Soil Mechanics
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Lecture – 44
Shear Strength of Soils
Lecture No.2

Dear students today we shall go through yet another lecture on shear strength of soil. This is the second lectures in a series of 5 or 6 lectures which I have planned to give. If you remember in the first lecture I gave you an idea about what shear strength is all about, why we need to study that. As usual let us first go through a quick review of what we already know before we go on to learn something more. So let me start with the first slide of today's lecture which is actually a review of what we had seen last time.

Take this slide for example. In the last lecture we had a brief over view of several aspects such as what's the important or significant of shear strength. In order to understand what shear strength is, what shear stresses are, we need to calculate the stress at any point on any plane. This aspect was emphasized in the last lecture and in order to do this computation of stresses at any point on any plane we also briefly looked into a concept known as the concept of Mohr's circle. We shall see a little more of that today. Then I also gave you an idea as to what kind of theories are available in order to see whether a soil will remain stable, will fail under the action of shear or not.

In order to know what is this shear strength of a soil, we need to have methods to measure them not only in labs but also if also directly in the field. So we also had a look at what kind of methods are available for this purpose. This is was what we have covered in the last lecture; a little more detail of what we saw in the last lecture is given in the next few slides. So what's the significant of shear strength for example? It is necessary primarily for the design of geotechnical structure against shear failure. What are the geotechnical structures? There can be many examples for example foundations, excavation.

Even an excavation is rightly called as a geotechnical structure because it is made in earth. Embankments, slopes, retaining walls and any number of earth structures they all come under the category of geotechnical structures and they are likely to fail if the shear stress along any plane at any point exceeds what the strength of the soil against shear is. That's the significance of learning shear strength. In order to know how to compute shear stress at any point so that we will know whether that strength is below or lower than the strength of the soil against shear. We need to have the method to compute the stresses at a point on any plane. So in order to check for shear failure it is necessary to have a method to determine the shear along any plane any point in a soil mass. For example suppose I take a point P in an infinite soil mass, there can be any number of planes passing through that point P, actually infinite number of planes. Take any plane CA and an element at the point P so called parallelepiped in two dimensions it becomes just a rectangle. Let the angle that CA makes with the horizontal B theta. Now this element can be exaggerated for visualization and represented as shown here in this key diagram.

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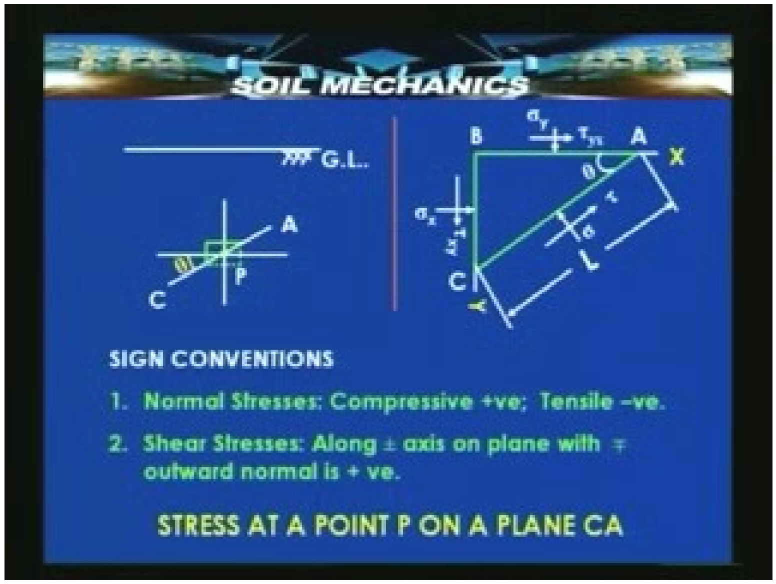

You see here this small element is expressed expanded in a set of 3 planes, actually all the 3 planes pass through this point P but for convenience we take them like this and in the limit when the dimensions of these planes reduced to or tend to zero the elements itself tend to be a point here. Now take this 3 planes AB, BC, CA in general on any plane there will be stresses which can always resolved into normal and shear components. So the normal component on this plane is sigma$_y$, shear stress is tow y x, on the other plane sigma$_x$ and tow xy. Now the notation used here follows a certain sign conversions. For normal stresses compressive positive, tensile negative is chosen. So here a compressive normal stress sigma$_y$ acts on the plane AB, the normal to the plane AB is in the direction y and according to the chosen coordinate axis and so the stress is called sigma$_y$ so also sigma$_x$. The tow x y or tow y x they represents shear stresses according to the sign convention along the positive axis on a plane with negative outward normal, the shear stress is positive.

For example take plane AB, the positive axis is in this directions and it's on a plane with a negative outward normal, the negative outward normal of plane AB is the negative y direction and therefore a positive shear stress acts in the positive direction of this plane, that's why tow y x in the direction x is taken as the positive shear stress so also tow x y. Now with this we will be in a position to calculate the stress on any arbitrary plane CA provided we know the stresses on AB and BC. So stresses on AB and BC are supposed to be known and usually they are known because for example take the plane AB that is horizontal. The stress there is essentially due to the weight of the column soil above it and the stress on the vertical plane is a fraction of that depending upon what's known as the coefficient of lateral earth pressure about which you must have definitely studying in some other lecture.

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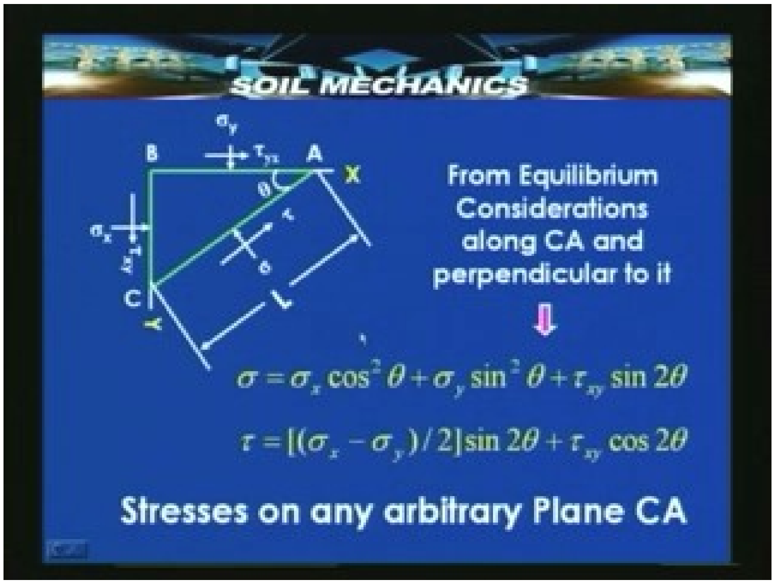

If we have this scheme of stresses on a set of planes, in terms of the stresses on AB and BC we can express the stress on CA from a very simple consideration that this element and therefore that point P must be in equilibrium. Now suppose the length of AC is capital L, correspondingly BC will be L sin theta, AB will be L cos theta and therefore we will be in a position to calculate the total forces on each one of these planes and find out what's the condition required for satisfying equilibrium of this forces. Force on AB for example can be calculated as sigma_y into L cos theta, force on BC can be calculated for example as tow xy into L sine theta. The force on AC will be sigma into L and tow into L.

In all for equilibrium to be maintained on this plane either parallel to it or perpendicular to it, we can show by resolving the forces that this stress sigma must satisfy the condition that sigma is equal to this and stress tow must be equal to this. This is what we discussed in some detail in the earlier lecture, so for purposes of this lecture, today's lecture we will take for granted that by resolving and satisfying equilibrium, we will be able to get these two stress values. So if all the terms on the right hand side are known and therefore it's possible to calculate the unknown stresses sigma and tow, particularly tow which is what we are interested in because that's the shear stress on any plane on CA.

If the shear stress exceeds the strength along this plane then shear failure will take place. Now in order to compute this stresses, there is also a geometrical method available which is based on a concept known as the Mohr circle concept. This is a circle which is devised so ingeniously devised that if you plot the known stresses and follow certain geometrical procedures, you can calculate the unknown stress sigma and tow on any arbitrary inclined plane CA. Let us take a look this key diagram once again which we are already familiar with AB BC CA and the stresses are sigma_x sigma_y tow xy sigma and L. Now let us see the geometrical procedure which is used for computing sigma and tow on the plane CA. First we choose a set of axis, one axis for all the normal stresses another axis for all the shear stresses. We plot the normal stress for sigma_x here as shown point J, sigma_y that is point k.

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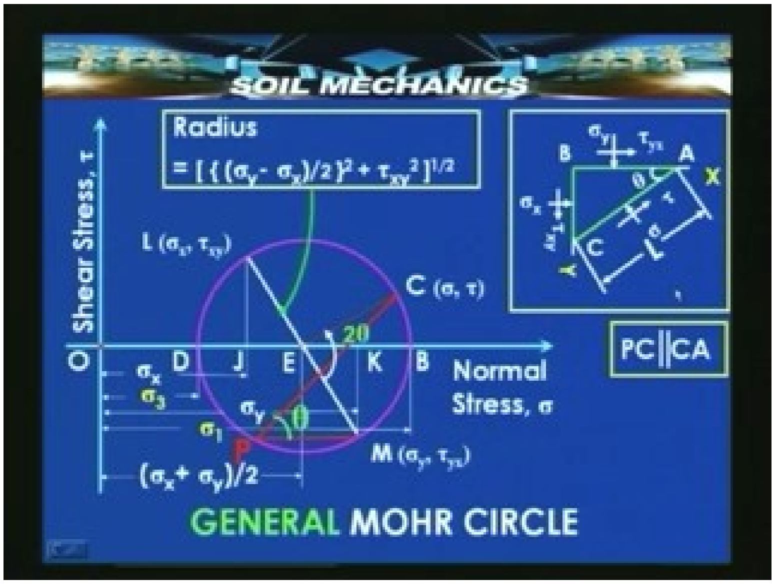

Then we draw so called Mohr's circle whose center will be at the midpoint between sigma$_x$ and sigma$_y$ and whose radius will be given by sigma$_y$- sigma$_x$ by 2 whole square plus tow xy by 2 whole square. Now with this radius if you draw a circle then this height obviously will be given by tow xy and a point will have a coordinate's sigma$_x$2 xy, correspondingly a point M will have sigma$_y$ tow yx. And the center will be having a coordinates sigma$_x$ plus sigma$_y$ by 2. In addition it can be shown that point B will have the maximum possible normal stress which is called as the major principle stress sigma$_1$ and point D will represent the minimum possible normal stress that is the minimum principle stress sigma$_3$. And any point C as shown here having stresses sigma and tow will represent the stresses on the plane CA.

