Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 38
Consolidation and Settlement
Lec No. 5

Hello students let us have a look at lecture number 5 on consolidation and settlement. As usual let me briefly recapitulate what we had seen in some of the previous lectures. Actually in the last lecture we made a beginning with the mathematical solution of the consolidation equation, the one dimensional consolidation phenomenon we found and it could be expressed mathematically and solved. The idea of solving is to be able to predict the amount of settlement that will take place at any given point of time and consequently to determine the over all percent of consolidation that will occur at any given point of time. When I say percent, what I mean is percentage of the total consolidation or the total settlement and we already know that on the basis of e log p curve by knowing the change in void ratio and the change in pressure, we can determine the total consolidation or for that matter the total settlement even with out knowing the rate at which the water is flowing out, the rate at which the void ratio is decreasing or the rate at which the stress transfer is taking place in the soil.

This means that the total consolidation can be computed directly even without going into the mathematical theory of one dimensional consolidation. One of the reasons is obviously because the total consolidation theoretically takes place over a time period equal to infinity and therefore at infinity what is the total consolidation? Remains the same irrespective of what the problem is and it doesn't therefore depend upon the rate. Now once we know how to determine the total consolidation and once we have a theory for determining the rate of consolidation we can say that we understood and solved the problem of consolidation completely.

Let us now take a look at what we saw in the last lecture how we introduced the theory of one dimensional consolidation. So the theory of one dimensional consolidation, we saw by means of some parameters which are used for representing the compressibility and in terms of an analogy which simulated the consolidation phenomenon in such a way that we could understand it very well. The simulation was so perfect that it was explaining practically every aspect of the time dependant nature of the consolidation phenomenon. The mathematical formulation closely follows this analogy as applied to a real field situation.

Let us see what we covered in the last lecture. We defined three compressibility parameters, the compression index, the coefficient of compressibility and the coefficient of volume decrease. When at a later date we solved some typical problems on consolidation and settlement, we will come to know the utility of each one of these coefficients. Why we need different compressibility parameters for different purposes we will be using each one of these depending upon what is the data available and depending upon what is required. So this will become evident when we have some problems at on hand to solve. What's important to notice however is that C_c is a non dimensional parameter and it is dependant upon void ratio change and the logarithm of the stress change where as a_v coefficient of compressibility depends upon the volumetric change or the void ratio change and the stress change, effective stress in particular and n_v is nothing but a_v upon the constant volume of the solids.

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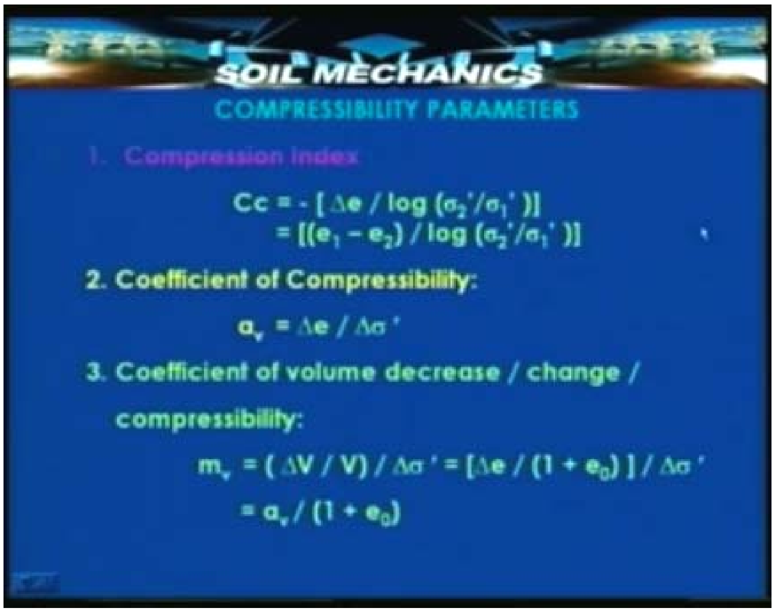

We know that if the total volume is 1+ e_0 the volume of solids is 1 upon 1+ e_0 at any point of time where e_0 is the initial void ratio and therefore a_v upon 1+ e_0 is a parameter that represents compressibility in terms of the original volume of the solids which is invariant. And therefore it serves as a convenient denominator, a parameter in the denominator for comparison of different clays or different soils with different a_v's or different degrees of compressibility.

Let us take a quick look at the analogy since we are going into the mathematical representation of this analogy we defined a cylinder with a piston and a spring with an outlet for water and we saw that the cylinder represented the soil, the spring represented the soil skeleton, water represented the pore fluid and the piston with its opening represented the pores and the valve represented the degree of opening of the voids or the permeability of the soil. Now we saw in the next stage that initially the soil is at rest, there is no so called excess hydrostatic pressure because there is no load.

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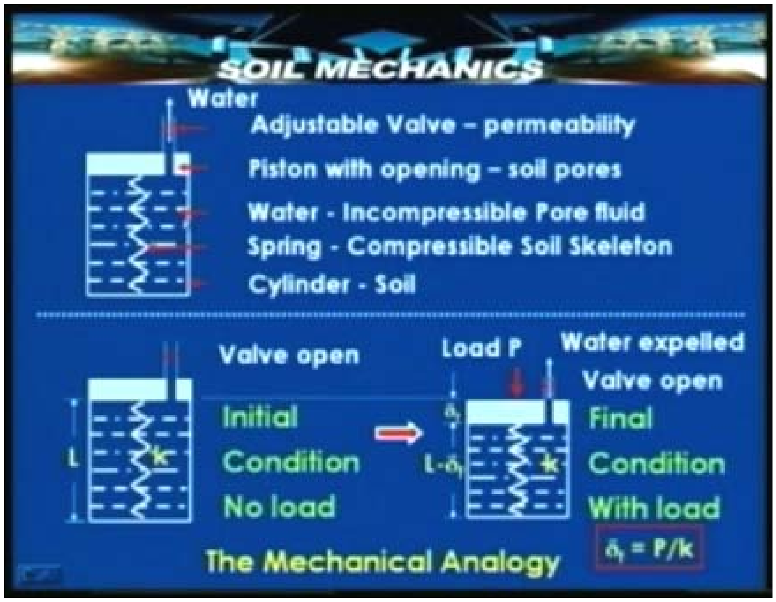

But the moment you apply a load at t =0, the water takes the load and then gradually it passes it on to solids and ultimately at the end of the consolidation phenomenon at t equal to infinity, you will find that water would have escaped to such an extent as to once again bring back hydrostatic equilibrium and the spring would have got compressed by an amount delta final which is obviously equal to the load that is now completely transferred to the spring p divided by the spring constant k where p is the applied load.

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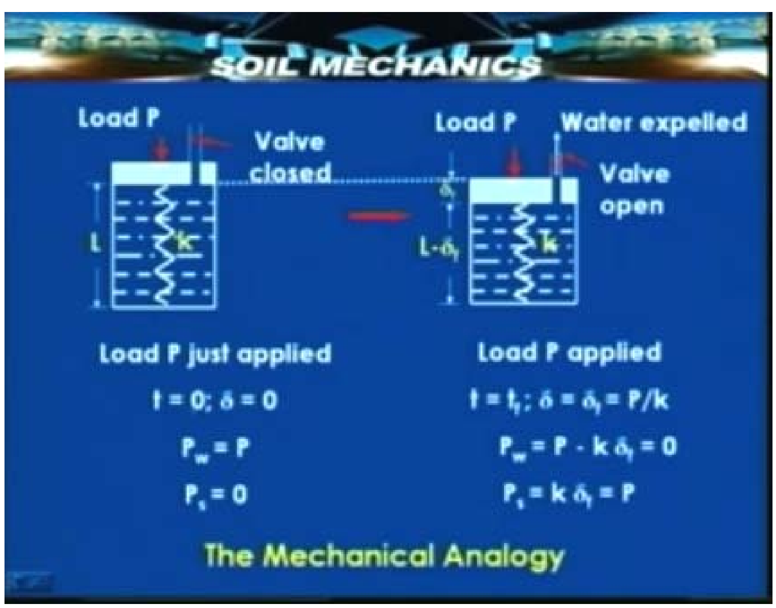

So if we take the consolidation part alone, as soon as we applied load at t =0 there is no deformation of the spring, the load carried by the water is the complete load p and the load carried by the solids is zero. But at the end of the consolidation phenomenon situation is different, the load carried by water is p minus the load carried by the spring and at time t equal to infinity after the consolidation phenomenon is completed k delta f becomes equal to p and therefore water carries no load where as the solids carry the entire load capital P.

