Soil Mechanics
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Lecture – 43
Shear Strength of Soils
Lecture No.1

You all have heard of compressive strength, tensile strength of material such as steel, wood, metals in general. Similarly we have also compressive and tensile strength for soils as well but what is important in the case of soil from point of view of failure of soil under load which is what is of interest in engineering. It is the shear strength that is of significance. Even when a soil is loaded under compression then along some clay internally inside the soil may experience a shear stress which exceeds what may be considered as the capacity of the soil. In such a case a slip or slide takes place along that plane and that in turn leads to failure of the soil and therefore there is a great need to understand what is shear strength of soils. Now as we did in the earlier lectures, in this lecture as well we shall ask ourselves a few pertinent simple questions and try to answer them for example. To begin with let us take a look at the first slide.

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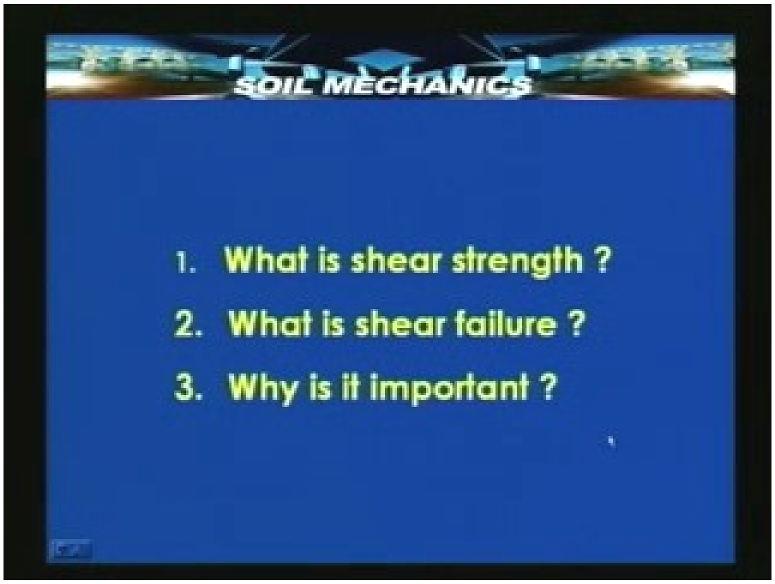

The first slide shows 3 questions. What is shear strength? What is shear failure? Why is it important? Let’s take the first question, what is shear strength? As I said that there will be an internal plane inside the soil mass along which shear stress may exceeds the capacity to resist. So shear strength is defined as the capacity to resist shear stress. The maximum capacity that a soil can mobilize in order to withstand without failure, any applied stress along a plane is what is known as shear strength.

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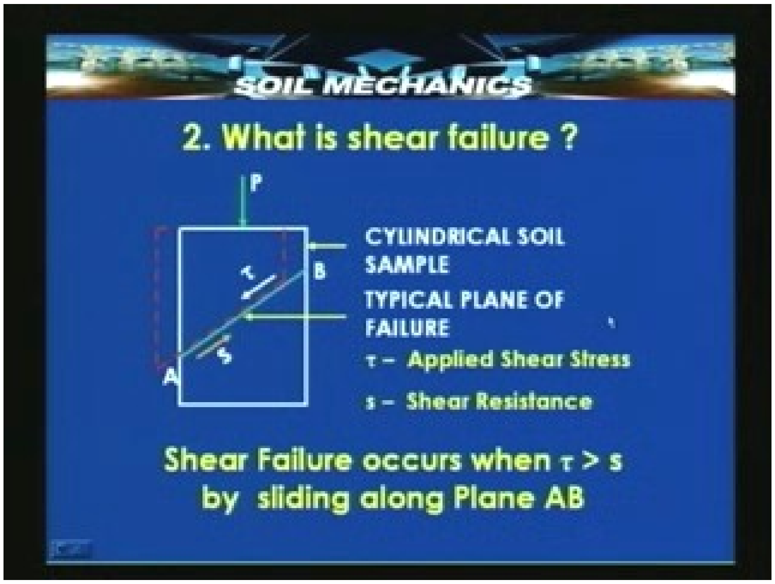

Now what is failure? In the case of shear, failure is by sliding or slipping along some plane. So as I said shear strength is therefore the resistance of a soil to shear failure and shear failure itself results from an applied load and it manifest itself in the form of sliding along some internal plane in the soil. Consider for example a cylindrical soil sample that is shown here. Suppose a load P is applied to this cylindrical soil sample. Then this cylindrical sample is likely to fail along a plane A B.

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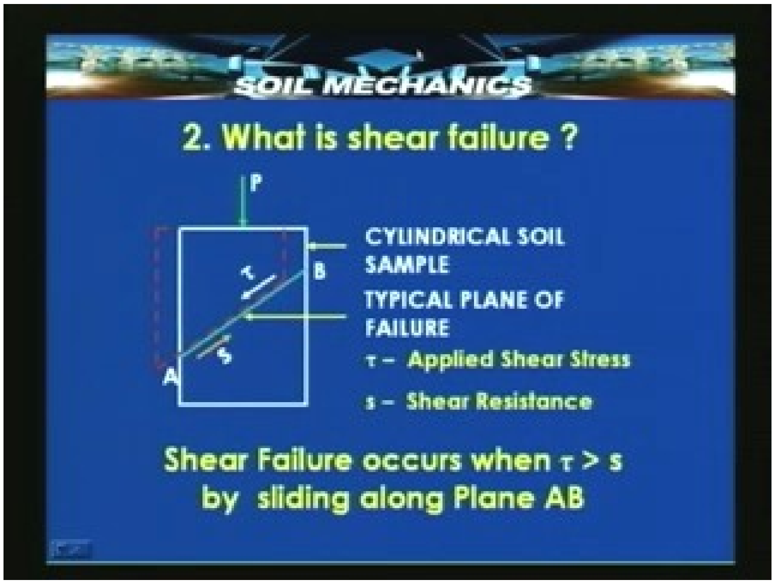

The reason being due to the applied force P, a stress is going to be mobilized or generated along the plane BA and it will act downwards and as the result of this when there is a slip there will be a resistance that will takes place and that resistance S, if that is equal to the tow then balance is maintained and no slide takes place but if the shear stress exceeds the shear strength S then failure takes place. So shear failure can be defined is that which occur when tow is greater then s and its manifest itself in the form of sliding along any plane such as that shown here plane AB. Now shear failure occurs in different forms in different problems or different types of problems which we come across in soil mechanics.

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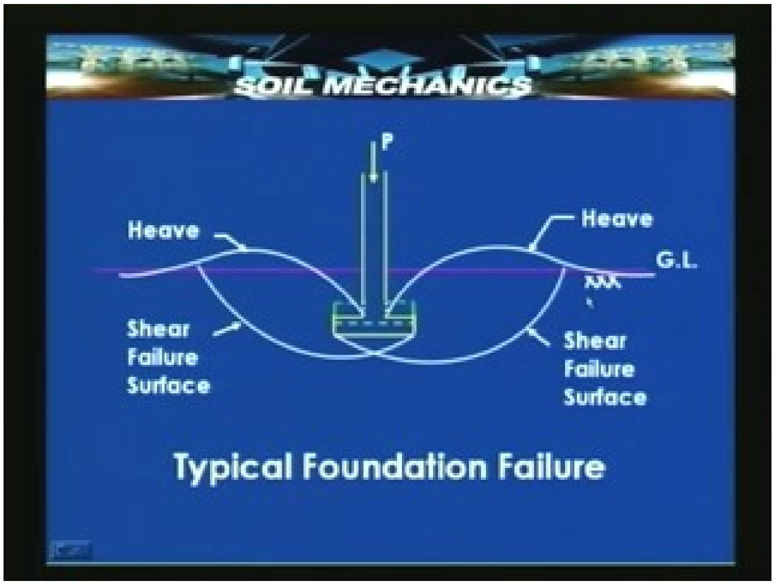

Take the example of a foundation; a typical shallow foundation will fail due to shear failure of the soil. So at this point itself let me make one clarification. I am going to talk about failure of foundation and when I say failure of foundation what I actually mean is the distress that is caused to the foundation due to shear failure of the soil. It is the soil which actually fails but the distress is felt by the structure. And since the structure no longer is serviceable, we say the structure has failed and we say the failure of the structure but actually what has caused it is the shear failure that occurs inside the soil. Take for example this typical foundation shown by these yellow lines. This is the position of the foundation when a load P is applied, original position is shown in form of these dotted green lines.

So when a load P is been applied the foundation is pushed into the soil as a result of this soil gets pushed out from below the foundation on both sides and there will be a typical plane or surfaces of failure along which the shear stress exceeds the shear resistance. And this pushing out of the soil in turn will result in a heave on the surface of the soil.

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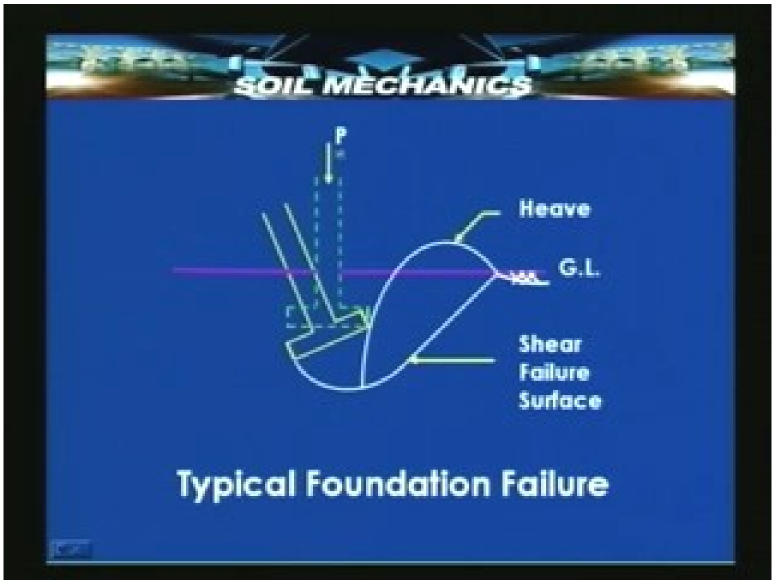

That means any typical shear failure will involve sliding along an internal surface as shown here. As the result of that a slight heaving of the soil surface. And this is the mode in which a typical shallow foundation fails. There is another type of foundation failure for example suppose you have this foundation again represented by green dotted lines, initial position then when a load P is applied if this load is not centric, if there is a rotation of the foundation that rotation will also lead to failure. And once again there will be a shear surface of failure developing inside the soil and correspondingly there will be a heave. So this is the way in which foundations fail. There are also other structures like retaining walls, embankments. So let us see how these fail.

