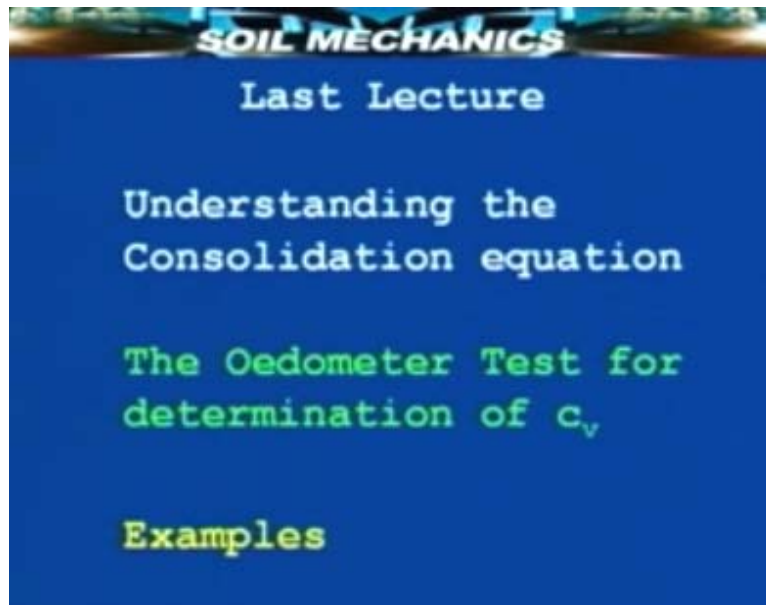


Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 40
Consolidation and Settlement
Lecture No. 7

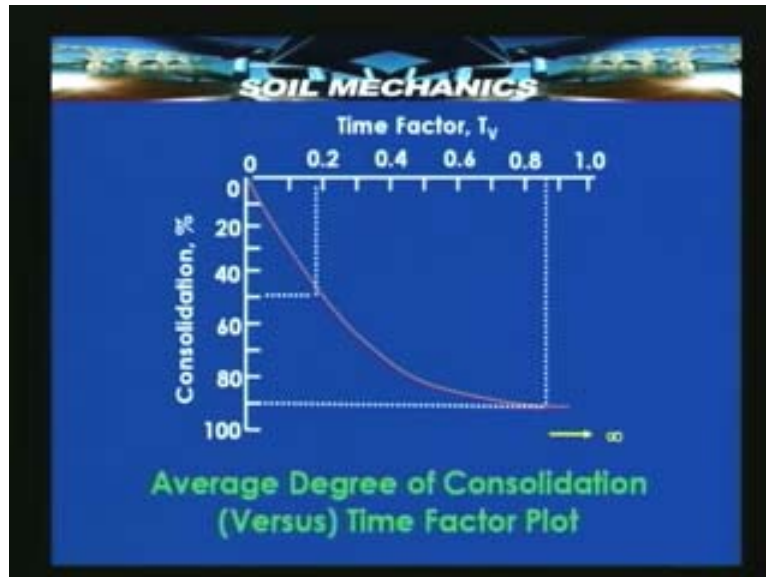
Students we now have the seventh lecture on consolidation and settlement. During the course of the last 6 lectures we had made considerable progress. We had discussed almost **third** bag the phenomenon of consolidation and we had seen how to compute several parameters which are used for expressing compressibility. We had understood the method that is being used so called the oedometer test for determining the void ratio changes and to determine the parameters which are important for us from point of view of understanding the phenomenon of consolidation. As we had seen, we had seen 2 methods, very broadly speaking methods for determining the total compression and methods for determining the time rate of compression. We had also seen 1 or 2 examples. We saw in the last lecture, the lecture that just preceded, how to compute the coefficient of consolidation from an oedometer test.

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Let us take a quick review of this so that we can put in perspective, the correct perspective of what is going to follow in today's lecture. Let us see the next slide, so as I said this is what we had a look at in the last lecture. That is we saw what isochrones were, what is the variation of excess hydrostatic pressure with respect to z and t in non dimensional terms and we saw that there are 2 values of capital T which are of prime importance (Refer Slide Time: 02:53). We saw what is an average degree of consolidation and how it varies with time (Refer Slide Time: 03:00).

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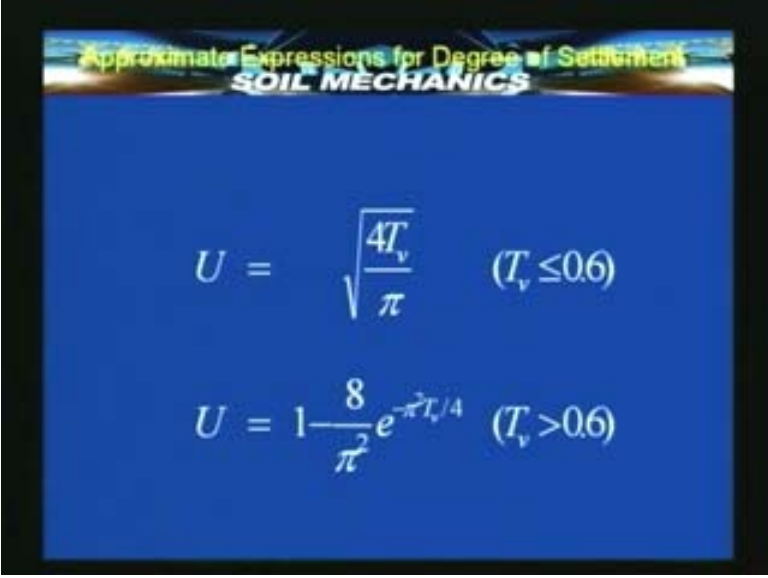
We saw this look up table, a very useful look up table where we have discrete values rather than a curve of T_v corresponding to u . And as I mentioned given you we will be able to calculate capital T and therefore small t in a given instance. That is given or rather for a stipulated value of degree of consolidation, we can find out what will be the probable time that will be taken or given a certain duration or time of performance of a building, we can find out over that period of time how the consolidation process would have improved or progressed.

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SOIL MECHANICS

U	T_v
0.1	0.008
0.2	0.031
0.3	0.071
0.4	0.126
0.5	0.197
0.6	0.287
0.7	0.403
0.8	0.567
0.9	0.848

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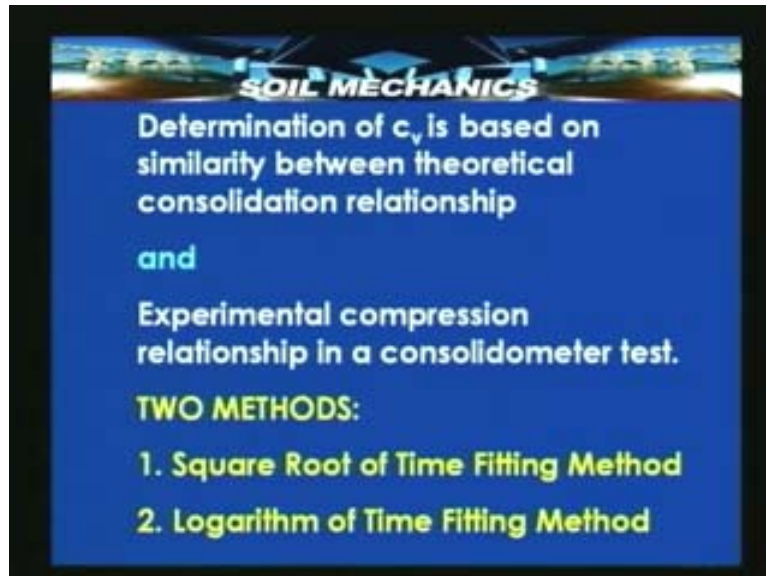


