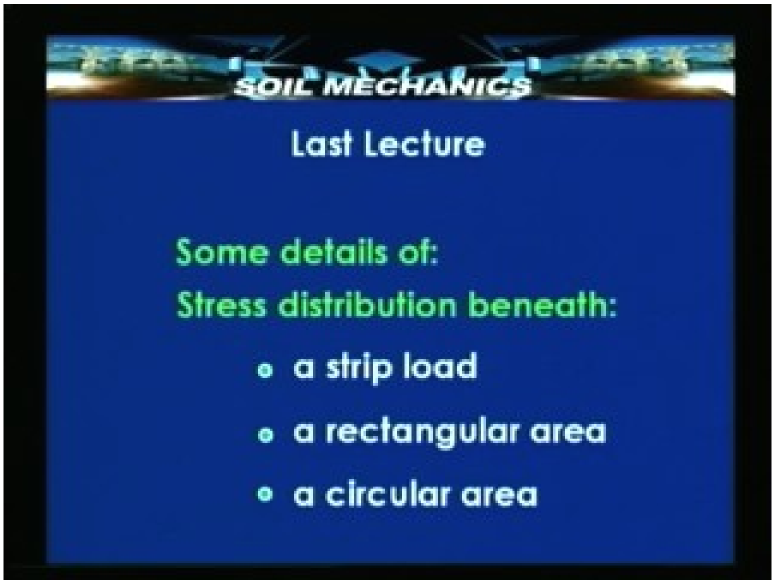
Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 32
Stress Distribution in soils
Lecture No.5

Dear students welcome to the next lecture on stress distribution in soils. This is the fifth in the series. In the last four lectures we had seen several important concepts and methods relating to stress distribution in soils. In particular we spent a lot of time in understanding what theory of elasticity is and how it has been ingeniously applied for determination of stresses in soils. Today we will proceed further; we will review briefly and add a few more points, cover a few more aspects of what ever we have already seen and also see how to apply this with help of a few examples.

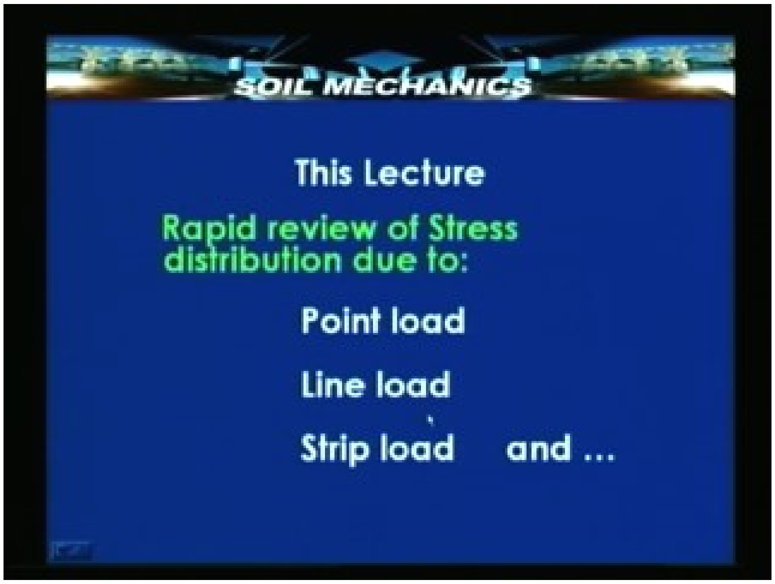
(Refer Slide Time 01:45)



So the scope of this lecture is going to be an extension of what we saw in the last lecture. If you look at this slide in the last lecture we covered, specifically speaking the method of determining stress distribution beneath a strip load, a rectangular area and a circular area. What we shall we do today is we will go through quickly and also add a few more points to our understanding of the effect of point loads, line loads and the strip loads and also the effect under a rectangular area, a circular area. In particular we will make use of a table called the newmarks influence table and we will see how best it can be applied for determination of stress distribution in soils.

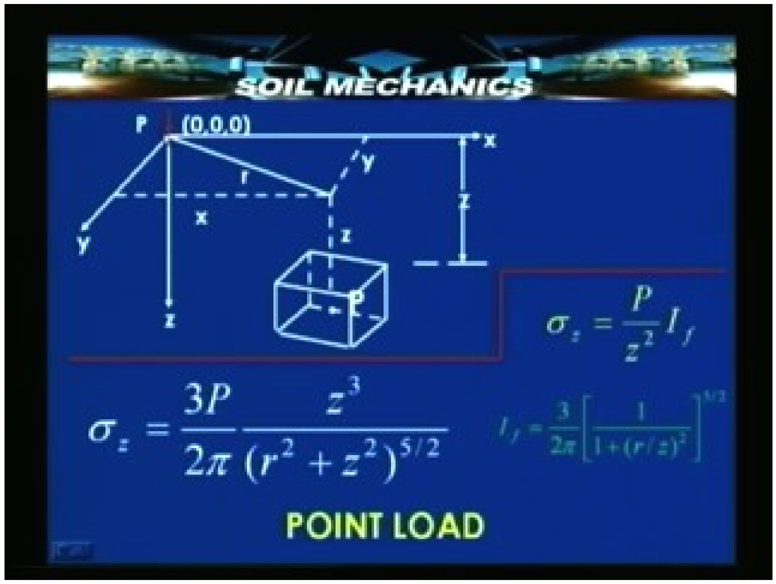
So let us go ahead with a quick review of some of the basic points that we had covered in the last lecture and also see additional details of what ever we had seen last time. Next slide shows in brief, in a nut shell what a point load does. You remember in the last lecture we discussed that the first and fore most contribution to stress distribution was by Boussinesq's for determining stresses under a concentrated load.

(Refer Slide Time 02:02)



If there is a surface ground surface, if there is a concentrated load which is also called a point load then using Boussinesq's theory we can calculate the vertical stress at any point p. As you know as we have discussed several times it is a vertical stress which is of primary interest to us. The scheme that is being used can be understood from this. This is the coordinate system, this is the load, and this is the element the so called parallelepiped envelop in the point at which we want the stress. The equation for the vertical stress as given by Boussinesq's is sigma z equal to 3 P z cube by 2 pi into r square plus z square to the power of 5/2 where all the coordinate axis and the coordinates are already mentioned in this figure, P is the load. As we saw last time this was very cleverly expressed in the form of P upon z square into a constant factor which in turn is known as the influence factor. You note here that this P is the load applied and z square has units of area where z is the depth, so P by z square is having the units of stress and this multiplied by a non dimensional quantity namely the influence factor gives you the stress at any point.

(Refer Slide Time 03:35)



So this is as far as point load is concerned, we even examined the application of this with the help of a small problem. This is a table which gives you in detail how the influence factor itself varies. Since this influence factor depends upon the coordinate ratio r upon z, I mean that is understandable, the influence factor has to be different for different points. The load remaining same and fixed at one point as you move over the medium in the lateral and the vertical direction downwards, at every point you will have a different stress and therefore a different value of influence factor.

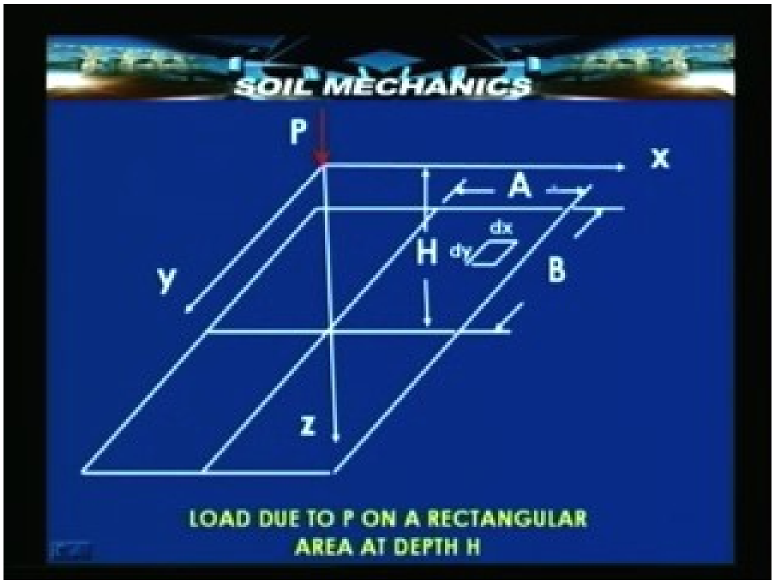
(Refer Slide Time 04:48)

r/z	I_f	r/z	I_f	r/z	I_f
0.00	0.478	1.00	0.0844	2.00	0.0085
0.10	0.466	1.10	0.0658	2.10	0.0070
0.20	0.433	1.20	0.0513	2.20	0.0058
0.30	0.385	1.30	0.0402	2.30	0.0048
0.40	0.329	1.40	0.0317	2.40	0.0040
0.50	0.273	1.50	0.0251	2.50	0.0034
0.60	0.221	1.60	0.0200	2.60	0.0029
0.70	0.176	1.70	0.0160	2.70	0.0024
0.80	0.139	1.80	0.0129	2.80	0.0021
0.90	0.108	1.90	0.0105	2.90	0.0018

And what is more convenient than a table which gives you all these influence factors values in a systematic way and that is what we have here. If you have a point load for vertical stresses the influence values for different r y z starting from zero and going up to a value of say 2.9 is given like this. And you can continue with these you can go for higher and higher values of r upon z. But this is an illustrative table which is not an exhaustive and in practice very often these ratios which are depicted in this table are found to be quite sufficient. So this table serves as a basis for us for calculating stress at any point due to a point load.

