Soil Mechanics
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Lecture – 33
Stress Distribution in Soils
Lecture No. 6

Students once again we meet. Today's lecture is on stress distribution in soils. This is the sixth in the series that we had so far. This shall also be the final lecture on stress distribution in soils. So in today's lecture we will be devoting much of our time to solve problems and that is to understand the application of the theory that we have seen so far, mainly the Boussinesq's theory. We shall also quickly review some of the points that we have discussed in the previous lectures. This will be something like a winding up lecture in which we shall not only quickly review or summarize what we have discussed so far. But also see the application of mainly the Boussinesq's theory.

So first let us take a quick review of what we did in the last lecture. The lecture that we had just prior to this, we discussed that the three different types of loads that we apply normally through a foundation on to the soils. The three types of loads are as you know very well point load, line load, strip load. You may also add if you like uniformly distributed load, this is actually nothing but a strip load but acting on a rectangular area of finite dimensions rather than at an area of very long length and short cross section.

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How ever basically there are only three different types of loads, the uniformly distributed load is just a variation of this. We also saw how these loads act on the various foundations of different shapes and how to determine the stresses beneath those foundations.

Let us take a look at next slide. We also discussed in the last lecture more details than that we had discussed in the earlier lecture about how to calculate stresses beneath a rectangular area, what is the theory, how to use an influence chart.

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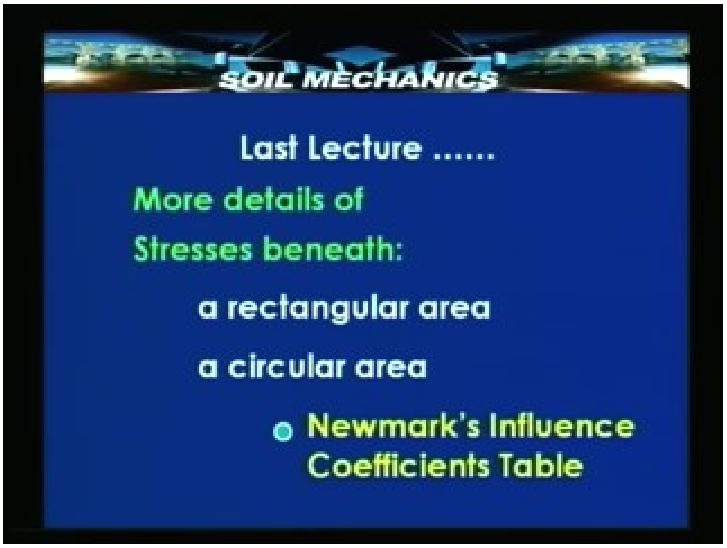

We saw for example different types of charts Fadum's chart, Newmark's chart and the Boussinesq's influence table. We also understood how this theory is applied to circular areas. This is the influence coefficients table, this is the basis of the calculation of stresses beneath a point load. And therefore we must understand this table very well and its use. If you look at this table we have a parameter R upon z in the first column and the corresponding influence coefficient on the second column and this is repeated for different R by z values in the subsequent columns. If you see here R is the distance measured radially from the point of action of the concentrated or the point load. So that must be remembered and z is the vertical depth at which the point, where we want the stress lies.

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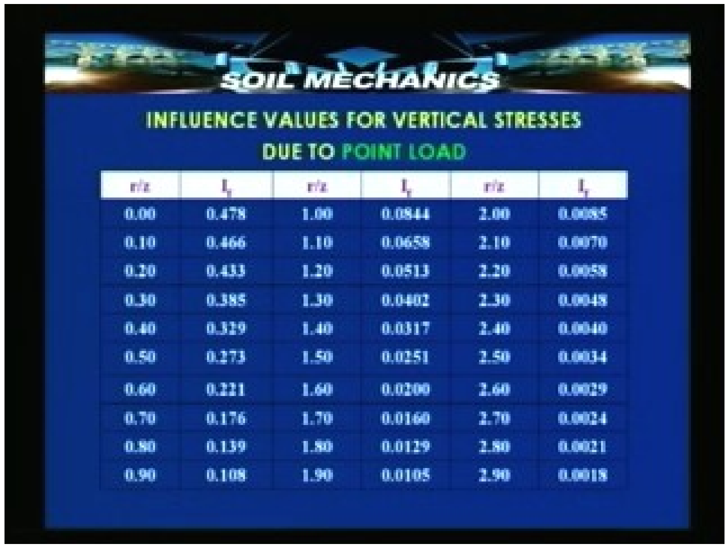

r/z	I_z	r/z	I_z	r/z	I_z
0.00	0.478	1.00	0.0844	2.00	0.0085
0.10	0.466	1.10	0.0658	2.10	0.0070
0.20	0.433	1.20	0.0513	2.20	0.0058
0.30	0.385	1.30	0.0402	2.30	0.0048
0.40	0.329	1.40	0.0317	2.40	0.0040
0.50	0.273	1.50	0.0251	2.50	0.0034
0.60	0.221	1.60	0.0200	2.60	0.0029
0.70	0.176	1.70	0.0160	2.70	0.0024
0.80	0.139	1.80	0.0129	2.80	0.0021
0.90	0.108	1.90	0.0105	2.90	0.0018

The ratio R upon z is directly related to the influence factor, the influence factor contains R upon z as a non dimensional term. So for different values of R by z we already have the influence factor. If how ever in any given problem the R by z value does not lie within this range, rather than using this table we shall directly use the closed formula that we had seen in the last lecture. How do we apply the influence table or the table containing the influence coefficients for point load? Take a look at this figure.

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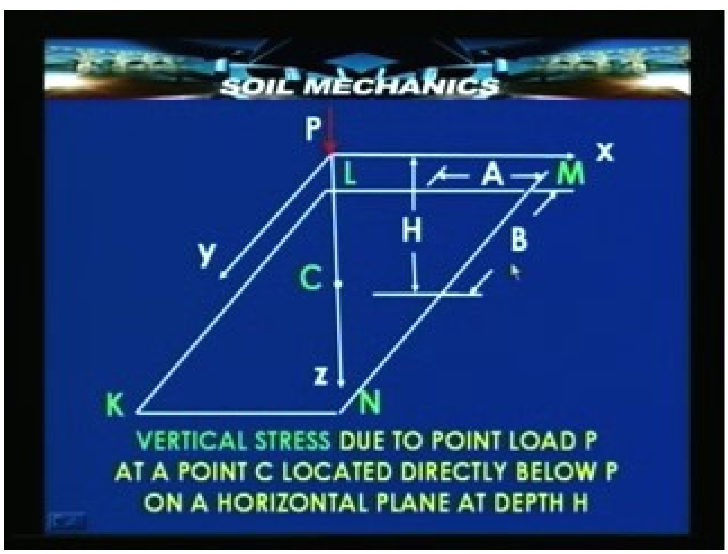

Suppose this is the point load we are normally interested in finding out the stress at any point such as C directly beneath the load P or some times away from this. If it is directly beneath the load P, if the centre C of a horizontal rectangular area is below the point P then the formula that we have seen in the case of point loads, the Boussinesq's theory based formula is directly applicable here. That means this is the schematic which can be used for determining the vertical stress due to a point load P at any point C directly beneath P but on a horizontal plane of finite dimensions. The point C as I said could lie away from the centre line also. In this case we have the reverse of that problem. Here we have the concentrated load, we have a finite area L M N K the centre of which is directly beneath the point P. We already know how to calculate the stress at a point C directly beneath P? Now if this happens to be the centre of a rectangular area of finite dimensions then we can apply this formula repeatedly or the integral of this to find out the total load on that finite area denoted by LMNK of dimension 2 A upon 2 B.

We had seen an example of how to do this in the last lecture. This diagram, the next one for example illustrates the case where we want to determine the load over again a rectangular area but this centre of this rectangular area is no longer under the line of action of the load or the point of action of the load P. In this case what is required is to apply the Boussinesq's theory either the formula directly or if the values concerned are within the range, then the point load influence coefficient table. We need to use this repeatedly to obtain the total load carried by rectangular area LMNK whose centre is not directly below P but away from it by a certain distance.

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In this figure for example C is away from the O which defines the line of action of the load P. In order to determine the total load carried by this LMNK, what we need to do is to divide this into a set of rectangles. We must remember that the formulae we have are applicable only for the corner of a rectangle. So we must divide the given rectangle in such a way that the given point or the point at which we want the stress lies at the corner of given area. For example if we divide this LMNK into an area a M N small b and subtract from that area small a capital L K small b, we have in effect 2 areas each one of them has the point O lying at the corner of two rectangles. So thus for example small a M N and this CO has the corner at O, for half the given area and the other half defined by NKb bounded by NKb also has a corner lying just directly below the point P.

So we can apply the known formula already available formula, once for this rectangle and once for the mirror image the other rectangle. And this can also be repeated for the smaller rectangles OALN similarly O this point K b and that is what we have written here aLKb. This way we can find the total load on any area, any rectangular area whose centre may be lying away from the point P or its line of action. In the above diagram we have taken C on a line parallel to the x axis but in general it need not necessarily be on a line parallel to x axis. The rectangle could be even rotated. We have to nevertheless divide this into equivalent rectangular areas and by using appropriate geometries and addition subtraction, we should be able to get total load on any desired area.

If we have a strip load then we use a different chart in which z and x, the coordinates in the two directions are given in non dimensional terms and influence coefficient values are available in this chart. The range of z in this chart is 0 to 2.5 and the range of x is from 0 to 3 which mean that the use of this table is restricted to a non dimensional width of 2.5 and a non dimensional x coordinate dimension of 3. That means the horizontal width is 3 meters in non dimensional terms and depth in non dimensional terms of the point at which we want the stress is 2.5.