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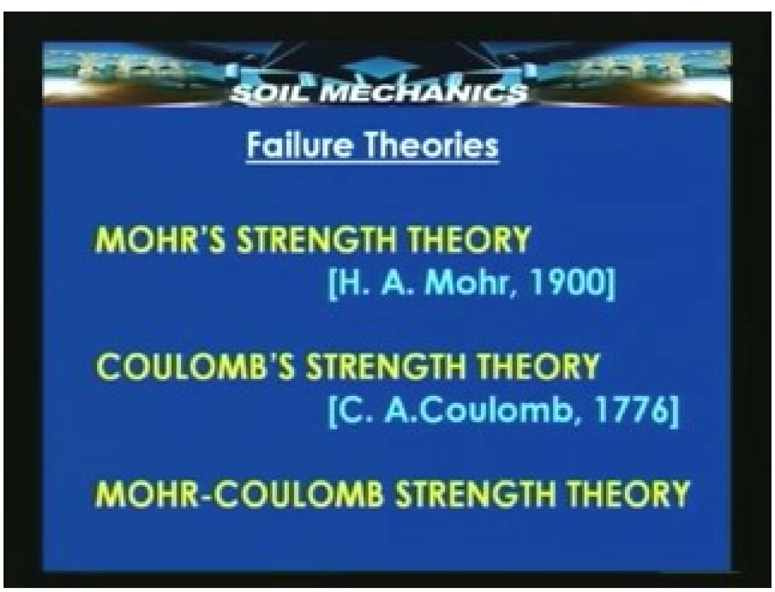

Since CA is at an angle theta with respect to the horizontal, on this drawing if we plot a plane EC at angle 2 theta with respect to the line joining L and M whose coordinates are sigma x tow xy and sigma y tow y x. Then we get the point C whose coordinates are sigma tow. There is also another method of obtaining a value of C through drawing lines MP and PC which we shall see in a subsequent slide. So this is what is known as the general Mohr circle which helps us to compute the normal and shear stress on any arbitrary plane provided we know stresses on any two perpendicular planes passing through the same point and the angle theta.

Now having a method to compute C, next step is to know whether failure will take place along particular plane, for this we have several failure theories. An idea about these failure theories had given in my earlier lecture. In today's lecture we shall try to see a little more about these theories. For all practical applications, suffice it to say that a linear relationship between stress and strength is sufficient to determine whether failure takes place or not. This is what is known as the Mohr coulomb failure plane, on the Mohr circle this can be expressed as a line having an equation s is equal to c plus sigma tan phi as we shall be seeing in little later.

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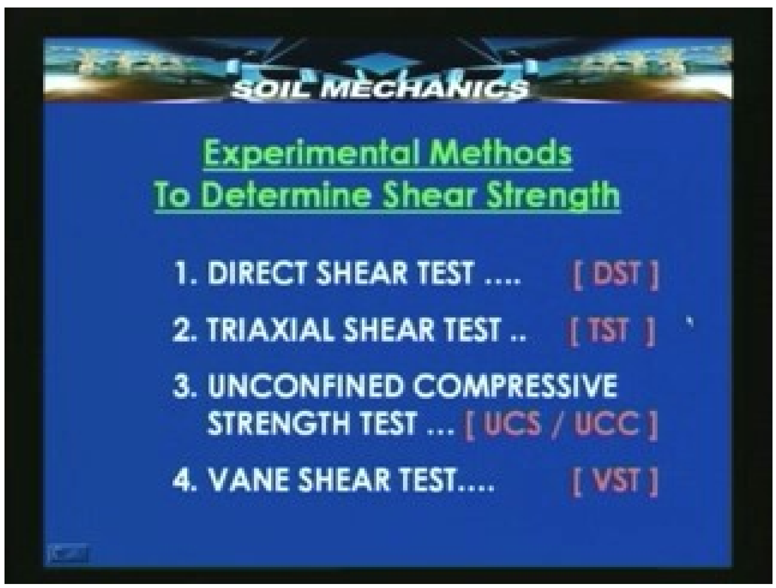

There are many methods to determine this shear strength, direct shear stress is one of them. This is a typically laboratory test although there are also large scale direct shear test meant for field application. Then we have triaxial shear test which is an improved variation of this shear stress called the direct shear test. Then we have certain quick test which can be conducted both in the laboratory and in the field, a little less rigorous than the first two test and they are also very useful and they are known as the unconfined compressive strength test UCS or UCC and the vane shear test VST. In a nutshell what we have seen so far is that there is a method or there needs to be developed a method to compute the normal and shear stresses on any arbitrary plane and it's also necessary to have a failure criterion or a strength theory to know whether failure will take place along any arbitrary plane or not. And then we need to know what the strength is in order to access whether failure will take place and for that we have methods.

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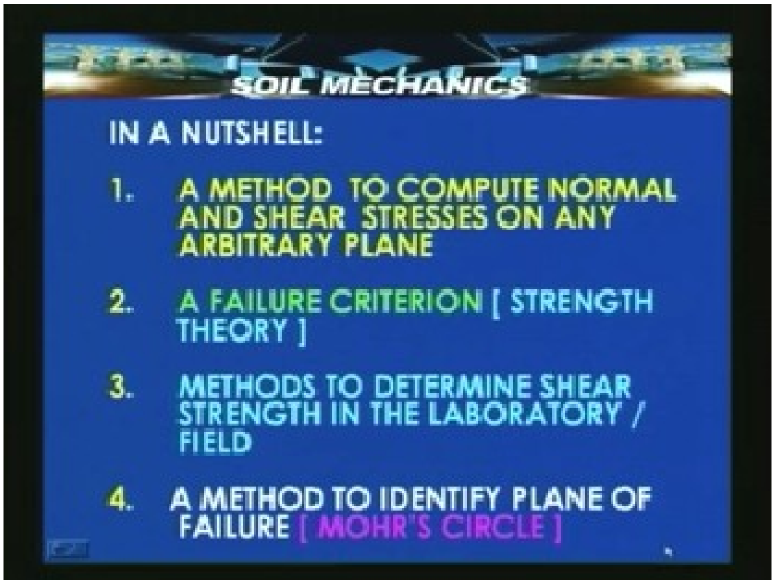

And then in order to compute the stress finally we have a nice method geometry based method known as the Mohr circle. Incidentally the Mohr circle itself is based on certain equations which we had developed rigorously and therefore either we can use the geometrical technique or directly the equations. We will see a few examples a little later and there both the concepts will be used to show that certain quantities which are associated with shear strength determination can be computed either from Mohr circle or by direct equations. In today’s lecture what we shall do is to take a closer look at this Mohr’s circle how it is constructed, different variation of this Mohr’s circle for different conditions and also a couple of examples to see how Mohr’s circle can be applied to compute certain parameters which are associated with shear failure.

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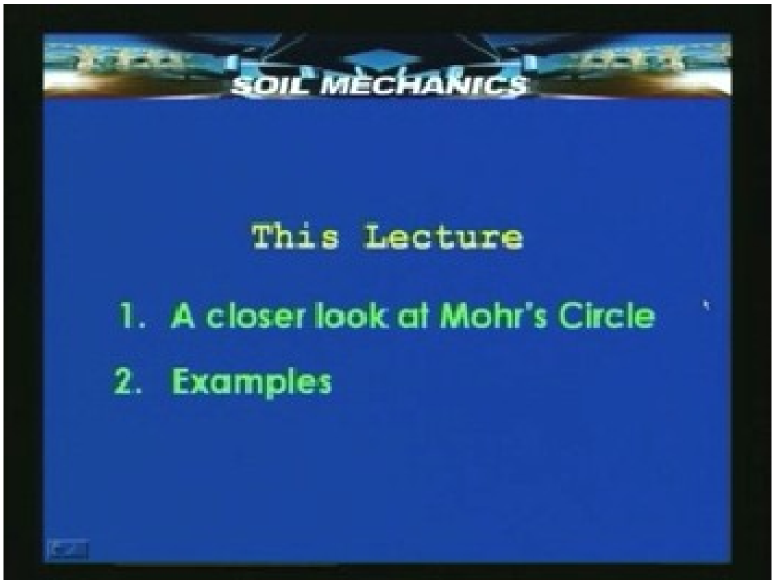

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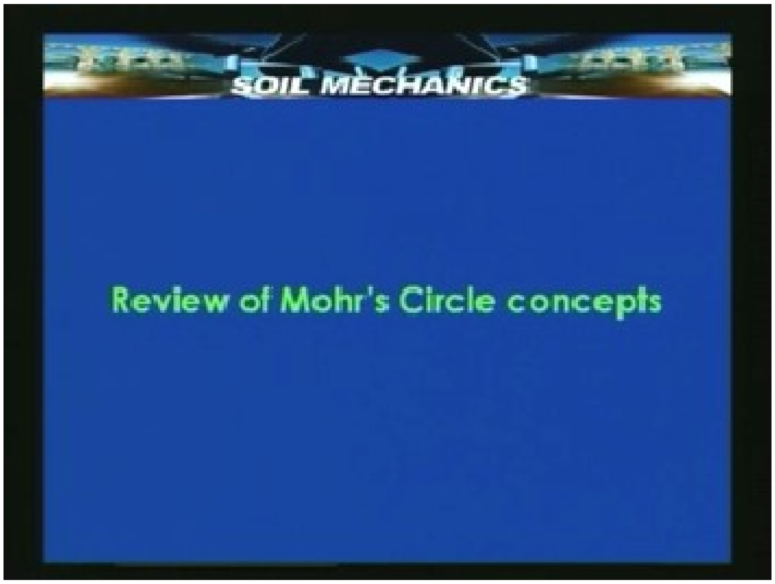

So first a review of the Mohr's circle concepts. What's Mohr's circle meant for? It's meant for determination of normal and shear stresses on any arbitrary plane at an angle theta to x axis and it makes use of the geometrical concept of the circle.

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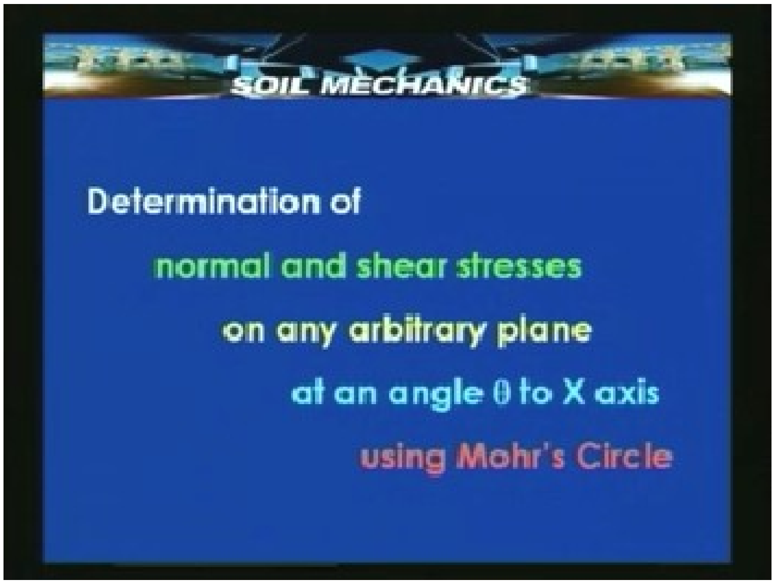

Let us see how a Mohr circle is constructed for a general set of stresses. What is meant by a general scheme or general system of stresses? Take a look at this familiar key diagram which is on the right. This shows as we have already seen 3 planes passing through a point but slightly exaggerated to indicate an element in equilibrium.