Consider a few more additional points which we have not discussed earlier regarding the effect of a concentrated load. We know that when a concentrated load is applied it is going to get dissipated according to some rule given by Boussinesq's theory. We saw in fact that it is varying inversely as the square of the distance as we go downward. It is of interest in practice to know what will be the total load that is transferred at any depth z and over what area. Suppose this is the area over which the load is getting dispersed, then what is the quantum of the load or what is the total load that is coming over this area where this is length and let us say this is the breadth. It is very interesting that this has been computed and in fact found to be very similar to the reverse problem which we had seen last time. That is if a rectangular area is loaded uniformly then what will be the stress at any point at the depth? This is a converse of that, if a load is applied at the surface, what will be the component of the load over a rectangular area at some known depth?
Let us see this a little more in detail. Suppose this is the load and this is the area, this rectangular area which is shown here. This is the area over which, we want the total load that is transferred and what fraction of P is transferred here. In order to do this, we can take a small element dx dy. Find out the stress at the centre of these by Boussinesq's theory by the same equation which we saw in the last slide. The equation meant for stress at any point below a concentrated load. This element being very small we can take this stress that we get at the centre of this to be stress on this area uniformly distributed.

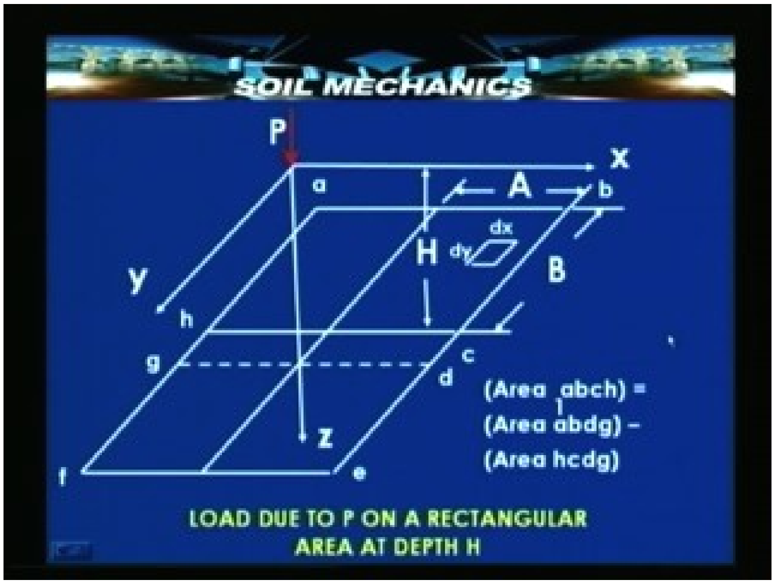
(Refer Slide Time 07:44)



So we get the total load on this elemental area as the stress at the centre multiplied by dx dy. It is very simple now to get the total load corresponding to the same depth over a rectangular area by simply integrating this over this entire area of dimensions say two times A in this direction and two times B in this direction. What you see here is one quadrant of the rectangular area has been isolated. You also must notice that the concentrated load is passing through the centre of the rectangular area. Therefore the load that we are going to calculate over this rectangular area will correspond to an area whose centre is directly below the load. If we have the total load in the small elemental area we can integrate this over an area given by dimension A and dimension B. And then since all other quadrants are also symmetrical in relation to this particular quadrant.

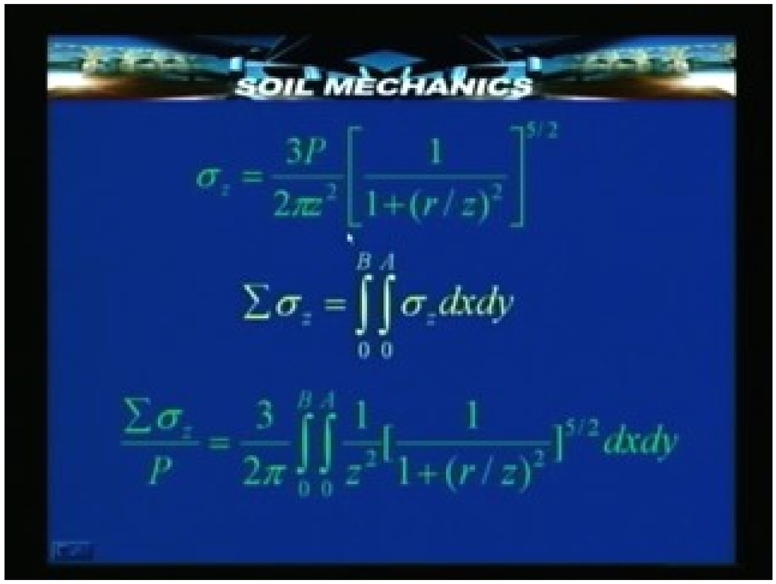
After having obtained the load over one quadrant by simply multiplying it by four, we can get the total load over the entire rectangular area. That makes the computations much simpler. So lets see if H is the depth at which we have a rectangular area and dx dy is a small element and A and B are the dimensions of one quadrant whose corner is directly below the load P, how to compute the total load over this area AB? Here are the details. The expression for the stress at any point is the same expression which we had been seeing all along. The total load at any point over a very small elemental area dx dy would be sigma of sigma z. That is the total area over a very small elemental area dx dy and it can be obtained by integrating the expression for stress sigma z over the limits zero to B and zero to A where A and B are the dimensions of the rectangular area.

(Refer Slide Time 10:02)

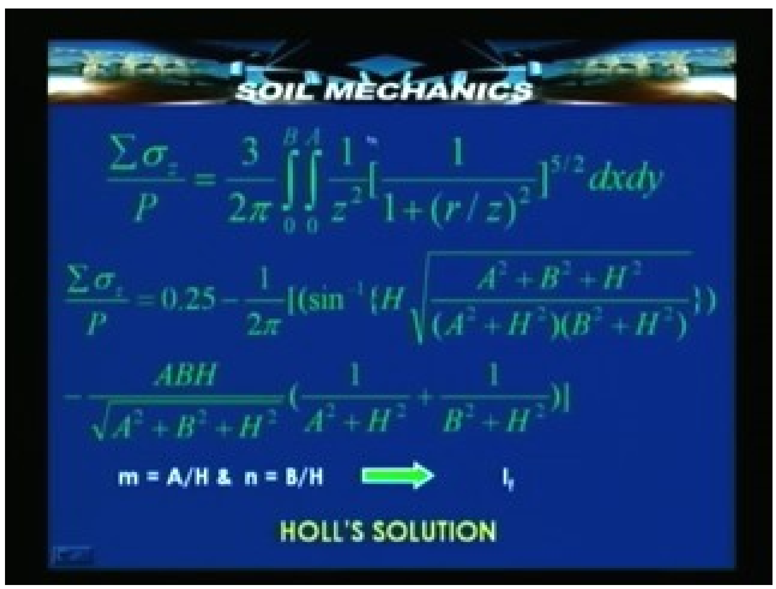


Now if we do integrate this, then divide this total loads by P we will find that this integration gives us an expression like this. That is we have simply substituted sigma z in place of here. This is the basic expression which gives what fraction of the load P is transmitted to any area at a depth z. If we proceed further this integration can be actually performed and you will then get, that the fraction of load transmitted over an area is equal to 0.25 minus this term minus this term (Refer Slide Time: 11:15). This is a very interesting expression and this is known as the Holl's solution. This is an interesting expression in the sense that this becomes something like an influence factor, this is what is known as an influence factor. That is what is the load transmitted to any area for a given load capital P and what is interesting is further H is the depth, A and B are the horizontal dimensions.