Now let us see the real situation as well. In the real situation if we apply a surcharge q there is an initial pore pressure u = q and when you just apply the load and at t =0 therefore u will be equal to q and sigma dash is equal to zero. Now if we take the situation at the end of the consolidation phenomenon then the pore pressure is equal to u final and sigma dash is equal to $sigma_f$ final and if we proceed further we find that ultimately the pore pressure gradually gets dissipated and u becomes equal to zero and entire sigma dash is carried by the solids in the form of q. Now this u the so called excess hydrostatic pressure is visibly a function of depth and time. This is the variation of u at any point of time.

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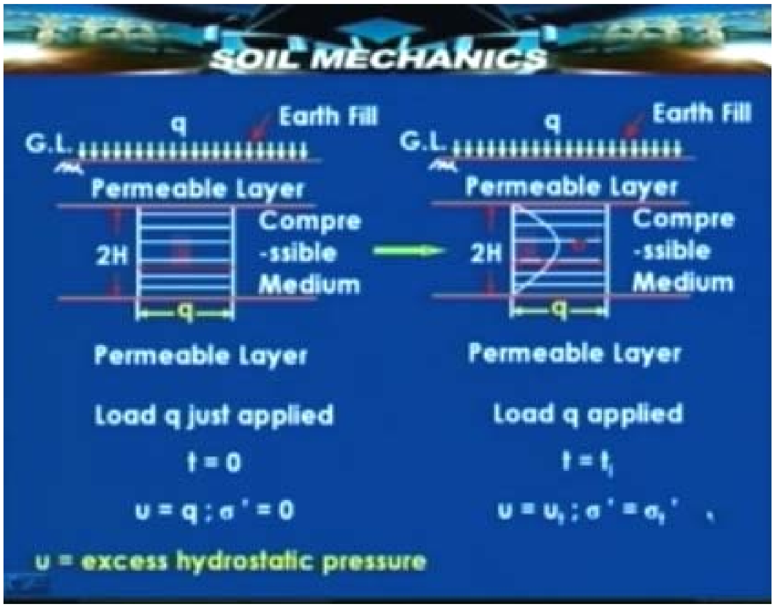

It is worth mentioning here that it is zero at this permeable layer and also zero at the bottom permeable layer. What happens is if we just go back one slide, what happens is at t =0, as soon as we apply the load entire applied pressure q is carried by the water. And therefore water takes a pore pressure u in the form of excess hydrostatic pressure that is pressure over and above the hydrostatic equal to u throughout its depth. Since this point upper layer is permeable and the lower layer is also permeable, at these two points inside the clay layer, in the compressible layer the pressure is u whereas just outside because it is a raining layer the pressure is zero.

That means there is a infinite hydraulic gradient which means that instantly the pore pressure u will get dissipated completely and that's why as shown in this diagram, we have pore pressure equal to zero at the two ends and maximum at the end at the centre and this goes on. The ends keep on constantly having a pressure u =0 whereas at the centre pressure the pore pressure is maximum but its value goes on changing with time. Ultimately it decreases to such an extent as to transfer the entire load to the solids in the form of sigma dash.

Let us take a look at mathematical formulation quickly to the extent we had introduced it last time. We defined a number of parameters, one is a_v. What is a_v? a_v is nothing but minus de by dp that is, it is the rate of change of void ratio with pressure, how the void ratio changes as pressure is increased. Since the void ratio decreases with increase in pressure a_v is minus de by dp. Then we also defined a parameter called degree of consolidation corresponding to any depth in terms of e. You may express it usually as is usual in the form of a percent.

This is denoted as an u_z that means the degree of consolidation at any depth z. It will be equal to at any given point of time, the initial void ratio minus the void ratio at that depth, at that time divided by the total change in void ratio. And now this void ratio has to be related to pressure and through the pressure it has to be related to the excess hydrostatic pressure and then we can relate that to the rate at which water flows out.

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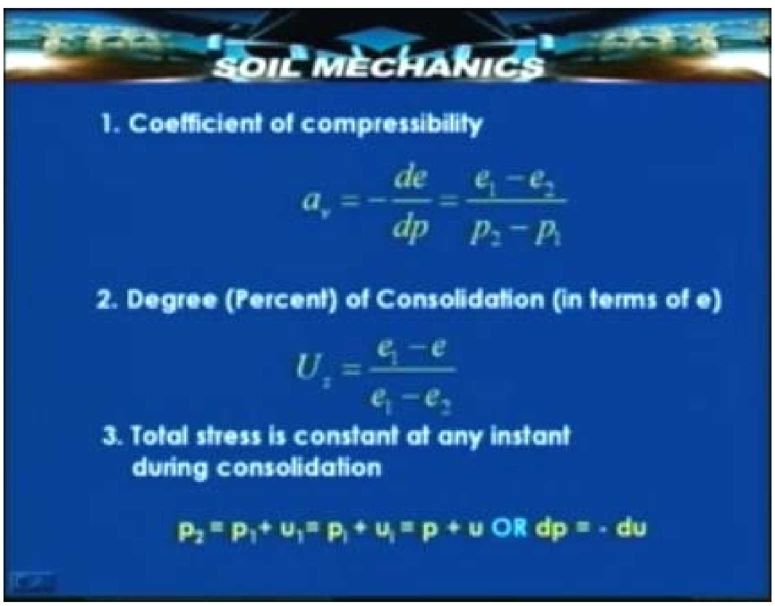

So that we can get the value of degree of consolidation at any point of time. So for this purpose suppose we take advantage of the fact, the total stress is always equal to the pore pressure plus the effective stress then p_2 at any instant, at any second instant you can say will be equal to the effective stress in the first instant plus the excess hydrostatic pressure in the first instant.

Or in general if I take any instant i, it will also be equal to p_i+u_i or it will be equal to p+u in general at any point of time or you can say p+ u is a constant and therefore dp is minus du. So we have a very interesting observation here to make, this dp is related to de here through minus a_v whereas it is related to pore pressure change as dp equal to minus du.

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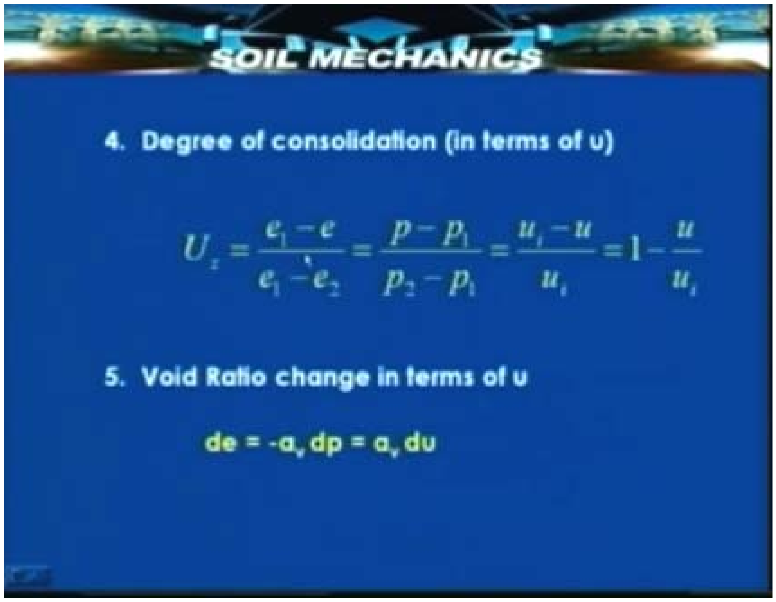

So this gives us a way to express this degree of consolidation through e_1- e and p- p_1 and finally u-u_i and finally we get the degree of consolidation is 1-u upon u_i where u_i is initial excess hydrostatic pressure and we know that the initial excess hydrostatic pressure is nothing but the external stress applied, that is in our case q. So now what this means is if we sum up all these, we say that de is equal to minus a_v dp and it is also equal to a_v du. So here we have the indication as to how to link the change in hydrostatic pressure, excess pressure for that matter through the change in stress to the degree of consolidation or change in void ratio.

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For this purpose we made a series of assumptions, all the assumptions are here. Homogenous, saturated, incompressible, one dimensional compression is valid, Darcy's law is valid and e-p idealized linear relationship is valid.