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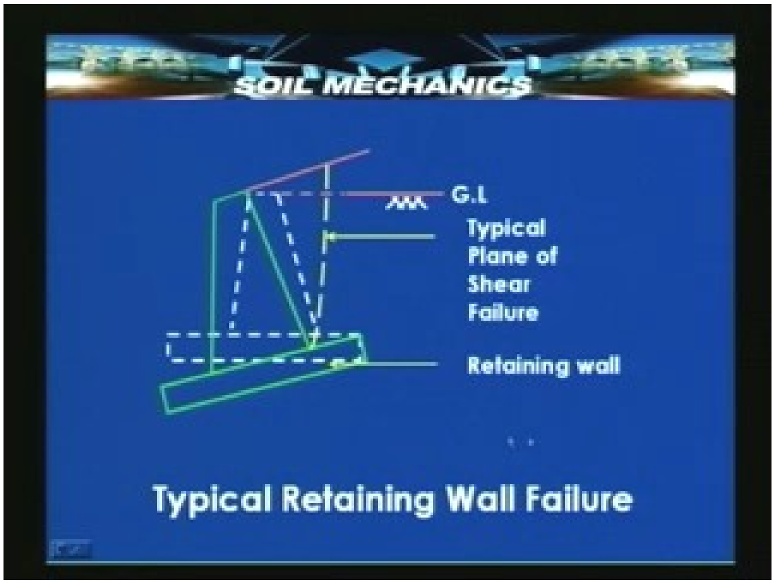

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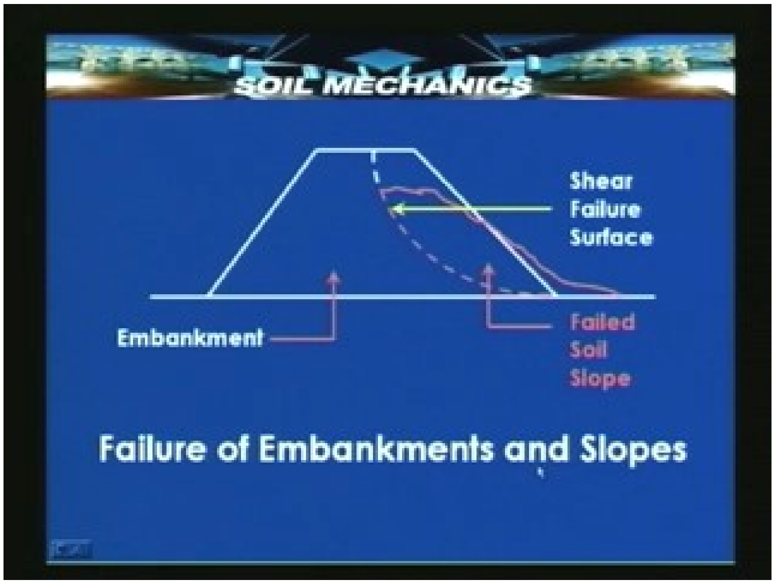

Here is an example of typical retaining wall failure. Suppose this white line, dotted lines is the initial position of the retaining wall. Then when the soil fails due to shear this retaining wall will take up a position like this. What really happens is due to the pressure acting on the retaining wall from the soil, the retaining wall tends to move. If this movement is in the form of rotation then correspondingly at certain degree of rotation after certain degree of rotation a well defined plane of failure due to shear will develop inside the soil as shown by these dotted yellow line. This is again a form of shear failure. Let us take a look at an embankment slope. Here the typical surface of failure along which an embankment slope will fail is this red line.

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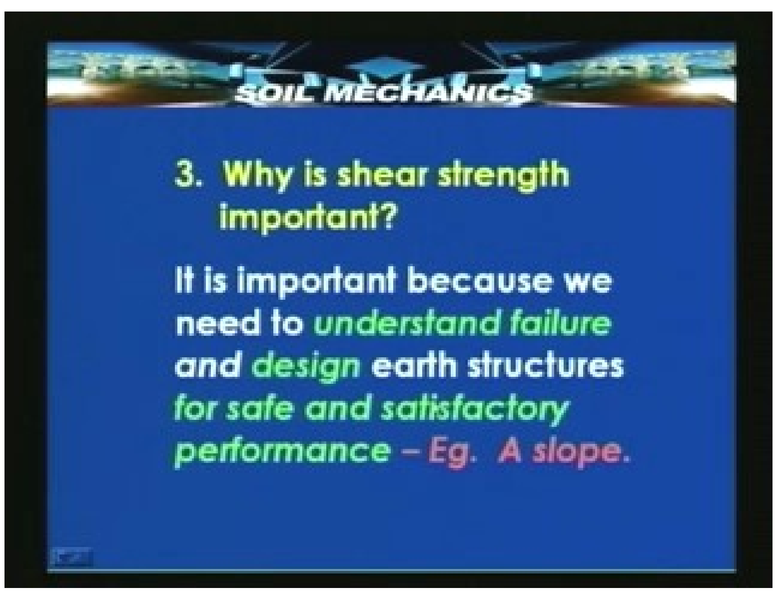

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This red line is usually found to be an arc of a circle or very close to that and you can see this soil surface after this slide has taken a new shape and position. This is the failed soiled slope or the mass of soil that has slip. Now this again is another form of failure this again is due to shear. Now the next question that we need to know or answer is what is the importance of shear strength? Why shear strength is important? It’s very simple we do not want the structure to fail. If the structure is not to fail due to shear failure of the soil then we need to understand what shear failure is all about, how is it caused, why is it caused?

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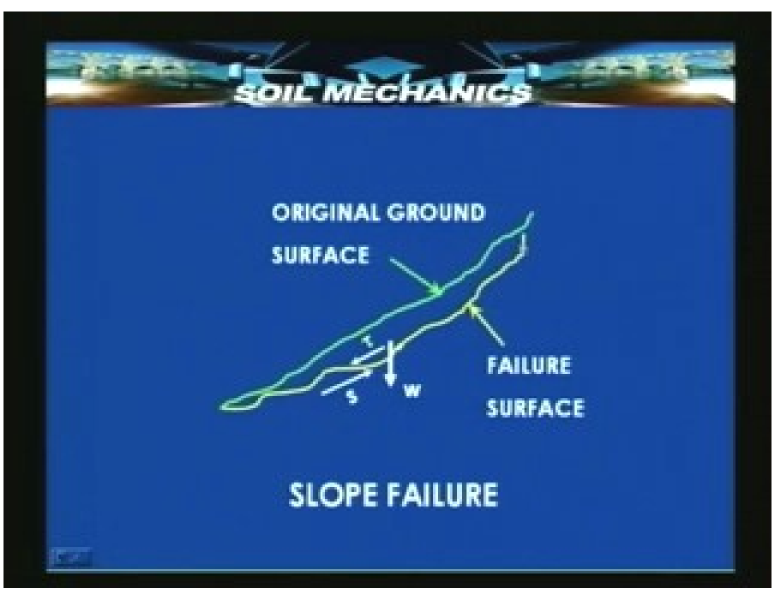

And then with that understanding we should be in a position to define a or rather design a structure in such a way that it won't fail, in such a fail that it will give safe and satisfactory performance. So understanding of the shear strength phenomenon, the phenomenon of shear failure of a soil is very essential. I am showing here now in this slide an example of shear failure of a slope, you can see that this is nothing but a landslide what we normally call as a land slide. This green line represents the original surface of the slope and this yellow line shows the surface that is now visible after the slope has experienced a landslip. The next slide will show you in detail about the original surface and the surface of failure, the surface along which the slope has slid and the surface which is now exposed as a scar (Refer Slide Time: 10:41 min).

This is a typical shear failure and it occurs because the soil mass that is contained between these failure plane and the original slope surface has a certain weight say w. This weight has two components, one downward in the form of shear along this yellow line or yellow surface and another upward resistance in the form of s that resist this shear. We all know that whenever there is a slip between two objects or two surfaces then friction is mobilized. So this shear strength s that we are taking about which has to be mobilized to resist the shear resistance tow is nothing but frictional strength basically but as you will see a little later in the case of soil that is another component of strength as well which is called cohesion.

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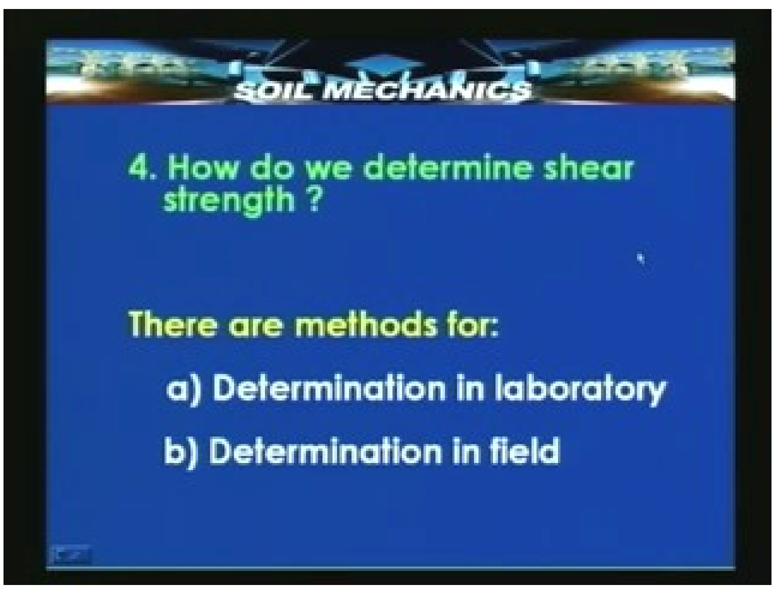

Let's proceed further we shall now ask 3 more questions. These questions are, how do we determine shear strength because as I said we need to understand what is shear failure, if we want to design a safe and satisfactorily performing structure. So we need to be able to determine the shear strength of a soil, so that we will be in a position to know whether the available strength at any point of time along any plane is sufficient to resist the shear stress that's going to be caused by any applied force. So what are the factors on which this shear strength will depend is therefore is also important because once we know whats the shear strength is under certain standard condition we can extrapolate and find out what the shear strength will be under different typical scenarios that exist in practice. For this purpose we need to know what are all the factors which affects the shear strength and how they affect and then lastly naturally we should know how to apply this knowledge in order to estimate and find out whether or not failure will take place and along which plane in a given situation and then how to avoid that failure. Now how do we determine the shear strength? The first question is there are methods for determining the shear strength, some methods are useful for the laboratory and some are useful in the field.

We have seen earlier also in the case of consolidation when we discuss the chapter of consolidation and settlement that it is very important to simulate the field condition and therefore extract samples of soil from the field which are simulating the field condition and test them under similar condition in laboratory as well. And the conditions under which the samples are tested in the laboratory should be as close as possible to the field condition and ideally it will be good to determine the shear strength in the field if possible but alternatively it is good to extract samples and determine the shear strength in the laboratory under simulated condition. There are many different methods for determining the shear strength of a soil sample in the laboratory and some of these are also useful in the field. Let's take a look at a list of such commonly used methods for determining the shear strength.