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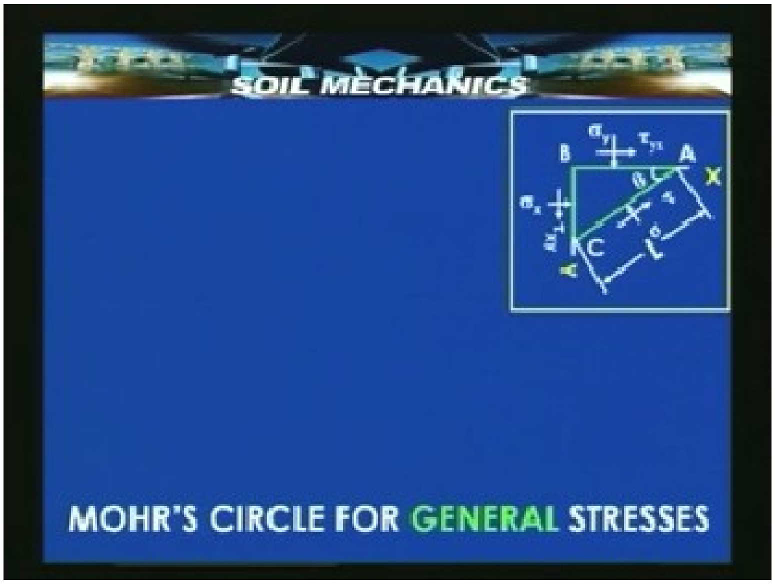

These set of stresses shown here is what I am referring to as the general set of stresses or general scheme of stresses. In the general case we have a normal and shear stress on each one of these planes. Now how do we construct a Mohr's circle and what do we construct it for? We are constructing the Mohr circle to determine sigma and tow on a plane AC in terms of sigma x sigma y tow x y and the angle theta. So how does this go? Step by step let us see. First step is to choose an x axis for normal stresses, all these normal stresses sigma sigma$_y$ sigma$_x$ they will be plotted along the x axis. Then we choose an origin O and a y axis meant for shear stresses. Having chosen this set of axis, now we choose the center of the Mohr's circle for that we start with plotting the known stresses.

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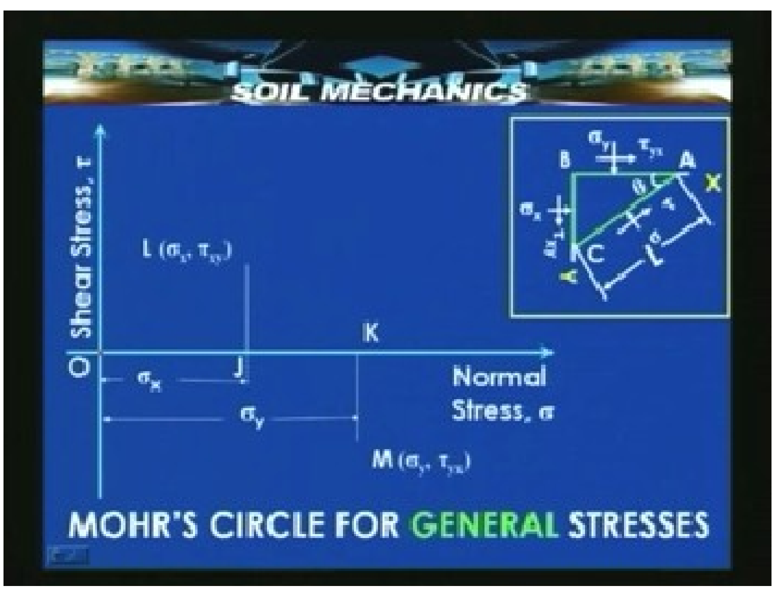

Take sigma$_x$ for example, find out the point J whose x coordinate will be sigma$_x$, sigma$_x$ is a known normal stress and at J plot the shear stress tow xy which is also known. Then you get the point L, similarly plot sigma$_y$ and get the point K and the point M whose coordinates will be sigma$_y$ tow yx.so now we have 2 extremities L and K of a typical diameter of the Mohr's circle LK. Now that intersects the x axis at point E and obviously E is the centre of the circle which will pass through both L and K. So now construct that circle.

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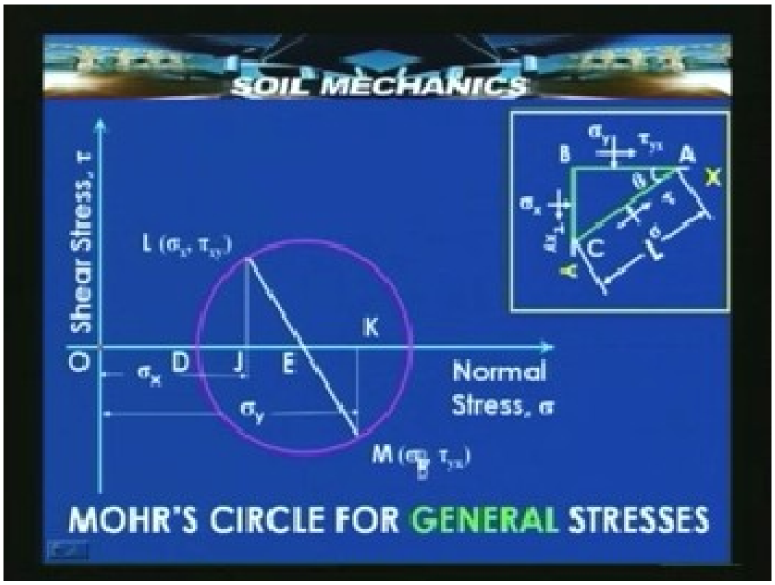

This is a circle, this circle intersects the x axis at point D and E, they have a significant as I briefly mentioned in 2, 3 slides ago. Point D has a least possible normal stress here and that's known as sigma$_3$.

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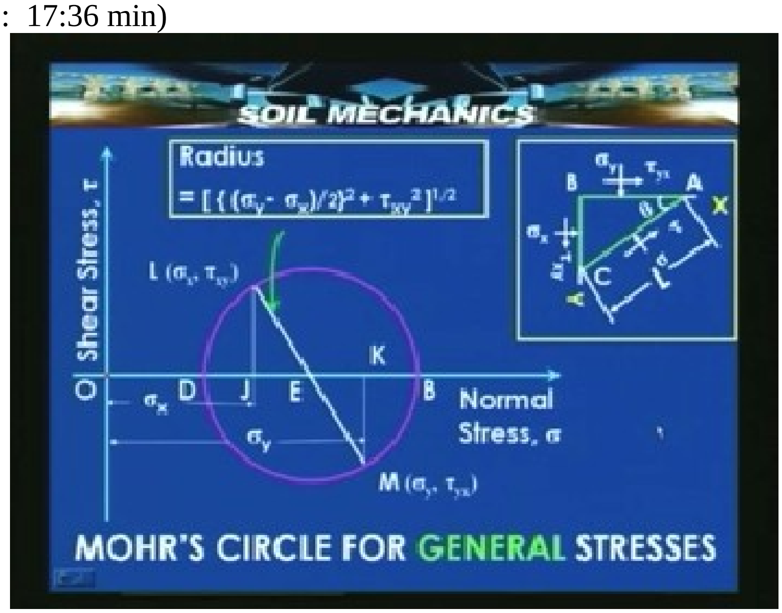

Now see here the radius of the Mohr s circle which we plotted with E as a centre and EL as the radius. The magnitude of the radius is $sigma_y$- $sigma_x$ by 2 whole square plus tow x y whole square raise to the power half. The point D will have a coordinate $sigma_3$ which can be either calculated or measured from the Mohr's circle. Similarly the point B will have the coordinate $sigma_1$ which can be measured or computed and this completes the Mohr's circle for a general stress of stress.

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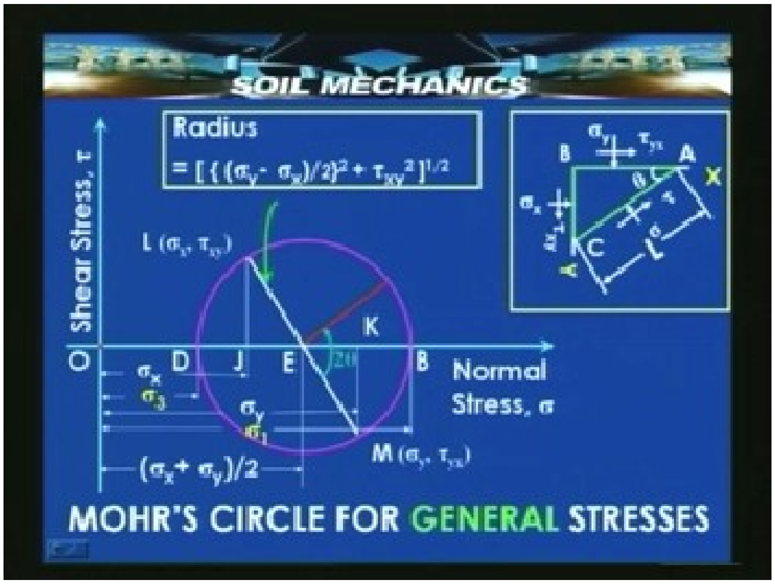

Now if we want the stress on the plane like CA, all that we need to do is to know the angle theta and then from this reference diameter draw a line at angle 2 theta taken in the anti-clockwise direction. And this red line typically represents the plane which we will get corresponding to the point C whose coordinates will be sigma and tow. So point C represents the stress level on the plane AC.

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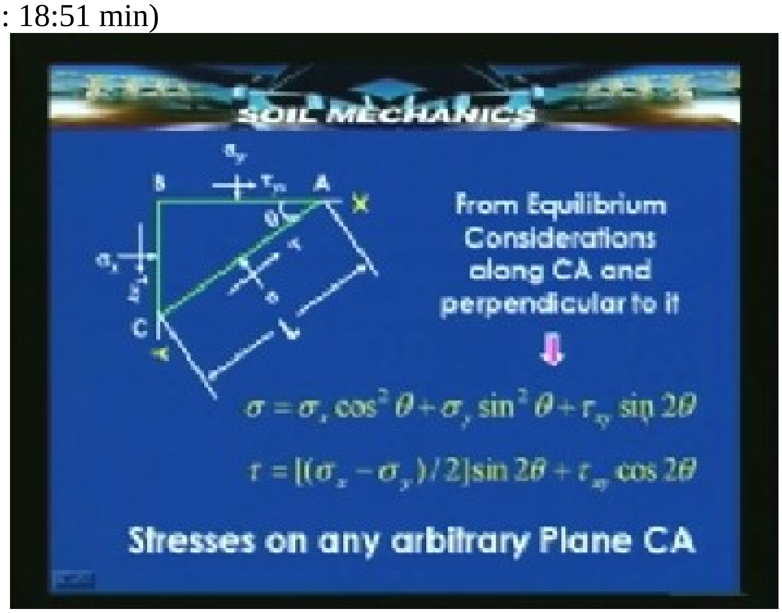

Now it's also possible to compute these stresses by these equations as we have seen earlier. If we compute them from the Mohr's circle or measure them off from the Mohr's circle. As the coordinate of the point C, the horizontal coordinate and vertical coordinate we will get the same values of C sigma and tow as from the numerical computation. So these expressions will be obtained also from the Mohr's circle and these are what are the stresses which are unknown on the plane AC.

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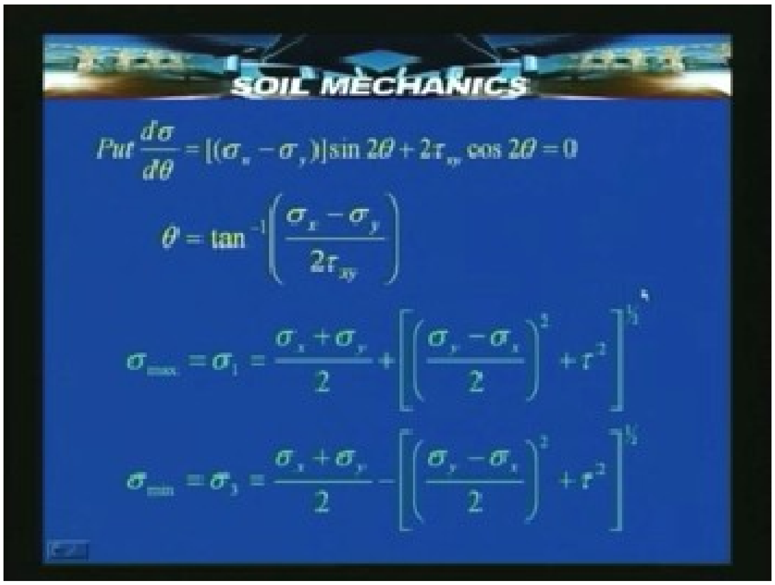

In addition now as we have seen, there are 2 points D and B which have the minimum and maximum stresses. How to compute these stresses? We can of course measure them off the Mohr which is plotted to scale but we can also compute that. For computing this we use the simple notion that sigma$_3$ is the minimum possible stress. Sigma$_1$is the maximum possible stress and we already know what sigma is we have an expression for sigma. This is the expression for sigma and if sigma has to be minimum or maximum it's simply means mathematically that d sigma by d theta must be zero. So take d sigma by d theta, put it equal to zero and this is the condition that you get.