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So for the purposes of mathematical formulation we will consider a typical compressible layer with 2 permeable layers on either side. This is known as a double drainage system. The importance of this double drainage system will become evident after we finish the formulation of this problem in mathematical terms or rather 2 directional or you can say double drainage system. This earth fill is nothing but the external load applied. We also saw last time that at the end of the mathematical analysis this is what we are going to have that is we are going to have the degree of consolidation as a function of depth and the time.

For convenience the degree of consolidation expressed as a function of a non dimensional depth and a non dimensional time. Non dimensional depth is the actual depth, absolute depth divided by the total thickness of the layer and the non dimensional time is nothing but c_v t by H square. I have still not defined what is c_v? And therefore at a later date we will again come back to this definition. These lines here which show the variation of u_z with depth and time are all similar in pattern but as time increases, you can see here that the degree of consolidation is going on increasing or the pore pressure is going on decreasing and these are therefore nothing but lines of equal degree of consolidation with depth and these are therefore known as isochrones.

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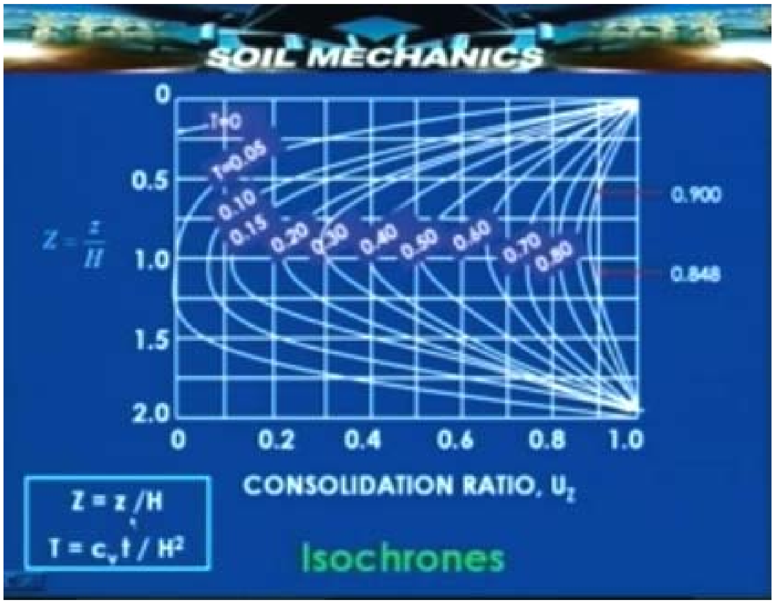

Now since the degree of consolidation varies with depth we preferably need to have a parameter which gives us an average degree of consolidation over the entire depth of the clay layer and we will be seeing how that is defined.

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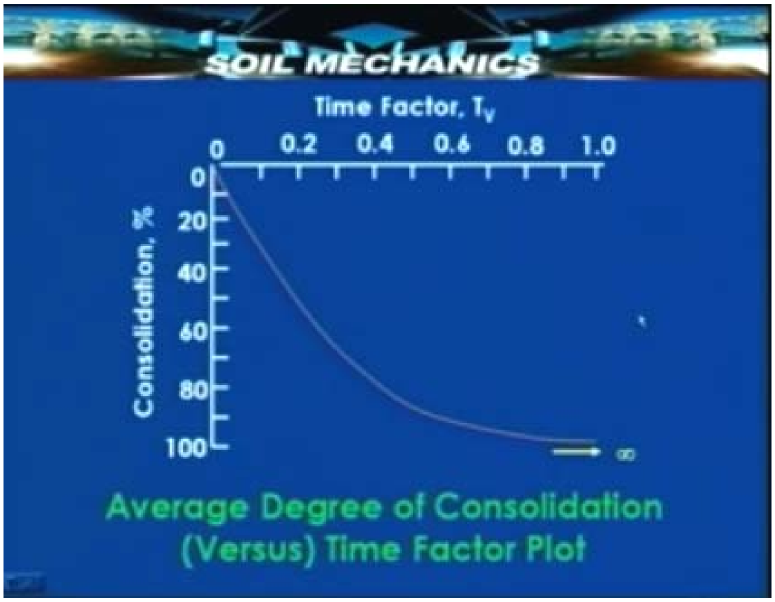

Suppose we know how the average consolidation over the entire depth can be determined. Then we will find that this average consolidation varies with time as shown in this diagram that is it goes on increasing with time and ultimately at time t equal to infinity it's completed that is 100% degree of consolidation is achieved.

So now with this background we will launch into the derivation of the mathematical expression for one dimensional consolidation that is what is going to be the subject matter of today's lecture. That is derivation of mathematical expression for one dimensional consolidation, let us see how. What we really need as I have told a couple of times already is basically an equation which relates the rate of out flow water through the voids and the rate at which the void ratio changes or the volume changes or the settlement is taking is place or the compression is taking place. So if we can relate these two quantities, we are in a position to model the hydro dynamics of a system and the resultant consolidation.

Let us take a look at this slide which shows a typical element in a homogenous soil medium, let me sketch it for convenience for better understanding. We know that we are dealing with a soil medium and we will be considering a typical element in the medium.

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We have assed homogeneity, so if we can analyze and understand any one element it's as good as understanding the phenomenon over the entire medium. Now what is the phenomenon that we are going to understand? We are interested in knowing how water is flowing out of this element. Let us see, it is very easy to utilize the knowledge that we have of flow through this element. Suppose a certain amount of water is flowing into this element and coming out, now the element is saturated.

Suppose this element is totally incompressible then what ever water flows inside will also come outside. Every drop of water that's entering this saturated element will push out a corresponding drop of water and therefore what ever flows in will also come out. But we have a system where the soil is saturated but as the water flows out the soil compresses. We have a system in which the element compresses so what this means is its volume decreases as q goes out there is small reduction of volume in the water.
And therefore there is a difference in the quantity of flow between the out flow and the in flow and it is in the problem of consolidation it is this net out flow that we are interested in. See how we can obtain is by simple basic considerations.

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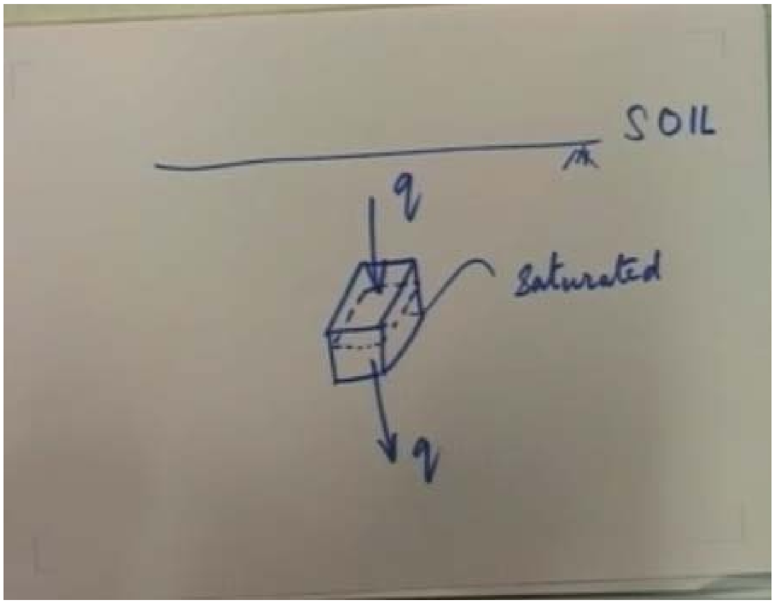

Suppose there is an in flow which can be represented as a v_z where v_z is the velocity. Velocity as we know is nothing but flow per unit time and in this case if we consider unit area then v_z will be nothing but velocity is nothing but flow per unit area per unit time and if the area is of finite dimensions delta x delta y as shown here where x y z axis are also shown. Then the inflow and out flow because they are going to be different in this medium.

Let us represent the inflow as v_z that is flow per unit area per unit time and out flow as v_z plus an incremental flow that is a rate of change of flow multiplied by the depth over which the flow takes place. That is v_z plus del v_z by del z into delta z. But however we know that this rate of out flow of element is going to be equivalent to the degree of compression that take place ultimately. So let us see what this rate of out flow is so that we can express it in terms of this volume and then relate it to the volumetric changes. The net out flow from the element, per unit time would obviously be this minus this. So del v_z by del z into delta z will give us the net out flow per unit time per unit area. So if I multiply this by the area delta x delta y then I will get the out flow per area delta x delta y over the whole element but still per unit time.