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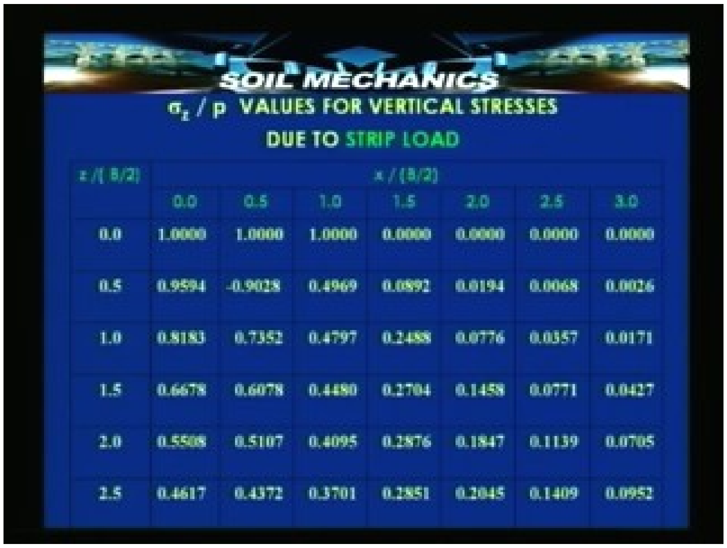

SOIL MECHANICS

σ_z / p VALUES FOR VERTICAL STRESSES DUE TO STRIP LOAD

z /(B/2)	x / (B/2)						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0.0	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9594	-0.9028	0.4969	0.0892	0.0194	0.0068	0.0026
1.0	0.8183	0.7352	0.4797	0.2488	0.0776	0.0357	0.0171
1.5	0.6678	0.6078	0.4480	0.2704	0.1458	0.0771	0.0427
2.0	0.5508	0.5107	0.4095	0.2876	0.1847	0.1139	0.0705
2.5	0.4617	0.4372	0.3701	0.2851	0.2045	0.1409	0.0952

This means that if we need to determine stresses at any point deeper than this or any point whose z non dimensional with respect to B/2 is larger than 2.5 or the x dimension is larger than 3 when non-dimensionalized. We need to go back to the actual formula, substitute appropriate values and get the results. If you remember we had also seen another diagram with respect to this, a cross sectional diagram which illustrates the angle beta, delta which are to be used in employing the formula that we have for stresses beneath a strip load. So either the formula or the table can be used for computing the stress. In today’s lecture we shall proceed further and we shall see some more details of how to compute stresses beneath rectangular and circular areas but we shall stop not only with that we shall proceed further and we shall see how to compute stresses beneath an area of any arbitrary shape.

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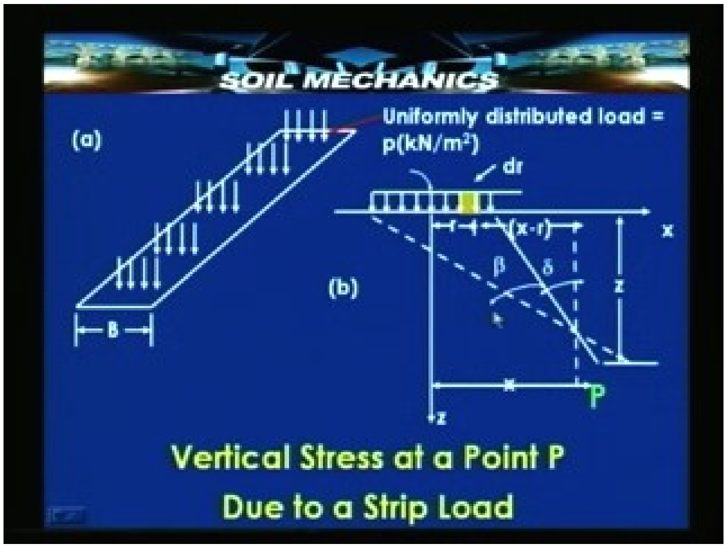

The last one requires the use of a graphical chart which is known as the Newmark's influence coefficient chart. That we shall see towards the end of this lecture. We already know how to determine the stress at any point below a rectangular area, when particularly the point at which we want the stress is located directly beneath the corner of the rectangular area. Let us see a diagram. This is the chart known as the Fadum's chart which gives the influence chart rather influence coefficients for a rectangular area of non dimensional values mz, nz at a point A, at a point depth z.

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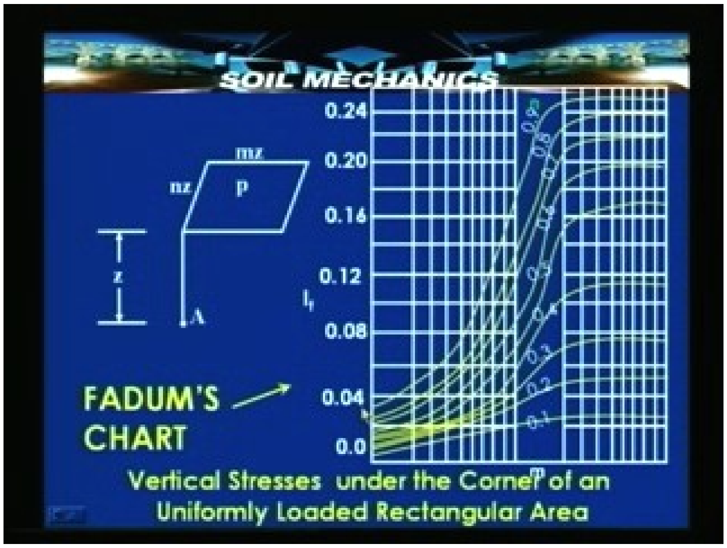

This requires that we should pay attention to the point, that the desired point at which we want the stress is directly below one of the corners. The table which also gives the influence coefficient for different non dimensional ratios n and m, where n and m have been defined even earlier or in this chart you can see that m is nothing but half the longitudinal dimension, non dimensionalized with respect to the depth and n is half the cross sectional dimension, non dimensionalized with respect to the depth and for different values of m upon n or rather m and n for a whole range of m from 0.1 to infinity and a whole range of n ranging from 0.1 to infinity. That means practically all values of m and n that is a rectangular area of very small dimensions to very large areas, very large loaded areas occupying very huge extents we can determine the stresses beneath such areas, remember for a uniformly distributed load.

This particular table has been evolved by Newmark based on Boussinesq's theory. Let us see the application of some of these formulae or these charts that we have been familiar with and let us start with application to rectangular areas. Let us read this problem and understand this. The centre of a rectangular area on the ground surface has Cartesian coordinate 0, 0, 0. So there is a ground surface, there is a rectangular area. On the ground surface its centre has coordinates 0, 0, 0 that means this is the origin which means that we can take the x axis like this and the y axis like this.

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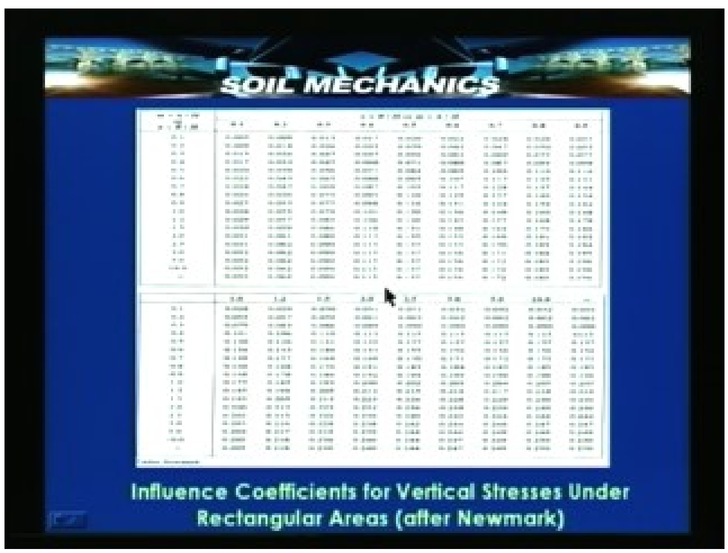

The next line is, the corners of this rectangle have coordinates. Let us take plus minus 6m. That means this point has got 6 meters, this point has got -6 meters dimension or rather coordinate. The y coordinate is 15, the y coordinate is 15 here as well, and then this depth is zero because this is on the surface. Then we also have +15 and -15, so -15 would come in this direction. So here we will have -15, 0 and we will have -6, -15, 0. These are the four corners of the rectangle, so the loaded area has been defined. But what we need is for a given loading on this rectangular area, uniformly distributed over the entire area of magnitude 1.5 kilo Newton per meter square, we need to estimate the stresses at several points. This being an illustration we have chosen points at several locations in order to thoroughly understand the application of this table. The points which we have chosen have coordinates, all of them being at 6 meters depth. We have taken one single horizontal plane of course the depths can vary. But in order to illustrates the relative effects due to various positions of the point we have taken the depth constant.