Approximate Expressions for Degree of Settlement
SOIL MECHANICS

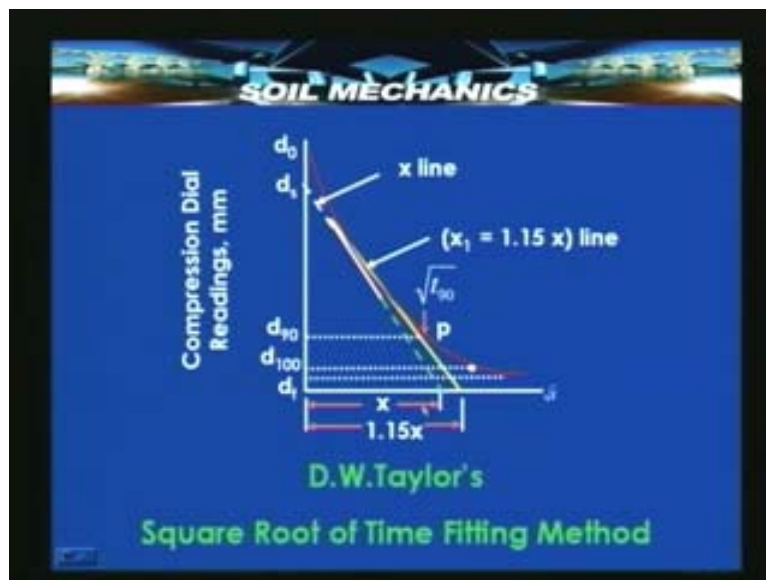
$$U = \sqrt{\frac{4T_v}{\pi}} \quad (T_v \leq 0.6)$$
$$U = 1 - \frac{8}{\pi^2} e^{-\pi^2 T_v / 4} \quad (T_v > 0.6)$$

There were 2 other expressions convenient for computing capital U rather than complicated expression that we had derived from one dimensional compression theory, we can use this expressions although they are not rigorously valid (Refer Slide Time: 04:02). Then we found that C_v can be determined from the oedometer test results either by the square root of the time fitting method or by the logarithm of the time fitting method both are basically curve fitting techniques. The first square root of time fitting method we use the condition that the straight line here starting from the zero point, zero consolidation point and going tangentially to the curve of dial gauge reading versus root of time.

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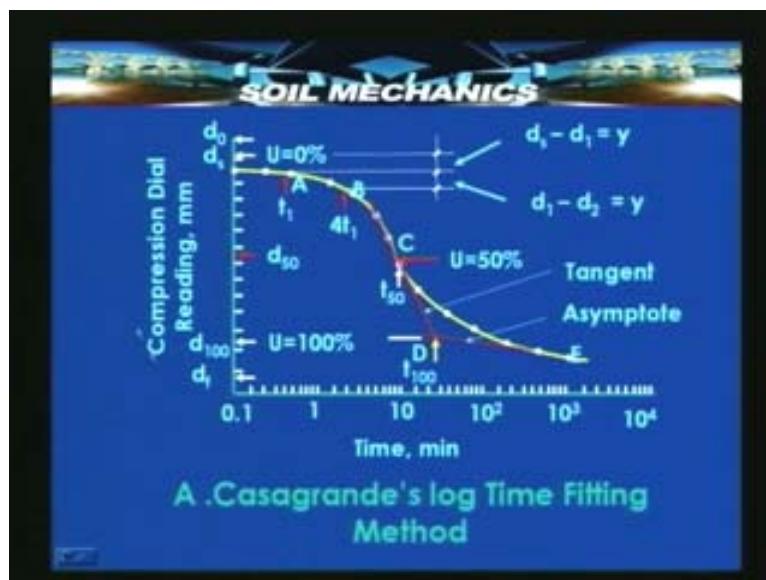


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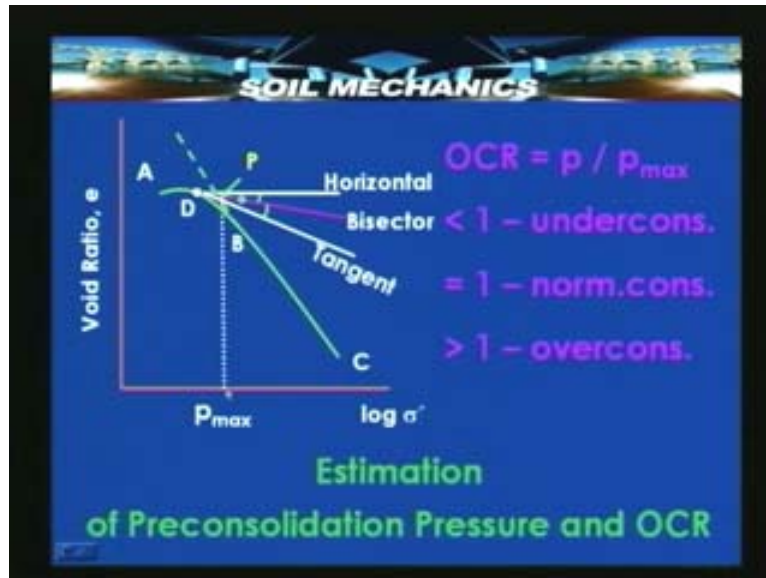


We will have certain ordinates and if we draw another line which we said tangent to this red line which will have may not be necessarily be a tangent. If we draw another line which has got ordinates equal to 1.15 times x then this point here where it intersects this line that corresponds to d_{90} and root of t_{90} from which we can now calculate the value of C_v by knowing that capital T_{90} equal to 0.848 (Refer Slide Time: 05:22). Similarly from this logarithm of the time fitting curve, we first determine d_s the starting point that is u equal to zero point by taking advantage of the fact that this curve is a parabola. So we set out with 2 points A and B, a distance y another distance y so that we can locate this point corresponding to d_s . Once we know d_s that is $u = 0$ and $u = 100$ the mid point gives you d_{50} and therefore t_{50} and therefore C_v on the basis of capital T_{50} which is 0.197 (Refer Slide Time: 06:02).

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We also saw another phenomenon known as pre consolidation phenomenon that is some soils which have undergone compression under larger loads than are existing today, will show the phenomenon of over consolidation. That is they will rather follow the recompression and curve beyond that recompression and this point corresponding to the maximum pressure to which a soil has been subjected to in the past can be obtained by a graphical procedure which involves construction of this tangent through the point of maximum curvature, drawing of a horizontal line then a bisector, then extending this straight line portion BC to meet the bisector at P, the pressure corresponding to P gives you p_{max} .

Now in this lecture we will continue to see some more examples then we will pass on to another important phenomenon known as the secondary consolidation and then lastly radial consolidation. Let us see a few examples. If you remember we had seen one example, the example number 4, it was in the previous lecture. For convenience I repeat the statement of that example again. Let us see. A 6 meter thick clay layer is subjected to a load of 60 kilo Newton per meter square. One year after loading average consolidation is 50% complete. So in this problem when we saw this earlier in the previous lecture based on the same data, we computed C_v and we also computed the settlement at the end of one year. You probably remember that we calculated the total settlement based on m_v and since we know that at the end of one year 50% consolidation is completed, we calculated settlement at the end of one year as 50% of the total settlement.

Now in this problem as a continuation, we shall see some interesting calculation. What we now see is how to compute the rate at which water is flowing out of this soil at the end of one year. We know that this phenomenon of consolidation goes on taking place at a slower and slower rate. Initially it takes place at a very faster rate; initially water is expelled out at a very faster rate but that rate goes on decreasing as more and more water goes out and as less and less water is remaining to be expelled.

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EXAMPLE 5


Consider Example 4 again.

A 6m thick clay layer is subjected to a load of 60 kN/m^2 . One year after loading, average consolidation is 50 % complete.

In addition to c_v and s @ 1 year, determine rate of outflow of water per unit area from the surface.

And since the water also getting dissipated, the excess hydro static pressure is dissipated water starts moving out relatively slowly and with time the rate at which water flows out goes on decreasing. So it will be of interest to know at the end of one year when half the consolidation is completed in this case, at what rate is water still flowing. So the question is determine the rate of out flow of water per unit area from the surface. Because this clay layer is a large huge layer, we are more interested in an idea of the flow. So we like to calculate the flow per unit area. So the solution is we will start with what we already know, we have already solved this problem and so we already know that the settlement corresponding to 50% consolidation and one year is 0.3045 meters, this we had computed earlier.

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EXAMPLE 5

SOLUTION

From solution to Example 4

We already know: $s (U=0.5) = 0.3045 \text{ m}$

It is known that for $U < 60\%$,

$$T_v = (\pi / 4) U^2 \text{ \& hence } t = a s^2$$

Now we know that this is less than 60% and therefore we can use this formula T_v equal to π by $4U$ square and we can incidentally compare this with an expression involving small t and we can see that since capital T is a constant into degree of consolidation square that is nothing but this parabola. Then t can also be expressed as constant a into the amount of settlement that has taken place at this t , up to this point of time. So T_v equal to π by $4U$ square can be taken to be similar or identical to $t = a s$ square. All that remains to be seen is what is a ? If we know a , then we know that this will strictly or exactly correspond to this expression. So let us see, since a is nothing but t upon s square. At least one value of a can be determined from the available data already. That is we know that when t is 1, we just now computed the settlement is 0.3045. So t upon s square is 10.82, this is the value of constant a which relates t with s square. Now since we have an expression for t in terms of s square through a known constant a , we can differentiate that and find out that at what rate this settlement takes place.