Now under what conditions this expression becomes zero. This expression becomes zero as you can verify easily when theta is equal to tan inverse of sigma$_x$- sigma$_y$ by 2 tow xy. When theta is equal to this, 2 theta will be equal to tan inverse of sigma$_x$- sigma$_y$ by tow xy and cos 2 theta can be correspondingly computed, sin 2 theta can be correspondingly computed and then you substitute in this equation you will find that this equation is satisfied. If that is the case then for this value of theta, corresponding to this value of theta what will be the maximum value of sigma and then minimum value of sigma which will correspond to d sigma by d theta equal to zero. Both of them can be computed and it can be shown that sigma maximum is sigma$_1$ equal to this, sigma minimum is this equal to all this prime quantities which is on the right within the brackets. Now how do we get 2 equations corresponding to theta? What happens is when tan theta is equal to this, it's possible to get this expression satisfied, both when sine 2 theta and cos 2 theta are positive and also when sine 2 theta and cos 2 theta are both negative.

When sine 2 theta and cos 2 theta are both positive this equation is satisfied and the corresponding sigma value we get from this expression will be equal to sigma$_1$. On the other hand when sin 2 theta and cos 2 theta are both negative and this equation is satisfied then this becomes zero and the corresponding sigma becomes the minimum value sigma$_3$ which is given by this expression. So it’s possible to calculate the minimum and maximum by a simple differentiation and substitution of condition under which the stress becomes minimum or maximum.

Now let us see once again the construction of a Mohr’s circle but this time for a specific case where the principle stresses are known or rather than the principle planes are known. Take the key diagram once again. This key diagram is similar to the earlier one but the only difference is here the horizontal plane AB only has a normal stress. The shear stress is zero on this because AB is a principle plane.

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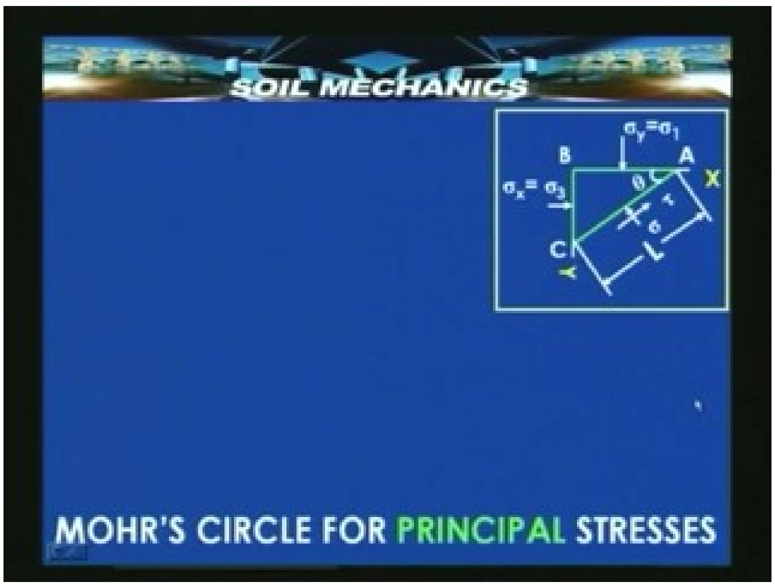

Let us take sigma$_y$ to be greater than sigma$_x$ and then sigma$_y$ becomes the major principle stress. So this is equal to sigma$_1$. On the other hand BC automatically becomes sigma$_x$ or the minor principle stress acting horizontally on the plane BC and correspondingly if you take a plane CA at an angle theta with respect to horizontal or the major principle stress. Then the normal and shear stresses on the plane AC sigma and tow can now be computed. In terms of sigma$_1$ and sigma$_3$ either from Mohr’s circle or from rigorous expressions.

Let us see the construction of a Mohr’s circle when the principle stresses are known. As before take the x axis that is normal stress axis, take the origin O then plot the y axis for shear stresses at right angle to the normal stress axis. Choose the point D, earlier D was not known. But now we know D because the value of sigma$_3$ the minor principle stress is known. Then choose the major principle stress as the x axis and get the point B.

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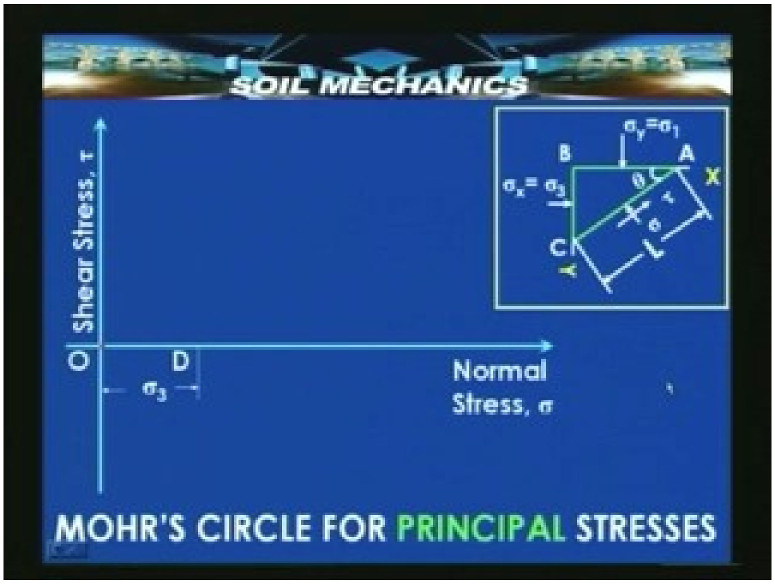

Obviously the center has to lie between D and B at its midpoint. So choose the origin E at the midpoint of the length DB. It will be equal to sigma$_1$ plus sigma$_3$ by 2. Plot the Mohr's circle with the radius as sigma$_1$- sigma$_3$ by 2 then take an angle 2 theta with respect to the horizontal that is the line EB.

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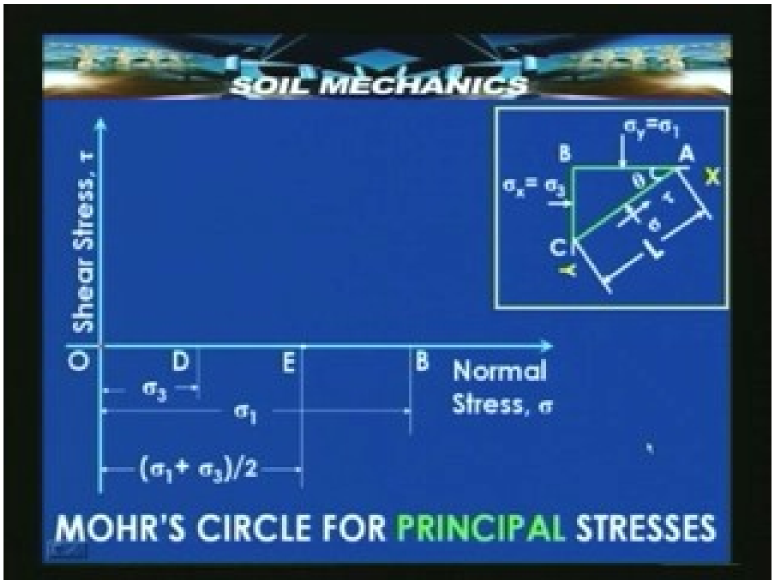

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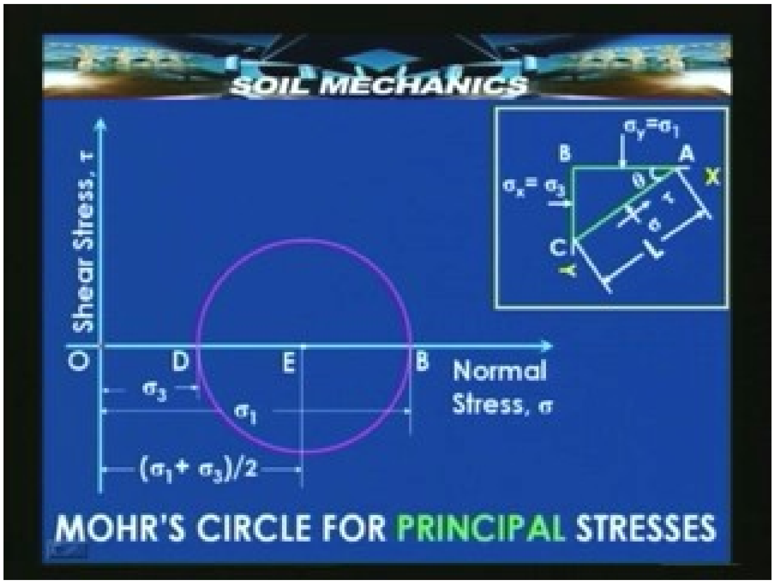

Get the red line, the coordinates of the points where the points intersect the Mohr's circle will be nothing but sigma and tow. So points C will represents the stress state on any arbitrary plane AC at angle theta and it can be shown that sigma will be equal to this, tow will be equal to $sigma_1$-$sigma_3$ sin 2 theta.

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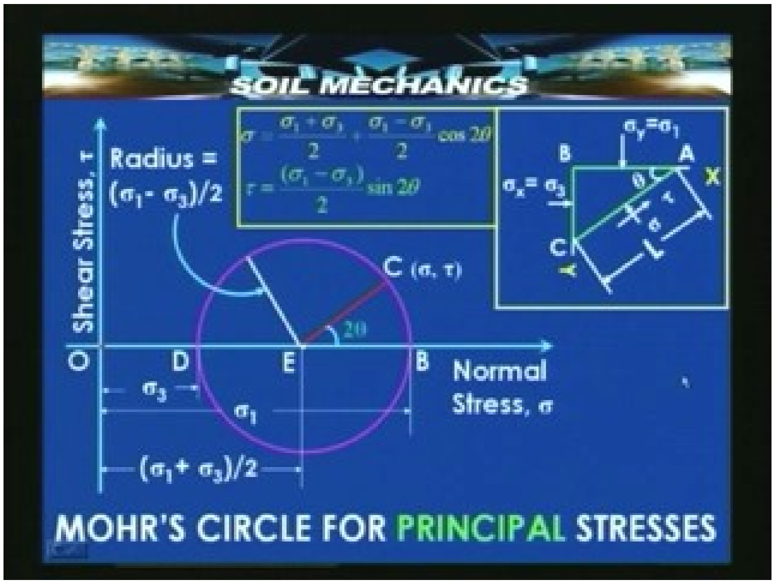

So once we know these 2 expressions we can compute sigma and tow from knowing the angle theta. So looking at this slide we can see that there is a minor different between general set of stresses and principle set of stresses.