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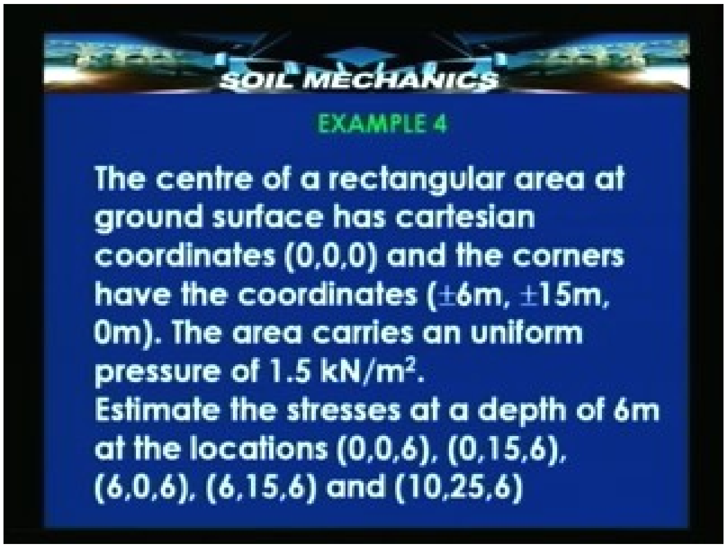

We have points such as 0, 0, 6 which would mean x coordinate is 0, y coordinate is 0, depth is 6 which mean it is this point. So this is point one, the next point is 0, 15, x that means x coordinate is 0, y coordinate is 15, depth is 6 that means this is the point. Let me just make a small correction in this. The y coordinate is positive in this direction and therefore this will be minus, this will be minus, these two will be plus (Refer Slide Time: 17:44). With this correction now point two P_2 will lie at 0, 15. The x coordinate is 0, y coordinate is 15, depth is 6 that means this is will be P_2 and P_3 will lie at 6, 0, 6 that means the x coordinate is 6, y coordinate is 0, depth is 6. That means this will be P_3, this mid point will be P_3.

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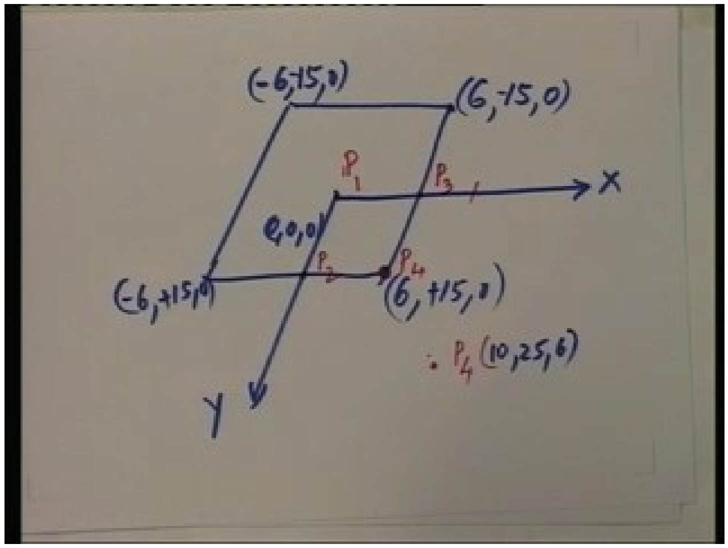

So we have centre, mid point one of the sides, mid point one of the other side and then we have 6, 15, 6 that means this corner, this is P_4. Lastly we have a point at 10 meters along the x axis, 25 meters along the y axis in the positive direction and 6 meters depth. So there is a point P_4 here whose coordinates are 10, 25, 6. Let us see how to determine the stresses beneath this. Remember we need to have the point at which we want the stress to be below the corner of a rectangle, that is the starting point. Let us take a look at the illustration that I have just now sketched on this paper. A clear illustration is available on the monitor. The rectangle is A B C D this point is C, as you can see here (-6, -15, 0), (6, -15, 0), (-6, 15, 0) and (6, 15, 0) that is this corner.

The point P_1, P_2, P_3 where the point P_1 is located directly below the origin here at a depth of 6 meters. P_2 is located below the mid point of this side, P_3 is located below the mid point of this side, P_4 is located at this corner and P_5 outside the area. In order to better comprehend, in order to understand better the relative locations of these points we shall sketch just the plan of this problem. Here is the plan. This is a plan view an illustration of the problem. A B C D is here, this A B C D is here. Then the points P_1, P_2, P_3, P_4 lie at the centre, mid side and at the corner whereas the point P_5 is out side the area at point G.

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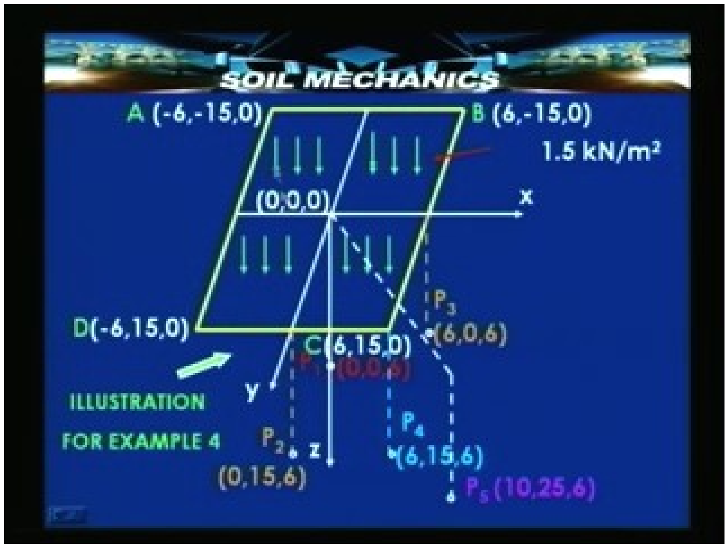

So these are the points P_1, P_2, P_3, P_4, P_5 at which we want the stresses. So now what we need to do is to successively put these points below the corner of a rectangle and calculate the influence coefficient and then compute the vertical stress. For example if I take P_1, if I consider these 4 small rectangles which meet at this point P_1, I can say that P_1 is already at the corner of four different rectangles. So if I can calculate the influence factor for one rectangle and the corresponding stress, then I will get the same influence factor and the same stress because the other three rectangles are also meeting at the same point P_1. They have the same dimensions, so the stress at point P_1 can be computed by multiplying by four the stress obtained due to one of these four rectangles. The dimensions of which are 6 meters in the horizontal x direction and 15 meters in the y direction.

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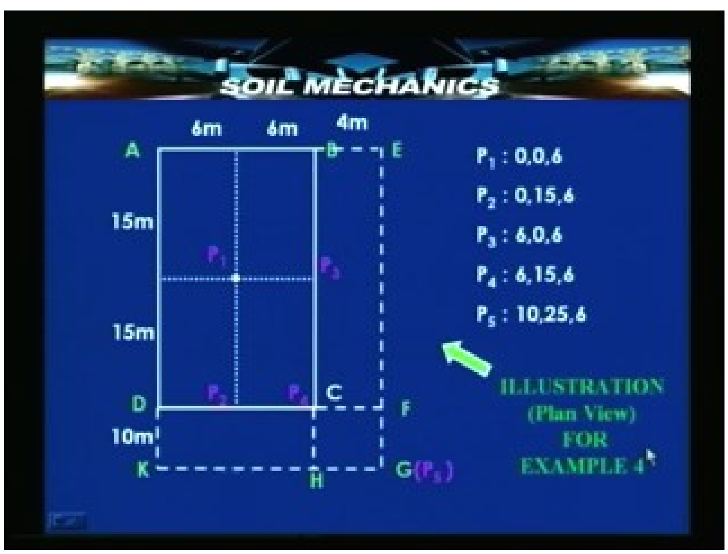

Whereas for P_2 we can take it to be lying at the corner of two larger rectangles. For example this rectangle of 6 meters width and 30 meters in the y direction has P_2 below one of its corners. Similarly for this rectangle ending in B here and C here also P_2 lies at one of the corners. Therefore the influence coefficient for P_2 can be obtained by considering this larger rectangle and its mirror image the other larger rectangle and by multiplying the influence coefficient by two and the load that is coming on the area, we can get the stress at the P_2. Similar logic applies for the point P_3 we can take two rectangles of 15 by 12 meter dimensions, one rectangle here one rectangle here. But for point P_4 since it is directly below the entire ABCD at the corner C, we can directly take the entire area as contributing to the stress and the dimensions of that area would be 12 by 30 meters. As far as point G is concerned we need to do this in two stages.

For example AEGK can be considered to be one rectangle where G is below one of its corner. However the given loaded area is only ABCD, so if we take a larger area AEGK we will be over estimating the stresses at the point G. So we need to subtract from this the stresses that we have obtained in addition because of considering a larger area. So we take rectangle DFGK which has a corner again above the point G and therefore this rectangle and its influence coefficient can be computed and the stress can be computed and subtracted from the stress that we obtained for the total area AEGK. Similarly BEGH also can be considered to be a rectangle with a corner at G and the influence coefficient for this and the stress corresponding to this can be computed and also subtracted. But however now if you see CFGH figures in both these rectangles which have been additionally added. Since this is getting subtracted twice, we need to add at once the stress due to CFGH. Since that also has G as one of its corner, the stress distribution can be computed by the same manner by taking an influence coefficient for the area CFGH.