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SOIL MECHANICS

EXAMPLE 5

SOLUTION

Hence:

$$a = t / s^2 = 1 / (0.3045)^2 = 10.82$$

$$(ds / dt) = 1 / (2 \times 10.82 \text{ s})$$

$$@ t = 1 \text{ yr} : (ds/dt) = 0.152 \text{ m / yr.}$$

For two-way drainage

$$\text{Rate of outflow} = 0.076 \text{ m}^3 / \text{year/m}^2.$$


As we know the rate at which this settlement is taking place will be related to the rate at which the water is flowing out. So when you differentiate ds by dt , it will be nothing but

1 upon 2 as and since t is equal to one year, at the end of t equal to one year ds by dt will be 1 divided by 2 into 10.8 into this settlement of 0.3045 which will come here. So the rate is 0.152 meter per year. At this rate, at the end of one year the water is getting expelled. Now if we assume that water is flowing in two directions that is towards the top and towards the bottom that is the so called two way drainage system where the clay layer is sandwiched between two pervious layers. Since this is the rate at which the settlement is taking place, this should be resulting because of half the water flowing out through one surface and half the water flowing out through the other surface. So the rate at which water will flow out will be half the rate at which this settlement is taking place, so it is 0.076 meter cube per year per unit area. This is the answer and this rate indicates how the compression is going to proceed with time in the subsequent period.

If we had known the rate at which water is flowing out or the settlement is taking place at the beginning of the phenomenon and if we compare now this with that, we would know how the settlement process has slowed down. Whether it has slowed down considerably or not and that will give us an idea as to whether it is progressing satisfactorily or not. If it is progressing very slowly compared to what we had computed perhaps, there is some practical problem associated with this and a situation that needs attention. Now we will take one more example. This example statement is necessarily rather long, so I will read it out slowly, let us take a look at this slide. There is a soil profile and it consists of a clay layer. For convenience I have said that it is sandwiched between two layers of sand that means double drainage situation exists.

Here the water table is at the top of the upper layer, so the entire system is saturated. A large tank is now being build on top of the layer the sand layer and it amounted to an application of a uniform pressure increment of 200 kilo Newton per meter square on the clay layer. And it's a rapid application, now at the end of one year the excess pore pressure measured with the help of some instruments was distributed as shown in a figure which will follow in the next slide. So what's asked is estimate the time required for 90% consolidation.

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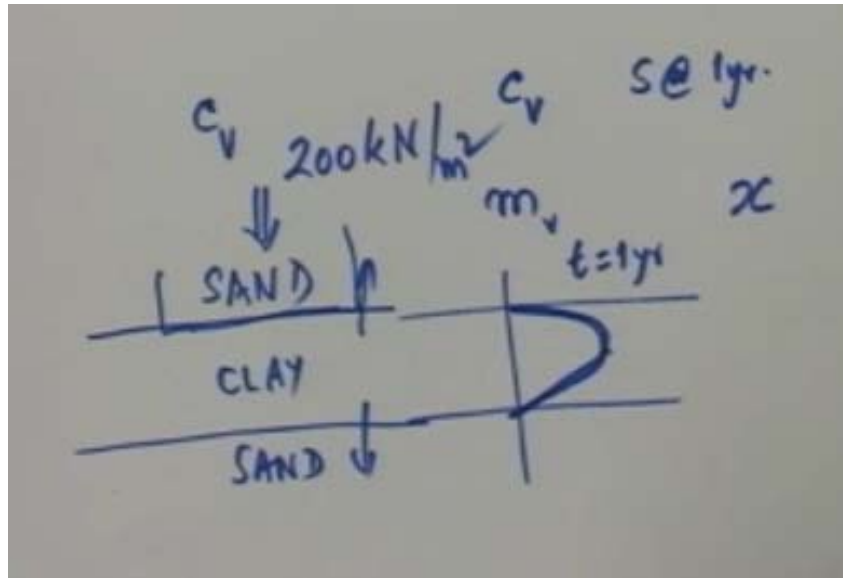
EXAMPLE 6

A soil profile consists of a clay layer sandwiched between two layers of sand, the water table being at the top of the upper sand layer. Construction of a large tank on the top layer amounted to the rapid application of a uniform pressure increment of 200 kN/m^2 on the clay layer. At the end of one year the excess pore pressure measured was distributed as shown in the figure below. Estimate the time required for 90% consolidation.

So this problem means that there is a clay layer bound between 2 sand layers and water is therefore expelled in both directions and on this there is a tank being constructed which is responsible for applying a uniform pressure intensity of 200 kilo Newton per meter square. And under this pressure excess pore pressure gets generated and what is given in the figure that is going to follow is how this excess pore pressure is varying with depth at t equal to one year. At the end of one year what is the variation of excess pore pressure?

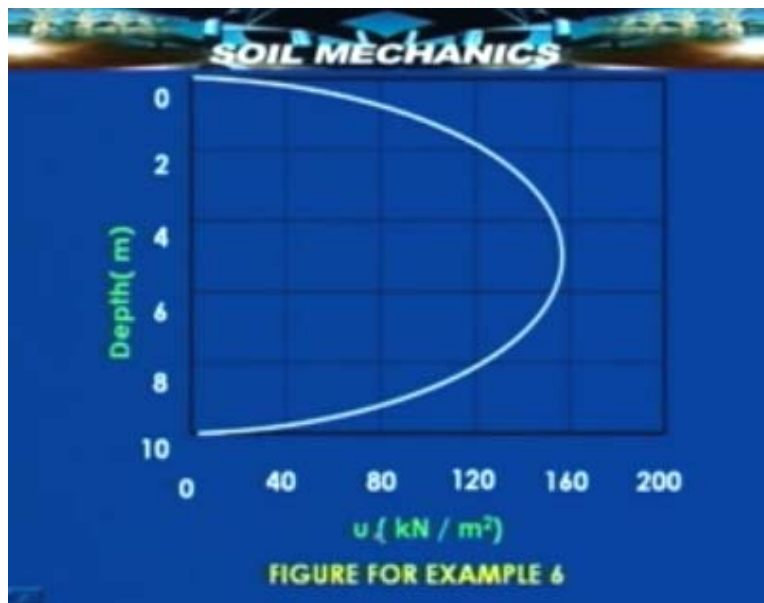
So from this graph we will know how much pressure is still remaining, how much pressure has been dissipated and from that we will be able to have an idea as to how much time has taken for what amount of consolidation. That is suppose x amount of consolidation has taken place in t equal to one year then we will know that required a time of one year. Then we can find out for 90% what will be the time required because we will know at this point of time how much pore pressure is still remaining to be dissipated.

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So let us take a look at the pore pressure distribution. This is a curve showing the pore pressure distribution. This is depth, this is the excess hydrostatic pressure and this is a curve of distribution.

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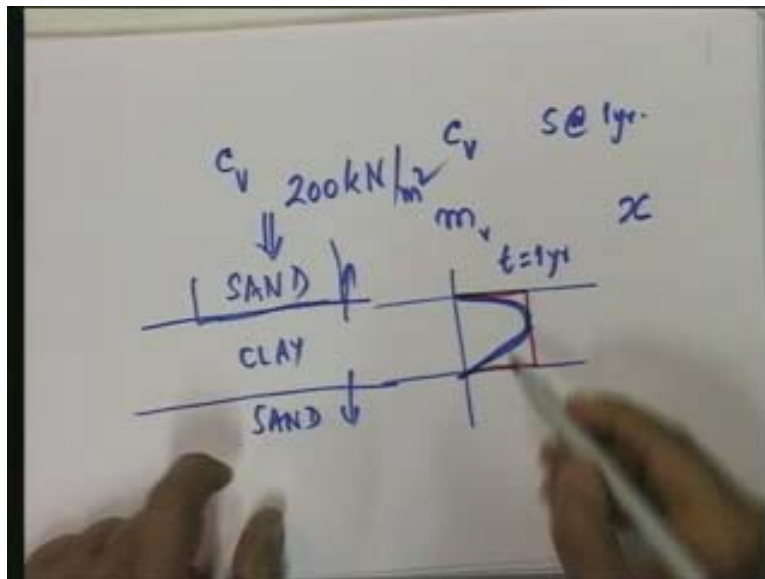


So what we see here from left to right, up to this curve this values. These are the values which represent the remaining excess hydrostatic pressure. We know that in our theory of one dimensional consolidation for solving the problem of one dimensional consolidation we had assumed at $t = 0$, $u = u_i$ uniform pressure distribution with depth. It's not really an assumption it is quite valid at the instant $t = 0$ as soon as we apply the load since the solids are incompressible, the red water takes over. It does take the load and that means the excess hydrostatic pressure u is equal to a value u_i and it is same through out the depth.