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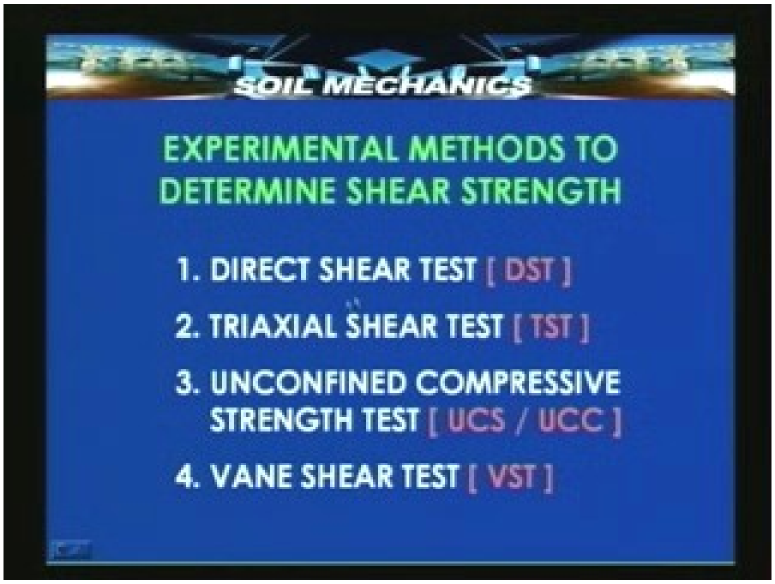

The commonly used methods for determining the shear strength are one direct shear stress DST for short, triaxial shear test TST, unconfined compressive strength test usually called as UCS or UCC, UCS standing for unconfined compressive strength, UCC standing for unconfined compression. Vane shear test, yet another test the fourth one which can also be used for determining the shear strength and incidentally the last two are also useful for determining the shear strength in the field. Let's take a look at these tests in brief. Let me illustrate what we mean by direct shear test, triaxial shear test and so on. In the direct shear test what we do is to take a cylindrical soil sample. We shall see the dimensions, typical dimensions of the soil sample and what length to diameter ratio it should satisfy and so on, when we see some details of the test. In the case of the direct shear test this sample is usually like this and we apply a normal load to this sample. This is the so called direct shear test. So along some predefined plane, we will directly shear the sample by applying a load T. So in this particular case the applied load T will tend to shear the soil sample along this plane say plane AB which turns out to be a plane of failure, a plane of slip nothing but the shear failure plane.

And for different normal loads if we can determine what is the value of T required then we will be in a position at a later date to find out at what value of N and what value of T, the soil has failed. So that we will know correspondingly what is the strength value at that time. The triaxial shear stress test on the other hand is considered to be a better test because it simulates the field situation particularly with respect to stress condition more precisely more faithfully. For example suppose this is the cylindrical sample you know in nature any typical element that is inside the soil medium will be experiencing not only stress from top and bottom but also from the slides, there will be lateral stress as well. And it will also be acting perpendicular to the plane of this paper.

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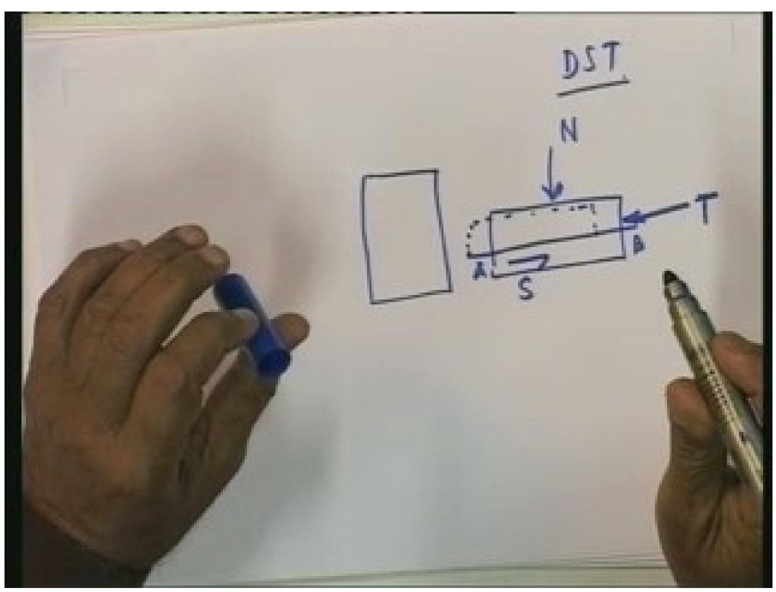

Therefore ideally speaking if we want to test a soil sample under field conditions, we should be in position to simulate this particular stress condition which is essentially a three dimensional stress condition. So in a triaxial test we apply stresses along three axes and that's the reason why it is called triaxial test. We apply initially to begin with loads on the sides, lateral pressure. After applying the lateral pressure we apply normal or axial pressure and gradually increase this axial pressure until shear failure takes place along some typical internal plane of failure. This is the triaxial shear stress. We also have a test known as the unconfined compressive test, in this the difference is that the soil is not confined in the lateral direction. So this is simply a cylindrical unconfined sample, no lateral pressure on the sides and this is subjected to axial load until failure takes along some typical weak plane inside the soil.

And as in the case of the direct shear stress we try to find out what is the load T at failure or here in this case what is the stress difference between the axial and the lateral stresses which is responsible for failure and at the point of failure we estimate what is the shear stress which has caused the failure and that happens to be also the strength of the soil. So in unconfined compressive strength you will find that failure takes place along this and this is the shear stress that is mobilized due to the applied load and at the time of failure, at the point of failure the shear resistance should be equal to this tow. And therefore the value of the tow at failure happens to be the shear strength and this is therefore the way we determine the shear strength of a soil.

Take a soil sample subjected to failure; determine the shear stress at failure. Shear stress at failure has to be equal to the shear strength of the soil. The vane shear test is a slightly different test; it's also used in the field although sometimes it can be also in the lab. It consists of a vane, a blade with a stem and this vane is inserted into the soil and rotated, torque is applied and then failure takes along the cylindrical shaped surface. We try to find out from incipient failure conditions what's the torque applied at failure.

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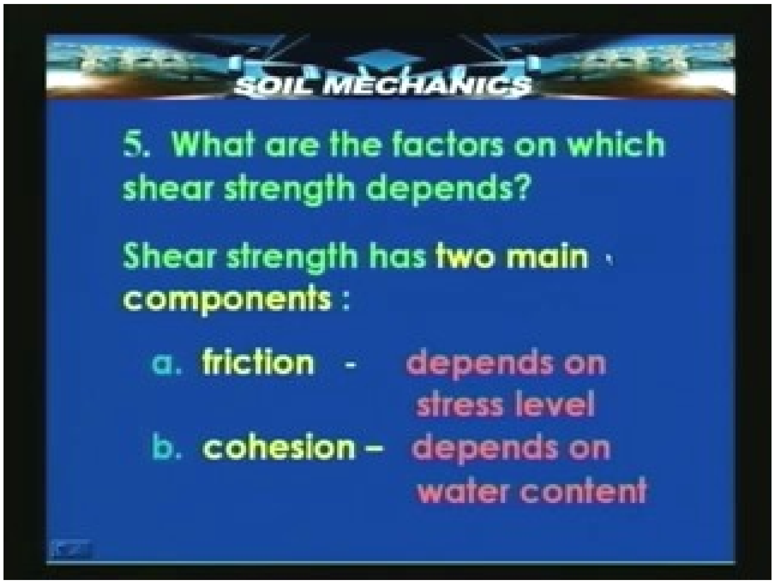

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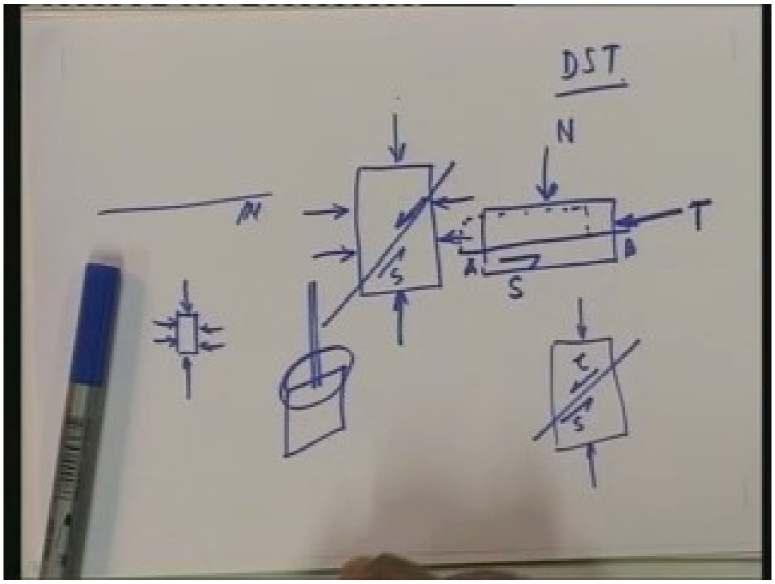

The torque applied must be equal to the total resistance mobilized from which we estimate what is the shear strength of the soil. Now the next question is what are the factors on which shear strength depends, although we will be seeing in very great detail each one of the factors and understand in what way they affect the shear strength. We shall at this point of time just say something about two major causes or factors. Now for this we must understand what shear stress is composed of. There are two constituents which make up shear strength, one is friction and another is cohesion. Now friction is a component that gets mobilized along the failure plane and it is a function of a stress that is applied. So friction depends upon the stress level.
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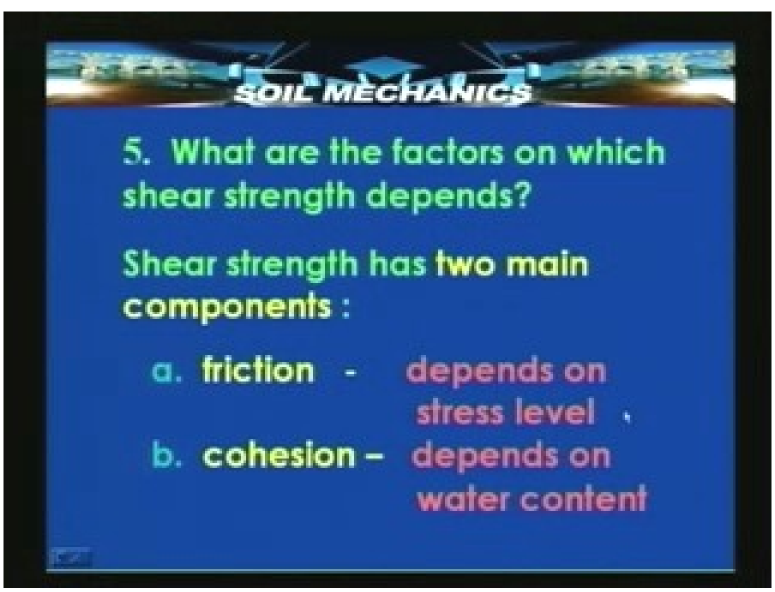

On the other hand there is another component called cohesion which is not dependent on the stress level but it depends upon the water content and it is independent of the stress level. Therefore shear strength has two factors or two components which in turn or affected by several factors. As far as friction is concerned the prime factor is stress level or as far as cohesion is concerned the most important factor is the water content or the amount of water which in turn as you will see will be related to the rate at which the water can flow out of the soil when shear failure takes place that is what is the drainage condition.