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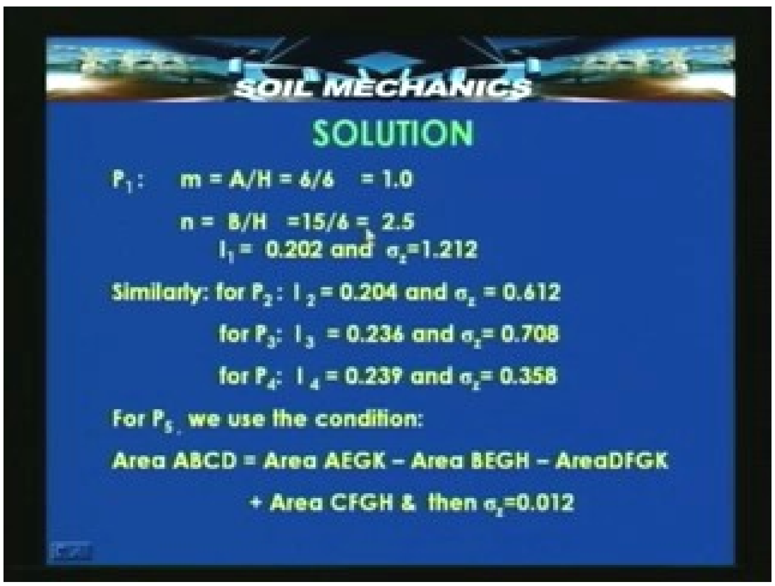

Let us see how this is done. For P_1 since it lies at the centre of the rectangular area, m will be 6 upon 6 where 6 is the half the dimension in the x direction and the next 6 is the height or the depth at which we want the stress. And n will correspond to half the dimension in the y direction that is 15 divided by the same depth H that is equal to 2.5. If you look at the influence coefficient table. Let us take a look at that table. If you take this table we have m equal to 1.0 and n equal to 15 upon C. Let us take a look, m =1 and n = 2.5, actually it should be 2.25. So if we go here and consider the influence coefficient chart we already have here, corresponding to m =1 and n = 2.25 which comes here we can work out the influence coefficient. That will be some where here (Refer Slide Time: 25:56) and that influence coefficient is what we need to consider and that comes out to be 0.202 and the corresponding stress will be four times this influence coefficient into the load and the load that is coming on this is also known, if you take a look at the problem. The load that is coming on this is 1.5 kilo Newton per meter square and therefore this influence coefficient into 4 into 1.5 will give you the stress.

Similar logic can be applied to P_2 and based on the explanation that I gave a little while ago, as to which rectangle should be considered for P_2, which rectangle must be considered for P_3 and similarly for P_4. We can compute the influence coefficients and the stress but for P_5 we need to consider the area AEGK, a larger area containing the point G and subtract from that, two of the smaller areas and add the areas CFGH which gets subtracted twice. And the corresponding influence coefficient and the corresponding stress can be computed and the final value of stress would come out to be 0.012 kilo Newtons per meter square.

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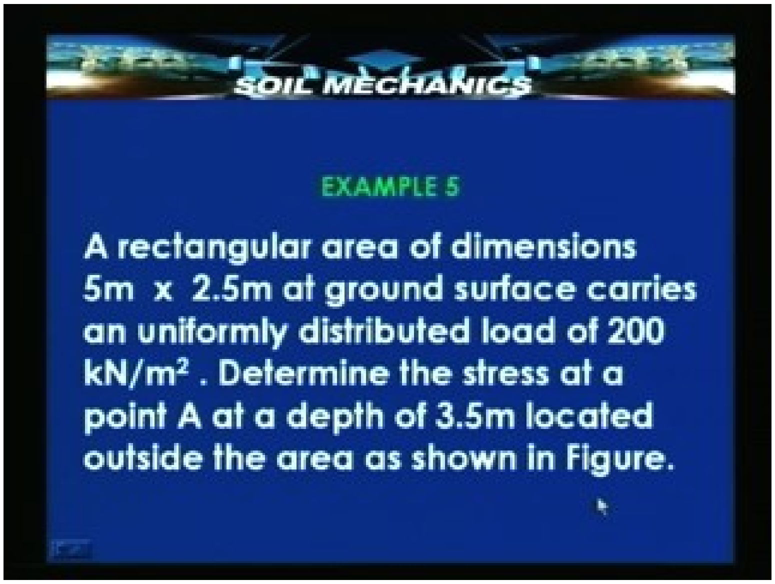

Let us take another example, this is again the example of a rectangular area. The dimensions are 5 meters by 2.5 meters. The problem is very similar to the previous one. The load is 200 kilo Newtons per meter square instead of 1.5. We again need to determine the stress at a point A at a depth of 3.5 meters rather than several points at several different locations as we did in the previous problem. We shall now consider only one point but it is located outside the given area. The method that we shall apply will be exactly the same as what we applied for the point P_5 in the earlier problem.

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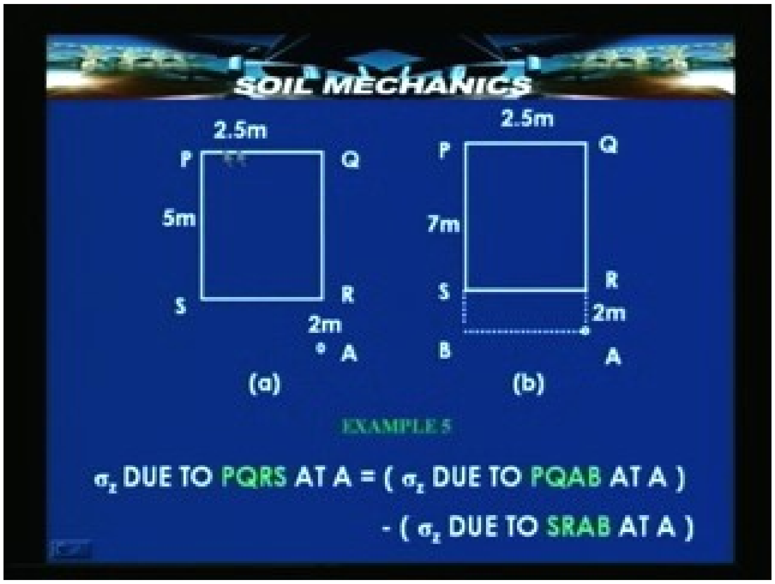

Let us take a look at this solution. The given area is 5 meters by 2.5 meters, the point A lies outside the given area and therefore in order to compute the stress at this point we take initially L larger area. The larger area would have dimensions 2.5 in the x direction and 7 in the y direction. All that we need to do is to compute the stress due to uniformly distributed load on PQAB which will be a larger area and subtract from that due to RABS and that is what we have. The sigma z due to PQRS which is the area of the interest is equal to sigma z due to PQAB and sigma z due to SRAB both at the point A, the later is subtracted from the formula.

The solution is, first take the larger rectangle compute m, compute n determine the influence coefficient from the influence coefficient table. Because the influence coefficient table that we use for rectangular areas have m and n values ranging from zero to infinity. And therefore they cover the entire range of m, n values that are feasible or practicable. Here therefore there is no problem in determining influence coefficient for any value of m and any value of n lying between zero and infinity.

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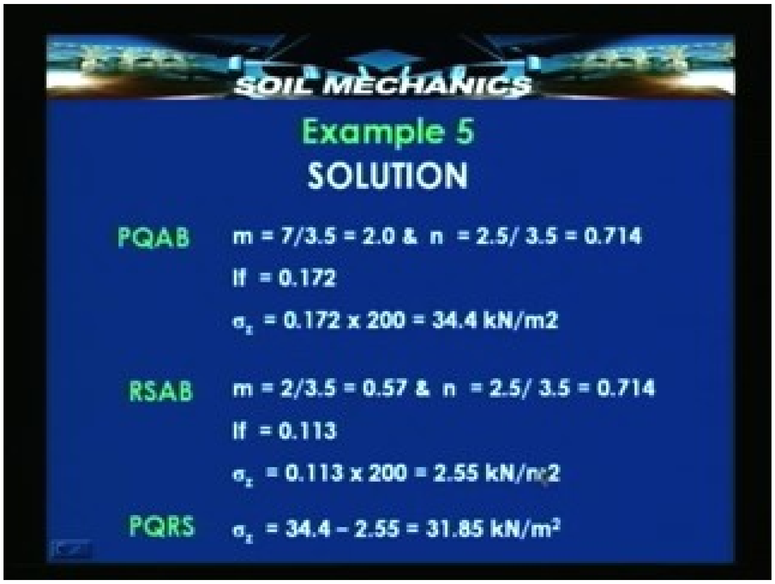

So we use that chart, for m and n we get equal to 2 and 0.714. We get influence factor as 0.172. The corresponding stress will now be this influence coefficient multiplied by the load equal to 34.4 kilo Newton's per meter square. We do not multiply this by either 4 or 2 because the rectangle is only one and we are determining the stress below one of its corners. Now RSAB is similar, m will be this much n will be 0.714 and the corresponding influence factor from the table would be rather 0.113 and sigma z will be 2.55. We note that because there is only one rectangle involved in this, as well as this. We are not multiplying this by either 2 or 4. We are only multiplying it by one and that is implicit and that is why, it is not mentioned. So also with PQRS, PQRS has a stress value sigma z corresponding to a value of m and n equal to 2 and 2 upon the depth which is also 2 and that will give us influence coefficient for that smaller area. And that needs to be added and then we can get PQRS, algebraically added and we get PQRS.

Now suppose we want to compute stresses below circular areas. Then we need to use a different table. The table that we can possibly use are two in number, either we can use a table in which z by R varies from say 0 to 5 and corresponding influence factor values are given. This is one of the possibilities; another possibility is where influence factors are given at regular intervals from 0 to 1 and the corresponding capital R by z ratios is given.