However since the ends of the clay layer are connected to the atmosphere, there the gradient is close to the infinity and the pore pressure gets dissipated instantaneously. So now the pore pressure that is dissipated so far is on the right side of this curve because this total area must represent the area corresponding to u equal to u_i this should correspond to u equal to u_i . This is the value of u that is existing now and whatever value is here up to this between u_i and this, that represents the value of excess hydrostatic pressure that has already been dissipated.

So what is the solution? So solution is we have to find out time taken for 90% consolidation that is what is t_{90} ? That's what we need to find out. It can be very comfortably done in this specific instance by simply studying, if you take a look at the picture again. What is the area occupied by the initial excess hydrostatic pressure curve because the initial excess hydrostatic pressure curve though it is not shown here would have been like this. This would have been the initial excess hydrostatic pressure whereas now the hydrostatic pressure has reduced to this blue line that means between these two there is an area which represents the area corresponding to pressure that has been dissipated. In other words the area under the curve is indicative of the excess pore pressure that is either being remaining or has been dissipated.

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SOIL MECHANICS

EXAMPLE 6

SOLUTION

$t_{90} = T_{90} H^2 / c_v = ?$

By considering the area under
excess pore pressure distribution
diagram by counting squares,

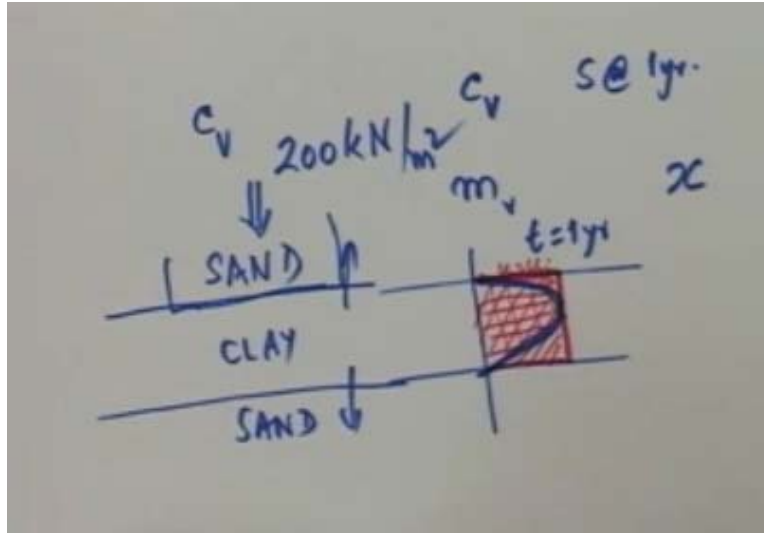
@ $t = 1$ year, $U = 11 / 25 = 0.44$

Since $U < 50\%$, $T = \pi U^2 / 4 = 0.152$

So let us see by considering the area under the excess pore pressure distribution diagram in terms of the number of squares into which we have divided the diagram. We can find out at t equal to one year to which this figure corresponds, what part of this area it is under this curve, out of the total area? That is what is the area occupied by this curve as a fraction of the total area occupied by the u equal to the u_i curve.

That will indicate the degree of consolidation actually the area which is between the 2 lines these red line and the blue line is the one which represents the degree of consolidation in terms of the pore pressure that has been dissipated or in terms of the area lying between the 2 curves. So if you count the squares, the small squares between the 2 curves then that will give us the degree of consolidation that has already taken place and that is number of squares in this previous curve.

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If you count this number of square you will find that this is very close to 11 and that means the number of squares 11 divide by the total 25 will tell you that 0.44 is the degree of consolidation that exists at the end of one year. Now since this is less than 50% we can use the approximate formula connecting T and U that is T equal to π by 4 U square. And from this we can calculate capital T as 0.152. So what does this mean? This means that at the end of T equal to one year, from the pressure distribution diagram we find that the area of the portion where pressure has been dissipated is 0.44 which means that is the degree of consolidation. And corresponding to that the capital value is 0.152. Now that means we know a value of capital T, we know that corresponding small t is 1.

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SOIL MECHANICS

EXAMPLE 6

SOLUTION

$t_{90} = T_{90} H^2 / c_v = ?$

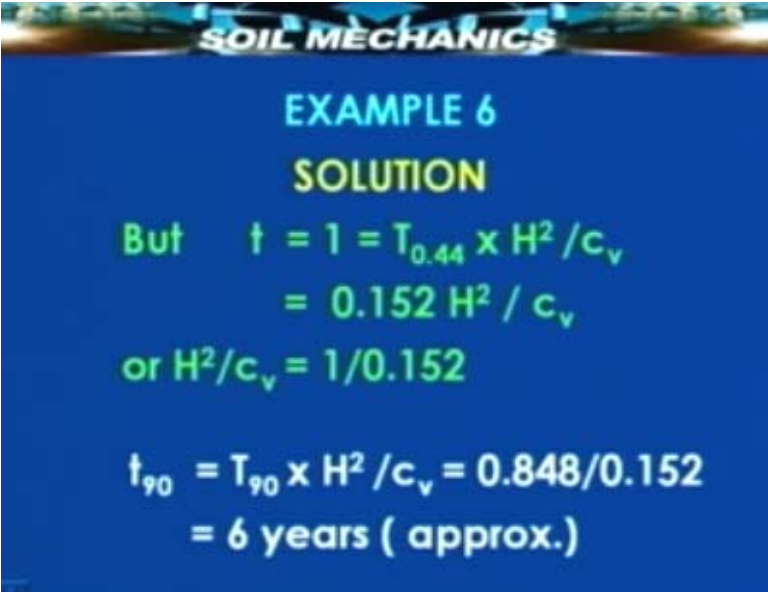
By considering the area under
excess pore pressure distribution
diagram by counting squares,
@ $t = 1$ year, $U = 11 / 25 = 0.44$
Since $U < 50\%$, $T = \pi U^2 / 4 = 0.152$

We know that the system is a double draining system and the thickness of the clay layer if we know then we can calculate C_v . What we actually need is the time corresponding to

90% consolidation and what we have is capital T corresponding to 44% consolidation. So what we will do is we will apply this expression in 2 stages. First we will apply this expression for capital T = 0.152 and small t equal to one year and find out what is H square upon C_v? And then we shall use this value of H square upon C_v and 0.848 in place of T₉₀ to find out the time taken small t for 90% consolidation.

So the solution is very simple, at t = 1 it must be equal to capital T into H square by C_v. So that is 0.152 H square by C_v or H square by C_v is 1 upon 0.152, substitute that in this and get the value of small t₉₀ and it works out to 6 years approximately. That means this particular structure is going to under go consolidation for 6 long years. The 90% of the consolidation is going to occur in 6 years of time period. So it's a very interesting way of calculating the amount of consolidation that has taken place, the amount of consolidation that is remaining and the corresponding time and whether compared to the life span of the building what this value is. If now what it shows is for 90% consolidation, time taken is 6 years where as for 44% of the consolidation time taken was only one year.

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SOIL MECHANICS

EXAMPLE 6

SOLUTION

But $t = 1 = T_{0.44} \times H^2 / c_v$
 $= 0.152 H^2 / c_v$
 or $H^2 / c_v = 1 / 0.152$

$t_{90} = T_{90} \times H^2 / c_v = 0.848 / 0.152$
 $= 6 \text{ years (approx.)}$

So after one year the consolidation has been progressing rather slowly gradually and at this rate it might take quite a bit of time may be 15 years, may be 12 years may be 10 years. For the total consolidation to be completed and in practical terms and that means if the life span of the building is about 40 years or 50 years almost 20% of the life time of the building is going to be the period over which the building or the structure is going to experience the effect of consolidation. After that when the consolidation process comes to an end, the building will remain more or less free from the effect of consolidation.