(Refer Slide Time 10:04)



(Refer Slide Time 11:05)



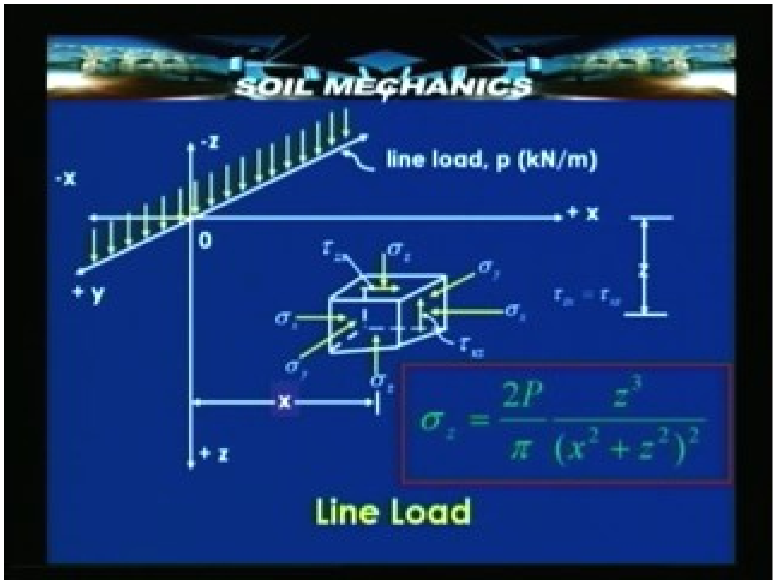
And if we non dimensionalize all the distances here by dividing throughout by H then we will have a term called small m which is A upon H, one of the dimensions divided by the depth and we will have another term small n which is equal to the other dimension B divided by H. And by taking different values of A/H and B/H and substituting in this expression we will be able to find out for a unit load capital P, what is the total load transmitted to an area AB and that is nothing but the influence factor. This Holl's solution can be conveniently used to find out the total load transmitted to any depth by a concentrated load.

Suppose we move on to another instance where a slight modification of this occurs. That is here is a rectangular area identical to what we saw earlier, only its centre is no longer coinciding with the z axis. In the previous case the centre of the rectangular area was directly below the load P. Here there is a small shift, the centre of the area is no longer directly under P in such a case we can still apply Holl's solution conveniently, imaginatively. All that we need to do is to take the area which is having one point directly below capital P. If I draw the additional line gd here then this point intersects the line of action P and now we can say that our given area whose centre is not coinciding with this is in fact equal to two rectangles of dimensions A in one direction B plus c d in the other direction.

And two more rectangles equal in dimension to two A in one direction and d e in the other direction which means that we have divided the given area into four quadrants again. Two of them are equal, the other two are also equal but different from these two and the concentrated load still passes through the corner of the rectangular area which means that the Holl's solution can once again be applied. All that we need to do is to remember that the influence value that we will be getting will be as before valid for one quadrant and to get the load over the whole area we will have to find out the influence factor for one quadrant here multiplied by two and for one quadrant here and multiplied by two and then add up.

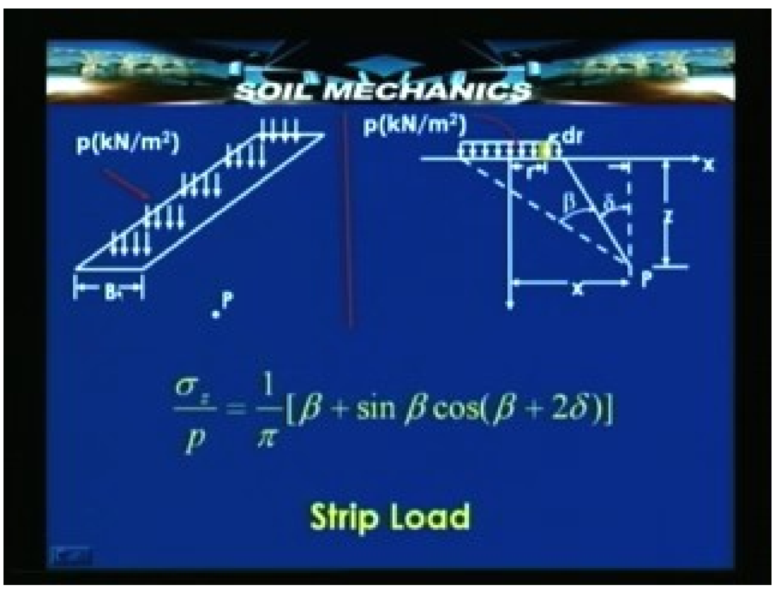
So in short area abch which is let us say the area over which we want the load can be expressed as area abdg whose corner is lying in fact below the concentrated load minus area hcdg whose corner is also lying under the load capital P. So this way any area including a small part of the total area can be conveniently analyzed for the total load. It is important to remember here that one dimension of the quadrant in the x direction for example is m which is equal to A upon H and the other direction is given as small n which is capital B upon H. Since H varies according to the point at which we want the stress, the scale A upon H and B by H enables us to non dimensionalize these dimensions of the area in such a way that one set of charts that we may be developing for m and n will be useful for all areas at all depths.

(Refer Slide Time 15:55)



See for example the area over which we want the stress can be easily split up into a number of parts and by applying the principle of super position we can get the total load either by subtraction or by addition depending upon the location of the centre of the area.

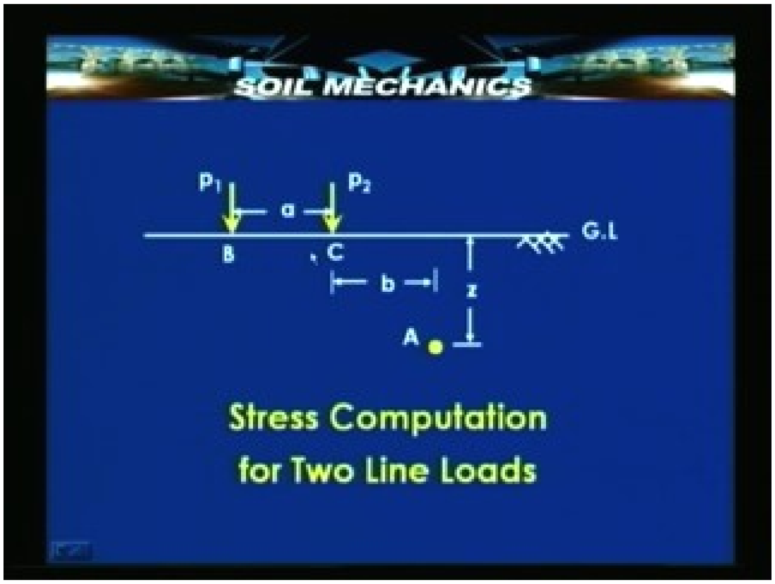
(Refer Slide Time 16:48)



Let us pass on to line loads. We will recollect that a line load produces a stress which is given by this expression. This again is merely an integration of the expression for a concentrated load over a long length. Further we can extend this to a strip load, line load expression can be integrated over an area like that shown here. And we can get an expression for the fraction of the stress as a function of the load per unit area small p and this is the expression in which we have two angles beta and delta which are marked here in this diagram. In this key diagram we have the angle beta and delta marked. So if we know these two angles if we know the load small p per unit area, we can calculate the stress at any typical point capital P at any known depth z due to a strip load. And once again as we did in the case of point load we can develop a chart of influence values. The advantage is that once we put in the non dimensional coordinates or dimensions z upon b by two and x upon b by two, we can get a chart which will be useful for any such area, any such strip loading for any depth irrespective of the dimensions.

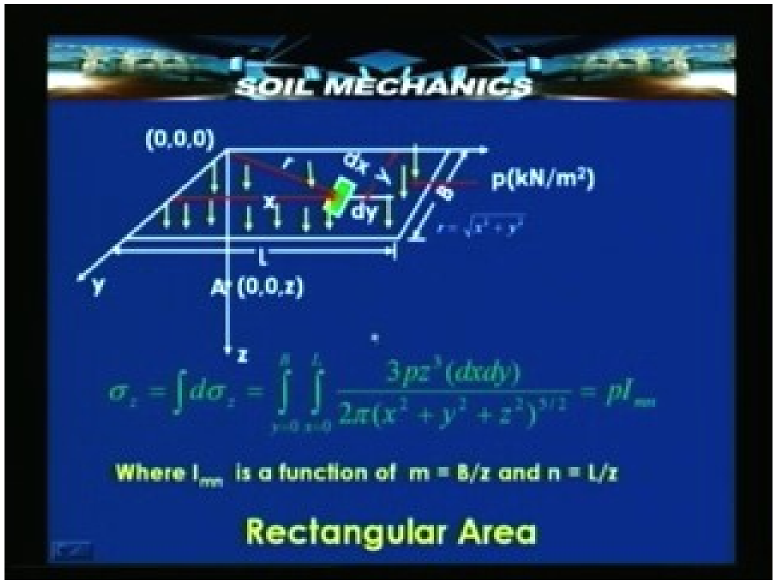
So this table for example gives you values ranging from z upon b/2 from zero to 2.5 and x upon b/2 varying from zero to 3. It is possible to enlarge this table and include values higher than these as well but in most practice this range is sufficient. We also saw in the last lecture that if we have two line loads, we can still employ the same influence factor chart that we saw in the previous slide. All that we need to do is to remember that still we are using the theory of elasticity. We have to note that Boussinesq's theory is based on the linear theory of the elasticity. And as I explained in one of my earlier lectures as long as the theory of the elasticity that we use assumes a linear relationship between stress and strain. That is modulus is constant, the principle of super position is always valid. That is the effect of two loads is the same as the effect of their sum.