Lastly the sixth question is where do we apply the knowledge of shear strength in practice. Knowledge of shear resistance is necessary for design of any structure in or on soil, for example foundations, excavations, embankments, slopes, retaining structures and all these structure ultimately fail by shear under certain applied loads.
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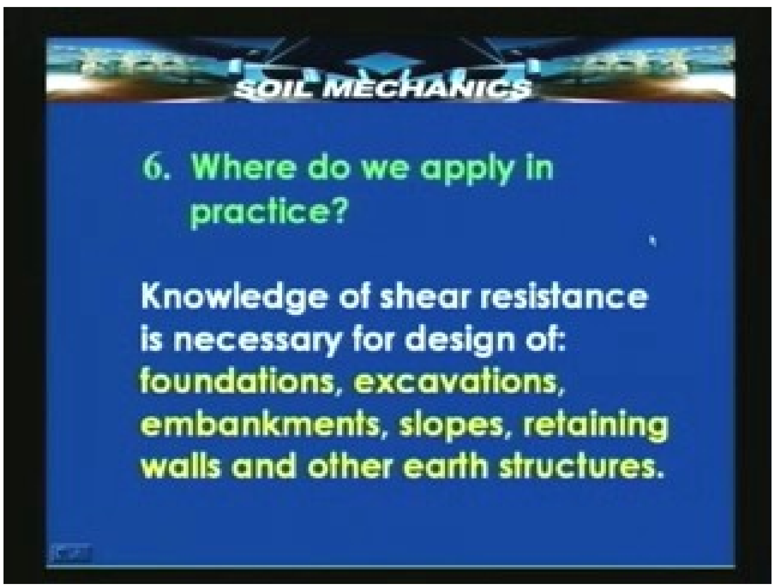

And it is necessary to understand how the shear failure takes place under each one of the cases. So that we will know how to design these for safe and satisfactory performance. Now we have discussed the importance of the mobilization of shear strength to resist an applied shear stress. As I fall out, it comes out automatically that we know need essentially a method to determine shear stress along any plane in a soil. If we have a method to compute the shear stress along any plane in a soil we will be in a position to estimate the shear strength as well. So let us spends sometime to see how stresses are distributed inside a soil and how we can determine the shear stress on any potential failure. So the remaining part of this lecture is going to be devoted to stresses at a point and on plane. The surface at a point and on a plane then related to distribution of stresses and determination of shear stress we also have a concept is known as the Mohr circle concept which is an integral part of understanding the distribution and determination of stresses at a point and on a plane.

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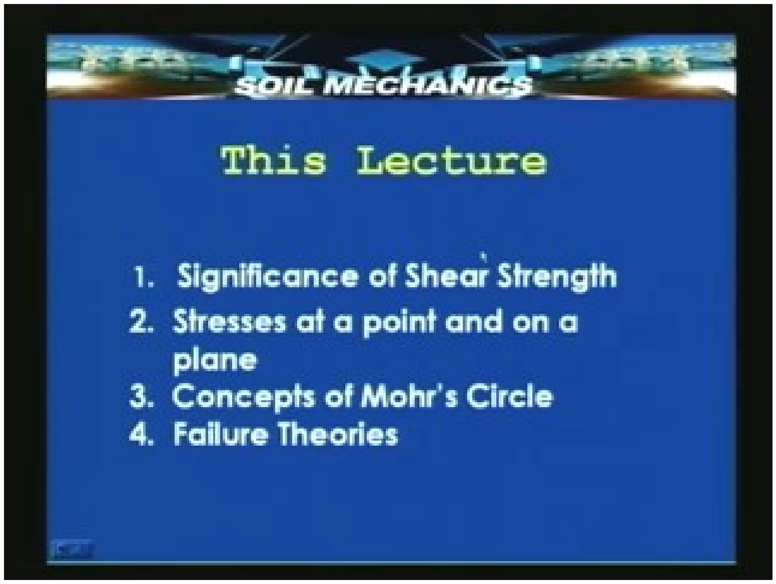

So we will also see this ultimately after all we are interested in failure therefore we shall also take a look at what are the theories that are available to help us to estimate what the shear stress will be at failure. So that we will know what will be the possible shear strength that a soil can mobilize at failure. Now this diagram shows how to estimate stress at a point and on plane this first diagram here shows a typical semi finite medium P is a typical point inside the medium and all this white straight lines here are nothing but different planes passing through this point P in the medium.

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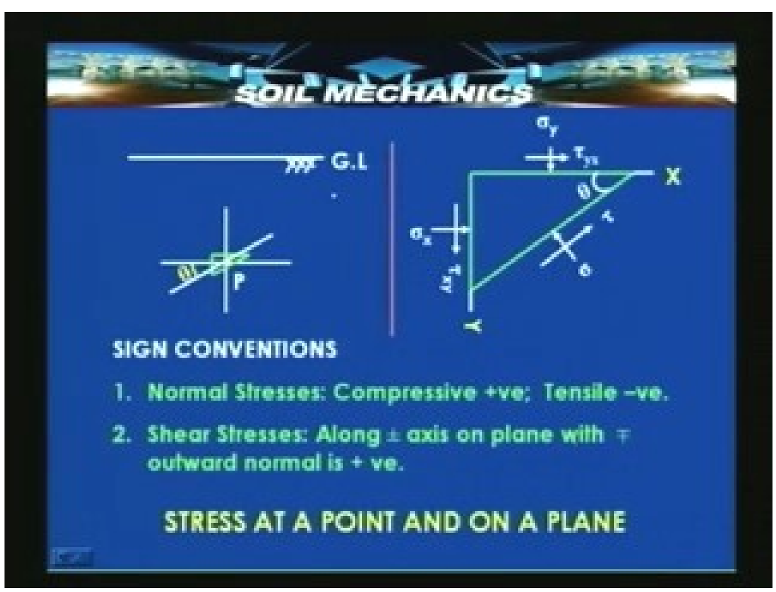

Let us consider a horizontal plane, a vertical plane and an arbitrary inclined plane, the angle of inclination being theta with respect to the horizontal. Let us take an infinitesimally small element at this point enclosed by these planes. So here we have the horizontal plane here, we have the vertical plane and here we have inclined plane at an angle theta. This we shall take as an infinitesimally small element which will tend to be this point when the dimensions tend to become zero. For convenience of understanding the stresses and their distribution on this element let us represent this in an enlarged manner as shown in this adjustment diagram.

Here you will find that we have this three planes which make up the planes at this point and on each one of these as you know in general there can be a normal stress and a shear stress. So on a plane directed in the x direction there will be a normal stress in the y direction. We will define now a set of positive stresses on each one of these planes according to a sine convention. For normal stresses we will take compressive stresses to be positive and tensile stresses to be negative. Here sigma y is a compressive force and therefore the positive normal stress as far as sigma y is concerned will be one which is directed downwards. As far as the sigma Y is concerned that is the stress in the X direction is concerned the positive stress will be one that is directed from left to right that is in the direction Y and acts on a plane Y.

The shear stress has a slightly more complex sine convention. Shear stress convention is defined like this. Along positive or negative axis on a plane with negative or positive outward normal the shear stress is positive that is suppose I take this plane X the outward normal for this is directed in the negative Y direction and on a plane like this a shear stress which is directed along the positive X is positive. Whereas on a plane Y the outward normal will be directed in the negative X direction and the positive shear stress will be again one which is directed in the positive X direction like this in the positive Y direction. So for the positive shear stress will be one which is directed in the positive Y direction. With this we shall now assume that the element has to be in equilibrium if the soil is not to fail and then try to find out on any typical plane what is the shear stress and of course what the normal stresses is.

So as you can see we are now converging on a method to determine shear stress on any plane one in the case of failure this plane could be the failure plane. Now how do we estimate the normal and the shear stress on any plane, any potential failure plane for that matter? What is known is the normal and shear stresses on the planes AB and BC two mutually perpendicular planes and what is required to maintain equilibrium is the knowledge of the normal and shear stresses on the plane AC. Suppose the length AC is capital L then AB will be L cos theta, BC will be L sine theta and assuming all these stresses to be distributed informally and this assumption is not an error because the element is infinitesimally small element and therefore on these small lengths AB BC AC it is perfectly reasonable to assume all the stresses to be uniformly distributed in which case the force on plane is AC for example in the normal direction would be sigma into L.
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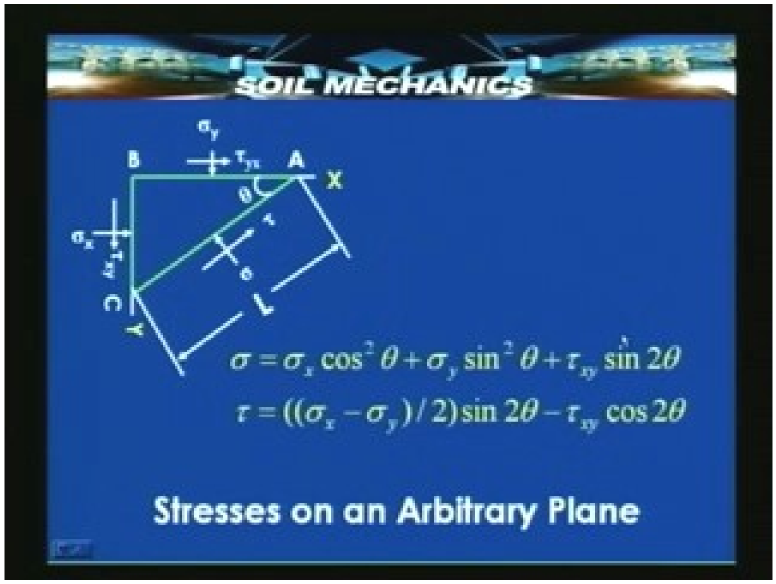

Similarly the sheer force will be tow into L and identically the forces on the plane AB will be sigma y into cos L theta and tow yx into L cos theta and similarly here. And now we can write down an equation for equilibrium for all these forces, the equations of equilibrium can be represented in terms of some of all the forces in the sigma direction equal to zero which will give us an expression for sigma like this and some of all the forces in the mutually perpendicular tow directions equal to zero which in turn will give us an expression for tow like this. So sigma is expressed here in terms of the known stresses on two mutually perpendicular planes.