Now we come to the second aspect of this lecture, that is secondary consolidation. The consolidation that we have been discussing in the past 6 lectures is known as primary consolidation. The reason why it is called the primary consolidation is that is the

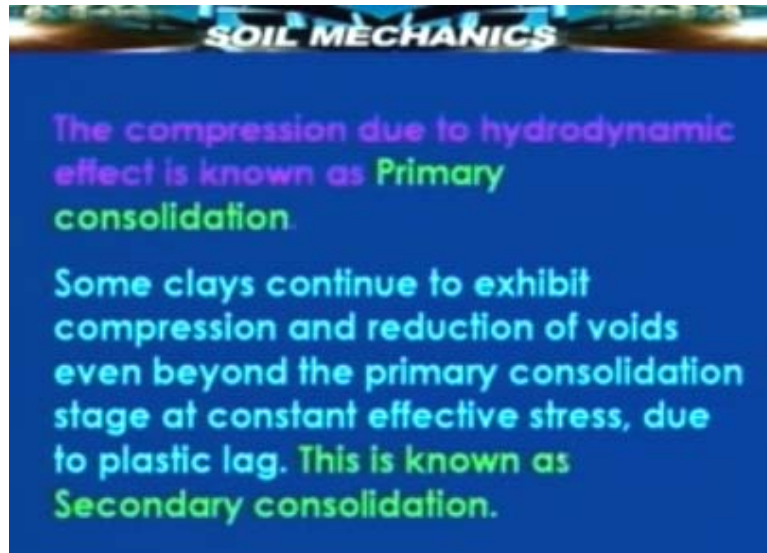
consolidation that primarily takes place when a load is applied to a saturated clay layer and we have seen all aspects of this, load is applied gradually, water flows out gradually total compression takes place theoretically in infinite time. The rate of compression can be computed using the theory of one dimensional consolidation and the rate of compression goes on changing with time usually the rate decreases with time. It is possible to evaluate how much time 90% consolidation is going to take place.

Now all these consolidation that we have been talking about is purely due to the so called hydro dynamic lag. That is water is going out slowly there is a lag between the application of the load and the movement of the water and the corresponding settlement and therefore that is known as hydro dynamic lag and this compression that is taking place primarily due to this phenomenon is known as primary consolidation. Now this primary consolidation gets over for any given load increment, primary consolidation gets over for all practical purposes, let us say in a few days then starts a secondary phenomenon.

The secondary phenomenon is a slightly different phenomenon, it is known as the plastic lag phenomenon. Here a different kind of activity takes place that is the particles have now adjusted themselves to the extent possible due to the movement of the water. The stress has already been transferred to them. The entire stress applied on to the clay layer has now been transferred to the solids in terms of effective stress, at the end of primary consolidation and now it is observed that this solids still undergo gradual compression. And this gradual compression due to the movement of the particles, further movement of the particles as a function of time leads to compression for continuing compression of the voids and that's known as secondary consolidation.

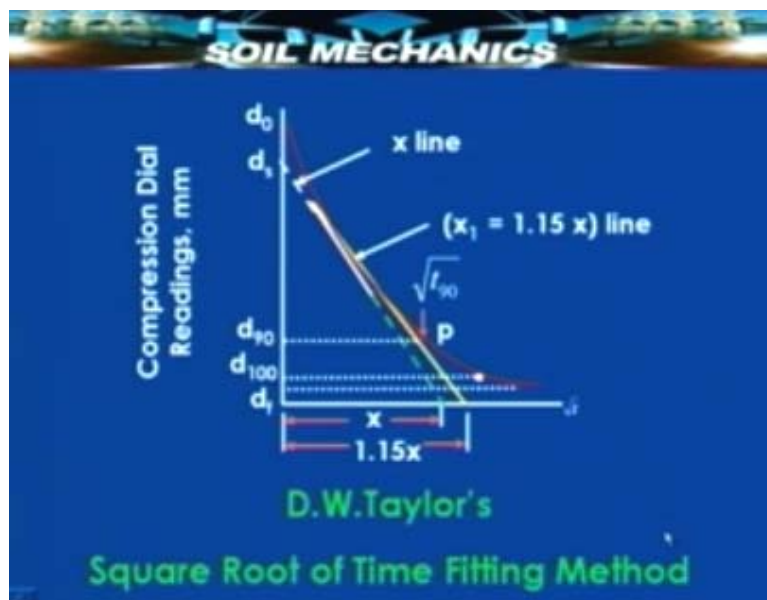
That is after the primary consolidation is over; the particles continue to still move gradually under constant stress to find final resting places for themselves and during this period the voids continue to compress and that is secondary consolidation. And this is known as the plastic lag because the phenomenon concerning the movement of these particles under constant effective stress is a plasticity phenomenon. So the compression that we had discussed in very great detail all this time is known as the primary consolidation and it is due to hydro dynamic lag and the compression that takes place beyond the primary at constant effective stress due to plastic lag known is the secondary consolidation.

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So you can see here that there are a number of dial gauge readings shown. This is the same graph which we have used earlier calculating C_v or studying the method for calculating C_v . Now you see here, the dial gauge reading versus root of time graph is starting at d_0 . This is that dial gauge reading corresponding to application of load but on application of load there is a compression of gases that takes place and the load gets seated properly.

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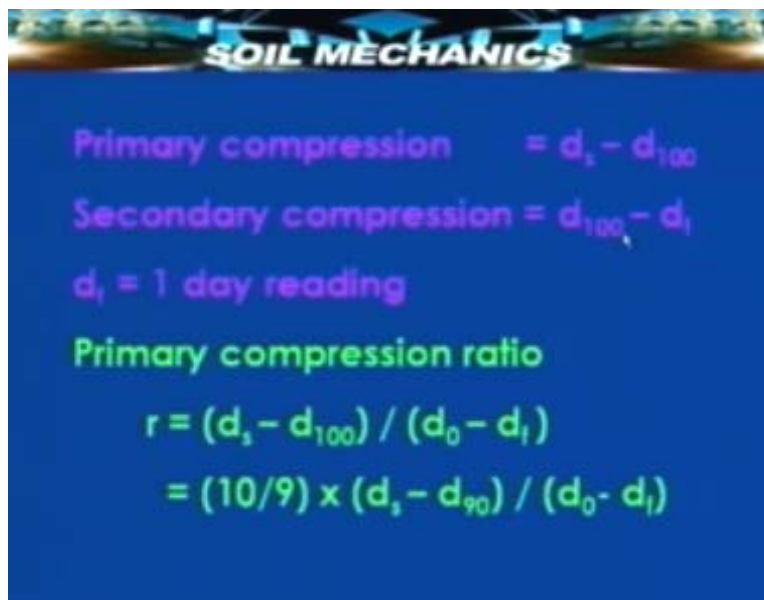
The dial gauge reading corresponding to that state is d_s and now starts the compression arising out of the hydrodynamic lag and it goes on until we reach the d_{90} point. If we allow this phenomenon to continue we will end up with after a few days say some times 24 hours some times 48 hours, we will find that the dial gauge readings reach a very

steady value and that can be for all practical purposes taken as corresponding to 100% point of primary consolidation, d_{100} point corresponding to primary consolidation may be taken as this point.

Now at this stage if we continue to watch the dial gauge readings, we will find that it doesn't stop here. It goes on increasing still with time because of the plastic lag phenomenon at constant stress and it reaches a real final value d_f after considerable amount of time. And it is this portion d_{100} to d_f which corresponds to secondary compression or secondary consolidation. This phenomenon of secondary consolidation starts at the end of the primary consolidation and as can be seen in this figure d_f corresponds to the point of secondary consolidation completion. So starting from d_s which corresponds to 0% consolidation we have 90% taking place up to this, the remaining 10% taking place up to this and then the plastic lag taking over and the final compression ending up here.

It is possible to estimate the total compression starting all the way from d_s to d_f and then express this primary consolidation as a proportion of this total consolidation for any given load increment. We assuming that all these dial gauge readings and versus time which is plotted here in this graph is for some load increment and therefore corresponding to that load increment how much is the primary consolidation, what part of the total consolidation is the primary consolidation that can be computed in terms of the dial gauge readings and the change in void ratios.

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SOIL MECHANICS

Primary compression $= d_s - d_{100}$

Secondary compression $= d_{100} - d_f$

$d_f \approx 1$ day reading

Primary compression ratio

$$r = (d_s - d_{100}) / (d_s - d_f)$$

$$= (10/9) \times (d_s - d_{90}) / (d_s - d_f)$$

So the dial reading is d_s to d_{100} that represents the primary consolidation where as d_{100} to d_f represents the secondary compression and if d_f usually is taken in after about a day after this 100% consolidation can be considered to be over. That is after the dial gauge readings have become practically stable, constant we take that has the point

corresponding to d_{100} and then wait for about 24 hours and what ever marginal changes that might have occurred during that period would lead to d_f . So based on this if we simply take the ratio of d_s minus d_{100} to d_0 minus d_f where d_0 is the point of application of the load and d_f is the point of transfer of the load completely to the solid grains including the portion corresponding to plastic adjustment, this is known as the primary compression ratio.