(Refer Slide Time 18:24)



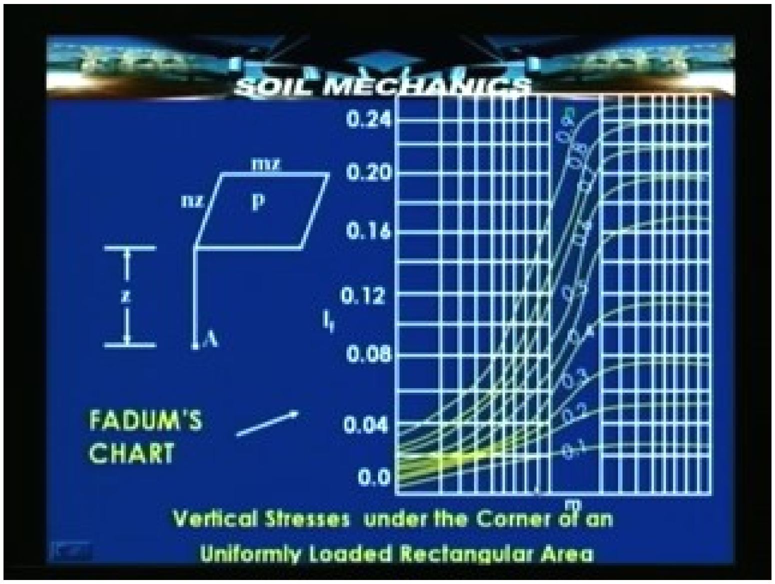
That is what we have, in order to find out the total effect of two loads P_1 and P_2. All that we need to do is to find out the effect of P_1 separately P_2 separately and add. And if we proceed further and see how to evaluate stress beneath a rectangular area of finite dimensions L and B. This is different from a strip loading in the sense that in a strip loading the length L is very large compared to the other dimension B. Here L and B are of comparable dimensions and therefore this is called a rectangular area.

(Refer Slide Time 19:17)



Suppose we want to find out the stress at any point beneath this. As we saw in one of our earlier lectures the value of sigma z can be obtained by taking a small element, finding out the stress in this element and then integrating it over the entire area. If we do that we find that the stress at any point P sigma z can be expressed as the unit load p per unit area, p per meter square which is applied over rectangular area into an influence value I_{mn} where m and n once again represents non dimensional distances or lengths. In this case m stands for B upon z, n stands for L upon z. Since L and B are relative, m and n are also relative and as you will see in the charts which we had seen earlier and as well as which are going to come after these which are going to follow. The m and n are interchangeable, it is just that they are two non dimensional sides of the same rectangle.

(Refer slide Time 20:54)



Going further this chart which also we briefly looked at last time gives you the influence value in graphical form. What does it give? Suppose this is the rectangular area, this chart gives you the influence value for calculating the stress at a point A beneath the corner of this rectangular area. The dimensions are m times z and n times z. What ever be the actual dimensions they can always be expressed as the non dimensional mz and nz. Here is this chart known as the FADUM'S chart which is based on the equation for stress which is written in the form p into I_{mn}. And this shows m along the x axis, I_f the influence factor long the y axis and there are several curves here corresponding to different n values. So all that we need to know is what is m corresponding to the z at which we want the stress. Remember that this area remains the same, its dimensions are L and B depending upon the depth at which we want the stress, these dimensions can be repeatedly expressed as mz and nz.

The actual absolute magnitudes of the dimensions remains the same, but the non dimensional values will be mz and nz where m and n go on varying will depend upon the z depth at which we want the stress. The advantage is when z becomes the scale factor one single graph showing m and n and corresponding I_f can be used for any depth. This is what one can employ to get the influence value at A and that multiplied by the stress per unit area here will give you what is the effect of this whole area, what is the stress corresponding to this entire loaded area at a point like A. If we have an area which is larger than this, we can always divide that into quadrants once again in such a way that a corner of the quadrant is always above the point at which we want the stress. That is what I mean here in this diagram.

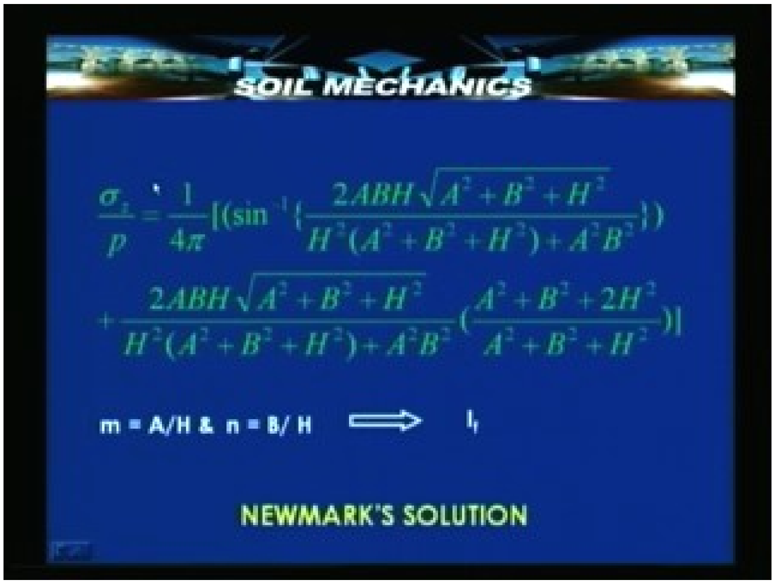
(Refer Slide Time 23:04)

Suppose this is the total area, this is the coordinate axis xy, these are the dimensions A and B, and this is the unit load small p per area unit area. What is important is that the point at which we want the stress is directly below the centre and therefore we employ the expression that we had derived for a point lying below the corner of a rectangle repeatedly for each one of the four rectangles into which we can divide the given area.

We can also modify this slightly in case we have an area whose centre is not above the point at which we want the stress. That we shall see shortly. Here Boussinesq's had derived the expression sigma z by p equal to this for the stress at any depth z due to the uniformly distributed load small p over a rectangular area. This is nothing but a different way of writing this same expression. This equation which we have here, where we have expressed sigma z in the form of an applied load p and the influence vector i, this again is identical to that just represented in a different way by newmark. So both of them based on Boussinesq's solution but they have been represented in a different way in order to non-dimensionalize them and facilitate calculation of influence factor.

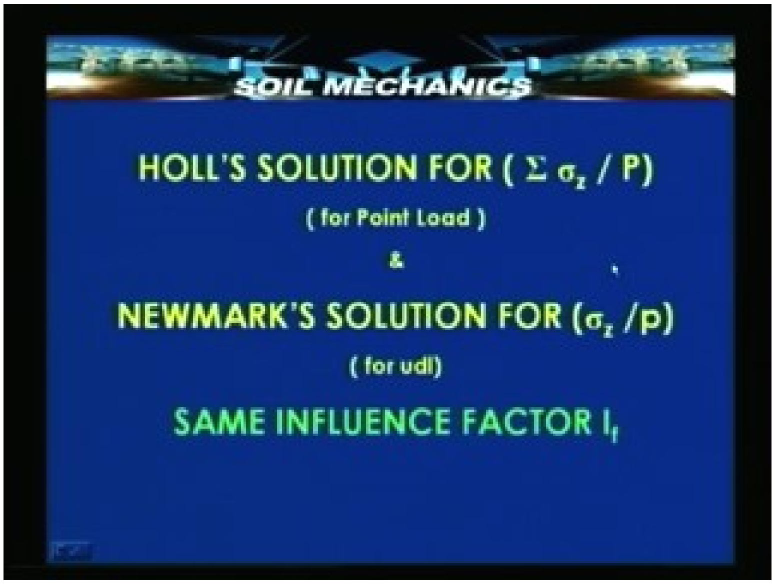
So once again if we express A upon H as m and B upon H as n, we have a table from which we can calculate the influence factor. And incidentally as I mentioned a little while ago whether it is calculating the stress at a point below due to a uniform load on the surface or finding the total load at depth over a rectangular area due to a concentrated load at the top, they are reverse of each other. The influence coefficient still remains the same. So given the same influence coefficient chart we can solve both problems.

(Refer Slide Time 23:49)



So the Holl's solution which we saw for computing load over a rectangular area at depth due to a point load on the surface and the newmarks solution which we saw in the last slide for stress at a point as a fraction of the applied unit load p, a uniformly distributed one udl, the same influence factor I_f is valid for both problems. This I_f can be computed therefore either from the expression given by Holl or from the expression given by newmark or expression given by Boussinesq's theory. In fact now we have therefore three different forms in which the influence factor is given.