And therefore this is nothing but the rate of out flow from the element. We know that velocity as per Darcy's law is nothing but coefficient of permeability into the hydraulic gradient. If we represent the head change from the top to bottom as h or delta h then delta h by delta z represents the hydraulic gradient.
And since the gradient or the head causing flow that is h decreases as the flow takes place in the direction of flow, v_z will be equal to minus k delta h by delta z. This is a very well known equation; you must have already covered this as Darcy's law in the chapter on permeability flow through soils flow nets and so on. Now what this means is we can represent ultimately the rate of out flow in terms of the permeability of the soil in terms of the rate of change of hydraulic gradient and in terms of the volume. It's a very interesting equation which completely captures the physical phenomenon that we know

in the soil, the rate of out flow of water is related very closely to the coefficient of permeability. And we know that is what is causing the entire phenomenon of consolidation in clays.

Then we know their flow of water depends upon the rate of change of gradient and that's also captured here and the quantity of flow obviously depends upon the volume of the element and therefore that is also entering into this equation. Now this rate of out flow must be equal to the rate at which compression takes place. Let us see at any instant during the consolidation phenomenon if the void ratio happens to be e, then the volume of the voids in the soil would be e upon 1+ e because volume of the solid skeletons remains constant and 1+ e is the total volume therefore volume of the voids is e upon 1+e. And if I take the entire soil element of dimensions delta x delta y delta z, then the volume of the voids in the element at any point of time to begin with will be e upon 1+ e delta x delta y delta z.

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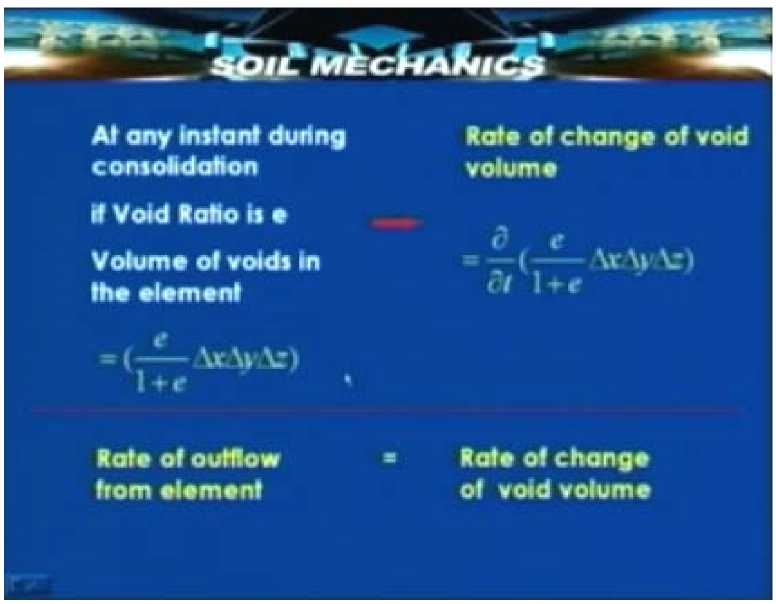

Now as the flow takes place from inside the element to the outside, these volume decreases. So the volumetric change in unit time that is the rate of change of volume which is nothing but the rate at which the compression takes place is going to be the derivative of this volume of voids with respect to time and that's what is the rate of change of voids.
Since this must be equal to the rate at which out flow is taking place, let us equate this and we then have k del square h by del z square into delta x delta y delta z is equal to all these which means that if we cancel out the volume of the element which obviously is not zero because we are dealing with a finite size dimensions of the element the volume of the element is not zero. If the volume of the element were zero, there would have no problem to discuss at all. Therefore in this case (Refer Slide Time: 25:42) these two can be cancelled out, we will have del square h by del z square is del by del t of e upon 1+e.

Now this h which is the head causing flow is nothing but u upon $gamma_w$ where u is the excess hydrostatic pressure.

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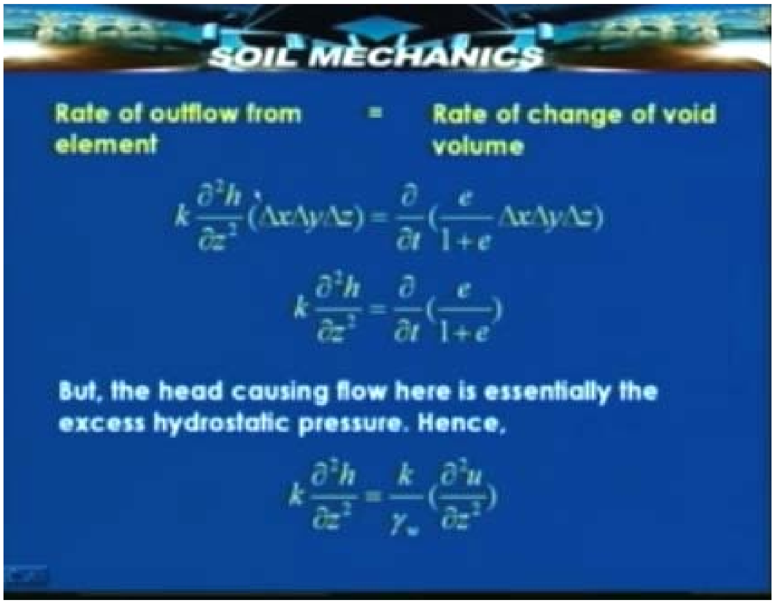

So this del square h by del z square can be written as u upon $gamma_w$ and so we will have k upon $gamma_w$ del square u by del z square where u now is the excess hydrostatic pressure. This must now be equal to del by del t of e upon 1+ e this, so let us equate them. If you equate, this (Refer Slide Time: 26:35) is what you get but before equating let us see what this del by del t e upon 1+ e is. We have just now stated that 1 upon 1+ e is the unchanging constant volume of the solids.

So volume of the solids is 1 upon 1+ e which remains constant and therefore del by del t of e upon 1+ e is nothing but 1 upon 1+ e of del e by del t, this being constant can be taken out. We have already seen that de is related to du and from the previous slide from this previous equation we can equate this change in void ratio with k del square u by del z square. That is if we equate the rate of flow with the rate of compression and transfer the terms containing u to the two sides of the equation then this is what we get (Refer Slide Time: 27:49). In this k, a_v these are all soil parameters, constants and so this whole left hand side can be written as c_v del square u by del z square equal to the right hand side which is del u by del t. In this we have now defined a new parameter c_v which is nothing but the coefficient of consolidation.

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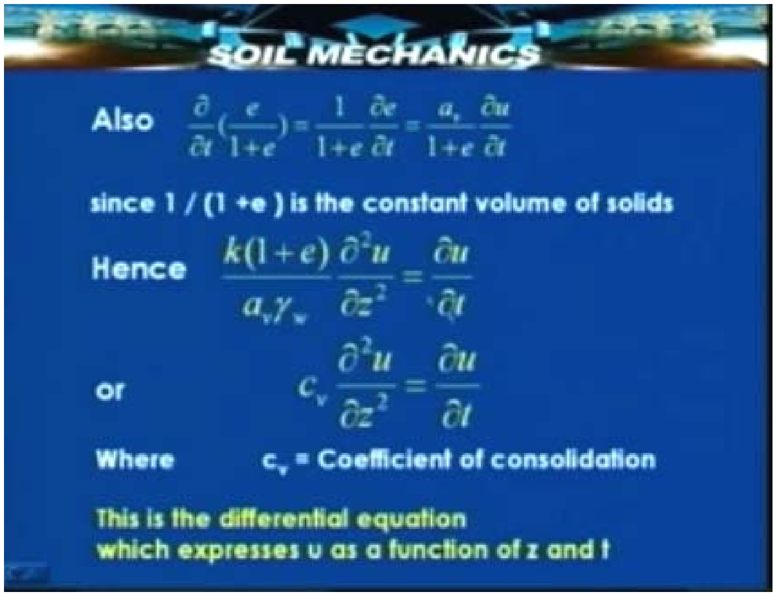

And this equation which we have is the differential equation which expresses u as a function of z and t then this is what we were after. We were in fact precisely interested in this equation because we wanted to know the rate of change of u with time so that we would be in a position to predict the compression corresponding to any time t. Now this coefficient of consolidation is yet another parameter which we have now defined which makes the total number of compressibility parameters that we have so far defined to four.