Now if we are using a method like this, square root of time method in which we get d_{90} rather than d_{100} precisely because that method helps us to calculate d_{90} rather than d_{100} where as the logarithm of time methods gives us d_{100} . This d_{90} once we know then d_s minus d_{90} multiplied by 10 upon 9 will give us the 100% primary consolidation and that divided by again $d_0 - d_f$ would give you the primary compression ratio. The secondary consolidation phenomenon also can be mathematically expressed; it's usually a logarithmic phenomenon because it's a plastic deformation that takes place in the solid grains after the application of the load and at constant load and generally therefore it is expressed in terms of a logarithmic equation. Now we pass on to 3 dimensional consolidation.

It is strange because actually consolidation is a three dimensional process but we started with the simpler version that is one dimensional consolidation by making simplifying assumptions and idealizations because that's what is generally necessary in practice under very large loaded areas. But there are instances where three dimensional consolidation becomes very relevant, very important.

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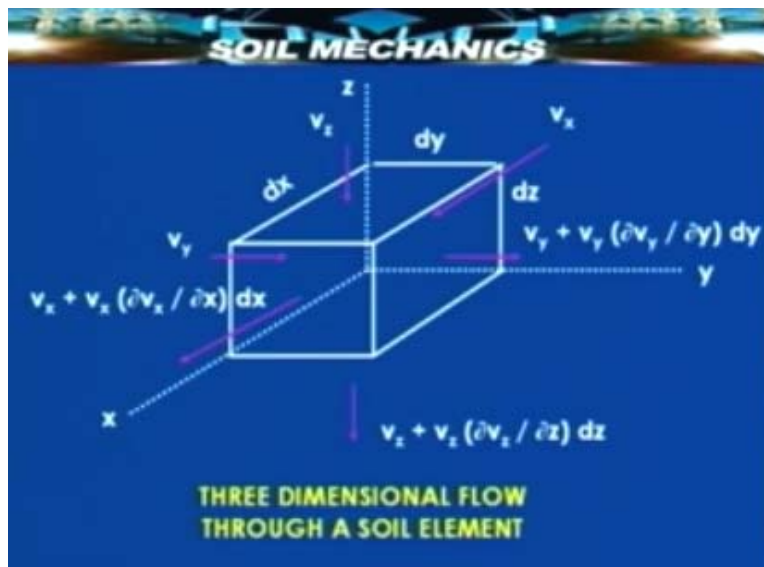
There is a method by which we can accelerate this consolidation phenomenon. After all on what does this consolidation phenomenon depend? The consolidation phenomenon its success its progress, the rate at which it progresses all these things depend upon the rate at which water is flowing out that is drainage. So if there is some way to improve the drainage in any given location, we can improve the consolidation process. We can

accelerate the consolidation process and there its convenient to use a technique known as sand drains and we will be seeing some description of that shortly in today's lecture and more details in the next lecture.

This technique as you will be seeing leads to a phenomenon where compression or consolidation due to flow of water takes place not only in the vertical direction but also because of the radial flow of water. We provide drains inside the soil which will serve as place points towards which the water can flow and that will induce in a movement of water into the drains in the horizontal direction as well and the resulting consolidation how ever compression how ever will be vertical. And in a way this becomes a three dimensional phenomenon and it needs to be analyzed very differently from the way the primary consolidation is analyzed and understood.

Let us see the next slide you will find here a typical soil element once again and I have shown here the flow of water in all 3 directions. If I take x direction V_x is the flow of water per unit time over this area $dy\ dz$, quantity of flow per unit time because velocity represents quantity of flow per unit time per unit area. And this is the outflow per unit time per unit area in the x direction. This kind of expression, this way of expressing the in flow and out flow we had used also while studying the one dimensional flow. All that we have done now is to extent that to all 3 dimensions. So if in the x direction, if V_x is the in flow V_x plus V_x into rate of change of V_x in the x direction into the distance V_x over which the change is taking place this distance, that becomes the out flow.

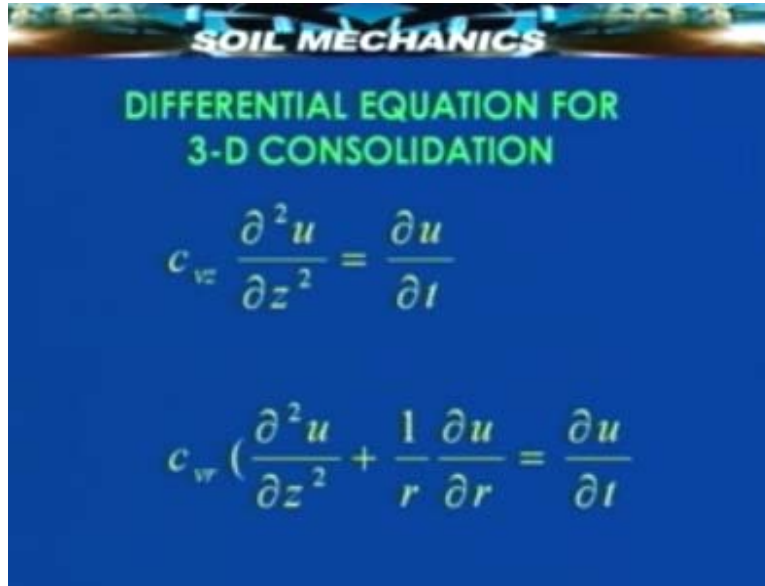
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So now you should take y direction, V_y is the in flow V_y plus this into dy becomes the outflow and for the z direction this is in flow and this is the outflow (Refer Slide Time: 37:33). So now the flow equation very similar to the one dimensional consolidation flow will lead to once again an expression in which there is a volumetric compression and the rate at which that volumetric compression takes place will be or can be related to the total

quantity of out flow and the rate at which the total out flow takes place. It is possible to extend the equation that we had used earlier to the three dimensional system and as a result of that we can once again get differential equations of consolidation.

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SOIL MECHANICS

**DIFFERENTIAL EQUATION FOR
3-D CONSOLIDATION**

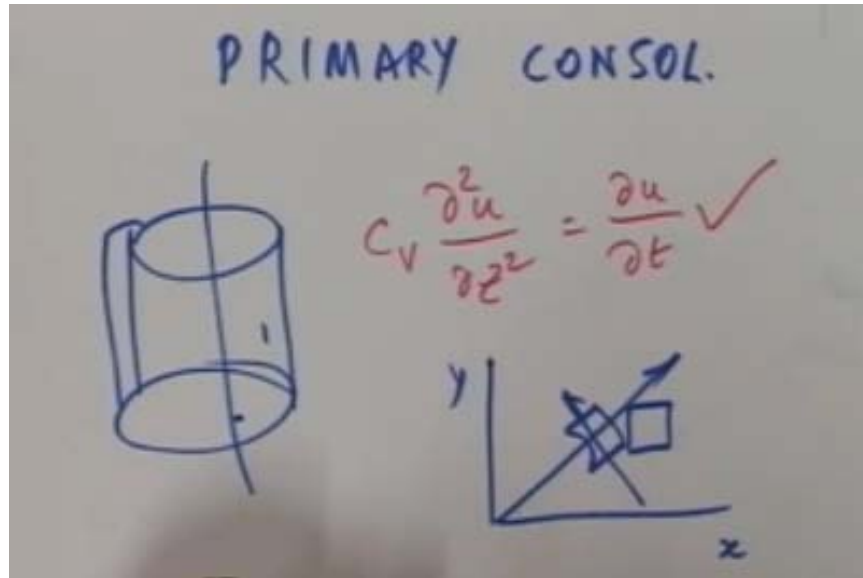
$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

$$c_v \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t}$$

We had derived, in great detail we had seen in fact the details of the derivation of the consolidation equation $C_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$. This is the so called consolidation equation or the diffusion equation for one dimensional phenomenon and we solved this using the theory of differential equations and got expressions for excess hydrostatic pressure. Now a similar of state of equations can be derived and this can be solved using appropriate boundary conditions and we can arrive at similar differential equations of motion and corresponding solutions in which average degree of consolidation will be related to the time factor t .