(Refer Slide Time 25:22)



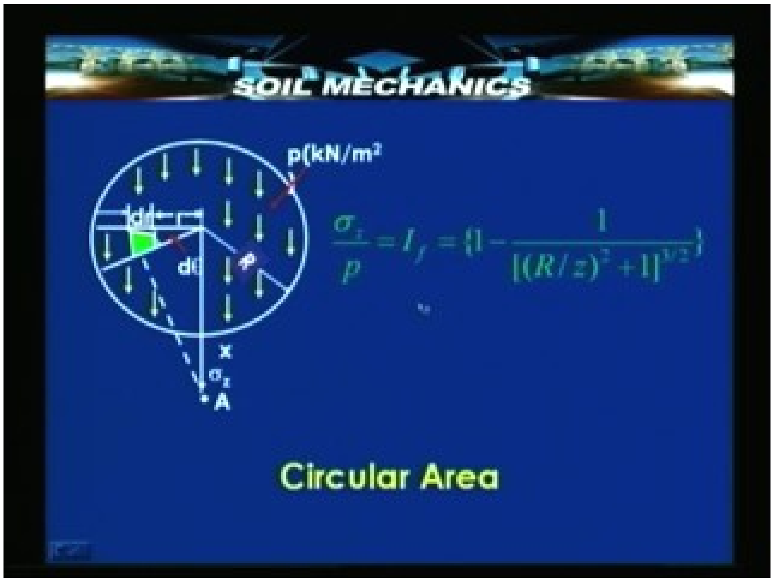
In one case we have influence factor corresponding to load on a rectangular area due to a concentrated load, in the other we have a graphical representation of the influence factor as a function of m and n. And here we have a table again which gives the influence factor as a function of m and n where m is A upon H and n is B upon H or vice versa. It all depends upon how we look at the area which dimension is length and which dimension is taken as B. So it does not really matter, the influence factor remains the same irrespective of whether we take this as m or this as n or and vice versa. In this chart we have a number of influence factor values given for m ranging from 1 to 10 or 0.1 to 10 and n ranging from also 0.1 to 10 or even infinity.

(Refer Slide Time 26:34)

This table is complete in itself, it covers all most all possible cases of m and n. This has been developed by Newmark or based on the Newmark's solution which we saw in the previous slide and it is an excellent tool for solving stress distribution problems. We can apply this influence value table to a rectangular area irrespective of whether the desired point is below the centre or away from the centre. Because in either case we can divide the given loaded area into either equal areas or into unequal areas and in both cases we can divide the given total area in such a way that the desired point lies below the point of intersection of the smaller areas, sub divisions. The reason as I have mentioned earlier is the expressions that we have for calculating stress are all based on a point lying directly below the corner of a rectangular area.

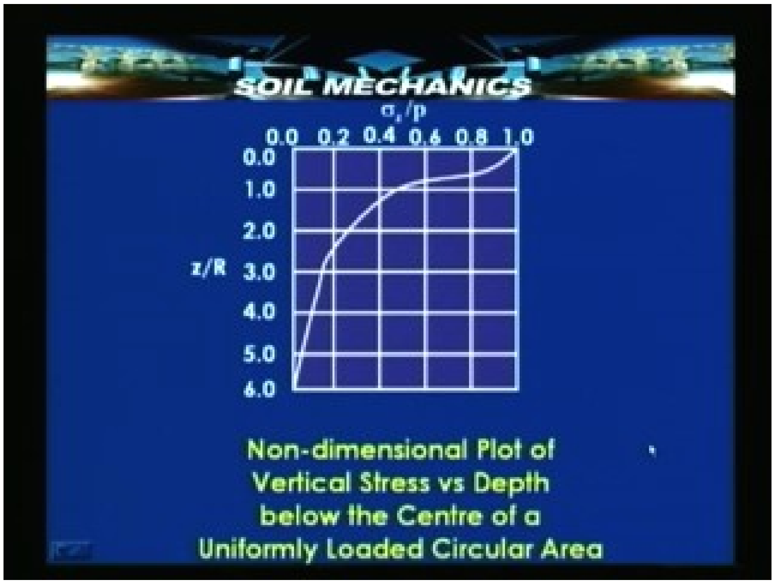
We can proceed further to understand stress distribution in the case of loads over a circular fitting or circular foundation. We had seen this also in the last lecture, the idea of presenting them again here now is to show that influence tables, influence charts are all available and they can advantageously used for computing the stress at any given point or the total load over any area at any depth. See here this is the expression for influence factor for stress at any point A below a loaded area which is carrying a uniform intensity of loading small p. This again has been solved by the same Boussinesq's theory, we take a small element here which has a length dr whose other dimension is r d theta where d theta is the small angle, just as we had taken a rectangle element dx dy and integrated over the whole given loaded area.

(Refer Slide Time 29:00)



Here again we can take the small element and integrate it for theta varying from zero to 360 and small r varying from zero to the diameter or radius capital R and thus we can get the stress at any point A directly below the centre of the circle. So this expression that we have here is valid for computing stress at any point directly below the centre of a uniformly loaded circular area. In case we want to do this with the help of charts, here is the non dimensional chart; this shows sigma z by p and z upon R. Why we take z upon R is here we have in the expression for sigma z upon p the term R upon z.

(Refer Slide Time 30:10)

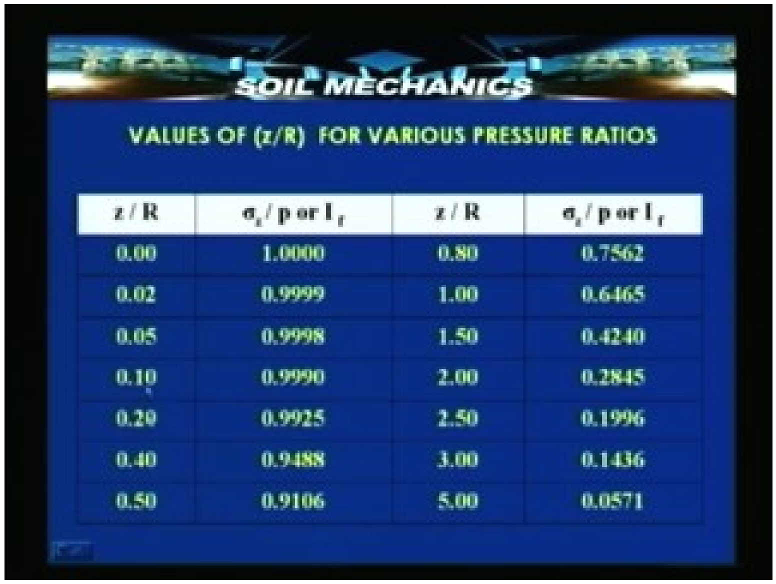


Since R is a constant, it is convenient to express the table or the chart or the graph in terms of a quantity z upon R where z is the variable and R is a factor which is a constant and which helps the non dimensionalized depth z. Here we have this non dimensionalized graph showing vertical stress versus depth below the centre of a uniformly loaded circular area. So this shows for example when z equal to zero, when we consider a point directly below the centre of the foundation on the surface of the soil you find that sigma z is exactly equal to small p. So this is the maximum value of stress ratio that is possible and as we go deeper and deeper, sigma z upon p goes on decreasing and at about 6 this sigma upon R, when it reaches a value of approximately 6 we find that sigma z upon p reaches a very low value which means that in practice the effect of any load small p is maximum at the top indicating that the stress at the surface is very high and could be equal to intensity of load applied.

And virtually at a depth of 5 to 6 times the radius of the loaded area the influence of the load p is practically vanishing. That means this is the depth over which the stresses really act. The loaded area on the surface that is here does not seem to have much influence beyond the depth of 6 times the radius capital R. This can be also plotted the same, whatever we saw in the previous table or whatever we saw in the graph can also be plotted or can also be presented in the form of a table. Here we have z by R values ranging from 0 to 5 and corresponding sigma z by p or I_f. What we need to notice or pay attention to and remember here is that when depth is zero, stress is highest that is it is equal to p that means influence factor is one.

And as we go deeper at a depth equal to 5 times the radius, the influence factor drops to 0.0571 compared to what it was on the surface this is a very small value. And again this table indicates that beyond a depth of about 5 times the radius, the influence factor and therefore the stress induced due to a surface load is not high, may be even negligible.