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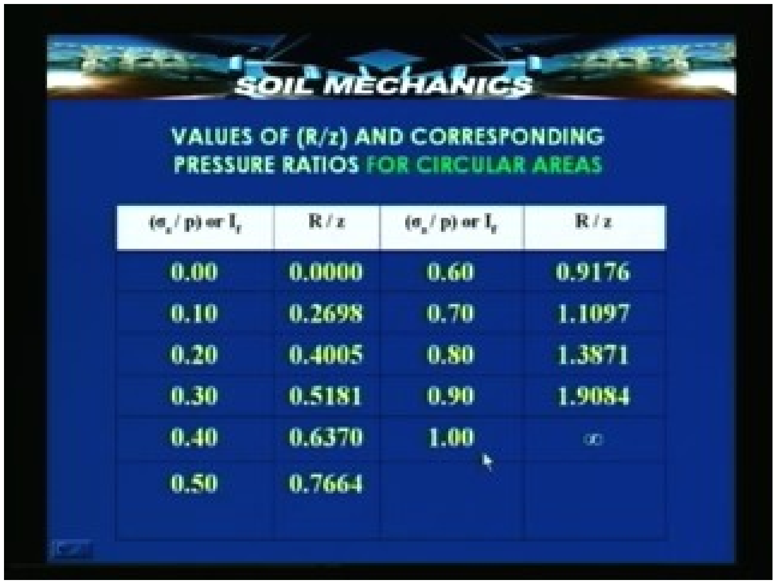

(σ_z / p) or I_σ	R / z	(σ_z / p) or I_σ	R / z
0.00	0.0000	0.60	0.9176
0.10	0.2698	0.70	1.1097
0.20	0.4005	0.80	1.3871
0.30	0.5181	0.90	1.9084
0.40	0.6370	1.00	∞
0.50	0.7664		

It is the former table which is useful for calculating stresses below a circular area. The range here is 0 to 5 which means that if in any given problem the depth is such that the ratio depth by radius of the circular area, loaded area is more than 5. Rather than using this table we shall have to use the formula directly. If we want to use this table which is the ideal one for computing influence factors for loaded areas which are circular and not the second table where we have influence factor corresponding to capital R by z. We will see shortly that the second table has its own advantage. The second table is useful for generalizing the problem of computing stresses and this generalization would eventually lead us to compute stresses not only beneath circular area for which this table has been evolved but also areas which are either rectangular or for that matter any arbitrarily shaped area.

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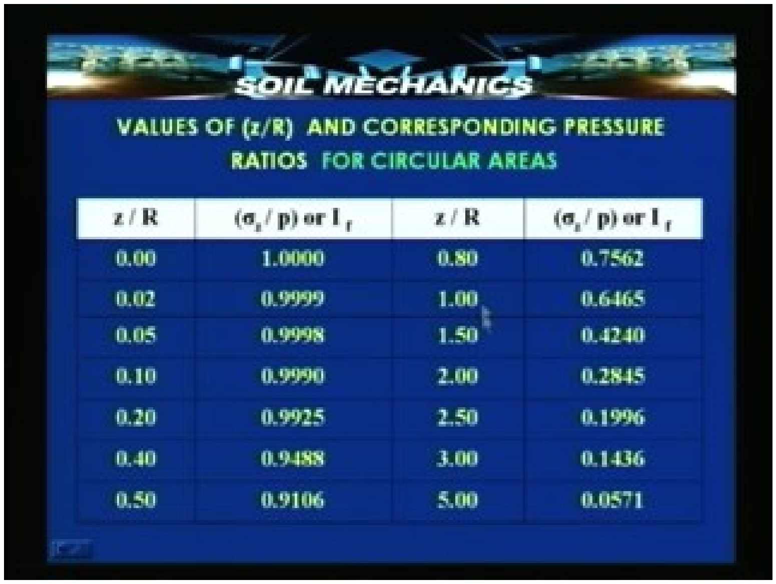

z / R	(σ_z / p) or I_f	z / R	(σ_z / p) or I_f
0.00	1.0000	0.80	0.7562
0.02	0.9999	1.00	0.6465
0.05	0.9998	1.50	0.4240
0.10	0.9990	2.00	0.2845
0.20	0.9925	2.50	0.1996
0.40	0.9488	3.00	0.1436
0.50	0.9106	5.00	0.0571

We shall come back to this second table later on or in a short while. It is the first table where we have z by R versus I_f that we shall be using for computing stresses below circular loaded areas.

Let us take the example of a circular loaded area and see how to compute the stresses. The statement of the problem is a circular foundation of diameter 3 meters. So diameter 3 meters, it stands on the horizontal surface of a semi infinite medium and it carries a load of 900 kilo Newtons. There is an area with diameter as 3 meters standing on a surface of a semi infinite medium and it carries a uniformly distributed load of 900 kilo Newtons over the entire area. That means the load which can be considered to be uniformly distributed over the entire area. That is load intensity per unit area will be this 900 divided by the area which is 900 divided by pi into d where d is 3, so d square upon 4.

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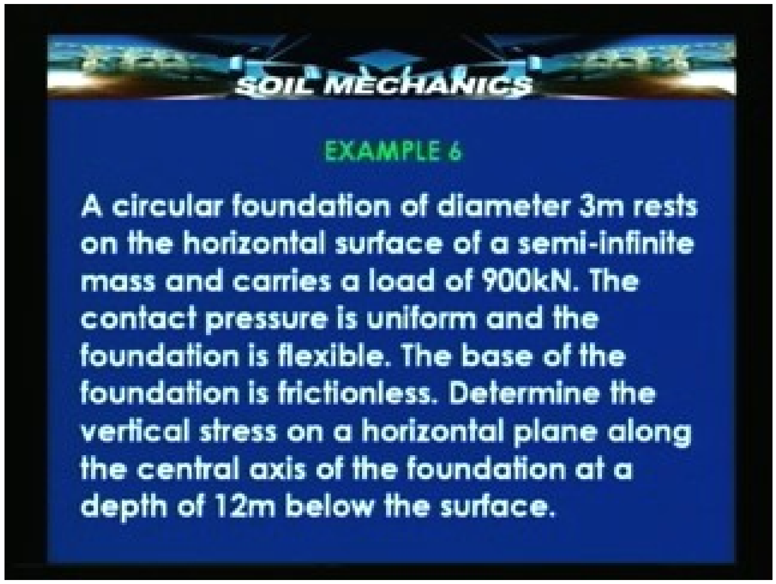

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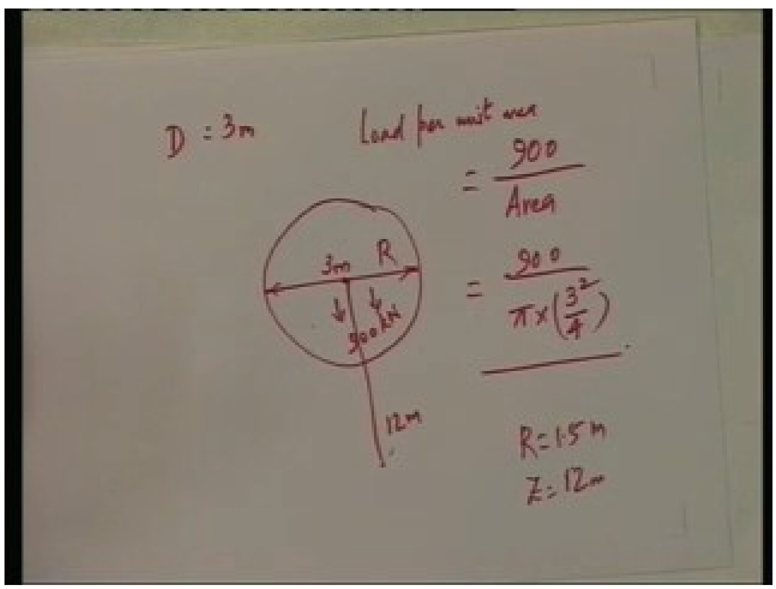

This will be the uniformly distributed load that comes on the circular area and for this we need to find out the stress at any point. The point in which we are interested is the vertical stress on a horizontal plane along the central axis of the foundation at a depth of 12 meters. That means the point we are interested in is located directly beneath the centre at a depth of 12 meters. The radius R is 1.5 meters, the depth z is 12 meters. So we need to use the influence coefficient charts or the formula depending upon whether this value of R and the ratio R by z lies within the range of table or not. This diagram show that this is a circular area of radius 1.5 meters uniformly loaded, the load per unit area is 900 by pi into d square by 4 and the formula that is to be used for computing influence factor is this (Refer Slide Time: 35:44) provided the ratio R upon z lies beyond the range of the table.

In fact here it does lie beyond the range of the table, so since the influence coefficients table includes values of z by R up to 5 only. We shall use the formula for computing I_f. According to this formula here, I_f is one minus one upon R upon z whole square plus one to the power of 3/2, R is 1.5, depth is 10 meters.

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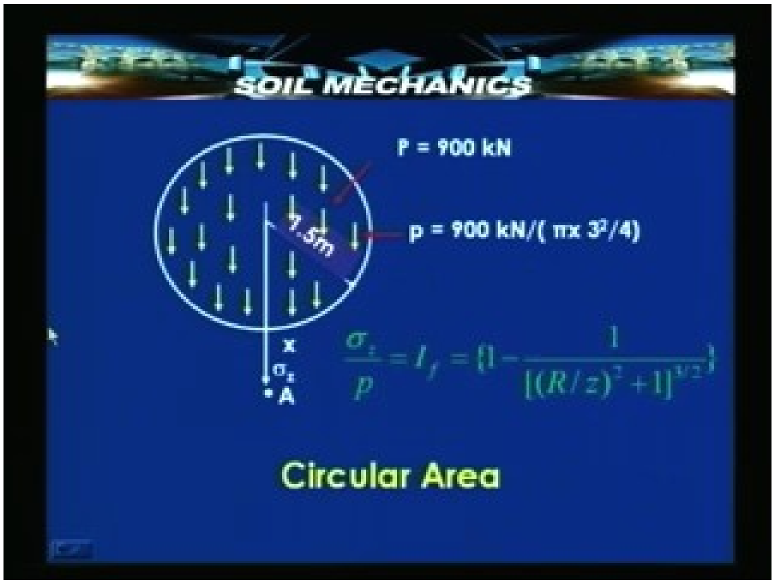

So with this it is possible to compute influence factor and you can get the influence factor as 0.023 and the stress as 0.023 that is the influence coefficient, into the stress per unit area or the load per unit area which is 900 by pi d square by 4 which is in fact equal to 127. This will give you a stress of 2.92 below the centre of the loaded area at a depth of 10 meters. We come to the last and one of the most important aspects of stress distribution problems. As I said it is possible to compute very easily, relatively using these influence charts, the stresses below rectangular and circular areas. But what about areas other than circular or other than rectangular. For these we use a graphical chart evolved by Newmark, that is known as the Newmark chart. Let us take a look at this. The principle of the Newmark's chart is based on the stress distribution beneath a circular loaded area.