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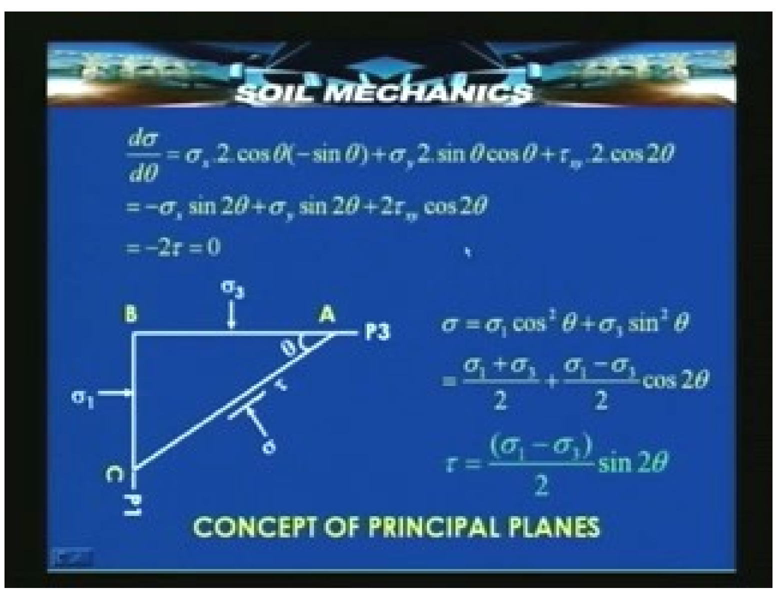

Similarly tow is also expressed in terms of stresses which are known on two mutually perpendicular planes. So here we have the two expressions which are very useful in determining the normal and shear stresses on any arbitrary plane if we know the stresses on two mutually perpendicular planes. Now let us take this normal stress in particular, if we want to find out where the normal will be stress be maximum or minimum we need to simply differentiate it with respect to theta. What is d sigma by d theta it will be equal to this and you when you simplify this you will find that it works out to an expression which is nothing but two times the expression which we have seen in the previous slide for tow.

So when sigma is maximum or minimum shear stress tow is zero and this happens at a particular value of theta and the planes AB and BC at that value of theta will have only normal stresses on them and no shear stress and one of the normal stresses will be the major principle stress sigma one and the other one will be known as the minor principle stress sigma_3 and any normal stress on a plane on which there is no shear stress is known as the principle stress. And these principle stresses can be obtained if we know what is the value of theta at which d sigma by d theta becomes zero and substituting that value of theta we can show that sigma is equal to this and tow is equal to this in terms of the principles stresses sigma_1 sigma_3. So this sigma and this tow can be also expressed in terms of sigma_1 and sigma_3 unlike in the previous slide we express them in terms of sigma x tow xy and sigma y tow yx. These two expressions are extremely convenient for graphical representation and understanding the processes of failure.

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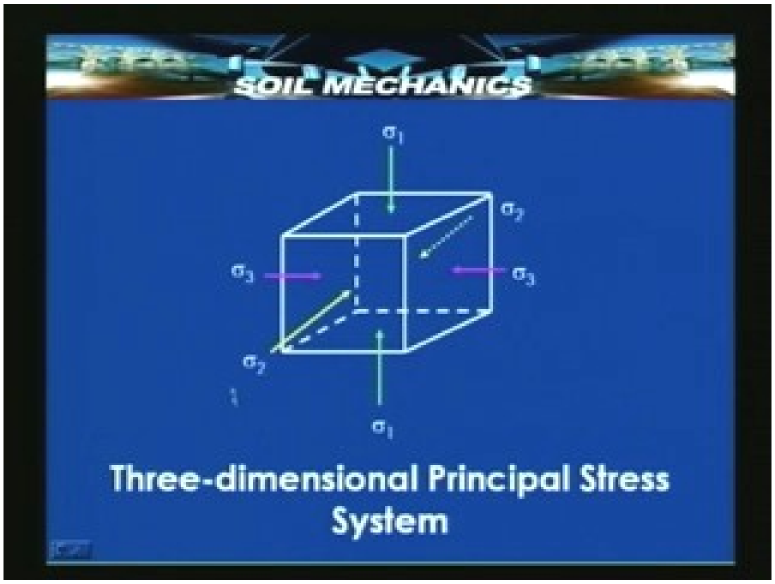

The principles stresses in a typical three dimensional case are three in number sigma$_1$, sigma$_2$, sigma$_3$we shall however simplify this problem and considered a planar two dimensional problem and deal with only the major principle stress sigma$_1$ and the minor principle stress sigma$_3$. Now we will see what is known as a Mohr's circle, we have just now seen that the normal stress and the shear stress can be expressed like this in terms of the principle stresses. These two expressions suggested to Mohr that it is possible to draw a circle in such a way that the radius of that will be nothing but sigma$_1$-sigma$_3$ by 2.

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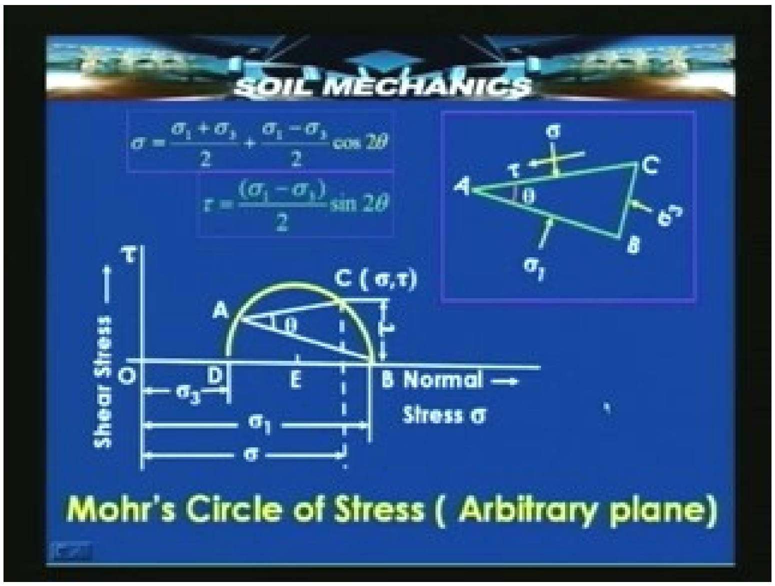

If you draw a circle with one end at sigma$_3$ and another end at sigma$_1$ and radius as sigma$_1$-sigma$_3$ by 2 and now from this point B if you draw a line parallel to the side of the infinitesimally small element. Then you find that BA represents one of the sides, the side on which the sigma$_1$ acts and AC represents the inclined plane at angle theta along which we want the stresses. Then the point C has coordinate sigma and tow which are given by these two expressions.

So here we have a method where if we know sigma$_3$ and sigma$_1$ and plot them in a forma of a circle as shown here and then plot the infinitesimally small element particularly the plane along which we want the normal and shear stress then we get the points C whose coordinates are sigma and tow which are nothing but the normal and shear stresses of the plane of interest. Now if the case, special case where this plane sigma$_1$ major principle plane is horizontal this diagram is slightly simplified and angle theta will be this. And once again the point C will have coordinates sigma and tow given by the same two expressions.

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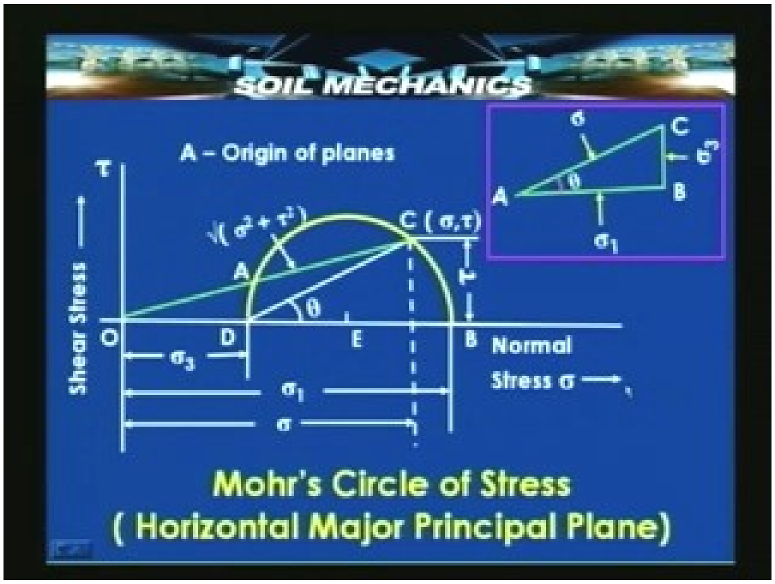

And this line OC will have an expression given by route of sigma square plus tow square because this is sigma and this is tow and therefore this length represents root of sigma square plus tow square and this point A is known as the origin of planes. Now our prime interest in plotting this Mohr circle of stress is to find out what is that plane AC whose orientation is such that the corresponding shear stress causes failure. Idea is once we have a method to find out sigma and tow, we want to find out what is that plane where tow causes failure because if we know the plane at which tow causes failure that would mean that is the plane along which the shear stress is just equal to the shear strength.

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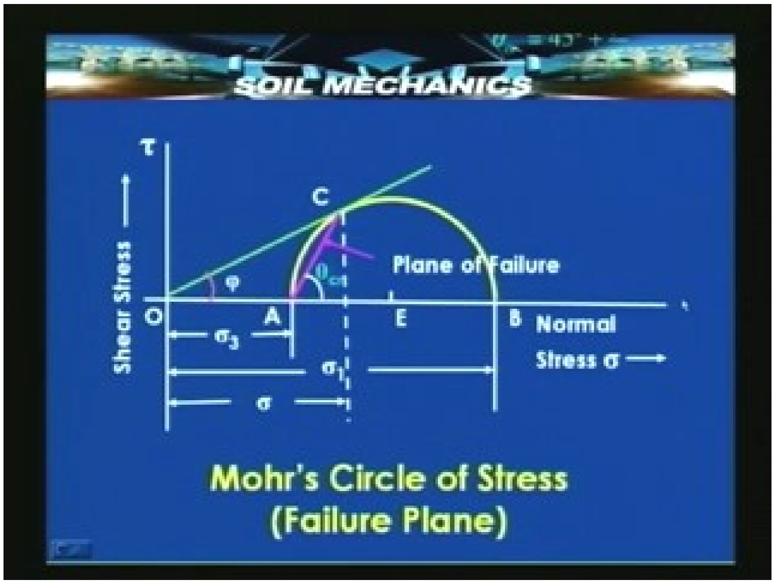

So we can determine the plane of failure and correspondingly we can determine the shear stress and that is nothing but the shear strength of the soil. So the Mohr circle of stress tells us a method or gives us a method to check whether or not failure will occur along a plane, inside the soil and that's what we are basically interested. Now if there is a plane of failure then the angle theta for that plane of failure will be given by 45 degrees plus 5 by 2.

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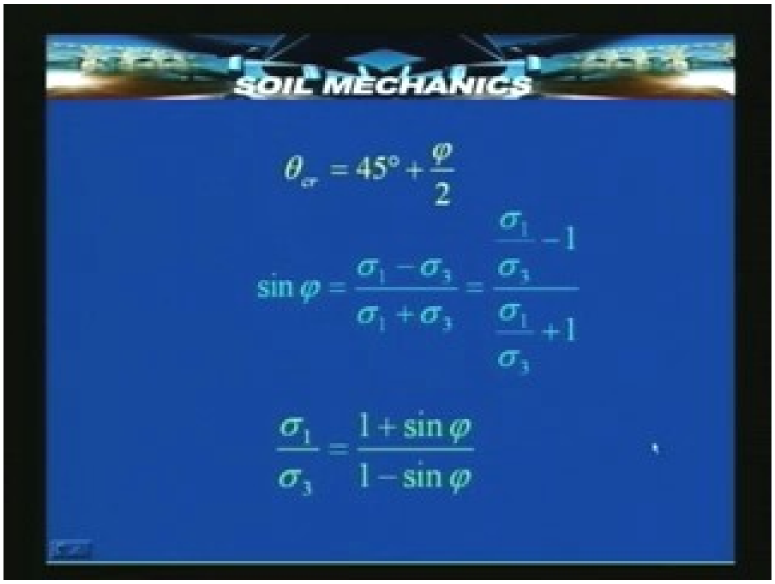

Let's take look at the diagram once again. This value of sigma and this value of tow give rise to a point C, corresponding to this point C is possible to draw a tangent and this angle let us say is phi. You can see here that this is a limiting value of this angle phi which is known as the angle of obliquity and it can be shown that it is this angle phi, limiting angle phi which represents failure which is a measure of the failure and therefore corresponding to this point C the plane, inclined plane will be having an angle which is the critical angle corresponding to the state of failure. And tow is the shear stress corresponding to the plane which is at an angle theta critical.