So let us take a quick look at these parameters. We have defined 4 parameters, the first one is minus de by de log p. It has no units because void ratio has no units and log of the ratio of pressures also has no unit where as a_v which is a linear relationship between e and p will have the units of inverse of stress. Because e has no unit, 1 upon dp will have the units of inverse of stress that means centimeter square per kg or meter square by kilo Newton or so and such units. Now another parameter which we have defined was a_v divided by the total volume 1+e or in other words you can say a_v multiplied by the constant volume of the solids. This obviously will have the same units as a_v that is meter square by kn because 1+ e has no units.

Now the last parameter c_v which we have just now defined is k upon m_v gamma$_w$. Since k has units of centimeter square per second and m_v has units of meter square per kilo Newton and gamma$_w$ has units of kilo Newton per meter cube. Effectively we will have here units centimeter square per second or meter square per year. We have come to a stage where we now have the governing equation for this phenomenon of consolidation. This equation c_v del square u by del z square =del u by t is the equation which controls or depicts very clearly the phenomenon of consolidation and not only the phenomenon of consolidation, it is seen that in general this represents any process where diffusion is the major phenomenon. So this equation is also known very often as the diffusion equation.

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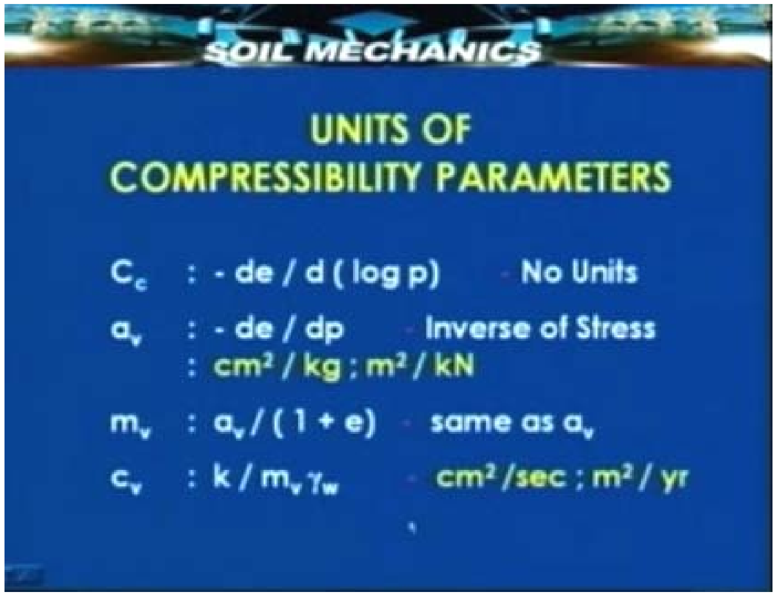

This equation has to be solved, if this equation is solved then we will get an expression for u in terms of z and t. Now what is important here to notice is on the left hand side, we have rate of change of pore pressure, hydrostatic excess pressure as a function of z and on the right hand side we have its rate of change with respect to time. So if we solve this obviously u as a function of z as well as t can be obtained. Then what is the best way to solve this. The best way to solve this would be to assume the solution as product of a function of only z and a function of only time, this is known as the principle of separation of variables in theory of differential equations.

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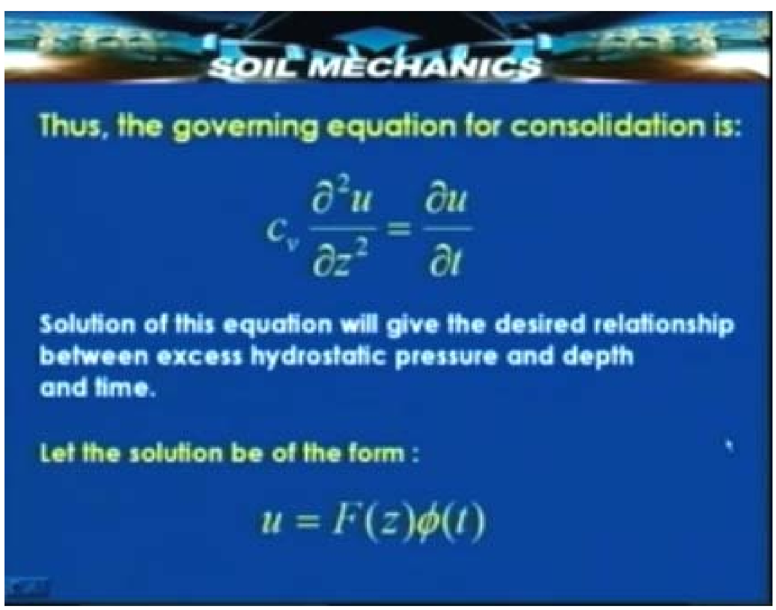

And suppose following that we express the excess hydrostatic pressure as a product of two independent functions. One a function of only z another a function of only t then there are certain advantages in solving them. We do not need to or rather we do not have to go through the process of treating simultaneously a function which is dependant upon z and t. At any given point of time one of the functions remains constant when we do any operation on the other function and vice versa. Since we have already written the differential equations c_v del square u by del z square equal to del u by del t and we have defined u as Fz ft.

Let us substitute this value of u by suitable differentiation in the consolidation equation that means del two u by del z square is required. So differentiate Fz phi t with respect to z since when you differentiate u with respect to z, phi t will remain constant. So we have c_v phi t F double prime z which is the second derivative of u with respect to z and on the right hand since we have del u upon del t and when you differentiate with respect to time, the function Fz remains constant. So we have Fz into phi dash t where phi dash is the first derivative of u with respect to time.

We can completely separate the two variables z and t by bringing this here and this here so that we will have an equality of this kind. Now let us a take a look at this equality, this equality suggests that on the left hand side we have no time parameter and on the right hand side we have no depth parameter. That means the left hand side is completely independent of time and right hand side is completely independent of depth.

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And that's only possible if this ratio is equal to some constant. One is a function of z and the other is a function of time and they are equal and that's only possible if the ratio is a constant and let us represent this constant as minus A square.
The reason why we are representing it as a square and as a negative quantity will become evident, it's for a getting an elegant equation which can be solved comfortably. If we take the ratio as the constant equal to minus A square then we get an equation like this which is a very elegant and convenient equation consisting of the derivatives of the function in terms of z and we also get another part where phi dash t is minus A square C_v phi t. So we have one equation which is consisting of the derivatives of z only and another equation consisting of the derivatives of only the function of time phi. This function which is a function of only z from theory of differential equations has a solution Fz given by C_1 cos Az plus C_2 sin Az it's a harmonic function and this solutions has two constants C_1 and C_2 and we need to determine these two constants c_1 and c_2 from known conditions of the problem in order to be able to evaluate Fz. We can continue like this with phi t as well.

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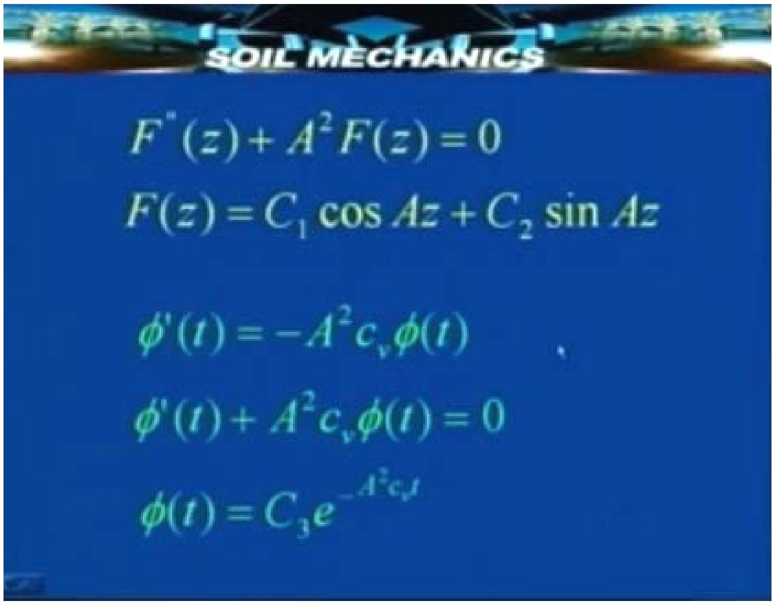

So continuing the second part of the equation would be phi dash t equals minus A square C_v phi t or in other words in a manner analogous to the equation which we wrote for z, we can say phi dash t plus A square C_v phi t equal to zero. Just as the expression with respect to z had a standard solution C_1 cos Az plus C_2 sin Az. This equation also has a standard solution phi t equal to a constant C_3 into exponential minus A square C_vt. So now we have two solutions one in terms of z and another in terms of t and these contain constants C_1, C_2 and C_3.