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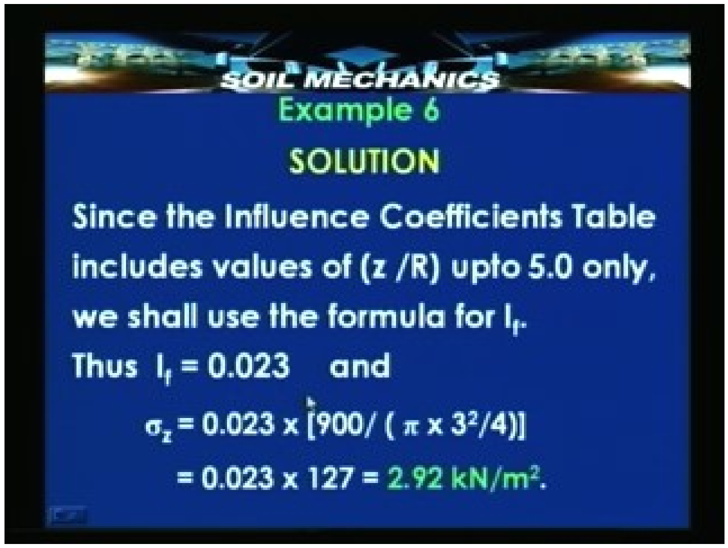

Suppose you take this circular loaded area, the influence factor is one upon rather one minus one upon R by z whole square plus one. You remember that we saw a table some time back in which we have the influence factor corresponding to different R upon z values. So that is what we make use of here. For different values of R by z we know therefore what are the corresponding influence factors or rather for a given influence factor we would know what are the R by z values which correspond. Obviously for the same influence factor there will be several R values and several corresponding z values which will same R upon z ratio. So what this signifies is that for different combinations of R and z, we will be able to develop areas which will have influence factors equal to a predefined value.

All these areas of different R upon z values giving the same influence factor will be concentric circles, if the depth remains constant. This gave an idea to newmark to convert or represent influence factors in terms of sectoral areas of circles as shown in the next figure or the table. This is the table which is used and this is the chart (Refer Slide Time: 39:49). This chart gives you for a predefined depth scale, different R values and corresponding areas with same influence factor.

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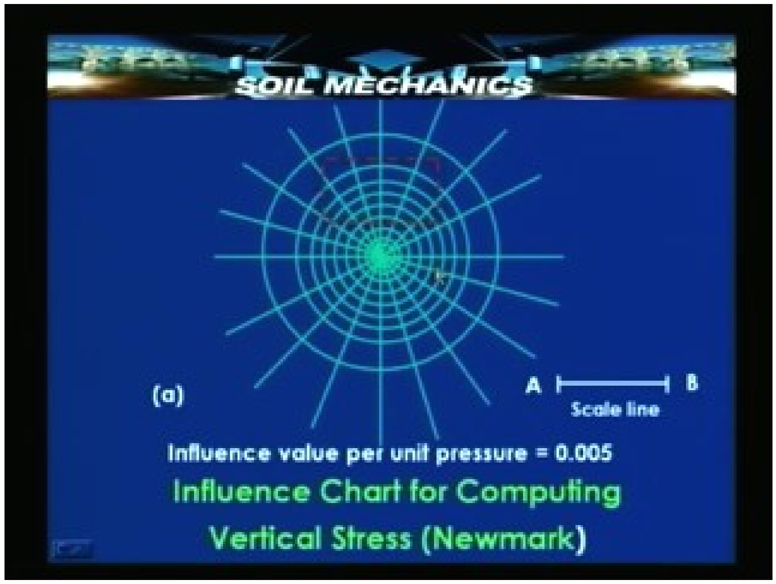

This means that every single sectoral area that we see here has got the same influence coefficient value. Because for each one of them R upon z remains the same. Then what is the value of each one of these small sectoral areas. It is possible to choose any given values of influence factor and plot these concentric circles to predefined scale AB. However Newmark has plotted after several trials and based on his experience, a chart with influence value equal to 0.005 which means that every small area here contributes in terms of influence factor a value of 0.005. This means that suppose we have a rectangular area similar to the red area that is depicted in this figure. Then every small area that comes within this rectangular area corresponds to an influence value of 0.005, which means that the total influence value for this entire rectangle will simply be equal to the area of all the sectoral elements put together. That means the sum of the areas of every small elemental sectoral area.

This gives us an interesting method of computing stresses beneath any area. All that we need to do is to choose a scale AB as the scale for the depth at which that is the value of z at which we want the stresses. This chart that is shown here has been plotted for a depth scale given by AB equal to z. Therefore in order to use the same chart for all values of loading, for all depths all that we need to do is always take the length AB of this chart as equal to the depth. Automatically the value of R here proportionately changes and therefore a circle of a given R or a given R upon z will continue to be representative of the same influence factor 0.005. That is the essence of this newmarks graphical chart for determining vertical stress. This means that if we have an area which is non rectangular which could be trapezoidal, which could be triangular, which could be simply a quadrilateral or even a circle, we can replot the drawing of the loaded area, the plan of the loaded area using a depth scale AB equal to the depth at which we want the stress. Now this shows that if we want a different depth we need to convert the dimensions of the loaded area correspondingly to a different scale. So the depth decides the scale and suppose we now have an area which is not rectangular, we can still determine the influence factor by just counting the number of elements that come within the given area.

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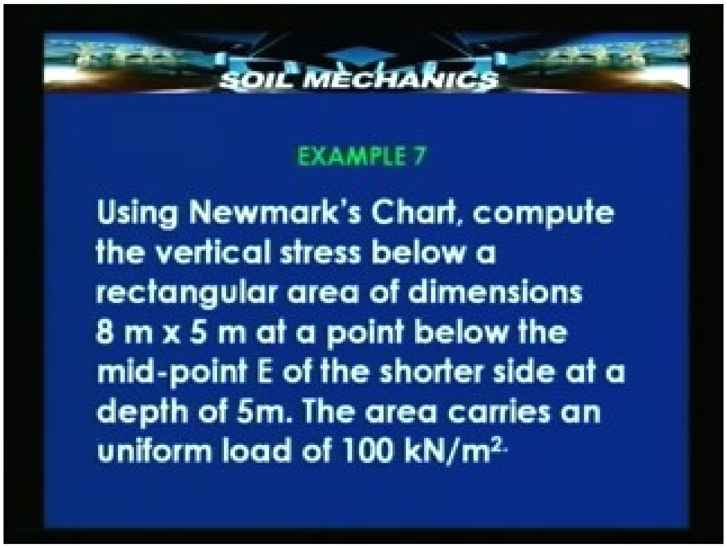

Let us take a numerical example. This numerical example reads like this. Using newmarks chart compute the vertical stress below a rectangular area of dimensions 8 meters by 5 meters. As I said this chart can be used for any shape of any area how ever we shall take the example of a rectangle which is much simpler to examine. And we shall verify our results that we obtained by the newmarks chart by computation using the influence coefficient table. If we were to plot this, then this area would be 8 meters by 5 meters. The depth scale for the influence coefficient chart is known a length AB, plot these 8 meters using this scale and also this dimension 5 meters using this scale. And then place this let us say A B C D area in such a way that its centre lies over the centre of the influence coefficient chart. Then we just measure the number of areas and get the influence factor. Influence factor multiplied by the load that is coming on the area that is 100 kilo Newton per meter square will give you the stress at this point.

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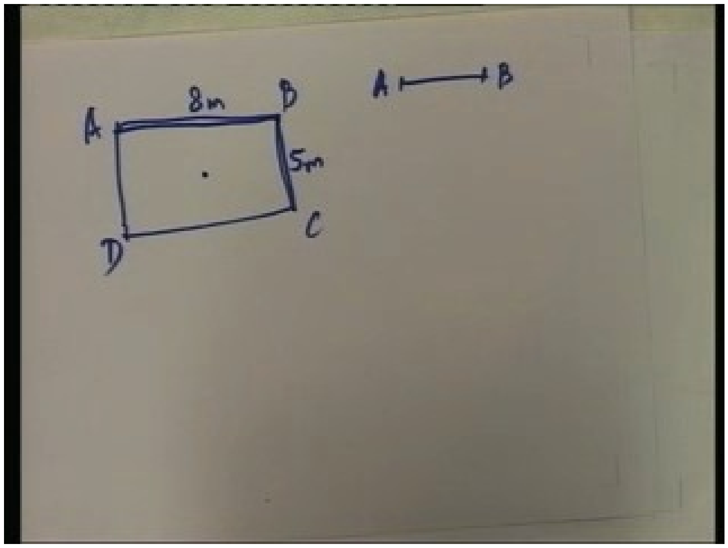

So our area is 8 meters by 5 meters, depth is also 5 meters and therefore the dimension LM, the scale of the diagram represents the depth of 5 meters. So LM defines the scale of the chart and LM is equal to depth equal to 5 meters. This means that if I plot to scale an area 8 meters by 5 meters in dimension it will look like this. And let us say I interested in stress at a point E which is in the middle of the shorter side. Then this is the shorter side, the middle of the shorter side lies directly over the centre of the chart.