Now if this angle is phi it is obvious that theta critical will be nothing but 45 degree plus phi by 2 and it also is obvious that the shear stress tow will be related to phi by our expression that tow is equal to sigma tan phi. Shear stress tow is sigma tan phi and therefore at failure we can say that the shear strength of the soil is nothing but sigma tan phi. And that's the basis of a number of theories which have emerged subsequently on the concept of failure. So once we know theta critical correspondingly we can write down some more expressions, one for sine phi another for the ratio of sigma_1 and sigma_3 in terms of sine phi.

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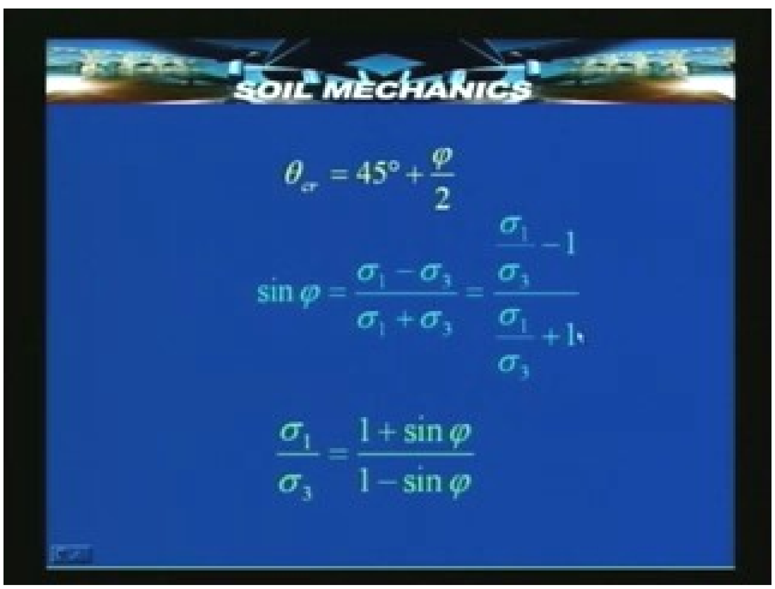

So these two expressions are actually similar here sine phi is expressed in terms of sigma_1 sigma_3. Here sigma_1 sigma_3 are expressed in terms of sine phi. So this basic expression will help us to find out what is the plane of failure and whats the angle of obliquity at that point phi which defines the potential failure plane.

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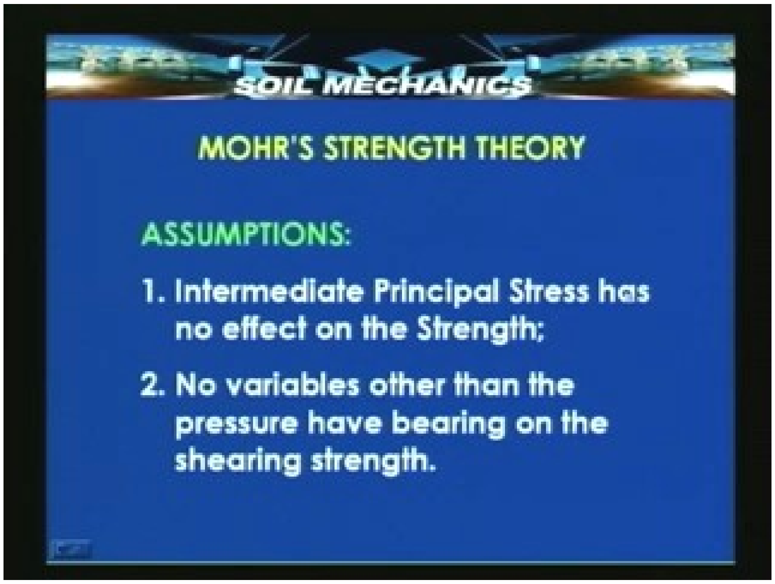

Now there are many theories of failure basically we shall be seeing three theories of failure. One is the Mohr's failure theory we call it the Mohr's strength theory because after all failure corresponds to the state at which the strength is fully mobilized. So the failure theory also can be referred to the strength theory. We have a strength theory known as the Mohr's strength theory there is another theory which also we shall see which is known as the coulomb's strength theory and then we shall also see a Mohr's general strength theory.

Let's take a look at these 3 theories. The Mohr's theory starts with the assumptions that the intermediate principal stress has no effect on the strength and no variable other than this stress have any bearing on the shearing strength. That means in effect the concept of cohesion is simply not included in this as we would have seen earlier itself that friction is a component of strength which is dependent on the stress level. So this theory indirectly is stating that strength is dependent on friction.

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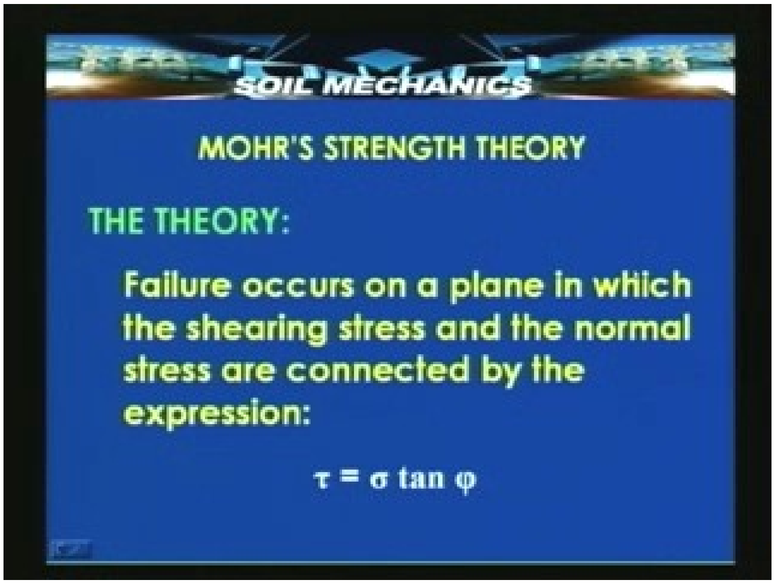

And as per this theory failure will occur on a plane on which shear stress is related to the angle of obliquity phi in terms of tow is equal to sigma tan phi. And this can be expressed in terms of a more envelop, here in fact I have 4 envelops and we know that at failure this green line where it touches the Mohr's circle defines the point at which tow will be equal to sigma tan phi. So that's the point which represents the stress state on the failure plane and if you join this with D, you will get the failure plane.

Now consider this first Mohr circle number one, this Mohr circle touches this green line which represents a line with an equation tow is equal to sigma tan phi. So this is referred to as the strength envelop and since this theory is that of Mohr this is referred to as Mohr strength envelop. So if this represents the liming strength then the first circle has stresses $sigma_1$ $sigma_3$ in such a way that it just touches this and this represents a state of stress which corresponds to failure.

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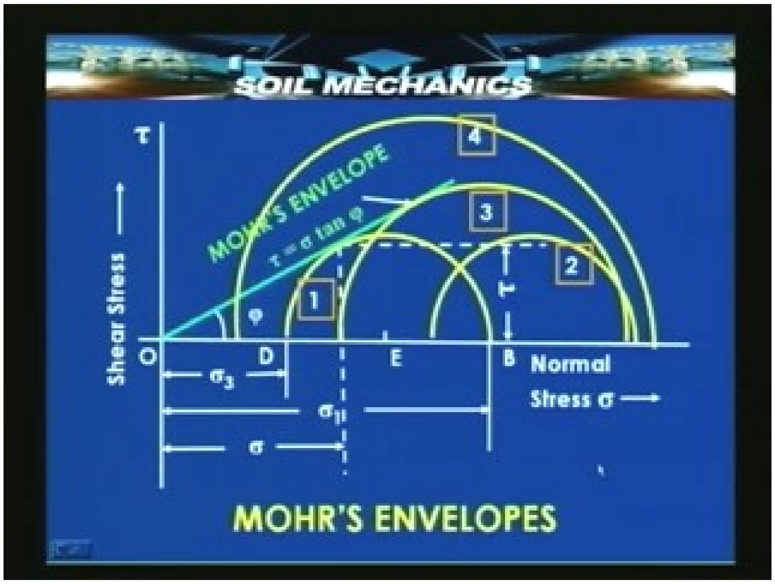

On the other hand if you take the circle number 2, this does not touch the envelop that means although the shear stress here is equal to this shear stress, the $sigma_1$ and $sigma_3$ values are such that if this circle does not touch the envelop and therefore this stress rate does not corresponds failure. On the other hand the larger envelop, the larger Mohr circle 3 again corresponds to the failure because it touches this whereas this circle 4 again represents a state of no failure. So Mohr's envelop can be taken as a guideline for checking whether any stress state defined by the Mohr s circle corresponds to failure stress state or not and that is what the Mohr's strength theory is all about.

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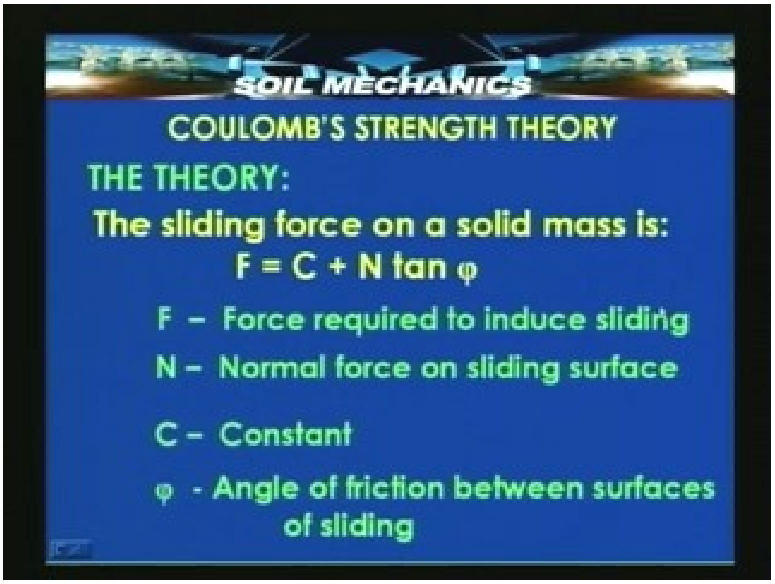

According to Mohr's strength theory shear strength is tow equal to sigma tan phi and plotting the Mohr's circle and the Mohr's envelop it is possible to find out whether or not given the given stress state corresponds to failure or not. And if it corresponds to failure then we can calculate phi theta critical from these expressions. Now there is the second theory, the coulomb strength theory. This theory is an empirical theory first of all and according to this the sliding force on a solid mass F is equal to C plus N tan phi.