The difference is that when there are principle stresses acting on the known planes there are no shear stresses and to that extent the plotting of the Mohr circle becomes a little easier and computation of this stresses on their plane AC also becomes a little easier. Now do we really need a geometrical method for determining this stresses? Since we know all the parameters on the right hand side we can always compute the stresses sigma and tow. Then where do this geometrical methods score? The advantage of geometrical methods as you will see a little later again is that it gives you a very nice concept or ability to visualize how the stresses vary over the plane. A geometrical concept is always easy to visualize and comprehend than an equation. Thus the reason why we have gone to such great length has to understand how a Mohr's circle is plotted. It helps to visualize how the planes are oriented. How the stresses are varying from plane to plane and which is the plane which is critical and why. So let us see further.

A method is known as the pole method which is also used for plotting the Mohr's circle. This is called a pole method because a specific point on the Mohr's circle which is known as the pole. For example to locate a pole you draw a line from a known point on the Mohr's circle parallel to the plane on which the state of stress represented by that point acts. What I mean is you have a Mohr's circle, you know the state of stress on a particular plane. Choose the point on the Mohr's circle which represents this state of stress and then draw a line parallel to the plane whose state of stress that point represents and then the point of intersection of this parallel line with the Mohr's circle is called the Pole is also sometimes referred to as the Origin of planes.

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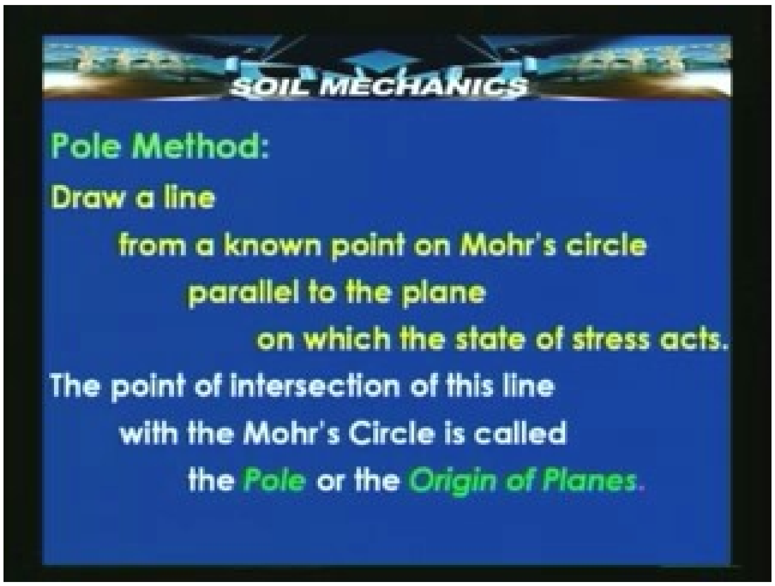

The reason why we see this is very simple. If you see the previous slide, the point C was located by taking an angle 2 theta, measuring of an angle 2 theta with respect to the normal stress axis is in the anti clockwise direction because the plane CA made an angle theta with respect to the horizontal axis rather than going by this concept. We can also proceed by simply knowing the orientation of the planes that's what the pole method is all about.

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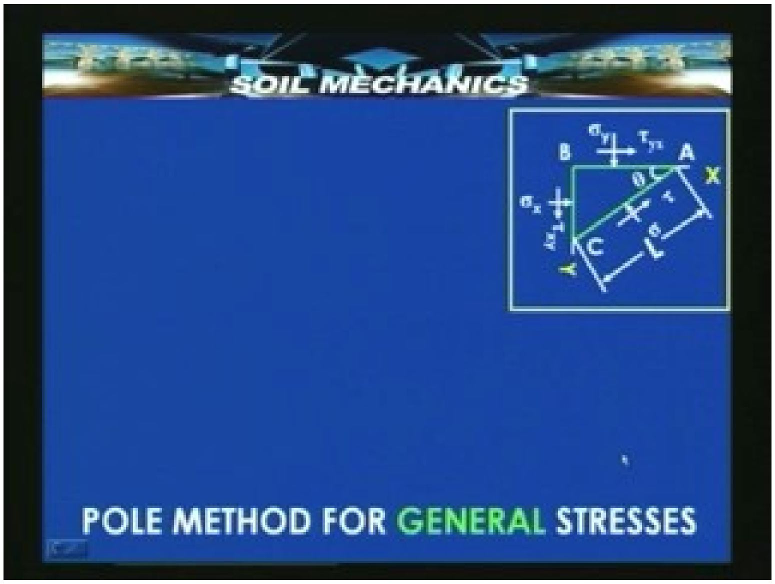

Start once again with this index or key diagram for general stresses. So now if we have general stresses how do we use the pole method for determining this sigma and tow? We follow a series of stress always as we have been doing in the earlier instances. First draw the normal stress axis, choose the origin and plot the vertical axis. Plot sigma$_x$, find out the points J and L, plot sigma$_y$ find the points K and N, join LM get E, draw a Mohr's circle get the point D and B the radius will be given by this expression. Midpoint E will have coordinates sigma$_x$ plus sigma$_y$ by 2, D will be equal to sigma$_3$, B will be representing sigma$_1$ and now comes the determination of the point C on the Mohr's circle which will corresponds to the state of stress sigma tow.

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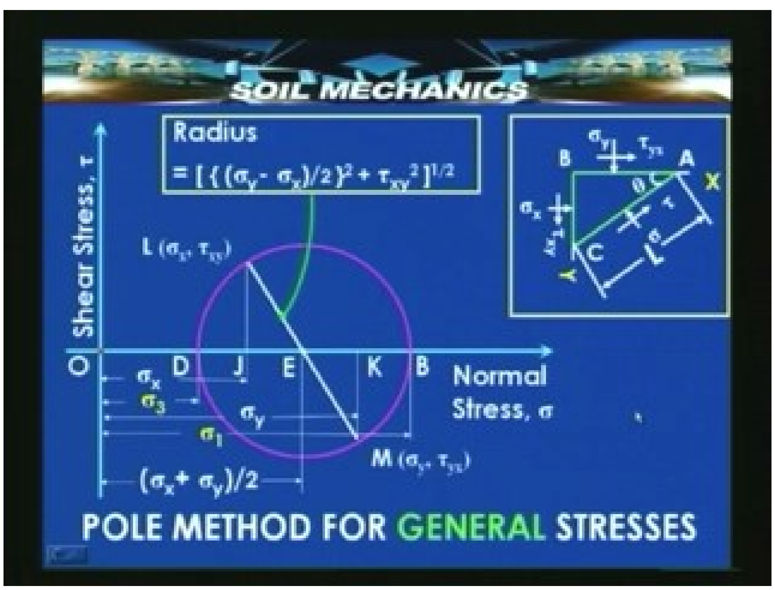

For this rather than plotting a line at an angle 2 theta as we did in the previous instance, we will now choose the point M and draw a line MP. Why do we choose point M? Because point M on the Mohr's circle is a point whose stress coordinate are sigma_y tow y x. That is nothing but the set of stresses, normal and shear acting on the plane AB which is horizontal. So point M represents the state of stress acting on the plane AB which is horizontal, so from point M if you draw a line MP parallel to the plane on which this state of stress acts, then the point P where it intersects the Mohr's circle is known as the pole. The property of the pole is any line that you draw through the point P the pole will have a corresponding point of intersection on the Mohr's circle and that point of intersection will have coordinates representing stresses which will be acting on the plane passing through the point P.

So for example in order to know the stress on the plane AC, from the pole draw line which represents the plane CA that is draw PC parallel to CA. Obviously then C which is the point of intersection of this red line with the Mohr's circle will have the state of stress given by the plane parallel to PC that is plane CA. Then it can also be easily shown that plotting an angle 2 theta from here and joining E with C will also give you a same point C as the red line drawn from the point P. So this is an alternative method so called pole method for determining the point C. It can also be applied to a situation where we don't know the general state of stress but we are fortunate enough to know the normal and tangential stresses on the planes AB BC and CA. The value of sigma and the value of tow are to be determined in terms of the sigma_1 and sigma_3. The plotting of the Mohr's circle does not require any explanation. You can just watch the progression of steps; we have so far plotted the minor principle stress and major principle stress.

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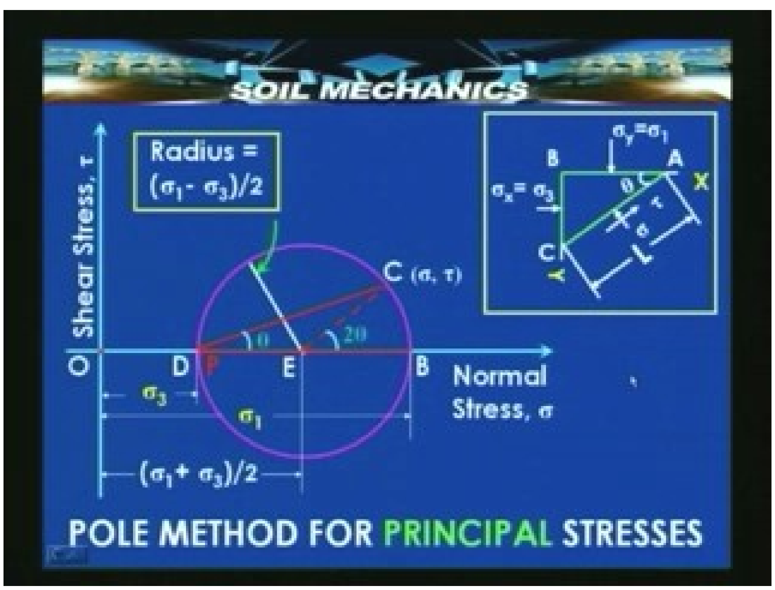

We have chosen the centre of the potential Mohr's circle; we have got the Mohr's circle and its radius. We have got the point C and how we have obtained is by choosing the point D which represents the state of stress corresponding to sigma_3. That means the plane DC will have to be a line which is horizontal passing through the point D.

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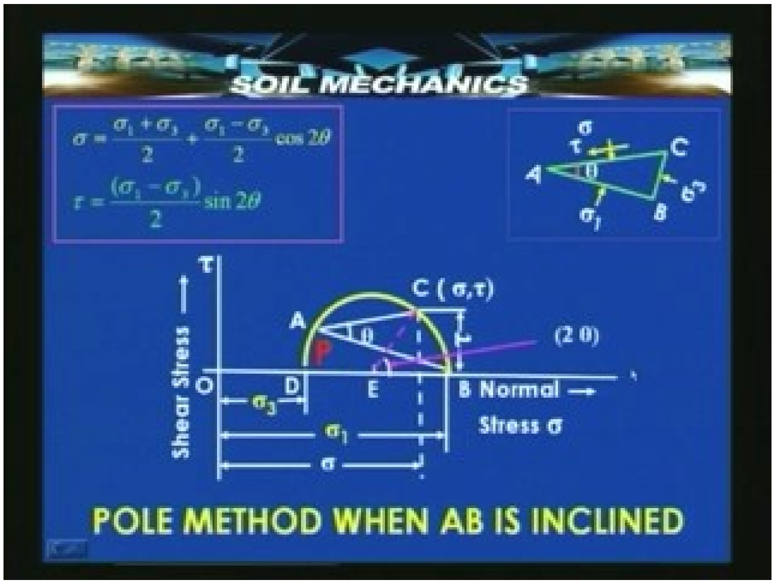

So from point B if you plot a line which is parallel to the plane AB on which this stress sigma$_1$ acts then the point P becomes the pole and PC at angle theta will give you sigma tow and PC is plotted parallel to CA or alternately plot 2 theta with respect to the line DB and get the point C. There is also a possibility that the line AB or the plane AB is not horizontal. We have been assuming that the 2 known planes are horizontal and vertical but that's only a special case. In the most general case the line AB representing the major principle plane could be an inclined plane. A line BC representing the minor principle plane could be a plane which is at right angled to AB and also therefore inclined to the vertical. In such a case if you have a plane AC inclined at angle theta to AB, we can follow the same Mohr's circle concepts and get the stresses on the plane AC, the difference will be that from the point B now rather than drawing a line horizontally to represent the plane on which stress is sigma$_1$.