(Refer Slide Time 32:30)

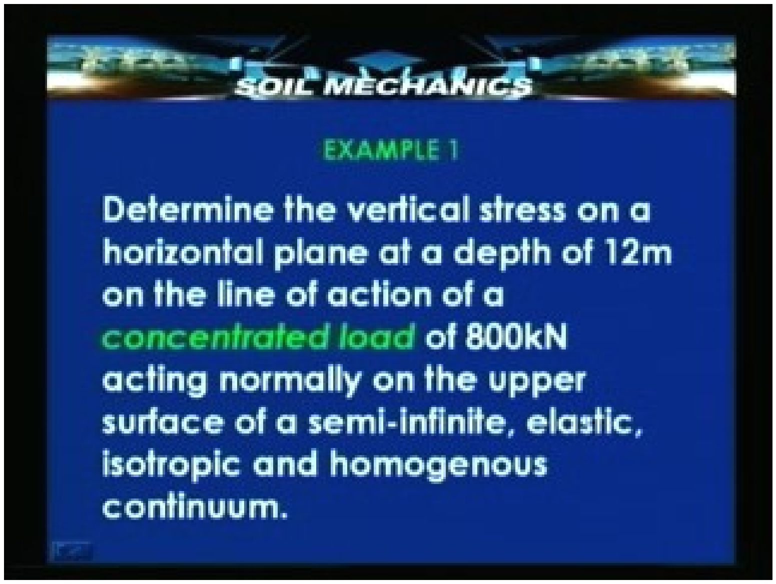


z / R	σ_z / p or I_f	z / R	σ_z / p or I_f
0.00	1.0000	0.80	0.7562
0.02	0.9999	1.00	0.6465
0.05	0.9998	1.50	0.4240
0.10	0.9990	2.00	0.2845
0.20	0.9925	2.50	0.1996
0.40	0.9488	3.00	0.1436
0.50	0.9106	5.00	0.0571

Here we have another table which is a representation of the same information that we had seen, in the case of circular area in the previous table and chart. That is in this table and in this chart what ever information we have that is the stress ratio due to an applied load p over a circular loaded area and the depth z by R non dimensional. This table also contains the same information but here it is slightly differently presented. We find here that is we got the influence factor against which the ratio r upon Z has been presented where as the previous table showed us at what depth what will be the influence factor. This table gives us what influence factor would correspond to what depth. And this also has its great relevance as you will see where this value has been successfully used to generate a chart by Newmark which is universally applicable to all loaded areas.

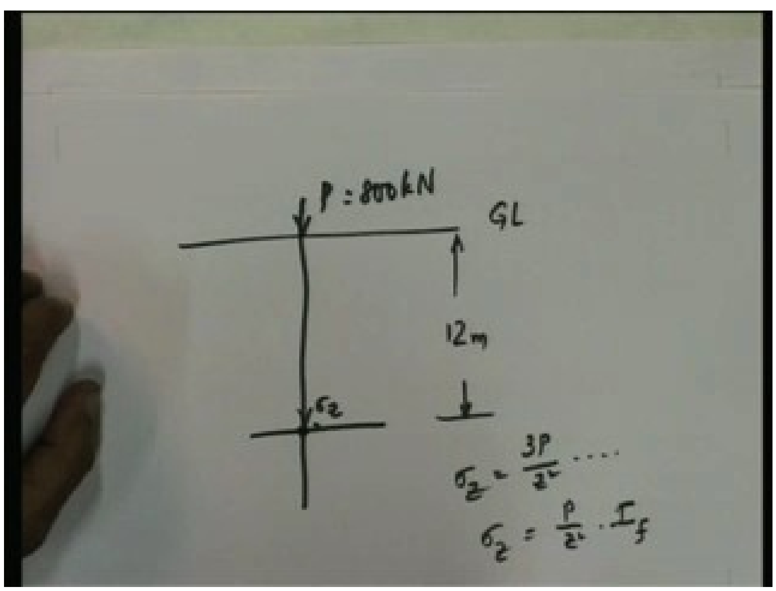
This contains nine rather eleven values. We can also introduce additional intermediate values or values beyond one in order to complete this table and make it exhaustive. But its unlikely to have a value of sigma z beyond one because the stress induced cannot exceed the load applied which is small p. So sigma z by p or I_f can only vary from zero to one. On the other hand R upon z can vary from zero that is R equal to zero means from surface to R equal to infinity or R by z equal to infinity. This covers a very wide range of R by z and stress ratio and this is more or less adequate for solving any stress distribution problem where we have circular loaded areas.

Let us take a look at an example of an application of the principle we saw so far. This example is something which we have already discussed but in the light of what we have learnt additionally today, in order to ensure continuity and better understanding let us take a look at the same example once again. What is this example? This example says compute the vertical stress; determine the vertical stress on what? On a horizontal plane, where is the horizontal plane located? At a depth of 12 meters and what is the location in the lateral direction? It is located at a depth of 12 meters on the line of action of a concentrated load.
(Refer Slide Time 35:58)



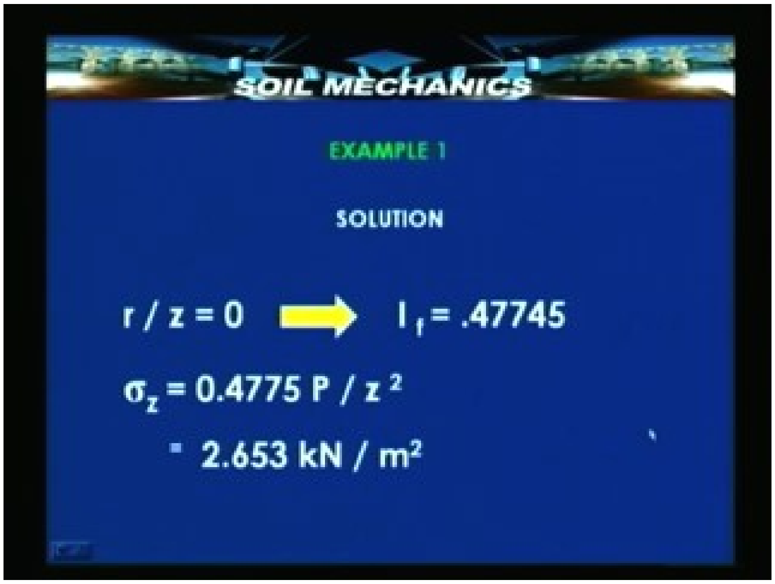
What is a concentrated load? 800 kilo Newtons and how does it acts? It acts normally on the upper surface of a semi infinite, elastic, isotropic and homogenous continuum. That means if we were to draw diagram to represent this problem, we will get the same thing that we had analyzed in the last class. That is here is the ground surface and at a depth of 12 meters there is again a horizontal plane and we want the vertical stress on a horizontal plane at a depth of 12 meters on the line of action of a concentrated load. So if this is a concentrated load and this is its line of action, at a depth of 12 meters at this point on a rectangular area or a horizontal plane we want the stress. The concentrated load is given on the surface as 800 kilo Newtons.

(Refer Slide Time 38:08)



And it is acting normally on the upper surface of a semi infinite, elastic, isotropic and homogenous continuum, which means that this satisfies the requirements of theory of elasticity and we can simply apply Boussinesq's theory to calculate the stress sigma z here. The method will be this being a simpler problem, relatively simple problem. The method is to simply take if the expression for sigma z which is 3 p by z square etc and that is what we have done.

(Refer Slide Time 38:24)



The solution can also be attempted by using the expression sigma z is equal to p by z square into influence factor. And so we have here the influence factor which is determined from a table which we had seen a little while earlier for point loading. Their for R by z equal to zero, we saw that influence factor is 0.4775. Let us take a quick look at the slide given below. Here we have the point load and corresponding to R by z here R by z equal to zero due to a point load we have I_f equal to 0.478 which is what we had found and used in the computation.

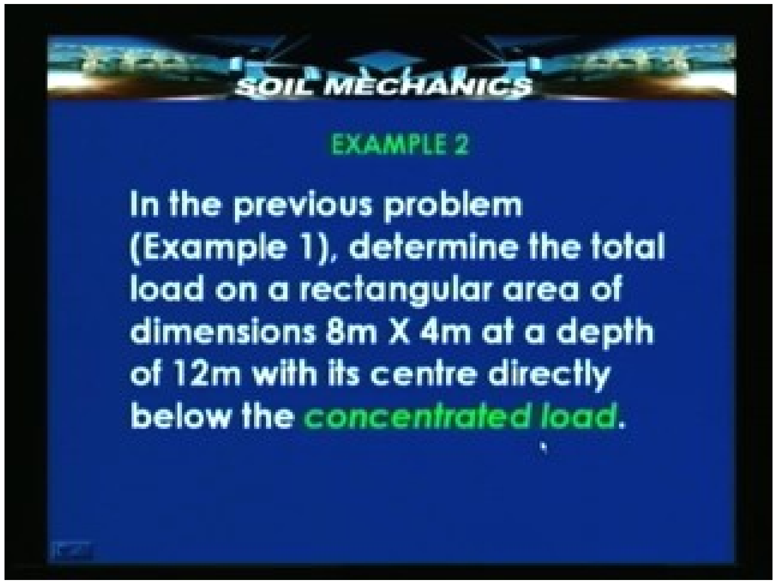
(Refer Slide Time 38:46)

r/z	I_f	r/z	I_f	r/z	I_f
0.00	0.478	1.00	0.0844	2.00	0.0085
0.10	0.466	1.10	0.0658	2.10	0.0070
0.20	0.433	1.20	0.0513	2.20	0.0058
0.30	0.385	1.30	0.0402	2.30	0.0048
0.40	0.329	1.40	0.0317	2.40	0.0040
0.50	0.273	1.50	0.0251	2.50	0.0034
0.60	0.221	1.60	0.0200	2.60	0.0029
0.70	0.176	1.70	0.0160	2.70	0.0024
0.80	0.139	1.80	0.0129	2.80	0.0021
0.90	0.108	1.90	0.0105	2.90	0.0018

We had 0.478, a more précised value is 0.47745 and that has been used. Its preferable to use an I_f value which is quite precise or rather than use the values which are given in the table. Therefore where ever it is possible or feasible to use the expression for stress directly and wherever it is not necessary to use the table or a chart, it is preferable to compute the influence factor more rigorously. It may be an important in certain instances and in certain problems. In problems where it is not going to make a significance difference we can as well use the graph or the table of influence values and compute the stress.