Now this is something which you know already from physics, coulombs law of friction is known to you already from physics. If there is a body which is sliding on a plane, if there is a normal force N then you know that the force F that is required to shear this will be equal to N tan phi plus some component which is independent of N. So that's what coulombs says that the force F that is required to cause instability sliding will be having two components one is proportional to N another is independent of N. The component which is proportional to N will be given by N into tan phi where this tan phi stands for the well-known coefficient of friction In this case the sliding takes place inside the soil along some plane and therefore this coefficient of friction corresponds to the friction coefficient along some internal failure inside the soil and so this is very often referred to as the angle of internal friction.

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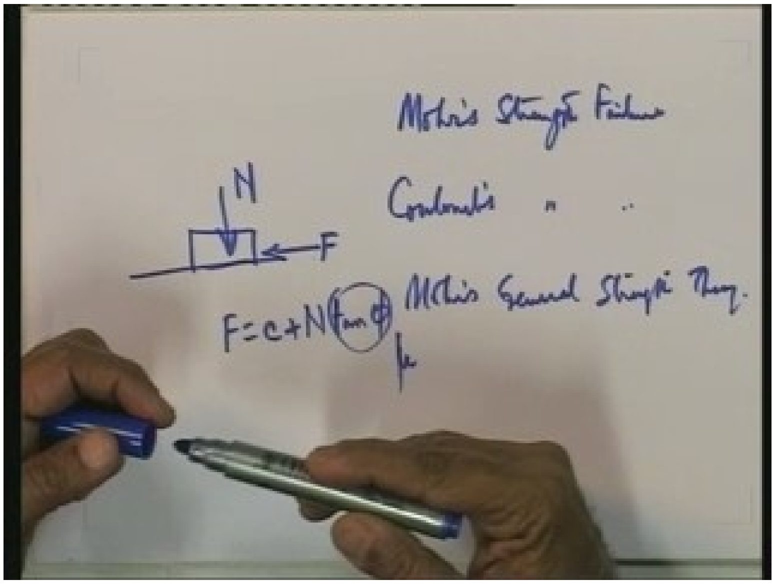

But nowadays it is more frequently referred to as the angle of shearing resistance because it is this angle which goes into computing the strength that is this defines what's the strength is. So this is known as the angle of shearing resistance. This is the coulombs theory where C is a constant value, N is a normal force, F is the force required to induce sliding and phi is the angle of friction between the surfaces of sliding in the soil.

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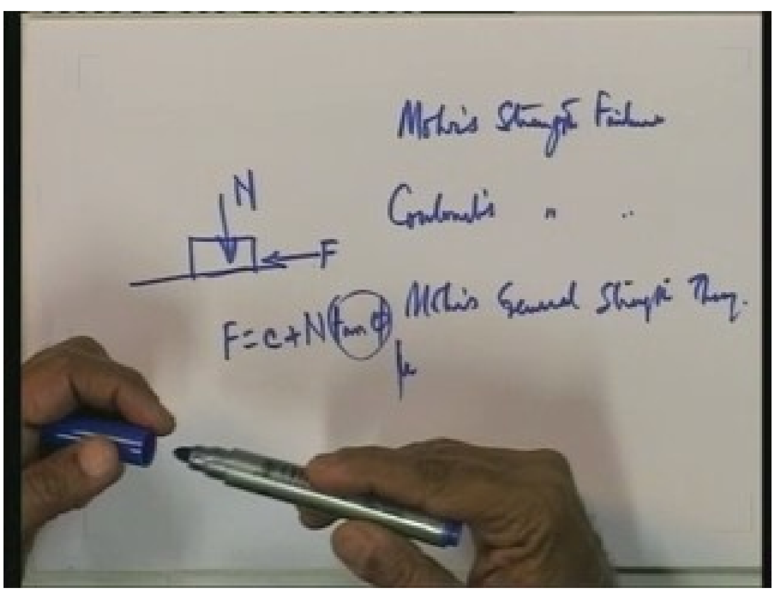

Now this can be applied to soil and if we take a finite area of contact a then this all the forces can be converted into corresponding stresses and we can say that the force required to fail is nothing but the strength that is s is equal to small c which is equal to nothing but capital C by a plus N upon a that is sigma into tan phi. This parameter c and phi which go into this equation are known as the shear strength parameter. So this means is that if in any situation if I know the normal stress on a plane and if I know the cohesion and coefficient of friction phi then I will be in a position to calculate the shear strength s using coulombs failure theory.

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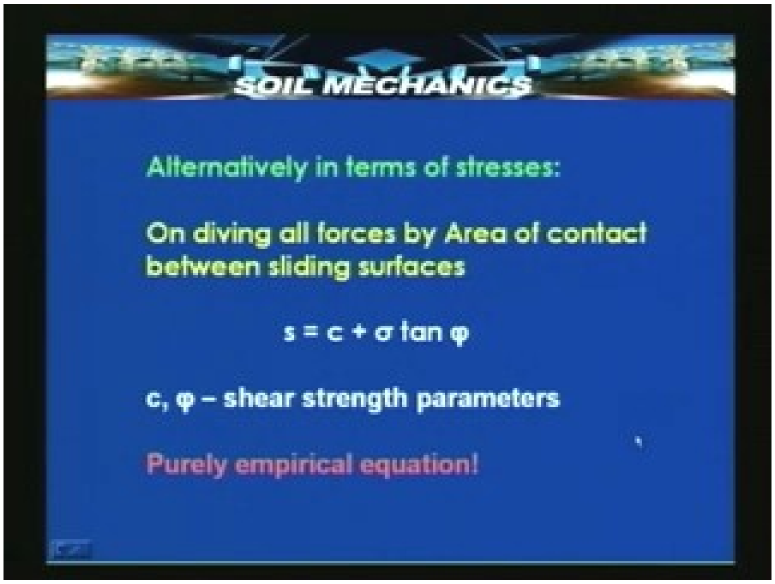

And incidentally this equation is purely empirical equation but practice as shown that it is widely applicable and quiet reliable and therefore even today in order to evaluate the strength of a soil we use the empirical equation of coulomb.

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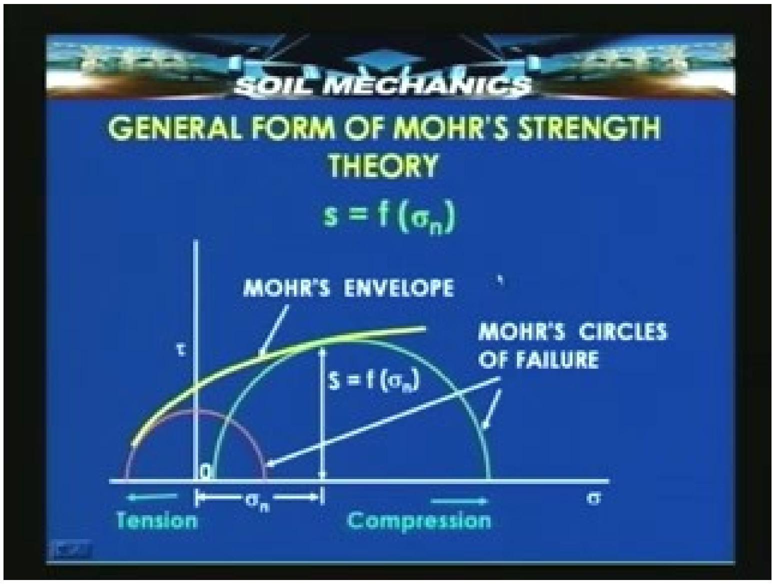

This third theory is nothing but a generalized form of Mohr strength theory. Here we say that the shear strength is not the sigma tan phi, we say it is some function of sigma N need not be sigma tan phi it could be some other in fact even nonlinear function of the normal stress sigma. So if we were to plot the Mohr circle for this general theory, you will find that there will be a Mohr circle corresponding to which there will be a yellow envelop like this which in the most general case will not be now a straight line. And it would also pass like this so that this will become an envelope even for stresses representing tension. So the Mohr's general strength theory is applicable to both compression and tension and the envelop that we get covers from tensile to compressive zones and this point as before is the point corresponding to failure and this is the point for which the shear stress is equal to the shear strength S given by f sigma n.

And what is this f sigma n? Once again practice as shown that s is equal to c plus sigma tan phi is sufficiently representative of this function of sigma N and once again therefore we will reinforce the statement that if we know c and phi the shear strength parameter and if we know the normal stress on a plane we can determine whether or not that plane is a plane of failure because we can get the shearing strength from this c sigma phi. We can calculate the shear stress from the equilibrium considerations if the shear stress exceeds the strength then there will be failure otherwise not. As I said there are two components contributing to shear strength one is friction another is cohesion and if we take a typical soil sample apply a normal load N on a plane of failure and apply a shear stress tow as we saw some time back in the case of a direction shear stress.

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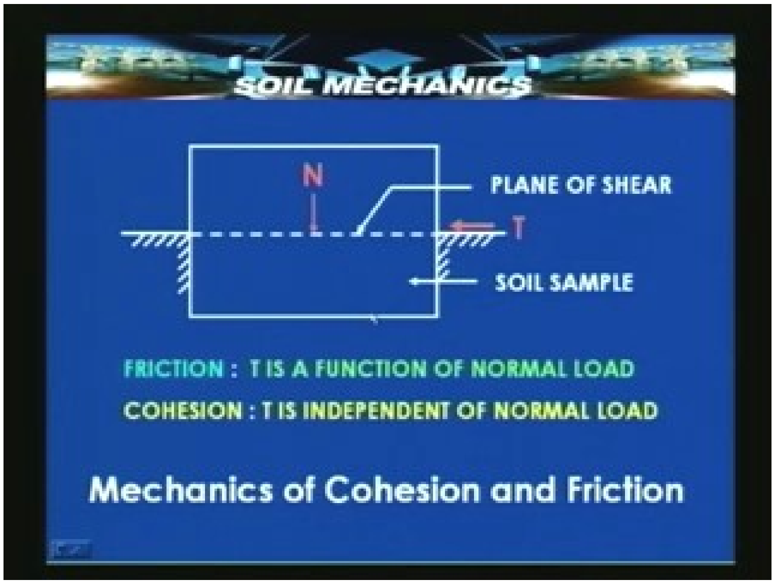

Then this T will be a function of the normal load if this material is a frictional material. That is material like sand, gravel, coarse grain materials. If on the other hand we have fine grain materials like clays then there will be a component contributing to T which is going to be independent of this normal load N. Of course the component proportional to normal load N will be there but in addition there will also be a component which is independent, which is what is known the cohesion component.