Now we will draw a line BA which is parallel to the known direction of BA on which sigma$_1$ is acting and where it ends that's the point P. And from P if you draw a line PC at angle theta that is parallel to AC then you get sigma tow. Sigma will be now equal to O to this point and tow will be from here to C. And this also is possible to obtain by plotting a line to theta through this point E with respect to the horizontal. The corresponding expressions for sigma and tow which we have already seen will be satisfied by this point C.

Now having heard an exposure to the method of determining the stresses on any plane let's see what is this concept of failure which will help us to access whether under given circumstances of shear stress and shear strength, failure can take place along a plane in a soil. So what's this concept of failure? Let us go back to physics and try to understand failure along a plane of contact between two bodies which are experiencing friction at the plane of contact. Let's say there is a rectangular block resting on a plane offering friction. This rectangular block let us say has a weight W, corresponding to W there will be a normal reaction N.

Now suppose you apply a shear force T, a tangential force T shown by this green line because there is friction between the bodies, a force F in the opposite direction will be automatically mobilized and that will resist T and for equilibrium T will be equal to F, N will be equal to W or in other words the resultant R will be equal to the resultant P.

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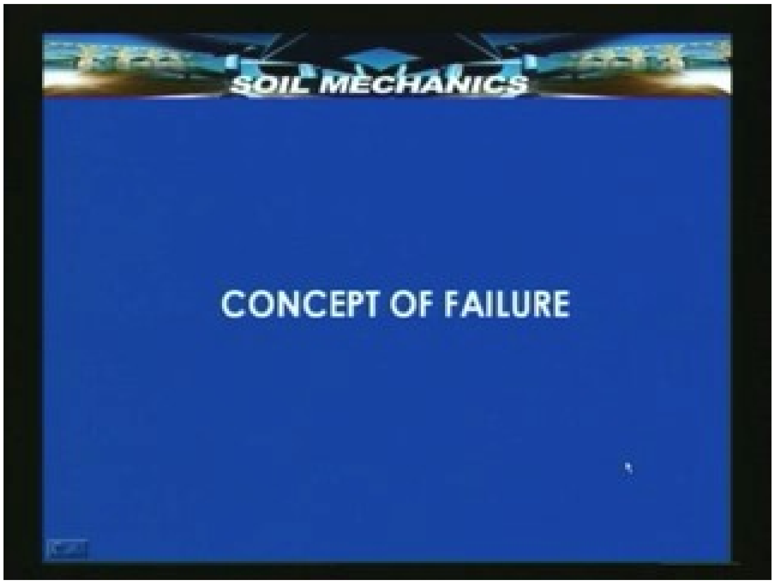

The angle which R makes with a vertical will be the angle which P makes with a vertical that is alpha. This is the situation when the box in equilibrium but let us increase the value of T until the block reaches the state of so called incipient seepage. That is T has now achieved its limiting value corresponding to this, this angle alpha has reached its maximum possible value.

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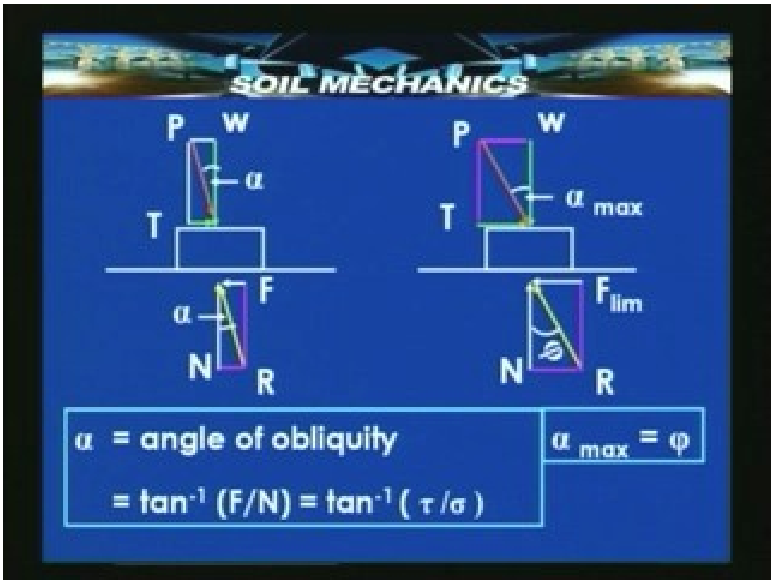

Under these circumstances the normal force N will balance the force W and F which is now going to have its limiting magnitude will once again be opposing capital T and be equal to capital T. And the angle which the resultant makes with a vertical now will be the maximum possible value of this angle alpha and that's referred to as phi. Then this alpha is known as the angle of obliquity and this serves as the condition representing failure. What is this angle alpha equal to? It is equal to tan inverse of the angle phi because alpha and phi are both equal. What's the tan inverse of angle phi is nothing but F upon N and therefore we have alpha equal to tan inverse of tow upon sigma because the area over which F and N act is the area of the plane of contact here and that remains the same.

If that plane has a magnitude A for area then shear stress on this contact plane is F limiting upon the area, normal stress on the contact plane is N by area of contact and therefore alpha is tan inverse of tow upon sigma or phi is tan inverse of tow upon sigma. Whereas this expression is valid for all values of alpha when it is equal to phi it becomes valid for the limiting state. And this is what is known as the state corresponding to limit equilibrium and its only possible to know the shear stress tow at this particular state because we know that even a slight increase in tow will cause shear failure at the plane of contact. Whereas at any other value of tow less than the limiting value we only know that equilibrium exists we do not know really what fraction of tow has been really mobilized.

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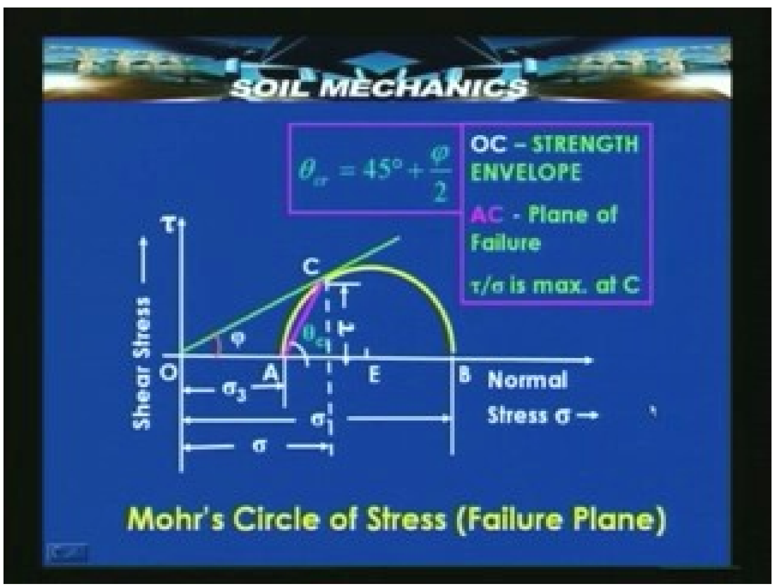

Having decided what criterion is satisfied during failure that is having known that for failure alpha has to be maximum or tan inverse of tow by sigma has to be maximum. We can now define failure in terms of the Mohr's circle concept also. For example it is very obvious that failure takes place when tow upon sigma is maximum because tow upon sigma represents tan of the maximum angle of obliquity that is phi. So we incidentally also have a criterion which is obvious from here that is tan phi, tan of the angle of obliquity with its maximum value is equal to tow upon sigma.

So tan of phi equal to tow upon sigma or rather tow equal to sigma tan phi becomes the criterion defining failure. Go to Mohr's circle, find out at what point tow becomes equal to sigma tan phi. Plot the normal stress, choose the origin, plot the vertical or shear stress, choose the point E which is the centre of the Mohr circle. Plot only half the Mohr's circle for convenience and clarity with EA as the radius, we know $sigma_1$ and $sigma_3$. So we get the Mohr's circle, sigma is the normal stress at any point on the Mohr's circle and tow is the shear stress of any point on the Mohr circle. And if we want to know what's that point on the Mohr's circle which will corresponds to this state of stress during failure. What we need to find out is at which point tow upon sigma becomes maximum because that will represent the condition sigma equal to or rather tow equal to sigma tan phi which defines failure.

So suppose I choose this pole, we have already seen that this has got to be the pole for a situation where major and minor principle stresses are known and plotted. From the pole draw a line to a point C which will represent the point where tow upon sigma will be maximum. Now we know that tow upon sigma maximum corresponds to tan inverse of the angle phi. So there exists an angle phi whose angle is tow upon sigma and tow upon sigma has its maximum possible value. Such an angle can only be this angle phi which can be obtained by drawing a tangent through the point C from point O. The tangent of this angle phi is tow upon sigma therefore this angle phi is nothing but the angle of maximum obliquity which defines failure. So the plane AC which has been plotted from the pole upto the point C is nothing but the plane on which acts the stresses tow and sigma corresponding to failure. So AC becomes the failure plane.

This failure plane AC makes an angle with the horizontal which we now call as theta critical because that corresponds to the critical possible angle. What's the value of theta critical at which the plane of failure will lie? Theta critical from simple geometry is obviously equal to half the angle OCA plus phi. So 45 degrees plus phi by 2 this means that if we know $sigma_1$, $sigma_3$ and plot a Mohr's circle with E as a center, all that we need to do is go to point A draw a line AC at an angle theta critical equal to 45 degrees plus phi by 2 and that point C will be representing the state of stress at the time of failure, incipient failure. Now this means that we need to know the angle phi, this means that we need to have a method to determine the angle phi. There are laboratory technique to determine the angle phi as we will be seeing it later.

Now how do we say that theta is equal to 45 degree plus phi by 2 from theoretical considerations? It's very simple, for a given value of $sigma_3$ and C the failure will occur when $sigma_1$ is minimum. Therefore take the expression for $sigma_1$ which we are already familiar with and maximize that, when that is maximized you will have d by d theta is equal to this, equal to zero. This when you simplify, you will get cos 2 theta plus sin 2 theta tan phi is equal to zero. That is nothing but theta is equal 45 plus phi by 2.

From this we can derive further relationships using the Mohr's circle, suppose this is the Mohr s circle where this angle is defined as the theta critical equal to 45 degrees plus phi by 2. We can also use the sine and cosine of this angle phi and show that sin of this angle phi is nothing but $sigma_1$- $sigma_3$ divided by $sigma_1$+sigma3. That's equal to this or from here we can show that $sigma_1$ by $sigma_3$ is equal to 1+ sin phi by 1- sin phi.