So here influence coefficient multiplied by p by z square is the stress that is 2.653 kilo Newton per meter square due to a point load of 800 kilo Newtons, p is 800 kilo Newtons and z is 12 meters. Take another example; this example relates to the previous one and that is the reason why I once again went through the previous example. Although we had seen it in on earlier occasion when we were talking about point loads, the method to compute a stress below a point load using the theory of elasticity.

(Refer Slide Time 40:13)

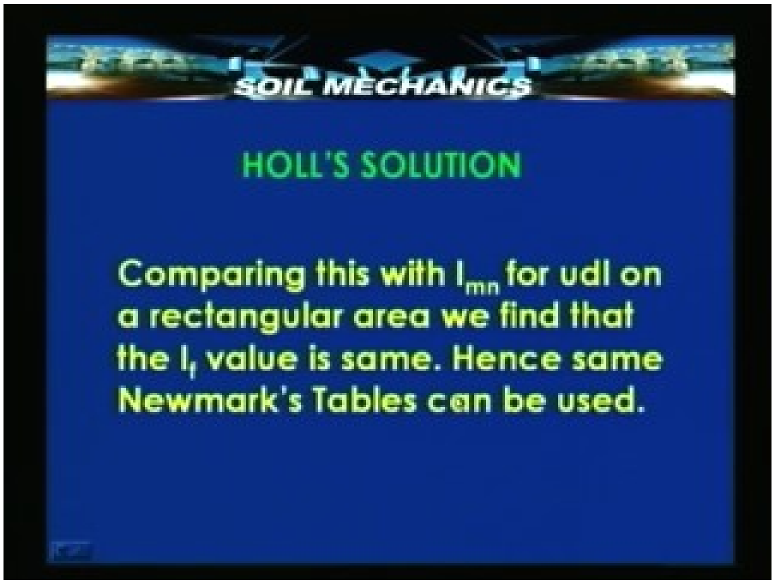


Suppose we once again consider a problem or example one. This question says determine the total load on a rectangular area. The dimensions of the rectangular area is given as 8 meters by 4 meters, we shall take 8 meters as the dimensional along the x axis and 4 meters as the dimensional along the y axis because that is the way normally length and breadth are taken with respect to our coordinate axis. The depth is as before same 12 meters and what is additionally given? A very significant piece of information, important point that is the centre of the rectangular area lies directly below the concentrated load. The centre of the rectangular area directly below the concentrated load therefore its confirming to the standard case where we have the concentrated load here, we have the rectangular area here at a depth z or at a depth H, depending upon the notation we used and the centre of that area is directly below p. So we want to calculate the load over this entire area due to the concentrated load p.

All that we need to do is to know what is A, what is B, what is H? Apply Holl's solution and the influence factor table suggested based on holl's solution. That will give us the load over an area A upon B where one corner of that area is directly below P. If we apply the same principle or the same solution to another area here, also A upon B whose corner is also directly below P and so also for a third quadrant and the forth quadrant, we will find that this total load on this dimension AB and the total load on each one of these other quadrants all together will add up to the total load on the entire rectangular area. This was the expression for sigma z, (Refer Slide Time: 42:55) using this expression for a small elemental area dx dy we get the expression for the total load dividing it by capital P after substituting this in this, we have part of the load, out of the load applied is equal to this at any depth z. The holl's solution, if we were recollect once again (Refer Slide Time 43:19) is this.

So in this now if I substitute A upon H as m small m, B upon H as small n and get the value of influence factor corresponding to this which is nothing but the ratio of this. Multiplying it by capital P, I can get the total load on one quadrant multiplied by four I will get the load on all the four quadrants together. So holl's solution can be used or it is the same thing as using the newmark table which was derived for a uniformly distributed load and its effect at depth. Both will give the same result.

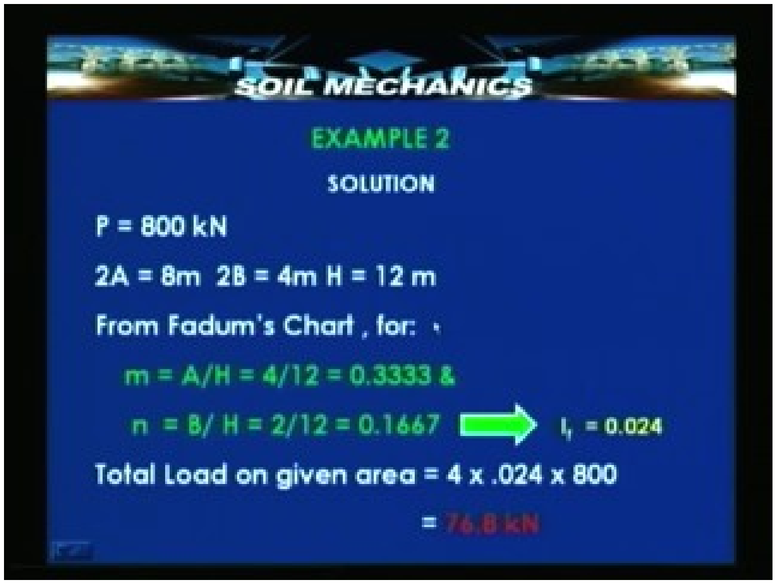
(Refer Slide Time 43:50)



Now the solution to this particular example is P is 800 kilo Newtons is just the part of the data, the length of the rectangle over which we want the total load. Total length is 8 meters which means one quadrant A has a length L equal to or a length A equal to 4 meters. Similarly one dimension of the quadrant in the y direction which is half of this that is 2 meters and the depth is 12 meters. So we can either use Fadum's chart or the influence coefficient tables. Suppose we use the Fadum's chart for a change or for illustration then m which is the ratio of the A upon H would be 4 upon the depth 12 that is 0.333 and B upon H or small n would be half of this that is 2 divided by the depth 12 that is 0.667.

Now if I enter Fadum's chart for m is equal to 0.333 and n equal to 0.167. Let us do that. We had m is equal to 0.33 along these and n is equal to 0.167 that is some where here, so our influence value will be some where here (Refer Slide Time: 45:30).

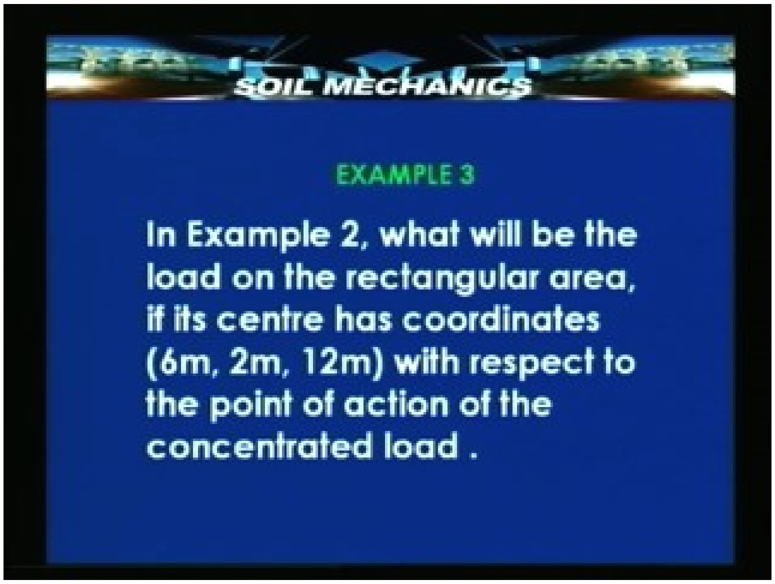
(Refer Slide Time 44:08)



So let us see what is that influence value is. It turns out to be 0.024, this means that the total load on the 8 by 4 meter area is going to be 4 times what ever value we have got here for one quadrant that is 4 into 0.024 into the load applied that is 800. This means that is 76.8 kilo Newtons is what the rectangular area carries at that depth of H equal to 12 meters. Let us say this is approximately 80 that means at 12 meters depth the load over the entire rectangle becomes virtually one tenth of the load applied at the surface. The total load carried at a depth of 12 meters is just one tenth or even a little less than that compared to the load applied which is 800 kilo Newtons that means for all practical purposes this concentrated load has an influence up to a depth of about 12 meters, beyond that its effect is not that significant.