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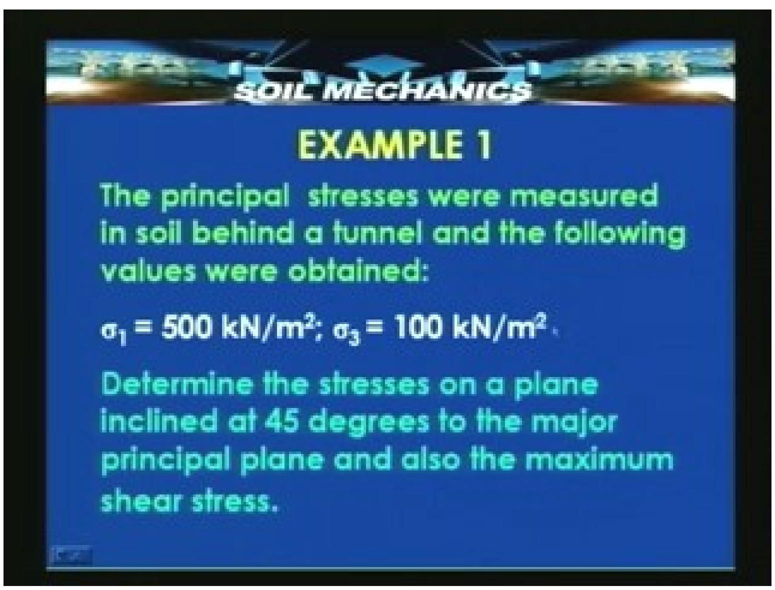

In general there can be soils which are having purely only cohesions, there can be soils which are purely frictional and real soils lie between these two extremes and most real soils have both cohesion and frictional component. Now let us see a quick example of how to use this Mohr's circle. This example says that if we know sigma$_1$ and sigma$_3$ how do we calculate the stress on a plane inclined at 45 degrees is sigma$_1$major principle plane and also we find out what is the maximum shear stress.

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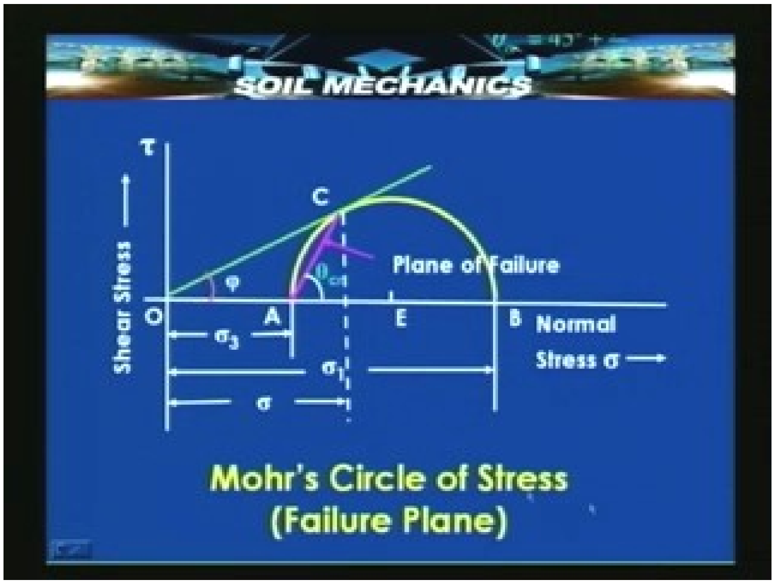

So this is the Mohr's circle, where does the maximum shear stress occur? Obviously the maximum shear stress occurs here just above the center of the circle, the shear stress whereas at failure the shear stress is only this much. Therefore maximum shear stress need not necessarily mean failure on the other hand failure depends not just on the shear stress but on the ratio of tow to sigma and when that ratio which is equal to c plus sigma tan phi is satisfied that only will correspond to this status of failure. Now we have been given in this example sigma$_1$ and sigma$_3$ the stress corresponding to point B and the stress corresponding to point A and value of theta is 45 degrees. So we need to find out what is the shear stress.

Let us see, the solution is that on a plane at 45 degrees if we use this two expression for normal stress and shear stress in terms of sigma$_1$ and sigma$_3$ and put the values of sigma$_1$ sigma$_3$ and theta equal to 45 then we simply get sigma equal to 300 and tow equal to 200. And incidentally it so happens that this plane of 45 degrees if you see the Mohr cycle again is nothing but a plane which passes through this point here that means the shear stress on this plane is nothing but tow max.

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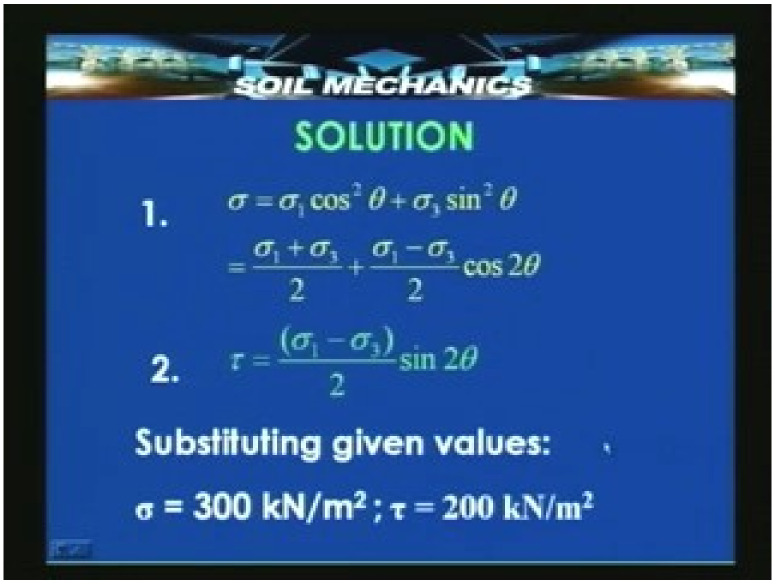

So this shear stress tow incidentally corresponds to the maximum shear stress as well which was asked in the example. So tow max occurs when theta is equal to 45 degrees and here since the given plane is at 45 degrees, it is also the plane of maximum shear stress.

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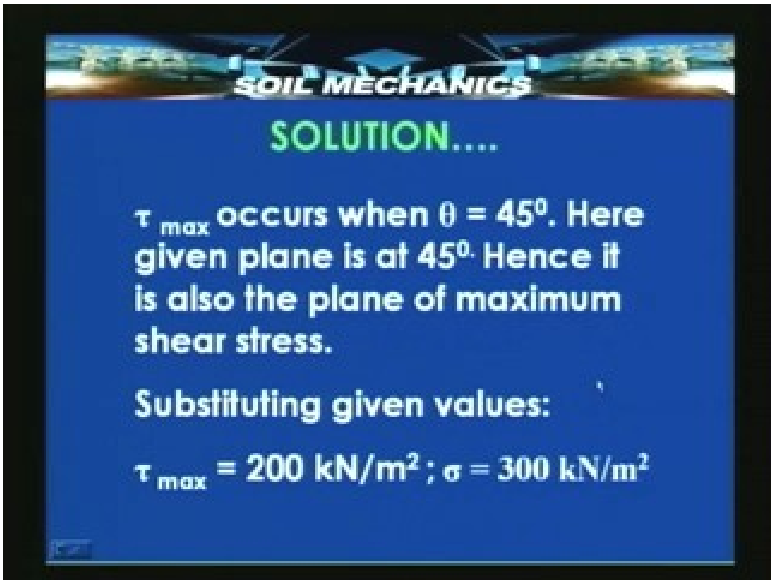

And what is this maximum shear stress? The maximum shear stress has to be the shear stress which we have just now estimated and that is 200 and the corresponding normal stress on that plane will be 300.

And so from the Mohr's circle given $sigma_1$ and $sigma_3$, we can always determine the normal and shear stress on any plane and then find out whether or not that plane is a failure plane by applying one of the failure theories. We determine tow, we determine sigma, if tow satisfies any of the failure theories then we know that corresponds to the state of failures.

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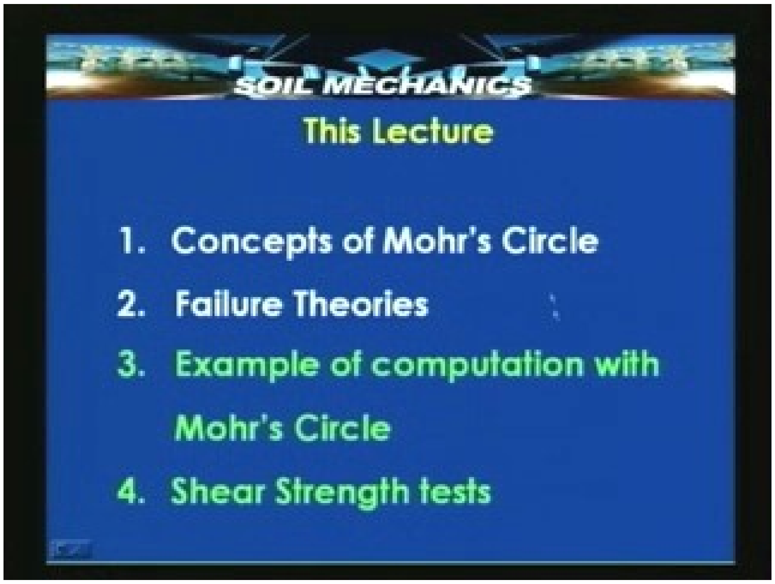

So in this lecture we have seen concepts of Mohr's circle, some failure theory, we also seen an example of how to use Mohr's circle and we have seen what are the test which are available for determining shear strength.

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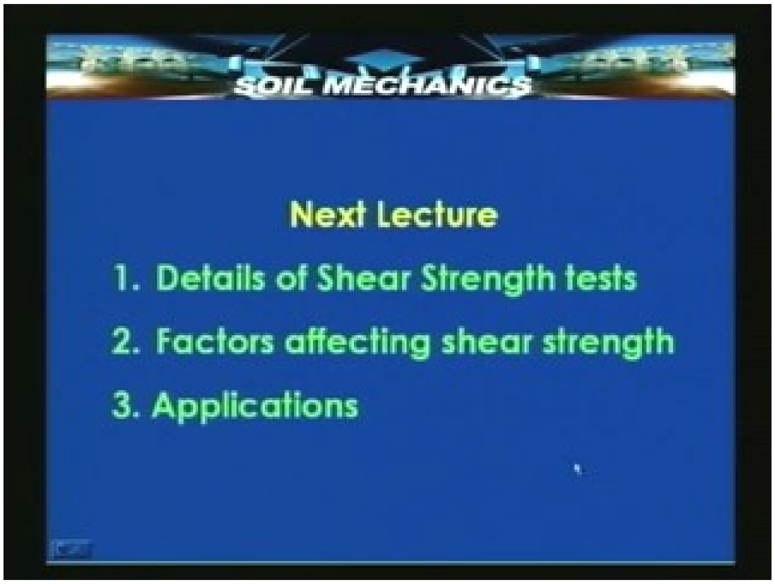

In the next lecture we will see some details of the various shear stress strength and their corresponding Mohr's circles. We will also see what are the factors affecting the shear strength and how they affect and then we will see some applications of these in the form of some examples.

Thank you.