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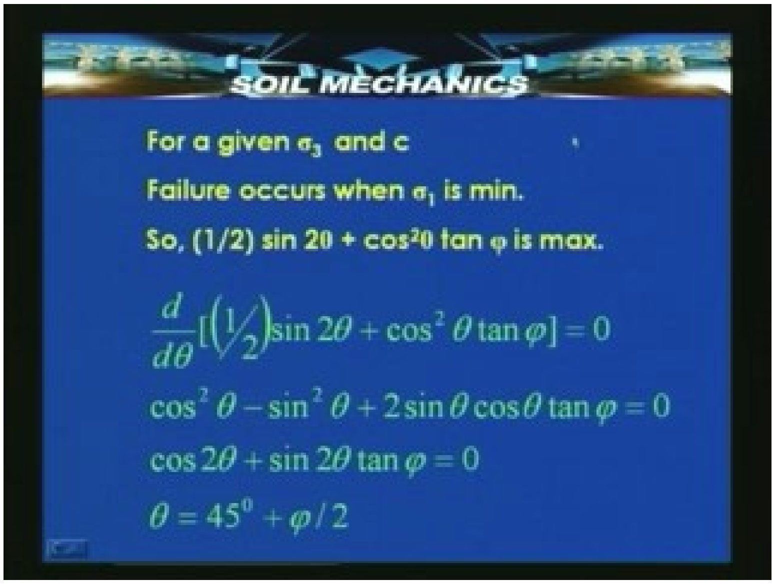

All these different expressions are useful in different context to relate the plane of failure and the states of stress sigma$_1$ and sigma$_3$. The failure criterion will also now look like this. We know that the failure criterion is shear stress is equal to shear strength equal to c plus sigma tan phi. The expression for c plus sigma tan phi and tow will be equal to this from the Mohr's circle which we have just now seen and this can be simplified into an expression sigma$_1$ equal to sigma$_3$ plus all this in which theta will corresponds to 45 degree plus phi by 2 the angle made by the plane of failure, the critical angle.

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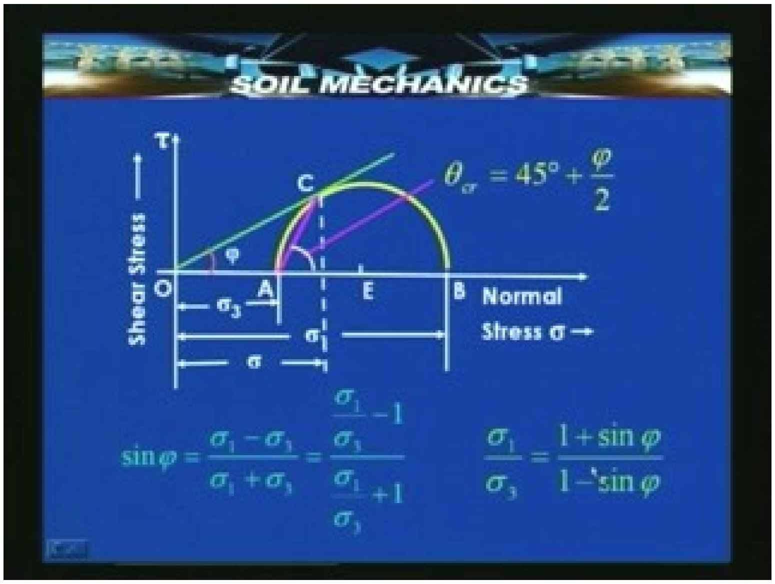

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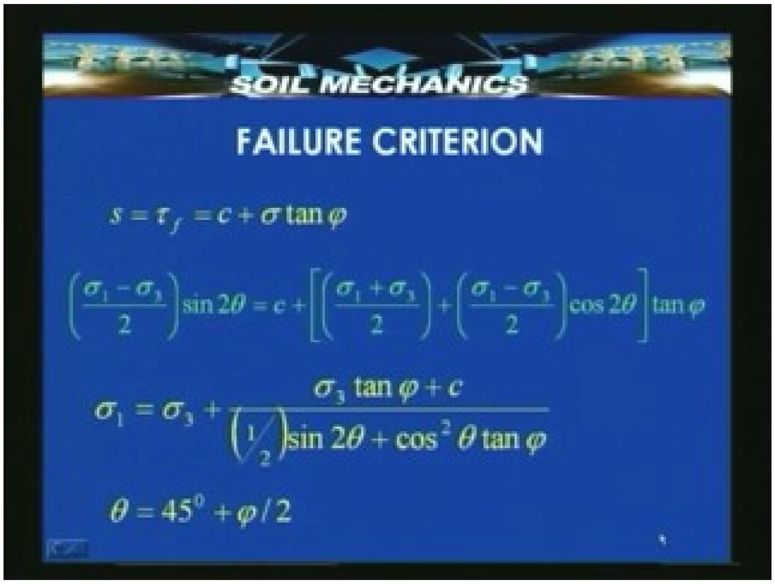

This means that failure will take place along a plane which makes this critical angle 45 degree plus phi by 2 provided the known states of stress sigma$_1$ and sigma$_3$e and the values of c and phi at the soil satisfies this relationship. The values of c and phi of the soil which are known as the shear parameters can be determined from the laboratory. Then knowing that the plane of failure makes an angle theta equal to 45 degrees plus phi by 2 and knowing sigma$_1$ and sigma$_3$ it is possible to check whether failure will takes place along a given plane or not.

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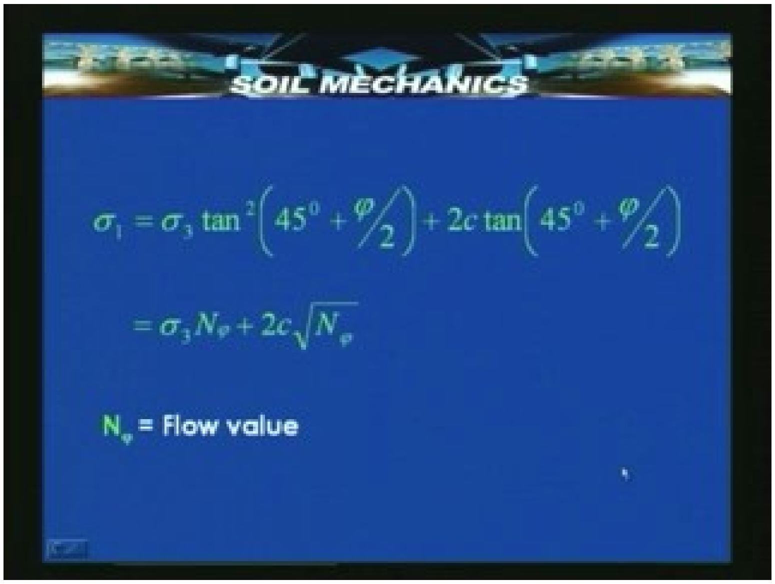

That same expression which we saw in the previous slide can also be rewritten in this form where we use now a notation N_{phi} known as the flow value to represent tan square 45 plus phi by 2. Let us see a couple of examples now. In the first example we have normal and shear stresses on two mutually perpendicular planes BC and BA on a soil element are given. This are plotted in a key diagram which appears on the next slide. The stresses $sigma_x$, $sigma_y$, tow xy are therefore known. What is required is determine the major principle stress, the minor principle stress, the normal and shear stress on the plane CA which makes an angle of 37 degrees with the horizontal and for this purpose use the pole method and the Mohr's diagram.

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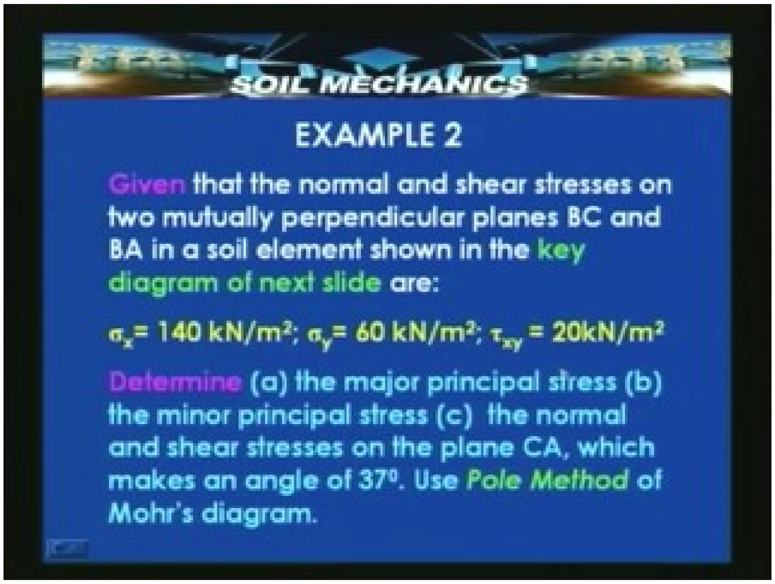

We have seen already in detail the step by step construction of the Mohr's circle, therefore we will not go through that again. All that we need to know is we have a system of general stresses here now where $sigma_x$, $sigma_y$ and tow yx are known. So we plot $sigma_x$, $sigma_y$, get the points J and K, points L and M, plot the Mohr's circle and obtained the point M corresponding to the state of stress on the plane AB. Once we know that since plane AB is horizontal, draw the line MP horizontally and determine the pole P. Once you know the pole P you can determine the point C whose coordinates are C tow sigma tow.

Once you know this point C corresponding to this state of stress, we can easily find out the values of normal stress and shear stress on these planes. Whatever is known is plotted first so normal stress on BA is plotted as 140, normal stress on BC is plotted is 60, shear stress on BA and BC are plotted as -20 and +20 to get the Mohr's circle which was shown on the previous slide.

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From the coordinates of the points D and E which correspond to major and minor principle stresses, we can measure off these values. We can also use the pole to get the point C and measure of the value sigma and tow from the Mohr diagram. If we calculate it rigorously from the formula also we will get the same stresses. The next example is one in which sigma_1 and sigma_3 are now known. So this is instead of a state of general stresses this is a state of specific set known stresses.

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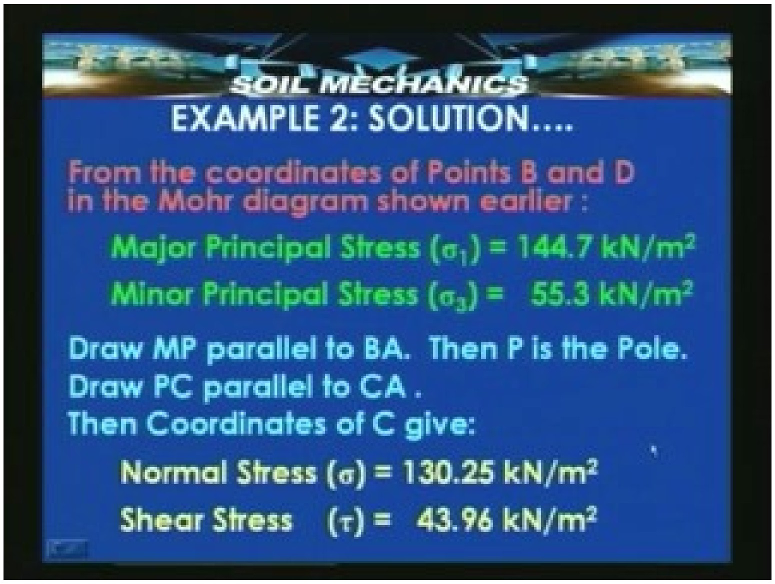

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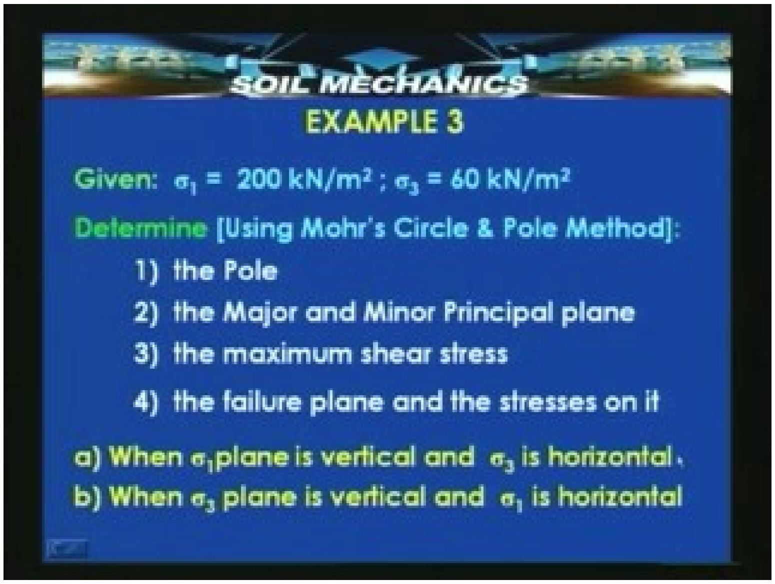

Again using the Mohr's circle and pole method determine first the pole then the major and minor principle planes, the maximum shear stress and the failure plane and the stresses on the failure plane. Under two conditions, one when sigma$_1$ plane that is the major principle plane vertical and the minor principle is horizontal and case b when the major principle is horizontal.