After drawing this rectangle to scale on a transparent sheet of paper and then super imposing it over this diagram in such a way that the point of interest lies exactly at the centre, we can get the influence factor the total influence factor corresponding to the whole area. The sum of all the elemental areas which come here is the one which corresponds to the total influence factor. We need to calculate and count each one of these elemental areas. We can make an allowance for areas which are not lying completely within the steady area shown in the rectangular form in red color. But we can count the number of elemental areas by giving due weightage to partial areas as well. Then the solution will be like this, the total number of elemental areas lying under ABCD, counted on the basis of the newmarks chart works out in this case to approximately 55.

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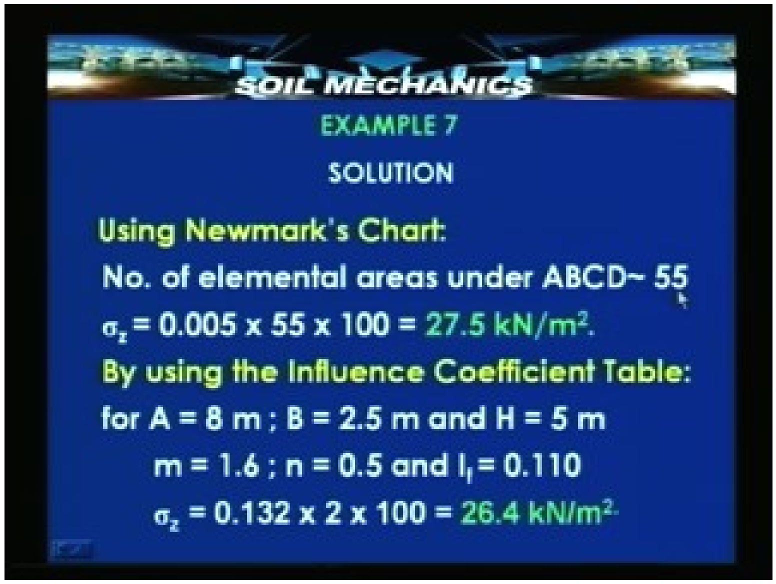

The vertical stress would obviously be the influence coefficient for one elemental area that is 0.005 multiplied by the total number of elemental areas that we observed under this chart for the given area, that is 55 and the loading intensity that is 100 kilo Newtons per meter square. So this works out to 27.5 kilo Newtons per meter square that is the vertical stress below the mid point of the shorter side, one of the shorter sides of the given rectangle. You can see here that these dimensions are such that we will get m and n values corresponding to this which will lie well within the influence coefficient table. So we can also calculate the vertical stress here using the influence coefficient table. If we were to use that then we need the dimensions A, B and the non dimensional values of m and n. We find here A is 8 meters, b is 2.5 meters the reason is the rectangle that we need to consider would be 8 meters in this direction and only 2.5 in this meter in this direction, because point E lies at the corner of this smaller rectangle.

It also lies at the corner of the other mirror image rectangle and therefore what we determine as the influence factor for one rectangle is also valid for the other rectangle. And the stress at E or the point below E due to both rectangles together will be simply twice what we get for one rectangle. So let us see. For one rectangle the length is 8 meters A therefore is 8 meters and B is only half the total width because the point E lies at the mid point of the shorter side. So B is only 2.5, H is already defined 5 with the stress we need is at a depth of 5 meters. So this gives us m = 1.6, n = 2.5 by 5 that is 0.5 and corresponding influence factor is 0.110.

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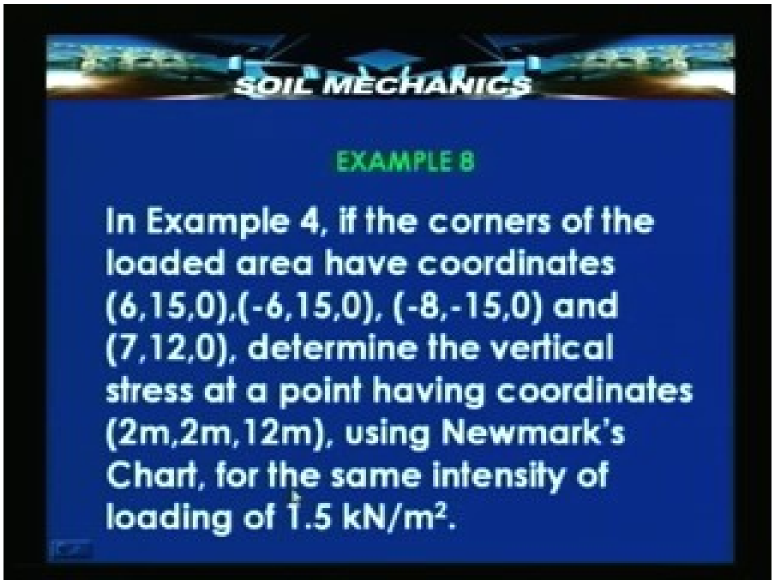

The stress would be the value that we get corresponding to the influence factors readings that we get from all these and that into 2 into 100 would give you the stress arising due to the area. There is a marginal difference between the influence factor that we get from newmarks chart and from the influence coefficient table. The chart involves an approximate counting of the total number of elemental areas coming within the given area and therefore it is slightly approximate. Whereas the value given by the influence coefficient table is far more accurate. Let us take one more example, this is also a loaded area but this is not rectangular. The corners of this loaded area are (6, 15, 0), (-6, 15, 0), (-8, -15, 0) and (7, 12, 0).

That means this loaded area is a quadrilateral. In order to determine the vertical stress at any point say a point with coordinates (2, 2, 12) that is at a depth of 12 meters a point having x coordinate 2 and y coordinate 2 meters. We can compute the stress using newmarks chart. The intensity of loading let us is again 1.5 kilo Newton per meter square. Then this would be the definition sketch of the problem. This is the rectangle, given rectangle, point G is the point (2, 2, 12).

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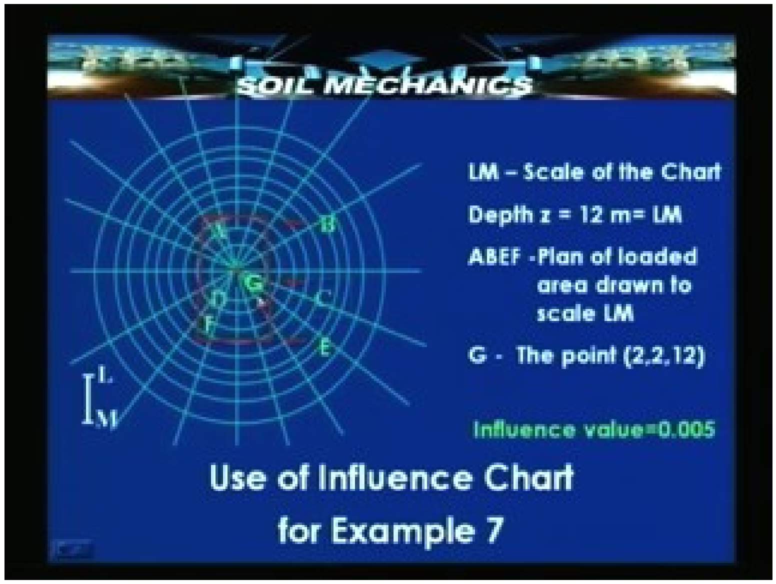

So we draw this rectangle to the scale LM that is shown here, LM being taken equal to the depth of 12 meters and then we draw this rectangle and super impose over the chart and count the number of squares and then we get the solution as using newmarks chart, 87 elemental areas multiplied by the influence coefficient value of one area that is 0.005 into the load 1.5 that is 0.653 kilo Newton per meter square.

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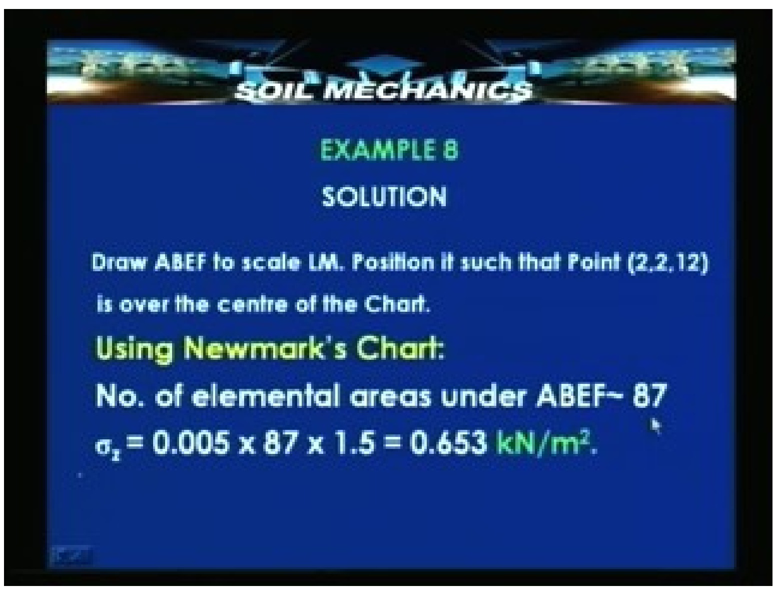

I have given here a few exercises that you can attempt, in order check for your self whether you have understood approximately or reasonably well, whether you need to work further, whether you need to go over this lecture over and over again until you get the matter clear in your head, depends upon how clearly you have understood these various problems and how you are able to solve them. Therefore this is an exercise for which I shall not be giving the full detailed solution; I shall only give the final answer. This is a rectangular area, 2 meter square carries a load of 1250 kilo newtons. What is the stress due to this at 2 meters? So this is a problem of a point load on a rectangular area. The answer is 105 kilo Newton per meter square.