Now since the function u, the expression for u is Fz phi t. The expression for u now will obviously be the product of this and this. So u will be equal to C_1 cos Az plus C_2 sin Az multiplied by C_3 e minus A square C_vt. In this there are two constants inside the bracket and one outside and in addition to that there is that constant A which we had introduced in the beginning.
If we unify these by expanding the bracket then we will have C_4 cos Az plus C_5 sin Az into e to the power of minus A square C_vt for the excess hydrostatic pressure u, which means now that in order to be able to compute u we should be able to determine the constants C_4 C_5 A. Although they were originally 5 constants, effectively we have now brought it down to 3 constants. Now these 3 constants C_4 C_5 and A have to be determined and in order to determine 3 constants we need 3 conditions. Since this is a problem in which the parameter which is to be determined that is u is a function of depth as well as time.

We need to have a boundary condition which depends upon the depth or which is related in some way to the depth as well as the so called initial conditions which is related to the time factor t. So we need in all 3 conditions some of them could be conditions relating to the depth and some of them could be conditions relating to time t. The conditions relating to depth z are known as the boundary conditions because we only know usually the conditions at the boundaries of the clay layer because what is happening inside the clay layer is not yet known and that’s what we are trying to look at or determine.

So at the boundary we can define certain conditions which are very obvious and which have to be obviously satisfied and these are at the boundaries of the given depth of or the thickness of the clay layer. So these are known as the boundary conditions where as the so called initial condition relates to the magnitude of the pore pressure at some point of time t if it is known. So what's the time t at which the pore pressure value is known, if we know at some point of time the pore pressure value that can serve as an initial condition for us. So we look for these conditions.

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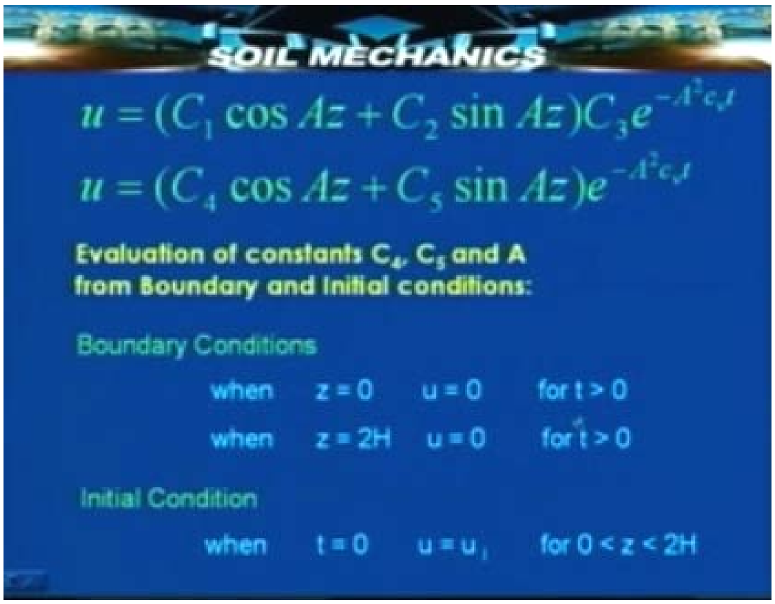

Let us take this clay layer and our already existing knowledge of pore pressure variations. This is our clay layer on top we have a permeable layer, at bottom we have a permeable layer and because of these. This is known as the double drainage case and what happens is because of these when we apply load, although initially the pore pressure from top to bottom is equal to u, equal to the applied load q. At the very next instant at the boundaries because pressure is u or equal to q on the inner side of the boundary and is equal to atmospheric or zero on the outer side of the boundary. The pressure u immediately gets dissipated it becomes zero here and which means that the pressure distribution as we already have seen becomes some what like this. It is zero here, zero here (Refer Slide Time: 41:22) and some pressure has already got dissipated at the centre and so it has reduced a little bit from the initial value how ever it is still the maximum at the centre of the clay layer.

So we can from here state or stimulate the boundary conditions like this. If z is the coordinate which describes the depth then at z =0 and at z =2 h where 2 h is the depth of the clay layer, we know that this pressure u the excess hydrostatic pressure u must be zero and not only at the initial instant but once the pressure got dissipated will continue to be zero at all times until the entire process is completed. We can say at z equal to zero and at z equal to 2 h, u the excess hydrostatic pressure is zero at all times for t greater than zero, except at the very first initial instant as soon as the load is applied the excess pore pressure is just equal to the applied pressure q. So now we already have two conditions and possibly we can determine 2 out of these 3 constants. One other condition that we need is an initial condition because since 2 of these are boundary conditions and are related to z, we must now look for a condition which tells us how the hydrostatic pressure excess hydrostatic pressure varies with time and that also can be easily stipulated when t is equal to zero, u the excess hydrostatic pressure will have some initial value.

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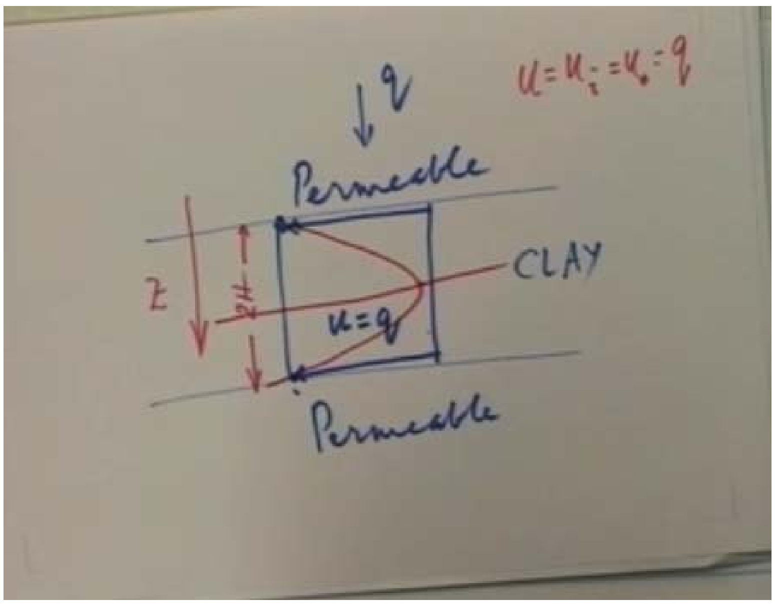

So this initial value of the excess hydrostatic pressure may be called as u initial or may be called as some fixed value u_0 or in any case it will always be equal to applied stress q. So at t equal to zero, u is equal to the initial excess hydrostatic pressure what ever be the value and this is valid through out depth of the clay layer, at t equal to zero through out depth of the clay layer we will find that u is equal to a fixed value equal to q that is uniformly distributed. So that is another condition which we can advantageously used in order to determine the constants. So if we do that, if we apply these boundary conditions then what will happen is we will get an expression for u.

Let us go back for a moment. This is the equation in which we want to determine the constants. Since we know the values of z or the value of u at certain values of z and at certain time t, what we can do is to substitute this values of z or this value of t here and write down the expression for u. And if we do that then successively by substituting these conditions one by one, first this condition then second and then this third condition, we will be able to get a series of equations which can then be solved and we can evaluate the constants and substitute them back in the expression for u, so that we get this expression which represents ultimately the variation of u with respect to depth and time.

Take a look at this expression. This is sigma of n =1 to infinity. What this means is that this pore pressure is a function of the total depth. It's a function of the initial pore pressure and it's a function of the parameter n pi z by 2 H. It's a function of C_v, it's a function of the depth total depth H. If we take this as the expression defining u then here we have a parameter $C_v t$ by H square which we have already defined in an earlier slide as the non dimensional time factor t. We can easily quickly verify mentally that $C_v t$ upon H square is not dimensional and since t is the only variable here, C_v and H are either geometrical and material constants. This is nothing but a non dimensionalized time.

