u and a friction angle ϕ_u equal to zero. But if I have measurements of the pore pressure value here then I can subtract the pore pressures values from these applied stresses and get the effective stress values which as I have shown in the previous table will be σ_3 dash equal to σ_0 dash minus $\Delta\sigma_1$ and σ_1 dash equal to σ_0 dash. And this being effective stress it will give an envelope which will give me an effective friction angle and zero cohesion, because that will correspond to fully drained condition.

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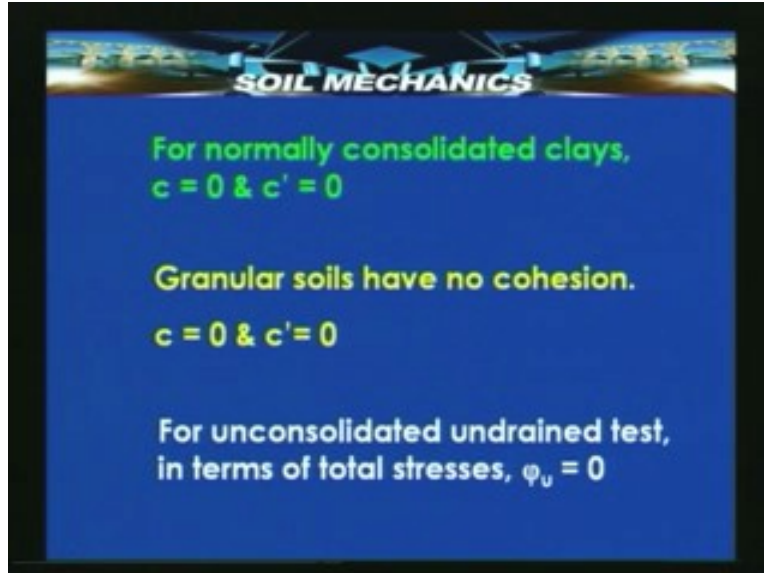
This is what the set of Mohr's circle will look and this is the final position as far as the effective stress conditions are concerned. Now let us see a few aspects corresponding to these tests. As $\Delta \sigma_3$ increases since no drainage is permitted, the pore pressure increases by the same amount and the effective stress remains unchanged. The change in pore pressure during shearing is a function of the initial effective stress and the moisture content; as these are identical for all the samples the strength obtained is also identical. Now if we are having a sample which is not fully saturated then ϕ_u equal to zero will not apply and c_u will not be constant, undrained analysis cannot be conducted.

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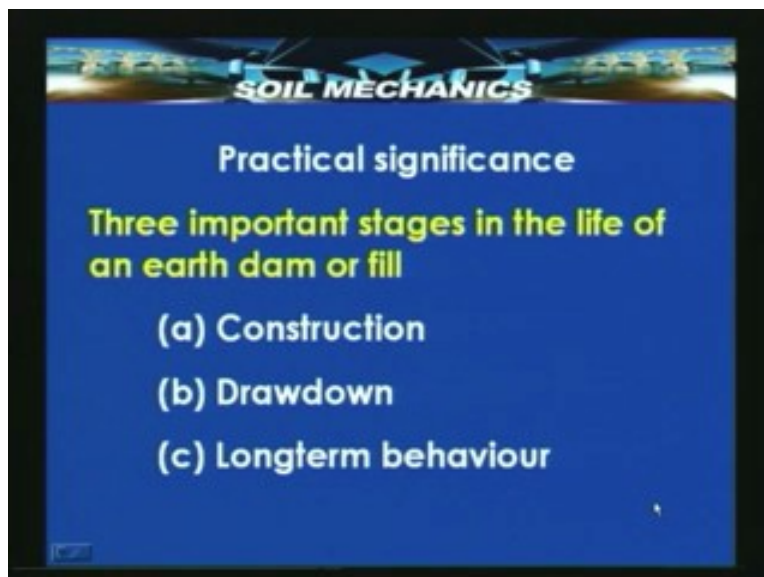
In an undrained test, totally undrained test it's a quick test so both confinement and shearing was done rapidly and no pore pressure is allowed to build up. And the values of c_u and ϕ_u which we get out of this can be used where similar conditions exists in the field that is condition of rapid construction. And therefore it is good for short term stability of an embankment or the stability of an embankment during the construction process itself, when the construction is rapid enough not to allow pore pressures to dissipate.

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Finally we can make a statement that c equal to zero and c' equal to zero for a normally consolidated soil in a fully undrained test and for granular soils also c equal to zero and c' equal to zero but in an unconsolidated undrained test ϕ_u equal to zero.

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We will see the practical significance of this in the next lecture. It's important to understand the significance of these tests and for that purpose we will take into account a typical earth dam and various stages like the construction stage, the drawdown stage and the long term stage of its life. Thank you.