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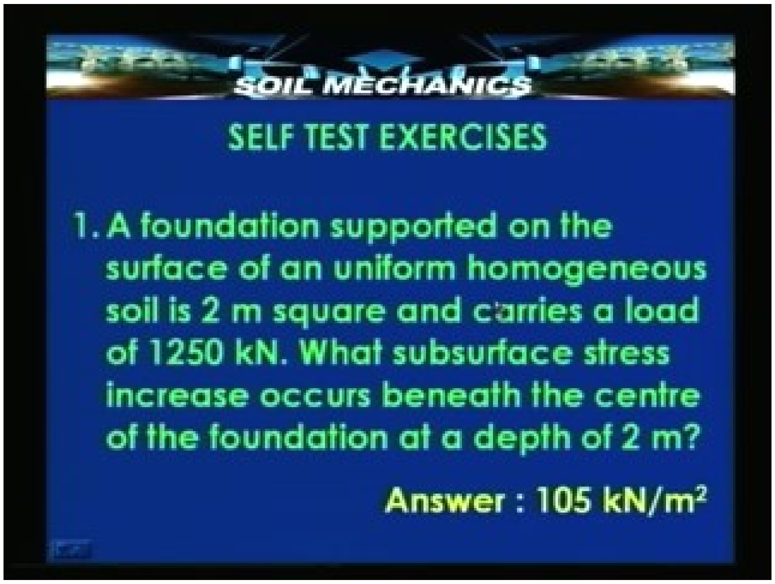

The next problem is an area of size 3 meters by 2 meters, we want the load that is total load being carried by this area. When its centre is at a depth of 5.5 meters and not directly below the concentrated load but at a distance of 4 meters from the line of action of the concentrated applied load. The applied load is 50 kilo newtons acting on the surface, the answer will be 1.80 kilo newtons. There are two more problems here. There is a rectangular area, the coordinates of which are given. The coordinates very clearly show that it’s a quadrilateral. The vertical stress is required at a point (5, 5, 10) meaning at a depth of 10 meters, at a point having x y coordinates both equal to 5 meters.

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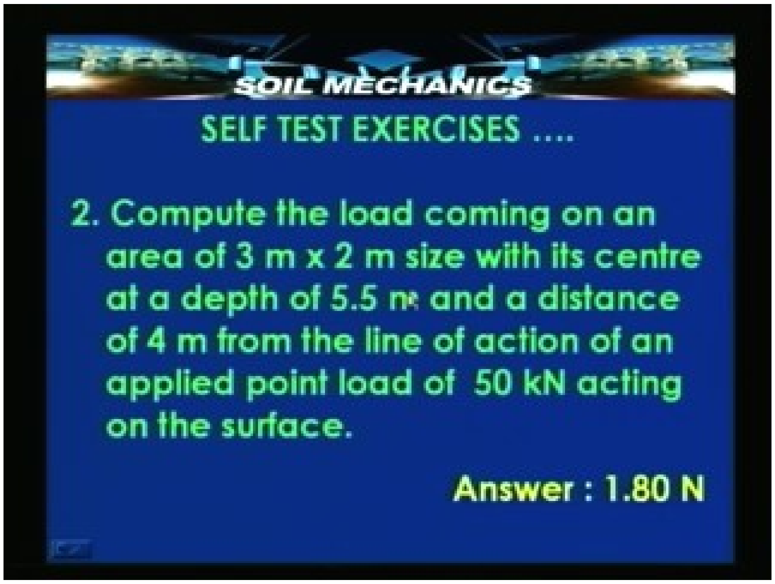

You can draw this to the scale of the newmarks chart, super impose this on the newmarks chart and get the total number of areas lying within this. That will give you the total influence factor that multiplied by the load 2 will give you the final answer of 0.95 kilo newtons per meters square. Lastly I would like you to solve example 6 that of the circular area using newmarks chart rather than using the formula as we did earlier.

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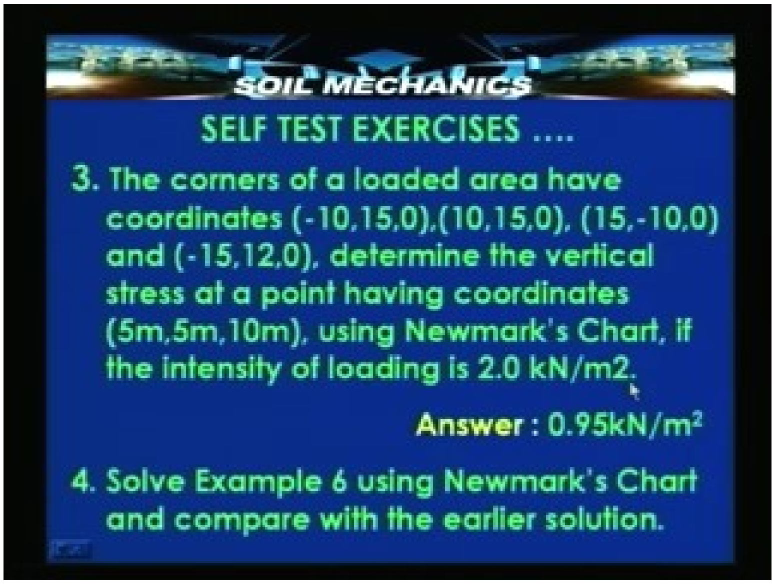

You can compare the two in order to understand how much difference is created by using the newmarks chart. The influence coefficient tables which contain influence coefficient values correct to the third forth decimal place always give you stresses a little more rigorously. Whereas the newmark chart give you the stresses some what approximately. The advantage of the newmark chart therefore lies mainly in areas which are not in regular shape that is neither rectangular nor circular. Now just to wind up let us summarize. You need to understand very clearly some of the terminology that we have used in the course of these 6 lectures. We have used terminologies like semi infinite mass and elastic material, different types of loads point, line and strip, uniformly distributed loads and then we have used terminologies such as influence coefficient tables, influence coefficient chart, influence coefficient factor, influence coefficient value, influence coefficient say just coefficient. We have also used the terminology critical depth, that is the depth at which the stress transmitted reduces to approximately 10 % of the uniformly distributed loads.

You need to understand these terminologies very well in order to appreciate, there importance in the assumptions that have been made in the application of the Boussinesq's theory and therefore the limitations of the Boussinesq's theory itself. Take a look at the limitations. The first limitation is that Boussinesq's theory assumes very ideal condition and impact is these ideal conditions are never met with and therefore that makes a lot of difference. The solutions obtained are deviant from the real stress values in the field. The initial stresses due to the weight or ignored because the medium is assumed to be weightless in Boussinesq's theory. This also adds to the error in computational of the vertical stress because the self weight of the material does impact on the vertical stress.

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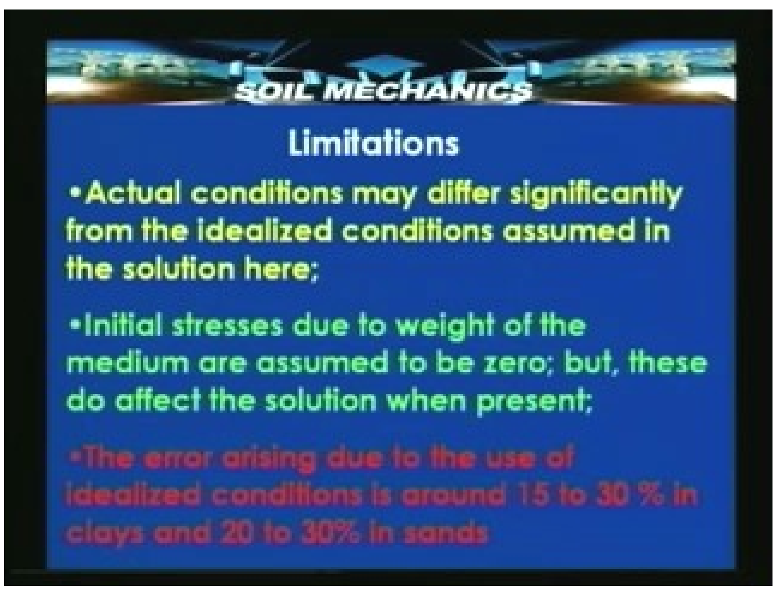

Lastly the error that arises due to the use of Boussinesq's theory due to the idealized conditions that we assume in Boussinesq's theory works out to approximately 15 to 30 % clays and 20 to 30 % in sands. The reason is, it is a little more in sands and a little less in clays that is because of the nature of the material. The more the idealized assumptions are closer to reality as in the case of clays, better will be the closeness of the results and lesser will be the errors.
So in this lecture we have covered three different areas and the influence coefficient chart and the limitations of the elastic theory. This completes our understanding and our

discussion of stress distributions in soils. We shall next pass on to another chapter called the theory of consolidation. With this I conclude, we shall meet again to discuss the theory of consolidation. Thank you