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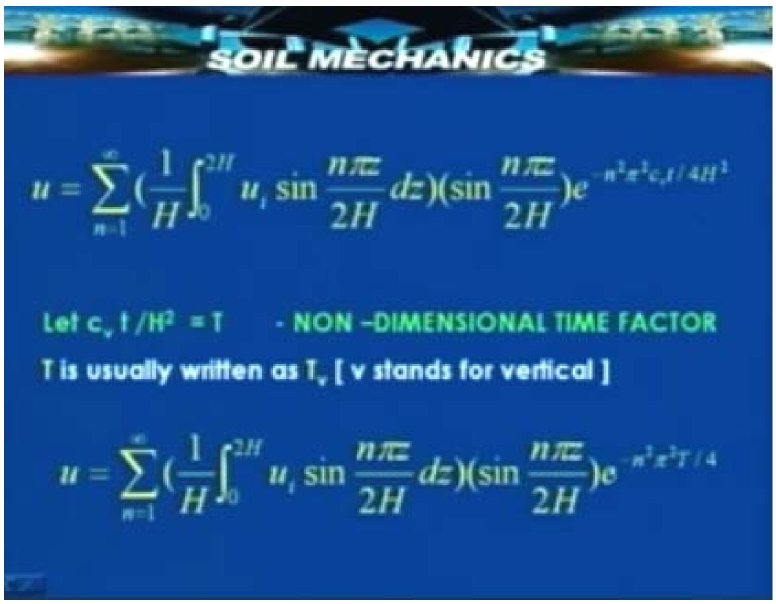

So it is known as the time factor, this T is usually written as T_v to indicate time factor for vertical consolidation at a slightly later time, we will see that we also have what is known as a radial consolidation phenomenon where also there is a time factor, a non dimensional time factor we will define analogous to this. And that we would like to denote at that point of time as T_r therefore this T_v stands for this non dimensional time, indicating time factor for vertical consolidation. If I substitute this non dimensional time factor here in place of this small t then I have an expression for excess hydrostatic pressure which becomes like this.

In this there is an integration, there is a summation. So if we do this then we will be in a position to get a final expression for u. Let us see how that is done. Now this solution of all this requires the initial condition which we have already seen and if we assume this initial condition that $u = u_i$ and u_i is equal to a constant value u_0 then we can substitute that and integrate and then we will have u is equal to m equal to zero to infinity 2 u_0 by M sin Mz by H e minus M square T.

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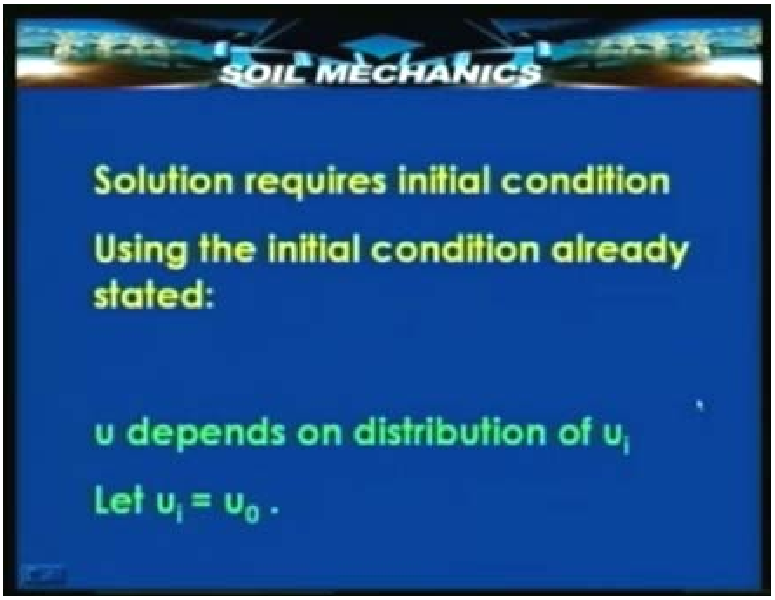

Here we have now introduced a parameter capital M, to understand this we need to go back a little bit. Here we find that in the summation sign there is a value of n which varies from 1 to infinity. The reason why we have brought this summation sign is the solution to this equation is in terms of sine and cosine and when we substitute the boundary conditions we found that the constant A works out to something which can be expressed in terms of pi, which is n pi to be précised. Then this equation is valid for all values of n.

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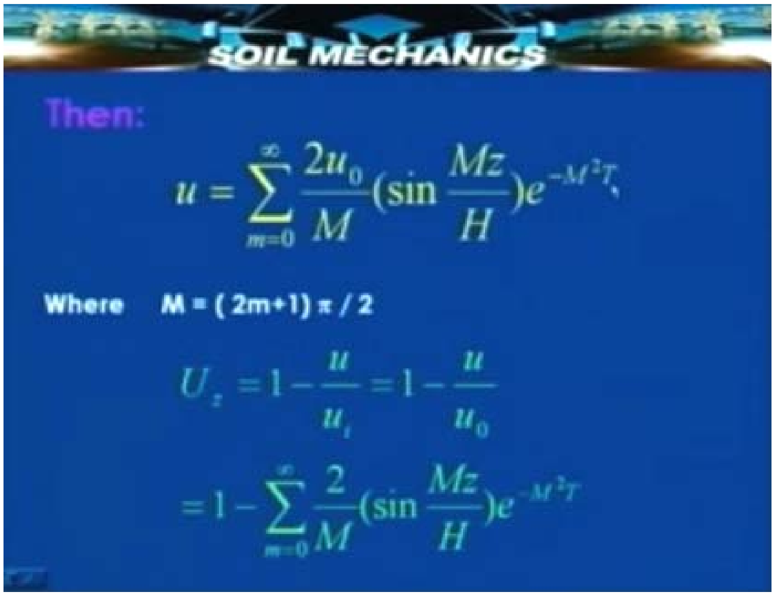

And therefore there is one solution for u corresponding to each value of n which means that the final real solution is the sum of all the solutions corresponding to all the values of n which are possible. The values of n that are possible are only 1 to infinity and therefore integer values which are possible for n are only 1 to infinity and that's why we have infinite number of solutions, n equal to 1 to infinity and this parameter or this entire factor within that summation sign and this will finally give real value of u taking into account all possible solutions.

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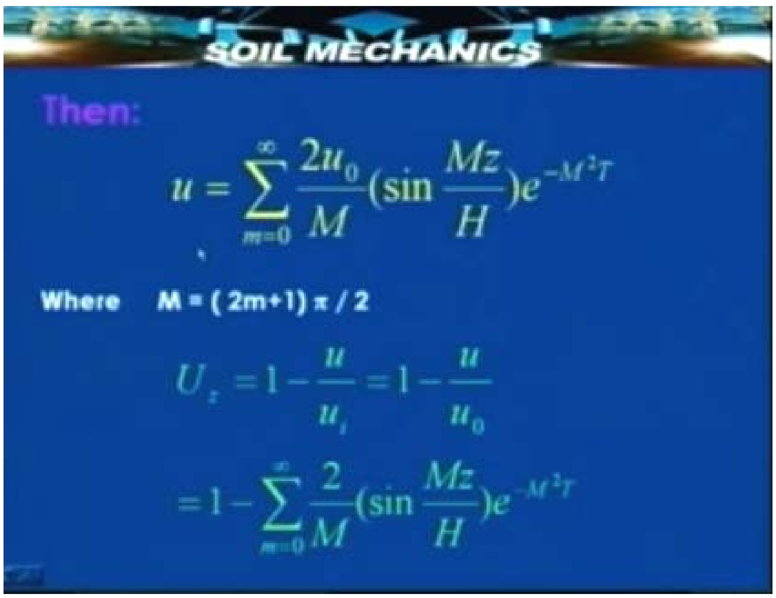

Continuing further we found that we can further simplify or further rewrite this expression for u in terms of another parameter capital M which depends upon a parameter small m which is related to capital M and as capital M is equal to 2 m +1 into pi by 2.

That is when we go into the details of the consolidation equation, we will find that we have this parameter m small m which could be even or odd and then we will find that values corresponding to even values of n do not exit or they vanish and finally the solution really contains only the terms corresponding to odd values of m which from a general point of u can always be expressed as 2 m +1 and that's how where we had m pi upon 2 since only odd terms are to be considered. Now we have 2 m +1 pi by 2 and this is for convenience represented as capital M.

So we have here a function in which u is dependant upon the initial pore pressure distribution, the parameter capital M, z upon H that is the non dimensional depth and the non dimensional time factor t or T_v. Once we have this, going back to our set of expressions which we had defined, the degree of consolidation u_z corresponding to any particular depth can be expressed as 1 minus u upon u_i but u_i is u_0. Therefore this is 1 minus u upon u_0. Since we already have evaluated small u, we can get the expression for capital U_z which is the degree of consolidation at any depth, as this. In this again we have non dimensional time factor, the parameter capital M. This a particular expression can be graphically represented in the form of isochrones as shown here.