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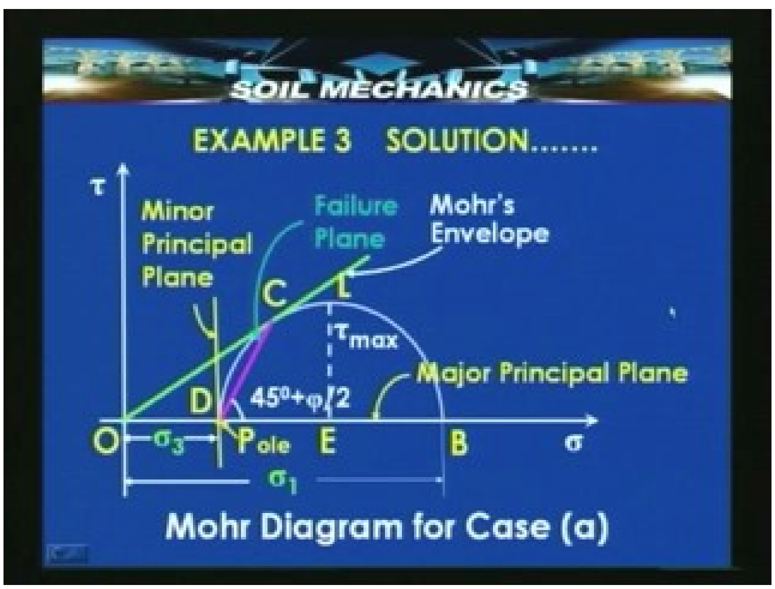

The solution is very simple as before plot the Mohr's circle and also after determining the pole P corresponding to the principle stress sigma$_1$, get the failure plane corresponding to the point C and get the stresses.

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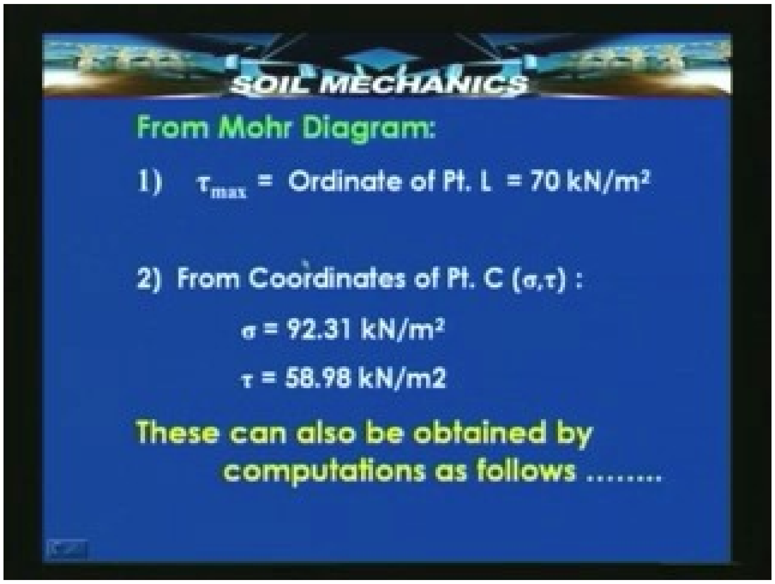

The tow max can be measured off as an ordinate of the point L that will come to 70. From the coordinates of the point C obtained from the pole method, sigma and tow can be calculated as 92 and 58 respectively or from rigorous computation we can get tow max must be equal to $sigma_1$-$sigma_3$ by 2, the radius of the Mohr's circle that's again coming to 70. The angle phi will be sin inverse of the quantities $sigma_1$ and $sigma_3$ which are known. Theta is again known, 45 degrees plus phi by 2. We can calculate sigma and tow from this quantities once again by using the same expressions.

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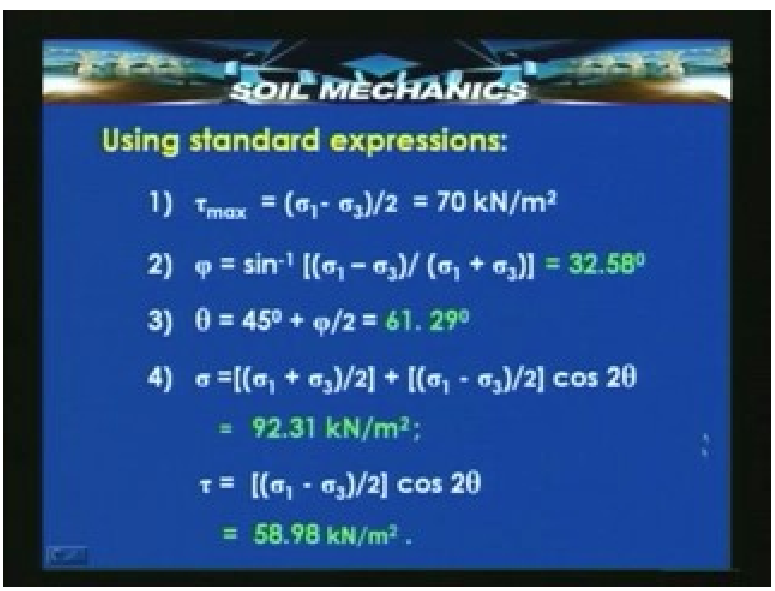

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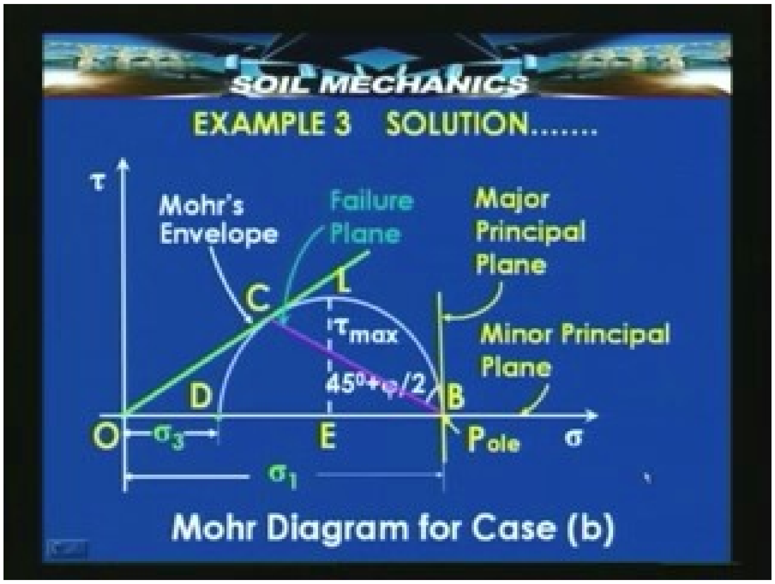

This is possible to compute the complete set of stresses either graphically or from rigorous computations and we will find that all that has happened is the principle planes and the pole have changed positions and points L and C however remain unchanged. And the maximum shear stress and the coordinates of the point C give rise to the values of tow max and sigma tow. Now in this lecture we have taken a close look at how to plot a Mohr's circle and what its cable of giving and we have seen a couple of examples to illustrate this concept and reinforce these.

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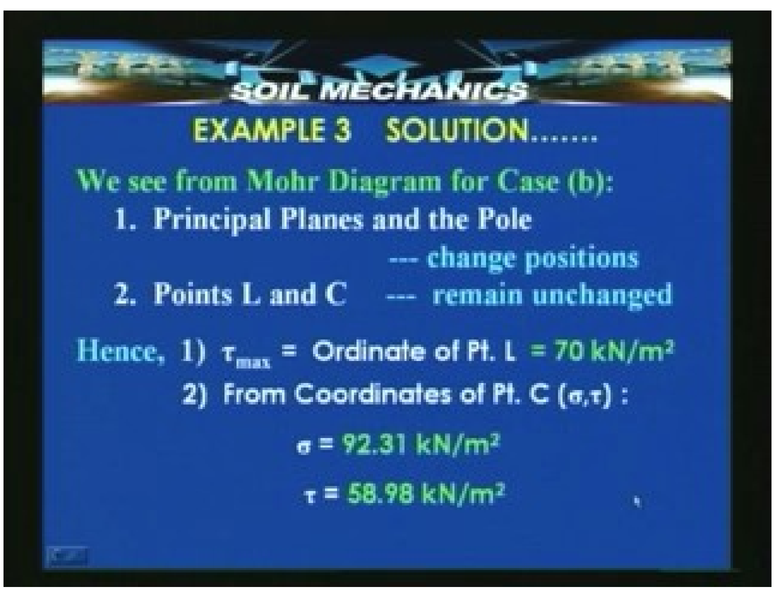

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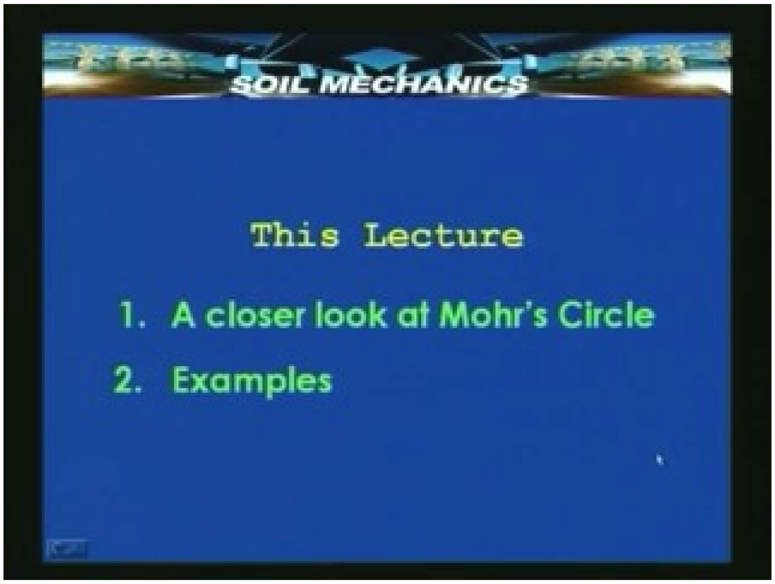

In the next lecture we shall be seeing the methods of determining the shear strength of a soil because that's what is going to help us to check whether this stresses which we have so far calculated are going to cause a failure or not. And we will also see a few examples during the next lecture and this way we will conclude today's lecture by saying that we now have a basis with a help of which we can check whether a given soil along a given plane is likely to undergo shear failure or not.

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And this will be useful in many instances such as foundations retaining walls etc.
Thank you