The differential equations for consolidation how ever will be now not only for vertical flow but also for the other directions of flow. Suppose we use polar coordinates rather than Cartesian coordinates then instead of x y and z directions we will have a typical element here like this suppose, I draw it in real polar terms rather than x y element. We will be having an element like this in which there will be flow in this direction and flow in this direction.

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So if I add to this the z dimension then it becomes a truly three dimensional phenomenon. What this means is our three dimensional problem in terms of polar coordinates can be treated as a vertical compression problem corresponding to the same z coordinate as in the Cartesian system plus a consolidation which is now a function of the radial direction which is a function of the radial coordinate r .

So let's take a look at the corresponding differential equations that we will obtain, if we follow the same procedure that we used for deriving this equation for primary consolidation. Following the same steps we will get 2 equations for three dimensional consolidation and they will be one, this we will find that this is in no way different from the equation that we had derived for one dimensional compression. All that the difference that we had made is we have added a notation z here to indicate that this is vertical direction C_v for vertical or z direction.

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SOIL MECHANICS

**DIFFERENTIAL EQUATION FOR
3-D CONSOLIDATION**

$$c_{vz} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

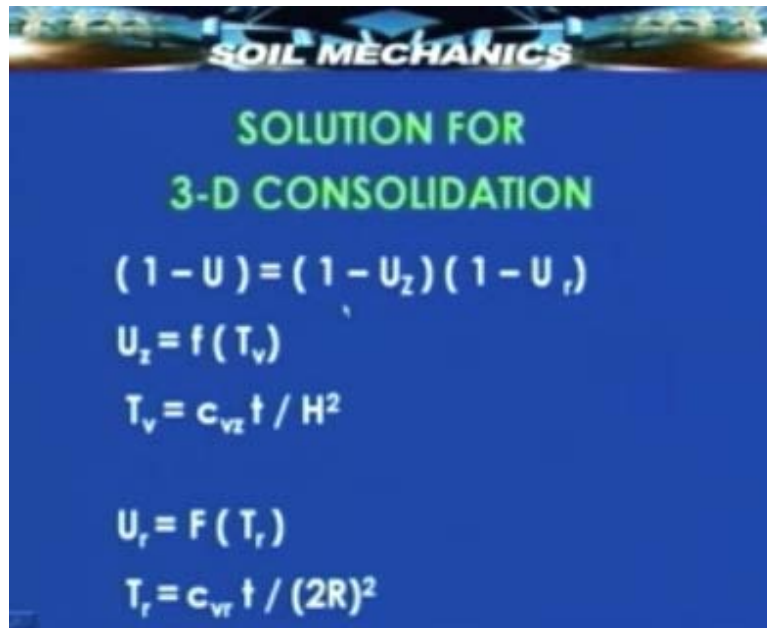
$$c_{vr} \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t}$$

In order to distinguish this from the value of C_v that we will get when for the differential equation involving the compression in the radial direction. So we have 2 C_v values and 2 equations these are the once which we need to be now solved in order to get the results of degree of consolidation and its relation to the time factor capital T. Next if we see this slide, we will come across in fact what we are seeing here is a method to solve the equation in order to get the values of u , u_z and u_r . I will not be going to the details of the derivation of the relationships between u , u_z , u_r . Suffice it to say that if I have a relationship between u_z and t_z or t_v and if I have a relationship between u_r and t_r , I should be in a position to get the average degree of consolidation for these total phenomenon of three dimensional compression both vertical and radial.

This has been done using again the product of variables concept that is 1- the total degree of consolidation is taken as 1- the degree of consolidation in the vertical direction multiplied by 1- the degree of consolidation in the radial direction. Now u_z is a function of t_v and we already know that it is C_{vT} by H square. If we can determine C_{vz} then there is no difficulty at all in getting capital T_v . On the other hand the degree of consolidation in the radial direction which is a function of t_r that is time factor for the radial process can be expressed to yield T_r as $C_{vr} t$ divided by a factor $2R$ square which corresponds to H square of the vertical case.

In order to have similarity, the time factor for vertical case and the time factor for radial case are expressed by similar equations. The term $2R$ in the radial case corresponds to the term H that is drainage path. So we can find out what the meaning of this radius capital R is, once we come to the phenomenon of sand drains and their application to accelerate consolidation.

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Right now as a prelude to that we are simply seeing how three dimensional consolidation can be split up into consolidation in the vertical direction and consolidation in the radial direction and they can be put together to get time factor for the vertical direction time factor for the radial direction. Now if we know how to determine the time factor for the vertical and the radial directions, we can get the degrees of consolidation for the vertical and radial direction from which we can get the total degree of three dimensional consolidation. Let us see the next one.

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In order to see what should be this value of capital R, we have to see 2 possibilities. There are two ways in which this three dimensional consolidation can take place.

Suppose there is a certain load that is resting on the soil, this is the soil and suppose there is a loaded area and this is some load resting on soil. Then depending up on the relative flexibility of this loaded area and the soil that is compared to the soil whether this loaded area is flexible or rigid will determine how the consolidation phenomenon is going to take place, whether it is going to take place under the so called free strain condition or under the fixed or rather equal strain condition. That is if this loaded area is flexible then we know that the strain of this is going to take place freely, if on the other hand it is rigid the strain induced in the soil will all be equal under the loaded area at all points. And based on these two extremes, solutions have been evolved on the basis of which we can calculate capital R for obtaining the value of T_r and U_r for radial consolidation.

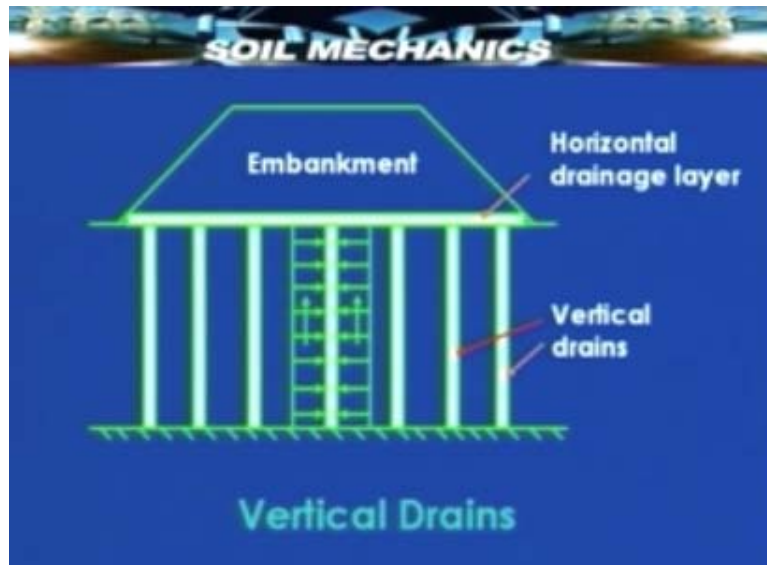
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As in the case of vertical consolidation, this R is nothing but a length. It is equivalent of the drainage path H and therefore it's also a length and this length will depend upon how this sand drains are arranged. Now assuming that we can solve this problem of three dimensional consolidation either under the so called free strain condition or the equal strain condition for which solutions have been given by various people, we can see how this can be applied to radial consolidation using sand drains.

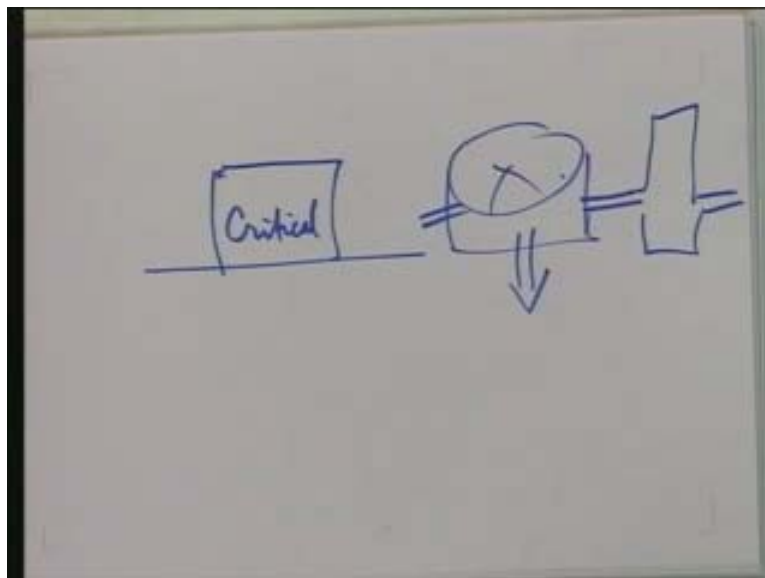
So here is a very typical set up that is used together with vertical drains in order to accelerate consolidation. Now this needs some explanation. See there are many instances in practice where we will be building rather critical structures which cannot be allowed to experience significant consolidation during their life time. Even an ordinary residential building if it undergoes serious consolidation, severe consolidation due its life time it is bound to suffer difficulties. Now more than that or more critical than these residential buildings could be, let us say buildings and factories, structures and factories, huge tanks which contain intermediate products during a chemical process, tanks in which raw material is stored may be cement, may be petrol, may be any chemical.