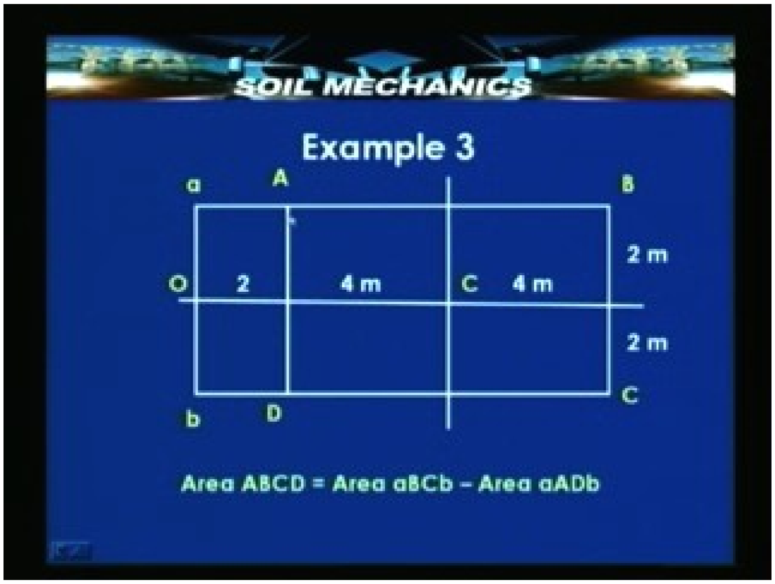
Let us take one more example. This example is an extension of the previous example number two. Here what is asked is what will be the load once again on a rectangular area and in fact the same rectangular area of 8 meters by 4 meters dimension but its centre has coordinates 6, 2, 12 meters with respect to the point of action of the concentrated load. If you remember the previous example, the dimensions were 8 by 4 meters, the depth was 12 and the centre of the rectangle was below the concentrated load directly which means the coordinates of the centre would have been half of the length 8 that is 4, x coordinate is 4 y coordinate will half of the y dimension means 2 meters, the depth was 12. So we had the centre at a point 4, 2, 12 where as now we have the centre at a point 6, 2, 12. The rectangular area is once again 8 meters by 4 meters that remains the same. So now how do we find out the load on the same area even though its centre is no longer under the concentrated load?

(Refer Slide Time 46:48)



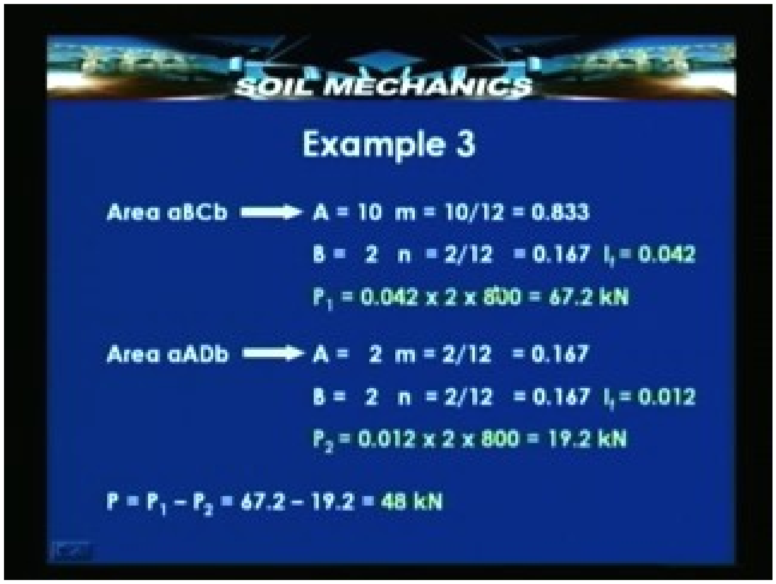
Here it is, this is a plan view. Let us say A B C D of dimensions 8 meters by 4 meters is our desired area over which we want the load. This is at the depth 12 meters. The centre or the point directly below the load is let say O then the centre of this 8 by 4 area is at 6 meters from O in plan. That is already given in the statement of the problem which means now effectively we have one rectangle small a, capital B, capital C, small b. Another area small a, capital A, capital D, small b and our own original given area capital ABCD. Since we want the load on capital ABCD and since our method enables us to calculate load only for quadrants or areas whose one corner is below the applied load.

(Refer Slide Time 48:12)



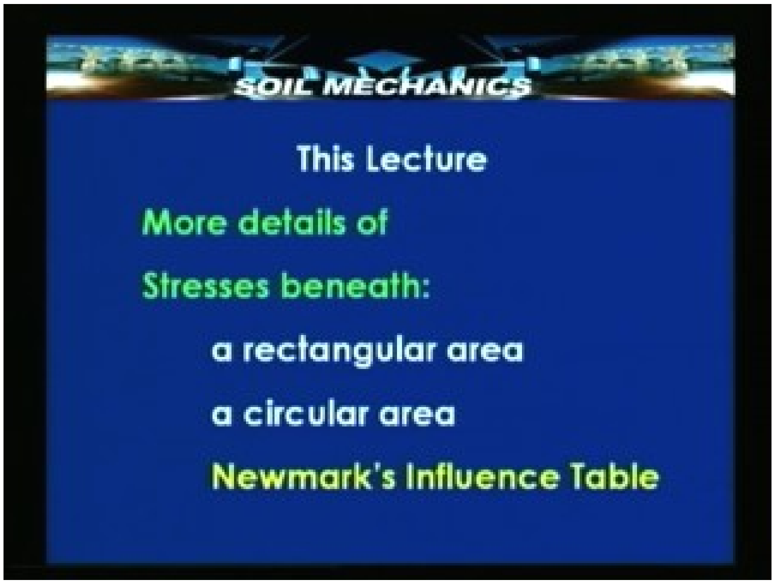
What we need to do now is to find out the total load over entire area small aBCb and subtract from that the load carried by small a capital A capital D b. This is possible because both these sets of rectangles have one corner directly below O.

(Refer Slide Time 49:38)



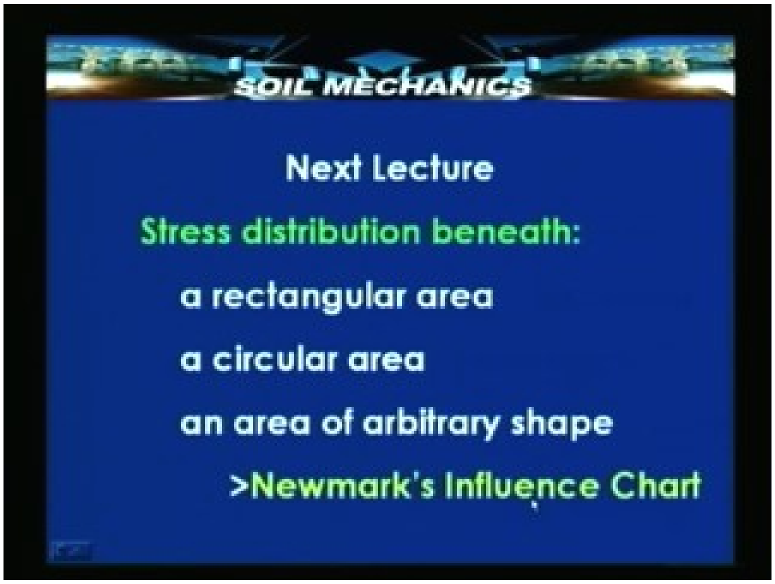
If we do that we find that for the large area A is 10 meters, B is 2 meters and so we have m equal to 0.833, n equal to 0.167, influence factor equal to 0.042 and the load is 67.2 kilo Newtons. For the small additional area which has been added is A 2 meters, B 2 meters, m is 2, n is 2 upon 12, both are 0.167, influence factor is 0.012 and the corresponding load is 19.2. Here you must note that we have multiplied the influence factor by the 2 and the applied load 800. The reason why we have multiplied by 2 is, there are two rectangles which constitutes this area. By subtracting 19.2 from 67.2 we get 48 which is the total load carried by the remaining area ABCD.

(Refer Slide time 50:46)



So now with all this, in this lecture we have gone into sufficient depth and details and done a quick review of how to compute stresses due to point line and strip loads and also gone into a little deeper into the use of influence table or the Fadum's chart, to find out the stresses beneath a rectangular area or a circular area. We shall proceed further and in one lecture we shall be covering subsequently a little more about a Newmark's influence chart and its application to rectangular, circular and areas of other shapes as well. What I have mentioned here is slightly different from what we have seen in today's lecture.

(Refer Slide Time 51:19)



What we are going to see in the next lecture is Newmark influence chart which is graphical or a diagrammatic representation of influence values. What we have used today is a tabulation of the influence values. What we are going to see in the next class the

Newmark's influence chart which is a very ingenious way of developing a chart or a diagram which helps us to compute the stress due to any area rectangular, circular. So with this I will conclude today's lecture. We shall meet again later.

Thank you.