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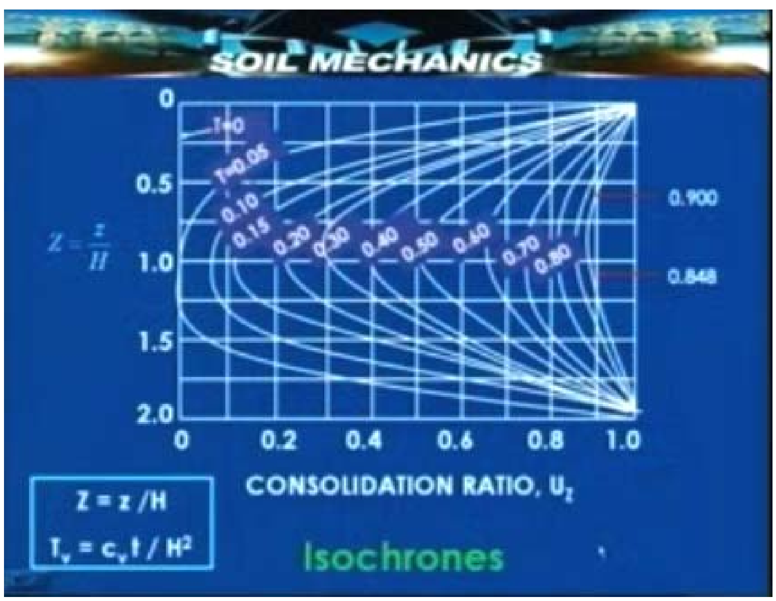

In that expression for U_z we had non dimensional depth z by H and the non dimensional time factor T_v. So suppose we substitute different values of T, T =0, 0.05, 0.1 and so on up to 0.9. Then we find that at different values of z by H we will get different U_z values and this is how the U_z values will vary, these are known as isochrones indicate meaning that these are the values of U_z corresponding to constant time or constant time factor.

Here we have a line for example corresponding to T =0.05 that means at this depth say a depth corresponding to a non dimensional value z upon H equal to let us say 0.1, there is a time T =0.5 certain U_z value, let us say this. Then at the same time at another depth, we have U_z=0. So the u_z values vary with depth and at any particular time T this will be the nature of distribution of U_z with depth. There are two particular values of time which are important the reason why they are important we will see a little later. The two values of capital T which are important from a practical point of view are T capital corresponding to 0.5 and T capital corresponding to 0.9. That is the reason why we take this is, corresponding to 0.9 factor U_z value is know to be 0.848 and similarly corresponding to U_z equal to 505, we also have a corresponding time factor known as T 50.

Now we can use this in the computation of the degree of consolidation corresponding to any time as we will see a little later. Now there is one more aspect, one more parameter which is of importance. We have been able to determine the degree of consolidation as a function of depth but in any given practical situation rather than looking for the degree of consolidation at any particular depth, we would like to know the average degree of consolidation over the entire depth of the clay layer. And now since we know what is U_z it is possible to determine the average U by simply integrating it with over the entire depth and averaging it over the depth 2 H.

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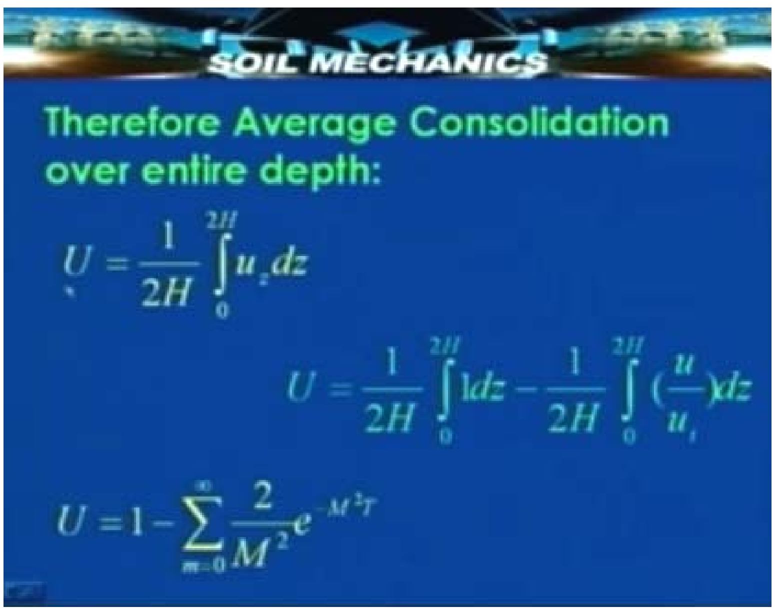

This can be done step by step to get this final expression and now if you take this final expression, the final average degree of consolidation and related to capital T and see its variation, you will find that the average consolidation varies with time factor T_v in such a way that it becomes maximum, it becomes 100 % at T equal to infinity.

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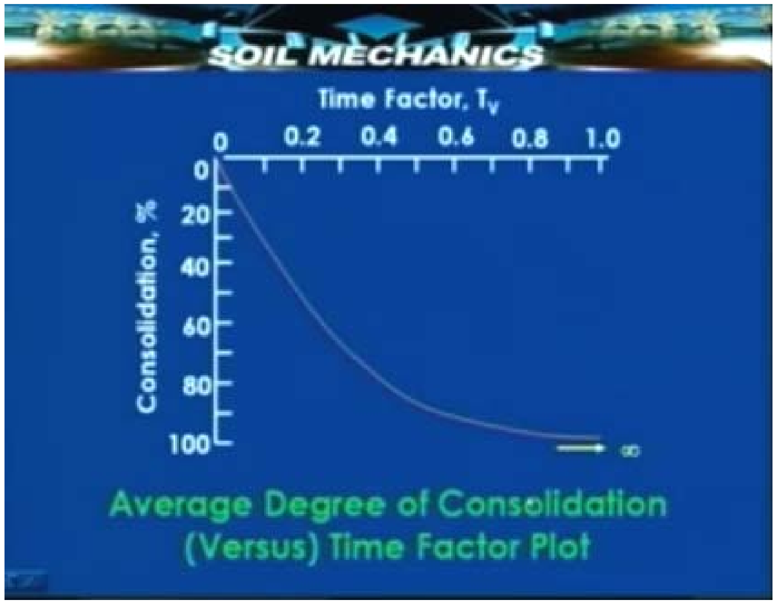

So now in today's lecture we have seen the details of the derivation of an expression for U which is nothing but the mathematical treatment of the phenomenon of one dimensional consolidation.

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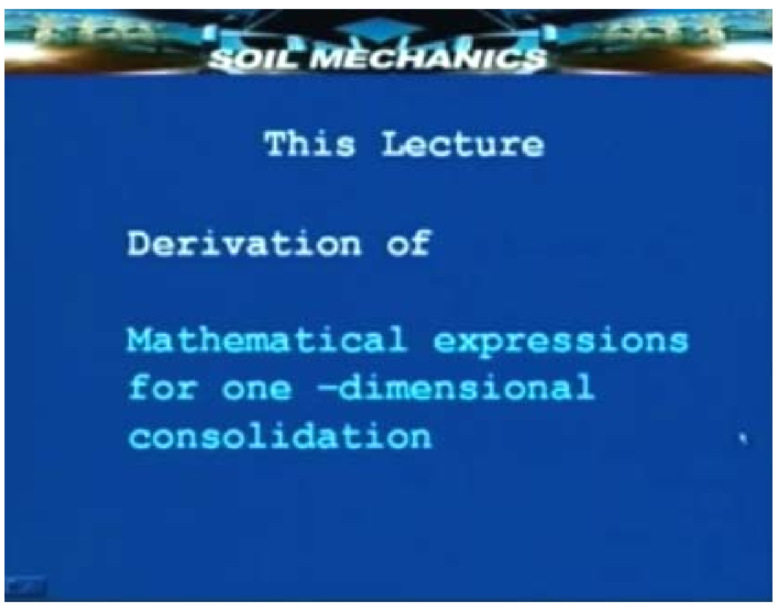

In the next lecture we will develop further understanding of the physical nature of this mathematical expression that we have derived. Then we will go into the details of determining the coefficient of consolidation C_v which is very important from point of view of determining the time factor from point of view determining the pore pressure and the degree of consolidation not to speak of the average degree of consolidation. We will reinforce all these concepts with the help of a few examples.

Thank you.