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These usually have pipe lines serving as inlets and outlets which are connected to these. Imagine a tank like this connected to a number of other systems through pipe lines undergoing consolidation. If this under goes consolidation during its life time and if that consolidation is a significant consolidation then imagine their effect on these pipe lines and imagine the effect of these on the adjoining structures. It doesn't need much explanation to see that this kind of consolidation. So imagine the effect of this kind of consolidation on the pipes to which they are connected and their effect on the other structures hence very easy to visualize as this tank settles if the settlement is exorbitant there could be a rupturing of these pipes.

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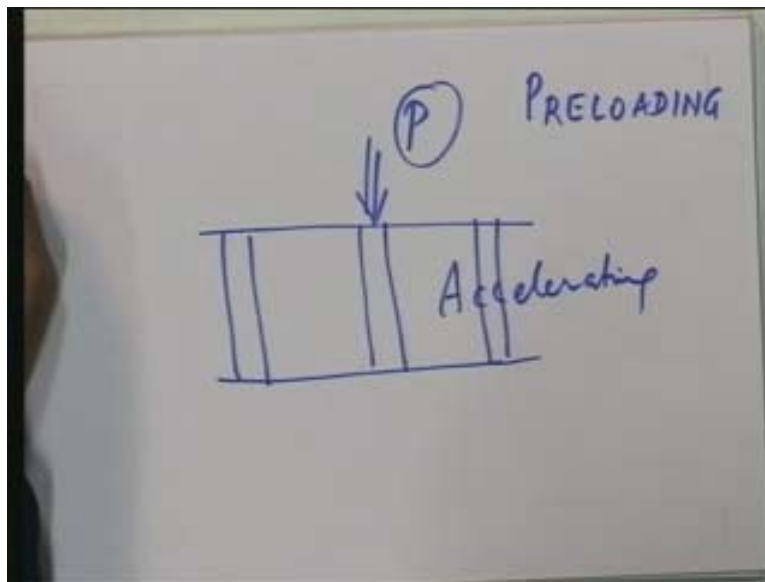


If these pipes are carrying dangerous chemicals then rupture of these pipes could lead to disasters. And therefore many critical structures it's preferable to consolidate the soil

before the structures are built by applying a load similar to what the structures are going to cause when they are constructed. Let us see suppose there is a clay layer, we want to build a structure which is going to apply a load capital P but this being a critical structure, we will rather like to consolidate this soil by applying a similar load before hand and afterwards erect the structure. The advantage is when the structure is erected if the soil below would have already undergone consolidation under a load say equivalent to what the structure is going to apply. We remember our discussion with respect to pre consolidation and over consolidation. This is precisely that we are in fact taking advantage of the knowledge of that phenomenon. We are consolidating the soil before hand then applying the load so that any further consolidation if it takes place will be minimal and will be normal consolidation.

Now this process of accelerating the consolidation by applying a load before hand is known as preloading. That is the clay layer is loaded before hand and in order to accelerate this consolidation we build what are known as sand drains which are nothing but drains or which are nothing but bore holes filled with some pervious material. We can see this diagram now (Refer Slide Time: 51:27).

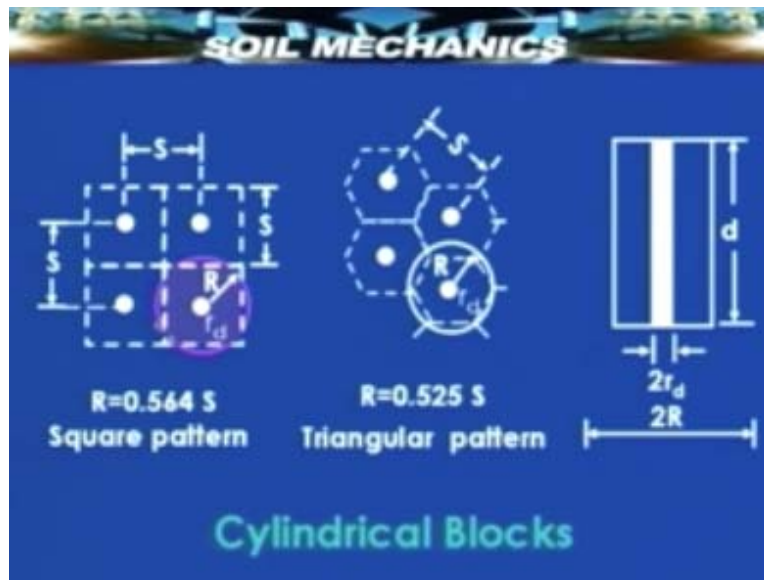
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The preloading phenomenon is usually in the form of construction of an embankment. So there is an embankment here, there is a horizontal drainage layer constructed below the embankment in order to provide easy exit for the water. And there are drains provided like this, all these vertical drains which are filled with pervious materials permit the flow of water in the radial direction towards them and there final out go through the horizontal drainage layer at the top. Now this embankment load will now be gradually transmitted to the clay layer and the clay layer will undergo consolidation. And since this embankment is not a critical structure it's a highly flexible structure and a temporary one.

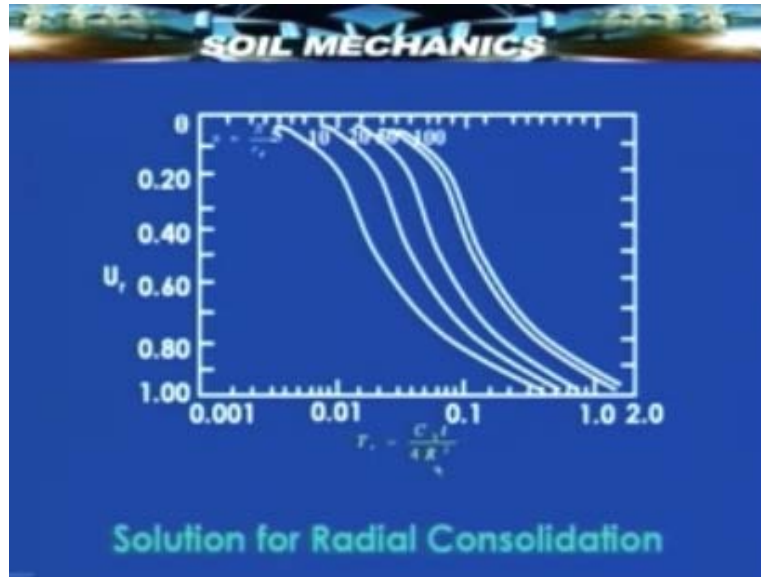
This consolidation can be permitted to take place at a rather rapid rate and that's why we afford to use these vertical drains. So with the help of these so called sand drains, we have accelerated consolidation which luckily will not affect the embankment in any way. And after the consolidation is over, we can in fact strip of these remove this embankment and put our own structure on top of these. This is very frequently used in highly compressible clays because during the life time of the structure we do not want any further compression to take place number one, number two in some clays some compressible clays the time required for a certain load increment could be very long which we cannot permit.

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Now this show two different arrangements of this sand drains. Sand drains can be arranged in plan either in a square pattern or in a triangular patter below the loaded embankment and accordingly the drainage path capital R will vary for both of them and it is this value of capital R which will go into the equation which we will be using for calculating capital T_r which we saw in a couple of slides earlier. So this capital R is related to the spacing of the sand drains and the pattern of the arrangement of the sand drains, based on this we get capital R which we can use for getting T_r and then U_r .

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So we can get T_r as a function of this R for equal strain condition and corresponding U_r using the so called the Barron's equation and this is a graph which represents this, we can also use a look up table for this purpose. So in this lecture we had seen a couple of examples then we have discussed the secondary consolidation phenomenon and we have seen the importance of radial consolidation. In the next lecture we will continue and see some more examples.

Thank you.